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RENATA FILIPOWSKA*

MODELLING OF THE OUTRUN PROFILE OF A SKI JUMPING HILL

MODELOWANIE PROFILU ZESKOKU SKOCZNI NARCIARSKIEJ

Abstract

This paper presents the way of using the available information for an analytical description of the elements of the outrun profile. It was shown that it is possible to describe a landing area track by means of third – order splines, which guarantees that the radius of curvature tends to infinity at the *K* point.

Keywords: outrun profile of a ski jumping hill, landing area

Streszczenie

W artykule pokazano sposób wykorzystania dostępnych informacji do analitycznego opisu elementów profilu zeskoku. Pokazano, że możliwy jest opis zarysu strefy bezpiecznego lądowania za pomocą funkcji sklejanych trzeciego stopnia, gwarantujących nieskończenie duży promień krzywizny w punkcie *K*.

Słowa kluczowe: profil zeskoku skoczni narciarskiej, strefa bezpiecznego lądowania

* PhD. Renata Filipowska, Institute of Applied Informatics, Faculty of Mechanical Engineering, Cracow University of Technology.

1. Introduction

The ski jumping hill, on which first jumps were performed didn't resemble nowadays precisely designed constructions. The former ski jumpers used the natural relief to perform their shows. Nowadays, the ski jumping hills are designed and constructed according to the rules established by FIS. The aim of the modernization of existing constructions is to extend flight and provide safety.

The profile of the present ski jumping hills is created on the basis of geometrical rules. A ski jumping hill consists of an in-run and an out-run, which profiles forms mostly rectilinear segments with accurately defined length and inclination angles and circular arcs that are tangent to them.

This paper concerns a modeling method of the outrun profile taking a landing area under special consideration.

2. The elements of the outrun profile of a ski jumping hill

The outrun profile (Fig. 1) consists of a *SP* circular arc (called a knoll), a *P–K–L* landing area with the *K* construction point, the *LM* transition arc with a r_2 radius and the *MN* segment.

Taking into consideration the available information [1], description of the outrun profile of a ski jumping hill isn't evident. The knoll constitutes a circular arc, with a radius, which value wasn't given. We know: the coordinates of the *K* point: $x_K = n$, $y_K = -h$, the length of the segments $|PK| = l_1$ and $|KL| = l_2$, the β_P, β_K, β_L inclination angles of the tangent towards the landing area profile at the *P*, *K* and *L* points and the value of the r_2 radius, the *a* length of the *MN* segment (Table 1).

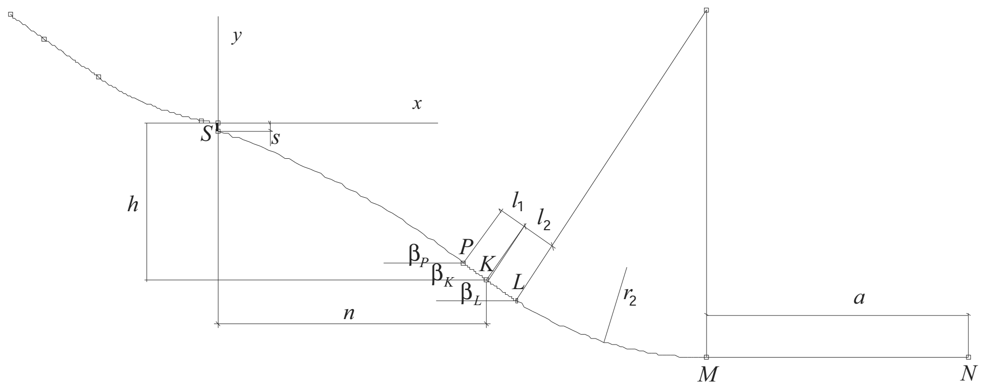

Fig. 1. The Wielka Krokiew profile in Zakopane

Rys. 1. Profil Wielkiej Krokwi w Zakopanem

Table 1

The dimensions of the outrun profile of the Wielka Krokiew in Zakopane

$K = 120$ [m]	$s = 3$ [m]	$\beta_P = 37.5°$
$v_0 = 25.5$ [m/s]	$l_1 = 11$ [m]	$\beta_K = 35.5°$
$h = 60.29$ [m]	$l_2 = 13.9$ [m]	$\beta_L = 33°$
$n = 103.06$ [m]	$r_2 = 133.5$ [m]	$a = 107$ [m]

3. The description of the landing area

In order to describe the outrun profile of a ski jumping hill, we should start with determining function of the P–K–L landing area as the coordinates of P and L points are unknown. It is possible to do it by means of an w interpolation polynomial of an appropriate order, which has to pass the P, K and L points or appropriate order splines.

3.1. The description of the landing area by means of an interpolation polynomial

In order to find demanded polynomial describing the P–K–L landing area passed the points with the coordinates: (x_P, y_P), (x_K, y_K), (x_L, y_L), the following conditions have to be fulfilled:

$$\begin{cases} w(x_P) = y_P \\ w(x_K) = y_K \\ w(x_L) = y_L \end{cases} \tag{1}$$

In addition the following three sequential conditions with appropriate inclination angles of the tangent towards at the P, K and L points have to be satisfied.

$$\begin{cases} w'(x_P) = -\mathrm{tg}\,\beta_P \\ w'(x_K) = -\mathrm{tg}\,\beta_K \\ w'(x_L) = -\mathrm{tg}\,\beta_L \end{cases} \tag{2}$$

The length of $|PK|$ and $|KL|$ segments define the following equations:

$$\begin{cases} \sqrt{(x_K - x_P)^2 + (y_K - y_P)^2} = l_1 \\ \sqrt{(x_L - x_K)^2 + (y_L - y_K)^2} = l_2 \end{cases} \tag{3}$$

The continuity of the radius of curvature at the L point will be obtained by:

$$w''\left(x_L\right)=\frac{\left(1+\left(\mathrm{tg}\,\beta_L\right)^2\right)^{\frac{3}{2}}}{r_2} \tag{4}$$

Then a polynomial, which fulfills the conditions mentioned above will have the following form:

$$w=a_w x^4+b_w x^3+c_w x^2+d_w x+e_w \tag{5}$$

The set of nine equations (1)–(4), where the values of coefficient of the w (5) polynomial and the coordinates of the P and L points are unknown and has to be found. The graphs of the polynomial describing the P–K–L landing area and its second derivative are presented in Fig. 2.

It is possible to observe (Fig. 2) that the second derivative is positive in all the P–K–L landing area and in the neighborhood of the K point the value of radius of curvature is relatively large at this point and equals $r_K = 702\ \mathrm{m}$. The safest ski jump ought to have the nominal length $120\ \mathrm{m}$, therefore the r_K radius has to be as large as possible. If $r_K \to \infty$, it would provide an ideal solution. In order to find it, the landing area will be described by means of splines.

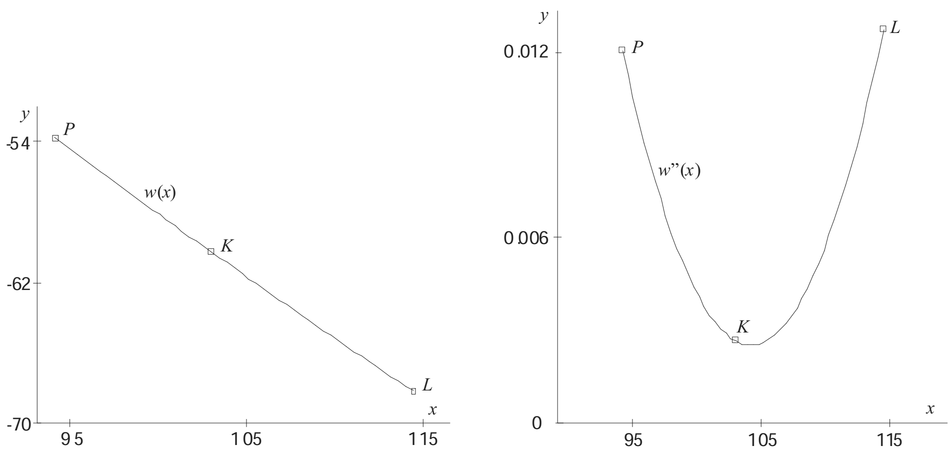

Fig. 2. The landing area profile described with w polynomial and its second derivative

Rys. 2. Profil strefy bezpieczeństwa opisanej wielomianem w oraz druga pochodna tego wielomianu

3.2. The description of the landing area by means of splines

In order to find two sought splines, which will describe the landing area, the following conditions have to be fulfilled:

$$\begin{cases} s_1(x_P) = y_P \\ s_1(x_K) = y_K \\ s_2(x_K) = y_K \\ s_2(x_L) = y_L \end{cases} \tag{6}$$

Tangents to the landing area profile at P, K and L points should be inclined by the defined angles

$$\begin{cases} s_1'(x_P) = -\mathrm{tg}\,\beta_P \\ s_1'(x_K) = -\mathrm{tg}\,\beta_K \\ s_2'(x_L) = -\mathrm{tg}\,\beta_L \end{cases} \tag{7}$$

The first and the second derivatives of both curves have to be the same at the K point

$$\begin{cases} s_1'(x_K) = s_2'(x_K) \\ s_1''(x_K) = s_2''(x_K) \end{cases} \tag{8}$$

The continuity of the radius of curvature at the L point will be obtained by

$$s_2''(x_L) = \frac{\left(1+(\mathrm{tg}\,\beta_L)^2\right)^{\frac{3}{2}}}{r_2} \tag{9}$$

To obtain the radius of curvature $r_K \to \infty$, the s_1'' and s_2'' second derivatives have to have the value 0 at the K point and the β_K angle has to be regarded as unknown

$$s_1''(x_K) = 0 \tag{10}$$

The both splines, which can fulfill the conditions described above are the polynomials in the form:

$$\begin{cases} s_1 = a_{s_1}x^3 + b_{s_1}x^2 + c_{s_1}x + d_{s_1} \\ s_2 = a_{s_2}x^3 + b_{s_2}x^2 + c_{s_2}x + d_{s_2} \end{cases} \tag{11}$$

As a result we obtain the set of thirteen equations (3), (6)–(10), where the values of coefficients of the s_1 i s_2 splines (10), the coordinates P and L points and the β_K inclination angle of the tangent towards the ski jumping trajectory at the K construction point are unknown. By solving this set of equations we obtain the value of the β_K = 35,8° angle. The graphs of the P–K–L landing area profile described with the s_1 i s_2 splines and their second derivatives are presented in Fig. 3.

By increasing the β_K inclination angle of the tangent towards the P–K–L landing area profile at the K point by 0,3°, was obtained $r_K \to \infty$, which leads to the increase of the safety of landing by elimination a normal inertia force in the neighborhood of the K point.

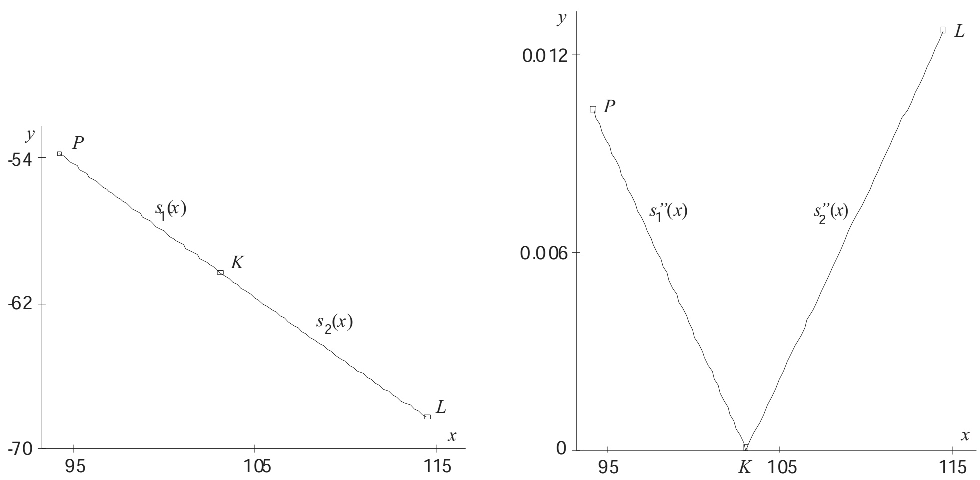

Fig. 3. The landing area described with the s_1 and s_2 splines and their second derivatives

Rys. 3. Strefa bezpieczeństwa opisana krzywymi sklejanymi s_1 i s_2 oraz ich drugie pochodne

4. Description of other elements of the outrun profile of a ski jumping hill

The knoll is described by means of the SP circular arc passing the P and S points with the following coordinates x_S = 0, $y_S = -s$ (s – the height of take-off), which is tangent towards the P–K–L landing area track at the P point.

Taking into consideration the fact that the β_L angle and the coordinates of the L point, which are already known, we can strike out the LM circular arc with the r_2 radius, which is tangent towards the P–K–L landing area at the L point and the MN segment at the M point.

These arcs are described by the following formula:

$$y_i = y_{O_i} - \sqrt{r_i^2 - \left(x - x_{O_i}\right)^2}, \quad i = 2,3 \qquad (12)$$

where: (x_{O_i}, y_{O_i}) – points which are the centers of the circles and equal: $x_{O_2} = 187{,}2$ m, $y_{O_2} = 43{,}7$ m, $x_{O_3} = -109{,}9$ m, $y_{O_3} = -319{,}8$ m.

The $r_3 = 335{,}3$ m radius of the *SP* arc was calculated on the basis of geometrical rules.

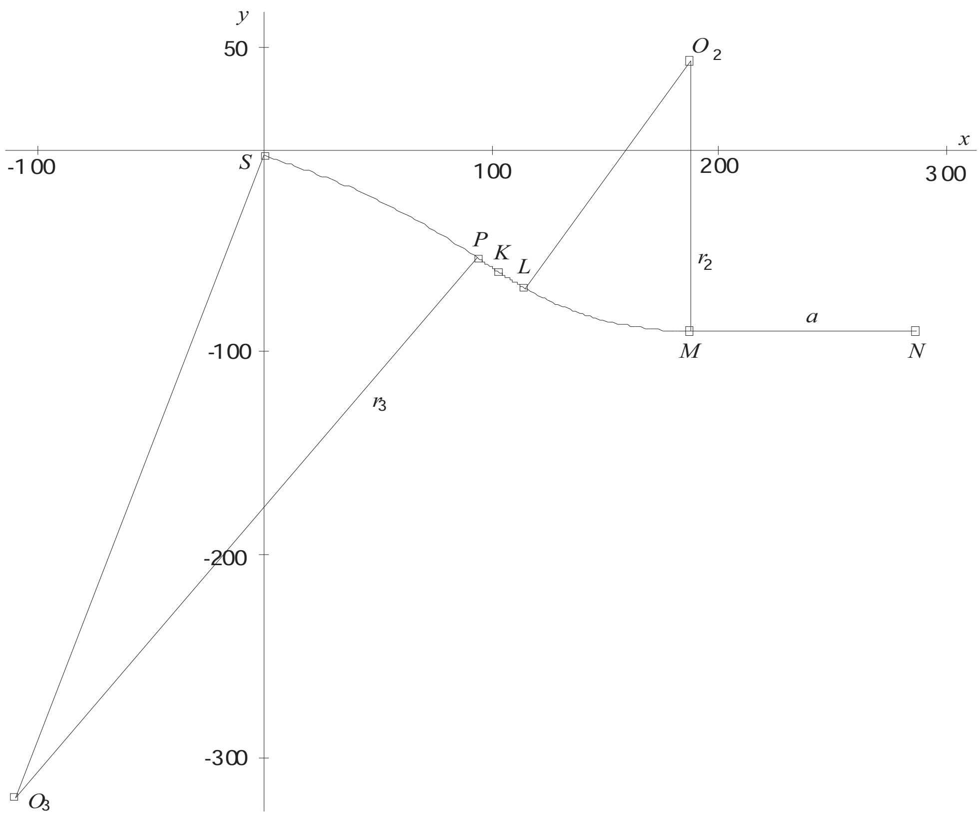

Fig. 4. The out-run profile of a ski jumping hill

Rys. 4. Profil zeskoku skoczni narciarskiej

5. Conclusions

The outrun profile of a ski jumping hill with parameters presented in Table 1, could be described in different ways. This paper presents the description of the landing area by means of the interpolation polynomial and the splines. In the case of an interpolation in the form of splines, we can use the radius of curvature $r_K \to \infty$ at the *K* construction point, which means that the *P–K–L* landing area is a straight line in the neighborhood of this point. Landing on a rectilinear segment is the safest for ski jumpers. This situation is possible on condition that we resign from the given value of β_K angle. As it can be figured out on the basis of the calculations, its value changes slightly, because it increases only by 0.3°, which in comparison with the values of the β_P and β_L angles seems to be admissible.

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