CZASOPISMO TECHNICZNE M
WYDAWNICTWO POLITECHNIKI KRAKOWSKIEJ

z. 3-M/2008
ISSN 0011-4561
ISSN 1897-6328

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NURBS CURVES AND SURFACES APPLICATION IN CAD SYSTEMS

ZASTOSOWANIE KRZYWYCH I POWIERZCHNI NURBS W SYSTEMACH CAD

Abstract

Currently, B-splines and NURBS (Non-Uniform Rational B-spline) are the most popular mathematical forms for curves and surfaces representation. NURBS offer a unified mathematical form not only for representation of free-form curves and surfaces, but also for the precise representation of close-form shapes such as lines, conics, quadrics, and surfaces of revolution. In the paper application of NURBS curves and surfaces in CAD systems like Pro/Engineer Wildfire 3.0 was presented. Animation of human expressions based on the NURBS representation based on human facial anatomy was illustrated. Facial muscle movements and expressions can be simulated by modification of the weights, or moving the control points of the NURBS curves.

Keywords: *the NURBS curves and surfaces*, *animation of human expressions*, *CAD systems*

Streszczenie

Obecnie najpopularniejszą matematyczną formą zapisu krzywych i powierzchni stały się krzywe B-Spline oraz NURBS (*Non Uniform Rational B-Spline*). Niejednorodne ułamkowe krzywe B-Spline oferują ujednoliconą formę matematycznego zapisu dla krzywych i powierzchni swobodnych, jak również dla linii, krzywych zamkniętych, powierzchni stożkowych oraz obrotowych. W artykule przedstawiono zastosowanie krzywych i powierzchni NURBS w systemach CAD w środowisku programu Pro/Engineer Wildefire 3.0 oraz przykład animacji mimiki ludzkiej twarzy za pomocą metody bazującej na punktach kontrolnych oraz niejednorodnych krzywych ułamkowych B-Spline zamkniętych w geometrycznej siatce modelu twarzy. Pokazano, jak ruch oraz wyraz mięśni twarzy mogą być modyfikowane za pomocą zmiany lub przemieszczenia punktów kontrolnych krzywej NURBS.

Słowa kluczowe: *krzywe i powierzchnie NURBS*, *animacja mimiki ludzkiej twarzy*, *systemy CAD*

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1. Introduction

Over the recent years, different curves and surfaces representation forms have been proposed. Currently, B-splines and NURBS (Non-Uniform Rational B-splines) are the most popular mathematical forms. NURBS offer a unified mathematical form not only for representation of free-form curves and surfaces, but also for precise representation of close-form shapes such as lines, conics, quadrics, and surfaces of revolution. It has been an IGES (Initial Graphics Exchange Specification) standard since 1983 has been approved by American National Standard Institute – ANSI, and many commercial CAD systems and other applications are based on the NURBS representation. IGES defines a neutral data format (independent from hardware) that allows digital exchange of information among Computer-aided design (CAD) systems. The standard ensures support for exchange of 2-D and 3-D technical drawings, specification and others needed to design and production, including geometrical and characteristic data relative materials and surfaces with their properties (shapes, dimensions, tolerances, usable feature, etc.).

In the present paper the NURBS curves and surfaces examples and applications will be presented together with novel method of animation of human expression using NURBS curves based on facial anatomy.

2. Definition and properties

2.1. NURBS Curves

A p-th-degree curves is defined by [1]

$$C(u) = \frac{\sum_{i=0}^{n} N_{i,p}(u) w_i P_i}{\sum_{i=0}^{n} N_{i,p}(u) w_i} \tag{1}$$

where:

P_i – control points (forming a control polygon),
w_i – weights of control points,
$N_{i,p}$ – the p-th-degree B-spline basis functions defined on the nonperiodic (and nonuniform) knot vector u

$$U = \left\{ \underbrace{a,\ldots,a}_{p+1}, u_{p+1},\ldots,u_{m-p-1}, \underbrace{b,\ldots,b}_{p+1} \right\} \tag{2}$$

where:

$a_i = 0,\ b_i = 1,\ \text{for}\ \ i \in \langle 0, p \rangle$.

Geometric properties of NURBS curves:

– End points of curves for $u \in [0,1]$ are $C(0) = P_0$ and $C(1) = P_n$,
– An affine transformation is applied to the curve by applying it to the control points,
– NURBS curves are invariant under perspective projections,

- If $u \in \langle u_i, u_{i+1} \rangle$, then $C(u)$ lies within the convex hull of control points P_{i-1}, ..., P_i,
- $C(u)$ is infinitely differentiable on the interior of knot spans and is $p-k$ times differentiable at a knot of multiplicity k,
- No plane has more intersection with the curve than control polygon,
- A NURBS curve with no interior knots is a rational Bezier curve. NURBS curves contain nonrational B-spline and rational and nonrational Bezier curves as a special cases,
- If the control point P_i is moved, or the weight w_i is changed, it affects only that portion of the curve on the interval $u \in \langle u_i, u_{i+p+1} \rangle$.

2.2. NURBS surfaces

A NURBS surface of degree p in the u direction and degree q in the v direction is a bivariante vector-valued piecewise rational function of the form

$$S(u) = \frac{\sum_{i=0}^{n}\sum_{j=0}^{m} N_{i,p}(u)N_{j,q}(v)w_{i,j}P_{i,j}}{\sum_{i=0}^{n}\sum_{j=0}^{m} N_{i,p}(u)N_{j,q}(v)w_{i,j}} \tag{3}$$

where:

$P_{i,j}$ – bidirectional control net,
$w_{i,j}$ – weights of control points,
$N_{i,p}(u)$, $N_{j,q}(u)$ – nonrational B-spline basis functions defined on the knot vectors u, v; and $0 \le u, v \le 1$

$$U = \left\{ \underbrace{0,...,0}_{p+1}, u_{p+1},...,u_{r-p-1}, \underbrace{1,...,1}_{p+1} \right\} \tag{4}$$

$$V = \left\{ \underbrace{0,...,0}_{q+1}, v_{q+1},...,v_{s-q-1}, \underbrace{1,...,1}_{q+1} \right\} \tag{5}$$

where:

$r = n + p + 1$ and $s = m + q + 1$.

3. Application of NURBS curves and surfaces in CAD system Pro/Engineer Wildefire 3.0

Pro/Engineer was applied to create a profile of the human face. The environment can model a surface with control points which are prepared in table data. An ASCII file with *.ibl extension was prepared to generate a NURBS surface. The file defined the kind of surface, and the coordinate system for all curves forming sequential sections.

In table 1 values of control points of curves forming NURBS surface have been given. Sections of curves that are placed near the middle of the modelled face. The file with *.ibl

extension was built from the data in table, from 20 curves, every curve with 29 control points. This number results from the dimensions of the modelled face. The curves are 1 cm apart, at the same distance that control points are apart. The mesh forming the outline of surface was created, to be further imported to the file with *.ibl extension, according to the requirements Pro/Engineer Wildefire 3.0.

Table 1

Part of table with values of control points marked out for sequential NURBS curves

VII curve			VIII curve			IX curve			X curve		
x	*y*	*z*	*x*	*y*	*z*	*x*	*y*	*z*	*x*	*y*	*z*
1.00	7.00	0.00	1.00	8.00	0.00	1.00	9.00	0.00	1.00	10.00	0.00
2.00	7.00	0.00	2.00	8.00	0.00	2.00	9.00	0.00	2.00	10.00	0.00
3.00	7.00	3.10	3.00	8.00	3.00	3.00	9.00	3.25	3.00	10.00	3.25
4.00	7.00	4.80	4.00	8.00	4.90	4.00	9.00	5.25	4.00	10.00	5.25
...	...	...	...	...	...	...	...	...	...	...	...
13.00	7.00	10.30	13.00	8.00	10.75	13.00	9.00	11.45	13.00	10.00	11.45
14.00	7.00	10.10	14.00	8.00	11.00	14.00	9.00	11.60	14.00	10.00	11.60
15.00	7.00	9.85	15.00	8.00	10.45	15.00	9.00	11.00	15.00	10.00	11.50
16.00	7.00	10.15	16.00	8.00	10.50	16.00	9.00	12.20	16.00	10.00	12.70
...	...	...	...	...	...	...	...	...	...	...	...

To import a surface from a file in Pro/Engineer [2]:

1) choose *INSERT* → *ADVANCED* → *BLEND FROM FILE* → *SURFACE,*
2) create or select a coordinate system that the curve will reference,
3) choose file name (with extension *.ibl),
4) choose direction of adding material. Change the direction by click *FLIP*.

An example of generated surfaces from appointed points with fragment ASCII file is presented in Fig. 1.

Fig. 1. Surface generated from 20 NURBS curves with 29 control point and part of ASCII file

Rys. 1. Powierzchnia wygenerowana na podstawie 20 krzywych NURBS z 29 punktami kontrolnymi dla każdej krzywej oraz fragment pliku ASCII

The following tests were carried out to optimise the obtained shape of the surface:

a) reduction of number of control points by about ⅓ with invariable number of NURBS curves,
b) reduction of number of control points by about ½ with invariable number of NURBS curves,
c) reduction of number of NURBS curves about ⅓ with invariable number of control points,
d) reduction of number of NURBS curves about ½ with invariable number of control points.

The results of research are presented in Fig. 2.

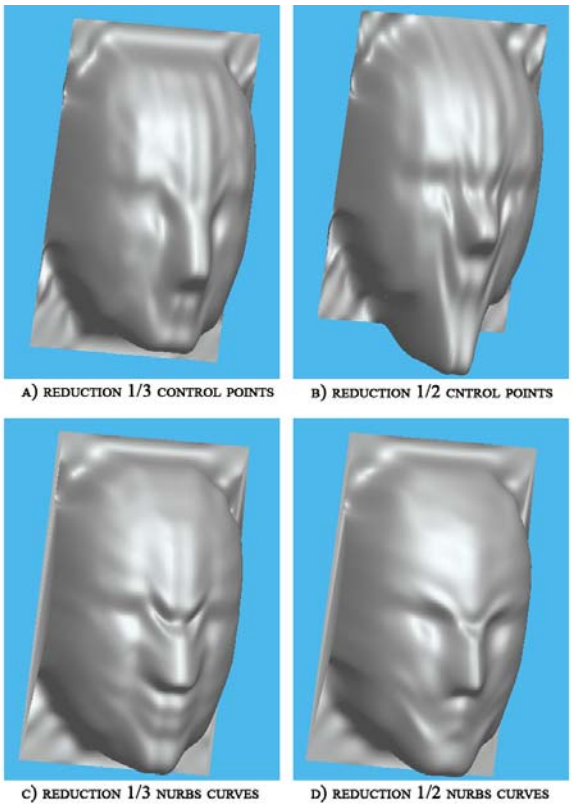

Fig. 2. Transformation of surfaces with reduction of number of control points or number of curves
Rys. 2. Transformacja powierzchni ze zmianą liczby punktów kontrolnych lub liczby krzywych

The decrease of the number of control points to 50% did not yield good results. The reduction of the number of curves that modelled the object by about 30% did not have a better influence on the simulated surface that was first simulated, then in case b) underwent big deformation. However, with the decreasing number of curves by about ½ it much more common model of the face was received than the first time. With 10 curves and 29 control points for each curve the surface of the face was received that could be transformed with modification and relocation of control points or curves.

4. Application of NURBS curves and surfaces to animation of human expression

The paper presents a novel method in facial modelling and animation based on human facial anatomy, control points are positioned and sample points of the non-uniform rational B-spline (NURBS) curves are associated with the facial mesh geometrically [3]. Facial muscle movements and expressions can be simulated by modifying the weights or moving the control points of the NURBS curves. The applications of NURBS curves are limited to creating surfaces, keyframing and setting motion paths. In addition, the method uses the NURBS curve to control a polygonal mesh.

Cubic curves are used in face modelling since they offer shape flexibility and also avoid unwanted oscillations. The positions of the control points are primarily decided by the locations where the simulated muscles are attached to facial bones. Fig. 3 shows the general positions of the control polygons of the NURBS curves.

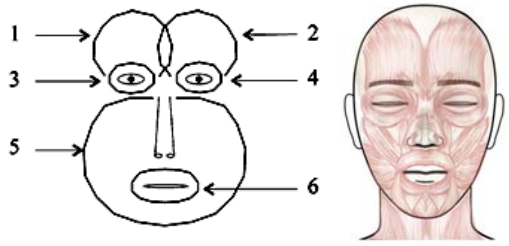

Fig. 3. Approximate locations of control polygons of NURBS curves with location of muscles [6] (Simulated muscles: 1, 2 – Frontalis, corrugator, 3, 4 – Orbicularis oculi, 5 – Zygomaticus major, levator labii superioris, traingularis, depressor anguli, depressor labii inferioris, mentalis, 6 – Orbicu-laris oris)

Rys. 3. Aproksymacja lokalizacji punktów kontrolnych krzywej NURBS wraz z rozmieszczeniem poszczególnych mięśni na ludzkiej twarzy [6] (Symulowane mięśnie: 1, 2 – Frontalis, corrugator, 3, 4 – Orbicularis oculi, 5 – Zygomaticus major, legator labii superioris, triangularis, depressor anguli, depressor labii inferioris, mentalis, 6 – Orbicularis oris)

Based on the distribution and movement of human facial muscles, vertices of the facial mesh can be separated into two top-level units, each with two sub-units:

a) vertices where muscles affect the eye region (*vertices on the forehead and vertices on the eyes*),
b) vertices where muscles affect the mouth region (*vertices on the cheeks and jaw and vertices on the mouth*).

To more realistically simulate the mutual influence of facial muscles and their gradual transition, fuzzy sets [5] can be used to assign any vertex a membership to a facial unit.

Figure 4 shows the simulation results. These are three primary expressions: happiness, anger and sadness. Wireframe and texture mapping are shown.

Three former expressions were simulated with weight changing and control point repositioning. Modelling facial movements using NURBS curves is a geometrical modelling method. The NURBS curves are used to control and coordinate the movement of vertices on the facial mesh.

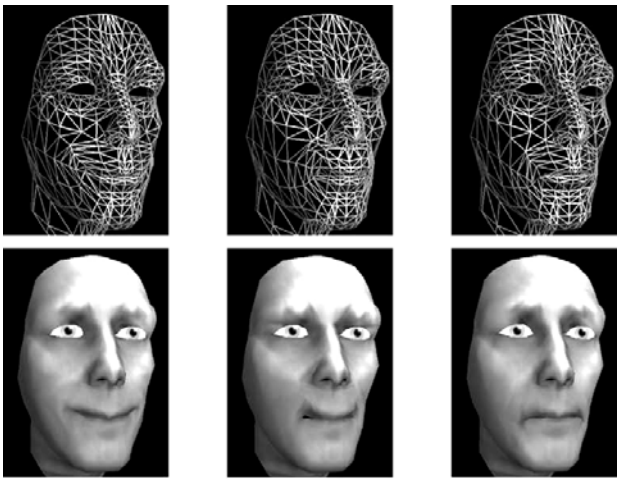

Fig. 4. Generated surfaces of human face (wireframe and texture mapping)
Rys. 4. Wygenerowane powierzchnie twarzy (obraz szkieletowy oraz odwzorowanie tekstury)

5. Summary

Non-uniform rational B-splines curves and surfaces, due to their useful geometric properties, are often applied in engineering drawing, computer graphics and wherever complicated curves with defined continuity is needed. NURBS curves were introduced in many CAD/CAM/CAE applications. Implementation of human facial surface and influence of reduction of the number of control points and the number of NURBS curves on the level of reality was presented. A novel method in facial modelling and animation based on human facial anatomy was presented, too. A unified mathematical form not only for representation of free-form curves and surfaces, but also for precise representation of close-form shapes such as lines, conics, quadrics, and surfaces of revolution has made NURBS curves popular and offered their wide applications spectrum.

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