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DECOHESIVE CARRYING CAPACITY OF STRONGLY CURVED I-BEAM

NOŚNOŚĆ ROZDZIELCZA SILNIE ZAKRZYWIONEGO PRĘTA O PRZEKROJU DWUTEOWYM

Abstract

In the paper certain mechanism of ending of process of elastic-plastic deformations for perfectly elastic-plastic, strongly curved I-beam is discussed. It is connected with separation of one of flanges from a web, due to infinite value of radial strain in the web. According to Szuwalski, Życzkowski [1] the corresponding value of external loading describes decohesive carrying capacity of the I-beam. The domain of validity for obtained solutions is defined.

Keywords: decohesive carrying capacity, perfect plasticity, I-beam

Streszczenie

W artykule rozważono pewien mechanizm zakończenia procesu odkształceń sprężysto-plastycznych dla sprężysto-idealnie plastycznego, mocno zakrzywionego pręta o przekroju dwuteowym. Polega on na oddzieleniu się jednej z półek od środnika w związku z nieskończenie wielką wartością odkształcenia promieniowego. Zgodnie z propozycją Szuwalskiego i Życzkowskiego [1] odpowiedni moment zginający określa nośność rozdzielczą pręta. Określony został zakres ważności otrzymanych rozwiązań.

Słowa kluczowe: nośność rozdzielcza, idealna plastyczność, pręt dwuteowy

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1. Historical Background

Among numerous scientific achievements of late Professor Michał Życzkowski, one of the most important seems to be introduction of new type of ending for process of elastic-plastic deformations. Some perfectly elastic-plastic structures cannot reach their limit carrying capacity, connected with mechanism of plastic collapse. Earlier some inadmissible discontinuities of displacement field may occur, resulting in termination of process of continuous deformations. Corresponding external loadings are called decohesive carrying capacity, as they cause separation of system into parts.

For the first time general concept of decohesive carrying capacity was presented by Życzkowski and Szuwalski on the XIII-th IUTAM Congress in Moscow (1972) and published in 1973 [1]. Especially this concept is useful in problems of thermal stresses, when limit carrying capacity does not exist and was presented by Życzkowski and Szuwalski in 1975 [2]. Szuwalski [3] in 1980 showed that effect of decohesive carrying capacity may be observed also for some types of asymptotically perfect plasticity. Termination of the continuous solution was analysed within the framework of finite strain theory by Życzkowski and Szuwalski [4] in 1982. Then termination of the process is caused by inadmissible discontinuity of the stress field.

Życzkowski (1981) in his book on combined loadings in plasticity devoted a large chapter to problems of decohesive carrying capacity. In monograph on decohesive carrying capacity Skrzypek, Szuwalski and Życzkowski (1998) are citing over hundred papers on this problem. The vast majority of them were prepared by the group in Cracow University of Technology under supervision, or with participation of Professor Michał Życzkowski.

The idea of the present paper was born while preparing this last monograph. It corresponds to some problems of incomplete toroidal shells solved by Skrzypek and Życzkowski (1983). Strongly curved I-beam subject to pure bending, may be treated as a simplified model of incomplete toroidal shell.

2. Formulation of the problem

In present paper the process of deformations for perfectly elastic-plastic strongly curved beam with ideal I-cross section will be analysed (Fig. 1). The beam and loadings satisfy assumption of axial symmetry.

The beam is subject to pure bending by moment *M*. In case of very stiff flanges and flabby web one may expect expansion of flanges, due to plastification of the web – mechanism of nonsymmetric displacement, similar to that observed in incomplete toroidal shells. The part of bending moment will be carried by flanges – M_{I}, while the rest by web – M_{II}. Moment in flanges results from normal forces

$$M_{\mathrm{I}} = N_2 b - N_1 a \tag{1}$$

causing their tension. As they are different, also web must carry normal force

$$N_{\mathrm{II}} = N_1 - N_2 \tag{2}$$

in order to satisfy the assumption of pure bending.

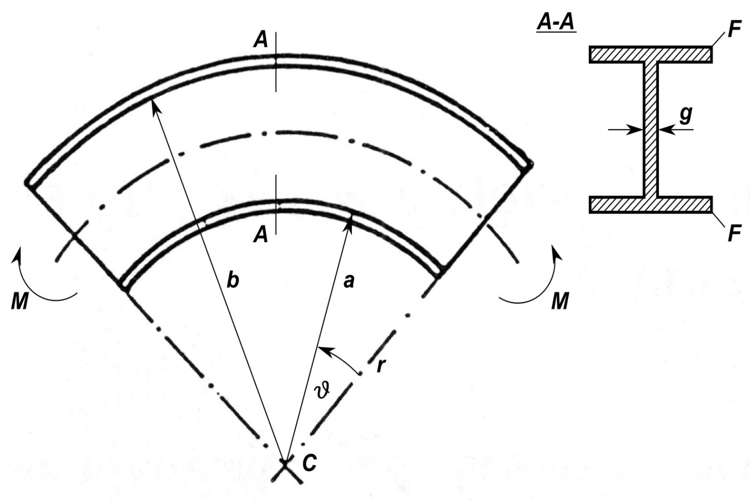

Fig. 1. Strongly curved I-beam

Rys. 1. Silnie zakrzywiony pręt o przekroju dwuteowym

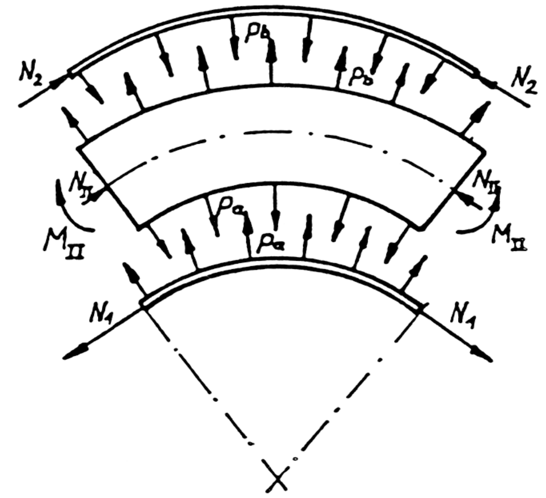

Fig. 2. Decomposition of the I-beam

Rys. 2. Podział pręta na elementy składowe

Interaction of flanges and web is realized in form of uniform pressure

$$p_b = \frac{N_2}{gb}; \quad p_a = \frac{N_1}{ga} \tag{3}$$

what results in momentless state in flanges and consequently stresses there are equal

$$\sigma_{\theta a} = \frac{N_1}{F}; \quad \sigma_{\theta b} = -\frac{N_2}{F} \tag{4}$$

Where F stands for cross sectional area of each flange.

Web may be treated as a segment of annular disk subject to combined axially symmetric loadings: bending moment M_{II}, normal force N_{II} and pressures p_a and p_b. Stress state and strain state caused by them, will be also axially symmetric, but displacements will not. They will depend on angular coordinate, as well.

In the elastic range stresses in the web are described

$$\sigma_r^{(e)} = \frac{A}{r^2} + B(1 + 2\ln r) + C \tag{5}$$

$$\sigma_\theta^{(e)} = -\frac{A}{r^2} + B(3 + 2\ln r) + C$$

while strains

$$\varepsilon_r^{(e)} = \frac{1}{E}\left[\frac{1+\nu}{r^2} A + 2B(1-\nu)\ln r + (1+3\nu)B + (1+\nu)C\right]$$

$$\varepsilon_r^{(e)} = \frac{1}{E}\left[\frac{1+\nu}{r^2} A + 2B(1-\nu)\ln r + (1+3\nu)B + (1+\nu)C\right] \tag{6}$$

The effective stress always takes the largest value at the inner radius a and there starts the plastic zone which will spread out towards external radius. In this plastic zone we will take advantage of the Nadai-Sokolovsky parametrization of yield condition

$$\sigma_r = \frac{2}{\sqrt{3}}\sigma_0 \sin\zeta; \quad \sigma_\theta = \frac{2}{\sqrt{3}}\sigma_0 \sin\left(\zeta + \frac{\pi}{3}\right) \tag{7}$$

Distribution of the parameter ζ found from internal equilibrium condition is described by equation

$$r = \frac{C_1 \exp\left(\frac{\sqrt{3}}{2}\zeta\right)}{\sqrt{\sin\left(\zeta - \frac{\pi}{3}\right)}} \tag{8}$$

Making use of the Hencky – Ilyushin theory of small elastic-plastic deformations, physical law takes form

$$(2\sigma_\theta - \sigma_r)\varepsilon_r - (2\sigma_r - \sigma_\theta)\varepsilon_\theta = \frac{1}{3K}\left(\sigma_\theta^2 - \sigma_r^2\right) \tag{9}$$

According to (6) compatibility condition takes form

$$\varepsilon_r = \varepsilon_\theta + r\frac{d\varepsilon_\theta}{dr} - \frac{4B}{E} \tag{10}$$

and calculated from it radial strain is equal:

$$\varepsilon_r^{(p)} = \frac{\sigma_0}{3\sqrt{3}K}\sin\left(\zeta+\frac{\pi}{3}\right) + C_2\exp\left(-\sqrt{3}\zeta\right)\frac{\sin\left(\zeta-\frac{\pi}{6}\right)}{\cos\zeta} +$$

$$+\frac{4B}{E}\exp\left(-\sqrt{3}\zeta\right)\frac{\sin\left(\zeta-\frac{\pi}{6}\right)}{\cos\zeta}\int_{\zeta_n}^{\zeta}\exp\left(\sqrt{3}\xi\right)\frac{\cos\xi}{\cos\left(\xi+\frac{\pi}{6}\right)}d\xi \tag{11}$$

Where ξ is the integral variable for parameter ζ.

3. Decohesive carrying capacity

In order to solve the general problem of elastic-plastic deformations we must take advantage of the complete set of boundary conditions

$$\begin{aligned} &\text{for } r = a \quad \sigma_r^{(p)} = p_a; \quad \varepsilon_\theta^{(p)} = \frac{N_1}{EF}; \quad \zeta = \zeta_a \\ &\text{for } r = b \quad \sigma_r^{(e)} = p_b; \quad \varepsilon_\theta^{(e)} = \frac{N_2}{EF} \end{aligned} \tag{12}$$

From those conditions the elastic constants *A*, *B*, *C* and integral constants C_1 and C_2 in plastic zone may be found. Moreover, the radius separating elastic and plastic zones r^* and value of parameter there ζ^* and parameter at the internal radius ζ_a will be calculated.

Let us notice, that when parameter ζ reaches value $\frac{\pi}{2}$ the radial strain tends to infinity. It leads to termination of the continuous solution, as the derivative of radial displacement becomes infinitely large, resulting in jump of this displacement (decohesion). This effect is possible only on the internal radius *a*.

At the onset of plastic zone, at radius *a*, parameters ζ_a and ζ^* are equal. With growth of external loading (bending moment *M*), radius separating zones r^* becomes larger than *a*, and parameters at these radii start to differ. The process lasts till parameter at inner radius becomes equal $\frac{\pi}{2}$, what means exhaustion of decohesive carrying capacity. Substitution of this value to the third of boundary condition eliminates one of unknown and makes it possible to evaluate decohesive carrying capacity of the beam $\hat{M}$.

Calculations are rather complicated and it is easier to carry them using dimensionless quantities such as: dimensionless radii

$$\alpha = \frac{ag}{F}; \quad \rho^* = \frac{r^* g}{F}; \quad \chi = \frac{b}{a} \tag{13}$$

and dimensionless normal forces in flanges

$$n_1 = \frac{N_1}{\sigma_0 F}; \quad n_2 = \frac{N_2}{\sigma_0 F} \tag{14}$$

With their help dimensionless bending moment causing infinitely large radial strain at inner radius a, i.e. decohesive carrying capacity, describes formula

$$\begin{aligned}\hat{m} = \frac{\hat{M}}{\sigma_0 F a} &= \frac{\alpha}{3}\exp\left(-\frac{\sqrt{3}}{2}\pi\right)\left[\exp(\sqrt{3}\zeta^*)\cot\left(\zeta^* - \frac{\pi}{3}\right) - \sqrt{3}\exp\left(\frac{\sqrt{3}}{2}\pi\right)\right] + \\ &+ \frac{\alpha}{12}\exp\left(-\frac{\sqrt{3}}{2}\pi\right)\int_{\frac{\pi}{2}}^{\zeta^*}\frac{\exp(\sqrt{3}\xi)}{\sin^2\left(\xi - \frac{\pi}{2}\right)}\mathrm{d}\xi + \frac{B}{\sigma_0}\alpha\left(\beta^2 - \rho^{*2}\right)\ln\frac{\chi}{\rho^*} + \\ &+ \frac{\alpha}{2}\left[(\chi^2 - \rho^{*2})\cos\left(\zeta^* - \frac{\pi}{3}\right) + \rho^{*2}\ln\frac{\chi}{\rho^*}\cos\zeta^*\right] + \chi n_2 - \frac{\alpha}{\sqrt{3}}\end{aligned} \tag{15}$$

where constant B is equal

$$B = \frac{\sigma_0}{4\ln\frac{\chi}{\rho^*}}\left[\mathrm{n}_2\left(\frac{1+\nu}{\alpha\chi} - 1\right) - 2\cos\left(\zeta^* - \frac{\pi}{3}\right)\right] \tag{16}$$

and dimensionless normal force in upper flange

$$n_2 = \frac{\left(\rho^{*2} - \chi^2\right)\cos\left(\zeta^* - \frac{\pi}{3}\right) - \frac{4}{\sqrt{3}}\rho^{*2}\ln\frac{\chi}{\rho^*}\sin\left(\zeta^* - \frac{\pi}{3}\right)}{\left(\frac{1+\nu}{\alpha\chi} - 1\right)\left(\rho^{*2} - \chi^2 + 2\chi^2\ln\frac{\chi}{\rho^*}\right) - 4\frac{\chi}{\alpha}\ln\chi\rho^*} \tag{17}$$

Dimensionless radius separating elastic and plastic zones

$$\rho^* = \frac{\exp\left[\frac{\sqrt{3}}{2}\left(\zeta^* - \frac{\pi}{2}\right)\right]}{\sqrt{2\sin\left(\zeta^* - \frac{\pi}{3}\right)}} \tag{18}$$

is defined with help of of parameter ζ^*, which must be calculated from equation

$$\begin{gathered}\frac{1}{2\sqrt{3}}(2\alpha + 2\nu - 1)\exp\left[\sqrt{3}\left(\frac{\pi}{2} - \zeta^*\right)\right] - \cos\zeta^* + \\ + \frac{4B}{\sigma_0}\exp\left(-\sqrt{3}\zeta^*\right)\int_{\zeta_a}^{\zeta^*}\exp\left(\sqrt{3}\xi\right)\frac{\cos\xi}{\cos\left(\xi + \frac{\pi}{6}\right)}d\xi = 0\end{gathered} \tag{19}$$

Obviously, solution of this equation, and further solution of the whole problem of decohesive carrying capacity, may be found only numerically. The obtained solution is valid, provided the second plastic zone, starting from external radius *b* will not occur earlier. This condition (effective stress at radius *b* must be smaller than yield stress σ_0) leads to inequality

$$\left[1 - \acute{\imath} + \acute{\imath}^2 + (1 - 2\acute{\imath})\alpha\chi + \alpha^2\chi^2\right]n_2^2 - \alpha^2\chi^2 \leq 0 \tag{20}$$

When it is not satisfied the second plastic zone must be taken into account, what results in significant complication of the problem – a system of fifteen equations (majority of them transcendental or integral) with fifteen unknowns. This problem will not be solved in the present paper.

The second condition necessary for occurance of decohesive carrying capacity is, that lower flange must remain elastic. In opposite case – after plastification, the lower flange can deform arbitrarily and will never separate from the web. This condition leads to

$$n_1 = \frac{2}{\sqrt{3}}\alpha \leq 1 \tag{21}$$

The domain of validity for obtained solutions in terms of geometry of the beam is shown in Fig. 3.

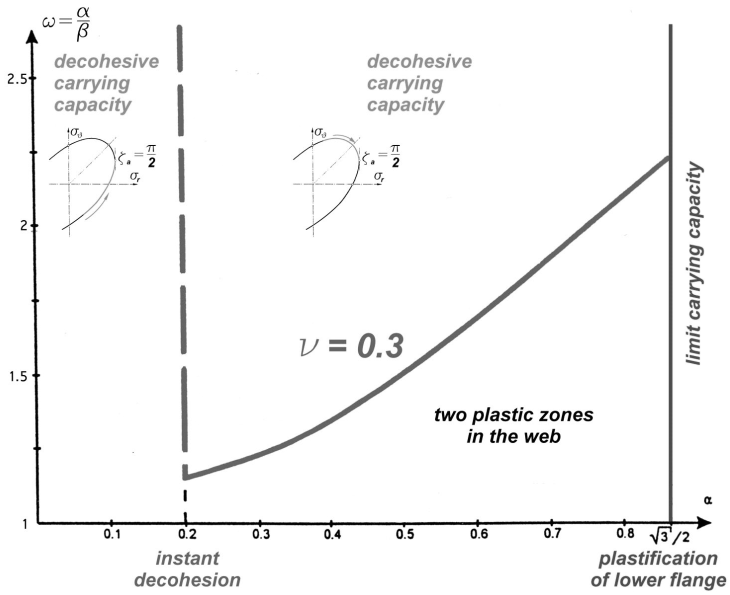

Fig. 3. Domain of validity for obtained solutions

Rys. 3. Zakresy stosowalności otrzymanych rozwiązań

In this diagram additionally a line of instant decohesion was introduced. When at the moment of first plastification, parameter ζ_a is equal $\frac{\pi}{2}$ elastic carrying capacity coincides with decohesive carrying capacity and plastic zone cannot spread out. It may happen for beams satisfying condition

$$\alpha = \frac{1}{2} - \nu \tag{22}$$

If not, as sho 1w the Huber–Mises–Hencky elipses on both sides of this vertical line, the parameter at the moment of first plastification may be either larger, or smaller than $\frac{\pi}{2}$. Then the plastic zone will spread out as long, as ζ at inner radius reaches value $\frac{\pi}{2}$ and decohesive carrying capacity occurs.

The values of decohesive carrying capacity in terms of α for various heights of the beam are shown in Fig. 4 for material with Poisson's ratio equal 0.3.

The decohesive carrying capacity is larger for higher beams with smaller initial curvature. It increases for beams with stiffer web, or suppler flanges. The solid thick line on the right is connected with plastification of the lower flange.

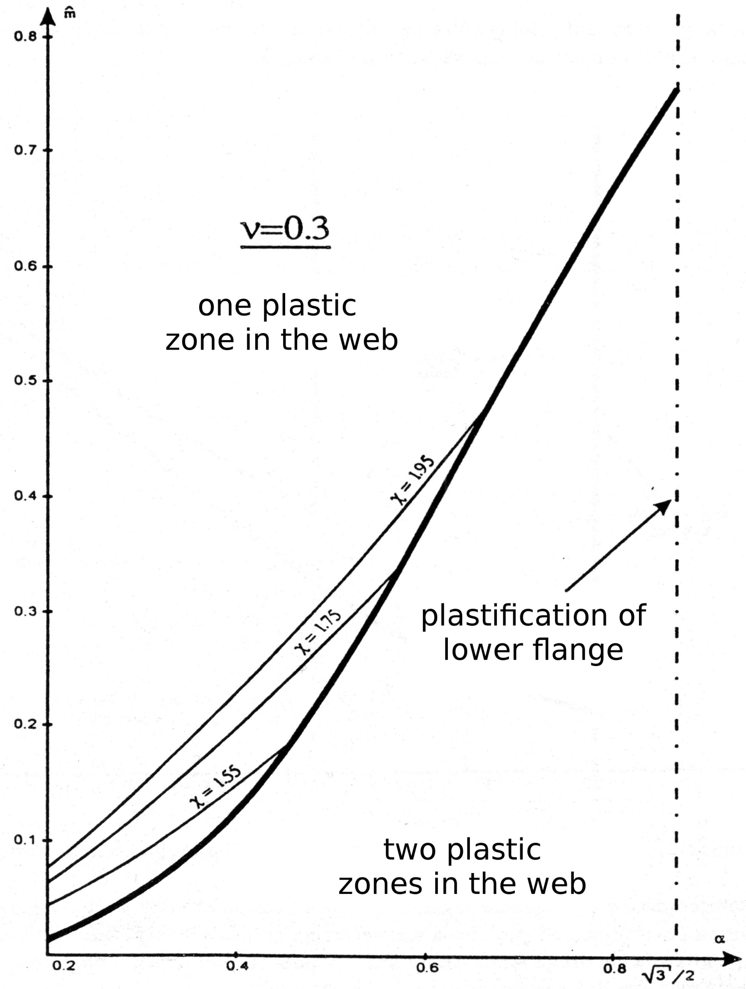

Fig. 4. Decohesive carrying capacity of strongly curved I-beam

Rys. 4. Nośność rozdzielcza silnie zakrzywionego pręta dwuteowego

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