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SOME EXAMPLES ON THE SIGNIFICANCE OF THE NOTIONS COMBINED LOADINGS AND NON-HOMOGENEITY IN THE MECHANICS OF SOLIDS AND STRUCTURES

WYBRANE PRZYKŁADY ZNACZENIA POJĘĆ OBCIĄŻENIA ZŁOŻONE I NIEJEDNORODNOŚĆ W MECHANICE CIAŁ ODKSZTAŁCALNYCH I MECHANICE KONSTRUKCJI

A b s t r a c t

In the following presentation, at first we shortly report about the visits and the fruitful scientific advice, which the late Professor Michał Życzkowski was so kind to spend to the Mechatronics group at the Johannes Kepler University of Linz (JKU Linz) in Austria. Afterwards, we review some own contributions at the Institute of Technical Mechanics of the Mechatronics group in Linz, which were motivated by the research thoughts of Professor Życzkowski. Particularly, we emphasize the usefulness of the notion of Combined Loadings, which we have learned from Professor Życzkowski. Professor Życzkowski introduced this extremely fruitful classification scheme in his famous book on *Combined Loadings in the Theory of Plasticity* [1], where he distinguished between an analysis at the point level, at the surface level and at the level of the body as a whole. Consequences that have followed from this classification for the scientific strategy of our group concerning elasto-plastic vibrations and vibrations of smart (piezoelectric) structures are shortly sketched. In the framework of a pull-out stability problem, nn interesting aspect noted in the celebrated article on the *Stability of Bars and Bar Structures* [17] by Professor Życzkowski is also addressed. As a whole, we demonstrate how Professor Zyczkowski's classification scheme serves for us as a highly powerful guide for choosing, explaining and justifying new directions of research in the Mechanics of solids and structures.

Keywords: Combined Loadings, elasto-plastic vibrations, piezoelectric Structures, stability of bars

S t r e s z c z e n i e

Na początku przedstawionego artykułu zamieszczono krótką informację na temat wizyt oraz istotnych sugestii naukowych, jakie Profesor Michał Życzkowski był uprzejmy przedstawić w trakcie swoich pobytów w Zespole Mechatroniki na Uniwersytecie Johannesa Keplera w Linz (JKU Linz) w Austrii. Następnie przedstawiono przegląd kilku własnych prac wykonanych w Instytucie Mechaniki Technicznej Zespołu Mechatroniki w Linz, które były inspirowane naukowymi ideami Profesora Życzkowskiego. Zwrócono szczególną uwagę na użyteczność pojęcia obciążenia złożone, które poznaliśmy dzięki Profesorowi Życzkowskiemu. Profesor Życzkowski, w swojej bardzo znanej monografii *Obciążenia złożone w teorii plastyczności* [1], wprowadził wyjątkowo wygodny rodzaj klasyfikacji obciążeń, dokonując bardzo użytecznego podziału analiz na poziomie punktu, przekroju i całego ciała odkształcalnego. Skutki, jakie przedstawiony podział wywarł na późniejszą strategię naukową Zespołu Mechatroniki w analizach zagadnień drgań w zakresie sprężysto-plastycznym oraz drgań struktur inteligentnych (piezoelektrycznych), zostały pokrótce przedstawione w artykule. Wiele istotnych aspektów leżących u podstaw problemu stateczności pręta rozciąganego zawarto w artykule Profesora Życzkowskiego *Stateczność prętów i układów prętowych* [17]. W niniejszym artykule, patrząc całościowo, przedstawiono jak zaproponowana przez Profesora Życzkowskiego klasyfikacja stała się dla naszej grupy niezwykle inspirującą w wytyczeniu, wyjaśnieniu i uzasadnieniu nowych kierunków badań w obszarze mechaniki ciał odkształcalnych i mechaniki konstrukcji.

Słowa kluczowe: obciążenia złożone, drgania sprężysto-plastyczne, struktury piezoelektryczne, stateczność prętów

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Dedicated to the memory of Professor Michał Życzkowski, presented at the Commemoration of His Oeuvre on the occasion of 80th anniversary of His birthday on April 12, 2010, at the Cracow University of Technology, Cracow, Poland.

1. Introduction

The late Professor Michał Życzkowski, Professor at the Cracow University of Technology, Full Member of the Polish Academy of Sciences, Corresponding Foreign Member of the Austrian Academy of Sciences, was an internationally leading researcher in various fields of Theoretical and Applied Mechanics, he was a dedicated and highly successful academic teacher, and, among his many international contacts, he served as a friendly, patient and careful advisor and supporter of Mechanics in Austria.

Professor Michał Życzkowski was so kind as to visit the Mechatronics group of the Johannes Kepler University (JKU) of Linz twice. This group was established in 1990, in order to follow the request of the Austrian industry for the worldwide first Master Curriculum in the novel field of Mechatronics, intending to produce a synergetic connection of Mechanics, Electrical Engineering and Computer Sciences. Due to financial limitations, but also in order to follow a philosophy of bringing together academia and industry literally, by sitting door to door, the Mechatronics Institutes were established in an old building in the Linz industrial area, hired from the largest steel factory in Austria, Voest-Alpine. This building was located about 11 kilometers apart from the Campus of the University of Linz, where the class-room lectures were to be held, but in an immediate neighborhood of the large steel producing devices of Voest-Alpine. Professor Michał Życzkowski did not hesitate a moment to undertake the adventure of accepting our invitation to travel from Cracow to this Institute building in Linz already in the year 1994, where he gave a lecture entitled *Optimal Plastic Shape Design via Boundary Perturbation Method*. The Mechatronics group in Linz at this time was in a stage of development, the graduation of the first Mechatronics students having been planned for the next years. Professor Michał Życzkowski was so kind as to come back to us five years later, in the year 1999, where he lectured about *Strain Singularities in Perfect Plasticity and the Concept of Decohesive Carrying Capacity*. That Professor Życzkowski, as a leading international expert, was willing to visit the young Mechatronics group in Linz and to present outstanding lectures, was not only a big honor and pleasure for us, it was an important sign for academia and industry in Austria, the value of which cannat be overestimated. Most important for the present author and his group, Professor Michał Życzkowski was so kind as to discuss in detail our work on Mechanics in the light of his unique scholarship, and to give us various highly valuable scientific advice, not only during his stays in Linz, but also during two visits of the present author in Cracow. He also was very helpful in discussions concerning questions of Mechatronics as a whole.

We are very sad that Professor Michał Życzkowski, due to a decease, against which he heroically did fight, was not able to come to Linz again. In 1999, he had mentioned that a 5 years period of visits would be reasonable, but this unfortunately could not be realized. The Mechatronics group in Linz nowadays consists of 11 Institutes, it is located in a somewhat futuristic new building directly at the University Campus of JKU, and it runs one of Austria's largest peer-reviewed research centers, the Austrian Center of Competence in Mechatronics

(ACCM), the pre-competitive research of which is funded by leading industries, as well as by the Austrian government. We are sure that Professor Michał Życzkowski would have been interested in these developments, and that he would have offered his highly valuable advice and support again. With Professor Michał Życzkowski, the present author and the Johannes Kepler University of Linz as a whole have lost a true friend of eminent scientific and personal importance.

In the following, we intend to review some own works at the Institute of Technical Mechanics of the Mechatronics Group of JKU Linz in the light of some thoughts by Professor Michał Życzkowski. Particularly, we wish to emphasize the significance of two ideas, which we have learned from him, namely of the notions of Combined Loadings and of Non-Homogeneity, and to shortly report about consequences that have followed from these considerations in the last years.

2. Some own studies motivated by the notions

Combined Loadings and *Non-Homogeneity*

In his famous book on *Combined Loadings in the Theory of Plasticity* [1], Professor Michał Życzkowski introduced an extremely fruitful classification scheme of modeling in the Mechanics of solids and structures, where he distinguished between an analysis at the point level, at the surface level and at the level of the body as a whole. He pointed out that, at any of these levels of analysis, the deformation process can be understood as being controlled by independent functions, which he called exertion factors. Knowledge of the latter factors as a function of time determines the deformation and loading processes under consideration. At the point level, stress components often are considered as exertion factors, while at the surface level one deals with stress resultants. Kinematical quantities may be used at the three levels, additionally or alternatively, and eigenstrains and their integrals may come into play, such as temperature loadings. We mention that in Mechatronics one often has to deal with analogous electrical entities, such as the electric field in a piezoelastic member and its cross-sectional integrals. Exertion factors at the level of a body as a whole are usually identified as independently acting external forces, sources of heat, sources of the electric field, etc., but, as Professor Życzkowski pointed out clearly, more complex exertion factors may occur also at this level, to which we will come back at the end of this presentation. Professor Życzkowski in [1] talked about a *Combined Loading*, if several exertion factors exist at any of the three levels, and he added the number of exertion factors to the respective levels for the sake of indication. As another point very important for classification, he mentioned the Homogeneity or *Non-Homogeneity* of the distribution of the exertion factors at the three levels, and their transitions between the different levels. Due to space limitations, we cannot go deeper into this scheme, which among other aspects also contains the notions of active and passive processes, or of interaction surfaces. On the basis of his ingenious scheme of classification, Professor Życzkowski in his book [1] gave a comprehensive and complete presentation of the stage of research on the theory of plasticity at the time, which in the list of references includes more than 3000 entries, containing also various important papers by himself and his co-workers. He remained a highly active researcher all his life. For an obituary on Professor Życzkowski and his work, see Mang [2].

The book [1] and the other important works of Professor Życzkowski and his group have influenced many researchers worldwide, among them the author of the present paper. As a contribution to the Commemoration of His Oeuvre, three examples for explaining the development of the work of the present author and his co-authors in the light of Professor Życzkowski's scheme of classification will be shortly reviewed in the following.

The first example deals with the occurrence of plastic strains in structures. The present author is interested in problems of structural elasto-plasticity under reversed loading, one reason for this being the extremely non-homogeneous stress-distribution, which may result from processes with loading, unloading, reversed loading, re-loading, etc. Such situations with various active and passive parts of the process particularly do occur when bending vibrations of structures are considered, in which the cyclic inertia effects lead to plastic parts of strains and stresses. Already after a few cycles of vibration, stresses and plastic parts of strain are distributed across and along the structure in an extremely non-homogeneous manner. Advanced techniques in such a case are needed in order to perform a sufficiently accurate transition from the point to the surface level, due to the non-homogeneity, while more classical structural dynamics formulations, such as Ritz-approximations, can be applied to compute the response at the body level with a good computational efficiency. For a review on the earlier work of the present author and his teacher Professor Franz Ziegler on such dynamic processes in structural plasticity, the reader is referred to [3], where also thermal loading and visco-plasticity were addressed. Here, in the light of the above classification scheme by Professor Życzkowski, we particularly mention the paper [4] on biaxial dynamic bending of elasto-plastic beams. In [4], a Bernoulli-Euler formulation was used, i.e. the modeling in the Życzkowski scheme can be classified as: Point 1 (axial stress), Surface 2 (biaxial bending moments), Body 2 (biaxial external loading). Afterwards, an extended formulation taking into account the influence of normal forces was performed by Brunner and Irschik [5] in the framework of a second-order theory, i.e. the number of exertion factors at the surface level was increased by one (normal force), and the non-homogeneity was increased by studying multi-layered beams. While the formulations in Refs. [3–5] dealt with vibrations about equilibrium positions, the present author and his co-workers more recently have studied elasto-plastic vibrations of structural elements caused by superimposed rigid-body motions. First, the studies again were undertaken at the levels Point 1 (axial stress), Surface 1 (bending moment), Body 2 (rigid body rotation, weight), see Refs. [6] and [7]. Afterwards, the number of exertion factors at the surface level was increased by one, by taking into account a non-homogeneous distribution of normal forces due to the influence of axial inertia stiffening, [8]. These considerations were extended by Johannes Gerstmayr [9] to whole elasto-plastic multi-body systems, for which the number of exertion factors at the body level are considerably increased, due to the rigid-body degrees-of-freedom of the various single members of the system, see also [10].

As a second example of the influence of the strategic influence of the Życzkowski scheme on the development of our work, we mention the studies of our group on piezoelastic solids and structures. A review on the various corresponding analytical, numerical and experimental studies has been recently presented in [11]. Here, we only mention the following aspects: First, such mechatronical structures are of course non-homogeneous due to the attached piezoelectric actuators and sensors, which are integrated within the structure. Moreover, the transition from the electric exertion factors at the body level (applied voltage or charge) to those at the surface and point level (electric field or electric displacement) has to be performed by considering coupling effects between mechanical and electric fields, see e.g. Krommer and Irschik [12]. Often, electric exertion factors at the body level are not given in advance, but they are to be determined in an inverse procedure, e.g. in order to enforce a force-loaded structure to maintain its undistorted shape, or to follow some prescribed field of trajectories, see [13–14] and [15]. As an exemplary study of interest in the light of Professor Życzkowski's scheme, we also mention Ref. [16] on "plane" piezoelectric bending of beams with a rectangular cross-section, the extension of which in the out-of-plane direction is larger than in the transverse direction. It is then necessary to go over from beam to plate theory, i.e. to increase the numbers of exertion factors at the point and surface level correspondingly, in order to account for the effect of bending of the cross-section about the longitudinal axis.

We finally would like to present an example, which will cast some light on Życzkowski's above cited remark, namely that more complex exertion factors can occur at the body level, see [1]. This example also refers to the following remark by Professor Życzkowski, which he made in his comprehensive and celebrated article on the *Stability of Bars and Bar Structures* [17]: "The influence of the behavior of loading on the loss of stability is very significant in the case of bars in tension. With the Eulerian behavior of loading (materially fixed point of application, direction fixed in space), the bar cannot lose stability at all. With a more general behavior of loading, loss of stability is possible (e.g. loading of a vessel by a liquid)." Motivated by problems of multi-body systems, we recently have studied the Reissner theory of large displacements and finite deformations of bars [18]. This theory is interesting also insofar, as it originally did not refer to the point level, but was formulated by Reissner at the surface and body level alone, i.e. one deals with a level Point 0 problem. This brings into play the problem of not being able to use constitutive relations at the point level; rather, experimentally obtained constitutive relations at the surface level, between stress-resultants and generalized strains, must be used instead, and this must be done for a geometrically non-linear problem. We first have tried to overcome this drawback in the framework of a geometrically non-linear Bernoulli-Euler type kinematical assumption, i.e. by bringing the theory to the levels Point 1 (axial stress), Surface 2 (normal force and bending moment) and Body (plane force excitation), see [19]; an extension to increase the number of exertion factors by one at both, the point level (shear stress) and the surface level (shear force) is in publication. Constitutive relations at the point level, such as the St.Venant-Kirchhoff or the Simo – Hughes hyperelastic laws can be consistently used, and the door seems now to be open to inelastic problems. For a first dynamic extension, see Humer and Irschik [20]. During these studies, the following stability problem for bars under tension came to our mind: Assume a horizontal bar, which is clamped and axially fixed at one end, and is also clamped at the other end, but can be pulled out freely at this end from a straight, force- and friction-free reservoir. The bar is under the action of its own weight in transverse direction. We seek for an equilibrium position in the framework of the above extended Reissner-type geometrically non-linear theory. This theory is essentially a Lagrange description, which refers to a straight reference configuration, and not an Eulerian one. The bar is pulled out from the reservoir, and is stretched and bent due to its weight, leading to a non-homogeneous distribution of tensile normal forces, which have to vanish at the ends and at the mid-span of the bar. Due to the pull-out process, the length of the reference configuration does not coincide with the span between the clamped ends, but is unknown and must be determined. The resultant loading is the weight per unit length times this unknown length and thus is also not known in advance, and thus is of a rather general type. In the framework of the above Bernoulli-Euler assumption, Professor Życzkowski's scheme here reads: Point 1 (axial stress), Surface 2 (normal force, bending moment), Body 2 (weight, unknown length of reference configuration). This increase of the exertion factor by one at the body level leads to a stability problem of the bar under tension: There is a critical weight of the beam per unit length, above which equilibrium is not possible, see Humer and Irschik [21] for details. This fact indeed is felt to demonstrate Professor Życzkowski's above cited remarks.

3. Concluding remark

Via the above examples, we hope to have demonstrated by own experience that Professor Życzkowski's classification scheme can serve as a highly powerful guide for choosing, or at least for explaining and justifying, new directions of research in the Mechanics of solids and structures. The latter field is not only extending at its borders, but it contains many undiscovered places in its inner region, also today. Professor Michał Życzkowski not only has found a highly valuable classification of this field, but, most important, he and his pupils and collaborators have found and explained many novel aspects, and they have driven further the frontiers of Mechanics in a manner representing a shining example for Science as a whole.

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