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MEASUREMENT OF STRUCTURAL PARAMETERS OF TRABECULAR BONE USING FUZZY LOGIC

POMIAR PARAMETRÓW STRUKTURALNYCH KOŚCI BELECZKOWEJ PRZY UŻYCIU LOGIKI ROZMYTEJ

A b s t r a c t

In this paper there was proposed the trabecular bone structure analysis using fuzzy logic. Theoretical and algorithmic basis method of measurement structural parameters there was presented. Application using FDT algorithm was adapted to analysis thickness of trabecular bone. Study methodics of measurements of gray level images allows using physical relation between mechanical and structural properties of images especially for limited resolution images, with data inaccuracies, graded object compositions and for structures having similar intensities for different parts.

Keywords: fuzzy logic, FDT algorithm, trabecular bone

S t r e s z c z e n i e

W artykule zaproponowano zastosowanie logiki rozmytej do analizy struktury kości beleczkowej. Przedstawione zostały teoretyczne i algorytmiczne podstawy metod pomiaru parametrów strukturalnych. Do analizy grubości beleczek zastosowano aplikację korzystającą z algorytmu FDT. Opracowanie metodyki pomiarów na obrazach w skali szarości pozwoli na wykorzystanie fizycznie uzasadnionych relacji wiążących własności mechaniczne i strukturalne obrazów w zastosowaniach medycznych, szczególnie w przypadku obrazów o niskiej rozdzielczości, z nieścisłymi danymi, stopniowanym układem obiektu oraz dla podobnej struktury różnych części.

Słowa kluczowe: logika rozmyta, algorytm FDT, kość beleczkowa

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1. Introduction

Lots of things on the world are fuzzy, especially digital images. As opposed to crisp memberships fuzziness quantifies vagueness and ambiguity. There are manifold types of uncertainty in images, from pixel-based grayness ambiguity over fuzziness in geometrical description up to uncertain knowledge in the highest processing level. Image features logically have to be considered fuzzy because of the problem with a gray-value slope, an edge, border of a blurred object and so on.

There will be presented the fuzzy logic theory and the feasibility of applying fuzzy logic. Fuzzy image processing is special in terms of its relation to other computer vision techniques. It describes a new class of image processing techniques and allow to characterized objects where data inaccuracies, graded object compositions or limited image resolutions are. As an example of using fuzzy logic it will be presented the dynamic programming-based algorithm for computing the FDT (fuzzy distance transform) to measure thickness of trabecular bone.

2. Background

Fuzzy image processing is based on fuzzy logic and uses its logical, set-theoretical, relational and epistemic aspects. Fuzzy geometry, measures of fuzziness/image information, rule-based approaches, fuzzy clustering algorithms, fuzzy mathematical morphology, fuzzy measure theory are the most important theoretical frameworks that can be used to construct the foundations of fuzzy image processing. All of these areas can be used either to develop new techniques or to extend the existing algorithms [1].

2.1. Fuzzy logic

The theory of fuzzy logic was introduced by Zadeh [2], in his innovative work *Fuzzy Sets*. A fuzzy set is a class of objects with continuum of grades of membership. Fuzzy sets are an extension of crisp sets. Crisp sets do not allow partial memberships; they only allow full or null membership of an element x to the set A, i.e.,

$$\mu_A(x) = \begin{cases} 1 & x \in A \\ 0 & x \notin A \end{cases} \tag{1}$$

where:

$\mu_A(x)$ – represents the membership of x to A.

Partial memberships are allowed in fuzzy sets. The range of $\mu_A(x)$ is [0, 1] instead of {0, 1} as for crisp sets, and the set A is defined as

$$A = \{(x, \mu_A(x)) | x \in U\} \tag{2}$$

where

U – is the universe of discourse.

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The notions of inclusion, union, intersection, complement, relation, convexity, etc., are extended to such sets, and various properties of these notions in the context of fuzzy sets are established.

2.2. Fuzzy geometry and information

Geometrical relationships between the image components play a key role in intermediate image processing. Many geometrical categories such as area, perimeter, and diameter, are already extended to fuzzy sets [3]. The geometrical fuzziness arising during segmentation tasks can be handled efficiently if we consider the image or its segments as fuzzy sets. The main application areas of fuzzy geometry are feature extraction (e. g., in image enhancement), image segmentation and image representation ([3], see Table 1). A more detailed description of other aspects of fuzzy geometry can be found in the literature.

Table 1

Theory of fuzzy geometry based on [3]

Aspects of fuzzy geometry	**Examples of subjects and features**
digital topology	connectedness, surroundedness, adjacency
metric	area, perimeter, diameter, distance between fuzzy sets
derived measures	compactness index of area coverage, elongatedness
convexity	convex/concave fuzzy image subsets
thinning/medial axes	shrinking, expanding, skeletonization
elementary shapes	fuzzy discs, fuzzy rectangles, fuzzy triangles

Fuzzy perimeter and compactness were investigated in this work as possible parameters to characterize trabecular bone structure. Considering an image *S* of size *KL*, containing one object with the membership values $\mu_{k,l}$. The area of the object – interpreted as a *fuzzy subset* of the image – was defined as:

$$\mathrm{area}(\mu) = \sum_{k=0}^{K} \sum_{l=0}^{L} \mu_{k,l} \tag{3}$$

and the perimeter of the object can be calculated as

$$\mathrm{perimeter}(\mu) = \sum_{k=1}^{K} \sum_{l=1}^{L-1} \left\| \mu_{k,l} - \mu_{k,l+1} \right\| + \sum_{k=1}^{K-1} \sum_{l=1}^{L} \left\| \mu_{k,l} - \mu_{k+1,l} \right\| \tag{4}$$

Then, the fuzzy compactness, introduced by Rosenfeld [4] was represented as

$$\mathrm{compactness}(\mu) = \frac{\mathrm{area}(\mu)}{[\mathrm{perimeter}(\mu)]^2} \,. \tag{5}$$

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2.3. Measures of fuzziness and image information

Fuzzy sets can be used to represent a variety of image information. Fuzziness refers to the level of uncertainty of an image feature given the corresponding membership function. In theory the intersection of a crisp set with its complement equals zero. However, for two fuzzy sets, this condition no longer holds. The more fuzzy a fuzzy set is, the more it intersects with its own complement [3]. This leads to the definition of *indices of fuzziness* γ.

In this work, there is given a fuzzy set A with the membership function μ_A defined over an image of size K x L so the linear (γ_l) and quadratic (γ_q) *indices of fuzziness*, which are given by Eqs. (6) and (7), respectively, and explored them as potential descriptors of the trabecular bone structure:

$$\gamma_l(S) = \frac{2}{\sqrt{KL}} \sum_{k,l} \min(\mu_{mn}, 1-\mu_{mn}) \tag{6}$$

$$\gamma_q(S) = \frac{1}{\sqrt{KL}} \left[\left(\sum_{k,l} \min(\mu_{mn}, 1-\mu_{mn}) \right)^2 \right]^{1/2} \tag{7}$$

Entropy is a theoretic measure quantifying the information content of an image. The counterpart in fuzzy set theory is given by the *fuzzy entropy*. As the indices of fuzziness, the logarithmic ($H_{\log}$) and exponential ($H_{\exp}$) which has been proposed by Pal and Pal [5]) *fuzzy entropies* were defined as:

$$H_{\log}(S) = \frac{1}{KL \ln 2} \sum_{k,l} B_l(\mu_{kl}) \tag{8}$$

where

$$B_l(\mu_{kl}) = -\mu_{kl} \ln(\mu_{kl}) - (1-\mu_{kl}) \ln(1-\mu_{kl}) \tag{9}$$

and

$$H_{\exp}(S) = \frac{1}{KL(\sqrt{e}-1)} \sum_{k,l} \{\mu_{kl} e^{(1-\mu_{kl})} + (1-\mu_{kl}) e^{\mu_{kl}} - 1\} \tag{10}$$

The measures of uncertainty ranging from zero to unity.

3. Fuzzy distance transform

Using fuzzy logic, described above, we can measure length between two points. First we have to defined the length of a path on a fuzzy subset and then find the infimum of the lengths of all paths between two points. The length of a path π in a fuzzy subset of the n-dimensional continuous space R^n is defined as the integral of fuzzy membership values

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along π [6]. There are many paths between any two points in a fuzzy subset and the shortest one may not exist. Infimum of the lengths of all paths between two points is the definition of the fuzzy distance between them. The shortest path between two points in a fuzzy convex subset is not necessarily a straight line segment, unlike in hard convex sets. For any positive number $\theta \leq 1$, the θ-support of a fuzzy subset is the set of all points in R^n with membership values greater than or equal to θ. For any fuzzy subset, for any nonzero $\theta \leq 1$, fuzzy distance is a metric for the interior of its θ-support. However for any smooth fuzzy subset, fuzzy distance is a metric for the interior of its 0-support (referred to as *support*). Therefore a process on a fuzzy subset that assigns to a point its fuzzy distance from the complement of the support is defined as FDT.

It is very important and effective tools for object shape analysis. It can be used in many imaging applications especially medical ones. DT for fuzzy objects becomes useful in many imaging applications where data inaccuracies, graded object compositions, or limited image resolution are. FDT will be useful in feature extraction [7], local thickness or scale computation [8, 9], skeletonization [10, 11, 12], and morphological [13] and shape-based object analyses [14].

3.1. Algorithm for computing the FDT

In this chapter, it will be presented the dynamic programming-based algorithm [6] for computing the FDT of digital objects. Assuming a uniform neighborhood, with respect to a point $p \in Z^n$, all its adjacent neighbors are ranked. A vector δ, called a resolution vector, is used whose ith element gives the continuous distance between a point and its ith adjacent neighbor. Here, we use the (3^n-1)-adjacency relation. Therefore, δ is a $(3n-1)$-dimensional vector. The information about the resolution vector δ may be directly obtained from the application imaging system. A digital object O is a fuzzy subset $\{(p, \mu_O(p)) \mid p \in Z^n\}$, where $\mu_O : Z^n \rightarrow [0, 1]$ specifies the membership value at each point in the object where Z is the set of all integers. A binary relation is α. The algorithm is following:

Input: $O = (Zn, \mu_O)$, α, and δ.

Auxiliary Data Structures: modified fuzzy object $O' = (Z^n, \mu_{O'})$, an image (Z^n, Ω) representing FDT of O', and a queue Q of points.

Output: an image (Z^n, Ω) representing FDT of O'.

1. compute $O' = (Z^n, \mu_{O'})$ from $O = (Z^n, \mu_O)$ following Steps A1 and A2;

   (***Step A1.*** $\mu_{O'}(2i+1,\ 2j+1) = \mu_O(i, j)$ for all $i, j \in Z^n$. The points in O' with coordinates of the form $(2i+1, 2j+1)$ *primary points* and among those, the points with nonzero membership *primary object points*.

   Step A2. For a non-primary point $p \in Z^n$, if p is adjacent to a primary non-object point, assign $\mu_{O'}(p) = 0$, otherwise assign $\mu_{O'}(p)$ = the mean of the membership of all adjacent primary object points.)

2. for all $p \in \Theta(O')$, set $\Omega(p) = 0$;
3. for all $p \in \Theta(O')$, set $\Omega(p) = \text{MAX}$; /**MAX is a large value**/
4. for all $p \in \Theta(O')$ such that $N^*(P) \cap \Theta(O')$ is non-empty
5. push p into Q;
6. while Q is not empty do
7. remove a point p from Q;

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8. find $dist_{\min} = \min_{q \in N^*(p)}[\ \Omega(q) + \delta_{\mathrm{rank}}(p,q) \times \frac{1}{2}(\mu_{O'}(p) + \mu_{O'}(q))]$;
/*$rank(p, q)$ gives the rank of q in the neighborhood of p*/
9. if $dist_{\min} < \Omega(p)$ then
10. set $\Omega(\mathrm{p}) = dist_{\min}$;
11. push all points $q \in N^*(P) \cap \Theta(O')$ into Q;
12. output the FDT image O';

3.2. Computation of structural parameters

Above was presented an algorithm for computing FDT of a fuzzy digital object. In this section, there will be discussed application of FDT to measure one of the structural parameters of trabecular bone e.g. local thickness in the regime of limited resolution where it becomes fuzzy [6].

In Fig. 1 and 2 there is illustrated an example of thickness computation (this is a important parameter in analyzing object shape). It is a high-resolution in vivo 3D magnetic resonance image (MRI) of the human trabecular bone. The image size is 512×256×32 and the voxel size is 137×137×350 μm^3.

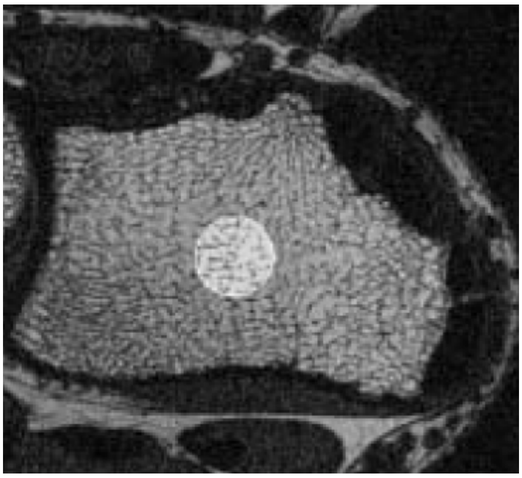

Fig. 1. 2D slice taken from the raw image

Rys. 1. Nieprzetworzony obraz 2D kości beleczkowej

In a Fig. 1 a slice from the raw image is shown. For analysis a cylindrical region of interest (ROI) was chosen. The central highlighted disk in Fig. 1 is the cross section of the ROI with that slice. To produce a bone volume fraction (BVF) map the image within the ROI was preprocessed by deshading and noise reduction using a histogram deconvolution method. The spatial resolution was enhanced to 68×68×88 μm^3 using subvoxel classification. A 3D projection of the final BVF image is presented in Fig. 2. From the resolution-enhanced BVF image the FDT image was computed. Thickness values were computed with use a surface skeleton of the bone mask. The mean deviation of thickness values along the skeleton was 102 and the standard deviation was 42 µm.

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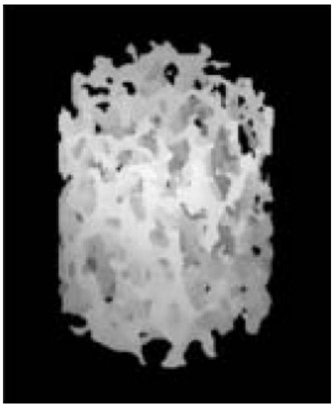

Fig. 2. 3D projection of the final BVF image

Rys. 2. Odwzorowanie 3D końcowego obrazu BVF

4. Conclusions

In this study, there was demonstrated the theory and the feasibility of applying fuzzy logic to measure the structure parameters of trabecular bone. Fuzzy logic image processing techniques allow to characterized objects where data inaccuracies, graded object compositions or limited image resolutions are. In many medical imaging applications such as computation of local thickness (an important parameter in analyzing object shape) of trabecular bone, or morphology-based separation of anatomic structures having similar intensities algorithm of fuzzy distance transform will be useful.

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