CZASOPISMO TECHNICZNE M
WYDAWNICTWO POLITECHNIKI KRAKOWSKIEJ

z. 5-M/2008
ISSN 0011-4561
ISSN 1897-6328

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DROP BREAKAGE MODELS FOR LIQUID–LIQUID DISPERSIONS IN TURBULENT FLOWS – COMPARISON

MODELE ROZPADU KROPEL DLA DYSPERSJI CIECZ–CIECZ W PRZEPŁYWACH BURZLIWYCH – PORÓWNANIE

Abstract

In the paper four models of drop breakage in turbulent flow were compared. Breakage models were included to the population balance equation to predict transient drop size distributions. Predicted distributions were compared with experimental data. It was shown that models that take into account the effect of the system scale on breakage (multifractal model and model based on the division of impeller region into isotropic turbulent region and nonisotropic turbulent region) predict drop sizes most properly.

Keywords: drop breakage model, intermittency, population balance equation, turbulence

Streszczenie

W artykule porównano cztery modele rozpadu kropel w polu burzliwym. Modele rozpadu zostały włączone do równania bilansu populacji w celu przewidywania zmian rozkładów wielkości kropel w czasie. Obliczone rozkłady zostały porównane z rozkładami zmierzonymi. Pokazano, że modele uwzględniające wpływ skali układu na rozpad kropel (model multifraktalny i model oparty na podziale strefy mieszadła na strefy burzliwości izotropowej i nieizotropowej) dają najlepsze rezultaty. Model oparty na podziale stref zawiera jednak aż cztery stałe, które muszą być dobierane dla każdego układu. Stosunkowo dobrze rozmiary kropel mogą być również przewidziane za pomocą modelu Coulaloglou i Tavlaridesa. Model Martineza–Bazan przewiduje krople o rząd wielkości za duże.

Słowa kluczowe: bilans populacji, burzliwość, intermitencja, rozpad kropel

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1. Mathematical description of breakage

Drop size distribution usually depends on droplet breakage and drop coalescence processes. However, in the case of low dispersed phase volume fraction coalescence can be neglected and pure breakage process can be considered. In this paper the discussion is confined to lean dispersions. In such dispersions drop size distribution is determined by breakage frequency as well as the character of mother drop fragmentation into daughter droplets. Therefore, drop size distribution can be predicted by solving the population balance equation in the form

$$\frac{\partial n(\upsilon,t)}{\partial t}=\int_{\upsilon}^{\infty}\beta(\upsilon,\upsilon')\cdot\nu(\upsilon')\cdot g(\upsilon')\cdot n(\upsilon',\mathrm{t})\mathrm{d}\upsilon'-g(\upsilon)\cdot n(\upsilon,t) \tag{1}$$

Only binary droplet breakage will be considered in this paper, so $\nu(\upsilon') = 2$.

In turbulent flows drop fragmentation is caused by turbulent velocity fluctuations. It is assumed that drop size is within the inertial subrange of turbulence ($\lambda_K < d < L$). Therefore, the mean square of relative velocity between points separated by a distance r can be described by

$$\overline{u^2(r)}=C_1\cdot\varepsilon^{2/3}\cdot r^{2/3} \tag{2}$$

according to classical Kolmogorov theory of turbulence [1]. Stresses acting on droplets of diameter d are given by

$$p(d)=C_p\cdot\rho_C\cdot\langle\varepsilon\rangle^{2/3}\cdot d^{2/3} \tag{3}$$

In fact Kolmogorov theory neglects fine-scale intermittency of turbulence. Employing the multifractal model of intermittency [2] one can show that the velocity increment over a distance r scales as

$$\frac{u_r}{u_L}=\left(\frac{r}{L}\right)^{\alpha/3} \tag{4}$$

where α is multifractal (scaling) exponent. Assuming $\varepsilon \cong u_r^3/r$ the velocity increment can be expressed in the following form

$$u_r=\left[\langle\varepsilon\rangle L\right]^{1/3}\left(\frac{r}{L}\right)^{\frac{\alpha}{3}}=\left[\langle\varepsilon\rangle r\right]^{1/3}\left(\frac{r}{L}\right)^{\frac{\alpha-1}{3}} \tag{5}$$

and stresses acting on droplets of size d [3]

$$p(d,\alpha)=C_p\cdot\rho_C\cdot\langle\varepsilon\rangle^{2/3}\cdot d^{2/3}\cdot\left(\frac{d}{L}\right)^{\frac{2(\alpha-1)}{3}} \tag{6}$$

Stabilizing stresses due to interfacial tension can be estimated for spherical droplets by

$$\tau_\sigma=\frac{4\cdot\sigma}{d} \tag{7}$$

One should remember that coefficient 4 indicates only the order of magnitude of the constant because the droplet is deformed [4]. Therefore, the constant C_x in the expression for the maximum stable drop diameter determined from the comparison of the dispersive and stabilizing stresses

$$d^o_{max} = C_x \cdot \frac{\sigma^{0,6}}{\langle\varepsilon\rangle^{0,4} \cdot \rho_C^{0,6}} \tag{8}$$

is not equal to $(4/C_p)^{0,6}$, but smaller. Maximum quasi-stable drop size resulting from multifractal model of intermittent turbulence is given by [5]

$$d_{max} = C_x^{\frac{5}{3+2\alpha}} \cdot L \cdot \left(\frac{\sigma}{\rho_C \langle\varepsilon\rangle^{2/3} \cdot L^{5/3}} \right)^{\frac{3}{3+2\alpha}} \tag{9}$$

and reduces to Eq. (8) for $\alpha = 1$ corresponding to the most probable turbulent events. The really stable drop size corresponding to the most vigorous eddies is characterized by $\alpha = \alpha_{\min}$. The smallest value of multifractal exponent determined by Meneveau and Srinivasan [6] is equal to 0,12.

Expressions for disruptive forces (Eq. (3) or (6)) and stabilizing forces, Eq. (7), are also used to derive the expressions for drop break-up rate $g(\upsilon)$. One of the most popular breakage models based on the classical theory of turbulence was proposed by Coulaloglou and Tavlarides [7]. They assumed that the oscillating drop deformed due to local pressure fluctuations breaks if the energy transmitted to the droplet by eddies exceeds the drop surface energy $E_c \propto \sigma \cdot d^2$. The fraction of eddies with kinetic energy greater than E_c is represented by $\exp\left(-E_c / (C_2 \cdot \rho \cdot d^3 \cdot \overline{u^2(d)})\right)$ and this fraction is equal to the fraction of droplets that break. The drop breakage frequency, $g(\upsilon)$, is inversely proportional to the breakage time t_b ($t_b \propto d^{2/3} \cdot \varepsilon^{-1/3}$) and proportional to fraction of drops breaking. Coulaloglou and Tavlarides [7] had taken into account damping effect on turbulence due to relatively high dispersed phase volume fraction (up to $\phi = 0,15$ in their experiments). The final form of their expression for breakage rate is as follows

$$g(\upsilon) = 0,4 \frac{D^{2/3} \cdot N}{\upsilon^{2/9} \cdot (1+\phi)} \cdot \exp\left(- \frac{0,08 \cdot (1+\phi)^2 \cdot \sigma}{\rho_D \cdot \upsilon^{5/9} \cdot D^{4/3} \cdot N^2} \right) \tag{10}$$

Model parameters 0,4 and 0,08 were obtained from the fit of their experimental drop size distributions with the estimated ones. They include not only constants C_1 or C_2, but also details of geometry of the stirred tank used in experiments. The above equation can be rewritten in terms of energy dissipation rate ε instead of impeller diameter D and impeller speed N. The parameters 0,4 and 0,08 and the knowledge about the place of breakage in the tank (the impeller zone) can be used to determine new parameters that should work for other tank geometries. The more general form of Eq. (10) reads

$$g(\upsilon) = C_{1C} \cdot \frac{\varepsilon^{1/3}}{\upsilon^{2/9} \cdot (1+\phi)} \cdot \exp\left(- \frac{C_{2C} \cdot (1+\phi)^2 \cdot \sigma}{\rho_D \cdot \upsilon^{5/9} \cdot \varepsilon^{2/3}} \right) \tag{11}$$

Following Valentas et al. [8] and treating daughter distribution function as a combined result of a large number of independent random events Coulaloglou and Tavlarides applied normal $\beta(\upsilon, \upsilon')$ distribution

$$\beta(\upsilon, \upsilon') = \frac{2{,}4}{\upsilon'} \cdot \exp\left[-4{,}5 \cdot \frac{(2\upsilon - \upsilon')^2}{\upsilon'^2}\right] \tag{12}$$

Model of Coulaloglou and Tavlarides do not predict the influence of the system scale on breakage process. Konno et al. [9] tried to include this effect in their model. They divided the break-up region near the impeller into two subregions of isotropic and nonisotropic turbulence. Therefore, the total breakup frequency was expressed as follows

$$g(d) = g_I(d) + g_{NI}(d) \tag{13}$$

Of course, $g(\upsilon) = g(d)$ for $\upsilon = \pi \cdot d^3/6$. The break-up frequency in the region of isotropic turbulence was derived under the assumptions similar to those used by Coulaloglou and Tavlarides [7]. However, Konno et al. [9] assumed that probability density function of $u(d)$ is given by Maxwell distribution. Their $g_I(d)$ function takes the form

$$g_I(d) = C_{1K} \cdot \frac{N \cdot D^{2/3}}{d^{2/3}} \cdot \int_{\xi}^{\infty} 3 \cdot \left(\frac{6}{\pi}\right)^{1/2} \cdot x^2 \cdot \exp\left(-\frac{3}{2} \cdot x^2\right) \mathrm{d}x \tag{14}$$

with

$$\xi = \frac{u_{\mathrm{crit}}(d)}{\sqrt{\overline{u^2(d)}}} = C_{2K} \cdot \left(\frac{\sigma^{1/2}}{\rho_C^{1/2} \cdot N^{2/3} \cdot d^{5/6}}\right) \tag{15}$$

and

$$x = \frac{u(d)}{\sqrt{\overline{u^2(d)}}} \tag{16}$$

Konno et al. [9] assumed that in the region close to the impeller surface (nonisotropic region) the break-up frequency is proportional to the product of the frequency of droplet circulation in the tank and the fraction of breaking up drops (equal to the probability that relative velocities $u(d)$ are greater than critical $u_{crit}(d)$)

$$g_{NI}(d) = C_{3K} \cdot N \cdot \mathrm{erfc}(\eta) \tag{17}$$

with

$$\eta = C_{4K} \cdot \left[\frac{\sigma^{3/2}}{N^3 \cdot D^3 \cdot \rho_C^{3/2} \cdot d^{3/2}}\right]^{1/3} \tag{18}$$

Konno et al. [9] also assumed that breakage into similar daughter droplets is more probable than breakage into small and large ones. Therefore, normal daughter distribution function, Eq. (12), will be used in calculations together with Konno et al. [9] breakage function $g(d)$.

Multifractal breakage model derived by Bałdyga and Podgórska [5] introduces the effect of the system scale in a natural way – through the integral scale of turbulence, L. The appearance of the integral scale of turbulence in the expression for the velocity difference u_r

($\lambda_K < r < L$) results from intermittent character of turbulence. Breakage function, $g(d)$, was derived by summing up the contributions to the break-up frequency from all vigorous eddies

$$g(d) = C_g \cdot \sqrt{\ln\left(\frac{L}{d}\right)} \cdot \langle\varepsilon\rangle^{1/3} \cdot d^{-2/3} \cdot \int_{\alpha_{\min}}^{\alpha_x} \left(\frac{d}{L}\right)^{\frac{\alpha+2-3\cdot f(\alpha)}{3}} \mathrm{d}\alpha \tag{19}$$

The upper bound α_x results from the comparison between turbulent disruptive stresses and stabilizing stresses

$$\alpha_x = \frac{2{,}5 \cdot \ln\left[\dfrac{L \cdot \langle\varepsilon\rangle^{0,4} \cdot \rho_C^{0,6}}{C_x \sigma^{0,6}}\right]}{\ln\left[\dfrac{L}{d}\right]} - 1{,}5 \tag{20}$$

Multifractal spectrum $f(\alpha)$ given by polynomial of the 8th order (see [5]) was fitted to the spectrum measured by Meneveau and Sreenivasan [6]. More detailed description of the multifractal breakage model one can find in Podgórska [10]. This model was combined with daughter distribution function $\beta(\upsilon, \upsilon')$ proposed by Tsouris and Tavlarides [11] and based on energetic considerations. From energetic point of view breakage into two droplets differing much in volume should be more probable than breakage into two equal drops, because in the last case larger surface is generated. Daughter distribution function expressed in volume domain takes the form

$$\beta(\upsilon, \upsilon') = \frac{E_{\max} + E_{\min} - E(\upsilon, \upsilon')}{\int_0^{\upsilon'} \left(E_{\max} + E_{\min} - E(\upsilon, \upsilon')\right) \mathrm{d}\upsilon} \tag{21}$$

$E(\upsilon, \upsilon')$is the increase in total surface energy for breakage into a pair of droplets of volumes υ and υ', $E_{\max}$ is the increase in surface energy for breakage into two equal droplets, and $E_{\min}$ for breakage into smallest drop and its complementary drop.

Martinez-Bazan et al. [12] postulated that the acceleration of the bubble or drop interface during deformation is proportional to the difference between the deformation and confinement forces. Deformation stress is given in their model by

$$p(d) = \frac{1}{2} \cdot \rho_C \cdot \overline{u^2(d)} = \frac{1}{2} \cdot \rho_c \cdot C_{MB} \cdot (\varepsilon \cdot d)^{2/3} \tag{22}$$

With $C_{MB} = 8{,}2$, while the surface restoring pressure is given by

$$\tau_\sigma = \frac{\pi \cdot \sigma \cdot d^2}{\left(\pi \cdot d^3 / 6\right)} = \frac{6 \cdot \sigma}{d} \tag{23}$$

Therefore, breakup frequency is estimated as

$$g(d) = K_g \cdot \frac{\sqrt{8{,}2 \cdot \langle \varepsilon \rangle^{2/3} \cdot d^{2/3} - 12 \cdot \sigma / \rho \cdot d}}{d} \tag{24}$$

Furthermore, Martinez-Bazan et al. [13] assumed that probability of the splitting of the daughter particle of diameter d from mother particle of size d' should be weighted by the difference in deformation and confinement stresses $\Delta\tau_1 = \frac{8{,}2}{2} \cdot \rho_c \cdot (\varepsilon \cdot d)^{2/3} - 6 \cdot \sigma / d'$. Together with the particle of size d, a complementary particle of size d_2 is generated. Therefore, probability of the formation of a pair of daughter particles (bubbles or droplets) of sizes d and d_2 from mother particle of diameter d' should be proportional to the product of $\Delta\tau_1$ and $\Delta\tau_2 = \frac{8{,}2}{2} \cdot \rho_c \cdot (\varepsilon \cdot d_2)^{2/3} - 6 \cdot \sigma / d'$. Martinez-Bazan et al. [13] proposed the following nondimensional daughter probability density function $f^*(d^*) = f^*(d/d')$

$$f^*(d^*) = \frac{\left(d^{*2/3} - \Lambda^{5/3}\right) \cdot \left(\left(1 - d^{*3}\right)^{2/9} - \Lambda^{5/3}\right)}{\int_{d^*_{\min}}^{d^*_{\max}} \left(d^{*2/3} - \Lambda^{5/3}\right) \cdot \left(\left(1 - d^{*3}\right)^{2/9} - \Lambda^{5/3}\right) \mathrm{d}(d^*)} \tag{25}$$

with $\Lambda = d_{\text{crit}} / d'$ and $d_{\text{crit}} = \left(\frac{12 \cdot \sigma}{8{,}2 \cdot \rho_c}\right)^{0,6} \cdot \varepsilon^{-0,4}$ (compare to Eq. (8). Assuming the arguments of Martinez-Bazan et al. [13] one can derive the following daughter distribution function $\beta(\upsilon, \upsilon')$

$$\beta(\upsilon, \upsilon') = \frac{\left(A \cdot \upsilon^{2/9} - B\right) \cdot \left(A \cdot (\upsilon' - \upsilon)^{2/9} - B\right)}{\int_{\upsilon_{\min}}^{\upsilon' - \upsilon_{\min}} \left(A \cdot \upsilon^{2/9} - B\right) \cdot \left(A \cdot (\upsilon' - \upsilon)^{2/9} - B\right) \mathrm{d}\upsilon} \tag{26}$$

with

$$A = \frac{8{,}2}{2} \cdot \left(\frac{6}{\pi}\right)^{2/9} \cdot \rho_c \cdot \varepsilon^{2/3} \tag{27}$$

and

$$B = \frac{6 \cdot \sigma}{(6/\pi)^{1/3} \cdot \upsilon'^{1/3}} \tag{28}$$

The smallest daughter droplet is estimated from the equality between the turbulent stresses acting between two points separated by the distance $d = d_{\min}$ and confinement pressure due to surface tension $\tau_\sigma = 6 \cdot \sigma / d'$, provided that $d_{\min} > \lambda_K$. Otherwise, $d_{\min}$ is equal to Kolmogorov microscale. The model of Martinez–Bazan et al. [12-13] was originally proposed for bubbles immersed in a turbulent water flow. However, it was recommended also for liquid–liquid systems [14-15].

2. Results and discussion

The population balance equation presented in the previous section was solved to predict the evolution of drop size distribution in time. Calculations were performed for four different breakage models. Calculated drop size distributions were compared with experimental data. It was decided to chose experiments that were not performed by any of the authors of considered breakage models to check the universality of the models. The experimental results of Lam et al. [16] were chosen for this purpose. Results of Lam et al. [16] have the virtue of long experimental time and can be a good test for breakage models. Lam et al. performed their experiments in the stirred tank of diameter $T = 0{,}14$ m equipped with Rushton turbine impeller of diameter $D = 0{,}05$ m and four baffles. The organic phase was a mixture of benzene and carbon tetrachloride to make a naturally buoyant dispersion ($\rho_D = \rho_C = 1000$ kg/m^3) and the aqueous phase was distilled and deionized water. Interfacial tension σ for this system is equal to 0,035 N/m. The viscosity of the dispersed phase $\mu_D = 0{,}00074$ Pa·s. It means that stabilizing forces due to dispersed phase viscosity can be neglected in calculations. The dispersed phase volume fraction used in experiments was also low $\phi = 0{,}0058$. For such low ϕ the influence of coalescence on drop size can be neglected. Furthermore, Lam et al. shown that negligible effect of coalescence can be inferred from the fact that transient drop size distributions collapse into a single curve under the similarity transformation for drop breakage.

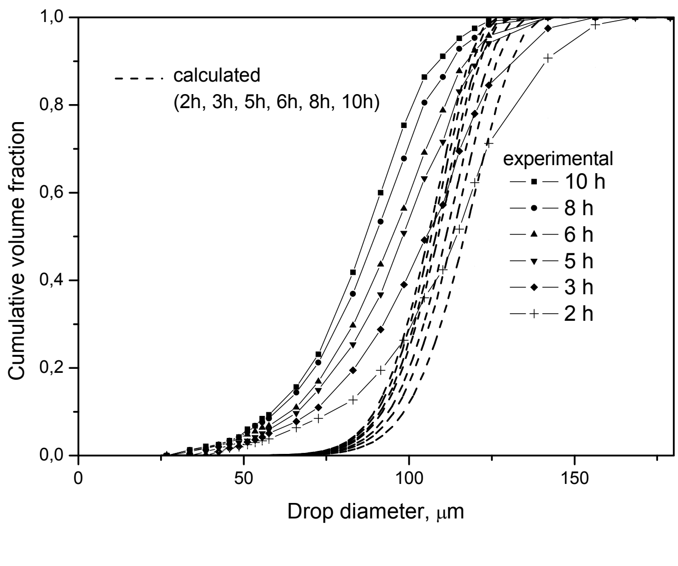

Fig. 1. Transient drop size distribution – comparison between Coulaloglou and Tavlarides [7] model predictions and experimental results of Lam et al. [16]

Rys. 1. Zmiany rozkładu wielkości kropel w czasie – porównanie przewidywań modelu Coulaloglou i Tavlaridesa [7] z wynikami doświadczalnymi Lama i in. [16]

Figure 1 presents the comparison between experimental results and predictions of Coulaloglou and Tavlarides model, Eqs. (11), (12). The model properly predicts drop size changes even after 10 hours of agitation, though, the predicted drop diameters are larger than the measured ones. There are also smaller differences between drop size distributions calculated for different agitation times, than those observed experimentally. Moreover, predicted drop size distributions are narrower than experimental ones. It results from the assumed larger probability for breakage into equal daughter droplets that into droplets differing in size. Besides, the authors formulated their model (so also estimated model parameters) on the base of experiments in which coalescence and breakage were not separated and, therefore, their model can work better for the conditions of dynamic equilibrium between breakage and coalescence, so for relatively short agitation times, when for example intermittent effects of turbulence are not very influential on drop breakage. One should, however, remember that the breakage rate model proposed by Coulaloglou and Tavlarides was very important and helpful for developing breakage models by other authors.

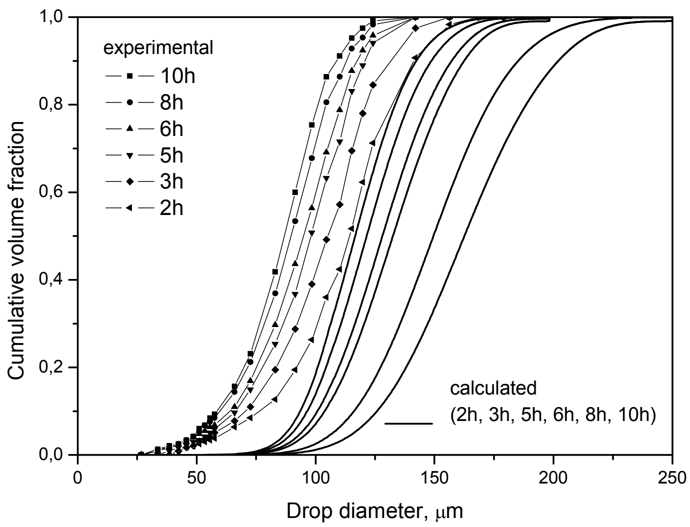

Fig. 2. Transient drop size distribution – comparison between Konno et al. [9] model predictions (C_{A1} = 0,07; C_{A2} = 1,45; C_{B1} = 0,00024; C_{B2} = 0,24) and experimental results of Lam et al. [16]

Rys. 2. Zmiany rozkładu wielkości kropel w czasie – porównanie przewidywań modelu Konno et al. [9] z wynikami doświadczalnymi Lama i in. [16]

Figures 2 and 3 show comparison between experimental transient drop size distributions and distributions predicted using Konno et al. [9] model of breakage, Eqs. (13)–(18), and normal distribution for daughter drop distribution function $\beta(\upsilon, \upsilon')$, Eq. (12). Results in

Fig. 2 were obtained for model parameters proposed by Konno et al. for impeller diameter to tank diameter ratio $D/T = 1/2$, while results in Fig. 3 were obtained for model parameters proposed originally for $D/T = 2/3$. One can see that drop size distributions presented in Fig. 3 are better predicted, though impeller diameter to tank diameter ratio in Lam et al. experiments, $D/T = 0{,}357$, is closer to $D/T = 1/2$ than to $D/T = 2/3$. Drop size distributions could be predicted better if new model parameters would be proposed. However, the trends of changes of these parameters are not clear. Konno et al. [9] decreased parameters C_{A1} and C_{B1} and increased parameters C_{A2} and C_{B2} when changing D/T from 2/3 to 1/2. Applying the same trend of parameter changes fails for the transition from $D/T = 1/2$ (as in Konno experiments) to $D/T = 0{,}357$ (as in Lam experiments). Such parameter changes would lead to droplet even larger than those shown in Fig. 2. The number of model parameters and first of all not universal character of these parameters is a large disadvantage of Konno et al. breakage model. The advantage of this model lies in possibility of prediction the effect of the system scale on drop size (shown in their paper) and prediction that the exponent on Weber number in the relation $d_{\max} \propto \mathrm{We}^a$ changes up to –1 (which was close to a = –0,93 observed experimentally).

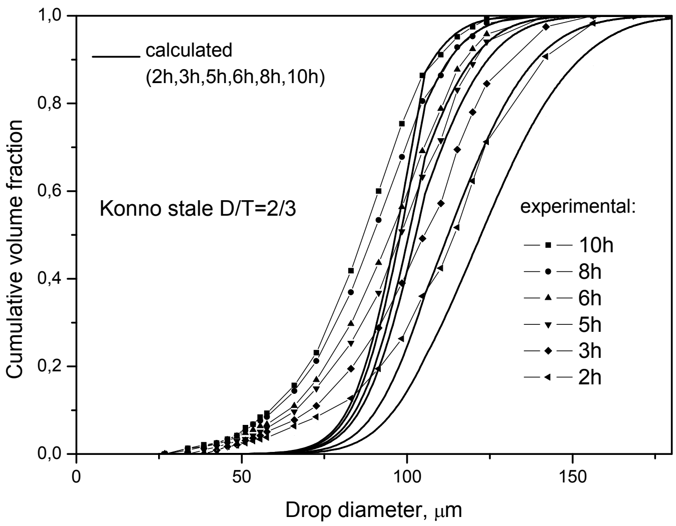

Fig. 3. Transient drop size distribution – comparison between Konno et al. [9] model predictions C_{A1} = 0,1; C_{A2} = 1,3; C_{B1} = 0,00026; C_{B2} = 0,205) and experimental results of Lam et al. [16]

Rys. 3. Zmiany rozkładu wielkości kropel w czasie
– porównanie przewidywań modelu Konno i in. [9] z wynikami doświadczalnymi Lama i in. [16]

Multifractal model predicts exponent –0,93 for the most vigorous turbulent events (see [5]). It gives also better agreement with experimental transient drop size distributions, Fig. 4. Model parameters C_g = 0,0035 and C_x = 0,23 used in calculations are universal and were successfully used to predict drop size distributions for different liquid–liquid systems (also for dispersed phase of high viscosity, when additional stabilizing stress has to be taken into account), for different levels of energy dissipation rate ε, and different system scales. More examples of predictions of this model one can find in [10].

The fourth breakage model considered here, Eqs. (24)–(25), completely failed in prediction of drop size. It predicts drop diameters one order of magnitude larger than observed experimentally, and no difference between drop size distributions for $t = 2{\cdot}h$ and $t = 10{\cdot}h$ (it can be changed by changing K_g constant, however, it cannot change the final drop size). One of the reasons of this failure is the neglect of drop deformation in estimation of model parameters, so also the critical drop size is much too high. Besides, Martinez–Bazan et al. model [12, 13] is based on experimental results in water jet and for very high energy dissipation rates. In such conditions the smallest daughter drop may strongly depend on the mother size. However, in small agitated tanks it is probably not true.

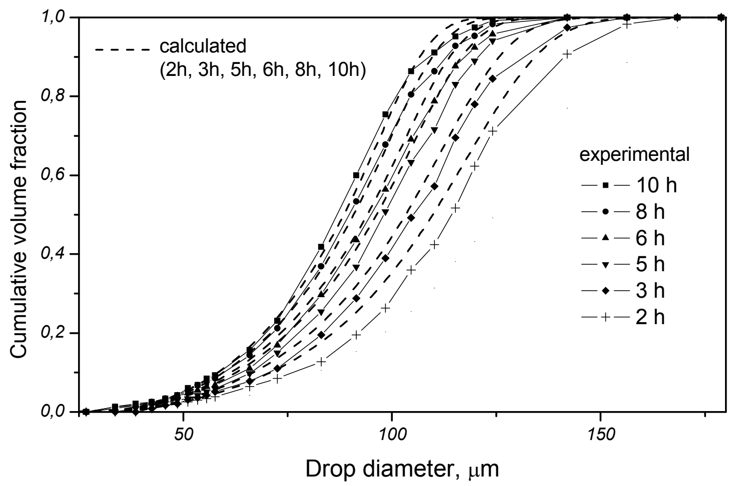

Fig. 4. Transient drop size distribution – comparison between Bałdyga and Podgórska [5] model predictions and experimental results of Lam et al. [16]

Rys. 4. Zmiany rozkładu wielkości kropel w czasie – porównanie przewidywań modelu Bałdygi i Podgórskiej [5] z wynikami doświadczalnymi Lama i in. [16]

Symbols

C_1	–	constants	
C_{1C}, C_{2C}	–	constant in Coulaloglou model, Eq. (11)	
$C_{1K}..C_{4K}$	–	constants in Konno model, Eqs. (14-18)	
C_g	–	constant in multifractal breakage model, Eq. (19)	
C_{MB}	–	constant in Martinez-Bazan model, Eq. (22)	
C_p	–	constant in Eqs. (3) and (6)	
C_x	–	constant in Eqs. (8), (9) and (20)	
d	–	drop diameter	[m]
d_{crit}	–	critical drop diameter	[m]
d_{max}	–	maximum stable drop size	[m]
D	–	impeller diameter	[m]
E	–	surface energy	[J]
$f(\alpha)$	–	multifractal spectrum	
f^*	–	nondimensional daughter probability density function	
g	–	breakage rate	[1/s]
K_g	–	constant in Martinez-Bazan model, Eq. (24)	
L	–	integral scale of turbulence	[m]
n	–	number density of drops	[1/m^6]
N	–	impeller rotational speed	[1/s]
p	–	turbulent stress	[N/m^2]
t	–	time	[s]
t_b	–	breakage time	[s]
T	–	tank diameter	[m]
u_L	–	velocity increment over a distance L	[m/s]
u_r	–	velocity increment over a distance r	[m/s]
α	–	multifractal exponent	
β	–	daughter distribution function (probability density function)	[1/m^3]
ε	–	energy dissipation rate	[m^2/s^3]
λ_K	–	Kolmogorov microscale	[m]
ν	–	kinematic viscosity	[m^2/s]
$\nu(\upsilon')$	–	number of daughter drops formed per breakage	
ρ	–	density	[kg/m^3]
σ	–	interfacial tension	[N/m]
τ	–	stress	[N/m^2]
υ	–	drop volume; daughter drop volume	[m^3]
υ'	–	mother drop volume	[m^3]
ϕ	–	dispersed phase volume fraction	

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