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## CONTACT INTERACTION OF RESILIENT AND CYLINDRICAL DIES WITH INITIAL (RESIDUAL) TENSION

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### ZAGADNIENIE KONTAKTOWE W SPRĘŻYSTYCH I CYLINDRYCZNYCH FORMACH W OBECNOŚCI WSTĘPNYCH (SZCZĄTKOWYCH) NAPRĘŻEŃ

#### Abstract

The article deals with a mixed type task of measuring pressure of an elastic cylinder die upon a layer with initial stresses within the framework of linear elasticity theory. In general, the research was carried out for the theory of great initial (ultimate) deformations and different variants of the theory of small initial deformations with arbitrary structure of elastic potential.

*Keywords: contact interaction, resilient die, cylindrical die, initial tension, residual stresses*

#### Streszczenie

W artykule podjęto zagadnienie pomiaru ciśnienia w elastycznej formie cylindrycznej w obecności wstępnych naprężeń, wykorzystując teorię sprężystości. Ogólnie, badania były prowadzone na podstawie teorii dużych odkształceń początkowych oraz różnych wariantów teorii małych odkształceń początkowych przy dowolnej wartości potencjału sprężystości.

*Słowa kluczowe: zagadnienie kontaktowe, forma sprężysta, forma cylindryczna, naprężenie wstępne, naprężenie szczątkowe*

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The article deals with a mixed type task of measuring pressure of an elastic cylinder die upon a layer with initial stresses within the framework of linear elasticity theory [1–3]. Two cases will be viewed in the article: 1) the layer is placed on an elastic surface without frictioning; 2) the layer is fastened to an elastic surface. In general, the research was carried out for the theory of great initial (ultimate) deformations and different variants of the theory of small initial deformations with arbitrary structure of elastic potential. It is assumed that elastic potentials are two continuously differentiated functions of algebraic invariants of Green tensor deformation (the initial state of the layer remains uniform). The research is carried out within the coordinates of the initial deformed state  $y_i$  which are interrelated with the Lagrangian coordinates (natural state). Their interrelation may be described by the following equation  $y_i = \lambda_i = \lambda_i x_i$  ( $i = 1, 2, 3$ ) where  $\lambda_i$  is a prolongation quotient; it determines the change of the general initial state.

Besides, it is also assumed that influence of the die causes small disorders of the main elastic deformed state, for which the following conditions are true

$$S_0^{11} = S_0^{22} \neq 0; \quad S_0^{33} = 0; \quad \lambda_1 = \lambda_2 \neq \lambda_3$$

Values relevant to the elastic die are recorded in accordance with elastic theory designations. Values relevant to the previous state of elastic layer with the initial stresses as in [1–4].

It is assumed that both the elastic die and the layer are made of different isotropous, transversely isotropic or composite materials and they interact on one of the die's surfaces.

**Task setting:** taken that an elastic cylindrical die with radius  $R$ , height  $H$  and force  $P$ , presses onto an elastic layer which has initial stress that already exists before the contact. Thickness of the layer after the initial deformation will be marked  $h$ , whilst  $h_1$  is thickness of the layer before the deformation. This relation may be translated with the help of the following equation:  $h = \lambda_3 h_1$ . Force applied to the elastic die so that its free end-face is deformed uniformly with  $\varepsilon$  value in the direction of  $Oy_3$  axis. Surface beyond the contact area remains stress-free. Besides, tangential stress within the contact area of the layer and the die is ignored. In the system of circular cylindrical coordinates  $(r, \theta, z_i)$  we get boundary conditions:

For the elastic die end-face:  $z_i = n_i^{-1/2} H$

$$u_z = -\varepsilon; \quad \tau_{rz} = 0 \quad (0 \leq r \leq R) \quad (1)$$

For the elastic die boundary within the contact area:  $z_i = 0$

$$u_3 = u_z; \quad \tilde{Q}_{33} = \sigma_{zz} \quad \tilde{Q}_{3r} = \tau_{rz} = 0 \quad (0 \leq r \leq R) \quad (2)$$

For the elastic die boundary beyond the contact area  $z_i = 0$

$$\tilde{Q}_{33} = 0 \quad \tilde{Q}_{3r} = 0 \quad (R \leq r < \infty) \quad (3)$$

For the side face of the elastic die:  $r = R$

$$\sigma_{rr} = 0; \quad \tau_{rz} = 0 \quad (0 \leq z_i \leq H) \quad (4)$$

For the layer lower face which is placed on the elastic surface and is fastened to it

$$z_i = -\frac{\lambda_3 H_2}{\sqrt{n_i}} = -\frac{H_i}{\sqrt{n_i}}$$

$$u_3 = 0 \quad \tilde{Q}_{3r} = 0 \quad (0 \leq r < \infty) \quad (5)$$

$$u_3 = 0 \quad u_r = 0 \quad (0 \leq r < \infty) \quad (6)$$

where:

- $z_i = n_i^{-1/2} y_3, H_2$  – is thickness of the layer in its natural (not deformed) state,
- $H_1$  – thickness of the layer in its initial deformed state,
- $R$  – die radius; boundary conditions (1–6) together with the condition of balance of outer loadings

$$P = -2\pi \int_0^R \rho [\tilde{Q}_{33}]_{y_3=0} d\rho$$

define task setting concerning the contact interaction of the elastic die with initial layer under pressure which rests without friction on a surface or is fastened to it;  $n_i$  – solutions of the equation [4, formula (2), (12)].

Mode of deformation in elastic layer with initial stress will be defined with the help of harmonic functions by way of Henkel integrals. We should note that although Henkel – method does not provide exact solutions it lets us reduce the task to Fredholm equations which let us use the method of consecutive approximations for  $\lambda_1 > \lambda_{kr}$  satisfying the equation (2), (3), (5) and (6). Consequently, we may get components of potential vector and tensor of deformations in the case of axis-symmetrical type task

$$u_3 = \frac{1}{\omega_1} \int_0^\infty \eta^{-1} F(\eta) J_0(\eta\rho) d\eta - \frac{1}{\omega_1} \int_0^\infty \eta^{-1} F(\eta) G(\eta h) J_0(\eta\rho) d\eta \quad (7)$$

$$\tilde{Q}_{33} = \frac{2\omega_2}{R} \int_0^\infty F(\eta) J_0(\eta\rho) d\eta; \quad \tilde{Q}_{3r} = 0$$

where:

$$\omega_1 = \frac{\sqrt{n_i}}{m_i} \begin{cases} (s_1 - s_0)^{-1} & n_1 = n_2 \\ (s_2 - s_3)^{-1} & n_1 \neq n_2 \end{cases}$$

$$\omega_2 = \begin{cases} c_{44}(1+m_1)l_1(s-s_0) & n_1 = n_2 \\ c_{44}(1+m_1)l_1(s-s_3) & n_1 \neq n_2 \end{cases}$$

$$s_0 = \frac{1+m_2}{1+m_1}; \quad s_1 = \frac{m_1-1}{m_1}; \quad s_2 = \frac{m_2}{m_1} \sqrt{\frac{n_1}{n_2}}; \quad s_3 = s_0 \sqrt{\frac{n_1}{n_2}}; \quad s = s_0 \frac{l_2}{l_1}$$

$$G(\eta h) = 1 - q_i^{-1}(\eta h); \quad h = H_1 / R; \quad \varphi_i = 2\eta \frac{h}{\sqrt{n_i}}$$

$$q_i = \begin{cases} (sh2\varphi_1 + 2\varphi_1/s - s_0)(ch2\varphi_1 - 1)^{-1} & n_1 = n_2 \\ s(cth\varphi_2 - s_3cth\varphi_2) & n_1 \neq n_2; \\ \frac{(1-s)(s_0 - s_1) + (1-s_1)(s_0 - s)sh^2\varphi_1 + \varphi_1^2}{\varphi_1 - (1-s_1)sh\varphi_1ch\varphi_1} & n_1 = n_2 \\ \frac{(s_2 + ss_3) - (s + s_2s_3)sh\varphi_2ch\varphi_1 + (s_3 + ss_2)ch\varphi_2sh\varphi_1}{(s_2sh\varphi_1sh\varphi_2 - ch\varphi_1ch\varphi_2)} & n_1 \neq n_2 \end{cases} \quad (8)$$

In (8)  $q_1$  and  $q_2$  are similar to (5),  $q_3$  and  $q_4$  are similar to (6).

Equations (7) are received in a general form for compressed bodies as well as for those which are not under pressure; they include quotients  $n_i$ ,  $m_i$ ,  $c_{44}$ ,  $l_i$ . Values of the quotients are given in [4].

The received solutions are defined by way of lines with the help of very many constants. These constants are defined with regular and linear algebraic systems. Two cases were viewed in the article: the layer is placed on an elastic surface without frictioning and the layer is fastened to an elastic surface. The research was carried out for the question about the influence of initial stresses on the law of distribution of contact disorders in elastic layer with initial (residual) stresses. And in all cases the solutions were defined with the help of Henkel integrals.

#### References

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