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NEW MODELS OF FRACTURE IN SOLIDS AT MESO- AND NANOSCALES

NOWE MODELE PROPAGACJI PĘKNIĘĆ W CIAŁACH STAŁYCH UWZGLĘDNIAJĄCE MESO- ORAZ NANOPOZIOMY STRUKTURY MATERIAŁU

A b s t r a c t

Novel properties of the present cohesive crack models provide a better insight and an effective tool to explain multiscale nature of fracture process and the associated transitions from macro- to meso- and nano-levels of material response to deformation and fracture. Fracture testing of materials with cementitious bonding such as concrete and certain types of ceramics demonstrates that fractal cracks are commonly observed. In the limit of vanishing fracture quantum and/ or reduced degree of fractality the quantized cohesive model of a fractal crack, as presented here, reduces to the well-known classic models of Dugdale–Barenblatt or to the linear elastic fracture mechanics or the quantized fracture mechanics theories. Therefore, the basic concepts of linear elastic fracture mechanics, quantized fracture mechanics and fractal geometry are all incorporated into the present theory.

Keywords: fracture, deformation, cohesion, fractals, discrete process, multiscale, nano, meso, macro

S t r e s z c z e n i e

Nowe właściwości materiału oraz jego zachowania w procesie deformacji i pękania zostały opisane teoretycznie na podstawie dyskretnego modelu kohezyjnego szczeliny, uwzględniającego również geometrię fraktalną. Okazuje się, że dla makroszczelin różnice między nowym opisem oraz klasycznymi teoriami zniszczenia, takimi jak teoria Griffitha oraz LEFM (liniowo-sprężysta mechanika zniszczenia), nie są zbyt istotne. Natomiast w zakresie nanoszczelin, kiedy długość szczeliny jest porównywalna z kwantum propagacji a_0, różnice te są istotne. Uwzględnienie geometrii fraktalnej oraz dyskretnej natury propagacji szczeliny ma znaczący wpływ na końcowe rezultaty teorii dotyczącej tzw. wytrzymałości rezydualnej materiału niedoskonałego zawierającego początkowe defekty.

Słowa kluczowe: propagacja szczeliny, naprężenie krytyczne, defekt początkowy, kwantum propagacji, model szczeliny fraktalnej wg. Wnuka i Yavariego, model dyskretny szczeliny, wykładnik fraktalny, skale nano, mezo i makro

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1. Introduction

Cohesive models of cracks (Fig. 1) have been remarkably successful in explaining certain essential features of fracture process such as a finite stress at the crack tip, a non-zero crack opening displacement at the tip of the crack, and a certain equilibrium length of cohesive zone. This length increases with the applied load up to the point of incipient fracture and it serves as a measure of the material resistance to crack propagation.

We shall begin with re-visiting this model and then go on to incorporate fractal and discrete nature of fracture occurring in real materials. Recent research on nanocracks [1, 2] indicates that the classic failure criteria break down for very small cracks. Such examples show the need for novel non-local failure criteria for design of structures, in which multiscale fracture mechanisms are present. In order to refine available mathematical tools and to extend their validity for nanoscale and for fractal (rough) cracks we will merge here the basic concepts of the cohesive crack model with the fractal view of the decohesion process. At the same time we shall employ the non-local criteria, which remain valid at atomistic or nanoscale levels. The present model is used to predict the upper and the lower bounds for the cohesion modulus, which is used as a measure of material resistance to initiation and to propagation of fracture.

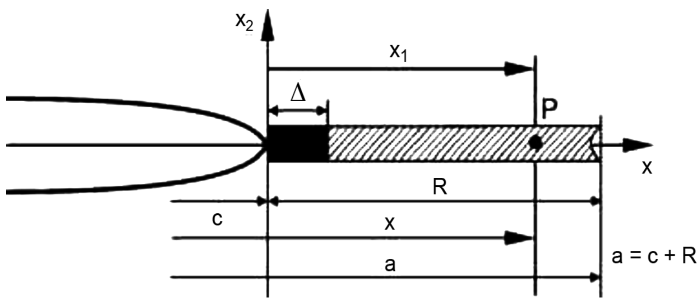

Fig. 1. Cohesive crack model created by extending the crack to a new length, which incorporates the length of the physical crack and the length the equilibrium cohesive zone. For quantized fracture mechanics the unit growth step or the fracture quantum Δ (or a_0 – as this quantity is referred in the text) is embedded within the cohesive zone and located next to the crack tip. This quantity enters all equations pertinent to the quantized fracture mechanics representation of fracture

Rys. 1. Model kohezyjny szczeliny, której propagacja zachodzi w sposób dyskretny. Kwantum propagacji oznaczono na rysunku greckim symbolem Δ, natomiast w równaniach przedstawionych w niniejszym artykule użyto symbolu a_0; oczywiście $a_0 = \Delta$. Należy zauważyć, że cząsteczka Neubera a_0 jest całkowicie wbudowana wewnątrz strefy kohezyjnej *R*. Jeśli Δ jest bliska wielkości *R*, to mamy do czynienia z pękaniem kruchym, jeśli natomiast $\Delta << R$, to opisujemy pękanie zachodzące w materiale ciągliwym

The concept of discrete nature of fracture has been investigated by many researchers, beginning with H. Neuber, V.V. Novozhilov, M.P. Wnuk, A. Seweryn, N. Pugno and R.S. Ruoff [3–7]. Comparison of various fracture criteria used to describe discrete fracture process suggests that a certain finite length parameter, determined either by the mi-crostructural or atomistic considerations, must be introduced into the basic equations underlying the theory of fracture. Even though various names and symbols have been used in describing this entity, such as "Neuber particle" by M.L. Williams [8], "unit growth" step by M.P. Wnuk [5] and "fracture quantum" by V.V. Novozhilov [4] and N. Pugno and R.S. Ruoff [7], the physical meaning of these length parameters is the same, and it can be accounted for mathematically by a discretization technique, cf. [9, 10].

A discretization procedure for the cohesive model of a fractal crack requires that all pertinent entities describing the influence of the cohesive stress that restrain opening of the crack, such as effective stress intensity factor, the modulus of cohesion, extent of the end zone and the opening displacement within the high-strain region adjacent to the crack tip are re-visited and replaced by certain averages over a finite length referred to as either "unit growth" step [5] or "fracture quantum" [4, 7]. Thus, two novel aspects of the model enter the theory: (1) degree of fractality related to the roughness of the newly created surface, and (2) discrete nature of the propagating crack.

The fractal crack model described here is based on a simplifying assumption, according to which the original problem is approximated by considerations of a smooth crack embedded in the stress field generated by a fractal crack [11]. The well-known concepts of the stress intensity factor and the Barenblatt cohesive modulus, which is a measure of material toughness, have been re-defined to accommodate the fractal view of fracture. Specifically, the cohesion modulus, in addition to its dependence on the distribution of the cohesion forces, is shown now to be a function of the "degree of fractality" reflected by the fractal dimension *D*, or by the Hausford fractal roughness parameter *H*. It is also a function of the unit growth step, the so-called fracture quantum and depends on the form of the "decohesion law". As expected in the limit of the fractal dimension *D* approaching 1 and for the disappearing magnitude of the fracture quantum, one recovers the cohesive crack model known from the fracture mechanics of smooth cracks.

In what follows we shall solve the problem assuming that we are dealing with a fractal crack represented by a cohesive model and that propagation of fracture is not continuous but discrete. Naturally, when the problem is reduced to that of a smooth crack and when the discrete nature of decohesion act is neglected; the present model yields the results identical to those known in classical fracture mechanics. This is analogous to the "correspondence principle" used in quantum mechanics to recover the solutions pertinent to continuum.

2. Essential results

Cohesion modulus for the Dugdale model

$$\begin{aligned} K_{\mathrm{coh}} &= 2\sqrt{\frac{a}{\pi}}\int_{c}^{c+\tilde{R}} \frac{S dx}{\sqrt{\left(c+\tilde{R}\right)^{2}-x^{2}}} = S\sqrt{\pi\left(c+\tilde{R}\right)}\left[\frac{2}{\pi}\cos^{-1}\left(\frac{c}{c+R}\right)\right] = \\ &= S\sqrt{\pi a_{0}}\sqrt{X+R}\left[\frac{2}{\pi}\cos^{-1}\left(\frac{X}{X+R}\right)\right] \end{aligned} \tag{1}$$

has been compared with the analogous result obtained from the discrete cohesive model ("Dugdale discrete"). Here, in addition to the quantities c, $\tilde{R}$ and S a new variable appears; and it is the fracture quantum a_0. The final result is

$$\langle K_{\mathrm{coh}} \rangle = K_{\mathrm{coh}}^{d} = S\sqrt{\pi a_0}\left\{ \int_X^{X+1} (Y+R)\left[\frac{2}{\pi}\cos^{-1}\left(\frac{Y}{Y+R}\right)\right]^2 dY \right\}^{1/2} \tag{2}$$

The length variables X and R represent the nondimensional crack length $X = c/a_0$ and the equilibrium length of the cohesive zone $R = \tilde{R}/a_0$, respectively. The expression (1) simplifies substantially for the physically important case of the relaxation length R being much smaller than the crack length X. This is so-called Barenblatt condition. Under an assumption of $R << X$ equation (1) reduces to an expression, which does not depend on the crack length, and in what follows will be used as the normalizing constant for the cohesion modulus.

If the inverse cosine function in (1) is denoted by y, then both sides of the resulting equality

$$\frac{1}{1+R/X} = \cos y \tag{3}$$

can be expanded into the Maclaurin series yielding a simple result $y = \sqrt{2R/X}$, or

$$\cos^{-1}\left(\frac{X}{X+R}\right) \approx \sqrt{\frac{2R}{X}} \tag{4}$$

Using this expression and neglecting R vs. X under the square root in (1) gives the desired formula for the Dugdale–Barenblatt cohesion modulus valid in the limiting case of $R << X$, namely

$$K_S = \left[K_{\mathrm{coh}}\right]_{R<<X} = \frac{2}{\pi} S\sqrt{\pi a_0}\sqrt{2R} \tag{5}$$

Next, the equilibrium length R is evaluated as a function of the loading parameter Q (= $\pi\sigma/2S$). When the stress intensity factors due to the applied load K_σ and $\langle K_\sigma \rangle$ are calculated according to both models

$$\begin{gathered} K_\sigma = \sigma\sqrt{\pi a_0}\,(X+R)^{1/2}, \\ \langle K_\sigma \rangle = \sigma\sqrt{\pi a_0}\left(X+R+\frac{1}{2}\right)^{1/2} \end{gathered} \tag{6}$$

and set equal to the respective cohesion moduli given in equations (1) and (2), the following relationships defining the equilibrium length R as a function of the loading parameter Q are obtained for both models

$$
\begin{aligned}
&Q_D = \cos^{-1}\left(\frac{X}{X+R}\right),\\
&Q_d^d = \frac{\left\{\int\limits_X^{X+1}(Y+R)\left[\cos^{-1}\left(\frac{Y}{Y+R}\right)\right]^2 dY\right\}^{1/2}}{\sqrt{X+R+1/2}}
\end{aligned}
\tag{7}
$$

It follows that for the same load the length of the equilibrium cohesive zone calculated from the discrete Dugdale model is greater that the one predicted by the classic model of Dugdale, namely

$$
R_D^d > R_D \text{ at an arbitrary } Q \tag{8}
$$

This length serves as a measure of material ability to relax stresses prior to the onset of fracture.

It is of interest to compare these results against analogous equation expressing the relation between the loading parameter Q and the equilibrium length of the relaxation zone R, as predicted from the cohesive discrete model with fractal geometry of the crack taken into account. The mathematics involved here is rather cumbersome, and the details were shown by M. Wnuk and A. Yavari [10]. To distinguish the quantities pertinent to so stated problem additional subscripts (or superscripts) D, d and f will be used; meaning "Dugdale", "discrete" and "fractal", correspondingly.

With auxiliary notation defined below let us state the essential results. To describe the discreteness and the roughness of the crack, we shall need the following functions

$$
\begin{aligned}
&\chi(\alpha) = \frac{1}{\pi^{2\alpha}}\int\limits_0^1 \frac{(1-s)^{2\alpha}+(1+s)^{2\alpha}}{(1-s^2)^{\alpha}}ds,\\
&H(X,R,\alpha) = \int\limits_{X/(X+R)}^{1} \frac{(1-s)^{2\alpha}+(1+s)^{2\alpha}}{\pi^{2\alpha}(1-s^2)^{\alpha}}ds
\end{aligned}
\tag{9}
$$

With such notation the relationship between the applied load and the equilibrium relaxation zone R associated with the crack of length X and characterized by the fractality exponent α reads

$$
Q_f = \frac{\pi}{2\chi(\alpha)}\left[\int\limits_X^{X+1}(Y+R)^{2\alpha}H^2(Y,R,\alpha)dY\right]^{1/2}\left[\frac{2\alpha+1}{(X+R+1)^{2\alpha+1}-(X+R)^{2\alpha+1}}\right]^{1/2} \tag{10}
$$

Now when formula is used in conjunction with two equations in (7) we are able to compare the equilibrium length of the cohesive zone R associated with three models discussed in the present work namely – classic Dugdale model (D) discrete cohesive model (D_d) and the discrete cohesive model which incorporates the fractal geometry (D_{df} or just Q_f). Due to the nature of equations (7) and equation (10) the functions Q_D, Q_{Dd} and Q_f

are plotted first against the length R (Fig. 2). Then, the inverse functions are determined numerically (Fig. 3). We note that only the first of the equations in (7) can be inverted in a closed form; this is the classic result of Dugdale. The other two results, the second equation in (7) and equation (10) can only be manipulated numerically to find the inverse functions $R = R(Q)$. This has been done and the results are shown in Fig. 3.

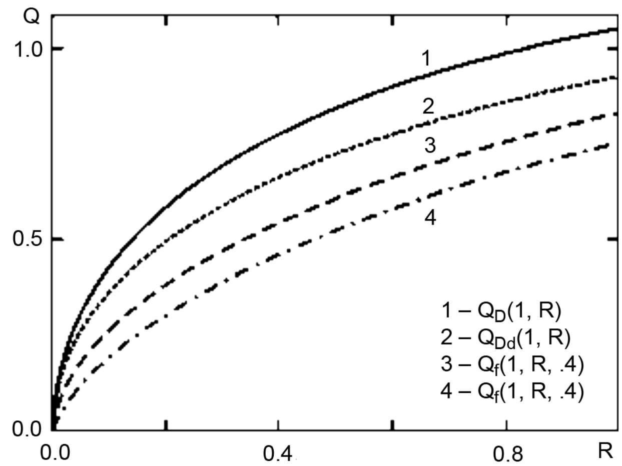

Fig. 2. Graphs representing the relationships between the applied loading parameter Q and the equilibrium length of the relaxation zone R. The top curve corresponds to the classic cohesive crack model of Dugdale–Barenblatt. The curve below the top one corresponds to the discretized cohesive crack model, cf. equations (7), while the other two curves were obtained from equation (10) pertinent to the fractal geometry of a discrete cohesive crack. In all four cases shown the nondimensional crack length X is assumed to be one

Rys. 2. Wykresy przedstawiają związki między paramtrem obciążenia zewnętrznego Q oraz równowagową długością strefy kohezyjnej R. Górna krzywa reprezentuje klasyczny wynik modelu Dugdale'a-Barenblatta, podczas gdy krzywe pokazane poniżej ilustrują wyniki otrzymane z dyskretnej teorii zniszczenia, zob. równanie (7) dla szczeliny gładkiej (2) oraz równanie (10) dla szczeliny fraktalnej, krzywe 3 i 4. We wszystkich rozważanych przypadkach długość szczeliny a równa jest kwantum propagacji a_0, czyli $X = 1$

As can be seen from Fig. 3, one obtains different predictions from each model. The equilibrium lengths, when considered for a fixed load Q, tends to increase in the following manner: the smallest value is obtained for the classic Dugdale model, a larger value is predicted by the discretized cohesive model, while incorporating the fractal geometry via index α leads to yet larger values of R. Since the length R and it is indicative of material ability to relax stresses prior to the onset of crack propagation it can be used as a measure of material fracture toughness. The physical interpretation of these results confirms a supposition that nature itself provides intricate mechanisms for enhancing material resistance to propagation of fracture; it is accomplished by a transition from continuous to a discrete mode of crack growth, and further enhanced by presence of roughness of the crack surface as reflected through the fractal geometry. These conclusions are illustrated graphically in the last section (cf. Fig. 7). It is noteworthy that the phenomena discussed here are essential for explaining fracture in the nano-range. The parametric study indicates that while the effects of the quantization procedure disappear for the longer cracks (much larger than the quantum fracture), the influence of the fractal geometry remains significant also for long cracks.

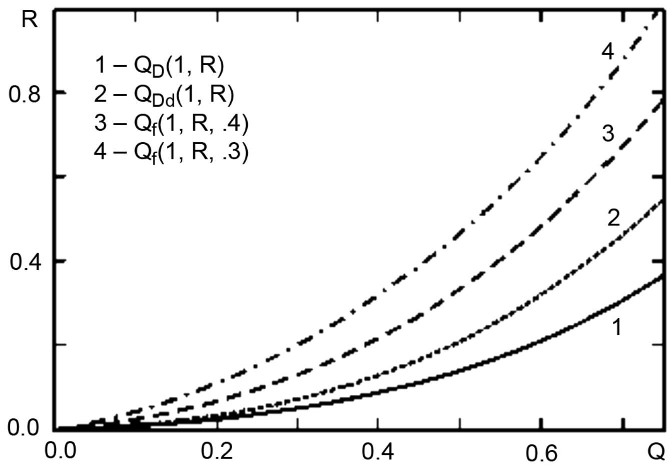

Fig. 3. The inverse functions corresponding to the same four cases shown in Fig. 2. Now the equilibrium lengths of the relaxation zone R are shown as functions of the applied load Q at a fixed crack length $X = 1$. It is seen that the length R observed for any constant load significantly increases when the discrete nature of fracture and/or fractal geometry of the crack is accounted for Top two curves correspond to fractal discrete model at α equal 0,3 and 0,4 The next curve resulted from discretized smooth cohesive crack model (it would be identical with the curve obtained for a fractal crack when α = 0,5). The lowest curve represents Dugdale's classic result. It can be shown that for $X >> 1$ the last two curves merge

Rys. 3. Funkcje odwrotne odpowiadające czterem przypadkom pokazanym na rys. 2. Tutaj długość strefy kohezyjnej R jest pokazana jako funkcja parametru obciążenia zewnętrznego Q. Wykresy otrzymano dla długości szczeliny $X = 1$; dwie krzywe od góry, 3 i 4, otrzymano dla fraktalnego wykładnika α = 0,3 oraz α = 0,4, natomiast krzywa 2 odpowiada gładkiej szczelinie kiedy α = 0,5. Najniższa krzywa 1 ilustruje wynik standardowego modelu Dugdale'a

One might ask a question – could the relations between load Q and the equilibrium length R be simplified for the limiting case of R being much smaller than the crack length X? For the first two models considered here, classic Dugdale and quantized Dugdale, see equations (7), the answer is "yes". Yet, for the discrete cohesive fractal crack, the answer is "no". Applying the condition $R << X$ in equations (7), one obtains the approximations valid for the $R << X$ case, namely

$$Q_D^{\lim} = \left[\frac{2R}{X}\right]^{1/2}, \qquad Q_{Dd}^{\lim} = \left[\frac{2R}{X + 1/2}\right]^{1/2} \tag{11}$$

They both can be readily inverted resulting in the R vs. Q relation. It is noted, however, that for the nanoscale fracture the Barenblatt condition $R << X$ is usually not satisfied, and then the calculations must be performed through application of the complete equations (7) and (10).

If the length of the relaxation zone R present in the equations (11) is replaced by its critical value, occurring at the onset of fracture, R_{cr}

$$R_{cr} = \frac{1}{a_0}\left(\frac{\pi}{8}\right)\frac{K_c^2}{S^2} \tag{12}$$

then the equations (11) can be used to express the critical stresses predicted by both models

$$\begin{aligned} Q_D^{cr} &= \left[\frac{2R_{cr}}{X}\right]^{1/2}, \\ Q_{Dd}^{cr} &= \left[\frac{2R_{cr}}{X+1/2}\right]^{1/2} \end{aligned} \tag{13}$$

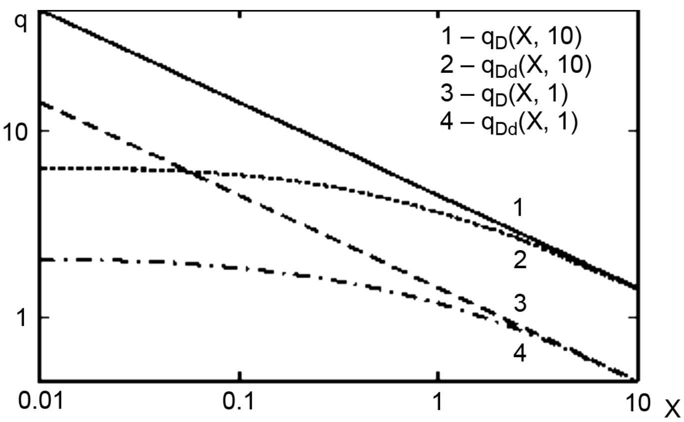

Fig. 4. Critical stresses predicted by the Griffith theory (straight lines) and the quantized cohesive crack model, shown for two different levels of material toughness, $R_{cr} = a_0$ and $R_{cr} = 10a_0$. All graphs show the critical stress as a function of crack length using the double logarithmic scale

Rys. 4. Naprężenia krytyczne wynikające z teorii Griffitha przedstawione w skali podwójnie logarytmicznej. Linie proste wynikają z teorii Griffitha, natomiast krzywe o warościach skończonych na osi pionowej reprezentują rozwiązania wynikające z dyskretnej mechaniki zniszczenia. Wykresy 1 i 2 zostały uzyskane dla odporności na propagację szczeliny $R_{cr} = 10a_0$, natomiast krzywe 3 i 4 obowiązują dla materiału o odporności na pękanie $R_{cr} = a_0$

The expression (12) resulted from setting the cohesion modulus defined by (5) to the material fracture toughness K_c. Graphical representations of these results are provided in Fig. 4. It is seen that the first of these equations predicts an infinite critical stress for vanishing crack length; as expected this is identical with the Griffith result. However, when the model is subjected to discretization, the situation changes drastically, see the curved lines shown in Figs. 4 and 5. In the nanocracks range these curves diverge substantially from the straight lines produced by the Griffith theory. The curves with finite values attained at $X = 0$ were generated by the second equation in (13), and they represent the predictions for the critical stresses valid for the nanocracks range. By "nanocracks" we understand the cracks with sizes comparable to the fracture quantum a_0. Most important is the finite strength predicted from the discrete fracture model for a virgin material with no pre-existing defects ($X = 0$). In a nondimensional form suggested by the definition of the parameter Q, the inherent strength reads

$$Q_0 = \left[Q_{Dd}\right]_{X\to 0}^{\mathrm{cr}} = \left[2\sqrt{R}\right]_{R=R_{\mathrm{cr}}} = 2\sqrt{R_{\mathrm{cr}}} \tag{14}$$

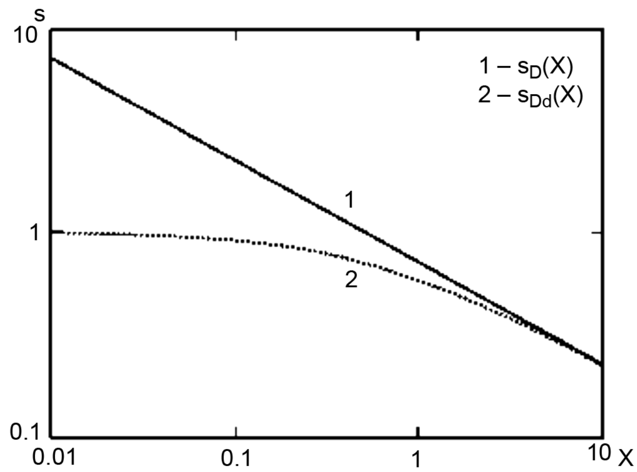

Fig. 5. Critical stresses normalized through the inherent material strength shown as functions of the crack length. The straight line corresponds to the Griffith theory, while the curve (shown in a double logarithmic scale) represents the prediction of the quantized cohesive crack model, as defined by the second equation in (16). The differences between the two results are significant only within the nanocracks range. For a material with no initial pre-existing defects the new model predicts finite critical stress, which is identified with the material "inherent strength"

Rys. 5. Naprężenie krytyczne znormalizowane poprzez naprężenie krytyczne dla materiału dziewiczego (bez początkowych defektów) jako funkcja długości szczeliny przedstawiona w skali podwójnie logarytmicznej. Linia ciągła przedstawia rozwiązanie Griffitha według klasycznej mechaniki zniszczenia, podczas gdy linią przerywaną oznaczono rozwiązanie wynikające z dyskretnej mechaniki zniszczenia, zob. równanie (16). Dla makroszczelin różnice między tymi dwoma reprezentacjami szczeliny są nieznaczne, natomiast dla zakresu nanoszczelin różnic tych nie można zaniedbać

We shall refer to this quantity as a dimensionless inherent material strength. With substitution of (12) and invoking the definition $Q = \pi\sigma/2S$, it can be readily shown to reduce to

$$\sigma_0 = \sqrt{\frac{2}{\pi a_0}} K_c \tag{15}$$

This expression for the inherent material strength is identical to the quantized fracture mechanics result presented by N. Pugno and R.S. Ruoff [7]. This was to be expected, as despite very different physical interpretations of the near-tip conditions, the Dugdale–Barenblatt model in its classic form generates the same critical stress as the Griffith theory. Such property of the model carries over to its quantized mode, since the quantized fracture mechanics (quantized Griffith model) is compared with the quantized cohesive crack model, or quantized Dugdale–Barenblatt model.

Finally, if both sides of the expressions (13) are divided by Q_0 defined in (14), the following simple results for the normalized critical stresses are obtained

$$s_D^{\mathrm{cr}} = \frac{Q_D^{\mathrm{cr}}}{Q_0} = \frac{\sigma_D^{\mathrm{cr}}}{\sigma_0} = \sqrt{\frac{1}{2X}},$$
$$s_{Dd}^{\mathrm{cr}} = \frac{Q_{Dd}^{\mathrm{cr}}}{Q_0} = \frac{\sigma_{Dd}^{\mathrm{cr}}}{\sigma_0} = \sqrt{\frac{1}{2X+1}} \tag{16}$$

This is perhaps the most concise and most appropriate way to normalize the critical stresses (note that the inherent material strength σ_0 is used here as the normalization constant). With such a choice of the normalization constant, all pertinent graphs representing dependence of the critical stresses on the crack length reduce to just two curves, shown in fig. 5. The differences between the Griffith theory and the quantized cohesive crack model are significant only in the nanocrack range. Note that in (16) the dependence on the toughness level disappears, as K_c is hidden now in the normalization constant σ_0.

Another result pertinent specifically to nanofracture initiated by small cracks, i.e., cracks on the order of magnitude of fracture quantum a_0. For this nanorange the nondimensional cohesion modulus for a discrete cohesive crack

$$F_{\mathrm{coh}}(X,R) = \frac{K_{\mathrm{coh}}^{d}}{K_S} = \frac{S\sqrt{\pi a_0}\left\{\int_X^{X+1} (Y+R)\left[(2/\pi)\cos^{-1}(Y/Y+R)\right]^2 dY\right\}^{1/2}}{(2/\pi)S\sqrt{\pi a_0}\sqrt{2R}} \tag{17}$$

shows a visible enhancement for $X \geq 0$. This effect becomes more pronounced when the Barenblatt condition $R << X$ is not satisfied, as is often the case in the nanocracks range. On the other hand, when the length R is much smaller than the length X, then the ratio (17) does not depend on the crack length and can be shown to approach unity (as then the integral inside the curly brackets of (17) reduces to $(4/\pi^2)2R$). This effect and the phenomenon of the enhancement of toughness for the nanocracks are illustrated in Fig. 6.

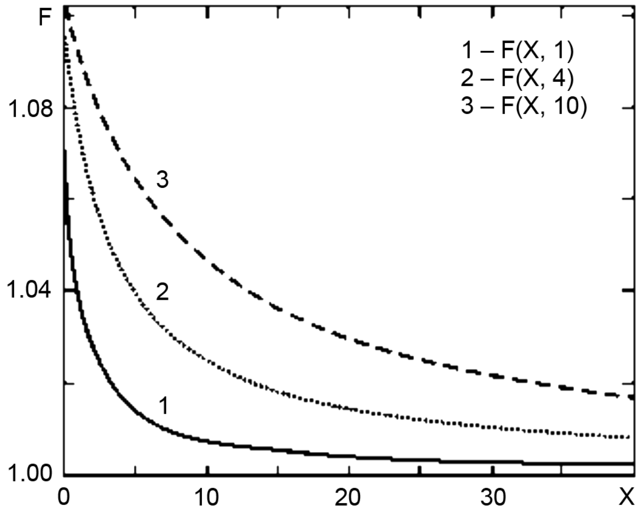

Fig. 6. Function F_{coh} defined by equation (17) demonstrates the enhancement of the cohesion modulus when the crack is represented by the discrete cohesive model and when the crack length is less or about equal the fracture quantum a_0. Note that this phenomenon is particularly visible when the Barenblatt condition of R being much less than X is not satisfied; that is when the length of the relaxation zone R may assume arbitrary values compared to the crack length X. This is often the case for the nanocracks accompanied by their associated relaxation zones

Rys. 6. Funkcja F_{coh} zdefiniowana równaniem (17) ilustruje wzbogacenie modułu spójności w przypadku dyskretyzacji modelu kohezyjnego szczeliny i dla zakresu długości szczeliny porównywalnej z kwantum propagacji a_0. Godny uwagi jest fakt, że zjawisko to jest szczególnie widoczne, kiedy warunek Barenblatta $R << X$ nie jest spełniony. Wówczas długość strefy kohezyjnej R może przyjmować dowolne wartości w porównaniu z długością szczeliny X; co często ma miejsce dla nanoszczelin

3. Conclusions

Relations between applied load and the equilibrium length of the cohesive zone have been established for three different mathematical representations of the crack, and these are:

a) cohesive crack model of Dugdale–Barenblatt for a smooth crack described by two K-factors, K_σ and K_S corresponding to the applied stress and the cohesive stress, respectively,
b) discrete cohesive crack model described by the averages $\langle K_\sigma \rangle_{c+a_0}$ and $\langle K_S \rangle_{c+a_0}$, and
c) discrete and fractal cohesive crack model involving the fractal equivalents of the averages used in second part, namely $\langle K^f_\sigma \rangle_{c+a_0}$ and $\langle K^f_S \rangle_{c+a_0}$.

For a classic linear elastic fracture mechanics crack model there is no cohesive zone, and the crack itself provides a mechanism for relaxing the high stresses in the vicinity of a stress concentrator. Therefore, the classic representation may be thought of as a limiting case of a more general and refined mathematical formulation involving cohesive zone associated with a crack. For such a representation the equilibrium between the driving force K^2_σ and material resistance K^2_S defines a unique relation between the length of the equilibrium cohesion zone say $\tilde{R}$ and the applied load say Q. The equilibrium between R (= $\tilde{R}$ /a_0) and Q is maintained during the loading process up to the point of incipient fracture.

Cohesive zone generated prior to fracture is related to the material fracture toughness. It also provides an important mechanism for relaxing stresses prior to fracture initiated in the immediate vicinity of a stress concentrator.

Figure 7 shows schematically four sketches of a crack represented by four mathematical models (a) the linear elastic fracture mechanics (LEFM) concept of a Griffith crack embedded in a linear elastic solid (b) Dugdale–Barenblatt cohesive model, (c) discretetized cohesive model and, finally, by (d) a fractal cohesive and discrete model. The very first crack shown in the figure has no cohesive zone at all, and the stresses are singular at the tip of the crack. In this case fracture toughness must be measured by employing the ASTM standards that make no mention of cohesion. The second crack in the figure corresponds to a cohesive model suggested independently by G.I. Barenblatt [12] and D.S. Dugdale [13]. With cohesion included we gain a better insight into material response to fracture by visualizing the effect of the cohesive zone. In the next two models considered we have generalized the cohesive crack model by incorporating material mesomechanical features related to discrete growth of the crack and with its fractal geometry taken into account. Therefore, in the most advanced model considered here two new variables enter the theory: fracture quantum a_0 and degree of fractality measured either by the fractal dimension D or by the fractal exponent α. Accounting for the discrete and fractal nature of fracture leads to a conclusion that the equilibrium length of the cohesive zone is indeed significantly influenced by both the quantum (discrete) and fractal aspects of the subcritical as well as the propagating crack.

At the micro- and nanoscale the size of the Neuber particle, or process zone in a more updated nomenclature, becomes important not only for mathematical treatment of the problem, but also for the physical interpretation of the decohesion phenomenon at the atomistic scale.

It is noteworthy that each of the successive models listed in Fig. 7 predicts for a given fixed level of the applied load successively larger cohesive zone, namely

$$R^{f}_{Dd} \geq R^{d}_{D} \geq R_{D} \geq R_{\mathrm{LEFM}}$$

The interpretation of the subscripts is as follows: f – fractal; Dd – Dug dale discrete, D – Dugdale. Of course the LEFM value of $\tilde{R}$ is zero, but using the K_c value obtained from the tests specified by ASTM one could, in hindsight, define an equivalent length $\tilde{R}$ that could be associated with a LEFM crack. It is noted that all equations given here reduce to the LEFM results when a fractal crack is replaced by a smooth crack and when the fracture quantum is allowed to vanish.

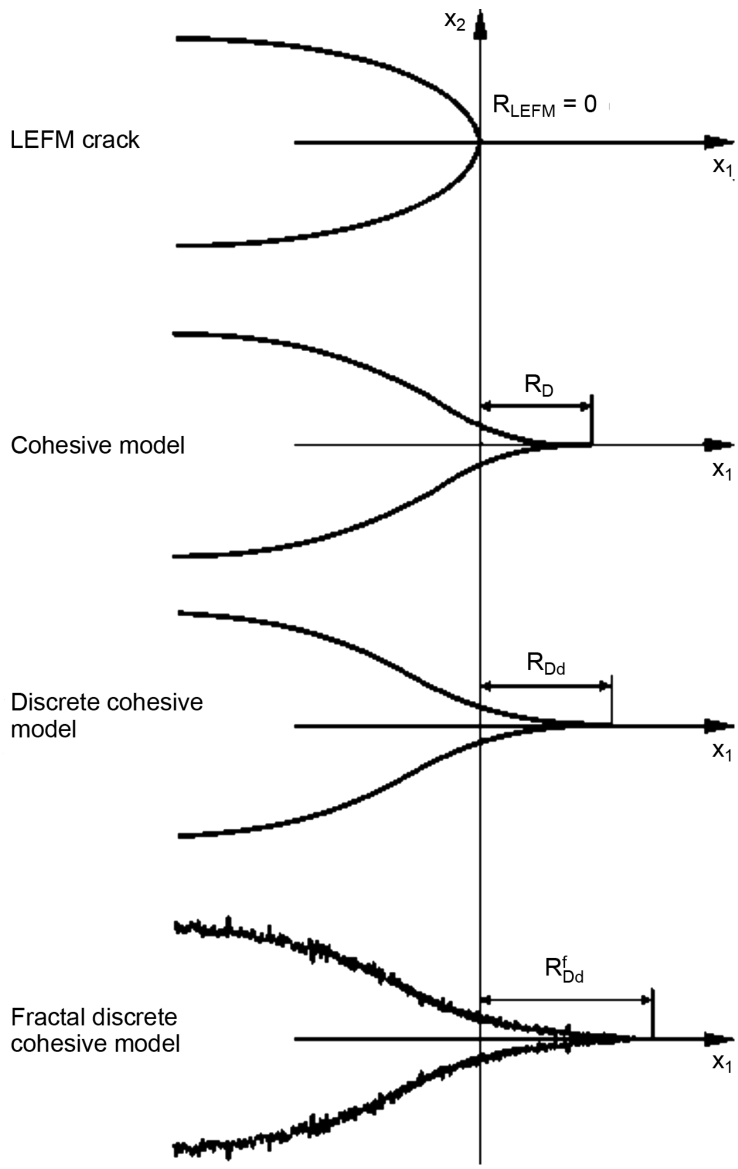

Fig. 7. Near tip region of a crack represented by four different models. The size of the equilibrium cohesive zone associated with a crack serves as a measure of the degree of stress relaxation occurring in the crack vicinity. It appears that the two features of the model considered here, discreteness and roughness of the newly created surface are inherent mechanisms of stress relaxation provided by nature

Rys. 7. Przemieszczenia powierzchni szczeliny w pobliżu wierzchołka szczeliny. Długość strefy kohezyjnej jest miarą odporności materiału na pękanie. Każdy z przedstawionych tutaj wyników wskazuje na zwiększoną długość R_{coh} dla przedstawianych kolejnych modeli szczeliny

References

[1] Isupov L.P., Mikhailov S.E., *A comparative analysis of several nonlocal fracture criteria*, Arch. Appl. Mech., 68, 1998, 597.

[2] Ippolito M., Mattoni A., Colombo L., Pugno N., *Role of lattice discreteness on brittle fracture: Atomistic simulations versus analytical models*, Phys. Rev. B, 73, 2006, 104-111.

[3] Neuber H., *Theory of Notch Stresses*, Springer-Verlag, Berlin, 1958.

[4] Novozhilov V.V., *On a necessary and sufficient criterion for brittle strength*, Prikl. Mat. Mekh., 33, 1969, 212 (in Russian).

[5] Wnuk M.P., *Quasi-static extension of a tensile crack contained in a viscoelastic-plastic solid*, J. Appl. Mech., 41, 1974, 234.

[6] Seweryn A., *Brittle-fracture criterion for structures with sharp notches*, Eng. Fract. Mech., 47, 1994, 673.

[7] Pugno N., Ruoff R.S., *Quantized fracture mechanics*, Phil. Mag., 84(27), 2004, 2829.

[8] Williams M.L., *Dealing with Singularities in Elastic Mechanics of Fracture*, in: 11th Polish Symposium on Mechanics of Solids, Krynica 1965.

[9] Wnuk M.P., Yavari A., *Discrete fractal fracture mechanics*, Eng. Fract. Mech., 75(5), 2008, 1127.

[10] Wnuk M., Yavari A., *A discrete cohesive model for fractal cracks*, Eng. Fract. Mech., 76, 2009, 548.

[11] Wnuk M.P., Yavari A., *On estimating stress intensity factors and modulus of cohesion for fractal cracks*, Eng. Fract. Mech., 70, 2003, 1659.

[12] Barenblatt G.I., *The mathematical theory of equilibrium of crack in brittle fracture*, Adv. Appl. Mech., 7, 1962, 55.

[13] Dugdale D.S., *Yielding of steel sheets containing slits*, J. Mech. Phys. Solids, 8, 1960, 100.