

ANDRZEJ DUDA, JERZY KAMIENSKI, JAN TALAGA\*

ON THE PUMPING EFFICIENCY  
IN AGITATED VESSEL  
EQUIPPED WITH DIFFERENT DUAL IMPELLERS

---

WYDAJNOŚĆ POMPOWANIA W MIESZALNIKU  
WYPOSAŻONYM  
W DWUSTOPNIOWY UKŁAD MIESZADEŁ

Abstract

The results of the pumping and circulation efficiencies for the different dual impeller configurations in agitated vessel were presented in the paper. The influence of the impeller spacing on the overall pumping flow rate and the circulation flow rate was evaluated. The values of the criteria flow number and the circulation flow number were discussed for different configurations of dual impellers.

*Keywords: dual impeller, pumping efficiency, circulation efficiency, impeller spacing*

Streszczenie

W niniejszej pracy przedstawiono wyniki obliczeń wydajności pompowania oraz wydajności cyrkulacyjnej dla różnych zestawów mieszadeł w mieszalniku dwustopniowym. Określono wpływ odległości pomiędzy dwoma mieszadłami na wartość sumarycznej wydajności pompowania oraz wydajności cyrkulacyjnej. Zaprezentowano wartości kryterialnej liczby wydajności pompowania oraz liczby wydajności cyrkulacyjnej, wyznaczone dla poszczególnych mieszadeł w układach dwustopniowych.

*Słowa kluczowe: mieszalnik dwustopniowy, wydajność pompowania, wydajność cyrkulacji, odległość mieszadeł*

---

\* Mgr inż. Andrzej Duda, prof. dr hab. inż. Jerzy Kamiński, dr inż. Jan Talaga,  
Instytut Aparatury Przemysłowej i Energetyki, Politechnika Krakowska.

## 1. Introduction

The agitated vessels, equipped with the dual impellers on common shaft, are applied in the slim apparatuses with the shape factor over unit. The filling height  $H$  in these agitators exceeds standard value equal to the tank diameter  $D$ . As the  $H \geq 1,2D$ , the second impeller has to be used [1, 2]. This solution provides sufficiently intensive liquid circulation in the whole volume of the vessel, as the result of the superposition of flows generated by two impellers. The efficiency of the circulation in agitated vessel describes the flow of liquid in the whole vessel volume while the pumping efficiency refers to the stream of liquid flowing across the blades of impellers, i.e. stream restricted to the impeller region. Furthermore, the circulation and the pumping efficiency are proportional to each other, as it was confirmed in several studies [3–5]. It can be found in these studies, for the dual agitated vessel with two Rushton turbines with six blades, the circulation efficiency is twice as big as the pumping efficiency. The detailed description of the pumping and the circulation efficiencies was published in [6–8]. However, the authors carried out the calculations for the agitated vessel with the single impeller. The geometry of the impellers and their configuration in dual packages have a huge influence on the liquid circulation as well as on the final effects of technological process carried out in the agitated vessel. When analyzing this problem, the spacing value between the impellers seems to be one of the most important geometric parameters. Not many studies have been made so far to characterize the influence of impellers' spacing on the pumping and the circulation efficiency in dual agitated vessel. The dual impellers were described for example in papers [9–12], but the measurements concerned the multiphase mixing processes mainly and the authors focused their attempts on the effects of process itself, such as end-dimension of the liquid drop or gas cavern or – in other cases – the mixing input power or the mixing time.

The aim of this work was to extend the current knowledge of the pumping and circulation efficiency and their dependence on the impeller spacing, for the different impeller types in dual configurations and the different spacing values between the impellers.

## 2. The experimental set-up

The calculations were carried out on the base of values of mean velocities in the agitated vessel. The radial and axial components of the velocity, measured in the vertical mid-plane of the vessel, were used for calculations. The measurements of the velocity were performed by means of Laser Doppler Anemometer. It was two component system of Dantec Measurement Technology, with single probe, build as a two-channel (four-beam), utilizing green (514,5 nm) and blue (488 nm) lines of Ar-Ion laser with power of 300 mW and operating in the backscattering mode.

The experiments were carried out in open cylindrical vessel,  $D = 286$  mm, equipped with a flat bottom and four baffles with the width of  $D/10$  each, equally spaced and extended to the bottom. The liquid level  $H$  was of  $1,5 \cdot D$ . The diameter of each used impeller was  $d = 1/3 \cdot D = 95$  mm and the impeller spacing  $\Delta h$  was changed in the range of  $(0,5-2) \cdot d$ . The off-bottom clearance was of  $0,5 \cdot d$ . The working liquid was dimethyl

sulphoxide ( $\rho_c = 1100 \text{ kg/m}^3$ ,  $\eta_c = 0,0023 \text{ Pa}\cdot\text{s}$ ). The rotation speed of the impeller was kept at constant value of  $5 \text{ s}^{-1}$ , as it guaranteed turbulent flow in the vessel.

Table 1

**The dual impeller combinations**

No of configuration	Lower impeller	Upper impeller
1	6-RT	6-RT
2	6-RT	6-PBT
3	6-PBT	6-PBT
4	6-PBT	6-RT
5	Lightnin A-315	6-RT
6	Chemineer HE-3	6-RT

The exact description and drawing of the experimental set-up can be found in [13]. Four impellers with the different geometries were used: the Rushton turbine with six blades – 6-TR, the six-Pitched Blade Turbine – 6-PBT and the hydrofoils: Lightnin A-315 and Chemineer HE-3. The combinations of dual impellers are given in Table 1.

### 3. The results and discussion

#### 3.1. The pumping efficiency

The pumping efficiency of the impeller is determined by the rate of the volumetric flow that enters  $Q_{in}$  or leaves  $Q_{out}$  the zone swept by rotating impeller, i.e. the flow rate that passes across the planes established by rotating blades of the impeller [1, 2, 6]. According to this, the two flow rates should be equal

$$|Q_{in}| = |Q_{out}| = |Q_p| \quad (1)$$

where  $Q_p$  is the pumping capacity of the impeller adequate to the capacity of the pump and  $Q_p$  is proportional to the rotational speed of the impeller and its diameter cubed

$$Q_p = K_p \cdot n \cdot d^3 \quad (2)$$

$K_p$  is the dimensionless flow number dependent on the impeller's geometry but independent of the Re number, for  $Re > 10^3$  [2]. The pumping capacity is more often described by the dimensionless flow number  $K_p$  as more universal and the Eq. (2) takes then the form

$$K_p = \frac{Q_p}{n \cdot d^3} \quad (3)$$

The flow numbers  $K_p$  for all examined dual configurations were evaluated on the base of flow rates  $Q_p$  calculated by integrating the profiles of the mean velocity components  $\overline{u_n}$ , normal to the planes  $A_s$  established by edges of the blades of the rotating impellers [1, 2, 6]

$$Q_{in} = \int_{A_S} \bar{u}_{n, in} dA_S \quad (4)$$

$$Q_{out} = \int_{A_S} \bar{u}_{n, out} dA_S \quad (5)$$

The calculation of  $Q_p$  was applied in this paper for six examined dual impeller configurations, so the integration of the velocity profiles was performed separately for the upper and the lower impeller. The overall pumping capacity is then the sum of the flow rates for the upper and lower impeller, calculated from Eqs (1), (4) and (5). Furthermore, the calculations were repeated this way for four values of  $\Delta h$ :  $0,5d$ ,  $1d$ ,  $1,5d$  and  $2d$  within each dual configuration. The values of the flow number  $K_p$  evaluated this way, versus  $\Delta h$  over  $d$  are given in Table 2.

For the small distances of  $\Delta h$  some dual impellers work as single impeller in agitated vessel. In the configuration of two 6-RT impellers, the flow number  $K_p$  for  $\Delta h = 0,5d$  is 0,8 and takes almost the same value for  $\Delta h = 2d$ , with increase by about 28% between 0,5 and  $2d$ . Similar tendency can be noticed for all configurations with the upper 6-RT impeller, and the increase of  $K_p$  occurs for  $\Delta h \approx 1,5d$ , as a possible effect of reinforcing of the circulations generated by the respective impellers. This increase depends on the type of used configuration and varies from 11,8% for configuration: 6-RT – Lightnin A-315, to 19,6% for configuration: 6-RT – 6-PBT. The results are partly in accordance with [6, 7], where the published flow numbers for single 6-RT, 6-PBT and HE-3 at similar experimental conditions were:  $K_p(6-RT) = 0,74$ ,  $K_p(6-PBT) = 0,73$  and  $K_p(HE-3) = 0,41$ . There is real difference only for 6-RT turbine, however it is still in accordance with [1].

Table 2

**The overall flow number  $K_p$  versus the ratio of impeller spacing  $\Delta h$  to impeller diameter**

$\Delta h/d$	No of the dual configuration					
	1	2	3	4	5	6
0,5	0,80	1,16	1,25	1,02	1,18	0,69
1	0,94	1,13	1,03	1,16	1,27	0,74
1,5	1,03	1,06	0,90	1,22	1,32	0,81
2	0,84	0,97	1,05	1,00	1,13	0,77

The trend is contrary if the 6-PBT turbine is the upper impeller (configuration of two 6-PBT and with lower 6-RT impeller). The impellers reinforce each other at small distances, so that the  $K_p$  number takes the highest values for  $\Delta h = 0,5d$ . This can be also confirmed, when looking at the results [6, 7]. As the impellers are being drawn aside, the flow number is getting decreased. For the configuration No. 2 with lower 6-RT and upper 6-PBT turbine, this decrease is constant and almost linear as far as  $\Delta h = 2d$ , and the percentage drop is of 16,4%. For the 6-PBT turbines, the  $K_p$  number indicates the lowest value for  $\Delta h \approx 1,5d$ . But it is opposite situation for configurations with upper 6-RT impeller. I can point to damping effect between impellers, occurring for  $\Delta h \approx 1,5d$ .

### 3.2. The circulation efficiency

The pumping efficiency described by the flow number  $K_p$  relates to circulation efficiency evaluated according to equation [6]

$$K_c = \frac{Q_c}{n \cdot d^3} \quad (6)$$

The  $Q_c$  is here the circulation flow rate referred to the whole agitated vessel. The  $Q_c$  includes the flow entrained by the impeller discharge, and therefore takes usually higher values than the  $Q_p$  [6]. This will be verified in the last part of this work. The  $K_c$  number is in Equation (6) the dimensionless circulation flow number, independent of the Re number, for  $Re > 10^3$ . The circulation flow  $Q_c$  in the agitated vessel can be defined in axial  $z$  and radial  $r$  direction respectively, and then the Equation (6) takes the form

$$K_z = \frac{Q_z}{n \cdot d^3} \quad (7)$$

$$K_r = \frac{Q_r}{n \cdot d^3} \quad (8)$$

Assuming the axial symmetry in the agitated vessel, the total axial-radial circulation flow rate  $Q_c$  is equal both to the maximum value of  $Q_r$  and the maximum value of  $Q_z$  [6]

$$Q_c = \max_r \{ |Q_r| \} = \max_z \{ |Q_z| \} \quad (9)$$

Consequently, the total axial-radial circulation flow number  $K_c$  is defined

$$K_c = \max_r \{ |K_r| \} = \max_z \{ |K_z| \} \quad (10)$$

Moreover, assuming additional angular symmetry in the agitated vessel [6, 7, 8], the circulation flow rates in axial  $Q_z$  and radial  $Q_r$  direction can be calculated by integrating of the mean velocity profiles, at the different vertical ( $r_i$ ) and horizontal ( $z_i$ ) cross-sections

$$Q_z(z_i) = 2 \cdot \pi \cdot \int_{r_1}^{r_2} (\bar{u}_z)_{z_i} \cdot r \, dr \quad (11)$$

$$Q_r(r_i) = 2 \cdot \pi \cdot r_i \cdot \int_{z_1}^{z_2} (\bar{u}_r)_{r_i} \, dz \quad (12)$$

The Equation (12) was applied in this study to calculate the circulation flow rates  $Q_c$  for the six examined dual impellers. It was decided to apply this methodology for the dual

impeller on the base of following assumption: if the circulation loops generated by dual impeller have similar shape and the centres of loops coincide, then the impellers should yield one maximum value of  $Q_c$ . The analysis of the circulation loops was made on the basis of own results of distribution of the axial-radial mean velocities in  $r$ - $z$  plane of the vessel. The example vector plot is shown in Fig. 1. According to Eq. (9), only the radial component of the mean velocity was used here for integration process. It was due to the number of examined configurations (24) and was justified by the time saved on the calculations. The position of the vertical cross-sections along  $r$  coordinate in the agitated vessel is shown in Fig. 2, as well as the selected mean velocity profiles (radial component).

If the integrating process is repeated at each selected cross-section  $r_i$ , both the positive and the negative values of  $Q_c$  are calculated, depending on the orientation of vector of the radial velocity component. Therefore, the  $Q_c$  was calculated at different distances from the vessel axis, along the radius of the vessel, towards its wall. On the base of evaluated  $Q_c$ , the positive and the negative values of  $K_c$  were calculated, using Eq. (6). An example of  $K_c$  values versus dimensionless  $2r/D$  is presented in Fig. 3. The maximum value of  $K_c$  (corresponding to maximum value of  $Q_c$ ) can be found easy by approximation of evaluated points, using the function with the maximum correlation coefficient, as was shown in the Figure 3. The approximation algorithm was performed with use of *Table Curve* software.

The circulation flow numbers  $K_c$  evaluated this way for all examined dual configurations and  $\Delta h = (0,5-2) d$ , are collected in Table 3. Additionally, the values of  $K_c$  versus impeller spacing  $\Delta h$  over the impeller diameter  $d$  are presented on the graph – Fig. 4. Moreover, the values of  $K_c$  and  $K_p$  were compared to each other for the respective configurations and the spacing values  $\Delta h$ . The results are presented in Table 4. When analysing the graph on Fig. 4, the circulation flow numbers  $K_c$  show similar trend as the  $K_p$  numbers do. All of the configurations with the upper 6-RT turbine indicate distinct increase at  $\Delta h/d = 1,5$ . The magnitude of this increase varies, depending on the kind of the dual impellers, from 19% – for two 6-RT turbines up to 69% – for package: lower 6-PBT, upper 6-RT turbine. The exception is the configuration with the lower 6-RT and the upper 6-PBT impeller. Its  $K_c$  number indicates increase by about 33% at  $\Delta h/d = 1,5$ , while the  $K_p$  number is constantly decreasing, by about 8,6% at  $\Delta h/d = 1,5$ . This effect of  $K_c$  increase in all mentioned dual impellers can be explained as a possible result of reinforcing of the circulation loops in the vessel, that occurs just for  $\Delta h = 1,5d$ . There is another feature of  $K_c$  for all of the above described dual impellers. Once  $\Delta h/d$  exceeds 1,5 and reaches value of 2, the  $K_c$  number is getting decreased. The biggest drop of  $K_c$  can be found for the configuration: lower HE-3 and upper 6-RT impeller, and it is about 11%, while the lowest decrease is for two 6-RT turbines – 3,7%. However, this last configuration looks rather average, if to compare its  $K_c$  number to other duals with upper 6-RT turbine, it indicates the most stabile magnitude of  $K_c$  versus  $\Delta h$ , as its characteristics is the most flat. This feature can be recognized as an advantage in the industrial applications of two Rushton turbines. The configuration with two 6-PBT impellers is the only one, that indicates evident decrease of  $K_c$ , as the spacing  $\Delta h$  is growing. However, at  $\Delta h/d = 1$  the slight increase of 1,6% can be noticed, there is the evident decrease of  $K_c$  for bigger values of  $\Delta h/d$ , that takes the total value of 31%. It would point to damping effect, that occurs between impellers from the spacing value  $\Delta h/d = 1$ .

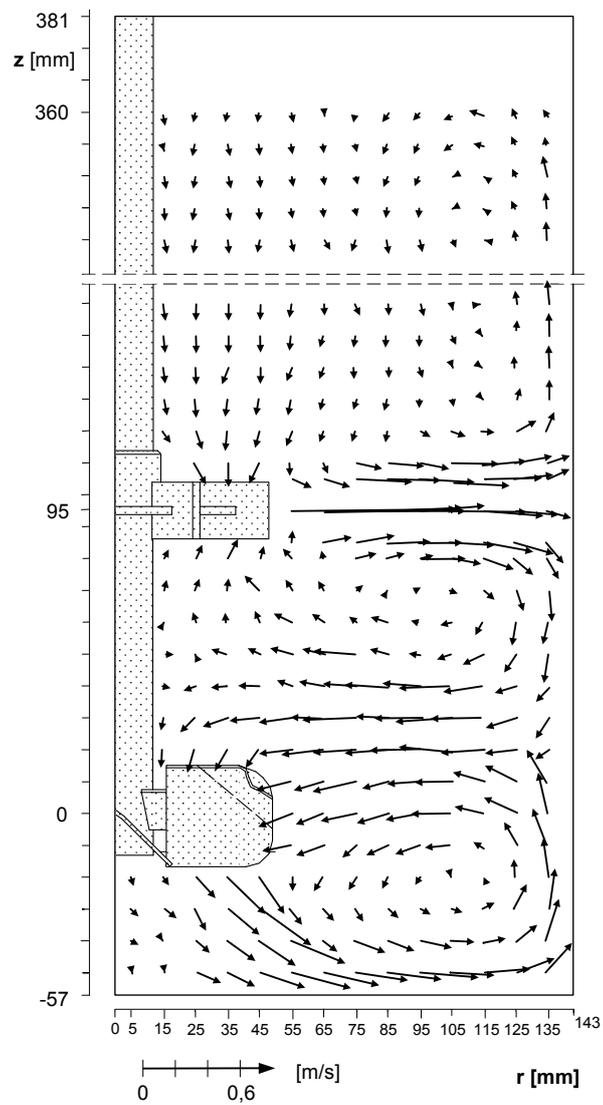


Fig. 1. The distribution of the axial-radial mean velocities in  $r$ - $z$  plane of the vessel for chosen dual configuration: lower A-315 and upper 6-RT impeller,  $\Delta h = 1d$

Rys. 1. Rozkład z wypadkowych promieniowych i osiowych składowych prędkości dla wybranej konfiguracji mieszadeł: dolne A-315, górne 6-RT,  $\Delta h = 1d$

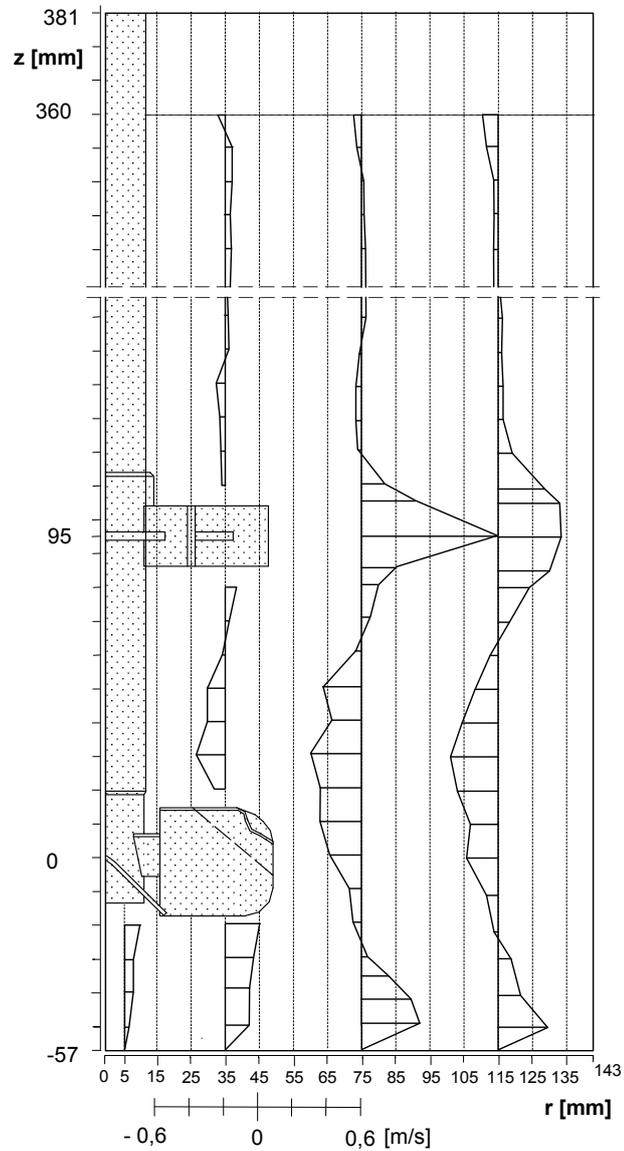


Fig. 2. The vertical cross-sections along  $r$  axis in the agitated vessel with the selected mean velocity profiles for the radial component, for chosen dual configuration: lower A-315 and upper 6-RT impeller,  $\Delta h = 1d$

Rys. 2. Linie przekrojów całkowania wraz z wybranymi profilami promieniowej składowej prędkości średniej dla wybranej konfiguracji mieszadeł: dolne A-315, górne 6-RT,  $\Delta h = 1d$

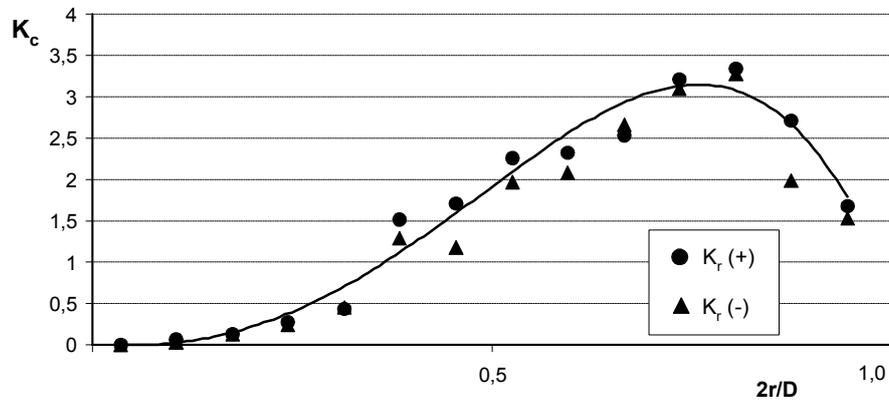


Fig. 3. The circulation flow number  $K_c$  versus dimensionless distance from the vessel axis  $2r/D$ . The dual configuration: lower A-315 and upper 6-RT impeller,  $\Delta h = 1d$

Rys. 3. Liczba wydajności cyrkulacyjnej  $K_c$  w funkcji bezwymiarowej odległości od osi mieszalnika  $2r/D$ . Zestaw mieszadeł: dolne A-315, górne 6-RT,  $\Delta h = 1d$

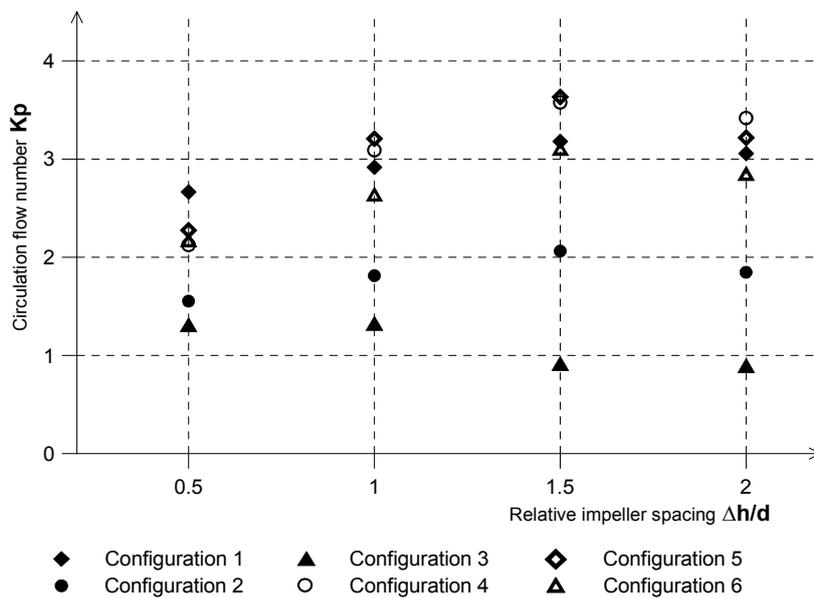


Fig. 4. The circulation flow number  $K_c$  versus dimensionless distance between impellers  $\Delta h$

Rys. 4. Liczba wydajności cyrkulacyjnej  $K_c$  w funkcji bezwymiarowej odległości między mieszadłami  $\Delta h$

Table 3

**The circulation flow number  $K_c$  versus the ratio of impeller spacing  $\Delta h$  to impeller diameter**

$\Delta h/d$	No of the dual configuration					
	1	2	3	4	5	6
0,5	2,67	1,55	1,31	2,12	2,28	2,15
1	2,92	1,81	1,33	3,09	3,21	2,62
1,5	3,18	2,06	0,92	3,58	3,63	3,08
2	3,06	1,85	0,90	3,42	3,22	2,83

All calculated values of  $K_c$  look quite reasonable, if to compare them to the results published in [6, 8]. The three values of  $K_c$ , for 6-PBT turbine, 6-RT turbine and for Chemineer HE-3, were evaluated at similar experimental conditions and the impeller geometries except HE-3 impeller (diameter was of 0,5 of the tank diameter). The  $K_c$  values for 6-PBT and 6-RT turbines were of 1,38 and 2,1 respectively. If to compare these values with the configurations of double 6-RT and 6-PBT turbines, there is one conclusion coming. From the hydrodynamics point of view in the agitated vessel, at small distances of  $\Delta h$  two impellers work as single one. The increase of  $\Delta h$  causes an increase of the  $K_c$  as far as some upper limit value that is probably the sum of the circulation numbers for two respective impellers, as if they worked separately. However, this claim has to be verified by evaluation of the circulation flow numbers  $K_c$  for four single impellers used in this work, i.e.: 6-PBT turbine, 6-RT turbine, Lightnin A-315 and Chemineer HE-3.

Finally, the  $K_c$  values were referred to the respective values of  $K_p$  in order to check the magnitude of the interval between them. The values of the ratio of  $K_c/K_p$  are presented in Table 4. The configuration of the two 6-PBT impellers yields the  $K_c$  number that is three times bigger than  $K_p$  (column 1), for all four distances  $\Delta h$ . It is comparable with [6–8], where this ratio was of 2,83. But it cannot be said about the configuration of two 6-PBT turbines. The  $K_c$  value is in this case almost of the  $K_p$  (column 3), except the distance  $\Delta h = 1$ . It does not compare to [6–8], where the  $K_c$  number exceeds  $K_p$  number by 1,89.

Table 4

**The comparison of  $K_c / K_p$  ratio for varied impeller spacing  $\Delta h$  relative to impeller diameter**

$\Delta h/d$	No of the dual configuration					
	1	2	3	4	5	6
0,5	3,32	1,33	1,05	2,09	1,89	3,12
1	3,10	1,60	1,29	2,78	2,54	3,54
1,5	3,09	1,95	1,01	2,93	2,76	3,81
2	3,62	1,91	0,85	3,42	2,84	3,67

#### 4. Conclusion

The selection of the most advantageous dual configuration in the agitated vessel, considering the mixing hydrodynamics characterized by the circulation and the flow numbers, is not an easy issue. The final solution depends both on the impellers' geometry and the spacing between them, as it was shown in the paper. Nevertheless, if taking only  $K_c$

numbers into consideration as this more important than  $K_p$  number [6], then the most advantageous seems to be dual configuration with upper 6-RT impeller and lower A-315 and 6-PBT turbine.

### Symbols

$A_s$	– plane established by edges of impeller in motion	$[m^2]$
$d$	– impeller diameter	$[m]$
$D$	– internal diameter of the tank	$[m]$
$\Delta h$	– impeller spacing	$[m]$
$H$	– liquid height in the agitated vessel	$[m]$
$K_c$	– circulation flow number	
$K_p$	– flow number	
$K_r$	– circulation flow number referred to $Q_r$	
$K_z$	– circulation flow number referred to $Q_z$	
$n$	– impeller rotational speed	$[s^{-1}]$
$Q_c$	– circulation flow rate	$[m^3/s]$
$Q_{in}$	– flow rate entering the impeller zone	$[m^3/s]$
$Q_{out}$	– flow rate leaving the impeller zone	$[m^3/s]$
$Q_p$	– pumping flow rate crossing the impeller zone	$[m^3/s]$
$Q_r$	– circulation flow rate in radial direction	$[m^3/s]$
$Q_z$	– circulation flow rate in axial direction	$[m^3/s]$
$r$	– vessel radius	$[m]$
$r_i$	– radius of integration	$[m]$
Re	– Reynolds number	
$\bar{u}_{n,in}$	– mean velocity component, normal to the plane $A_s$ referred to the $Q_{in}$	$[m/s]$
$\bar{u}_{n,out}$	– mean velocity component, normal to the plane $A_s$ referred to the $Q_{out}$	$[m/s]$
$\bar{u}_r$	– radial component of the mean velocity in vessel	$[m/s]$
$\bar{u}_z$	– axial component of the mean velocity in vessel	$[m/s]$
$z$	– vessel axis	$[m]$
$z_i$	– z-level of integration	$[m]$
$\rho_c$	– density of working liquid	$[kg/m^3]$
$\eta_c$	– viscosity of working liquid	$[Pa \cdot s]$

### References

- [1] Stręk F.: *Mieszanie i mieszalniki*, WNT, Warszawa 1981.
- [2] Kamiński J.: *Mieszanie układów wielofazowych*, WNT, Warszawa 2004.
- [3] Magni J., Costes J., Bertrand J., Couderc J. P.: Proceedings of the 6<sup>th</sup> European Conference on Mixing, BHRA, Pavia 1988, 7-14.

- [4] Mahouast M., Fontaine P., Mallet J.: Proceedings of the 7<sup>th</sup> European Conference on Mixing, BHRA, Brügge, 181-185.
- [5] Vasconcelos J. M. T., Alves S. S., Barata J. M.: Chem. Eng. Sci., **50**, 1995, 2343-2354.
- [6] Jaworski Z., Nienow A.W., Dyster K.W.: Can. J. Chem. Eng., **74**, 1996, 3-15.
- [7] Jaworski Z., Nienow A.W., Koutsakos E., Dyster K., Bujalski W.: Trans. IChemE., Part A, **69**, 1991, 313-320.
- [8] Jaworski Z., Nienow A.W.: Eighth European Conference on Mixing, , No 136, Cambridge 1994.
- [9] Buchmann M., Mewes D.: Chem. Eng. J., **77**, 2000, 3-9.
- [10] Bombač A., Žun I.: Chem. Eng. Sci., **55**, 2000, 2995-3001.
- [11] Gao Z., Smith J. M., Müller-Steinhagen H.: Chem. Eng. Proc., **40**, (2001), 489-497.
- [12] Bouaifi M., Roustan M.: Chem. Eng. Proc., **40**, 2001, 87-95.
- [13] Talaga J., Duda A.: 16th International Conference "Chemical Engineering and Plant Design", Berlin 2006, 159-168.