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CONTOUR-NODE COORDINATES FOR SOLVING LINEAR ELECTRICAL CIRCUITS INCLUDING INDUCTIVE COUPLINGS

MIESZANA WĘZŁOWO-OBWODOWA METODA OBLICZANIA PRĄDÓW I NAPIĘĆ DLA MAGNETYCZNIE SPRZĘŻONYCH OBWODÓW LINIOWYCH

A b s t r a c t

In this paper, the contour-node coordinates were used for the analysis of the ramified, non connected electrical circuits with arbitrary situated in their structure mutual inductances between separate circuit branches. The concept of the application of the proposed coordinates is shown on the practical calculations applied to the chosen example of the circuit.

Keywords: *contour-node coordinates*, *mutual interaction of the circuits*

S t r e s z c z e n i e

Artykuł opisuje zastosowanie mieszanej węzłowo-obwodowej metody opisu obwodów elektrycznych do analizy obwodów nie połączonych galwanicznie, ale sprzężonych magnetycznie, przy czym sprzężenie może dotyczyć każdych dwóch gałęzi zarówno pomiędzy obwodami, jak i wewnątrz obwodów. Koncepcja zastosowania takiej mieszanej metody została pokazana na praktycznych obliczeniach dla podanego schematu obwodu.

Słowa kluczowe: *metoda węzłowo-obwodowa*, *oddziaływanie wzajemne obwodów elektrycznych*

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1. Introduction

During the development of new electrical devices, the questions considering the mutual influence between the systems and their electromagnetic compatibility become more and more important. In practice, the evaluation of the compatibility is usually done using mathematical modelling methods which are considered most appropriate for a specific task. The accuracy, convergence and calculating speed of the selected method depends on the formulation of the equation structures describing investigated system. Usually, for solving current flow in complex circuits with electromagnetic couplings, the mesh currents methods are used and are considered to be more reliable than nodes potential methods [1].

2. The purpose of work

The aim of this paper is to show the advantages of contour-nodal coordinate's methods for solving electrically not connected circuits coupled by mutual inductances located in arbitrary chosen branches of the circuits over the methods of contour currents and nodal voltages [2]. The main purpose of this new approach is to derive the general scheme, in which the circuit can be divided into separate subsystems connected only via magnetic ties and which can be built in the form of the classical mathematical model. The approach, which use developed type of scheme would significantly reduce the order of initial equation set in comparison to the method of contour currents. Moreover, the sets of equations obtained using this method will be in the form much easier to solve than the equations obtained for other used methods.

3. Statement of the problem and it's solving method

Dividing the total system to the subsystems through electrical connections and establishing a mathematical model of contour-nodal coordinates does not lead to much lower order of system equations in comparison with the contour currents method. In this case mutual magnetic relations usually remain in subsystems, which are described by the method of contour currents.

In practice, complex circuits (electrical systems) usually consist of several galvanic unrelated subsystems, connected only via mutual inductances ties. The idea is to describe some subsystems using the contour currents method and for the rest use the method of nodal voltages. The system of equations describing given circuits has to be supplemented by analysis of their structure for the optimal division to the subsystems that can give appropriate structure of matrices, vectors of parameters and coordinates of the scheme as well as gives the possibility of lowering the order of system equations.

Electrical circuit consisting of two complex electrical subsystems linked together via mutual inductances (Figure 1) was considered in further example. For the first subsystem, the subsystem equations were formed using nodal coordinates, and for the second one – the subsystem equations were created using the contour coordinates. All vector voltages and currents were divided into sub-vectors and impedance and admittance matrixes into blocks corresponding to the division into subsystems. In this case, a linear circuit is considered, what simplifies the solution of the problem, although this method is also suitable for analysis and nonlinear circuits.

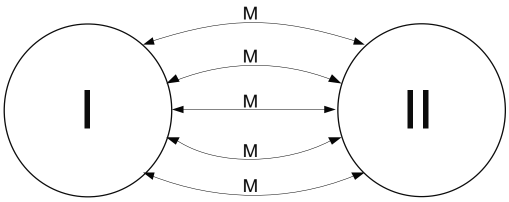

Fig. 1. Generalized scheme of two electrical circuits coupled only via mutual inductances
Rys. 1. Schemat ogólny dwóch obwodów połączonych tylko poprzez sprzężenia magnetyczne

Vector equation of voltage branches in matrix form for the first and second subsystem which include mutual inductive coupling can be stated as:

$$\underline{\dot{E}}_{\mathrm{I}} - \underline{Z}_{\mathrm{I-I}}\underline{\dot{I}}_{\mathrm{I}} - \underline{Z}_{\mathrm{I-II}}\underline{\dot{I}}_{\mathrm{II}} = \underline{\dot{U}}_{\mathrm{I}} \tag{1}$$

$$\underline{\dot{E}}_{\mathrm{II}} - \underline{Z}_{\mathrm{II-II}}\underline{\dot{I}}_{\mathrm{II}} - \underline{Z}_{\mathrm{II-I}}\underline{\dot{I}}_{\mathrm{I}} = \underline{\dot{U}}_{\mathrm{II}}, \tag{2}$$

where:

$\underline{Z}_{\mathrm{I-I}}$ $\underline{Z}_{\mathrm{II-II}}$ – matrixes of complex impedances, describing individual branches and mutual inductances between them within first and second subsystem respectively;

$\underline{Z}_{\mathrm{I-II}}$, $\underline{Z}_{\mathrm{II-I}}$ – matrixes of complex impedances, describing the inductive ties between subsystems.

In expanded form the general matrix of circuit's parameters achieve following form:

$$\underline{\underline{Z}}_{\mathrm{I-I}} = \begin{Vmatrix} \underline{Z}_{1,1} & \underline{Z}_{1,2} & \cdot & \underline{Z}_{1,n} \\ \underline{Z}_{2,1} & \underline{Z}_{2,2} & \cdot & \underline{Z}_{2,n} \\ \cdot & \cdot & \cdot & \cdot \\ \underline{Z}_{n,1} & \underline{Z}_{n,2} & \cdot & \underline{Z}_{n,n} \end{Vmatrix}; \qquad \underline{\underline{Z}}_{\mathrm{I-II}} = \begin{Vmatrix} \underline{Z}_{1,n+1} & \underline{Z}_{1,n+2} & \cdot & \underline{Z}_{1,n+m} \\ \underline{Z}_{2,n+1} & \underline{Z}_{2,n+2} & \cdot & \underline{Z}_{2,n+m} \\ \cdot & \cdot & \cdot & \cdot \\ \underline{Z}_{n,n+1} & \underline{Z}_{n,n+2} & \cdot & \underline{Z}_{n,n+m} \end{Vmatrix};$$

$$\underline{\underline{Z}}_{\mathrm{II-I}} = \begin{Vmatrix} \underline{Z}_{n+1,1} & \underline{Z}_{n+1,2} & \cdot & \underline{Z}_{n+1,n} \\ \underline{Z}_{n+2,1} & \underline{Z}_{n+2,2} & \cdot & \underline{Z}_{n+2,n} \\ \cdot & \cdot & \cdot & \cdot \\ \underline{Z}_{n+m,1} & \underline{Z}_{n+m,2} & \cdot & \underline{Z}_{n+m,n} \end{Vmatrix}; \qquad \underline{\underline{Z}}_{\mathrm{II-II}} = \begin{Vmatrix} \underline{Z}_{n+1,n+1} & \underline{Z}_{n+1,n+2} & \cdot & \underline{Z}_{n+1,n+m} \\ \underline{Z}_{n+2,n+1} & \underline{Z}_{n+2,n+2} & \cdot & \underline{Z}_{n+2,n+m} \\ \cdot & \cdot & \cdot & \cdot \\ \underline{Z}_{n+m,n+1} & \underline{Z}_{n+m,n+2} & \cdot & \underline{Z}_{n+m,n+m} \end{Vmatrix},$$

where:

n and m – number of branches in first and second subsystem espectively.

Non-diagonal elements of the matrixes $\underline{Z}_{\mathrm{I-I}}$, $\underline{Z}_{\mathrm{II-II}}$ represent the mutual relations between the branches inside subsystems respectively, the elements of $\underline{Z}_{\mathrm{I-II}}$, $\underline{Z}_{\mathrm{II-I}}$ matrixes represent the mutual relations between the branches of subsystems.

These mutual relationships were determined through mutual coupling coefficient between ith branch of the first subsystem and jth branch of the second subsystem defined as:

$$k_{i,j} = \frac{M_{i,j}}{\sqrt{L_i \cdot L_j}}$$

Let’s express the voltages across the branches of the first subsystem using nodal voltages – $\underline{U}_{\mathrm{I}} = \Pi_{\mathrm{I}t}\,\underline{U}_{N\mathrm{I}}$, where $\Pi_{\mathrm{I}t}$ matrix of 0,1, or –1 created to express voltages across branches by nodal voltages , and substitute it to (1) and similarly branch currents of the second subsystem can be expressed as functions of contour currents $\underline{I}_{\mathrm{II}} = \Gamma_{\mathrm{II}t}\,\underline{I}_{C\mathrm{II}}$ and substituted to equations (1) and (2). It can be also nodded that, from the Kirchhoff’s law, for the second subsystem $\Gamma_{\mathrm{II}t}\,\underline{U}_{\mathrm{II}} = 0$. After the substitution the equations (1) and (2) accomplish following form:

$$\underline{E}_{\mathrm{I}} - \underline{Z}_{\mathrm{I-I}}\,\underline{I}_{\mathrm{I}} - \underline{Z}_{\mathrm{I-II}}\,\Gamma_{\mathrm{II}t}\,\underline{I}_{C\mathrm{II}} = \Pi_{\mathrm{I}t}\,\underline{U}_{N\mathrm{I}}; \tag{3}$$

$$\Gamma_{\mathrm{II}}\,\underline{E}_{\mathrm{II}} - \Gamma_{\mathrm{II}}\,\underline{Z}_{\mathrm{II-II}}\,\Gamma_{\mathrm{II}t}\,\underline{I}_{C\mathrm{II}} - \Gamma_{\mathrm{II}}\,\underline{Z}_{\mathrm{II-I}}\,\underline{I}_{\mathrm{I}} = 0. \tag{4}$$

Final form of a mathematical model of contour-nodal coordinates is obtained from equations (3) and (4) by solving equation (3) with respect to the vector of branch currents of the first subsystem $\underline{I}_{\mathrm{I}}$ and substitute the result into equation (4) and taking into account that, $\Pi_{\mathrm{I}}\,\underline{I}_{\mathrm{I}} = 0$:

$$\begin{Vmatrix} -\Pi_{\mathrm{I}}\,\underline{Z}_{\mathrm{I-I}}^{-1}\,\Pi_{\mathrm{I}t} & -\Pi_{\mathrm{I}}\,\underline{Z}_{\mathrm{I-I}}^{-1}\,\underline{Z}_{\mathrm{I-II}}\,\Gamma_{\mathrm{II}t} \\ -\Gamma_{\mathrm{II}}\,\underline{Z}_{\mathrm{II-I}}\,\underline{Z}_{\mathrm{I-I}}^{-1}\,\Pi_{\mathrm{I}t} & \Gamma_{\mathrm{II}}\,\underline{Z}_{\mathrm{II-II}}\,\Gamma_{\mathrm{II}t} - \Gamma_{\mathrm{II}}\,\underline{Z}_{\mathrm{II-I}}\,\underline{Z}_{\mathrm{I-I}}^{-1}\,\underline{Z}_{\mathrm{I-II}}\,\Gamma_{\mathrm{II}t} \end{Vmatrix} \times \begin{Vmatrix} \underline{U}_{N\mathrm{I}} \\ \underline{I}_{C\mathrm{II}} \end{Vmatrix} = \\ = \begin{Vmatrix} -\Pi_{\mathrm{I}}\,\underline{Z}_{\mathrm{I-I}}^{-1} & 0 \\ -\Gamma_{\mathrm{II}}\,\underline{Z}_{\mathrm{II-I}}\,\underline{Z}_{\mathrm{I-I}}^{-1} & \Gamma_{\mathrm{II}} \end{Vmatrix} \times \begin{Vmatrix} \underline{E}_{\mathrm{I}} \\ \underline{E}_{\mathrm{II}} \end{Vmatrix} \tag{5}$$

From the obtained system of equations (5) nodal voltages of the first subsystem and contour currents of the second subsystem can be calculated. These quantities can be later used to obtain branch currents and voltages across branches.

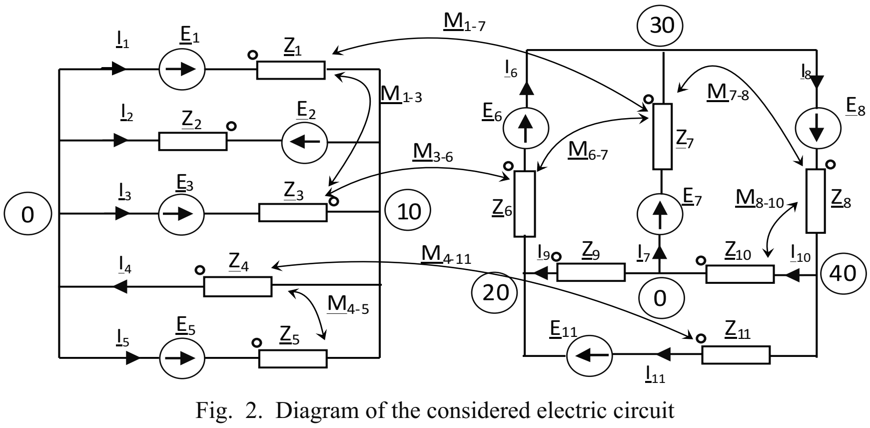

Fig. 2. Diagram of the considered electric circuit
Rys. 2. Schemat rozpatrywanego obwodu elektrycznego

To confirm the correctness and adequacy of received mathematical model, an example calculation for the complex electric circuits, which consists of two galvanic isolated subsystems tied only via mutual inductances (Fig. 2) was provided in this work.

This scheme (Fig. 2) can be initially described in contour-nodal coordinates and nodal voltages by four equations and using the method of contour currents by seven equations. In general, the number of equations for the three considered methods compare as follows:

$$n_{CC} \geq n_{NVCC} \geq n_{NV}.$$

where:

n_{CC} – number of equations for contour currents methods,
n_{NVCC} – number of equations for contour-nodal coordinates methods,
n_{NV} – number of equations for nodal voltages based methods.

Of course final description of the circuit will consist of the same number of the independent equations, but in real life it is often difficult to perform simple reduction of the number of equations.

The diagram from (Fig. 2) is a complex diagram consisting of close loops with the elements which are numbered in random order (the direction from left to right was chosen), and as a first element can be selected any element from any subsystem and any node can be chosen as reference one.

The goal is to form the impedance matrixes for a given equation scheme from given values of circuit parameters taking under consideration voltage polarity of the mutual inductance of each two coupled components (on the scheme polarity of coupling is shown as "**o**").When the coupling coefficient was chosen arbitrary – $k = 0.8$, and for given parameters set, the proposed solution scheme returned, for each subsystem, the vectors and matrices of parameters shown in Figure 3.

The matrices and vectors shown below were calculated for the diagram from Fig. 2. and were substituted to the equation (5), which was then solved using MathCAD-14 software. The results of these calculations are shown in Table 1.

Table 1

Calculated values of nodal voltages and contour currents for a given solution scheme and given circuit diagram

$\underline{U}_{10\_0}$, V	$\underline{I}_{C5}$, A	$\underline{I}_{C6}$, A	$\underline{I}_{C7}$, A
137,2 + j29,465	1,889 – j5,503	16,078 – j0,306	8,98 – j4,191

The contour currents from table 1 ($\underline{I}_{C5}$, $\underline{I}_{C6}$, $\underline{I}_{C7}$) were chosen as the currents of three contours from left diagram in figure 2.

Using obtaining nodal values of voltage and contour currents, the values of voltages across branches and branch currents were calculated and listed in Table 2

In order to assess the adequacy of the results, the currents and voltages of the circuit from the diagram shown in Fig. 2. were also calculated using the method of contour currents and utilizing the same set of the parameters. Calculation results are presented in Table 3.

$$\underline{\dot{E}}_{\mathrm{II}}=\begin{pmatrix}180\\260\\150\\0\\0\\130\end{pmatrix}\qquad \underline{\dot{E}}_{\mathrm{I}}=\begin{pmatrix}170\\85\\250\\0\\190\end{pmatrix}\qquad \underline{Z}_{\mathrm{II-I}}=\begin{pmatrix}0&0&\mathrm{j}3{,}919&0&0\\-\mathrm{j}4{,}8&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&\mathrm{j}2{,}53&0\end{pmatrix}$$

$$\underline{Z}_{\mathrm{I-II}}=\begin{pmatrix}0&-\mathrm{j}4{,}8&0&0&0&0\\0&0&0&0&0&0\\\mathrm{j}3{,}919&0&0&0&0&0\\0&0&0&0&0&\mathrm{j}2{,}53\\0&0&0&0&0&0\end{pmatrix}$$

$$\underline{Z}_{\mathrm{I-I}}=\begin{pmatrix}5+\mathrm{j}4&0&-\mathrm{j}4{,}525&0&0\\0&11+\mathrm{j}7&0&0&0\\-\mathrm{j}4{,}525&0&7+\mathrm{j}8&0&0\\0&0&0&12+\mathrm{j}5&-\mathrm{j}2{,}53\\0&0&0&-\mathrm{j}2{,}53&3+\mathrm{j}2\end{pmatrix}$$

$$\underline{Z}_{\mathrm{II-II}}=\begin{pmatrix}8+\mathrm{j}3&\mathrm{j}4{,}157&0&0&0&0\\\mathrm{j}4{,}157&6+\mathrm{j}9&-\mathrm{j}5{,}879&0&0&0\\0&-\mathrm{j}5{,}879&15+\mathrm{j}6&0&-\mathrm{j}5{,}185&0\\0&0&0&4+\mathrm{j}&0&0\\0&0&-\mathrm{j}5{,}185&0&9+\mathrm{j}7&0\\0&0&0&0&0&14+\mathrm{j}2\end{pmatrix}$$

Fig. 3. Matrixes formed for subsystems from given diagram
Rys. 3. Macierze otrzymane dla sub-systemów z rozpatrywanego obwodu

The comparison of the results obtained by the method of contour-nodal coordinates and the method of contour currents for the second subsystem ($\underline{I}_{C5}$, $\underline{I}_{C6}$, $\underline{I}_{C7}$) – Table 1 and Table 3 shows complete match, what indicates the adequacy and appropriate structure of developed mathematical model. The contour-nodal coordinates method was also used for the analysis of electrical circuits, which consist not only of the two, but three, four and more isolated subsystems. Results of the calculations confirmed correctness of the proposed approach.

Table 2

The calculated values of branch currents and voltages across

Number of the branch p / currents and branch voltages	1	2	3	4	5
$\underline{I}_p$, A	9,40 +j4,53	15,59 –j7,24	4,79 –j4,65	11,73 –j1,56	13,13 –j8,68
$\underline{U}_p$, V	137,2 +j29,46	–137,2 –j29,465	137,2 +j29,46	–137,2 –j29,465	137,2 +j29,47

Number of the branch p / currents and branch voltages	6	7	8	9	10	11
$\underline{I}_p$, A	1,89 –j5,50	14,19 +j5,19	16,08 –j0,31	–7,09 –j1,31	7,09 +j3,88	8,98 –j4,19
$\underline{U}_p$, V	151,75 –j39,39	178,8 –j27,05	–143,70 +j28,34	27,05 +j12,33	–35,1 –j1,29	–8,05 +j11,0

Table 3

The calculated contour current values of contour current method

$\underline{I}_{C1}$, A	$\underline{I}_{C2}$, A	$\underline{I}_{C3}$, A	$\underline{I}_{C4}$, A	$\underline{I}_{C5}$, A	$\underline{I}_{C6}$, A	$\underline{I}_{C7}$, A
9,409 +j4,533	6,182 –j11,776	–1,396 +j7,123	13,127 –j8,681	1,889 –j5,503	16,078 –j0,306	8,98 –j4,191

4. Conclusions

1. Order of the equation in the method of contour-nodal coordinates is always less or equal then order of the equation system obtained using the method of contour currents and usually equal to the order of the equations obtained using the method of nodal voltages.
2. Adequacy of the derived mathematical model was confirmed by comparison of the results obtained by the proposed method with the results obtained using contour currents method. A comparison showed that, for the same considered circuit structure and for the same parameter sets, the results are the same.
3. The derived mathematical model can be used for the analysis of circuits in power supply system that contains transformers and AC machines thus containing inductively coupled

elements and performing dynamic processes. The method becomes especially effective for systems with a significant difference between the number of independent circuits and number of the system components.

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