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VISCOUS HEATING GENERATION IN ROTATIONAL VISCOMETER – MAXIMUM SHEAR RATE DETERMINATION

WISKOTYCZNA GENERACJA CIEPŁA W REOMETRZE ROTACYJNYM – WYZNACZANIE MAKSYMALNEJ SZYBKOŚCI ŚCINANIA

A b s t r a c t

The present paper deals with viscous heating of liquid in rotational viscometer with coaxial cylinders, which is often used for measurement of rheological behaviour. Due to viscous energy dissipation the temperature of measured liquid can be considerably higher than tempering temperature, which can lead to significant experimental error. For this reason the maximum shear rate corresponding to acceptable maximum temperature difference should be controlled.

Keywords: viscometer with coaxial cylinders, viscous heating, power-law fluids, Bingham plastics

S t r e s z c z e n i e

Niniejsza praca dotyczy lepkiego nagrzewania się cieczy w reometrze rotacyjnym o współosiowych cylindrach, który często stosowany jest do pomiaru własności reologicznych. W wyniku lepkiego rozproszenia energii temperatura badanej cieczy może być istotnie wyższa od pierwotnej temperatury układu, co prowadzić może do znacznych błędów pomiaru. Dlatego należy kontrolować maksymalną szybkość ścinania związaną z maksymalną dopuszczalną różnicą temperatur.

Słowa kluczowe: wiskozymetr z cylindrami osiowymi, lepkie ogrzewanie, płyny potęgowe, ciecze binghamowskie

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1. Introduction

The rotational viscometer with coaxial cylinders is often used for measurement of rheological behaviour. If the inner to outer cylinder diameter ratio does not differ significantly from 1, the curvature can be neglected and the flow reduces to flow between moving and stationary plates.

The temperature distribution can be obtained by solution of Fourier-Kirchhoff equation

$$\lambda \cdot \frac{d^2T}{dy^2} = -\dot{Q}^g \quad (1)$$

where y is distance from stationary plate and

$$\dot{Q}^g = \tau \cdot \dot{\gamma} \quad (2)$$

is the heat generated by viscous dissipation in unit volume per unit time. The equation (1) will be solved with the following boundary conditions

$$\begin{aligned} &y = H, \quad dT/dy = 0 \\ &y = 0, \quad \lambda dT/dy = \alpha \cdot \left(T - T_f\right) \end{aligned} \quad (3)$$

it means we suppose insulated moving plate (rotating cylinder) and stationary plate (cylinder) tempered to temperature T_f.

The power-law and Bingham models are often used for description of rheological behaviour.

The power-law is the simplest model mostly used for description of rheological behavior of pseudoplastic fluids. Using this model, the dependence of shear stress τ on shear rate $\dot{\gamma}$ can be expressed by the following relation

$$\tau = K \cdot \dot{\gamma}^n \quad (4)$$

where K is the coefficient of consistency and n stands for the flow behaviour index. Inserting (4) into (2) the following relation can be received

$$\dot{Q}^g = \tau \cdot \dot{\gamma} = K \cdot \dot{\gamma}^{n+1} \quad (5)$$

The Bingham model is the simplest model used for rheological behaviour description of viscoplastic materials. Using this model, the relation of shear stress τ and shear rate $\dot{\gamma}$ can be expressed by the following relation

$$\tau = \mu_p \cdot \dot{\gamma} + \tau_0 \quad \text{for} \quad |\tau| \geq \tau_0 \quad (6)$$

where τ_0 is the yield stress and μ_p stands for plastic viscosity. Inserting (6) into (2) the following relation can be obtained

$$\dot{Q}^g = \tau \cdot \dot{\gamma} = \mu_p \cdot \dot{\gamma}^2 + \tau_0 \cdot \dot{\gamma} \quad (7)$$

2. Solution

Integrating (1), the following equation is received

$$T - T_f = \frac{H^2 \cdot \dot{Q}^g}{\lambda} \cdot \left(\frac{y}{H} - \frac{1}{2} \cdot \left(\frac{y}{H} \right)^2 + \frac{1}{\mathrm{Bi}} \right) \tag{8}$$

where Bi = $\alpha \cdot H/\lambda$. The graphical form of solution is shown in Fig.1, where

$$T^* = \frac{(T - T_f) \cdot \lambda}{H^2 \cdot \dot{Q}^g}, \quad y^* = \frac{y}{H} \tag{9}$$

From Eqs. (8) it can be seen that maximum temperature T_m is on moving plate (inner rotating cylinder) at $y = H$ and maximum temperature difference can be calculated from equation

$$\Delta T_m = T_m - T_f = \frac{H^2 \cdot \dot{Q}^g}{\lambda} \left(\frac{1}{2} + \frac{1}{\mathrm{Bi}} \right) \tag{10}$$

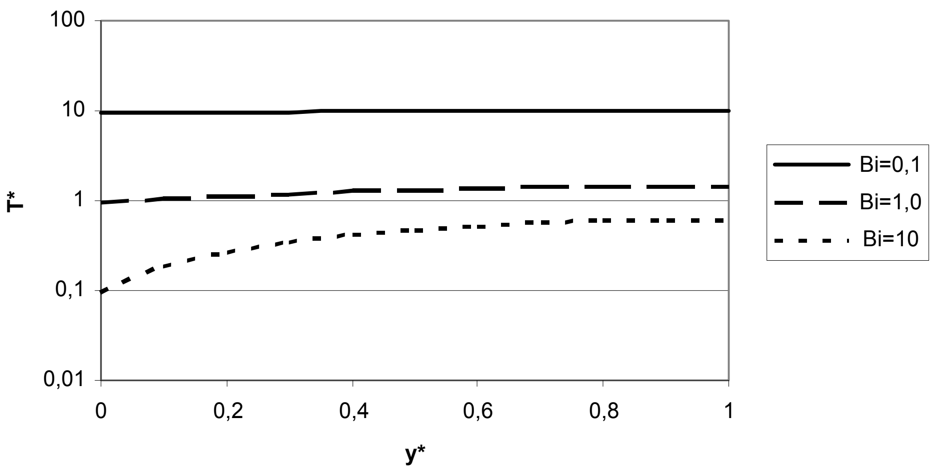

Fig. 1. Dimensionless temperature profiles

Rys. 1. Profile bezwymiarowej temperatury

From this equation the maximum $\dot{Q}^g$ corresponding to maximum acceptable ΔT_m can be expressed

$$\dot{Q}_m^g = \frac{\lambda \cdot \Delta T_m}{H^2 \cdot (1/2 + 1/\mathrm{Bi})} \tag{11}$$

Considerably greater heat generation could be accepted if temperature of outer cylinder T_s is controlled instead of tempering liquid temperature T_f. The maximum temperature difference can then be calculated from equation

$$\Delta T'_m = T_m - T_s = \frac{H^2 \cdot \dot{Q}^g}{2 \cdot \lambda} \tag{12}$$

From this equation the maximum $\dot{Q}^g$ corresponding to maximum acceptable $\Delta T'_m$ can be calculated

$$\dot{Q}_m^g = \frac{2 \cdot \lambda \cdot \Delta T'_m}{H^2} \tag{13}$$

3. Example

Rotational viscometer with inner cylinder diameter 48 mm and outer cylinder diameter 50 mm contains Newtonian liquid with density $\rho = 1000\ \mathrm{kg \cdot m^{-3}}$, viscosity $\mu = 1\ \mathrm{Pa \cdot s}$ and heat conductivity $\lambda = 0{,}5\ \mathrm{W \cdot m^{-1} \cdot K^{-1}}$. Calculate the maximum shear rate corresponding to the maximum acceptable temperature difference $\Delta T_m = 0{,}1\ ^\circ\mathrm{C}$:

1) when heat transfer coefficient on the side of tempering water $\alpha = 100\ \mathrm{W \cdot m^{-2} \cdot K^{-1}}$,
2) at no tempering and $\alpha = 5\ \mathrm{W \cdot m^{-2} \cdot K^{-1}}$,
3) calculate also the maximum shear rate corresponding to the maximum acceptable temperature difference $\Delta T'_m = 0{,}1\ ^\circ\mathrm{C}$.

Solution

At first Biot number will be calculated

1)

$$\mathrm{Bi} = \frac{\alpha \cdot H}{\lambda} = \frac{100 \cdot 0{,}001}{0{,}5} = 0{,}2$$

The maximum $\dot{Q}^g$ corresponding to maximum acceptable ΔT_m can be calculated from (11)

$$\dot{Q}_m^g = \frac{\lambda \cdot \Delta T_m}{H^2 \cdot (1/2 + 1/\mathrm{Bi})} = \frac{0{,}5 \cdot 0{,}1}{0{,}001^2 \cdot (0{,}5 + 5)} = 9{,}09 \cdot 10^3\ \mathrm{W \cdot m^{-3}}$$

Inserting $n = 1$ and $K = \mu$ into Eq. (5) or inserting $\tau_0 = 0$ and $\mu_p = \mu$ into Eq. (7) the maximum shear rate will be calculated from equation

$$\dot{\gamma}_m = \sqrt{\frac{Q_m^g}{\mu}} = 95{,}3\ s^{-1}$$

If measurement at temperature $T \pm 0{,}1$ °C is required, it is recommended to choose tempering temperature $T - 0{,}1$ °C and interval $\Delta T_m = 0{,}2$ °C. It causes increasing value of maximum shear rate to

$$\dot{\gamma}_m = \sqrt{\frac{0{,}5 \cdot 0{,}2}{1 \cdot 0{,}001^2 \cdot (0{,}5 + 5)}} = 135\ s^{-1}$$

2)

$$\mathrm{Bi} = \frac{\alpha \cdot H}{\lambda} = \frac{100 \cdot 0{,}001}{0{,}5} = 0{,}2$$

and the maximum shear rate will be calculated from the same equation

$$\dot{Q}_m^g = \frac{\lambda \cdot \Delta T_m}{H^2 \cdot (1/2 + 1/\mathrm{Bi})} = \frac{0{,}5 \cdot 0{,}1}{0{,}001^2 \cdot (0{,}5 + 100)} = 497{,}5\ \mathrm{W \cdot m^{-3}}$$

and maximum shear rate

$$\dot{\gamma}_m = \sqrt{497{,}5} = 22{,}3\ s^{-1}$$

3) The maximum $\dot{Q}^g$ corresponding to maximum acceptable $\Delta T'_m$ can be calculated from (13)

$$\dot{Q}_m^g = \frac{2 \cdot \lambda \cdot \Delta T'_m}{H^2} = \frac{2 \cdot 0{,}5 \cdot 0{,}1}{0{,}001^2} = 10^5\ \mathrm{W \cdot m^{-3}}$$

maximum shear rate

$$\dot{\gamma}_m = \sqrt{10^5} = 316\ s^{-1}$$

If measurements at temperature $T \pm 0{,}1$ °C are required, it is recommended to choose outer cylinder temperature $T - 0{,}1$ °C and interval $\Delta T'_m$ =0,2 °C. It causes increasing value of maximum shear rate to

$$\dot{\gamma}_m = \sqrt{2 \cdot 10^5} = 447\ s^{-1}$$

4. Conclusions

It was shown that the dissipative heating can play an important role at highly viscous fluid measurement. The temperature of measured liquid can be significantly higher than tempering temperature, which can cause significant experimental error. For this reason the maximum shear rate corresponding to acceptable maximum temperature difference should be calculated. Considerably greater shear rate can be accepted if temperature of outer cylinder T_s is controlled instead of tempering liquid temperature T_f.

Symbols

Bi	–	Biot number	
H	–	distance of planes (cylinder walls)	[m]
K	–	consistency coefficient	[Pas^n]
n	–	flow index	
$\dot{Q}^g$	–	heat generated by viscous dissipation in unit volume per unit time	[W/m^3]
T	–	temperature	[K]
y	–	coordinate	[m]
α	–	heat transfer coefficient	[W/m^2K]
$\dot{\gamma}$	–	shear rate	[1/s]
λ	–	heat conductivity	[W/mK]
μ	–	viscosity	[Pas]
μ_p	–	plastic viscosity	[Pas]
ρ	–	density	[kg/m^3]
τ	–	shear stress	[Pa]
τ_0	–	yield stress	[Pa]
		lower indexes	
f	–	fluid	
m	–	maximum	
s	–	wall	
		upper index	
*	–	dimensionless	

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