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FEM ANALYSIS OF A BEAM FOR PIEZOELECTRIC
PASSIVE VIBRATION CONTROL SYSTEMANALIZA MES BELKI DO PASYWNEGO TŁUMIENIA
DRGAŃ PRZY UŻYCIU ELEMENTÓW
PIEZOELEKTRYCZNYCH

Abstract

One of the most significant problems nowadays is an issue of vibrations suppression. Vibrations can have negative effects on proper work of industrial systems as well as on operators health. In recent years many researchers have devoted a lot of attention to smart materials, which can be implemented in vibration suppression systems. This paper presents a beam structure with piezoelectric stripes glued to the surface. The model was made in Ansys application utilizing Finite Element Method (FEM). The aim of the analysis was choosing the correct model and quality of the mesh which will ensure that the model can be used for wide spectrum of analysis. It was shown that only 3D model represents the behavior of vibrating beam in all the aspects and gives results very similar to analytical ones. Presented model may be utilized for multimodal passive damping by connecting the correct electric circuit with calculated values of inductance and resistance.

Keywords: piezoelectric materials, passive damping, FEM, modal analysis

Streszczenie

W dzisiejszych czasach ważnym problemem stało się zagadnienie tłumienia drgań, które mogą mieć negatywne skutki dla pracy maszyn, jak również dla zdrowia operatora. W ostatnich latach wiele uwagi poświęca się materiałom inteligentnym, które można wykorzystać do systemów tłumienia drgań. W pracy przeanalizowano strukturę belkową na powierzchni której przyklejono przetworniki piezoelektryczne. Struktura została zamodelowana w programie Ansys przy pomocy metody elementów skończonych. Celem analizy było dobranie odpowiedniego modelu oraz gęstości siatki, dzięki czemu w przyszłości model będzie mógł służyć do szerokiego spektrum analiz. Pokazano, iż tylko model 3D pokazuje zjawisko we wszystkich jego aspektach dając wyniki najbardziej zbliżone do rzeczywistych. Tak przygotowany model może być użyty do wielomodalnego pasywnego tłumienia drgań przez połączenie odpowiedniego obwodu z dobraną cewką i rezystancją.

Słowa kluczowe: materiały piezoelektryczne, tłumienie pasywne, MES, analiza modalna

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1. Introduction

In last few years the topic of investigation the most optimal methods of vibrations suppression is being widely researched. Those vibrations observed in natural environment and in human work environment can be a reason of serious health problems. Latest development in materials science shows that especially smart materials are being discussed as a possible implementation for vibration mitigation systems. Implementation of those controllable materials demands using expert knowledge from areas of vibration theory, dynamics of machinery and structures, materials science, computer science and control theory. The following paper attracts especially attention to piezoelectric materials as example of controllable materials.

1.1. Piezoelectric materials

1.1.1. Vibration Control using piezoelectric transducers

The first scientific publication describing the phenomenon, later termed as piezoelectricity, appeared in 1880 by Pierre and Jacques Curie. They discovered a class of materials that when pressured, generate electrical charge, and when placed inside an electric field, strain mechanically. Piezoelectricity, which literally means “electricity generated from pressure” naturally occurs in many monocrystalline materials. However, these materials are generally not suitable as actuators for vibration control applications. Instead, man-made polycrystalline ceramic materials, such as lead zirconate titanate (*PZT*), can be processed to exhibit significant piezoelectric properties. In recent years piezoelectric transducers have been extensively used in structural vibration control applications. Their wide utilization in this specific application can be attributed to their excellent actuation and sensing abilities which stems from their high electro-mechanical coupling coefficient, as well as their non-intrusive nature. One of the most promising damping methods is so called piezoelectric *shunt damping*. The technique is characterized by the connection of electrical impedance to a structurally bonded piezoelectric transducer. Such methods do not require an external sensor, and if designed properly, may guarantee stability of the shunted system.

1.1.2. Passive shunt dumping

Electrical shunt impedance is said to be passive if and only if it does not supply power to the system, which can be noted as:

$$\int_0^{\infty} v(t) \cdot i(t) \geq 0 \quad (1)$$

where:

$v(t)$ and $i(t)$ – are the voltage and current.

The foremost benefit of passivity is that stability of the shunted system is guaranteed.

One of the popular piezoelectric passive damping techniques is so called piezoelectric resonant shunt damping. Such a system is considered as connection of electric impedance and piezoelectric transducer structurally bounded to the structure. This method doesn't

require any external sensors, assure good stability of shunt system and during design process parametric model is not required. One of the easiest methods to create a passive piezoelectric shunt damping is to connect resistance R to the transducer. Mechanical energy is being converted (by the transducer) into electric energy which is being dissipated by the resistor. Although simple resistors are rarely used as they offer only a small amount of damping; typically only a few dB . Similar technique is adding capacitance to the terminals of a piezoelectric transducer varies its effective stiffness. Such a circuit is easy to build but effects are similar to resonant technique described above.

Much more efficient are single-mode shunt circuits. Especially circuits presented by Forward and Hagood and von Flotow, which may be widely used in vibration control systems. Forward's has proposed the idea of inductive (LC) shunting for narrow band reduction of resonant mechanical response. Hagood and von Flotow's idea was to add resistive element to the shunt network, resulting in an RLC tuned circuit. The resulting RLC circuit is tuned to a specific resonance frequency of the composite system. That is, if the vibration associated with the i -th mode is to be reduced, then L has to be chosen. By adopting a proper value for R , the resonant response at and in the vicinity of ω_i can be reduced. Although Hagood and von Flotow algorithm is easy to adapt in many systems but due to its passive nature can offer only few dB more than methods described before. Different (much more effective) method is a serial connection (to transducers electrodes) of resistance R and inductance L , which together with internal capacity of the transducer C_p form piezoelectric resonant circuit. Such a circuit is tuned using particular values of inductance to resonance frequency of the system and resistance is selected to assure whole electric energy being converted into heat. When more elements are glued to the structure it is possible to tuned every one of them to particular vibration mode. However, it is not easy to design such a system. Very often the problem with lack of space for piezoelectric elements may occur. Better way seems to be connection of such a circuit which allows controlling vibrations in many sets of frequencies. Such a system may be designed as parallel connection of many serial RLC circuits and first branch of RL circuit. Other solutions were discussed where additional LC circuits were used some blocking and some allowing current flow.

2. Numerical modelling of piezoelectric vibration control system

The objective of numerical analysis was to design a model of a beam structure restrained from one side and size as it is shown in Fig. 1. The examined structure exists at the test stand in the laboratory so in the future all results of numerical analysis can be checked. For particular model wide spectrum of analysis can be carried out. Two basic realizations are possible: active and passive damping of vibrations.

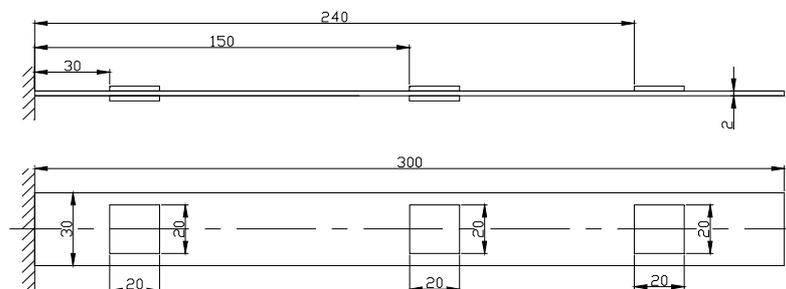


Fig. 1. The model of investigated beam presented with accepted dimensions and placement of piezoelectric elements

Rys. 1. Model badanej belki wraz z przyjętymi wymiarami i rozmieszczeniem elementów piezoelektrycznych

To ensure optimal damping of particular modes the place of piezoelectric elements attachment was discussed. The placement of piezoelectric elements has to be chosen in the area of high curvature which arises from particular vibration form. In particular those places cannot be in the nodes of the form we are planning to damp. The paper presents results of system vibration simulation using *Finite Element Method* in *Ansys* application. Two approaches were discussed: modelling of the beam structure as a two dimensional and a three dimensional problems. The reason for this kind of analysis was to double check results of the analysis and to proof that 3D model is more accurate and can be use for wider spectrum of analyses.

2.1. FEM 2D modelling of a beam

Important problem during *FEM* analysis is choosing the right elements for modelling particular parts of the structure. In described problem we have two types of elements: piezoelectric stripes and aluminium beam structure. In case of a beam, structure is regular without any curves and can be modelled with any kind of 2D element. However, it is a good idea to use higher order element type. During discussed analyses *PLANE82*, which is eight-node element, was used.

The problem occurs when element type of piezoelectric part of the structure has to be chosen. The aim is to handle a wide spectrum of analysis (passive or active damping) so piezoelectric element has to be modelled accurately. In case of researched beam piezoelectric stripes were modelled by *PLANE13* element which has a 2D piezoelectric capability with limited coupling between the fields. *PLANE13* is defined by four nodes with up to four degrees of freedom per node. Of course those element types are not the only sets available for modelling particular structure.

Table 1

Types of elements which can be used for modelling 2D structure of a beam with piezoelectric transducers glued to the surface of the beam

| Other elements types for 2D modelling | | | | | |
|---------------------------------------|--------------|----|---|----------|------|
| For beam structure (structural solid) | Nodes nr | | For piezoelectric (coupled field solid) | Nodes nr | TYPE |
| PLANE42 | 4 nodes | ST | PLANE13 | 4 nodes | CF |
| PLANE82 | 8 nodes | ST | PLANE223 | 8 nodes | CF |
| PLANE182 | 4 nodes | ST | | | |
| PLANE183 | 6 or 8 nodes | ST | | | |
| ST – structural solid type | | | CF – coupled field solid type | | |

For element types as above following structure can be obtained:



Fig. 2. The 2D *FEM* model of the beam in *Ansys* application

Rys. 2. Model *MES* belki 2D otrzymany w programie *Ansys*

To ensure that model is accurate modal analysis can be carried out and the results can be later checked with the analytical solution. First three vibration forms can be drawn as in Fig. 3.

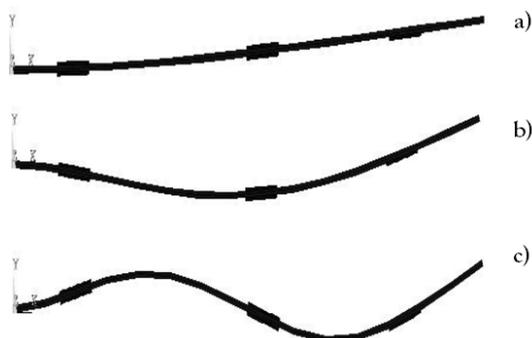


Fig. 3. Shape of free vibration forms for 2D model

Rys. 3. Wygląd pierwszych trzech form własnych modelu 2D

2.1. FEM 3D modelling of a beam

Model of the discussed structure was modelled using *SOLID45* elements for beam and *SOLID5* for piezoelectric elements. *SOLID45* is eight nodes element used for the 3D modelling of solid structures. *SOLID5* is Coupled-Field Solid element with eight nodes and has a 3D piezoelectric capability with limited coupling between the fields.

Types of elements which can be used for modelling 3D structure of a beam with piezoelectric transducers glued to the surface of the beam

| Other elements types for 3D modelling | | | | | |
|--|----------|----|--|----------|------|
| For beam structure (structural solid) | Nodes nr | | For piezoelectric (coupled field solid) | Nodes nr | TYPE |
| SOLID45 | 8 nodes | ST | SOLID5 | 8 nodes | CF |
| SOLID95 | 20 nodes | ST | SOLID98 | 10 nodes | CF |
| SOLID186 | 20 nodes | ST | SOLID226 | 20 nodes | CF |
| | | | SOLID227 | 10 nodes | CF |
| ST – structural solid type | | | CF – coupled field solid type | | |

The following 3D model of a beam was created in *Ansys* application (Fig. 4).

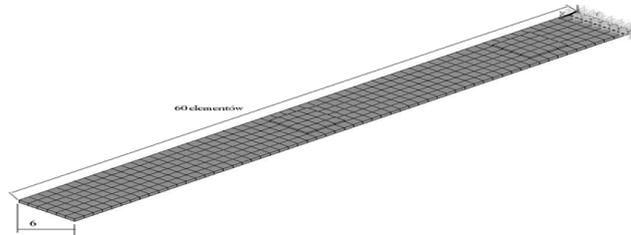


Fig. 4. The 3D *FEM* model of the beam in *Ansys* application

Rys. 4. Model *MES* belki 3D otrzymany w programie *Ansys*

Important matter during *FEM* analysis is choosing the correct mesh size. Not enough elements can cause the results to be not accurate but too high concentration will result in long processing. Three cases were discussed. First one where structure was divided into cubicooid elements with square base which side length was 5 mm and height 2 mm (Fig. 4). Second case where all elements lines were divided by 3, and third case where line division was equal to 4. Division Of the lines was possible using *Ansys* *ESIZE* command.

Table 3

Results of modal analysis for particular line divisions

| MOD | FREQUENCY [HZ] | | |
|-----|----------------|--------|--------|
| | ESIZE1 | ESIZE3 | ESIZE4 |
| 1. | 18.711 | 18.662 | 18.649 |
| 2. | 115.25 | 114.91 | 114.83 |
| 3. | 256.91 | 255.99 | 255.9 |
| 4. | 324.16 | 323.28 | 323.1 |
| 5. | 388.25 | 382.09 | 381.04 |
| 6. | 628.34 | 623.77 | 623.08 |

Line division into 4 ensured the good accuracy of results. Smaller elements didn't give much increase in results accuracy but made calculations last very long.

2.3. Discussion of the results

In order to check the correctness of the model and results obtained from modal analysis analytical solution was made. According to the solution for one-dimensional model of bended beam frequencies of free vibrations, have following equations:

$$\omega_1 = 3.5256 \cdot \frac{1}{l^2} \cdot \sqrt{\frac{EJ}{\rho A}}, \quad \omega_2 = 22.0346 \cdot \frac{1}{l^2} \cdot \sqrt{\frac{EJ}{\rho A}}$$

$$\omega_i = \left[\frac{(2i-1)\pi}{2} \right]^2 \cdot \frac{1}{l^2} \cdot \sqrt{\frac{EJ}{\rho A}} \quad \text{where } i = 3, 4, \dots \quad (2)$$

where:

- E – Young's modulus,
- ρ – Poisson ratio,
- J – moment of inertia of face area relative to neutral axis of bending.

Using equations (2) free vibration frequencies in planes the xz and yz can be calculated. First three frequencies of vibrations are:

$$f_1 = 18.522 \text{ [Hz]} \quad f_2 = 116.993 \text{ [Hz]} \quad f_3 = 325.147 \text{ [Hz]} \quad (3)$$

Obtained values are very similar to values obtained from 2D analysis using *FEM* but are slightly different than in 3D model. This difference occurred not because of model incorrectness but because 2D model (and analytical solution too) includes only occurrence of transverse vibrations. 3D *FEM* model allows determining frequencies and forms of vibrations regardless of what vibrations nature is. It includes transverse, torsional, longitudinal, complex nature of vibrations. This is a reason why frequencies (3) correspond to first, second and forth form of vibrations (Results from *FEM* 3D model).

$$f_1 = 18.649 \text{ [Hz]}, \quad f_2 = 114.83 \text{ [Hz]}, \quad f_3 = 114.83 \text{ [Hz]}, \quad f_4 = 323.1 \text{ [Hz]}, \quad (4)$$

During modal analysis in *Ansys* in 3D model, a transverse- torsional form of vibrations occurs.

3. Conclusions

The paper presents how important problem is an issue of vibrations mitigation. Special attention was paid to the use of smart materials called piezoelectric which can be used for creation of highly specialized damping systems.

The main aim of the simulation was to create a numeric model which can be used in future not only for simulation of passive but active damping as well. To approaches were

presented: 2D model and 3D model. Results were compared with each other and with analytical solution. It was shown that two dimensional model includes only bending and not include influence of shearing. Although results from 2D, 3D modelling and analytical solution were similar, only 3D model can be used for wide spectrum of analysis.

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