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STABILIZATION OF POST-BUCKLING PATH BY AXIAL LOADING CONTROLLED BY DISPLECMENTS FOR CYLINDRICAL SHELLS UNDER TORSION

STABILIZACJA POKRYTYCZNEJ ŚCIEŻKI RÓWNOWAGI OBCIĄŻENIEM OSIOWYM STEROWANYM PRZEMIESZCZENIOWO DLA POWŁOK WALCOWYCH OBCIĄŻONYCH MOMENTEM SKRĘCAJĄCYM

A b s t r a c t

A post-buckling path for a cylindrical shell under torsional moment is unstable one. It means that the loss of stability of a shell can be associated with a snap-through, which can lead to very large displacements and, finally, to destruction of the structure.The effect of modification of the post-buckling behavior in most cases has been obtained by changing of geometry of a structure. In this paper an alternative concept is applied, namely stabilization of the post-buckling path is obtained by application of an additional loading acting to the structure without changing its geometry. This additional loading is applied to a structure by imposing a certain axial displacement to the ends of the shell. It causes an axial tensional force, which can stabilize the post-buckling path. The problem was formulated as a problem of optimization, namely the minimum value of the axial load - the initial pretension *u*, which leads, together with the passive force, to stabilization of the post-buckling path is looked for. Calculations were performed using ANSYS code.

Keywords: post-buckling path, cylindrical shell, torsion, optimization

S t r e s z c z e n i e

Powłoka walcowa obciążona momentem skręcającym posiada niestateczną ścieżkę równowagi. Oznacza to, że utrata stateczności powłoki następuje przez przeskok, co prowadzić może do bardzo dużych przemieszczeń i w konsekwencji do zniszczenia konstrukcji. W większości wypadków pokrytyczną ścieżkę równowagi stabilizuje się przez zmianę geometrii powłoki. W obecnej pracy zaproponowano alternatywną koncepcję, mianowicie stabilizację ścieżki pokrytycznej uzyskuje się przez zastosowanie dodatkowego obciążenia konstrukcji bez zmiany geometrii konstrukcji To dodatkowe obciążenie realizowane jest poprzez zadanie końcom konstrukcji pewnej wartości osiowego przemieszczenia. Zadane przemieszczenie wywołuje w powłoce rozciągającą siłę osiową stabilizując pokrytyczną ścieżkę równowagi. Zadanie zostało sformułowane jako problem optymalizacji, w którym poszukiwano minimalnej wartości zadanego osiowego przemieszczenia końców powłoki stabilizującego ścieżkę równowagi. Obliczenia przeprowadzono przy wykorzystaniu systemu ANSYS.

Słowa kluczowe: ścieżka pokrytyczna, powłoka walcowa, moment skręcający, optymalizacja

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1. Introduction

A post-buckling path for a cylindrical shell under twisting moment is unstable one. It means that loss of stability of a shell can be associated with a snap-through, [1, 2]. This type of buckling seems to be very dangerous because it can lead to very large displacements and, finally, to destruction of a structure.

A standard problem of structural optimization under stability constraints is usually formulated as maximization of the critical load for a prescribed volume of a design element. Very often a standard optimal structure has unstable post-buckling behaviour and it is very sensitive to imperfections. That is a weakness of the design and it indicates that the combination of geometrically nonlinear analysis with the design becomes necessary, especially from the practical point of view. Post-buckling constraints of a special form added to formulation of the optimization problem allow to modify the post-buckling path and the stable post-buckling path can be created, even in the case of unstable behaviour of a reference structure.

The effect of modification of the post-buckling behaviour in most cases has been obtained by changing sizing variables, which are usually dimensions of the design elements. These type of problems were considered, for example, by Bochenek [3, 4], Cardoso et al. [5], Jasion [6], Mróz and Piekarski [7], Pietrzak [8], Suasa et al [9]. In this paper an alternative concept is applied, namely a stabilization of the post-buckling path is obtained by application of additional loadings acting on a shell without change of a shape and sizes of an optimized structure. These loadings can be either active forces applied to a structure or passive ones (reactions of additional supports), or mixed variant consisting of both active and passive forces acting simultaneously. The latter possibility is applied here.

Such problems, for finite-degree-of-freedom of rod system that models the behaviour of a real shell structure under external pressure, were considered by Bochenek and Krużelecki [10]. On the other hand Krużelecki and Król [11] and Król, Krużelecki and Trybuła [12] examined the real cylindrical shells with different geometrical parameters. Some solutions were also presented by Bochenek and Krużelecki [13]. These papers showed that axial loadings can stabilize an initially unstable post-critical path. Mathon and Limam [14] experimentally examined influence of internal pressure on buckling and equilibrium path for cylindrical shell under bending. They showed that internal pressure and axial tension due to this pressure applied to a cylindrical shell can significantly increase the critical bending moment and even can cause stabilization of the post-critical path. Krużelecki and Trybuła [15] examined post-buckling behaviour of conical shells and showed that axial active force can stabilize the post-buckling path for conical shells under external pressure.

In the presented paper stabilization of a post-buckling path for a simply supported cylindrical shell under torsional moment is formulated as a certain modified non-standard problem of optimization. The most interesting type of stabilizing loading is investigated here, namely mixed variant of additional loading. It consists both an axial pretension by imposing an axial displacement U and a passive axial force which occurs due to blocked ends of a structure loaded by a twisting moment. Calculations are performed using ANSYS code for elastic shells of different length and thickness.

2. Formulation of the optimization problem

Let’s consider a simply supported at both ends cylindrical shell of the length L, radius R, constant thickness h, loaded by torsional moment M applied to both ends of the structure Such structure has an unstable post-buckling path. In this paper it is assumed that stabilization of an unstable post-buckling path can be obtained by application of axial additional loadings to a cylindrical shell without changing its geometry Stabilization of the post-buckling path can be achieved, at least, in three different ways, namely: by direct application of additional tensile loadings (active loadings), by imposing on the structure additional constraints connected with displacements (passive loadings) and by application of ‘mixed variant’ consisting of both types of loadings. These three variants of stabilization are discussed in details by Król, Krużelecki and Trybuła [12] for cylindrical shells under external pressure. In this paper the mixed variant of stabilization is considered. First, an axial pretension of a certain value by imposing axial displacement U is applied and next the axial movement of the both ends of a shell is blocked. It means that additional loading is controlled by the axial displacement. Then, the torsional moment is applied to a shell under the initial pretension. Under this basic loading the shell sustains additional passive force caused by deformations of the structure. During deformations of a shell under twisting moment the axial force N (reaction) occurs and its, value depends on deformations of the structure. For larger deformations, connected with larger value of the moment, the effect of global extension, referring to elongation of individual “fibres” caused by bending of the wall of a shell, predominates. It should be emphasized that passive force is not uniformly distributed along the circumference of a cross-section, namely there are regions of ‘less’ and ‘large’ axial tension. These regions are connected with number n of circumferential waves but the global force, caused by deformations, is a tensile one. Global and local effects of tension can increase the critical twisting moment and finally, result in possible stabilization of the post-buckling path.

The problem of optimization can be stated as follows. The minimum value of the axial load - the initial pretension u, which leads, together with the passive force, to stabilization of the post-buckling path is looked for minimize U subject to:

$$\frac{\partial M}{\partial f}(f^*,U) = \frac{\partial^2 M}{\partial f^2}(f^*,U) = 0 \tag{1}$$

where:

- f – characteristic displactment of a wall at a chosen point of a shell,
- M – torsional moment,
- U – imposed dispacement causing pretension.

The displacement f^*, in Eq.(1), refers to the horizontal inflexion point at the equilibrium path, fig. 1. The conditions (1) lead to elimination of the snap-through and finally, one obtains the stable post-buckling path even the original equilibrium path is unstable one. It is shown in fig. 1, where the thick line refers the stable equilibrium path under the minimum axial displacement U_{min}.

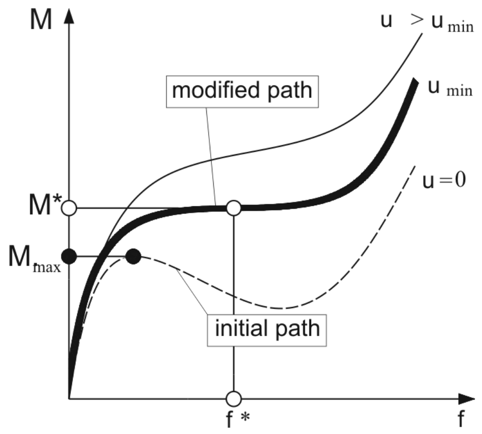

Fig. 1. Post-buckling paths for $U = 0$ and $U \neq 0$

Rys. 1. Ścieżki równowagi dla $U = 0$ i $U \neq 0$

This formulation of the optimization problem contains only one design variable u and two constraints, Eq.(1), which are imposed on the post-buckling state. They ensure the stable behavior of the cylindrical shell under torsional moment. A condition of a constant volume of the structure is automatically fulfilled because that formulation does not take into account modification of the shell geometry.

2. Results of calculations

Calculations were performed using ANSYS 11 code for elastic deformations of shells of different length and thickness. Considered shells with the length parameters: *L/R* = 1.0, 2.0 and the thickness parameter *h/R* = 0.005, taking that the radius is constant, *R* = 1 [m]. The material is defined by the following material constants: the Young modulus $E = 2 \cdot 10^5$ [MPa], the Poisson ratio ν=0.3. The reference stress $\sigma_0 = 225$ [MPa], which can be interpreted as the yield stress, was applied to define a dimensionless axial stress $s = \sigma_z / \sigma_0$ used as a measure of a stabilizing axial force, where σ_z is an axial stress due to an axial force.

To obtain the equilibrium path using ANSYS code it is necessary to introduce small initial geometrical imperfections into a perfect structure. The initial imperfections in the form of a buckling mode connected with geometry of the considered structure and with type of loadings were applied. It was found that possibility of stabilization of the post-buckling path does not depend on the magnitude Δ of small imperfections but they are necessary to perform calculations using the ANSYS code. Convergence of the applied ‘Arc-Length’ procedure, originally developed by Riks [16], strongly depends on the value of Δ. On the other hand values of stabilizing loads can depend on Δ. It was decided that in each considered case, the magnitude of the imperfection Δ was chosen in such way that a relative small decrease of the critical torsional moment, in comparison to the perfect structure, was in the same assumed small tolerance. It gives possibility to compare the results. It occurred that value of Δ, which causes decrease of critical twisting moment not larger than 5% – it means 5% tolerance – is a good compromise between conditions mentioned above.

The equilibrium path, which is obtained by numerical geometrically non-linear analysis, is given by discrete points. Therefore the constraints (1) were verified numerically (for any *u*) using the finite differences method. To obtain the horizontal inflexion point at the equilibrium path, which refers to stabilization of the equilibrium path, one finds $M_{i-1} = M_i = M_{i+1}$, where M_i denote values of torgue moment in three neighbouring points in distant Δf from each other. That equality of moments was satisfied numerically with very good accuracy under additional conditions: $M_{i-1} \le M_i \le M_{i+1}$ and $(M_{i+1} - M_{i-1}) / M_i \le \varepsilon$, where ε is an assumed small tolerance.

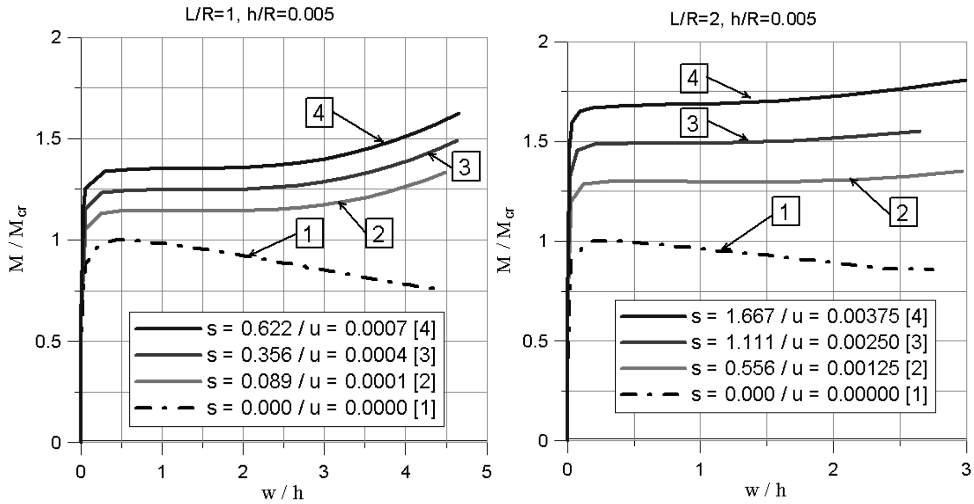

Fig. 2. Post-buckling paths for shells h/R = 0.005, L/R = 1 and 2

Rys. 2. Ścieżki pokrytyczne dla powłok o wymiarach h/R = 0.005, L/R =1 i 2

Fig. 2 presents the post-buckling paths for all considered shells. The post-buckling paths are shown in dimensionless coordinates: M/M_{cr}, w/h, where M_{cr} denotes the critical torsional moment (maximum moment at the equilibrium path for $U = 0$ ($N = 0$)) and *w* stands for the maximum radial displacement of a wall of a shell. Fig. 2, both contain four curves. The dashed lines (1) represent the equilibrium paths for shells under torsional moment only, it means the classical post-buckling paths for a simply supported elastic cylindrical shells of a constant thickness under torsion. The lines (2), which show still unstable equilibrium paths, were obtain for the axial displacement *u*, where $u = U/L$ (axial forces *N* occurs due to the axial displacement U) smaller than stabilizing ones. On the other hand, the lines (3) represent the stable post-buckling paths obtained for the minimal needed axial displacement u_{min}. The dimensionless axial forces *s*, referring to u_{min}, are shown in both figures. It means that these equilibrium paths meet the conditions (1) and the horizontal inflexion points occurred at these curves. The lines (4) were obtained for the axial displacement larger than stabilizing one. It should be stressed that axial tension obtained due to imposed the axial displacement improves resistance against buckling for shells under torsion and it can stabilize the post-buckling paths for such structures. The stabilization of the post-buckling paths by imposing of the axial displacement was obtained for all considered shells.

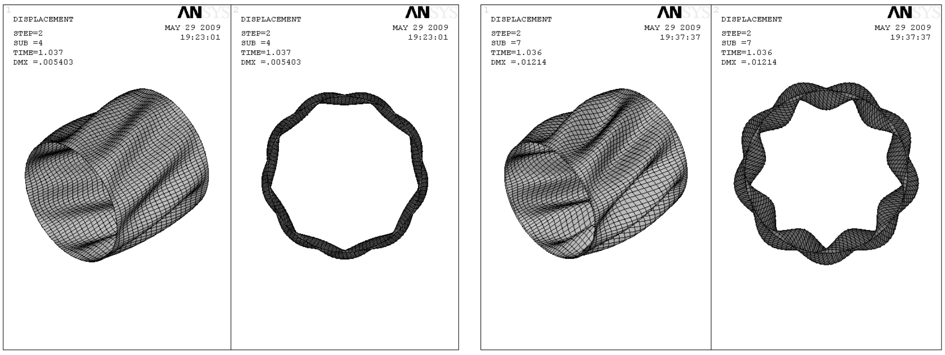

Fig. 3 Shape of the deformed shell for L/R = 0.5, h/R = 0.005; for the critical moment (on left) and under stabilizing umin (on right)

Rys. 3. Kształt zdeformowanej powłoki dla L/R = 0.5, *h/R* = 0.005; dla momentu krytycznego (z lewej) i po obciążeniu powłoki stabilizującym (z prawej)

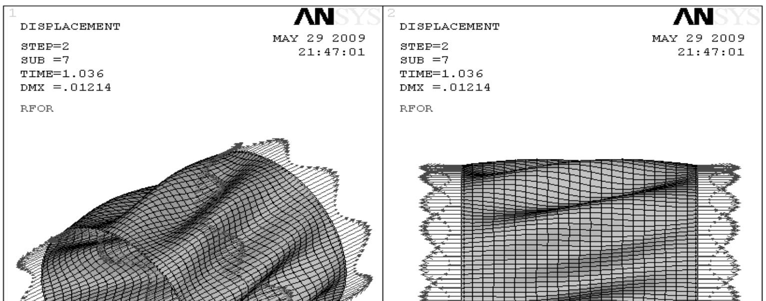

Fig. 4. Distribution of (axial reaction) for a shell with L/R = 0.5, h/R = 0.005

Rys. 4. Rozkład naprężenia σ_z (reakcji na podporach) dla powłoki o wymiarach L/R = 0.5, h/R = 0.005

In Fig. 3 the example shapes of the deformed structures are shown. The left hand side picture refers to the shell loaded by the critical torsional moment (u = 0) and the deformed shape is presented for the maximum moment at the equilibrium path. The right hand side figure refers to the shell loaded also by stabilizing $u_{\min}$ – the deformed shape is presented for displacement w referring to the horizontal inflexion point at the equilibrium path.

In this variant of stabilization the distribution of the axial stress σ_z along the circumference of the cross-section is not uniform one. It refers to a varying, along the circumference of a cross-section, the axial stiffness of the deformed structure due to bucking. Fig. 4 shows the axial stress σ_z (axial reaction) obtained as the result of the initial pretension and the passive force.

3. Conclusions

The numerical analysis showed that application of the axial tension displacement (the mixed variant of stabilization) to the ends of a cylindrical elastic shell under torsion can improve the resistance of the structure against buckling. It can also stabilize the post-buckling path without any modification of geometry of a structure. Because of rather high level of stresses obtained in elastic analysis possibility of stabilization of the post-buckling path for elastic-plastic deformations of cylindrical shells under torsion should be verified.

References

[1] Chryssanthopoulos M.K., Paggi C., Spagnoli A., *Buckling design of conical shells based on validated numerical models*, Thin-Walled Structures, vol. 31, 1998, 257–270.

[2] Volmir A.S., *Ustojchivost'deformiruemych system*, Nauka, Moskva 1967.

[3] Bochenek B., *Problematyka optymalnego kształtowania elementów konstrukcyjnych z uwagi na zachowanie pokrytyczne*, Zeszyty Naukowe Podstawowe Nauki Techniczne No. 22, Politechnika Krakowska, Kraków 2001.

[4] Bochenek B., *Optimization of geometrically nonlinear structures with respect to buckling and post-buckling constraints*, Engineering Optimization, vol. 29, 1997, 401–415.

[5] Cardoso J.B., Sausa L.G., Castro J.A. and Valido A.J., *Optimal design of nonlinear structures and mechanical systems*, Engineering Optimzation, vol. 29, 1997, 277–329.

[6] Jasion P., Stability analysis of shells of revolution under pressure conditions. Thin Walled Structures, 47, 2009, 311–317.

[7] Mróz Z., Piekarski J., *Sensitivity analysis and optimal design of non-linear structures*, Int. J. Numer. Engng., vol. 42, 1998, 1231–1262.

[8] Pietrzak J., *An alternative approach to optimization of structures prone to instability*, Structural Optimization, vol. 11, 1996, 88–94.

[9] Suasa L.G., Valido A.J., Cardoso J.B., *Optimal design of elastic-plastic structures with post-critical behaviour*, Structural Optimization, vol. 17, 1999, 147–154.

[10] Bochenek B., Krużelecki J., *A new concept of optimization for post-buckling behaviour*, Engineering Optimzation, vol. 33, 2001, 503–522.

[11] Król M., Krużelecki J., *Optimization of post-buckling path for cylindrical shells under external pressure*, Proc. III European Conference on Computational Mechanics; Solids, Structures and Coupled Problems in Engineering, (Eds. C.A. Mota Soares, J.A.C. Martin, H.C. Rodrigues, J.A.C. Ambrosio), Springer, (CD-ROM only), 2006.

[12] Król M., Krużelecki J., Trybuła D., *Optimal Stabilization of posbuckling path for cylindrical shells under external pressure*, Engineering Optimzation, vol. 41, 2009, 59–72.

[13] Bochenek B., Krużelecki J., *Optymalizacja stateczności konstrukcji. Współczesne problemy*, Wydawnictwo Politechniki Krakowskiej, Kraków 2007.

[14] Mathon C., Limam A., *Experimantal collapse of thin cylindrical shells submitted to internal pressure and pure bending*, Thin-Walled Structures, vol. 44, 2006, 39–50.

[15] Krużelecki J., Trybuła D. *Stabilizacja siłą osiową pokrytycznej ścieżki równowagi dla powłok stożkowych obciążonych ciśnieniem zewnętrznym*, III Konferencja Naukowo-Techniczna Doktorantów i Młodych Naukowców, Warszawa 2008, 92–97.

[16] Riks E., *An incremental approach to the solution of snapping and buckling problems*, International Journal of Solids and Structures, 15, 1979, 529–551.