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FAILURE CRITERIA USED IN STRUCTURAL OPTIMISATION

KRYTERIA ZNISZCZENIA STOSOWANE W OPTYMALIZACJI KONSTRUKCJI

A b s t r a c t

Mechanisms of the loss of structural integrity, i.e. bifurcation, snap-through, first-cycle shakedown, burst pressure, brittle failure, etc. are discussed for internally and for externally pressurised domed closures onto cylindrical pressure vessels. The concept of a plastic load and its suitability for optimal design of a mild steel end closure is discussed with some experimental results backing the concept.

Keywords: *domes*, *buckling*, *plastic loads*, *first-cycle shakedown*, *burst*, *internal*, *external pressure*

S t r e s z c z e n i e

W artykule przedstawiono typowe mechanizmy utraty funkcjonalności zamknięć zbiorników cylindrycznych obciążonych ciśnieniem wewnętrznym lub zewnętrznym. Rozważono bifurkację, przeskok, sprężystą adaptację jednocyklową, wyczerpanie nośności, kruche zniszczenie itd. Wprowadzono też koncepcję obciążenia plastycznego oraz jego zastosowanie do optymalizacji dennic ze stali miękkiej. Przytoczono wyniki doświadczeń weryfikujących tę koncepcję.

Słowa kluczowe: *dennice*, *wyboczenie*, *obciążenia plastyczne*, *jednocyklowa adaptacja sprężysta*, *nośność*, *ciśnienie wewnętrzne*, *zewnętrzne*

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1. Introduction

Efficient design of domed enclosures onto cylindrical pressure vessels requires reliable knowledge about mechanisms leading to the loss of domes' structural integrity. For externally pressurised domes these mechanisms may include bifurcation buckling, snap--through, collapse or brittle failure. These forms of the loss of load bearing capacity are well understood and an adequate comparison between theoretical predictions of buckling loads and experimental data exists. It is then the designer's choice as to what safety factor to apply at the design stage in order to achieve a safe performance, Spence and Tooth [22]. The well-defined forms of the loss of load bearing capacity make room for their structural optimisation as illustrated, for example, by Błachut [1, 2].

Structural integrity of internally pressurised domes, on the other hand, where buckling is of no concern remains a subject of an on-going research effort. Safe design of such heads will usually be achieved with the use of the theory of plasticity. One of the approaches which has been advocated by Gerdeen [17], Kalnins and Updike [19], and Błachut [3] is based on the concept of plastic loads. However under single incremental loading the plastic loads, per se, do not constitute failure of the closure. Burst pressures, on the other hand, constitute the ultimate measure of structural integrity of domed ends. Burst pressures usually fall outside the normal operating envelope but their reliable prediction is of great practical/safety importance.

The current paper examines some of the mechanisms affecting the load bearing capacity of externally and internally pressurised domed closures onto cylindrical vessels with a view of their applicability to structural optimisation. The paper illustrates typical modes of failure, provides a brief comparisons between theory and experimental data. The role of 'first cycle shakedown' is outlined in the context of plastic loads (for elastic, perfectly plastic material, e.g., for dished ends from mild steel). Recent work into burst pressures and results of optimal shaping of dished ends subject to plastic load constraints, are discussed.

2. Failure mechanisms in externally pressurised domed closures

Consider a torispherical end closure made from three segments, i.e. spherical, toroidal (knuckle) and cylindrical flange, as sketched in Fig. 1. Assume that the wall thickness, t, remains the same in all three segments and that the radius of the spherical portion is R_s,

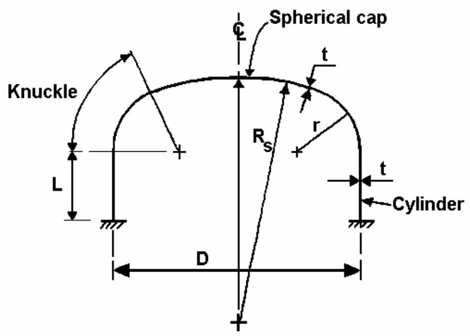

Fig. 1. Geometry of torispherical pressure vessel end closure
Rys. 1. Geometria typowej dennicy w zbiorniku ciśnieniowym

radius of the knuckle is r, length of the cylindrical flange is L, and dome's diameter is, D. Stability loss under static external pressure is usually associated with asymmetric bifurcation buckling or symmetric snap-through buckling. These two forms are rarely seen in practice, primarily due to initial geometrical imperfections. Hence, a localised collapse ends the load bearing capacity of a domed closure. A typical bifurcation pattern for a torispherical shell, obtained under laboratory conditions, is shown in Fig. 2. The torisphere was made using injection moulding of ABS plastic material, Błachut and Galletly [6]. The buckling, due to differential pressure between internal and external atmospheric values, happens suddenly and without any warning. Fig. 2 depicts a number of circumferential lobes which appear at asymmetric bifurcation buckling. There is no post buckling strength at all, and all domes tested without a supporting mandrel were completely destroyed. Another case of externally pressurised torisphere is depicted in Fig. 3. It illustrates an axisymmetric snap-through of a 0,2 m diameter CNC machined steel dome, Błachut et al [12]. Again failure was sudden, and with no warning. Table 1 provides a comparison of theoretical, i.e., numerical and experimental predictions. It can be seen here that theoretical predictions compare well with experimental data. It needs to be stressed that experimental data, for two cases quoted above, has been obtained under laboratory conditions – which in practice may be difficult to assure. As noted earlier, none of the above two modes, shown in Fig. 2 and in Fig. 3, can be seen in real components. This is illustrated in Fig. 4. It shows a collapsed, 0,5 m diameter, steel hemisphere which was manufactured using an industrial technique of metal spinning, Błachut and Galletly [7]. Here again the failure was sudden and with no prior warning.

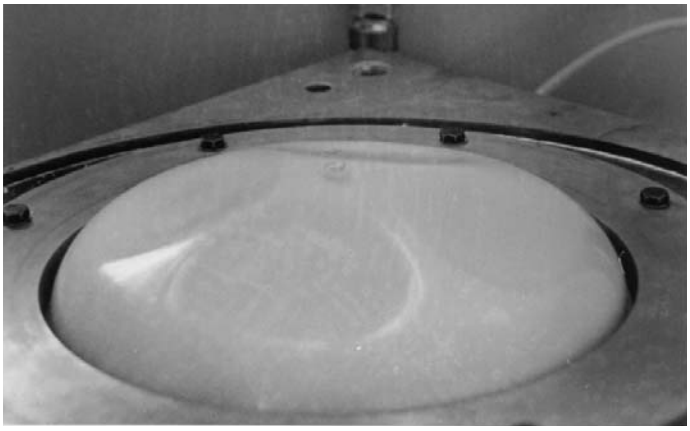

Fig. 2. View of an asymmetric bifurcation buckling mode, with n = 4 lobes, in an externally pressurised torisphere

Rys. 2. Widok asymetrycznej formy utraty stateczności dla dennicy poddanej działaniu ciśnienia zewnętrznego (widoczne n = 4 wklęśniecia)

Whilst theoretical and experimental predictions of static stability load seem to be adequate for geometrically perfect models, e.g., Refs [6, 12]. The same, by and large, does not apply to industrially manufactured domes, e.g., Ref. [7]. Strict shape tolerances have to be adhered to in order to ensure reasonable comparison between theoretical predictions and experimental results.

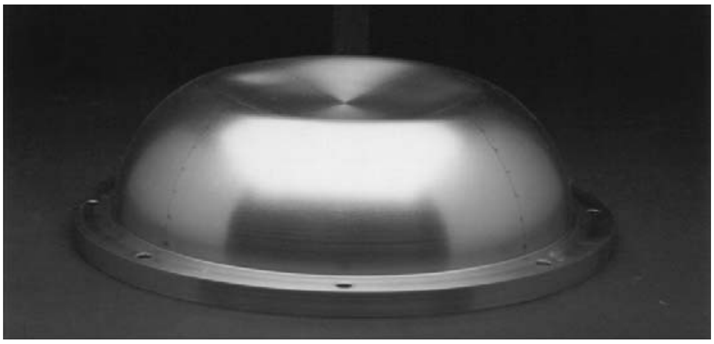

Fig. 3. View of symmetric snap-through buckling mode
in an externally pressurised torisphere
Rys. 3. Widok symetrycznej formy utraty stateczności (przeskoku) dla dennicy
obciążonej ciśnieniem zewnętrznym

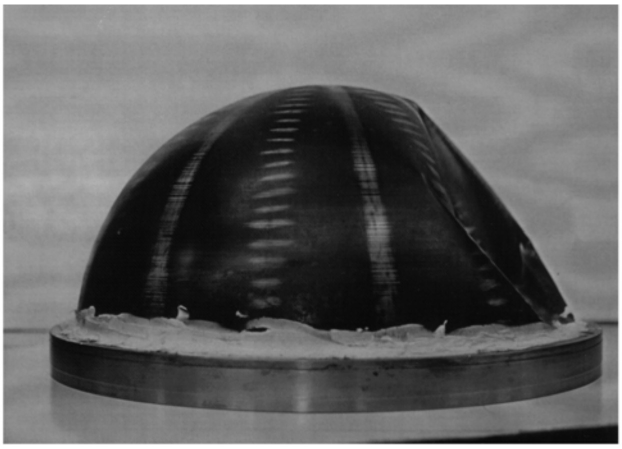

Fig. 4. Photograph of a collapsed hemisphere subjected to external pressure
Rys. 4. Fotografia stalowej powłoki półsferycznej po zniszczeniu
ciśnieniem zewnętrznym

The use of composite materials for manufacturing of domed closures can, in addition to bifurcation and snap-through, lead to the loss of structural integrity through brittle failure of the wall. Fig. 5 shows a torispherical dome made from 30 plies of carbon fibre material using a technique known as vacuum bagging – see Błachut et al [11]. All plies were cut from a single cloth of woven pre-preg. They were then draped into a female moulding tool before being cured. As it is seen from Fig. 5, the dome failed at the spherical-cap/knuckle junction through a large, brittle circumferential cracking. Composite materials offer a number of different ways for better tailoring of components' strength. One such way is the use of pieces of woven cloth to build a single ply into a multi-ply domed closure. Stacking sequence and orientation of these individual pieces can be manipulated in order to increase the magnitude of failure load. Fig. 6 illustrates a failed composite torisphere in which butt-jointing of pre-preg cloth pieces was used to make individual plies in a 30-ply torispheres, see Galletly and Błachut [16].

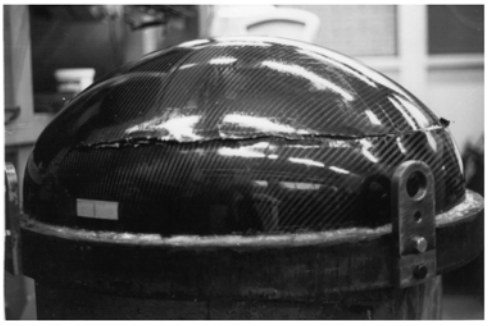

Fig. 5. Photograph of a failed torisphere made from CFRP and subjected to external pressure. Individual plies, 30 in total, were made from a single, non-cut, pre-preg woven carbon cloth

Rys. 5. Widok zniszczonej ciśnieniem zewnętrznym dennicy zbudowanej z kompozytu węglowego. Ścianka powłoki ma 30 ciągłych warstw włókna plecionego

Fig. 6. Photograph of a failed CFRP torispheres subjected to external pressure. Individual plies are assembled from butt-jointed pieces of woven pre-preg carbon cloth

Rys. 6. Fotografia zniszczonej ciśnieniem zewnętrznym dennicy z materiału kompozytowego. Poszczególne warstwy zostały złożone z małych segmentów włókna plecionego zestawionych „na styk"

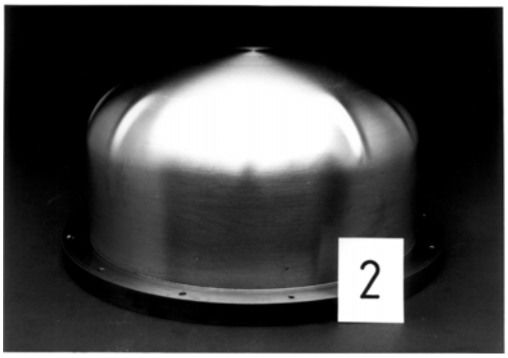

Fig. 7. Photograph of asymmetric bifurcation buckling in an internally pressurised steel torisphere

Rys. 7. Widok asymetrycznej formy wyboczenia powłoki stalowej obciążonej ciśnieniem wewnętrznym

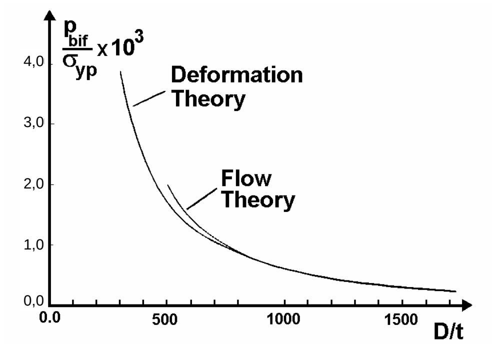

Fig. 8. Bifurcation buckling pressure versus the wall thickness for a mild steel torispheres (from Błachut et al. [8])

Rys. 8. Zależność ciśnienia bifurkacji od grubości ścianki w powłoce stalowej (wg Błachut et al. [8])

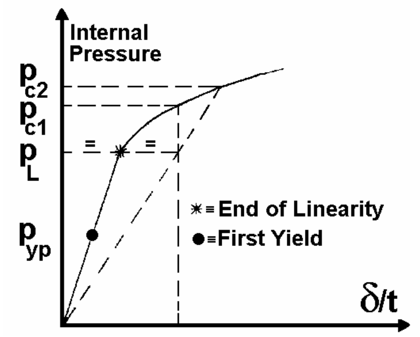

Fig. 9. Internal pressure versus apex deflection curve for a dished end. Also, definitions of first yield pressure, p_{yp}, and plastic pressures, p_{c1} and p_{c2}

Rys. 9. Zależność ugięcia szczytu dennicy od przyłożonego ciśnienia wewnętrznego. Definicje obciążeń p_{yp}, p_{c1} oraz p_{c2}

Table 1

Comparison of experimental and numerical predictions of bifurcation, snap-through, collapse and brittle cracking loads

Domed closure	r/D	R_s/D	D/t	L/D	Failure Pressure expt [MPa]	Failure Pressure theory [MPa]	Ref.
Torisphere	0,15	1,0	200	0,0	0,0497	0,0476	[6]
Torisphere	0,30	1,0	305	0,0	1,379	1,339	[12]
Hemisphere	–	0,5	935	0,0	0,386	0,326	[7]
Torisphere	0,24	0,6	87	0,03	9,45	9,13	[11]
Torisphere	0,24	0,6	86	0,03	10,69	8,49	[16]
Torisphere	0,15	1,0	518	0,30	0,67	0,80	[15]

The above examples illustrate different mechanisms leading to the loss of structural integrity. One common denominator in all of these examples is that the corresponding failure loads are the ultimate loads and they can serve as unique reference load levels for the design and optimisation purposes of domed closures subjected to external pressure, as discussed by Błachut [1, 2].

3. Internally pressurised steel domes

Internally pressurised end closures appear in far more varied applications than externally pressurised heads. The thinner heads are prone to buckling and there is a sizeable literature addressing bifurcation buckling of thin ends. The transition region between bifurcation buckling and axisymmetric yielding has also been actively researched. Studies to develop inelastic and limit analyses have also been carried out.

The results of 190 experimental tests on internally pressurised torispherical heads were collated and analyzed by Miller [20, 21]. Most analysed torispheres failed by asymmetric, elastic or inelastic, bifurcation buckling. In the paper attention was also paid to the plastic collapse and burst pressures. There is a continuing discussion to identify the best failure criterion for torispherical heads which do not buckle. Bifurcation buckling which occurs in internally pressurised torispherical closures is illustrated in Fig. 7. It shows a steel dome with a number of circumferential wrinkles – see Galletly et al. [15]. The magnitude of the bifurcation buckling pressure is dependent on the wall thickness and Fig. 8 shows this influence for heads with $R_s/D = 1$, $r/D = 0{,}10$, and for heads made from mild steel.

For mild steel torispheres with ratios $D/t \cong 50$, the failure mechanism has to be related to methods based on the theory of plasticity. The use of plastic loads, as recommended by Gerdeen [17] is a frequently adopted approach – see also Kalnins and Updike [19], Błachut [3], Kalnins and Rana [18]. The above papers adopt a common approach and evaluate plastic load, using a graphical representation of the relationship between internal pressure and apex deflection. This concept of plastic load is discussed next.

3.1. Plastic Loads

A typical plot of internal pressure, p, versus the crown deflection, δ, is shown in Fig. 9. As may be seen, there is no pressure which could definitely be called the plastic pressure, hence an arbitrary definition has been adopted for loadings p_{c1} and p_{c2}. The first one, i.e. p_{c1}, corresponds to the internal pressure at which the crown deflection equals twice the deflection point p_L in Fig. 9. Similarly, p_{c2}, corresponds to twice the elastic slope of the internal pressure versus the apex deflection, see Fig. 9 and Błachut [3]. A number of meridional profiles were tested experimentally up to burst and Fig. 10 shows a typical view of a tested dome. Comparison of numerical predictions with experimental data is given in Fig. 11. Plastic loads, for constant mass closures, varied by 80% for meridional profiles that

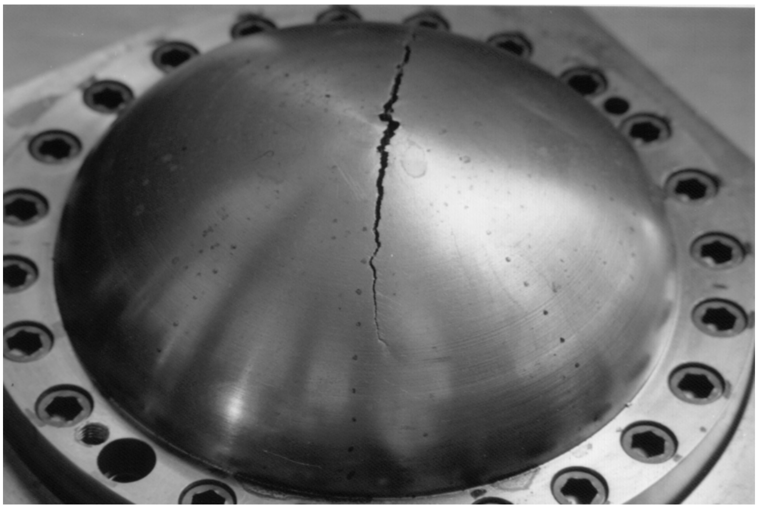

Fig. 10. View of a typical failure at the burst pressure
Rys. 10. Widok dennicy po zniszczeniu spowodowanym ciśnieniem wewnętrznym

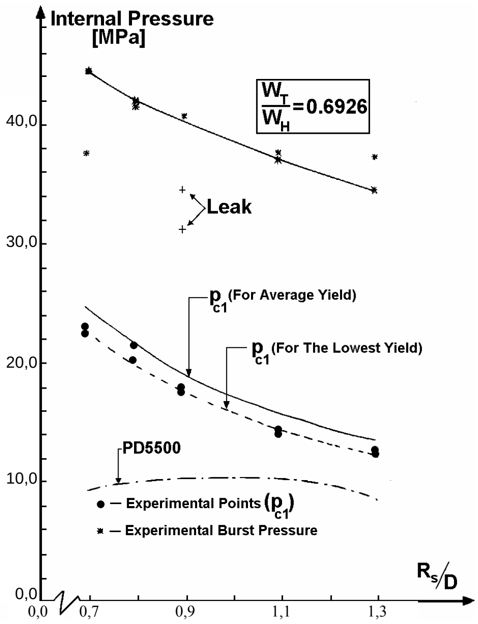

Fig. 11. Comparison of numerical and experimental plastic loads for constant mass domes. The burst pressures are also shown (from Błachut [3])

Rys. 11. Porównanie wartości numerycznych z wynikami doświadczalnymi dla powłok o stałej masie. Doświadczalne wartości ciśnienia zniszczenia przedstawiono na podstawie pracy Błachuta [3]

differed in height by no more than 5%. This sensitivity of plastic load to meridional shape will be examined later in this paper. As long as the meridional profile remains torispherical the quantities p_{c1} and p_{c2} adequately describe the plastic loads. Once the meridional profiles are sought outside the class of torispherical shapes, the above definitions can become inapplicable – see Błachut and Ramachandra [8]. Gerdeen [17] pointed out that the evaluation of the plastic load should employ a more objective and justified by physics criterion, i.e., for a pressure test the relevant quantities are pressure and change of internal volume. As a result a more universal definition, based on change of shell's volume, has been adopted in many studies. The corresponding plastic loads, i.e., p_{v1} and p_{v2}, are defined in Fig. 12. This approach was adopted by Błachut et al. [13] in order to obtain plastic loads for torispherical and ellipsoidal mild steel domes. Reasonable comparison of the experimental

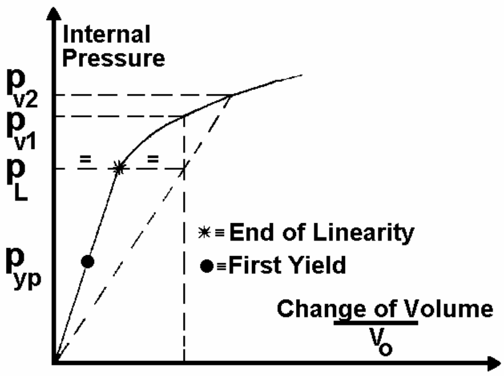

Fig. 12. Definition of plastic pressures p_{v1} and p_{v2}

Rys. 12. Definicje obciążeń plastycznych p_{v1} oraz p_{v2}

and numerical results was obtained. But if these domes were to operate in the post-yield range where cyclic load could be present, the role of the plastic load is less clear than the shakedown load. In the above study, the concept of a first-cycle shakedown pressure was adopted and the relative values of plastic and shakedown loads were compared. Plastic and shakedown loads for domes made from strain-hardening material were studied by Błachut et al. [14].

3.2. First cycle shakedown loads

Concept of the first cycle shakedown is outlined in Fig. 13. The first cycle shakedown pressure, p_{sh}, corresponds to the loading associated with point B_3 for which, after unloading, the state of stress reaches the failure envelope, i.e. point C in Fig. 13, see Błachut and Ramachandra [8]. A number of calculations were carried out and the results are summarised in Fig. 14. It is seen that the plastic load, p_{y1}, is a conservative estimate of the shakedown pressure for a range of torispherical profiles. It would, therefore, be sufficient to optimise end closures with respect to the plastic load and this would automatically ensure

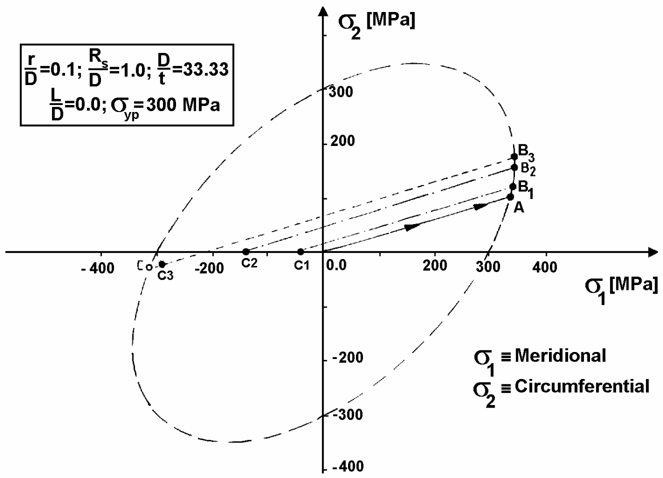

Fig. 13. The failure envelope for a torispherical dome made from an elastic perfectly plastic steel

Rys. 13. Ilustracja procedury prowadzącej do jednocyklowej sprężystej adaptacji stalowej dennicy idealnie sprężysto-plastycznej

that the ends adapt to the elastic state. However, some further studies are required for variable meridional profiles, i.e., for different than torispherical profiles and for ends with the variable wall thickness.

3.3. Burst pressures

While the role of plastic loads and first cycle shakedown loads for internally pressurised heads is still being research, it is the burst pressure which is of great value from a practical point of view as it gives an indication of the margin of safety. This is an important quantity, especially, at a design stage or at an emergency situation. A recent study into this problem

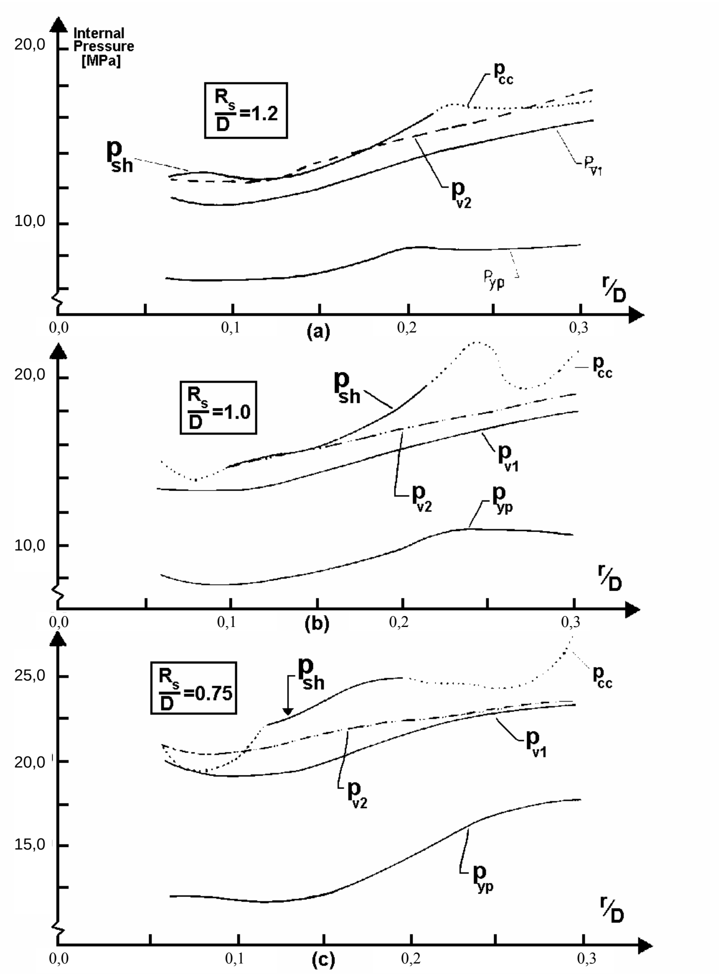

Fig. 14. Relative position of plastic loads and first cycle shakedown pressures, p_{sh}, for a range of torispherical shapes
Rys. 14. Wzajemna relacja obciążeń plastycznych oraz sprężystego obciążenia adaptacyjnego

is by Błachut and Vu [10]. It was postulated to use the true plastic strain, ε_p^u, corresponding to the ultimate tensile strength, UTS, for computing the magnitude of burst pressure. This is illustrated in Fig. 15 for the case of aluminium alloy AA6061-T6, where it is seen that $\varepsilon_p^u = 0{,}0815$. One needs not only the magnitude of plastic strain but also a place where this strain is to be attained. Based on the above criterion for admissible magnitude of plastic strain, it was further postulated to define the burst pressure as follows: p_{burst} is the pressure at which the equivalent plastic strain, PEQQ, reaches the ultimate plastic strain, ε_p^u, anywhere at the mid-surface of a structure.

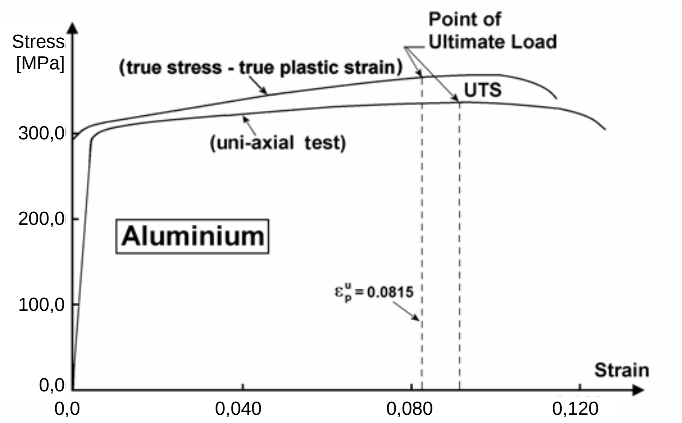

Fig. 15. Illustration showing how the ultimate plastic strain, ε_p^u,
was established for aluminium alloy 6061

Rys. 15. Schemat ukazujący sposób wyznaczania maksymalnego odkształcenia plastycznego ε_p^u w stopie aluminium 6061

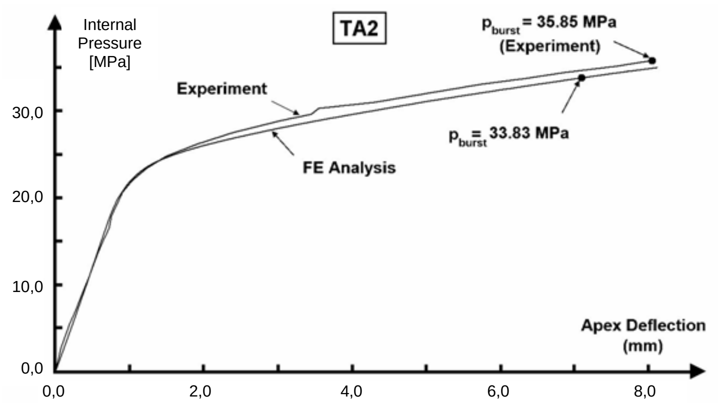

Fig. 16. Comparison of load versus apex deflection curves for internally pressurised aluminium torisphere ($r/D = 0{,}08$, $R_s/D = 1{,}0$ and $D/t = 26$). Location of experimental and computed burst pressures is also shown

Rys. 16. Porównanie ugięcia doświadczalnego z ugięciem wyznaczonym numerycznie dla dennicy aluminiowej ($r/D = 0{,}08$, $R_s/D = 1{,}0$ i $D/t = 26$). Pokazano również wartości teoretyczne i doświadczalne ciśnienia zniszczenia

A series of calculations have been performed for mild steel shallow spherical caps, tested previously for buckling by Błachut [5], and for torispherical heads. The latter were from mild steel and aluminium alloy. This was followed by experimental burst tests. Typical load versus change of internal volume for aluminium torisphere, T2, is reproduced here as Fig. 16. It is seen that the ratio $\left(p_{\text{burst}}^{\text{exptl}} / p_{\text{burst}}^{\text{theoretical}}\right)$ is 1,06 for this dome. Comparison with predictions by other known methods favours the proposed approach.

4. Optimisation of steel closures subject to plastic load constraint

Strong dependency of the plastic load to the meridional shape, as shown in Fig. 11, was behind the motivation for minimisation of the weight of a domed closure subject to a constraint on the magnitude of plastic load. Meridional profiles were sought within the class of (a) torispherical shells, (b) elliptical shells, (c) elliptical shells with variable wall thickness and (d) profiles described by a generalised ellipse and variable wall thickness. The fully stressed design was also considered. One of frequently used torispheres in practice is that of Korbbogen profile for the meridian (very close to a 2:1 ellipse). Due to this reason the optimal solutions were benchmarked against this geometry. It was shown

Fig. 17. Photograph of optimally shaped elliptical closures after the burst
Rys. 17. Fotografia dwóch nominalnie jednakowych powłok optymalnych po zniszczeniu

numerically, and verified experimentally, that approximately 31% mass reduction is possible through the appropriate choice of the meridional shape and variable wall thickness, Błachut [4]. Fig. 17 shows ruptured torispheres corresponding to the case (c). The ratio of the experimental burst pressure to the experimental plastic load was 1,7.

5. Conclusions

‘End of load bearing capacity’ in externally pressurised domed closures is sudden and the existing means of predicting these failures appear adequate. Hence the optimisation of externally pressurised end closures seems reasonable and feasible. The concept of plastic loads in internally pressurised domed closures seems to be fairly arbitrary, but it can offer a focal point for optimal design of thicker domes. Some preliminary calculations show that the plastic load can also be used as a conservative estimate of shakedown pressure in domes made from elastic perfectly plastic material, i.e., from mild steel. Results of a recent research into burst pressure offer another possibility, i.e., the use of burst pressure as the cost function, especially as this becomes rapidly affordable due the increasing speed of computations.

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