CZASOPISMO TECHNICZNE
E
WYDAWNICTWO POLITECHNIKI KRAKOWSKIEJ

z. 1-E/2008
ISSN 0011-4561
ISSN 1897-6301

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THE MODEL OF PERMANENT MAGNET SYNCHRONOUS MACHINES FOR DIAGNOSTIC PURPOSE

MODEL MASZYNY SYNCHRONICZNEJ Z MAGNESAMI TRWAŁYMI DLA POTRZEB DIAGNOSTYKI

A b s t r a c t

In this paper an analytical method of analyzing the flux density distribution in the air gap and the mathematical model for permanent magnet synchronous machines with non-uniform air gap are presented. The paper contains a way of calculating flux density distribution in the air gap for machine with eccentricity and shows the comparison results of calculation for simplified model and FEM. This approach is presented by an example of 3-phase PM synchronous machine.

Keywords: *permanent magnets machines, magnetic flux distribution, diagnostics, eccentricity*

S t r e s z c z e n i e

W artykule przedstawiono analityczną metodykę analizy rozkładu pola w szczelinie powietrznej maszyny z magnesami trwałymi oraz model matematyczny dla maszyny synchronicznej z magnesami o nieregularnej szczelinie. W artykule zaprezentowano sposób obliczeń pola w szczelinie powietrznej maszyny z ekscentrycznym wirnikiem oraz podano wyniki obliczeń rozkładu pola w szczelinie powietrznej maszyny uzyskane na podstawie modelowania w sposób uproszczony i metodą elementów skończonych (FEM). Podejście takie zostało przedstawione na przykładzie 3-fazowej maszyny synchronicznej z magnesami trwałymi.

Słowa kluczowe: *maszyny z magnesami trwałymi, rozkład pola magnetycznego, diagnostyka, ekscentryczność*

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Symbols

B_r – residual flux density
H_c – magnetic coercive force
x – angular position on stator surface
φ – rotor displacement
y – angular position on rotor surface ($y = x - \varphi$)
l_c – an equivalent length of a machine core
w_s – total turn numbers of phase stator winding

1. Introduction

In the paper the method creation of so called "circuit models" for diagnostic purpose of synchronous machines with permanent magnets (PM) located on the surface of the rotor is presented. It is very important for users to know mathematical model for diagnostic application, for example with taking into account slotting and rotor eccentricity, therefore that problem is discussed in the paper.

This method uses Lagrange's formalism and is based on obtained coil characteristics and the magnetic co-energy function of the whole system. Both of these cases require to know the flux density distribution in the air-gap of the machine. This problem can be solved by FEM or by simplified analysis. In this paper the simplified analysis of the flux density distribution in the air-gap of permanent magnet machines is proposed and applied.

2. Analytical description of flux density distribution in the air gap of PM machine

2.1. Flux density distribution for model 1D

Let us take into consideration, for example, a model of a synchronous machine with permanent magnets shown in Fig. 1.

For this machine the consideration is done under the following assumptions:
- the stator and rotor iron have infinite permeability,
- the permanent magnets are mounted at the surface of the rotor,
- the equivalent length of the machine core is so long that magnetic field has the 2D form and only the radial component of the flux density is considered,
- the demagnetization characteristics of the magnets are linear (Fig. 2).

This linear approximation of the PM characteristic is allowable for Rare Earth PM. Applying Ampere's law around this closed path (solid line in Fig. 1)

$$H_g(x,\varphi)l_g(x,\varphi)+H_m(x,\varphi)l_m(x-\varphi)-H_g(x_0,\varphi)l_g(x_0,\varphi)-H_m(x_0,\varphi)l_m(x_0-\varphi)=\int_{x_0}^{x}a_s(x')dx'=\Theta_s(x) \qquad (1)$$

where:

$H_g(x, \varphi)$ – magnetic field intensity in the air-gap at angle x,
$H_m(x, \varphi)$ – magnetic field intensity in the magnet,

$l_g(x, \varphi)$ and $l_m(x - \varphi)$ – substitutional functions of the line length of the magnetic field in the air-gap and permanent magnet respectively,
$a_s(x)$ – total ampere-turns at point x,
$\Theta_s(x)$ – MMF of stator windings.

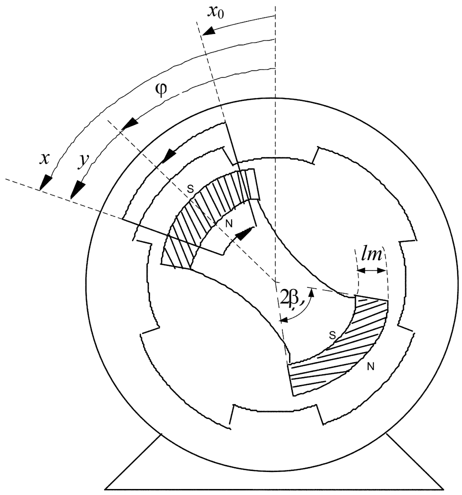

Fig. 1. Model of a permanent magnet machine

Rys. 1. Model maszyny z magnesami trwałymi

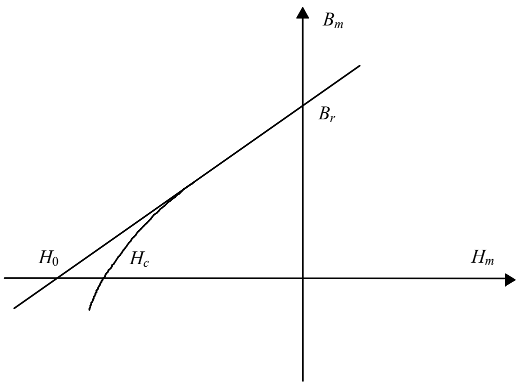

Fig. 2. Demagnetization characteristics of PM

Rys. 2. Charakterystyka demagnesowania magnesów trwałych

Assuming that the magnets have linear characteristic, it is possible to write formula for flux density in the magnet

$$B_m(x, \varphi) = B_r\left(1 - \frac{H_m(x, \varphi)}{H_0}\right) = B_r(x - \varphi) + \mu_0 \mu_r H_m(x, \varphi) \quad (2)$$

Defining in addition the function of residual flux density which is similar to rectangular function of PM magnetization it is possible to take into account the proper sign of the residual flux density

$$B_r(x-\varphi) = B_r \operatorname{sgn} NS(x-\varphi) \tag{3}$$

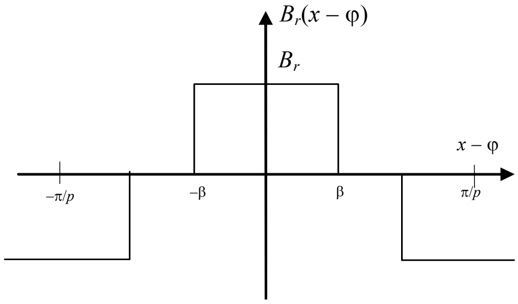

Fig. 3. Special function of residual flux density

Rys. 3. Specjalna funkcja indukcji remanentu

For the air-gap the flux density formula is as follows

$$B_g(x,\varphi) = \mu_0 H_g(x,\varphi) \tag{4}$$

Taking into consideration only radial components of flux density in the air-gap and PM (1D analysis)

$$B_g(x,\varphi) = B_m(x,\varphi) = B(x,\varphi) \tag{5}$$

Introducing into consideration substitutional function of the length line of the magnetic field in PM $l'_m(x-\varphi)$

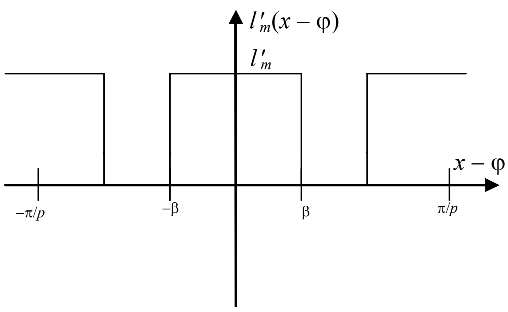

Fig. 4. Substitutional function of the line length of the magnetic field in PM

Rys. 4. Zastępcza funkcja długości linii sił pola magnetycznego w magnesie where value $l'_m = \frac{l_m}{\mu_r}$, the function of permeance can be obtain as [2]

$$\lambda(x, \varphi) = \frac{1}{l_g(x, \varphi) + l'_m(x - \varphi)} \tag{6}$$

and, the flux density on the stator surface can be expresed as [2]

$$B(x, \varphi) = B_\Theta(x, \varphi) + B_{PM}(x, \varphi) \tag{7}$$

where

$$B_\Theta(x, \varphi) = \lambda(x, \varphi)\left\{\mu_0\Theta_s(x) - \frac{\int_x^{x+2\pi}\lambda(x', \varphi)\mu_0\Theta_s(x')dx'}{\int_x^{x+2\pi}\lambda(x', \varphi)dx'}\right\} \tag{8}$$

$$B_{PM}(x, \varphi) = \lambda(x, \varphi)\left\{B_r(x-\varphi)\, l'_m - \frac{\int_x^{x+2\pi}\lambda(x', \varphi)\, B_r(x'-\varphi)\, l'_m\, dx'}{\int_x^{x+2\pi}\lambda(x', \varphi)\, dx'}\right\} \tag{9}$$

$B_\Theta(x, \varphi)$ – term of flux density produced by the stator MMF (classical form),
$B_{PM}(x, \varphi)$ – term of flux density produced by the PM.

For cylindrical symmetrical machine with smooth stator end rotor yoke (Fig. 5) and rare earth PM (P.U. value of permeability $\mu_r \cong 1{,}03\cdots1{,}08$)

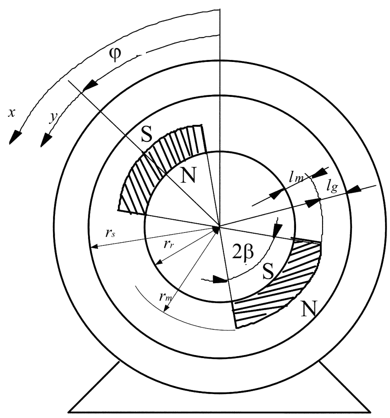

Fig. 5. Cross section of a cylindrical PM machine with internal rotor

Rys. 5. Przekrój poprzeczny cylindrycznej maszyny z magnesami trwałymi formulas (8) and (9) can be modified and reduced to the forms

$$B_{\Theta}(x, \varphi) = B_{\Theta}(x) = \lambda \mu_0 \Theta_s(x) \tag{10}$$

$$B_{PM}(x, \varphi) = B_{PM}^{1D}(x-\varphi) = \lambda B_r(x-\varphi)\, l'_m \tag{11}$$

where

$$\lambda = \frac{1}{l_g + l'_m}$$

2.2. Flux density distribution for model 2D

Equations (8) and (9) present, the general expressions for the radial component of the flux density created on the base of 1D model analytical model. Zhu and Howe [5] proposed an analytical model prediction of the two-dimensional air gap field distribution $B_{PM}^{2D}(x-\varphi)$ for PM machines with smooth cylindrical shape of stator and rotor joke (PM machine model – Fig. 5). Therefore, general solutions for the flux density distribution $B_{PM}(x, \varphi)$ produced by permanent magnets for the points located on the stator surface (smooth machine with number of pole pairs $p > 1$) according to [5] has the form

$$B_{PM}(x,\varphi) = B_{PM}^{2D}(x-\varphi) = \sum_{\varsigma \in Q} B_{\varsigma}^{PM} \cdot e^{j\varsigma(x-\varphi)} \tag{12}$$

where

$$B_{\varsigma}^{PM} = \frac{4}{\pi}\frac{B_r}{\mu_r}\sin(|\varsigma|\beta)\,\frac{p}{\varsigma^2-1}\left(\frac{r_m}{r_s}\right)^{|\varsigma|+1}\left\{\frac{(|\varsigma|-1)+2\left(\frac{r_r}{r_m}\right)^{|\varsigma|+1}-(n+1)\left(\frac{r_r}{r_m}\right)^{2|\varsigma|}}{\frac{\mu_r+1}{\mu_r}\left[1-\left(\frac{r_r}{r_s}\right)^{2|\varsigma|}\right]-\frac{\mu_r-1}{\mu_r}\left[\left(\frac{r_m}{r_s}\right)^{2|\varsigma|}-\left(\frac{r_r}{r_m}\right)^{2|\varsigma|}\right]}\right\} \tag{13}$$

and $Q = \{-\varsigma_{max} \ldots -5p, -3p, -p, p, 3p, 5p \ldots \varsigma_{max}\}$.

Introducing into formula (9) the results of considerations (11) and (12) is possible to modify this expression to general form

$$B_{PM}(x,\varphi) = \frac{\lambda(x,\varphi)}{\lambda}\left\{B_{PM}^{2D}(x-\varphi) - \frac{\int_{x}^{x+\pi 2} \lambda(x',\varphi)\, B_{PM}^{2D}(x'-\varphi)\, dx'}{\int_{x}^{x+\pi 2} \lambda(x',\varphi)\, dx'}\right\} \tag{14}$$

Moreover, the effects of rotor eccentricity and stator slot openings on air gap field distribution may be added by introducing suitable air gape permeance function $\Lambda(x, \varphi)$. Consequently, a 2D analytical model can be used to calculate field distribution in PM machine with real shape of air gap.

Assuming that geometry of the air-gap and magnets is represented in (14) by the permeance function

$$\Lambda(x, \varphi) = \sum_{m\in M} \sum_{n\in N} \Lambda_{m,n} e^{jmx} e^{jn\varphi} \tag{15}$$

where sets M and N contain successive whole numbers $M = \{-m_{\max} \cdots -1, 0, 1 \cdots m_{\max}\}$; $N = \{-n_{\max} \cdots -1, 0, 1 \cdots n_{\max}\}$, integrals in (14) can be written in following forms

$$\int_{x}^{x+2\pi} \lambda(x', \varphi)\, B_{\mathrm{PM}}^{2D}(x' - \varphi)\, dx' = \begin{cases} 2\pi \sum_{\varsigma\in Q} \sum_{m\in M} \sum_{n\in N} B_{\varsigma}^{\mathrm{PM}} \Lambda_{m,n} e^{j(-\varsigma+n)\varphi} & \text{for} \quad \varsigma + m = 0 \\ 0 & \text{for} \quad \varsigma + m \neq 0 \end{cases} \tag{16}$$

$$\int_{x}^{x+2\pi} \lambda(x', \varphi)\, dx' = \begin{cases} 2\pi \sum_{n\in N} \Lambda_{0,n} e^{jn\varphi} & \text{for} \quad m = 0 \\ 0 & \text{for} \quad m \neq 0 \end{cases} \tag{17}$$

3. Mathematical model of a 3-phase PM synchronous machine

Defining coil characteristics which formulate PM in an integral form in the magnetic circuit is the main problem of creating a mathematical model.

Let consider these symmetrical coils a and b which are represented by their MMFs and angular positions of their symmetry axes x_a and x_b.

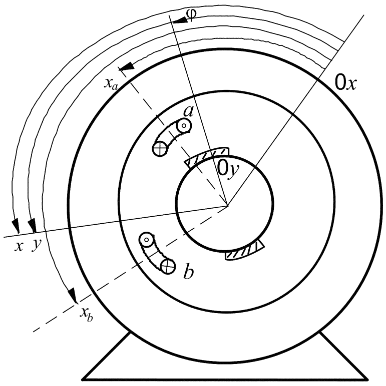

Fig. 6. Location of two coils magnetizing an air-gap PM machine

Rys. 6. Położenie dwóch uzwojeń magnesujących szczelinę powietrzną maszyny

The MMFs of coils express by the Fourier series are done by the sets of harmonics order numbers $P = \{-\nu_{\max} \ldots -5p, -3p, -p, p, 3p, 5p \ldots \nu_{\max}\}$ and a winding factor $k_w^{|\varsigma|}$ of each harmonic. On the basis of (7) and formulas for coil characteristics [1] the linked flux for winding a is defined as follows

$$\psi_a = r_s l_c \sum_{\nu\in P}\left(\int\limits_{x_a-\frac{\pi}{|\nu|}}^{x_a}\left(\sum_{k=1}^{|\nu|}\left(\frac{1}{\pi} j\,\nu\, W_\nu\, e^{j\nu(x-x_a)}\right)\int\limits_{x+(k-1)\frac{2\pi}{|\nu|}}^{x+\frac{\pi}{|\nu|}+(k-1)\frac{2\pi}{|\nu|}} B(x',\varphi)dx'\right)dx\right) \tag{18}$$

where $W_\nu = \dfrac{w_s k_w^{|\nu|}}{|\nu|}$ is an equivalent turn number.

The coil's linked flux (18) has two components

$$\psi_a(\varphi, i_1, i_2, i_3) = \psi_{\Theta a}(\varphi, i_1, i_2, i_3) + \psi_{\mathrm{PM}a}(\varphi) \tag{19}$$

where:

$\psi_{\Theta a}$ – the linked flux of winding *a*, produced by the stator MMF,

$\psi_{\mathrm{PM}a}$ – the linked flux of winding *a*, produced by the PM.

Assuming additionally that inductances are defined, what can be expressed for phase *a* as

$$\psi_{\Theta a}(\varphi, i_1, i_2, i_3) = \sum_{b=1}^{3} L_{ab}(\varphi) i_b \tag{20}$$

Simplified approach, accounting for a radial component of flux density in the air-gap where $B(x,\varphi) = B_\Theta(x,\varphi)$, presented in [1] leads to an analytical expression for the mutual inductance

$$L_{ab}(\varphi) = \frac{2\mu_0 r_s l_c}{\pi}\left\{\sum_{\nu\in P}\sum_{m\in M} Q_1 W_\nu W_{\nu+m}\cdot\left[\sum_{n_1\in N}\Lambda_{m,n_1} e^{jn_1\varphi} - Q_2 \frac{\sum\limits_{n_2\in N}\sum\limits_{n_3\in N}\Lambda_{-\nu,n_2}\Lambda_{\nu+m,n_3} e^{j(n_2+n_3)\varphi}}{\sum\limits_{n_4\in N}\Lambda_{0,n_4} e^{jn_4\varphi}}\right] e^{j\nu(x_a-x_b)} e^{jmx_a}\right\} \tag{21}$$

where parameters Q_1 and Q_2 depend on the sets *P*, *M*, *N* and are defined as follow

$$Q_1 = \begin{cases} 1 \Leftrightarrow \forall\nu, \quad \forall m, \quad \nu\in P \wedge m\in M \wedge (-\nu-m)\in P \\ 0 \quad \text{for opposed condition} \end{cases} \tag{22}$$

$$Q_2 = \begin{cases} 1 \Leftrightarrow Q_1 = 1 \wedge \nu\in M \wedge (-\nu-m)\in M \\ 0 \quad \text{for opposed condition} \end{cases} \tag{23}$$

Generally, a mutual inductance L_{ab} is aproximated by a triple Fourier series

$$L_{ab}(\varphi) = \sum_\nu \sum_m \sum_n L_{\nu,m,n} e^{j\nu(x_a-x_b)} e^{jmx_a} e^{jn\varphi} \tag{24}$$

where

$$x_a = (a-1)\frac{2\pi}{3p} \qquad a = 1, 2, 3, \qquad x_b = (b-1)\frac{2\pi}{3p}, \qquad b = 1, 2, 3$$

A self-inductance due to the main flux can be obtained by substituting $x_a = x_b$.

The linked flux of winding a $\psi_{\mathrm{PM}\,a}$, produced by the PM can be also derived based on formula (18) for substitution $B(x, \varphi) = B_{\mathrm{PM}}(x, \varphi)$. After mathematical transformations, it can be slightly modified to relation (21) and expressed as

$$\psi_{\mathrm{PM}a}(\varphi)=\frac{2\,r_s\,l_c}{\lambda}\left\{\sum_{\varsigma\in Q}\sum_{m\in M}D_1B_{\varsigma}^{\mathrm{PM}}W_{\varsigma+m}\cdot\left[\sum_{n_1\in N}\Lambda_{m,n_1}e^{jn_1\varphi}-D_2\frac{\sum\limits_{n_2\in N}\sum\limits_{n_3\in N}\Lambda_{-\varsigma,n_2}\Lambda_{\varsigma+m,n_3}e^{j(n_2+n_3)\varphi}}{\sum\limits_{n_4\in N}\Lambda_{0,n_4}e^{jn_4\varphi}}\right]e^{j\varsigma(x_a-\varphi)}e^{jmx_a}\right\} \tag{25}$$

where parameters D_1 and D_2 in (25) depend on the sets P, Q, M, N and are defined as follow

$$D_1=\begin{cases}1\Leftrightarrow \forall\varsigma,\quad \forall m,\quad \varsigma\in Q\wedge m\in M\wedge(-\varsigma-m)\in P\\ 0\quad \text{for opposed condition}\end{cases} \tag{26}$$

$$D_2=\begin{cases}1\Leftrightarrow D_1=1\wedge\varsigma\in M\wedge(-\varsigma-m)\in M\\ 0\quad \text{for opposed condition}\end{cases} \tag{27}$$

Generally, the linked flux of winding a produced by the PM $\psi_{\mathrm{PM}\,a}$ can be expressed by a triple Fourier series

$$\psi_{\mathrm{PM}a}(\varphi)=\sum_{\varsigma}\sum_{m}\sum_{n}\psi_{\varsigma,m,n}^{\mathrm{PM}}e^{j(\varsigma+m)x_a}e^{j(-\varsigma+n)\varphi} \tag{28}$$

For application of the Lagrenge's formalism for the class of PM machines, it's not only necessary to modify the characteristics of coil set but also co-energy function, in order to take into account the presence of PM in the machine magnetic circuit. Because the iron core has an infinite permeability, the conservation of energy of the magnetic field is only in the volume of the air-gap and permanent magnets, therefore, the co-energy expression can be described by

$$E_{c0}=\frac{r_m\,l_c}{2\mu_0}\int_0^{2\pi}\frac{\left(B(x,\varphi)\right)^2}{\lambda(x,\varphi)}dx=\frac{r_m\,l_c}{2\mu_0}\int_0^{2\pi}\frac{\left(B_{\Theta}(x,\varphi)+B_{\mathrm{PM}}(x,\varphi)\right)^2}{\lambda(x,\varphi)}dx \tag{29}$$

Using formal mathematical notation, for the most general case, the flux density distribution is in the form (7), (8) and (14), the co-energy function can be transformed and expressed as

$$E_{c0}(\varphi, i_1, i_2, i_3) = E'_{c0}(\varphi, i_1, i_2, i_3) + E''(\varphi) \tag{30}$$

where the co-energy expression depending on the stator MMF can be obtained as

$$E'_{c0}(\varphi,i_1,i_2,i_3)=\frac{1}{2}\sum_{a=1}^{3}\sum_{b=1}^{3}L_{ab}(\varphi)\,i_a\,i_b+\sum_{a=1}^{3}\psi_{\mathrm{PM}a}(\varphi)\,i_a \tag{31}$$

and the second component of the co-energy expression independent of the stator MMF has the form

$$E''_{c0}(\varphi) = \frac{r_m l_c}{2\mu_0} \int_0^{2\pi} \frac{\left(B_{\mathrm{PM}}(x,\varphi)\right)^2}{\lambda(x,\varphi)} dx \tag{32}$$

Lagrange's equations for 3-phase machines excited by permanent magnets can be written in the standard matrix form

$$\begin{aligned} &\frac{d}{dt}[L(\varphi)][i] + \frac{d}{dt}[\Psi_{\mathrm{PM}}(\varphi)] = [u] - [R][i] \\ &J\frac{d^2\varphi}{dt^2} = T_{em} + T_l - D(\dot{\varphi}) \end{aligned} \tag{33}$$

where

$$T_{em} = T_e + T_m + T_{\mathrm{cog}} = \frac{1}{2}[i]^T \left(\frac{\partial}{\partial\varphi}[L(\varphi)]\right)[i] + [i]^T \left(\frac{\partial}{\partial\varphi}[\Psi_{\mathrm{PM}}(\varphi)]\right) + \frac{\partial E''_{c0}(\varphi)}{\partial\varphi} \tag{34}$$

and

$$[L(\varphi)] = \begin{bmatrix} L_{\sigma s} & & \\ & L_{\sigma s} & \\ & & L_{\sigma s} \end{bmatrix} + \begin{bmatrix} L_{11}(\varphi) & L_{12}(\varphi) & L_{13}(\varphi) \\ L_{21}(\varphi) & L_{22}(\varphi) & L_{23}(\varphi) \\ L_{31}(\varphi) & L_{32}(\varphi) & L_{33}(\varphi) \end{bmatrix}$$

$$[i] = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} \quad [u] = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \quad [\Psi_{\mathrm{PM}}(\varphi)] = \begin{bmatrix} \psi_{\mathrm{PM1}}(\varphi) \\ \psi_{\mathrm{PM2}}(\varphi) \\ \psi_{\mathrm{PM3}}(\varphi) \end{bmatrix} \quad [R] = \begin{bmatrix} R_s & & \\ & R_s & \\ & & R_s \end{bmatrix}$$

The electromagnetic torque formula (34) includes reluctance torque T_e, main torque T_m produced by interaction between the PM flux and stator currents, and also torque at zero current so called cogging torque T_{cog} produced by the tangential forces on the slot or pole walls.

4. Permeance function for PM machine with rotor eccentricity

The geometry of the air-gap is represented in (8) and (14) by an air-gap permeance function which depends on air-gap geometry and especially a type of eccentricity. The permeance function of the air-gap is an inverse of the length of magnetic field force lines in the air-gap and PM. Good results for a smooth stator and rotor yoke can be obtained using the approach, which keeps perpendicular crossing of the boundary between iron and air by magnetic field force lines. It is connected to an assumption that $\mu_r \cong 1$ what is appropriate approximation for the Rare Earth PM.

$$\delta(x) = l_g(x,\varphi) + l_m(x-\varphi) \tag{35}$$

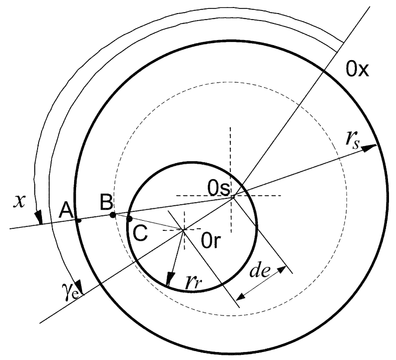

Fig. 7. Simplified cross-sections of a motor with rotor eccentricities

Rys. 7. Uproszczony przekrój maszyny z ekscentrycznym wirnikiem

The length of a magnetic field force line in the air-gap and PM $\delta(x)$ are equal to the sum of segments AB and BC in Fig. 7 and is given by the formula

$$\delta(x) = r_s - 2r_r - d_e + \sqrt{[(r_r + d_e)\cos x - d_e \cos\gamma_e]^2 + [(r_r + d_e)\sin x - d_e \sin\gamma_e]^2} \tag{36}$$

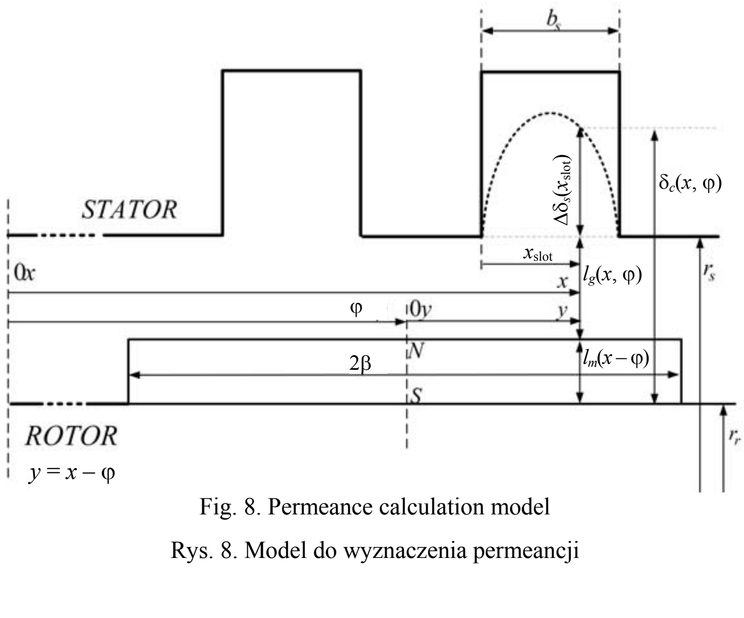

Fig. 8. Permeance calculation model

Rys. 8. Model do wyznaczenia permeancji

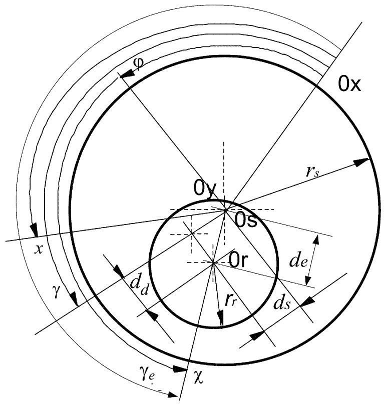

Fig. 9. Illustration cases of rotor eccentricities

Rys. 9. Ilustracja przypadków ekscentryczności

The slots on the stator side modify locally the length of the magnetic field force lines in the air-gap.

Then, correction value $\Delta\delta_s(x)$ should be added in respective places

$$\delta_c(x,\varphi) = \delta(x) + \Delta\delta_s(x) \tag{37}$$

These corrections can be determined using the conformal mapping approach as it is done for field calculations in a slot similar to Carter coefficient determination

$$\Delta\delta_s(x) = \begin{cases} \Delta\delta_{\max}(x)\ \sin\left(\dfrac{\pi}{b_s} x_{\text{slot}}\right) & \text{for staror slots} \\ 0 & \text{for stator teeth} \end{cases} \tag{38}$$

where:

$\Delta\delta_{\max}(x)$ – is calculated according to [6, 7],
b_s – is an equivalent slot opening,
x_{slot} – is a local variable over a slot; $x_{\text{slot}} \in (0, b_s)$.

An equivalent air-gap length $\delta_c(x, \varphi)$ can be locally determined for any rotor position angle φ. Then, it creates a function of two variables x and φ, which is periodic with respect to each of them. Inverse values give the permeance function.

$$\lambda(x,\varphi) = \frac{1}{l_g(x,\varphi) + l_m(x-\varphi) + \Delta\delta_s(x)} = \frac{1}{\delta_c(x,\varphi)} \tag{39}$$

For the static eccentricity $d_e = d_s$, $d_d = 0$, $\gamma_e = \gamma = \text{const}$ this function can be written as

$$\Lambda(x,\varphi) = \Lambda(x) = \sum_{m\in M} \Lambda_{m,0} e^{jm(x-\gamma)} \tag{40}$$

For dynamic eccentricity $d_e = d_d$, $d_s = 0$, $\gamma_e = \varphi + \chi$ it takes the form

$$\Lambda(x,\varphi) = \Lambda(x-\varphi) = \sum_{m\in M} \Lambda_{m,-m}\, e^{jm(x-\varphi-\chi)} \tag{41}$$

and only for mixed eccentricity where $d_s \neq 0$, $d_d \neq 0$, γ = const and $d_e = d(\varphi) =$ $= \sqrt{d_s^2 + d_d^2 + 2d_s d_d \cos(\varphi-\gamma)}$; $\gamma_e = \arcsin\left(\frac{d_d}{d_e}\sin(\varphi-\gamma)\right)$ for $d_e \neq 0$ is given by a double series

$$\Lambda(x,\varphi) = \sum_{m\in M}\sum_{n\in N} \Lambda_{m,n} e^{jm(x-\gamma)} e^{jn(\varphi-\gamma-\chi)} \tag{42}$$

The permeance function, can be represented by the double Fourier series. The Fourier coefficients $\Lambda_{m,n}$, can be obtained by the 2D FFT algorithm. It should be noted, that the permeance function depends on two variables x and φ only at dynamic and mixed eccentricities, whereas for an even slotted stator and a smooth rotor yoke the permeance function for symmetry case and static eccentricity depends periodically on one variable.

5. The comparison with the FEM results

The analytical model developed in this paper was compared with FEM computation. As an example, the calculations of flux density distribution in case of symmetry and rotor eccentricity of the PM synchronous machine SGPM with rated values: P_N = 2,5 kW, U_N = 230 V, I_N = 7,67 A, p = 3, n_N = 1000 rpm have been carried out. Necessary data for computation were fixed on the base of design data. For this machine the geometrical dimensions and parameters of permanent magnets and air gap are following: l_g = 3 mm, l_m = 11 mm, r_s = 66 mm, β = 22° (0,38 rd), B_r = 1,06 T, H_c = 720 kA/m.

For the PM machine with above parameters, the calculations of flux density distribution are presented in the figures bellow. The FEM computations have been done using the MAGNET software. Computations have been led at current-less state of the PM machine with smooth stator for case symmetry and rotor dynamic eccentricity. The level of eccentricity was 66%, it means that the relation between the symmetry, when length of air-gap over PM was 3 mm, in case of the eccentricity minimum air-gap was 1 mm.

Solid line – FEM flux density distribution

Dashed line – results obtained from the simplified model of the flux density distribution

The FEM cross section of the considered PM machines are shown in Figs 10 and 11.

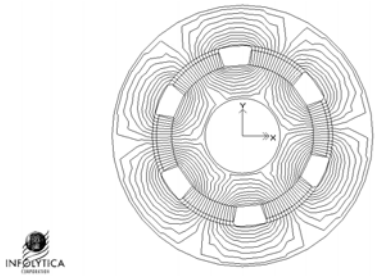

Fig. 10. FEM calculation for symmetrical machine

Rys. 10. Obliczenia polowe dla maszyny symetrycznej

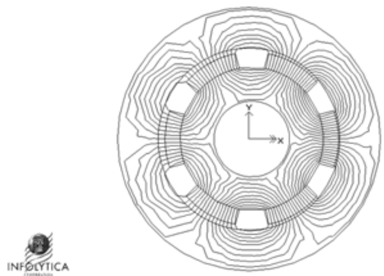

Fig. 11. FEM calculation for machine with rotor eccentricity

Rys. 11. Obliczenia polowe dla maszyny z ekscentrycznością

The effects of flux density distribution in the air gap for various coil angles obtained from FEM computations compared with effects obtained from simplified analysis are illustrated in Figs 12 and 13.

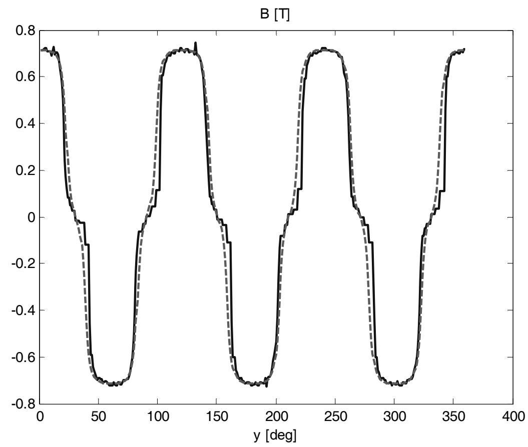

Fig. 12. Flux density distribution at stator surface for case of air-gap symmetry

Rys. 12. Rozkład pola magnetycznego na powierzchni stojana dla symetrii szczeliny powietrznej

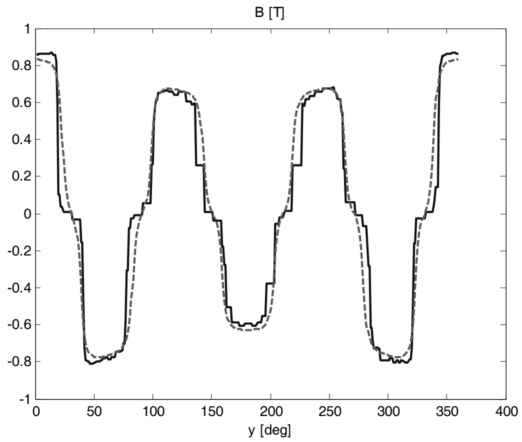

Fig. 13. Flux density distribution at stator surface for case rotor dynamic eccentricity

Rys. 13. Rozkład pola magnetycznego na powierzchni stojana dla przypadku ekscentryczności dynamicznej wirnika

This computation presented above shows a good agreement of the FEM analysis with the analytical calculation.

6. Conclusion

A method of modeling permanent magnet motors has been developed in this paper. The model is based on the co-energy function and magnetic field density in the machine air-gap. The model allows the calculation of such effects as rotor eccentricity, cogging torque and other word effects usually neglected in commonly used analytical models. The model reduce the necessity of using the FEM analysis and described the PM machine by a very simple but precise way.

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