Abstract

Mechanisms of the loss of structural integrity, i.e. bifurcation, snap-through, first-cycle shakedown, burst pressure, brittle failure, etc. are discussed for internally and for externally pressurised domed closures onto cylindrical pressure vessels. The concept of a plastic load and its suitability for optimal design of a mild steel end closure is discussed with some experimental results backing the concept.

Keywords: domes, buckling, plastic loads, first-cycle shakedown, burst, internal, external pressure

Streszczenie


Słowa kluczowe: dennice, wybochenie, obciążenia plastyczne, jednocyklowa adaptacja sprężysta, nośność, ciśnienie wewnętrzne, zewnętrzne

*Dr hab. Jan Błachut, Mechanical Engineering, The University of Liverpool, United Kingdom.
1. Introduction

Efficient design of domed enclosures onto cylindrical pressure vessels requires reliable knowledge about mechanisms leading to the loss of domes’ structural integrity. For externally pressurised domes these mechanisms may include bifurcation buckling, snap-through, collapse or brittle failure. These forms of the loss of load bearing capacity are well understood and an adequate comparison between theoretical predictions of buckling loads and experimental data exists. It is then the designer’s choice as to what safety factor to apply at the design stage in order to achieve a safe performance, Spence and Tooth [22]. The well-defined forms of the loss of load bearing capacity make room for their structural optimisation as illustrated, for example, by Blachut [1, 2].

Structural integrity of internally pressurised domes, on the other hand, where buckling is of no concern remains a subject of an on-going research effort. Safe design of such heads will usually be achieved with the use of the theory of plasticity. One of the approaches which has been advocated by Gerdeen [17], Kalnins and Updike [19], and Blachut [3] is based on the concept of plastic loads. However under single incremental loading the plastic loads, per se, do not constitute failure of the closure. Burst pressures, on the other hand, constitute the ultimate measure of structural integrity of domed ends. Burst pressures usually fall outside the normal operating envelope but their reliable prediction is of great practical/safety importance.

The current paper examines some of the mechanisms affecting the load bearing capacity of externally and internally pressurised domed closures onto cylindrical vessels with a view of their applicability to structural optimisation. The paper illustrates typical modes of failure, provides a brief comparisons between theory and experimental data. The role of ‘first cycle shakedown’ is outlined in the context of plastic loads (for elastic, perfectly plastic material, e.g., for dished ends from mild steel). Recent work into burst pressures and results of optimal shaping of dished ends subject to plastic load constraints, are discussed.

2. Failure mechanisms in externally pressurised domed closures

Consider a torispherical end closure made from three segments, i.e. spherical, toroidal (knuckle) and cylindrical flange, as sketched in Fig. 1. Assume that the wall thickness, \( t \), remains the same in all three segments and that the radius of the spherical portion is \( R_s \).

Fig. 1. Geometry of torispherical pressure vessel end closure
Rys. 1. Geometria typowej dennicy w zbiorniku ciśnieniowym
radius of the knuckle is \( r \), length of the cylindrical flange is \( L \), and dome’s diameter is, \( D \). Stability loss under static external pressure is usually associated with asymmetric bifurcation buckling or symmetric snap-through buckling. These two forms are rarely seen in practice, primarily due to initial geometrical imperfections. Hence, a localised collapse ends the load bearing capacity of a domed closure. A typical bifurcation pattern for a torispherical shell, obtained under laboratory conditions, is shown in Fig. 2. The torisphere was made using injection moulding of ABS plastic material, Blachut and Galletly [6]. The buckling, due to differential pressure between internal and external atmospheric values, happens suddenly and without any warning. Fig. 2 depicts a number of circumferential lobes which appear at asymmetric bifurcation buckling. There is no post buckling strength at all, and all domes tested without a supporting mandrel were completely destroyed. Another case of externally pressurised torisphere is depicted in Fig. 3. It illustrates an axisymmetric snap-through of a 0.2 m diameter CNC machined steel dome, Blachut et al [12]. Again failure was sudden, and with no warning. Table 1 provides a comparison of theoretical, i.e., numerical and experimental predictions. It can be seen here that theoretical predictions compare well with experimental data. It needs to be stressed that experimental data, for two cases quoted above, has been obtained under laboratory conditions – which in practice may be difficult to assure. As noted earlier, none of the above two modes, shown in Fig. 2 and in Fig. 3, can be seen in real components. This is illustrated in Fig. 4. It shows a collapsed, 0.5 m diameter, steel hemisphere which was manufactured using an industrial technique of metal spinning, Blachut and Galletly [7]. Here again the failure was sudden and with no prior warning.

Fig. 2. View of an asymmetric bifurcation buckling mode, with \( n = 4 \) lobes, in an externally pressurised torisphere
Rys. 2. Widok asymetrycznej formy utraty stateczności dla dennicy poddanej działaniu ciśnienia zewnętrznego (widoczne \( n = 4 \) wklęśnieć)

Whilst theoretical and experimental predictions of static stability load seem to be adequate for geometrically perfect models, e.g., Refs [6, 12]. The same, by and large, does not apply to industrially manufactured domes, e.g., Ref. [7]. Strict shape tolerances have to be adhered to in order to ensure reasonable comparison between theoretical predictions and experimental results.
The use of composite materials for manufacturing of domed closures can, in addition to bifurcation and snap-through, lead to the loss of structural integrity through brittle failure of the wall. Fig. 5 shows a torispherical dome made from 30 plies of carbon fibre material using a technique known as vacuum bagging – see Blachut et al [11]. All plies were cut from a single cloth of woven pre-preg. They were then draped into a female moulding tool before being cured. As it is seen from Fig. 5, the dome failed at the spherical-cap/knuckle junction through a large, brittle circumferential cracking. Composite materials offer a number of different ways for better tailoring of components’ strength. One such way is the use of pieces of woven cloth to build a single ply into a multi-ply domed closure. Stacking sequence and orientation of these individual pieces can be manipulated in order to increase the magnitude of failure load. Fig. 6 illustrates a failed composite torisphere in which butt-jointing of pre-preg cloth pieces was used to make individual plies in a 30-ply torispheres, see Galletly and Blachut [16].
Fig. 5. Photograph of a failed torisphere made from CFRP and subjected to external pressure. Individual plies, 30 in total, were made from a single, non-cut, pre-preg woven carbon cloth

Rys. 5. Widok zniszczonej ciśnieniem zewnętrznym dennicy zbudowanej z kompozytu węglowego. Ścianka powłoki ma 30 ciągłych warstw włókna plecionego

Fig. 6. Photograph of a failed CFRP torisphere subjected to external pressure. Individual plies are assembled from butt-joined pieces of woven pre-preg carbon cloth

Rys. 6. Fotografia zniszczonej ciśnieniem zewnętrznym dennicy z materiału kompozytowego. Poszczególne warstwy zostały złożone z małych segmentów włókna plecionego zestawionych „na styk”

Fig. 7. Photograph of asymmetric bifurcation buckling in an internally pressurised steel torisphere

Rys. 7. Widok asymetrycznej formy wyboczenia powłoki stalowej obciążonej ciśnieniem wewnętrzny
The above examples illustrate different mechanisms leading to the loss of structural integrity. One common denominator in all of these examples is that the corresponding failure loads are the ultimate loads and they can serve as unique reference load levels for the design and optimisation purposes of domed closures subjected to external pressure, as discussed by Blačut [1, 2].

3. Internally pressurised steel domes

Internally pressurised end closures appear in far more varied applications than externally pressurised heads. The thinner heads are prone to buckling and there is a sizeable literature addressing bifurcation buckling of thin ends. The transition region between bifurcation buckling and axisymmetric yielding has also been actively researched. Studies to develop inelastic and limit analyses have also been carried out.

The results of 190 experimental tests on internally pressurised torispherical heads were collated and analyzed by Miller [20, 21]. Most analysed torispheres failed by asymmetric,
elastic or inelastic, bifurcation buckling. In the paper attention was also paid to the plastic collapse and burst pressures. There is a continuing discussion to identify the best failure criterion for torispherical heads which do not buckle. Bifurcation buckling which occurs in internally pressurised torispherical closures is illustrated in Fig. 7. It shows a steel dome with a number of circumferential wrinkles – see Galletly et al. [15]. The magnitude of the bifurcation buckling pressure is dependent on the wall thickness and Fig. 8 shows this influence for heads with $R_s/D = 1$, $r/D = 0.10$, and for heads made from mild steel.

For mild steel torispheres with ratios $D/t \geq 50$, the failure mechanism has to be related to methods based on the theory of plasticity. The use of plastic loads, as recommended by Gerdeen [17] is a frequently adopted approach – see also Kalnins and Updike [19], Blachut [3], Kalnins and Rana [18]. The above papers adopt a common approach and evaluate plastic load, using a graphical representation of the relationship between internal pressure and apex deflection. This concept of plastic load is discussed next.

3.1. Plastic Loads

A typical plot of internal pressure, $p$, versus the crown deflection, $\delta$, is shown in Fig. 9. As may be seen, there is no pressure which could definitely be called the plastic pressure, hence an arbitrary definition has been adopted for loadings $p_{c1}$ and $p_{c2}$. The first one, i.e. $p_{c1}$, corresponds to the internal pressure at which the crown deflection equals twice the deflection point $p_L$ in Fig. 9. Similarly, $p_{c2}$, corresponds to twice the elastic slope of the internal pressure versus the apex deflection, see Fig. 9 and Blachut [3]. A number of meridional profiles were tested experimentally up to burst and Fig. 10 shows a typical view of a tested dome. Comparison of numerical predictions with experimental data is given in Fig. 11. Plastic loads, for constant mass closures, varied by 80% for meridional profiles that...
differed in height by no more than 5%. This sensitivity of plastic load to meridional shape will be examined later in this paper. As long as the meridional profile remains torispherical the quantities $p_{c1}$ and $p_{c2}$ adequately describe the plastic loads. Once the meridional profiles are sought outside the class of torispherical shapes, the above definitions can become inapplicable – see Blachut and Ramachandra [8]. Gerdeen [17] pointed out that the evaluation of the plastic load should employ a more objective and justified by physics criterion, i.e., for a pressure test the relevant quantities are pressure and change of internal volume. As a result a more universal definition, based on change of shell’s volume, has been adopted in many studies. The corresponding plastic loads, i.e., $p_{v1}$ and $p_{v2}$, are defined in Fig. 12. This approach was adopted by Blachut et al. [13] in order to obtain plastic loads for torispherical and ellipsoidal mild steel domes. Reasonable comparison of the experimental

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**Fig. 11.** Comparison of numerical and experimental plastic loads for constant mass domes. The burst pressures are also shown (from Blachut [3])

Rys. 11. Porównanie wartości numerycznych z wynikami doświadczalnymi dla powłoek o stałej masie. Doświadczalne wartości ciśnienia zniszczenia przedstawiono na podstawie pracy Blachuta [3]

**Fig. 12.** Definition of plastic pressures $p_{v1}$ and $p_{v2}$

Rys. 12. Definicje obciążeń plastycznych $p_{v1}$ oraz $p_{v2}$
and numerical results was obtained. But if these domes were to operate in the post-yield range where cyclic load could be present, the role of the plastic load is less clear than the shakedown load. In the above study, the concept of a first-cycle shakedown pressure was adopted and the relative values of plastic and shakedown loads were compared. Plastic and shakedown loads for domes made from strain-hardening material were studied by Blachut et al. [14].

3.2. First cycle shakedown loads

Concept of the first cycle shakedown is outlined in Fig. 13. The first cycle shakedown pressure, $p_{sh}$, corresponds to the loading associated with point B for which, after unloading, the state of stress reaches the failure envelope, i.e. point C in Fig. 13, see Blachut and Ramachandra [8]. A number of calculations were carried out and the results are summarised in Fig. 14. It is seen that the plastic load, $p_{pl}$, is a conservative estimate of the shakedown pressure for a range of torispherical profiles. It would, therefore, be sufficient to optimise end closures with respect to the plastic load and this would automatically ensure that the ends adapt to the elastic state. However, some further studies are required for variable meridional profiles, i.e., for different than torispherical profiles and for ends with the variable wall thickness.

3.3. Burst pressures

While the role of plastic loads and first cycle shakedown loads for internally pressurised heads is still being research, it is the burst pressure which is of great value from a practical point of view as it gives an indication of the margin of safety. This is an important quantity, especially, at a design stage or at an emergency situation. A recent study into this problem
is by Blachut and Vu [10]. It was postulated to use the true plastic strain, $\varepsilon_{\text{true}}^p$, corresponding to the ultimate tensile strength, UTS, for computing the magnitude of burst pressure. This is illustrated in Fig. 15 for the case of aluminium alloy AA6061-T6, where it is seen that $\varepsilon_{\text{true}}^p = 0.0815$. One needs not only the magnitude of plastic strain but also a place where this strain is to be attained. Based on the above criterion for admissible magnitude of plastic strain, it was further postulated to define the burst pressure as follows: $p_{\text{burst}}$ is the pressure at which the equivalent plastic strain, PEQQ, reaches the ultimate plastic strain, $\varepsilon_{\text{true}}^p$, anywhere at the mid-surface of a structure.
A series of calculations have been performed for mild steel shallow spherical caps, tested previously for buckling by Blachut [5], and for torispherical heads. The latter were from mild steel and aluminium alloy. This was followed by experimental burst tests. Typical load versus change of internal volume for aluminium torisphere, T2, is reproduced here as Fig. 16. It is seen that the ratio $\left( \frac{p_{\text{burst}}^{\text{exp}}}{p_{\text{theoretical}}} \right)$ is 1,06 for this dome. Comparison with predictions by other known methods favours the proposed approach.
4. Optimisation of steel closures subject to plastic load constraint

Strong dependency of the plastic load to the meridional shape, as shown in Fig. 11, was behind the motivation for minimisation of the weight of a domed closure subject to a constraint on the magnitude of plastic load. Meridional profiles were sought within the class of (a) torispherical shells, (b) elliptical shells, (c) elliptical shells with variable wall thickness and (d) profiles described by a generalised ellipse and variable wall thickness. The fully stressed design was also considered. One of frequently used torispheres in practice is that of Korrbogen profile for the meridian (very close to a 2:1 ellipse). Due to this reason the optimal solutions were benchmarked against this geometry. It was shown numerically, and verified experimentally, that approximately 31% mass reduction is possible through the appropriate choice of the meridional shape and variable wall thickness, Blachut [4]. Fig. 17 shows ruptured torispheres corresponding to the case (c). The ratio of the experimental burst pressure to the experimental plastic load was 1.7.

5. Conclusions

‘End of load bearing capacity’ in externally pressurised domed closures is sudden and the existing means of predicting these failures appear adequate. Hence the optimisation of externally pressurised end closures seems reasonable and feasible. The concept of plastic loads in internally pressurised domed closures seems to be fairly arbitrary, but it can offer a focal point for optimal design of thicker domes. Some preliminary calculations show that the plastic load can also be used as a conservative estimate of shakedown pressure in domes made from elastic perfectly plastic material, i.e., from mild steel. Results of a recent research into burst pressure offer another possibility, i.e., the use of burst pressure as the cost function, especially as this becomes rapidly affordable due the increasing speed of computations.
References


