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DESCRIPTION OF A SKI JUMPER'S FLIGHT

OPIS LOTU SKOCZKA NARCIARSKIEGO

S t r e s z c z e n i e

W artykule zaproponowano model siły aerodynamicznej opisującej oddziaływanie powietrza na skoczka podczas lotu. Wyznaczono trajektorię lotu przy pogodzie bezwietrznej oraz zbadano wpływ wiatru wiejącego z przodu i z tyłu na długość skoku.

Słowa kluczowe: model siły aerodynamicznej, trajektoria lotu skoczka

A b s t r a c t

In this paper the model of an aerodynamic force was proposed which describes the influence of the air on a ski jumper during his flight. The flight trajectory in windless weather conditions was given and the influence of a headwind and tailwind on a length of a ski jump's was analyzed.

Keywords: model of an aerodynamic force, trajectory of a ski jumper's flight

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1. Introduction

A ski jump consists of three fundamental phases: a downhill ride, a flight and a landing, which is connected with a slowing down. In order to obtain the highest speed at a take-off, a ski jumper takes a figure, which minimizes the air resistance force during a downhill ride. At the moment of taking off a ski jumper changes his downhill figure for a flight one, he inclines towards the skis and forms the letter “V”. The landing should finish by a telemark, which allows him to keep a balance and is assessed by the jury. In the case of long ski jumps, ski jumpers land on two legs. In this way they amortize bigger crash force with the ground.

The aim of this paper is to describe ski jumper’s flight taking into consideration the impact of a wind on the length of a ski jump.

2. A ski jumping hill profile

A ski jumping hill consists of an inrun and an outrun (Fig. 1). The inrun profile consists of an *AC* rectilinear segment (it includes the *AB* segment in the length of which the starting gate is situated) which is inclined at an α_1 angle, a *CD* circular arc with a r_1 radius and a *DE* take-off which is inclined at an α_2 angle. The outrun profile consists of a *SP* circular arc with a r_3 radius (called a knoll), a *P–K–L* landing area, a *LM* circular arc with a r_2 radius and a *MN* segment.

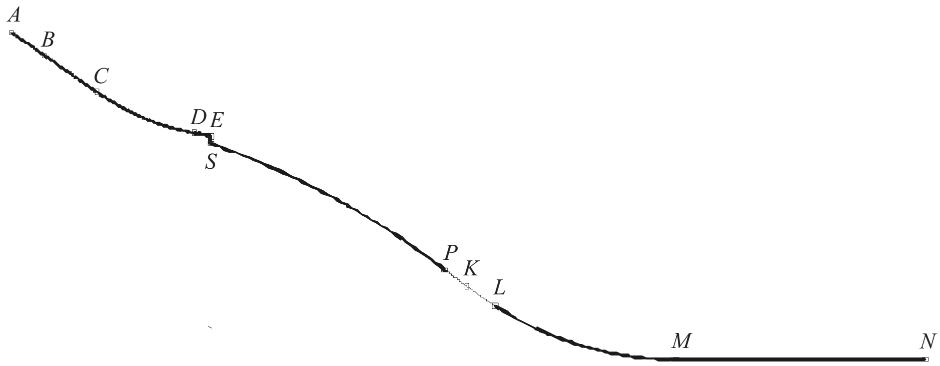

Fig. 1. The Great Krokiew profile in Zakopane (*K*–120)

Rys. 1. Profil Wielkiej Krokwi w Zakopanem (*K*–120)

The *K* point, which is situated in the landing area, is the construction point defining ski jump’s size.

3. Model of aerodynamic force

A ski jumper who has a flight figure (Fig. 2) is influenced by:

1) An aerodynamic force $W = k\,v^2$ which is a resultant of air resistance and lift force
2) A force of gravity $G = mg$

In order to describe the flight of a ski jumper, we should make a k coefficient dependent on a position of a ski jumper. If k_1 means a values of the coefficient of the aerodynamic force during the downhill ride and k_2 – at the end of the flight, we can make the k coefficient dependent on the velocity by the following formula:

$$k = k_1 + \frac{k_2 - k_1}{\cos\delta_0}\left(-\frac{\dot{y}}{v}\right) \tag{1}$$

An δ angle defining a direction of the aerodynamic force is connected with the speed of taking a flight figure and can be described by the following formula:

$$\delta = \delta_0\left(1 - \mathrm{e}^{-\frac{t}{\tau}}\right) \tag{2}$$

δ_0 – the maximum inclination angle of the W aerodynamic force from a tangent towards the trajectory of a ski jumper's flight,

τ – parameter defining the speed of taking a flight figure.

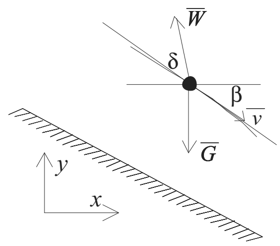

Fig. 2. The forces which influence a ski jumper during his flight

Rys. 2. Siły działające na skoczka podczas lotu

After taking off, when the ski jumper's task is to take the fastest flight position, the aerodynamic force is parallel to the ski jumper's velocity, because the angle $\delta = 0$. Using (1) and taking into consideration the fact that the vertical velocity is very small at the beginning of the flight, we obtain the following form of the aerodynamic force:

$$W_1 = k_1 v^2 \tag{3}$$

In „steady state", in the final phase of the flight, the direction of the aerodynamic force is similar to direction of the force of gravity, therefore the ski jumper's vertical velocity amounts $\dot{y} = -v\cos\delta_0$. The aerodynamic force at the end of the flight is described by the following formula:

$$W_2 = k_2 v^2 \tag{4}$$

The k_1 coefficient we can be calculated on the basis of measurement of real velocity of the ski jumper in the take–off, while the k_2 coefficient on the basis of equality between the aerodynamic force and the force of gravity in "steady state".

4. The differential equations of the ski jumper's flight

Taking into consideration the forces that have an influence on a ski jumper (Fig. 2) we obtain the following set of equations describing a ski jumper's flight by windless weather:

$$\begin{cases} m\ddot{x} = -W\cos(\delta+\beta) \\ m\ddot{y} = W\sin(\delta+\beta) - mg \end{cases} \tag{5}$$

Using (1), (2) and the following substitutions:

$$\begin{cases} v\cos\beta = \dot{x} \\ v\sin\beta = \dot{y} \end{cases} \tag{6}$$

the set of equations (5) will have the following formula:

$$\begin{cases} m\ddot{x} = -\left[k_1 + \dfrac{k_2-k_1}{\cos\delta_0}\left(-\dfrac{\dot{y}}{v}\right)\right] v\,\dot{x}\cos\delta - \left[k_1 + \dfrac{k_2-k_1}{\cos\delta_0}\left(-\dfrac{\dot{y}}{v}\right)\right] v\,\dot{y}\sin\delta \\ m\ddot{y} = -\left[k_1 + \dfrac{k_2-k_1}{\cos\delta_0}\left(-\dfrac{\dot{y}}{v}\right)\right] v\,\dot{y}\cos\delta - \left[k_1 + \dfrac{k_2-k_1}{\cos\delta_0}\left(-\dfrac{\dot{y}}{v}\right)\right] v\,\dot{x}\sin\delta - mg \end{cases} \tag{7}$$

The ski jumper's flight described by the equations (7) ends at the *K* point for the following data: $v_1 = 26.5\,\text{m/s}$ (velocity part tangent towards the take–off), $v_2 = 2.8\,\text{m/s}$ (velocity part perpendicular towards the take-off), $\delta_0 = 46.5^{\circ}$, $\tau = 0.32\,\text{s}$, $k_1 = 0.001\,\text{kg/m}$, $k_2 = 0.65\,\text{kg/m}$. Time of this flight amount $t_{sk} = 4.42\,\text{s}$, the limit velocity $v_{gr} = 30.09\,\text{m/s}$ and the landing velocity $v_k = 29.13\,\text{m/s}$. The graph of the velocity and the trajectory of the ski jumper's flight to the *K* point are presented on Fig. 3.

It is possible to observe that the ski jumper's velocity slightly decreases after taking off and after some time tends to the v_{gr} limit velocity and the flight trajectory resembles a straight line. If the aerodynamic force is the same as the force of gravity, a ski jumper will go along a straight line with a constant speed (Newton's first law). Each ski jumper tries to attain this situation as fast as possible, because the faster he achieves it the farther his ski jump will be. Unfortunately when atmospheric weather conditions are normal, the aerodynamic force and the force of gravity are never equal ([4], [6]).

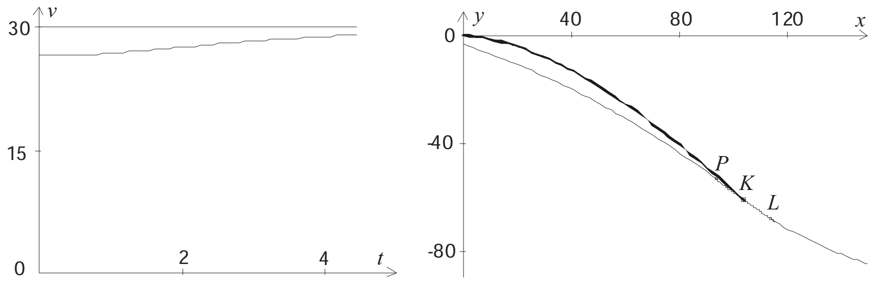

Fig. 3. The velocity and the trajectory of a ski jumper's flight against a background of the Great Krokiew profile

Rys. 3. Prędkość i trajektoria lotu skoczka na tle zarysu Wielkiej Krokwi

5. The influence of wind on a ski jumper's flight

Many conditions influences the length of a ski jumper's flight, such as: the speed of taking–off, the time of taking a flight figure, the wind's direction and others. The maximum permissible wind's speed on a ski jump amount $v_w = 2\,\text{m/s}$. Taking into consideration in (5) the v_w wind's velocity we obtain the following set of equations:

$$\begin{cases} m\ddot{x} + k\, v_{sw}^2 \dfrac{1}{v_{st}} \left(\dot{x}\cos\delta + \dot{y}\sin\delta \right) = 0 \\ m\ddot{y} + k\, v_{sw}^2 \dfrac{1}{v_{st}} \left(\dot{y}\cos\delta - \dot{x}\sin\delta \right) + mg = 0 \end{cases}, \quad v_{sw} = v_{st} \pm v_w \tag{8}$$

where:

v_{sw} – is ski jumper's speed with respect to a wind,

v_{st} – is ski jumper's speed with respect to a ring. A headwind increases v_{sw} and lengthens a ski jump, however a tailwind reduces v_{sw} and shortens a flight.
Using the maximum permissible wind's speed, Fig. 4 and Fig. 5 prove this rule.

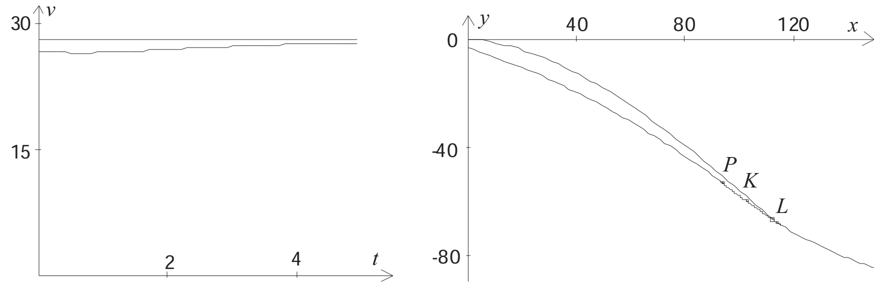

Fig. 4. The velocity and the trajectory of a ski jumper's flight taking headwind into consideration

Rys. 4. Przebieg prędkości i trajektoria lotu skoczka z uwzględnieniem wiatru wiejącego z przodu

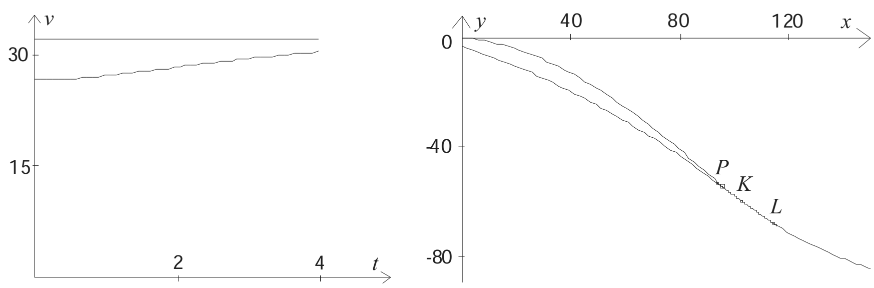

Fig. 5. The velocity and the trajectory of a ski jumper's flight taking tailwind into consideration

Rys. 5. Przebieg prędkości i trajektoria lotu skoczka z uwzględnieniem wiatru wiejącego z tyłu

Taking into consideration the same value v_1, v_2 of velocity and δ_0, τ, k_1, k_2 parameters as in the case of a flight in windless weather conditions and wind's velocity, we can observe the following changes the value of the landing speed (v_k), the value of the limits velocity (v_{gr}), the length of a ski jump (L_{sk}) and the duration of the flight (t_{sk}). For the permissible headwind's speed we obtain $v_k = 27.71\,\text{m/s}$, $v_{gr} = 28.09\,\text{m/s}$, $L_{sk} = 130.29\,\text{m}$, $t_{sk} = 4.92\,\text{s}$, however, for tailwind we obtain: $v_k = 30.33\,\text{m/s}$, $v_{gr} = 32.09\,\text{m/s}$, $L_{sk} = 109.76\,\text{m}$, $t_{sk} = 3.97\,\text{s}$.

6. Conclusions

The aerodynamic force's model given in this paper appropriately describes the influence of the air on a ski jumper during a flight, because the time and the length of a ski jump and the trajectory of flight correspond with real values. The role of the wind is crucial for the length of ski jump; the headwind is conducive for the ski jumpers, however the tailwind shortens a flight. Ski jumpers don't have an influence on a direction and the intensity of wind but they are able to use the headwind in order to jump as far as it is possible.

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