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COSTS AND COST-EFFECTIVENESS OF WASTE RECYCLING – THEORETICAL INTRODUCTION

KOSZTY ORAZ EFEKTYWNOŚCI EKONOMICZNE RECYKLIGU – WSTĘP TEORETYCZNY

A b s t r a c t

Any product has a possibility of turning into waste when it loses its economic value with the passing of time. Waste accumulates over time unless decomposed in the ecosystem or recycled. Calculation of the actual level of the costs and cost-effectiveness is one of the most serious and, at the same time, one of the most difficult issues. The disposal of municipal solid waste (MSW) is a problem confronting all nations around the world. Much research has been done throughout the world to determine the economic and ecological profitability of waste recycling in the substitution of raw materials. The purpose of this paper is to present the theoretical introduction of an additive and multiplicative model for analyzing the costs and cost-effectiveness of the recycling of waste.

Keywords: *costs, cost-effectiveness, waste, municipal, recycling*

S t r e s z c z e n i e

Każdy produkt staje się odpadem w momencie kiedy traci swoje wartości ekonomiczne. Akumulowane odpady z upływem czasu są rozkładane w ekosystemie albo mogą być używane jako surowce wtórne. Obliczenie rzeczywistego poziomu kosztów oraz efektywności recyklingu jest jednym z istotniejszych i zarazem trudniejszych zagadnień recyklingu odpadów komunalnych. Problem unieszkodliwiania odpadów komunalnych stał się jednym z podstawowych zagadnień ochrony środowiska i gospodarki odpadami. Prace na temat określenia efektów ekologicznych oraz ekonomicznych recyklingu odpadów jako surowców wtórnych były prowadzone przez różnych autorów. Artykuł ten stanowi wstęp teoretyczny do multiplikatywnego modelu do analizy kosztów oraz efektywności recyklingu odpadów.

Słowa kluczowe: *koszty, efektywność ekonomiczna, odpady, recykling*

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1. Introduction

Any product has a possibility of turning into waste when it loses its economic value with the passing of time [1]. Waste accumulates over time unless decomposed in the ecosystem or recycled. Today, the accumulation of waste has reached such magnitude that it can become a real threat to the existence of the whole ecosystem.

The promotion of environmental management and the mission of sustainable development worldwide have exerted the pressure for the adoption of proper methods to protect the environment. The hierarchy of disposal options categorizes environmental impacts into six levels, from low to high: to reduce, reuse, recycle, compost, incinerate, and landfill. Recycling, being one of the strategies of minimizing waste, offers three benefits: reduce the demand for new resources, cut down on transport and production energy costs, utilize waste which would otherwise be lost to landfill sites [2, 3].

Calculation of the actual level of the costs and co-effectiveness is one of the most serious and, at the same time, one of the most difficult issues. Most studies on waste management and recycling technology has been done by such authors as Bor Y. J., Hsu E., Kou C.-M., and Jenkins R. R., [4–6].

2. Theoretical introduction for application of multiplicative and additive functions for cost-effectiveness maximization

Both the costs and economic cost-effectiveness of a recycling plant can be presented by the function

$$f(x_1, x_2, \cdots, x_n) \qquad (1)$$

where x_1 determines the amount of waste to be processed, whereas x_2, x_3, etc the number of factors that are used in waste processing x_1 into raw materials (input). Function 1 presents the amount of the produced commodity (output) out of the factors of the production.

Unit prices of the factors of the production are as follows: q_1, q_2, q_3...; unit price of the produced commodity equals p.

Nevertheless, the prices of the factors of the production are $q_i > 0$, $i = 2, ..., n$.

In the model, (p_1) determines the unit price of waste delivered to the plant. It means that if the recycling plant pays for waste, then p will be $p > 0$. If the plant does not pay for the provided waste, p will equal $p = 0$, if the producer pays for the utilization of waste, then $p < 0$.

In the model, the issue of the economic cost-effectiveness of the plant which will pay for the delivered waste is discussed ($p_1 > 0$).

If the plant is going to pay for waste, the profit obtained from the recycling amounts to

$$Y = pf(x_1, x_2, ..., x_n) - K$$

where $q_1 x_1 + q_2 x_2 + \ldots + q_n x_n = K$ means that K presents the amount of money to be invested in processing waste into raw materials.

If the recycling plant is not going to pay for the delivered goods ($q < 0$), the profit received from the recycling is as follows

$$Y = -q_1 x_1 + pf(x_1, x_2, ..., x_n) - K$$

Owing to the fact that $\frac{\partial Y}{\partial x_i} = -q_1 + \gamma \frac{\partial f}{\partial x_i} \rangle 0$, we need to assume that the maximal amount of raw material that could be processed by the plant is $x_1 = a$. As a result, the profit is as follows: $Y = -qa_1 + pf(x_1, x_2, ..., x_n) - K$.

Having sum K destined for waste processing, we get the condition

$$q_2 x_2 + q_3 x_3 + \ldots + q_n x_n = K.$$

The function (both the costs and the economic cost-effectiveness of recycling) complies with the conditions that reflect the basic economic rules. Thanks to these rules the production makes sense (plant functioning).

Function 1 is positive, increasing with regard to every variable, i.e. the increase in the consumption of every initial factor causes the increase in the amount of the production.

Function 1 is concave, i.e. the effectiveness of the production decreases along with its increase.

Function 1 is homogeneous of degree α, i.e. $f(sx) = s^{\alpha} f(x) \;\; x \in R_+^n,\; s \in R_+, \alpha \in (0,1)$.

Consequently, if the production obtained from the means x amounts to $f(x)$, then the production received from the means sx will come to $sf(x)$ for $s < 1$.

Apart from the function 1, unit prices of the initial raw materials and the unit price of the produced commodity (product) are needed.

Assuming that $q_i > 0$ determines the unit price of raw material x_i for $i = 1, \ldots, n$, the unit price of the produced commodity equals p. K denotes the sum destined for the production. The profit obtained from the production is

$$pf(x_1, x_2, \cdots, x_n) - K \tag{2}$$

if the given condition is fulfilled.

$$q_1 x_1 + q_2 + \cdots + q_n x_n = K \tag{3}$$

Having the condition (3) given, we will examine the issue of profit maximization $\max(pf(x_1, x_2, ..., x_n) - K)$.

The issue can be solved on the basis of the Kuhn-Tucker methods. The Lagrange's function is presented as follows

$$L(x_1, ..., x_n, \lambda) = p\, f(x_1, ..., x_n) - K - \lambda(q_1 x_1 + \ldots + q_n x_n - K)$$

As a result, we have the system of equations to be solved

$$\begin{cases} \dfrac{\partial L}{\partial x_i} = p\dfrac{\partial f}{\partial x_i} - \lambda q_i = 0 \\ \dfrac{\partial L}{\partial \lambda} = q_1 x_1 + q_2 x_2 + \ldots + q_n x_n = K \end{cases}$$

Having rearranged the system of equations, we obtain

$$\begin{cases} q_i = \dfrac{p}{\lambda}\dfrac{\partial f}{\partial x_i} \qquad i = 1, 2, \ldots, n & (4) \\ q_1 x_1 + q_2 x_2 + \ldots + q_n x_n = K & (4a) \end{cases}$$

Substituting (4) to (4a) we get

$$\frac{p}{\lambda}\left(x_1 \frac{\partial f}{\partial x_1} + x_2 \frac{\partial f}{\partial x_2} + \ldots + x_n \frac{\partial f}{\partial x_n} \right) = K \tag{5}$$

The function $f(x_1, \ldots x_n)$ is a homogeneous function of degree a where 0 < α <1. Therefore, $f(tx_1, \ldots tx_n) = t^{\alpha} f(x_1, \cdots, x_n)$. Differentiating the last identity with respect to s, we obtain $x_1 \frac{\partial f}{\partial x_1} + x_2 \frac{\partial f}{\partial x_2} + \ldots + x_n \frac{\partial f}{\partial x_n} = \alpha t^{\alpha - 1} f(x_1, \ldots, x_n)$.
Hence, after substituting 1 to s, we get

$$x_1 \frac{\partial f}{\partial x_1} + x_2 \frac{\partial f}{\partial x_2} + \ldots + x_n \frac{\partial f}{\partial x_n} = \alpha f(x_1, \ldots, x_n) \tag{6}$$

Substituting the last equality (6) to the condition (5), we have $\frac{p}{\lambda} \alpha f(x_1, \ldots, x_n) = K$. Therefore

$$\lambda = \frac{p}{K} \alpha f(x_1, \ldots, x_n) \tag{7}$$

Substituting (7) to the equation (4), we get $q_i = \dfrac{p}{\frac{p}{K}\alpha f(x_i, \ldots, x_n)} \dfrac{\partial f}{\partial x_i}$. Consequently

$$\frac{\partial f}{\partial x_i} = \frac{\alpha}{K} f(x_1, \ldots, x_n) \cdot q_i, \quad i = 1, \ldots, n \tag{8}$$

Having solved the system of equations (8), we have the optimum distribution of the sum K for the means of production determined. This distribution does not depend on the price p of the produced product.

The system of equations (8) is presented in the vector form $\left[\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \ldots, \frac{\partial f}{\partial x_n}\right] = \frac{\alpha}{K} f(x_1, \ldots, x_n) \cdot [q_1, q_2, \ldots, q_n]$. As a result, the vector of the extreme utility of raw materials $\left[\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \ldots, \frac{\partial f}{\partial x_n}\right]$ is proportional to the prices of raw materials.

The issues were discussed in such a situation in which $q_i > 0$ for $i = 1, 2, ..., n$. Nevertheless, when $x_1 = a$ for $q_i < 0$, we obtain the issue of function maximization $Y = -q_1 a_1 + pf(xa, x_2, \ldots, x_n) - K$.

The condition is given $q_2 x_2 + q_3 x_3 + \ldots + q_n x_n = K$ which can be solved in the same way as $q_i > 0$, $i = 1, 2, ..., n$. However, we need to replace the function $f(x_1, x_2, ..., x_n)$ with $f(a, x_3, x_3, x_n)$.

3. Discussion

In recent years recycling has become one of the basic issues of environment protection and waste management. Recycling of waste reduces the demand for virgin materials, the amount of waste to be incinerated and/or landfilled, and it also reduces emissions from these sources; however, it generates waste and emissions of its own.

Much research has been done throughout the world to determine the economic and ecological profitability of waste recycling in the substitution of raw materials.

These are the results of our theoretical introduction for application of multiplicative and additive functions; at the moment no empirical analyses have been carried out on this issue. We think that it might be a good topic for further applied studies.

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