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KATARZYNA TAJS-ZIELIŃSKA*

LOCAL APPROACH TO OPTIMAL STRUCTURAL DESIGN USING CELLULAR AUTOMATA

LOKALNE SFORMUŁOWANIE PROBLEMU OPTYMALNEGO KSZTAŁTOWANIA W UJĘCIU AUTOMATÓW KOMÓRKOWYCH

A b s t r a c t

Cellular Automata is mathematical idealization of a physical system in which space and time are discrete. The design domain is divided into a lattice of cells, states of which are updated synchronously in discrete time steps according to some local rules. The behavior of each cell depends on its present state and states of its neighbors. This paper presents a topology optimization for 2D elastic structures by using the concept of Cellular Automata (CA). The novel local update rule is proposed. The presented concept is illustrated by results of numerical examples.

Keywords: Cellular Automata, local update rules, structural optimization

S t r e s z c z e n i e

Automat komórkowy jest matematyczną idealizacją fizycznego systemu, w którym przestrzeń i czas są dyskretne. Rozważany obiekt dzielony jest na komórki, których stany są uaktualniane jednocześnie w procesie iteracyjnym według pewnych lokalnych reguł. Zachowanie każdej komórki zależy od jej aktualnego stanu oraz od stanów jej sąsiadów. W artykule pokazano zastosowanie metody automatu komórkowego do optymalizacji topologicznej dwuwymiarowych struktur sprężystych. Zaproponowana została nowa reguła aktualizacji, a jej działanie zostało zilustrowane przykładami numerycznymi.

Słowa kluczowe: automaty komórkowe, lokalne reguły uaktualniania, optymalizacja konstrukcji

* MSc. Katarzyna Tajs-Zielińska, Institute of Applied Mechanics, Faculty of Mechanical Engineering, Cracow University of Technology.

1. Introduction

The concept of Cellular Automata (CA) was proposed in the late forties of the 20th century by Von Neumann [14] and Ulam [13], and developed afterwards by Wolfram [15]. Cellular Automata is a mathematical idealization of a physical system in which the design domain is divided into a lattice of cells, states of which are updated synchronously in discrete time steps according to some local rules. By applying the rules repetitively to locally updated physical quantities (states of cells) the process converges to a description of the global behavior of the system. The characteristic feature of Cellular Automata which is modeling of complex systems by simple local rules has attracted researchers from various disciplines like physics, computer science, biology, social sciences, transport and recently engineering.

The first application of CA to optimal structural design was proposed in the mid nineties by Inou et al. [6] and later on by Kita and Toyoda [7]. In [6] the design domain was divided into small cells, and the Young modulus of the cell material is used as the design variable. A local rule iteratively updated the value of the elastic moduli of cells based on the difference between a current and a target stress value. The cells with a low elastic modulus were removed . The stress analysis was carried out by the finite element method.

The design problem in [8] has originally been formulated as optimal sizing. The cell thicknesses were used as the design variables and a combination of weight and deviation of current stress value from admissible one was chosen as the objective. The local rule, which was derived by setting the first variation of the objective function equal to zero updates design variables values. The so-called "CA-constraint" responsible for exchange of local information between cells has been implemented into design problem formulation. One of the most important papers is [10]. A new cellular automata technique presented there was inspired by phenomenological approaches adopted to simulate bone functional adaptation. This method was extended e.g. in [11] and [12].

In the above papers local design rules have been applied simultaneously to each cell for a fixed displacement field. State variables are updated sequentially after optimization iterations. In [9] authors proposed a new scheme, in which analysis and design are performed simultaneously. This technique called simultaneous analysis and design (SAND) was practically realized by combining local design rules with local update of field variables. As a result both updates were implemented sequentially, what eliminated the need for finite element analyses being performed at each iteration step as in the typical optimization schemes. A global equilibrium was reached only for the optimal design.
A broad discussion on CA application to structural optimization and more details can be found in [12].

This paper presents Cellular Automata as the structural optimization tool, describes briefly the main principles of CA and proposes a novel update rule. The presented concept is illustrated by numerical examples. The present findings complement discussion regarding CA application to structural optimization was carried out in former papers [4, 8].

2. Cellular Automata basics

2.1. Neighborhood and boundary conditions

The CA formulation of optimal design problem requires decomposition of considered domain into a set of cells which form uniform lattice. A particular cell together with cells to which it is connected is called neighborhood. In updating the state of cell the local parameters are considered as well as the values related to the cells in its neighborhood. It is therefore assumed that the cells interact only within their neighborhood. The examples of neighborhoods are presented in the Fig. 1.

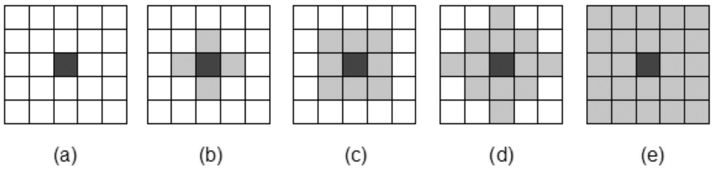

Fig. 1. Neighborhoods in plane problems: (a – empty, (b – von Neumann, (c – Moore, (d – MvonN – e)-extended Moore

Rys. 1. Rodzaje sąsiedztwa w zagadnieniach dwuwymiarowych: (a – puste, (b – von Neumann, (c – Moore, (d – MvonN, (e – rozszerzone Moore'a

Each cell has the same neighborhood, therefore those at the boundary have neighboring cells that lay outside the design domain. It is important to specify for them values of the design variables. The simplest and the most often used approach sets all these values to zero, but as the alternatives periodic, reflecting or adiabatic boundary conditions can also be adopted (see [11]).

2.2. Update rules and iteration schemes

States of cells are represented by state variables (e.g. displacements) and design variables (e.g. thickness for optimal sizing or the Young modulus for topology optimization). These quantities are modified during subsequent iterations according to local rules. The rules are identical for all cells and are applied simultaneously to each of them. By applying the rules repetitively to locally updated physical quantities the process converges to a description of the global behavior of the system. New values can be calculated based on already updated values found for cell neighbors (Gauss-Seidel iteration mode) or cell updates its state based on the states of the surrounding cells obtained in the previous iteration, no information is passed from one cell to another within the same iteration (Jacobi iteration mode). A local design update rule can be applied simultaneously to each cell for a fixed displacement field (sequential approach), or both design update and structural analysis are performed locally simultaneously for each cell (simultaneous analysis and design). In the latter case global equilibrium is reached only for the optimal design.

3. Structural optimization using Cellular Automata approach

3.1. Topology optimization and optimal sizing

The principle of the topology optimization started by Bendsoe and Kikuchi [3] and Bendsoe [2] is to find within a design domain the distribution of material that is optimal in some sense. The design domain is represented by finite elements and design variables are selected for these elements. The distribution of the optimal design variables is used to create the resulting topology. During optimization process material is redistributed and parts that are not necessary from objective point of view are selectively removed. This fulfills a typical constraint which limits the amount of material that may be present in the final solution. Topology optimization usually ends up in finding material distribution that is visualized by black and white regions over the design domain.

The relative density of material is the typical design variable for topology optimization. In this case the elastic modulus E_i and density ρ_i of each cell are modeled as functions of relative density d_i

$$E_i = d_i^p E_0, \quad \rho_0 = d_i \rho_0, \quad 0 \le d_i \le 1 \tag{1}$$

where:

E_0 – elastic modulus of solid material,
ρ_0 – density of solid material,
p – penalization power.

The penalization power penalizes intermediate densities and drives solution to black and white structure. If instead of varying the Young modulus thickness is chosen as the design variable, the optimization problems changes into optimal sizing. This can easily be achieved for p = 1. Optimal sizing is a parametrical design technique oriented on finding optimal values of a set of parameters describing structure dimensions. The typical examples of parameter optimization are searching for cross-sectional area of truss elements or optimizing stiffness distribution along beam or column axis (e.g. [1]).

3.2. CA formulation of topology optimization

The application of CA to structural optimization requires local formulation. The optimization problem for the whole structure is reformulated as the set of individual optimization problems set for each cell. The topology optimization can by formulated as minimization of strain energy of structure (set locally for each cell) under applied load. The local optimization problem for each cell can be formulated as follows:

minimize
$$U(\mathbf{d}) = \sum_{i=1}^{n} d_i^p \mathbf{u}_i^T \mathbf{k}_i \mathbf{u}_i \tag{2}$$

subject to
$$\begin{gathered} V(\mathbf{d}) = \kappa V_0 \\ d_{\min} \le d_i \le 1 \end{gathered} \tag{3}$$

where:

$\mathbf{u}_i$ – element displacement vector,

$\mathbf{k}_i$ – element stiffness matrix,

κ – prescribed volume fraction,

V_0 – design domain volume.

The lower bound imposed on d_i is necessary to avoid singularity while performing FEM analysis. That globally set constraint (i.e. total volume constraint) is checked after each iteration.

3.3. The novel local design update rule

This paper proposes the novel local design update rule for minimization of compliance design, which is modification and extension of a typical rule used for a fully stressed design approach, presented in Eq (4)

$$d_i^{(t+1)} = d_i^{(t)} \frac{\sigma_i^{(t)}}{\sigma_0} \tag{4}$$

where:

$\sigma_i^{(t)}$ – equivalent stress calculated for cell (i) at present iteration (t),

σ_0 – target stress.

The above approach has been expanded and adapted to minimization of strain energy of structure, what leads to the following formula:

$$d_i^{(t+1)} = d_i^{(t)} \left(1 + \alpha \left(\frac{\overline{U}_i^{(t)} - U_i^{(t)}}{U_i^{(t)}} + \beta \frac{U_i^{(t)} - \overline{U}^{(t)}}{U_i^{(t)}} \right) \right) \tag{5}$$

where:

$\overline{U}^{(t)}$ – an average strain energy density of the whole structure at present iteration t,

$U_i^{(t)}$ – strain energy density of cell (i) at present iteration (t),

$\overline{U}_i^{(t)}$ – strain energy density of N neighboring cells expressed by

$$\overline{U}_i = \frac{U_i + \sum_{k=1}^{N} U_k}{N+1} \tag{6}$$

α, β – constant values.

In Eq.(5) the first term following α represents information gathered within an individual cell neighborhood, whereas the second one carries global information influenced by all neighborhoods. This is similar to local and global formulation of the Particle Swarm

Optimization algorithm. By modifying values of parameters α and β it is possible to balance these two pieces of information and influence and improve performance of the design process. In particular case, for β = 1, the design variables d_i associated with central cells are updated according to:

$$d_i^{(t+1)} = d_i^{(t)}\left(1+\alpha\left(\frac{\bar{U}_i^{(t)}-\bar{U}^{(t)}}{U_i^{(t)}}\right)\right) \tag{7}$$

4. Numerical examples

4.1. Minimization of compliance with a total volume constraint– 2D structures

As the examples of application of the novel approach to topology optimization design of the Michell-type structure – 80×40 cells (Fig. 2), cantilevered structure – 40×20 cells (Fig. 3), and MBB beam – 150×25 cells (Fig. 4) has been performed. The Young modulus E_0 = 20 GPa, the Poisson ratio ν = 0.3, and volume fraction κ = 0.5 have been assumed. A vertical load P=100N has been applied to all of the above elements. The initial structures and the final results are shown below.

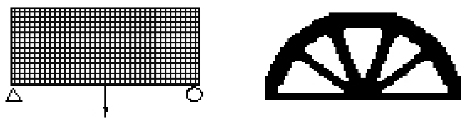

Fig. 2. The initial structure and the final topology of a Michell-type structure, α = 0.3 , β = 0, p = 1.5

Rys. 2. Wyjściowa struktura i otrzymana topologia dla struktury Michell'a, α = 0.3, β = 0, p = 1.5

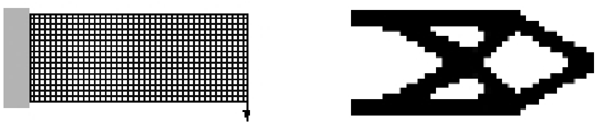

Fig. 3. The initial structure and the final topology for cantilevered structure, α = 0.3, β = 1, p = 3

Rys. 3. Wyjściowa struktura i otrzymana topologia dla wspornika, α = 0.3, β = 1, p = 3

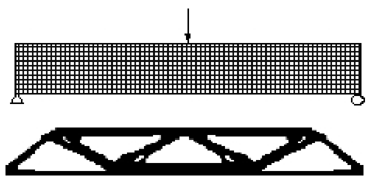

Fig. 4. The initial structure and the final topology for MBB beam, α = 0.3, β = 1.2, p = 3

Rys. 4. Wyjściowa struktura i otrzymana topologia dla elementu 'MBB beam', α = 0.3, β = 1.2, p = 3

4.2. Minimization of compliance with a total volume constraint – 3D structure

As the next example the 3D cantilevered structure has been chosen. The design domain has been divided into 20×20×6 cells. The Young modulus E_0 = 20 GPa, the Poisson ratio ν = 0.3, volume fraction κ = 0.5, α = 0.3, β = 0 and penalty parameter p = 1.5 have been selected. A vertical load P = 100 N has been applied. The initial structure and the final result are shown in the Fig. 5.

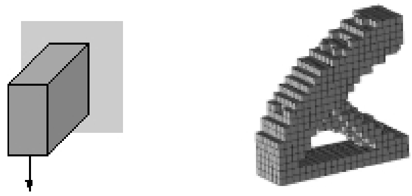

Fig. 5. The initial structure and the final topology for 3D cantilevered structure

Rys. 5. Wyjściowa struktura i otrzymana topologia dla trójwymiarowego wspornika

5. Conclusions

This paper briefly describes Cellular Automata concept and presents its application to topology optimization for 2D and 3D elastic structures. The novel local rule has been proposed and then applied to optimization of selected plane and three dimensional structures. The final profiles of the final structures are closely similar to the results presented in existing studies (e.g. [5],[11]). The described approach is still under development, it can be extended and the further improvement can be done. Nevertheless the results of numerical calculations confirm effectiveness of the proposed simple rule as the optimization tool for topology optimization problems. It can be therefore an alternative to the existing approaches.

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