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MARTA ŻAKOWSKA*

MESHING USING *P*-METHOD

TWORZENIE MODELI DYSKRETNYCH ZA POMOCĄ MODELI TYPU *P*

A b s t r a c t

Accuracy and efficiency of analysis carried out thanks to the FEM method mainly depends on the quality of discrete model. That is why there are many tools used in practice to create discrete models. The most often are used tools using models of *h*- and *p*-elements. The characteristic feature of discrete models using *h*-elements is approximation of continuous field by linear functions, or second order polynomials. In case of discrete models using *p*-elements approximation is carried out using higher order polynomials, which allows automatic and good mesh adaptation on these models.

In the article there were presented examples of creating discrete models using *h*- and *p*-elements and illustrated differences between discrete models created using both kinds of methods. Additionally, there was presented mesh adaptation process on discrete models made by *p*-elements.

Keywords: *p- and h-version of the finite element method mesh generation, p-method of finite element analysis*

S t r e s z c z e n i e

Efektywność i dokładność analiz prowadzonych z zastosowaniem MES zależy głównie od jakości modelu dyskretnego. Dlatego też jest wiele narzędzi służących do tworzenia modeli dyskretnych stosowanych w praktyce, z których najczęściej używane są narzędzia wykorzystujące modele elementów typu *h* oraz typu *p*. Cechą charakterystyczną modeli dyskretnych wykorzystujących elementy typu *h* jest to, że aproksymują ciągłe pole funkcjami liniowymi bądź też wielomianami stopnia drugiego. W przypadku modeli dyskretnych wykorzystujących elementy typu *p* aproksymacja przeprowadzana jest z zastosowaniem wielomianów wyższego rzędu, co pozwala na automatyczną i dobrą adaptację siatki na tych modelach.

W niniejszym artykule przedstawiono przykłady tworzenia modeli dyskretnych z użyciem elementów typu *h* i *p* oraz pokazano różnice występujące w modelach dyskretnych wykonanych za pomocą tych metod. Pokazano również proces adaptacji siatki na modelach dyskretnych wykonanych elementami typu *p*.

Słowa kluczowe: *MES, elementy typu h i typu p*

* Marta Żakowska, IV year student, Institute of Applied Informatics, Cracow University of Technology.

1. Introduction

Competition and costs of creating a new product force design engineers to analyse projects on designing stage instead of traditional prototyping and testing approach. Abilities of analysis by design engineers during the product development process strongly depend on the choice of tools that offer easy integration with CAD. One of this kind of tools is the finite element method. The origins of the finite element method reach back to the 1950s and 1960s, when it became very popular in engineering use. Fast development of FEM can be seen in following years together with computer industry development.

The accuracy and efficiency of analysis carried out using the FEM method mainly depends on the quality of discrete model. We can distinguish two main methods to create discrete models in practice: *p*- and *h*-method. Discrete models using *h*-elements approximate the continuous field by a function in the linear form (or second order polynomials) inside the element. However, in case of models using *p*-elements, the continuous field is approximated by higher order polynomials.

Additionally, the process of adaptation of the discrete model built using *p*-elements might be easily automated. *P*-methods have been worked out recently. However, only since the mid-1980s these methods have been used in commercial systems.

In the present article examples of creating discrete models using *h*- and *p*-elements are presented. Differences between discrete models created using *h*- and *p*-elements are illustrated. Additionally, the paper presents examples of discrete models made using *p*-elements, which were exposed to the process of mesh adaptation. An example used in this article is presented, thanks to the Pro/Mechanica – Pro/Engineer Wildfire 3.0.

2. Idea of FEM

The finite element method is one of discretization method of geometrical continuous arrangements, i.e. division of continuum into a finite number of subdomain. The idea of FEM method is based on modelling multiple constructions through their representation with the use of component elements of geometrically simple shape. The main assumption of the finite element method is that a continuous physical domain is divided (Fig. 1) into a finite

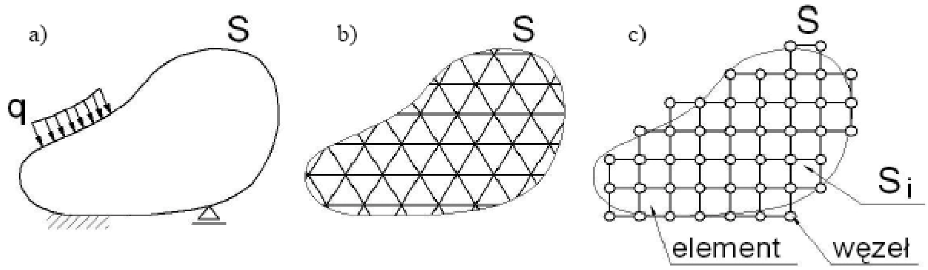

Fig. 1. Meshing of continuous model – transformation of finite elements into a set (mesh):
a) geometrical continuous model, b) discrete ideal model, c) discrete analytical model

Rys. 1. Dyskretyzacja modelu ciągłego – transformacja w zbiór (siatkę) elementów skończonych:
a) model geometryczny ciągły, b) model dyskretny idealny, c) model dyskretny obliczeniowy

number of elements which are joined in nodes. This results in creating a discrete model. It can be written in a simple form as

$$S = \sum_{1}^{n} S_i, \quad n \to +\infty \tag{1}$$

where:

S – field of model which is meshed,
S_i – field of element in discrete.

3. Meshing using *h*-method

There are two main ways to create discrete models in FEM method: using *h*-elements or *p*-elements. In both *h*- and *p*-methods the geometrical object under analysis is divided into a finite number of fragments called elements. In solids mechanics, continuous displacement field is approximated in the simplest case by the functions in linear form (*h*-elements) and displacements in the nodes of elements. The element displacement fields, $\{u\}$, can be written as

$$\{u\} = \sum_{i=1}^{n} N_i d_i = [N_i]\{d_i\} \tag{2}$$

where:

N_i – shape functions of order 1,
d_i – vector of unknown nodal displacements,
i – number of corner nodes.

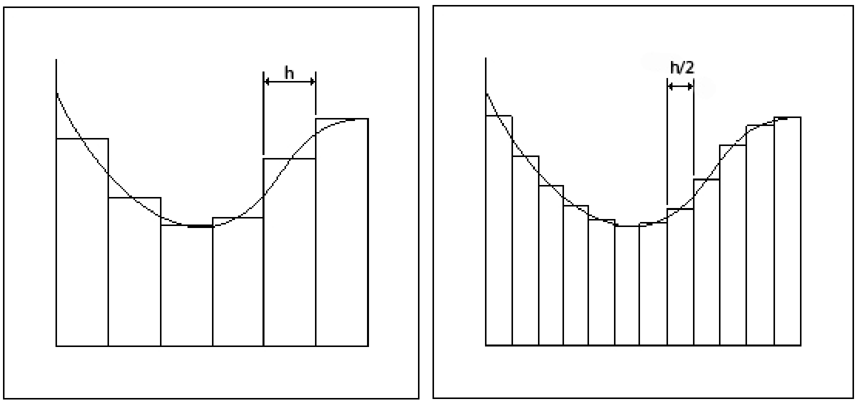

Fig. 2. Adaptation using *h*-elements: decreasing the sizes of telements

Rys. 2. Adaptacja typu *h*: zmniejszanie wymiarów elementów

Using the principle of virtual works, the approximate displacement fields and the material behaviour relations, the finite element equation can be written as

$$[k_{ii}]\{d_i\} = \{f_i\} \tag{3}$$

where:

k_{ii} – stiffness matrix,
f_i – load vector.

The continuous displacement field described by approximation functions cause a discretization error in the finite element solution. This error can be reduced by mesh adaptation, which can be done by decreasing the sizes of the elements and increasing their number.

4. Meshing using *p*-method

Meshing using *p*-method means to use elements with shape functions in a polynomial form and increase the order of this polynomial during calculation. There are a few solutions of adaptation using *p*-elements. One of them can be done by adding higher order shape functions into the existing shape functions present in the element. The element displacement fields in this case can be written as

$$\{u\} = \sum_{i=1}^{n} N_i d_i + \sum_{j=1}^{s} N_{h,j} d_{h,j} = [N_i]\{d_i\} + [N_h]\{d_h\} \quad (4)$$

where:

N_h – newly introduced shape functions,
d_h – the vector of p displacements,
s – the number of shape functions.

In case of elements with shape function in the second order polynomial, stresses in the element described by a linear dependence (Fig. 3) and adaptation using *p*-elements can be done by increasing the order of polynomial without decreasing the sizes of elements.

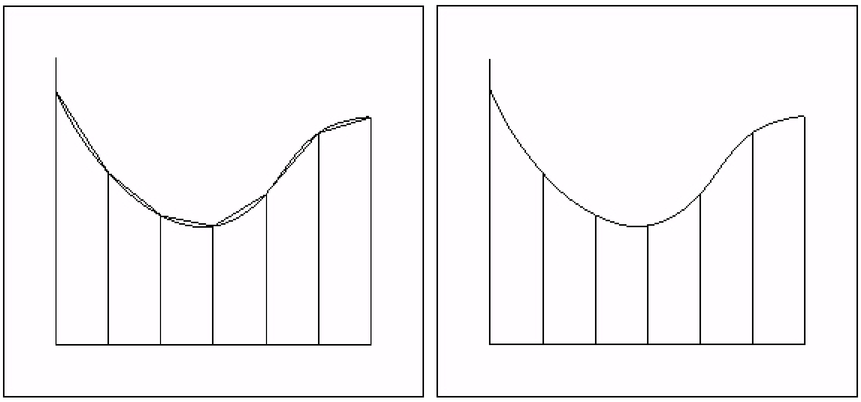

Fig. 3. Adaptation using *p*-elements: increasing order of polynomial shape function

Rys. 3. Adaptacja typu *p*: zwiększanie stopnia wielomianu funkcji kształtu

5. Discretization of geometrical models

The accuracy of results analysis depends on the elements shape of the mesh. *H*-method of FEA uses elements with the displacement field described by the first or second order polynomials. This limits the shapes of elements to simple geometric primitives (Fig. 2). Because of that, to minimise the discretization error and to get satisfactory results a mesh with a large number of small elements is usually created. Geometry must be extensively defeatured, geometry details removed, which are deemed unimportant for analysis, idealised and cleaned up before it can be meshed.

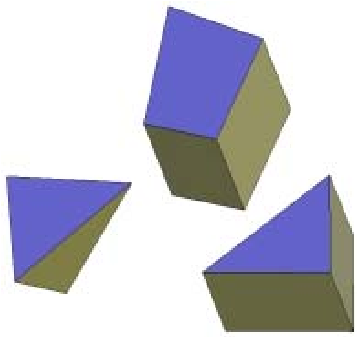

Fig. 4. Three-dimensional *h*-elements

Rys. 4. Trójwymiarowe elementy typu *h*

P-method of FEA uses elements of more complex shapes (Fig. 5) with the displacement field described by higher order polynomials (up to 9th order). This allows for larger elements that map precisely to geometry and correctly represent thin solid features. There is an advantage of automatic creating elements of acceptable shapes of the model in meshing process. Adaptation process of a discrete model using *p*-elements with little interference of the user is more effective.

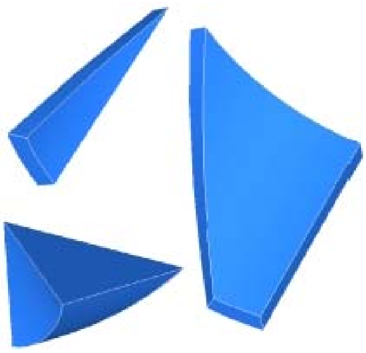

Fig. 5. Three-dimensional *p*-elements

Rys. 5. Trójwymiarowe elementy typu *p*

Examples of creating discrete models using *h*- and *p*-elements.

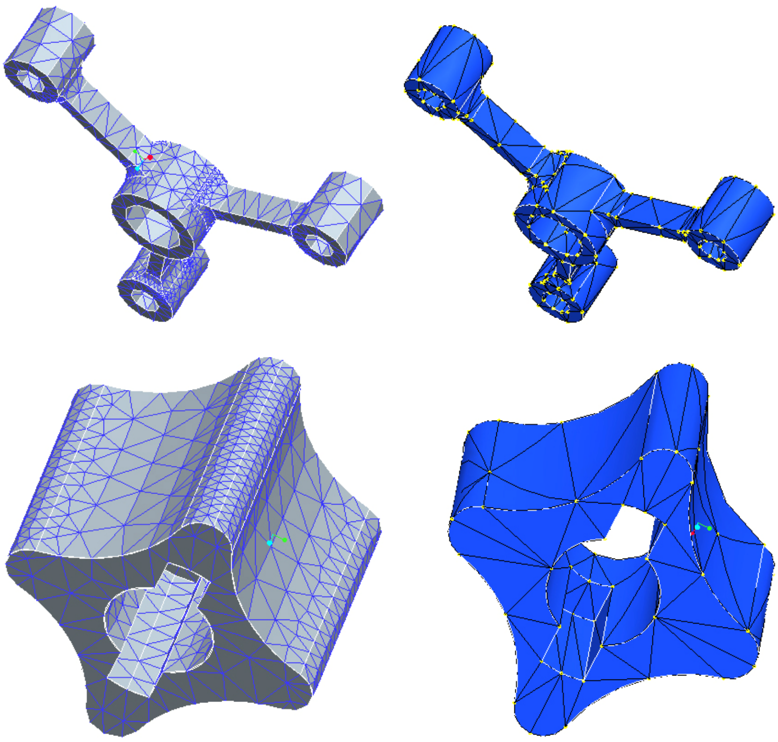

Fig. 6. Comparison of discrete models made using *h*- and *p*- method

Rys. 6. Porównanie modeli dyskretnych wykonanych z użyciem elementów typu *h* i *p*

Mesh adaptation of a model using *p*-elements can be carried out automatically in selected elements of the model. In Fig. 7 there are presented discrete models with *p*-elements with various order polynomials created as a result of mesh adaptation in Pro/Mechanica programme.

6. Conclusions

The aim of the paper was to compare discretization methods of geometrical models used in FEM systems. There were presented two basic methods of creating discrete models using *h*- and *p*-elements. There are also illustrated differences in models using both kinds of elements together with examples of adaptation process of discrete models using *p*-elements, made in Pro/Mechanica programme.

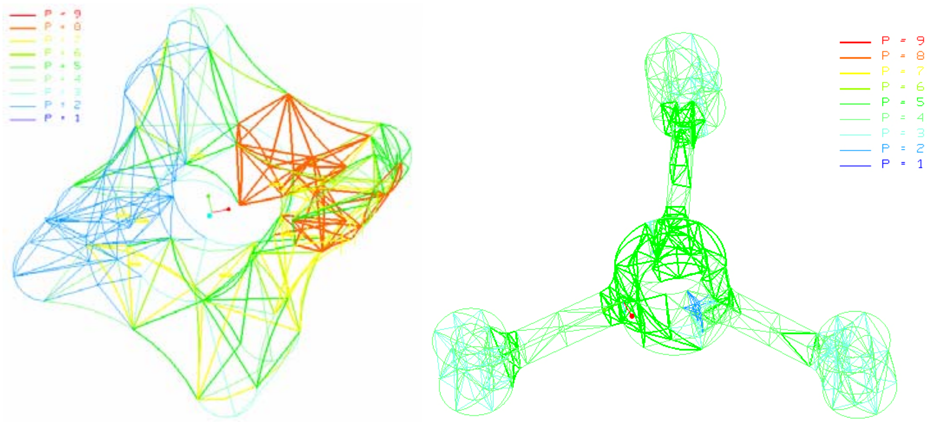

Fig. 7. Mesh adaptation of models using *p*-elements

Rys. 7. Adaptacja siatki modeli wykorzystujących elementy typu *p*

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