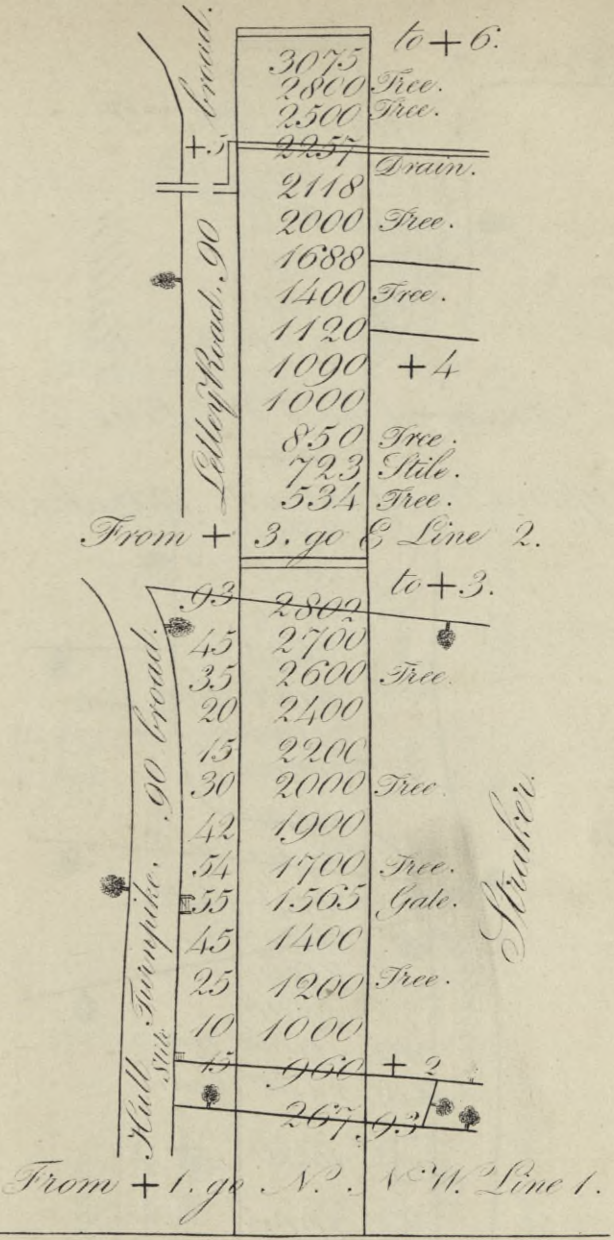
J. F. Graf Pfeil
October 1883.
Hirsdorf.

NESBIT'S

PRACTICAL LAND-SURVEYING.

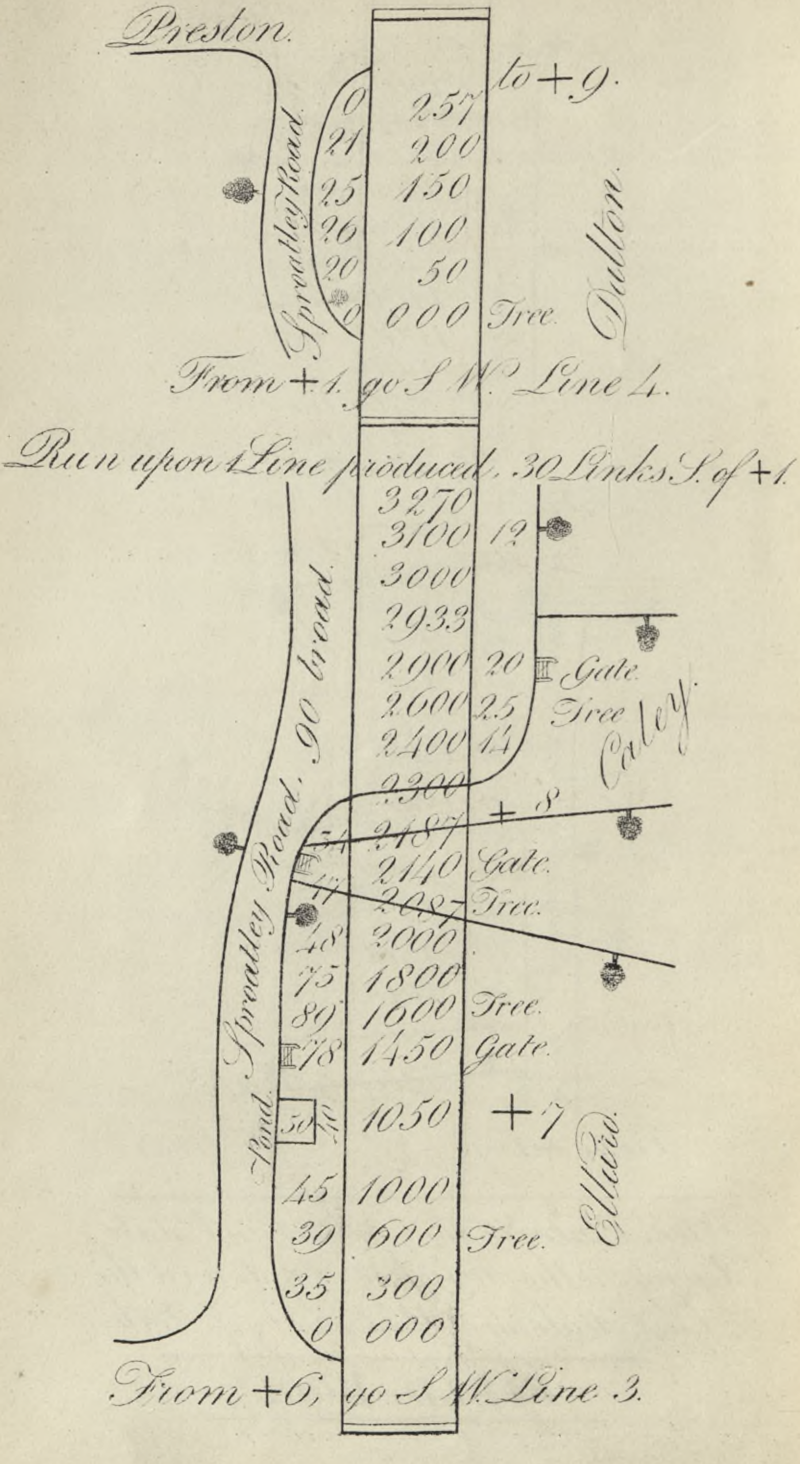
PRINTED BY BALLANTYNE, HANSON AND CO.
EDINBURGH AND LONDON

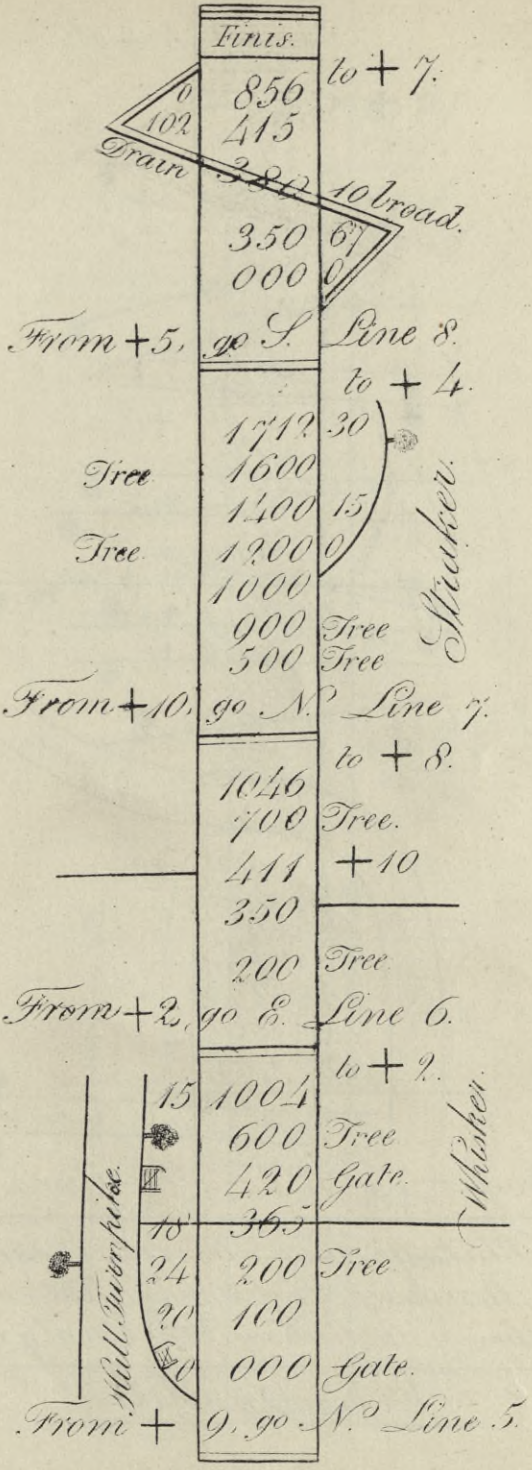
J. F. Graf Pfeil

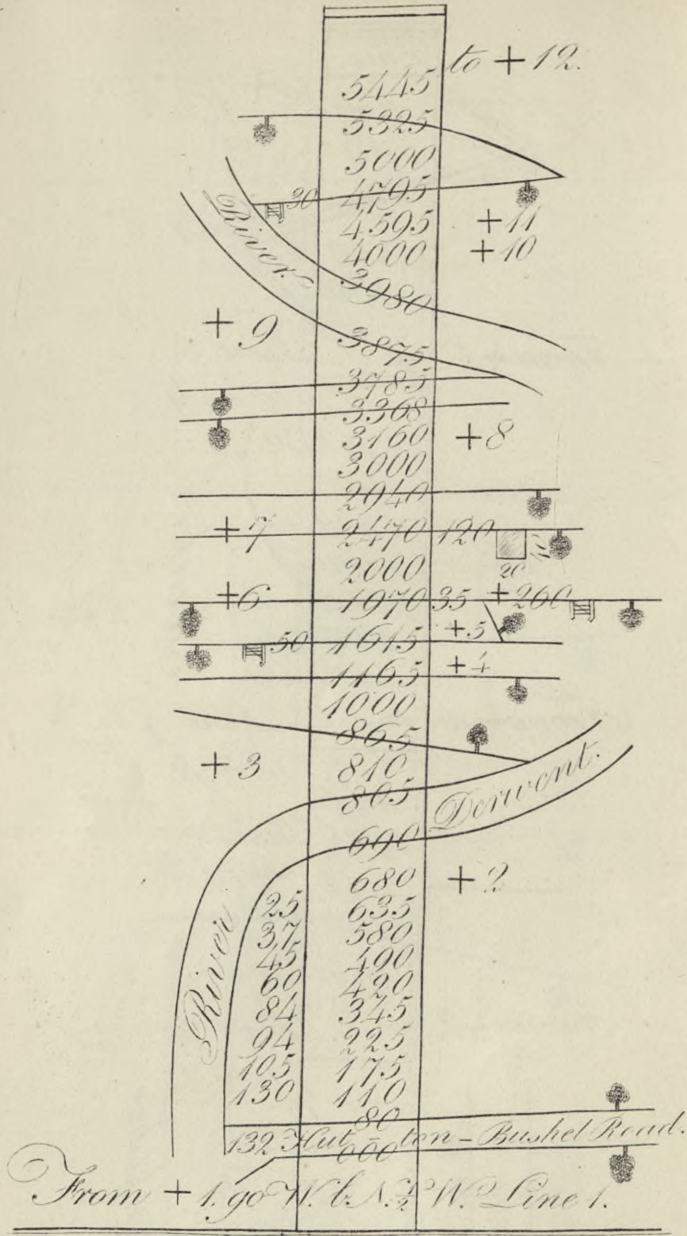


See Plate 8.–The. Notes of an Estate lying in the Parish of Preston, in the East-Riding of the County of York.

FIELD-BOOK.

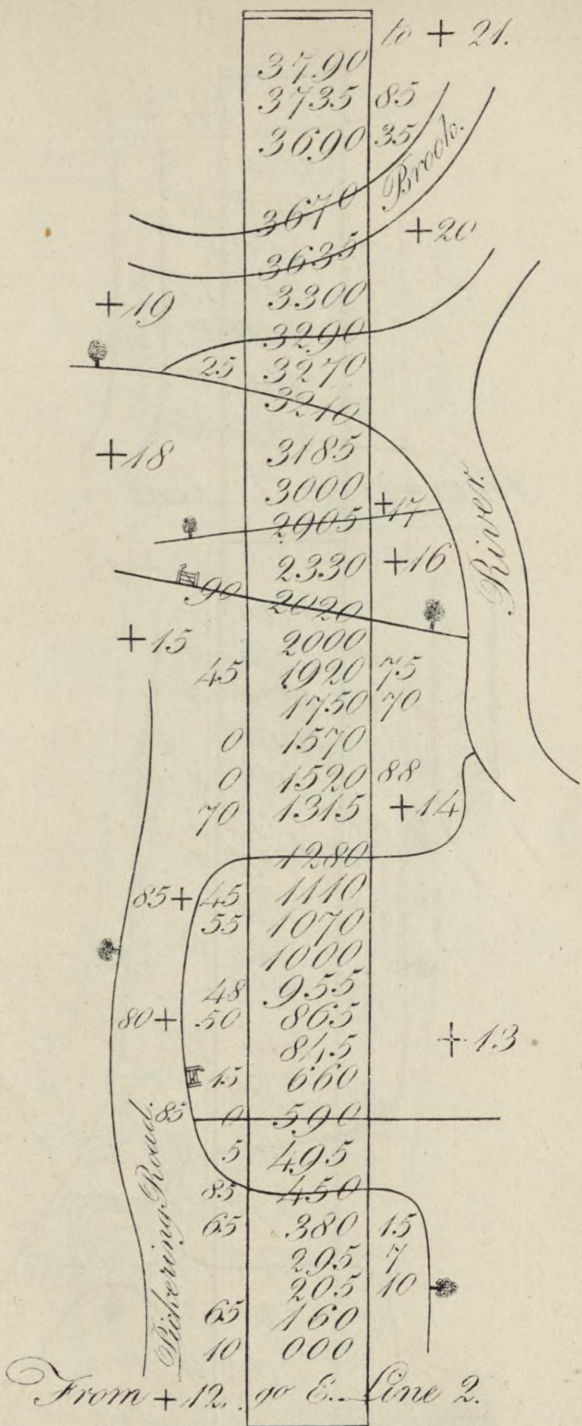

Preston.
to +9.
Spratley Road
Dutton.
0 257
21 200
25 150
26 100
20 50
0 000 Tree.
From +1. go S. W. Line 4.
Run upon 1 Line produced. 30 Links S. of +1.
3270
3100 12
3000
2933
2900 20 Gate.
2600 25 Tree
2400 14
Caley
2300
+ 8
Spratley Road. 90 broad.
34 2187
2140 Gate.
47 2087 Tree.
48 2000
75 1800
89 1600 Tree.
78 1450 Gate.
Pond. 50 70 1050 +7
Ellard.
45 1000
39 600 Tree.
35 300
0 000
From +6, go S. W. Line 3.

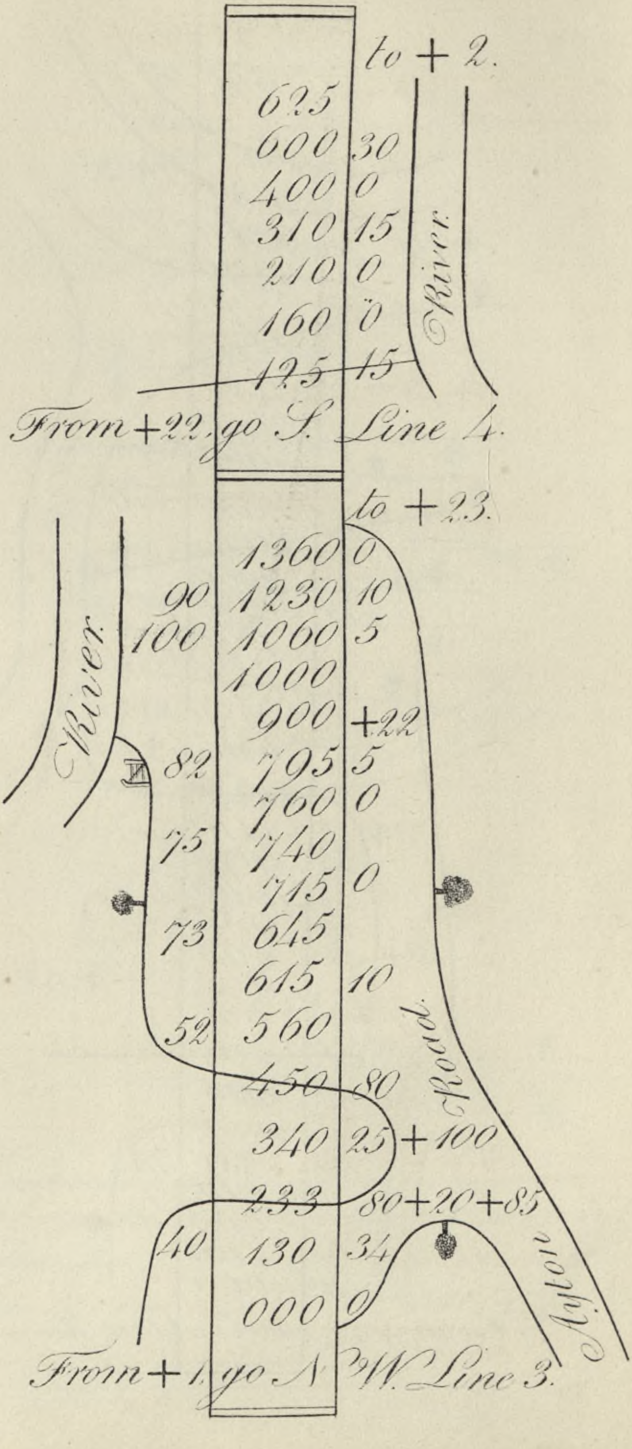

Finis.
to + 7.
0
102
856
415
Drain
380
10 broad.
350
67
000
0
From + 5, go S. Line 8.
to + 4.
1712
30
Tree
1600
1400
15
Tree
1200
0
1000
900 Tree
500 Tree
Stricker.
From + 10, go N. Line 7.
to + 8.
1046
700 Tree.
411
+ 10
350
200 Tree
From + 2, go E. Line 6.
to + 2.
15
1004
600 Tree
420 Gate.
18
365
Whisker
24
200 Tree
20
100
0
000 Gate.
Hull Turnpike.
From + 9, go N. Line 5.

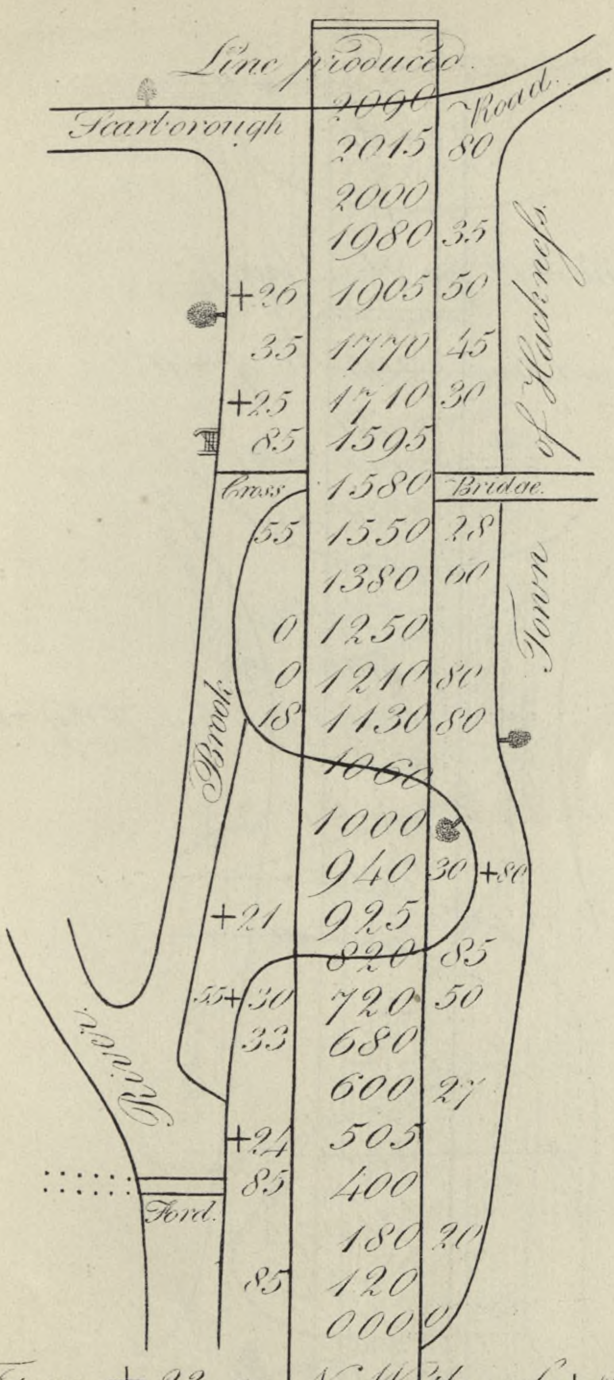


See Plate 10. The Notes of an Estate lying in the Parishes of Hackness and Hutton-Bushel, in the North-Riding of the County of York.

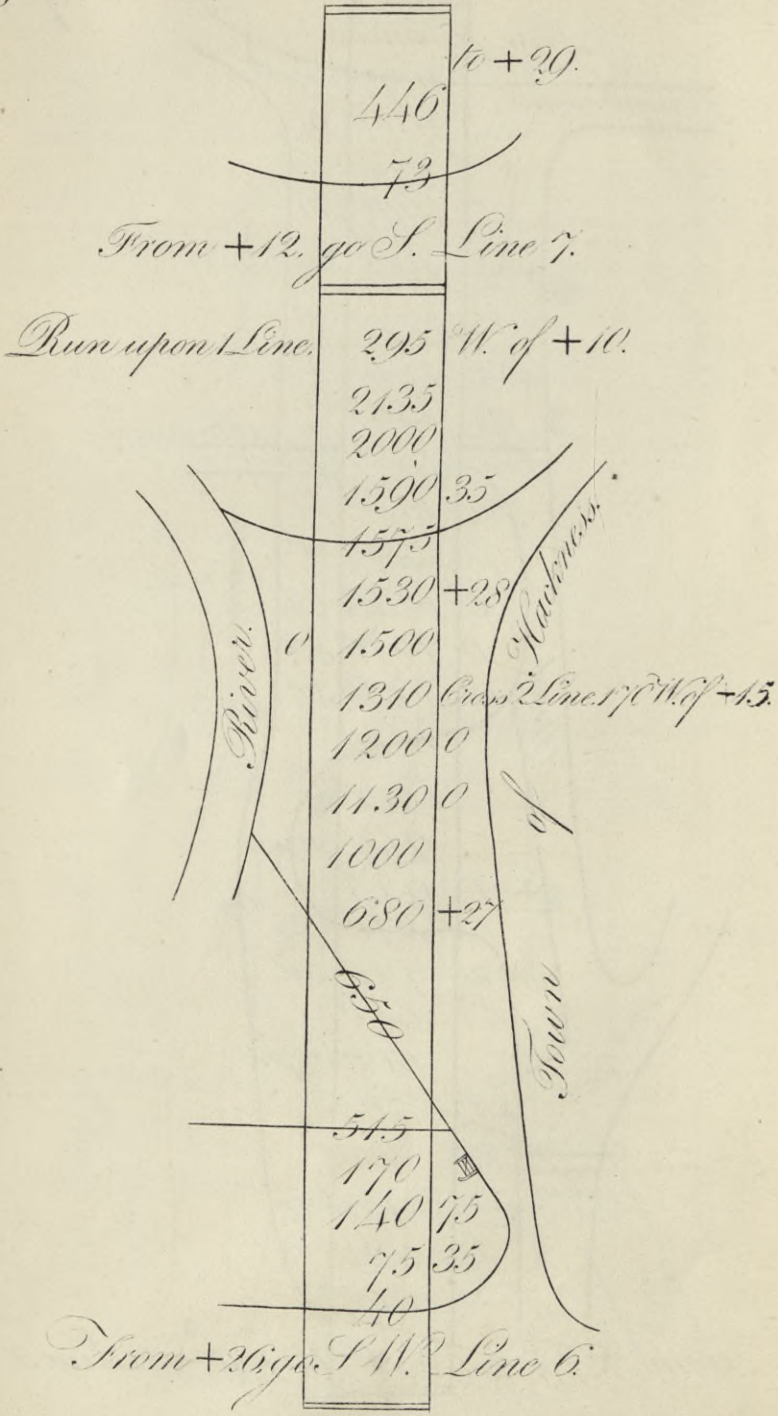
FIELD-BOOK.

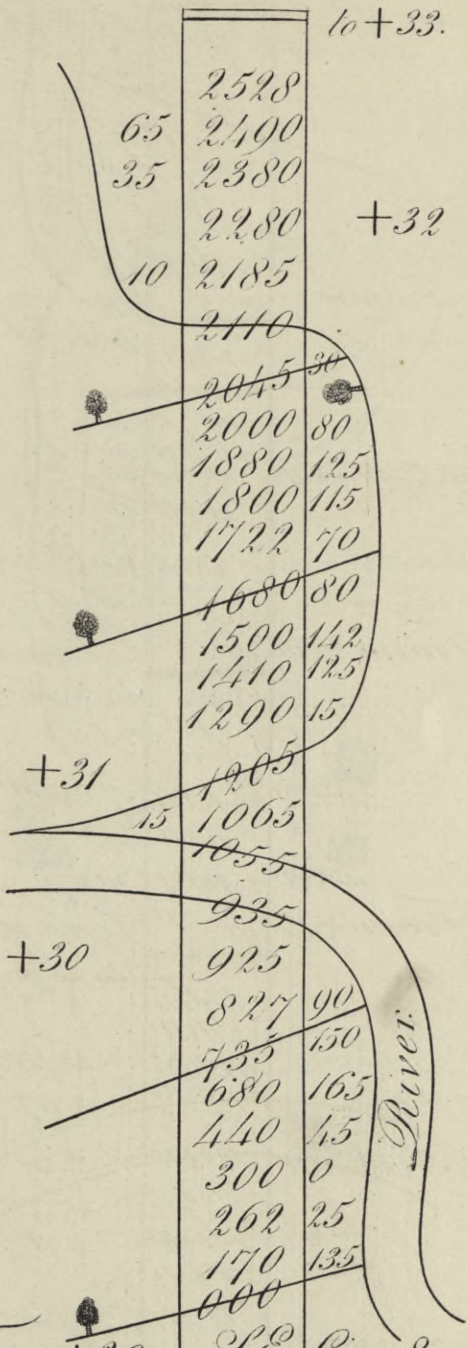

to + 21.
3790
3735 85
3690 35
Brook.
3670
+20
3635
3300
+19
3290
25 3270
3210
+18
3185
River.
3000
+17
2905
2330 +16
90 2020
+15
2000
45 1920 75
1750 70
0 1570
0 1520 88
70 1315 +14
1280
85+ 45 1110
55 1070
1000
48 955
80+ 50 865
845
+13
15 660
85 0 590
5 495
85 450
65 380 15
295 7
205 10
65 160
10 000
Pickering Road.
From + 12. go E. Line 2.


to + 2.
625
600 30
400 0
310 15
210 0
160 0
125 15
River.
From + 22, go S. Line 4.
to + 23.
1360 0
90 1230 10
100 1060 5
1000
900 +22
82 795 5
760 0
75 740
715 0
73 645
615 10
52 560
450 80
340 25 + 100
233 80 + 20 + 85
40 130 34
000 0
River.
Road.
Ayton
From + 1, go N W. Line 3.

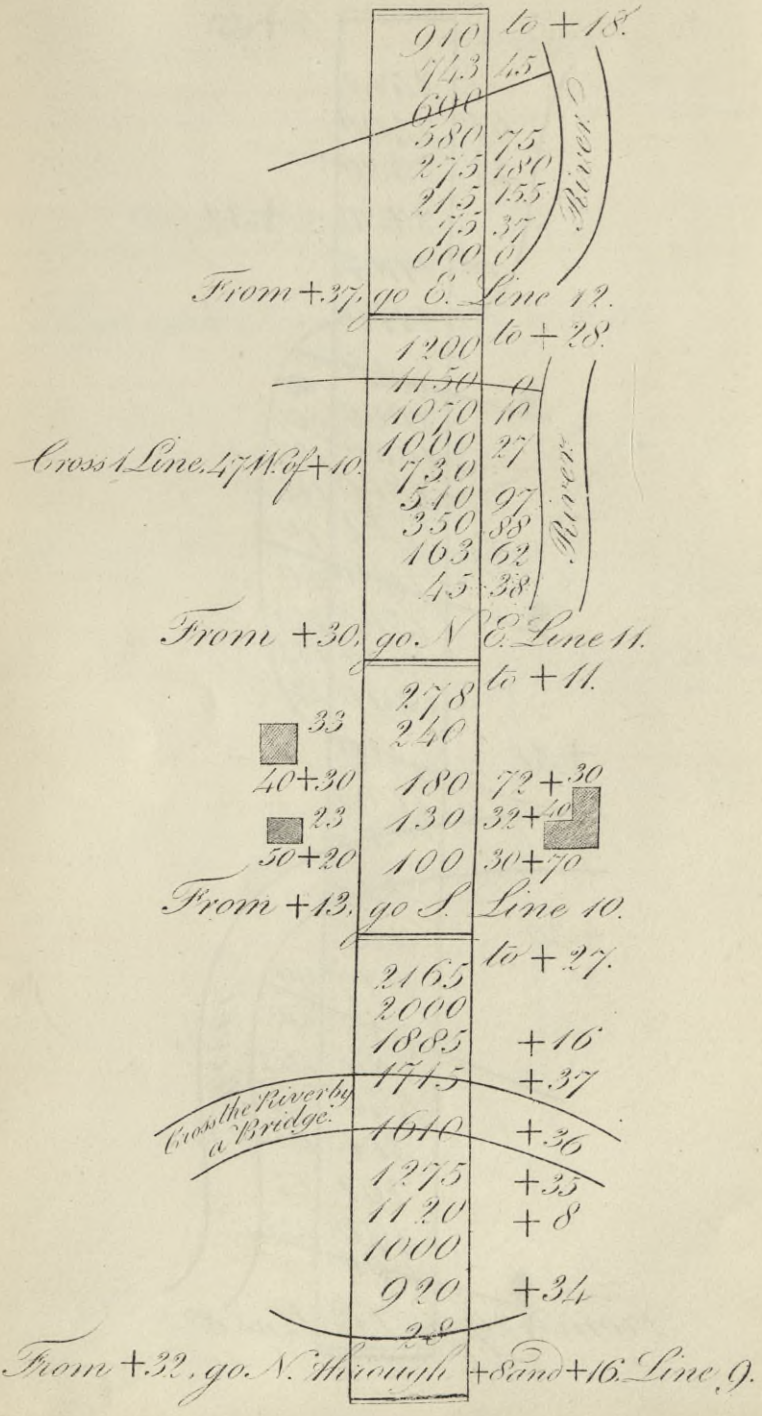


From + 23, go N W. through + 21 Line 5.

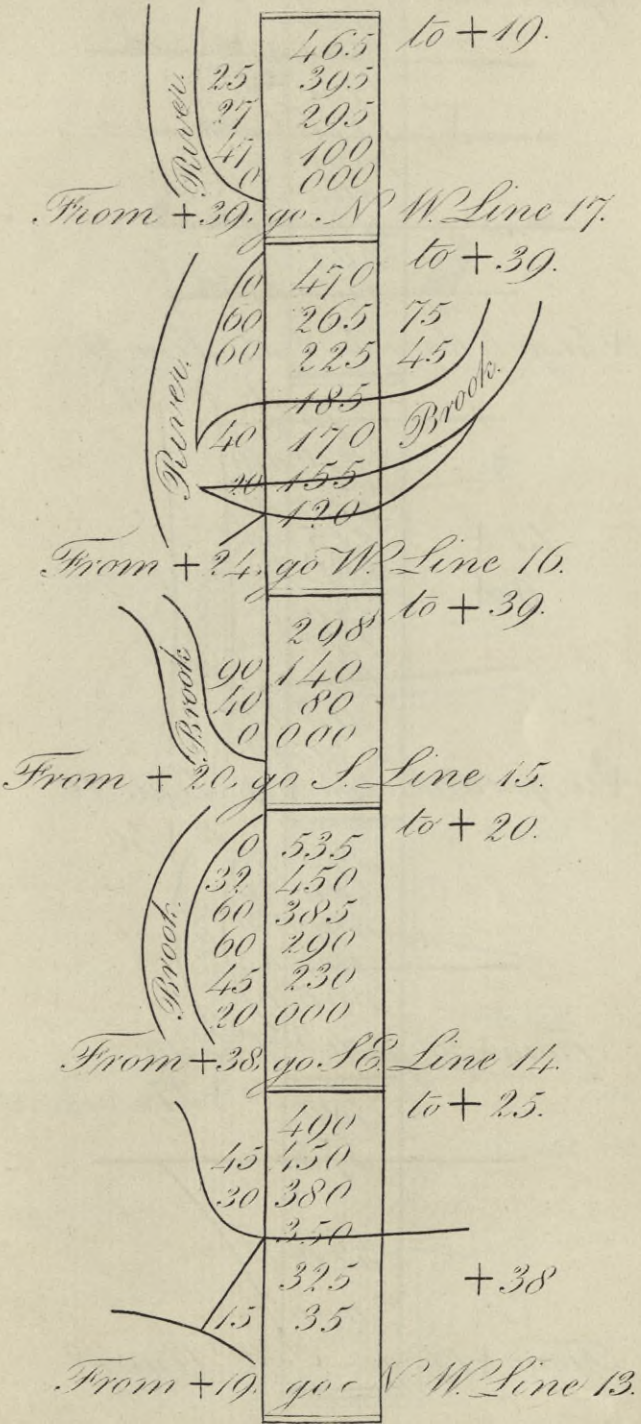

to +29.
446
73
From +12. go S. Line 7.
Run upon 1 Line.
295 W. of +10.
2135
2000
1590 35
1575
1530 +28
Hackness
0 1500
1310 Cross 2 Line 170 W. of +15.
River.
1200 0
1130 0
of
1000
680 +27
650
Town
515
170
140 75
75 35
40
From +26. go S.W. Line 6.



From +29. go S.E. Line 8.



to + 18.
910
743 45
600
580 75
275 180
215 155
75 37
000 0
River.
From + 37, go E. Line 12.
to + 28.
1200
1150 0
1070 10
1000 27
Cross 1 Line, 47 W. of + 10.
730
540 97
350 88
163 62
45 38
River.
From + 30, go N E. Line 11.
to + 11.
278
33
240
40 + 30
180
72 + 30
23
130
32 + 40
50 + 20
100
30 + 70
From + 13. go S. Line 10.
to + 27.
2165
2000
1885 + 16
1715 + 37
Cross the River by a Bridge.
1610 + 36
1275 + 35
1120 + 8
1000
920 + 34
28
From + 32, go N. through + 8 and + 16. Line 9.


to + 19.
465
25 395
27 295
47 100
0 000
River.
From + 39. go N. W. Line 17.
to + 39.
0 470
60 265 75
60 225 45
185
River.
Brook.
40 170
20 155
120
From + 24. go W. Line 16.
to + 39.
298
90 140
40 80
0 000
Brook.
From + 20. go S. Line 15.
to + 20.
0 535
32 450
60 385
60 290
45 230
20 000
Brook.
From + 38. go S.E. Line 14.
to + 25.
490
45 450
30 380
350
325 + 38
15 35
From + 19. go N. W. Line 13.

Run upon 1 Line. 72 E. of + 7.

1635

1562 20 + 30

1150

1110 50

20

1000

865 + 34

40 + 5 655 40

305

265 35

20

From + 41, go E. through + 34. Line 21.

to + 31.

1475

1070 110

1000

980 80

41 + 815 35

660 0

9 + 385

320 0

250 5

190 5

River.

95 15

25 15

From + 40, go S.W. through + 9. Line 20.

to + 40.

575 5

480 26

390 40

205 25

River.

000 0

From + 36, go W. Line 19.

to + 27.

580

575

320 15

000

From + 17, go N. W. Line 18.

to + 3.

	805
	765
50	700
110	600
140	510
180	420
160	320
130	260
50	80
0	000

River.

From + 45. go S. Line 25.

to + 45.

0	1415
50	1310
130	1070
	1000
110	960
60	855
10	660
0	575
0	480
100	278
130	160
130	125

\+ 44

River.

From + 43. go E. Line 24.

to + 43.

15	955
20	835
25	720
0	600
0	520
	425
	208

\+ 42

River.

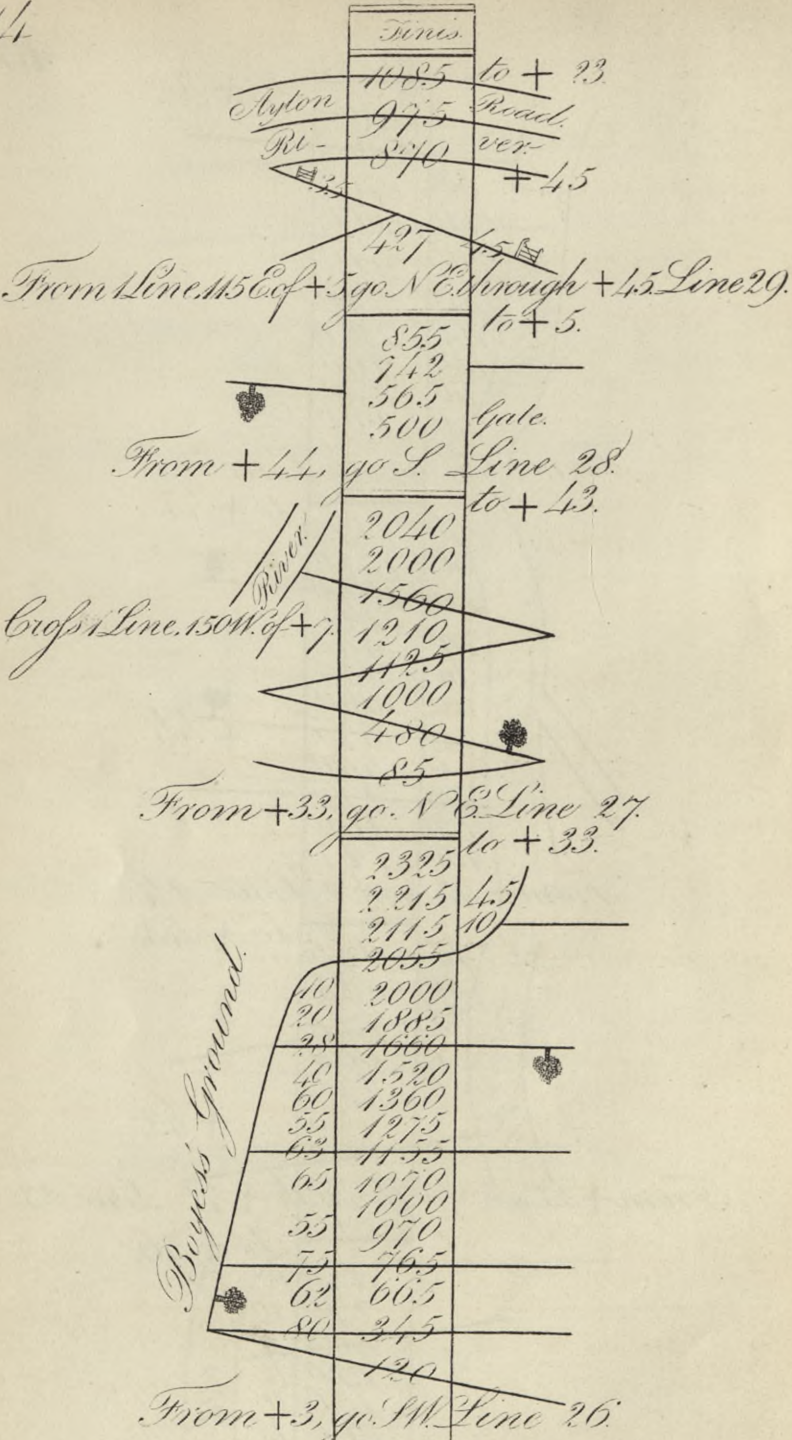
From + 35. go N E. through + 42 Line 23.

to + 36.

340	0
270	10
205	8
150	0
100	0
000	15

River.

From + 42. go W. Line 22.


Lines.
1085 to + 23.
Ayton 975 Road.
Ri- 870 ver.
+ 45
35
427 45
From 1 Line 115 E. of + 5 go N E. through + 45 Line 29.
to + 5.
855
742
565
500 Gate.
From + 44, go S. Line 28.
to + 43.
River.
2040
2000
1560
Cross 1 Line. 150 W. of + 7. 1210
1125
1000
480
85
From + 33. go. N E. Line 27.
to + 33.
2325
2215 45
2115 10
2055
10 2000
20 1885
28 1660
40 1520
60 1360
55 1275
63 1155
65 1070
1000
55 970
75 765
62 665
80 345
120
Boyess' Ground
From + 3, go. SW. Line 26.

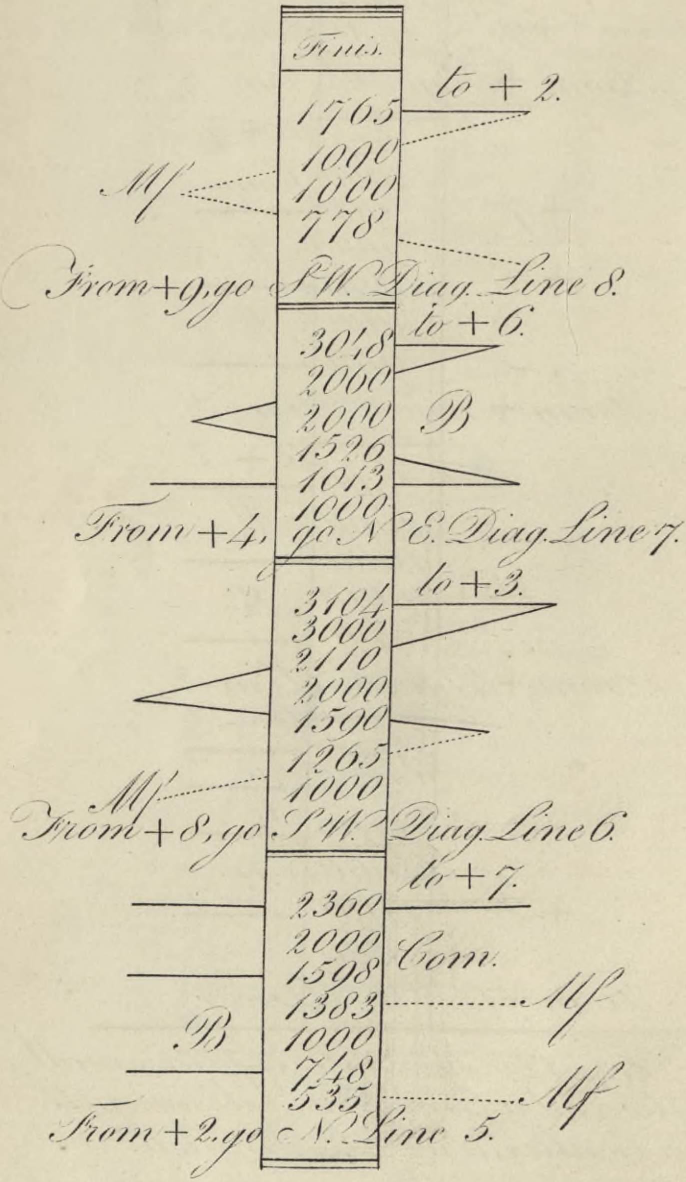
	2353	to +1.
	2000	
	1790	430
	1000	Mf.
+9	930	640
From +8.	go S.	Line 4.
	3950	to + 8
	3000	Com.
+7	2900	
	2000	A
+ 6	1950	
	1000	A
	950	
From +5.	go E.	Line 3.
	2400	to + 5.
	2000	A
	1600	
	1000	B
	800	
From +4,	go N.	Line 2.
	4000	to + 4.
	3000	C
+3	2100	C
	2000	C
+ 2	1100	
	1000	
From +1.	go W.	Line 1.

See Plate 12.– The Notes of three small Estates and a Common, belonging to three Gentlemen. A. B. and C.

FIELD-BOOK.

FINIS.

Mf denotes a quality-line, and a tree that side of the ditch on which the fence stands.



NESBIT'S

PRACTICAL LAND-SURVEYING.

EDITED BY

WILLIAM BURNESS, F.R.A.S.
PROFESSOR OF PRACTICAL LAND-SURVEYING.

Thirteenth Edition.

LONDON:
LONGMANS, GREEN, READER, AND DYER.
1877.

ADVERTISEMENT

TO

THE ELEVENTH EDITION.

THE thorough revisal and enlargement of NESBIT'S PRACTICAL LAND-SURVEYING cannot fail to continue for it the very prominent position which it has long held as a standard work. The sale of TEN LARGE EDITIONS, and the demand for the present one, is a very gratifying testimony of the patronage it has hitherto merited of land-surveyors, civil engineers, masters of seminaries, and the general public.

The subdivision of the work into Ten Parts, as in the last edition, has been continued. The *First Part* is elementary, and contains definitions, with a few select problems and theorems. The *Second* gives a description of the chain, offset-staff, compass, &c., with directions for their use. The *Third* and *Fourth* embrace the mensuration of single fields, the former with the common cross-staff and chain, the latter with the chain only. Part IV. also teaches how to survey hilly and uneven ground. The surface of the earth consists of a series of hypotenusal planes joined by curved lines, and before the dimensions of these can be plotted on to a plan, they have to be reduced to base-measure or that of a common plane; and this part gives various methods of doing so by the chain, quadrant-level, and theodolite. The *Fifth*, *Sixth*, and *Seventh* teach how to survey large estates, apportion land, plot, finish, transfer and reduce plans. Part VI. belongs to the higher branch of land-surveying, and is divided into *four sections*, the *first* of which is elementary, the *second* practical; the *third* gives general directions for field operations, and the *fourth* the reduction of customary to statute measure. Part VII. includes the surveying of farm-buildings, cities, towns, villages, and lands for building purposes. The *Eight* and *Ninth* teach the elements of trigonometry, and their application by examples. *Railway*

Surveying follows, and includes illustrated general directions for taking a preliminary survey of a projected railway. The *Tenth* includes *Railway Engineering* under *seven sections*, viz. :—

Section I. treats of levels and levelling, including trial, check, and cross levels; check-levelling; correction for curvature and refraction.

Section II. contains a Parliamentary survey, with plan and section; the method of keeping the level-book and of plotting the sections; also the method of laying out gradients, &c.

Section III. shows the general principles and practice of laying out railway curves, including compound, serpentine, and deviation examples, with the necessary constructions and formulas required.

Section IV. embraces railway earthworks, and gives formulas for setting out the width of the ground, for finding the areas of the railway estate, together with auxiliary tables for finding the contents of sectional areas.

Section V. gives the methods of setting out the ground for tunnels and tunnelling, including tables of dimensions.

Section VI. is on viaducts, aqueducts, oblique railway bridges, and includes examples with dimensions.

Section VII. comprises the survey and applications of rules for the superelevation of the outside rails at railway curves.

The following is a brief summary of the principal additions and alterations made to the present edition.

Part II.—Two new articles have been written for this Part: the one on *the variation of the needle of the compass;* and the other, directions for correcting the compass so as *to find out the true meridian.*

Parts III. and IV.—In these two Parts one proposition (page 85) has been corrected, and two new ones, with a diagram, added, in order to supply the necessary instruction required in ranging and measuring lines in surveys with the chain when sight is obstructed. The methods of measuring *hilly ground* have been re-written; so have the articles under a Quadrant, a brief description of several additional instruments of this kind being given. *General directions* for surveys on the vertical plane have likewise been added in Part III.

Parts V. and VI.—Very many corrections have been made in these two divisions, paragraphs out of date suppressed, and new ones written to supply their places. General directions for eight additional surveys have been added to the latter (Part VI.), while the statutory matter which it contained in previous editions has been transferred to the end of the work in the form of an Appendix. The two first surveys are on the horizontal plane, and embrace *lands intersected by a railway.* The remaining six include surveys of rivers, tidal, mountainous, and intermediate; canals, embankments, lands reclaimed from the sea, &c. They are principally on the vertical plane, and form together a large section, viz., Section III.

Part VII.—General directions for surveying and designing mansions, farm homesteads, stackyards, &c., have been added to this Part; other portions of it have been re-written.

Part VIII. has been divided into *five Sections.* Sect. I. consists of definitions and corollaries. Sect. II. treats of the species, constructions, ratios, resolution and solution, of triangles. Sect. III. gives the trigonometrical solution of plane triangles, right-angled, and oblique. Sect. IV. the measurement of angles, with an illustrated description of the theodolite. Sect. V. is on the trigonometrical survey of *heights and distances.* The first two sections have been added to this edition of the work. They have been written expressly to illustrate, by means of the several diagrams annexed, the elementary principles and practice of trigonometrical surveying: it is hoped they will prove not only instrumental in facilitating the education of private students, but also in promoting that of articled pupils in field operations with the theodolite, and in the office work of plotting the bearings. Three of the cases in Sect. III. have been re-written, and numerous corrections made in this and the remaining two sections of this Part.

Part IX. embraces examples of trigonometrical surveying; it has been divided into *seven Sections.* Sect. I. contains rules for finding the areas of triangles by logarithms. Sect. II. To survey a wood. Sect. III. To survey a wood or river. Sect. IV. gives the practice of taking bearings by two stations. Sect. V. To survey a town or city. Sect. VI. To range lines and take bearings in extensive surveys. Sect. VII. To survey with the circumferentor. Of these, Sect. VI. has been written for the present edition, and also the Introduction containing the new subdivision of this Part.

The section under Railway Surveying has been re-written in the form of illustrated general directions for taking a preliminary survey of a portion of a projected railway. In the previous edition the illustration or plan (p. 387), and the subject-matter of the section generally, was in Part IX. under a different heading; but as the field notes for the plan were wanting, and as the subject-matter was otherwise very defective, and as the former omission could not be supplied, or the latter defects corrected, without an actual survey of the ground,—conditions almost wholly impracticable,—it was considered advisable to make this change, as preliminary surveys of the kind in question are frequently demanded for reasons that will be found in the section to which we refer.

In this branch of land-surveying it is a standing rule that the measure of every line and angle shall be determined both by linear and angular instruments; and when an error or an omission is detected in any of the field books open, the error must be corrected, and the omission supplied in the field. The reason of this is, because the exceptions are few where a single line in a survey can be accurately measured with the chain; consequently, if a continuous check is not kept upon linear measure by angular, the field notes, both for plotting and setting out curve lines and inclinations or gradients, cannot be obtained with that degree of accuracy which the nature of the survey demands. If the station poles are accurately ranged, then the lines in the field close; and in plotting either plan or section, the only proof of accuracy admissible is when the lines formed by the station-points also close, for unless they do so, the subsequent operations of the survey cannot be correctly performed.

W. B.

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EXPLANATION

OF THE PRINCIPAL

MATHEMATICAL CHARACTERS.

The sign or character = (called *equality*) denotes that the respective quantities between which it is placed are equal; as 4 poles = 22 yards = 1 chain = 100 links.

The sign + (called *plus*, or more) signifies that the numbers between which it is placed are to be added together; as $9 + 6$ (read 9 *plus* 6) = 15. Geometrical lines are generally represented by capital letters; thus AB + CD signifies that the line CD is to be added to the line AB.

The sign − (called *minus*, or less) denotes that the quantity which it precedes is to be subtracted; as $15 - 6$ (read 15 *minus* 6) = 9. In geometrical lines also, AB − CD signifies that the line CD is to be subtracted from the line AB.

The sign × denotes that the numbers between which it is placed are to be multiplied together; as 5×3 (read 5 multiplied by 3) = 15.

The sign ÷ signifies division; as $15 \div 3$ (read 15 divided by 3) = 5. Numbers placed like a vulgar fraction also denote division; the upper number being the dividend, and the lower the divisor; as $\frac{15}{3} = 5$.

The signs : :: : (called *proportionals*) denote proportionality; as 2 : 5 :: 6 : 15, signifying that the number 2 bears the same proportion to 5, as 6 does to 15; or, in other words, as 2 is to 5, so is 6 to 15.

The sign $\overline{\qquad\qquad\qquad}$ (called *vinculum*) is used to connect several quantities together; as $\overline{\overline{9 + 3} - 6} \times 2 = \overline{12 - 6} \times 2 = 6 \times 2 = 12$.

The sign ^{2}, placed above a quantity, represents the square of that quantity; as $\overline{5 + 3}^2 = 8^2 = 8 \times 8 = 64$.

The sign ^{3}, placed above a quantity, denotes the cube of that quantity; as $\overline{\overline{9 + 3} \quad 8}^3 = \overline{12 - 8}^3 = 4^3 = 4 \times 4 \times 4 = 64$.

The sign $\sqrt{}$ or $\sqrt[2]{}$, placed before a quantity, denotes the square root of that quantity; as $\sqrt{9 \times 4} = \sqrt{36} = 6$.

The sign $\sqrt[3]{}$, placed before a quantity, represents the cube root of that quantity; as $\sqrt[3]{6 \times \overline{4 \times 3 -} 8} = \sqrt[3]{24 \times \overline{3 - 8}} = \sqrt[3]{72 - 8} = \sqrt[3]{64} = 4$.

DIRECTIONS TO BOOKBINDER.

Plans VIII., IX., X., XI., XIII., and Earthwork Tables at the end of the work, in the order thus given:

Fold Plans VIII. and IX. to face each other when open.
Fold Plans X. and XI. to face each other when open.
Fold Plan XIII. to face, when open, the work.
Fold Earthwork Tables to face, when open, the work.

LAND-SURVEYING.

PART I.

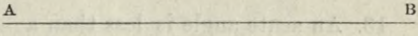
DEFINITIONS, PROBLEMS, AND THEOREMS IN GEOMETRY.

GEOMETRY originally signified the art of MEASURING the EARTH, or any distance or dimensions upon, or within it; but it is now used for the science of quantity, extension, or magnitude.

GEOMETRICAL DEFINITIONS.

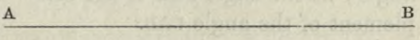
1. A point is considered as having neither length, breadth, nor thickness.

2. A line has length, but is considered as having neither breadth nor thickness; as AB.

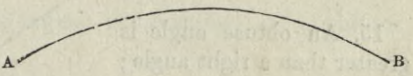


3. Lines are either right, curved, or parallel.

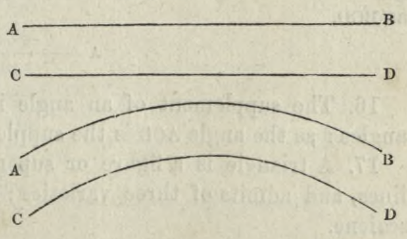
4. A right or straight line lies wholly in the same direction between its extremities, and is the shortest distance between two points; as AB.



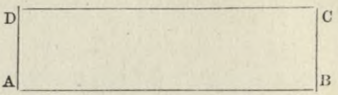
5. A curved line continually changes its direction between its extremities; as AB.



6. Parallel lines always remain at the same distance from each other, and, though continually produced, would never meet; as AB and CD.



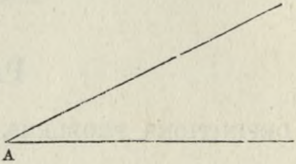
7. A surface or superficies has length and breadth, but is considered as having no thickness; as ABCD.



8. A superficies may be contained within one curved line, but cannot be contained within fewer than three straight lines.

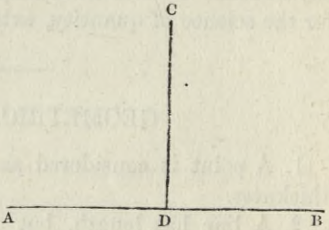
9. The area of a figure is its superficial content, or the measurement of its surface.

10. An angle is the inclination or opening of two lines having different directions and meeting in a point, as at A, which is called the angular point; and, when three letters are used, the middle one denotes that point.



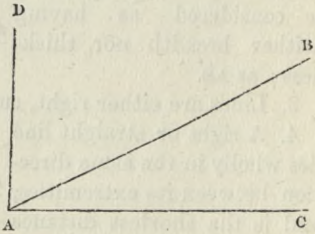
11. Angles are of three kinds, viz., right, acute, and obtuse.

12. A right angle is made by one right line standing perpendicularly upon another; thus, if DC be perpendicular to AB, the angles ADC and CDB are both right angles.

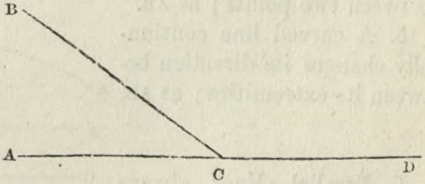


13. An acute angle is less than a right angle; as CAB.

14. The complement of an angle is what it wants to complete a right angle; as the angle DAB is the complement of the angle CAB.



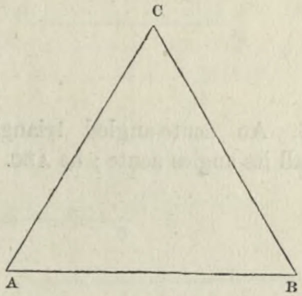
15. An obtuse angle is greater than a right angle; as BCD.



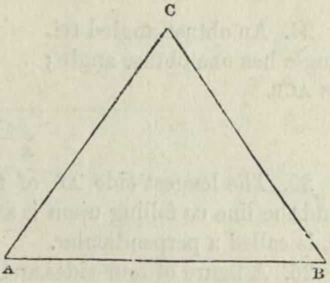
16. The supplement of an angle is what it wants of two right angles; as the angle ACB is the supplement of the angle BCD.

17. A triangle is a figure or superficies bounded by three right lines, and admits of three varieties; viz., equilateral, isosceles, and scalene.

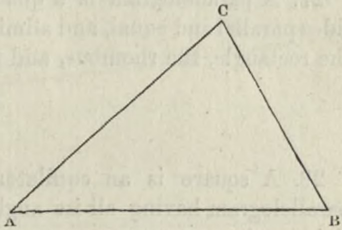
18. An equilateral triangle has all its sides equal; as ABC.



19. An isosceles triangle has only two of its sides equal; as ABC.

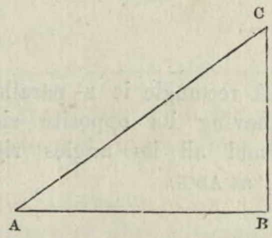


20. A scalene triangle has all its sides unequal; as ABC.

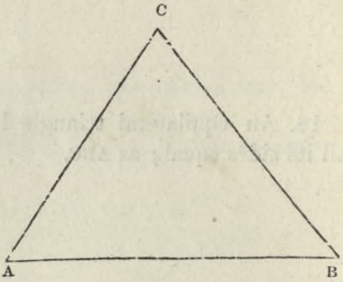


21. Triangles are also right-angled, acute-angled, and obtuse-angled.

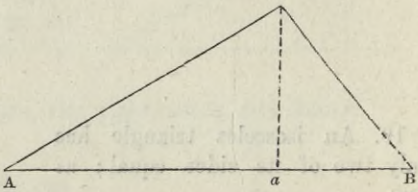
22. A right-angled triangle has one right angle, the side opposite to which is called the hypotenuse, the other two being termed legs, or one the perpendicular, and the other the base; thus AC is the hypotenuse, BC the perpendicular, and AB the base.



23. An acute-angled triangle has all its angles acute; as ABC.



24. An obtuse-angled triangle has one obtuse angle; as ACB.

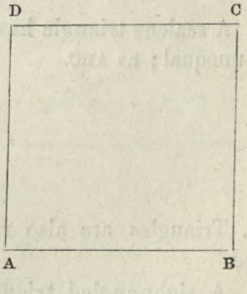


25. The longest side AB of the triangle ABC is called the base; and the line C*a* falling upon it at right angles from the opposite angle C, is called a perpendicular.

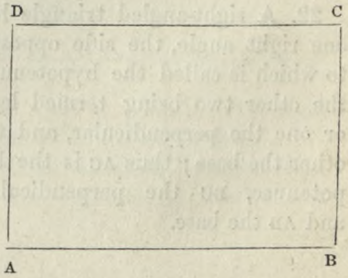
26. A figure of four sides and angles is denominated a quadrangle or quadrilateral figure.

27. A parallelogram is a quadrilateral figure, having its opposite sides parallel and equal, and admits of four varieties; viz., the square, the rectangle, the rhombus, and the rhomboid.

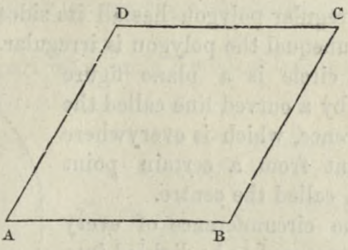
28. A square is an equilateral parallelogram having all its angles right angles; as ABCD.



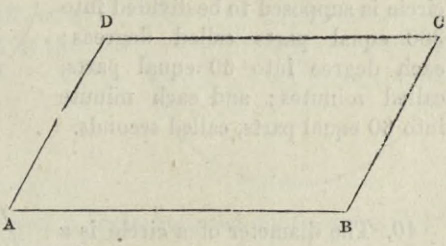
29. A rectangle is a parallelogram having its opposite sides equal and all its angles right angles; as ABCD.



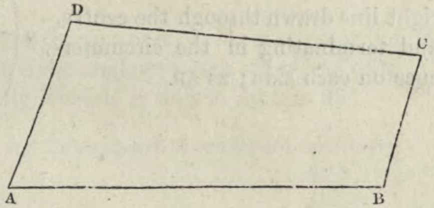
30. A rhombus is an equilateral parallelogram having its opposite angles equal; as ABCD.



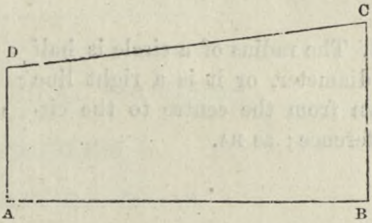
31. A rhomboid is a parallelogram having its opposite sides and angles equal; as ABCD.



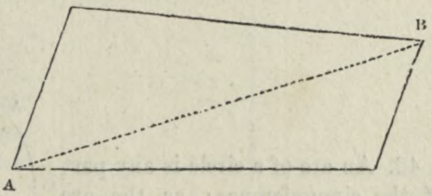
32. A trapezium is a quadrilateral figure whose opposite sides are not parallel to each other; as ABCD.



33. A trapezoid is a quadrilateral figure having two of its opposite sides parallel, and its angles, one acute, one obtuse, and two right-angled; as AD is parallel to BC, the angles A and B are right angles; the angle D is obtuse, and C acute.



34. A diagonal is a right line joining the two opposite angles of a quadrilateral figure; as AB.

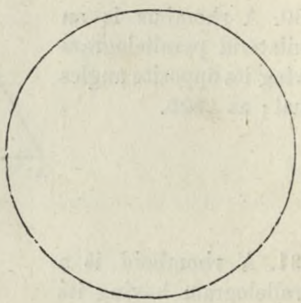


35. Plane figures having more than four sides are generally called polygons, and receive their particular denominations from the number of their sides or angles.

36. A pentagon is a polygon of five, a hexagon of six, a heptagon of seven, an octagon of eight, a nonagon of nine, a decagon of ten, an undecagon of eleven, and a duodecagon of twelve sides.

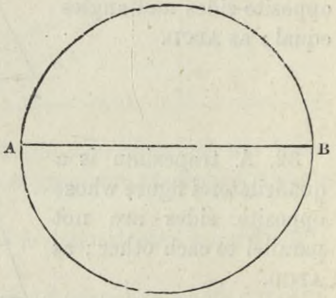
37. A regular polygon has all its sides and angles equal. When they are unequal the polygon is irregular.

38. A circle is a plane figure bounded by a curved line called the circumference, which is everywhere equidistant from a certain point within it, called the centre.

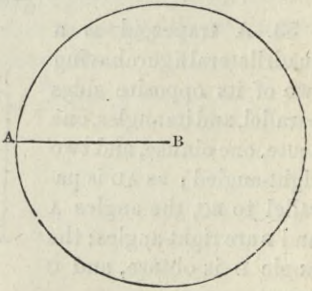


39. The circumference of every circle is supposed to be divided into 360 equal parts called degrees; each degree into 60 equal parts, called minutes; and each minute into 60 equal parts, called seconds.

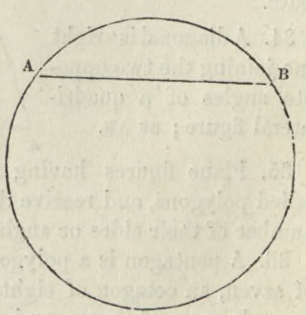
40. The diameter of a circle is a right line drawn through the centre, and terminating in the circumference on each side; as AB.



41. The radius of a circle is half the diameter, or it is a right line drawn from the centre to the circumference; as BA.



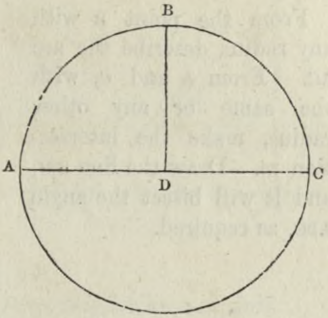
42. An arc of a circle is any part of the circumference; as the arc AB.



43. A chord is a right line joining the extremities of an arc; as the line AB.

44. A segment is any part of a circle bounded by an arc and its chord.

45. A semicircle is half of a circle, or a segment cut off by the diameter; as ABC.



46. A sector is any part of a circle bounded by an arc and two radii.

47. A quadrant is the fourth part of a circle, or a sector bounded by an arc and two radii at right angles to each other; as CDB.

Corol. Hence a right angle is said to contain 90°.

NOTE.—All Definitions and Rules should be committed to memory.

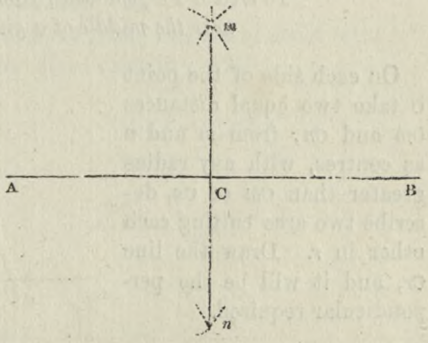
GEOMETRICAL PROBLEMS.

PROBLEM I.

To bisect a given Line AB.

From A and B as centres, with any radius greater than half AB, in your compasses, describe arcs cutting each other in *m* and *n*.

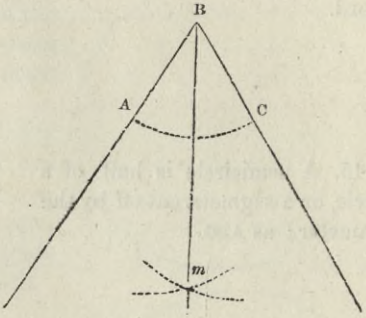
Draw the line *m*C*n*, and it will bisect AB in C.



PROBLEM II.

To bisect a given Angle ABC.

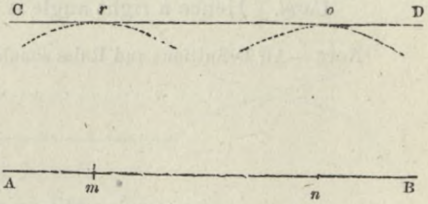
From the point B with any radius describe the arc AC. From A and C, with the same or any other radius, make the intersection *m*. Draw the line B*m*, and it will bisect the angle ABC, as required.



PROBLEM III.

To draw a Line parallel to a given Line AB *at a given Distance.*

From any two points, *m* and *n*, in the given line, with the given distance as a radius, describe the arcs *r* and *o*. Draw CD to touch these arcs, without cutting them, and it will be parallel to AB.

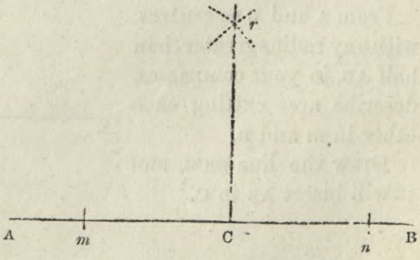


NOTE.—This problem may be more readily performed by a parallel ruler.

PROBLEM IV.

To erect a Perpendicular from a given Point C, *near the middle of a given Line* AB.

On each side of the point C take two equal distances C*m* and C*n*; from *m* and *n* as centres, with any radius greater than C*m* or C*n*, describe two arcs cutting each other in *r*. Draw the line C*r*, and it will be the perpendicular required.

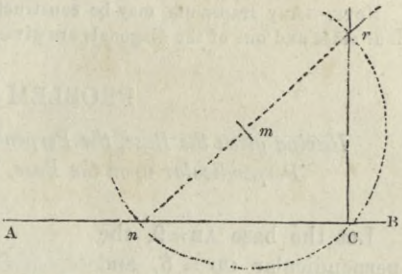


PROBLEM V.

To erect a Perpendicular from a given Point C, *near the End of a given Line* AB.

From any point *m*, as a centre, with the radius or distance C*m*, describe an arc cutting the given line in C and *n*.

Through *n* and *m* draw a line cutting the arc in *r*. Draw the line C*r*, and it will be the perpendicular required.

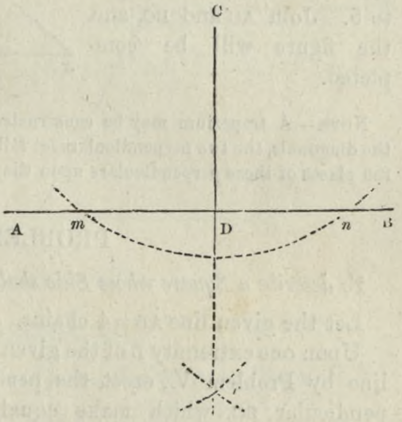


PROBLEM VI.

From a given Point C, *to let fall a Perpendicular upon a given Line* AB.

With C as a centre, and any radius a little exceeding the distance of the given line, describe an arc cutting AB in *m* and *n*. With the centres *m* and *n*, and the same or any radius exceeding half their distance, describe arcs intersecting each other in *r*. Draw the line C*r*, and CD will be the perpendicular required.

NOTE.—The last three problems may be easily performed by a square or a plotting scale.

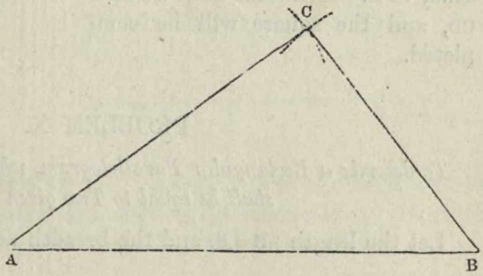


PROBLEM VII.

To make a Triangle with Three given Lines, any two of which must be greater than the Third (Euclid i. 22).

Let the given lines be AB = 10, AC = 8, and BC = 6 chains.

From any scale of equal parts (which is to be understood as employed likewise in all the following problems) lay off the base AB.



With the centre A and radius AC describe an arc. With the centre B and radius BC describe another arc cutting the former in C. Draw the lines AC and BC, and the triangle will be completed.

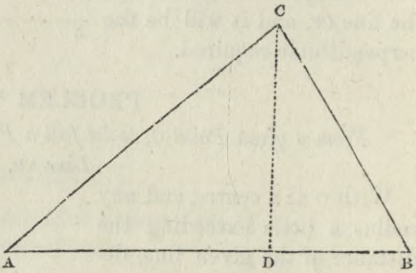
NOTE.—Any trapezium may be constructed in the same manner, when the four sides and one of the diagonals are given.

PROBLEM VIII.

Having given the Base, the Perpendicular, and the Place of the Perpendicular upon the Base, to construct a Triangle.

Let the base $AB = 9$, the perpendicular $CD = 5$, and the distance $AD = 6$ chains.

Make $AB = 9$, and $AD = 6$. At D erect the perpendicular DC, which make equal to 5. Join AC and BC, and the figure will be completed.



NOTE.—A trapezium may be constructed in a similar manner, when one of the diagonals, the two perpendiculars let fall thereon from the opposite angles, and the places of these perpendiculars upon the diagonal are given.

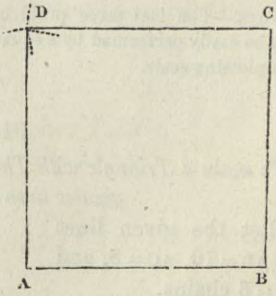
PROBLEM IX.

To describe a Square whose Side shall be equal to a given Right Line.

Let the given line $AB = 4$ chains.

Upon one extremity B of the given line by Problem V., erect the perpendicular BC, which make equal to AB.

With A and C as centres, and the radius AB, describe arcs cutting each other in D. Draw the lines AD and CD, and the square will be completed.

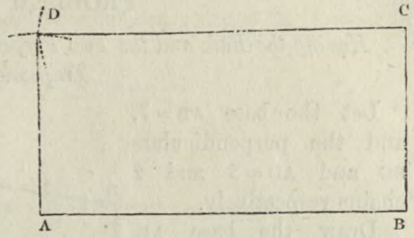


PROBLEM X.

Tc describe a Rectangular Parallelogram, whose Length and Breadth shall be equal to Two given Lines.

Let the length $AB = 8$, and the breadth $BC = 4$ chains.

At B erect the perpendicular BC, which make equal to 4. With A as a centre, and the radius BC, describe an arc; and with C as a centre, and the radius AB, describe another arc, cutting the former in D. Draw the lines AD and CD, and the rectangle will be completed.



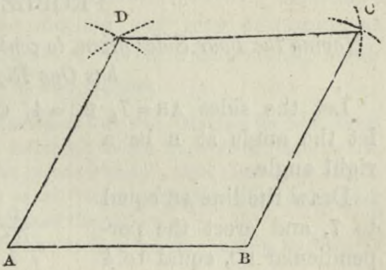
PROBLEM XI.

Upon a given Right Line to construct a Regular Rhombus.

Let the given line AB = 4 chains.

Draw the line AB equal to 4. With A and B as centres, and the radius AB, describe arcs cutting each other in D; then with B and D as centres, and the same radius, make the intersection C.

Draw the lines AD, DC, and BC, and the rhombus will be completed.

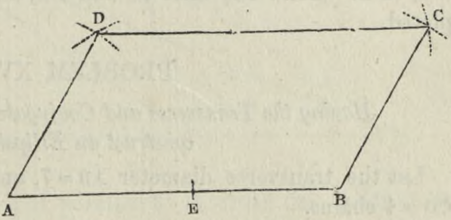


PROBLEM XII.

Having any Two Right Lines given, to construct a Rhomboid.

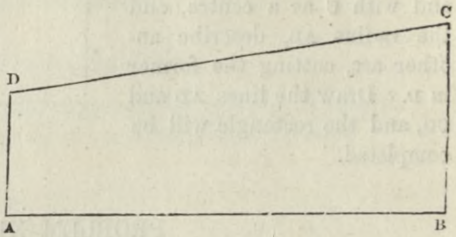
Let the given lines be AB = 7, and BC = 4 chains.

Draw the line AB equal to 7. Take in your compasses the line BC and lay it from A to E. With A and E as centres, and the radius AE, make the intersection D. Then with B as a centre, and the same radius, describe an arc; and with D as a centre, and the radius AB, describe another arc, cutting the former in C. Draw the lines AD, DC, and BC, and the rhomboid will be completed.



PROBLEM XIII.

Having the Base and the Two Perpendiculars given, to construct a Trapezoid.



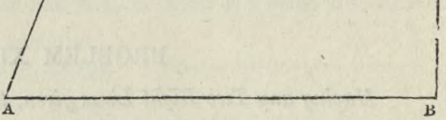
Let the base AB = 7, and the perpendiculars BC and AD = 3 and 2 chains respectively.

Draw the base AB, equal to 7, and erect the perpendiculars, BC equal to 3, and AD equal to 2 chains. Then join DC, and the trapezoid will be completed.

PROBLEM XIV.

Having the Four Sides given, to construct a Quadrilateral Figure which has One Right Angle.

Let the sides AB = 7, BC = 4, CD = 6, and DA = 3 chains; and let the angle at B be a right angle.



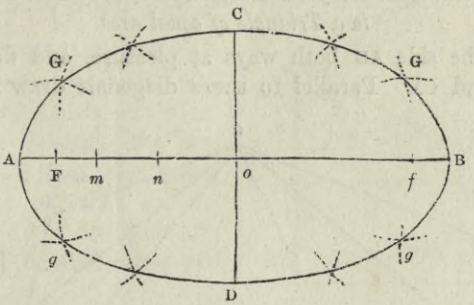
Draw the line AB equal to 7, and erect the perpendicular BC, equal to 4 chains. With C as a centre, and the radius CD, describe an arc; and with A as a centre, and the radius DA, describe another arc, cutting the former in D. Draw the lines CD and DA, and the figure will be completed.

PROBLEM XV.

Having the Transverse and Conjugate Diameters given, to construct an Ellipsis.

Let the transverse diameter AB = 7, and the conjugate diameter CD = 4 chains.

Draw the two diameters to bisect each other perpendicularly in the centre *o*. With the radius A*o*, and the centre C or D, intersect AB in F and *f*. These points will be the foci of the ellipse. Take any point *m*, in the transverse diameter, and with F and *f* as centres and the radius Am, describe the arcs G, G and g, g. Then with the same centres and the radius Bm, describe arcs cutting the former in



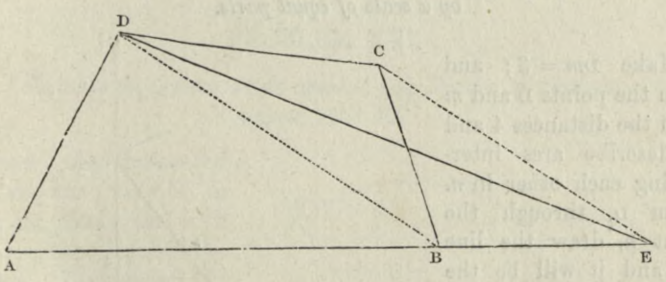
the points G, G, g and g: thus will you have four points in the circumference of the ellipse. After this, take a second point n, in the transverse diameter, and, proceeding as before, you will determine other four points. By the same method you may determine as many more as you please; through all of which, with a steady hand you must draw the circumference of the ellipse.

NOTE.—An ellipse may also be constructed as follows:—Having found the foci F and f, as before, take a thread equal in length to the transverse diameter AB, and fasten its ends, with two pins, in the points F and f; then stretch the thread to its greatest extent; and by moving a pencil round, within the thread, keeping it always tight, you will trace out the curve of the ellipse.

The principle upon which this construction is founded may be seen in Prob. X. Part VI.

PROBLEM XVI.

To reduce a given Trapezium ABCD *to a Triangle of equal area.*



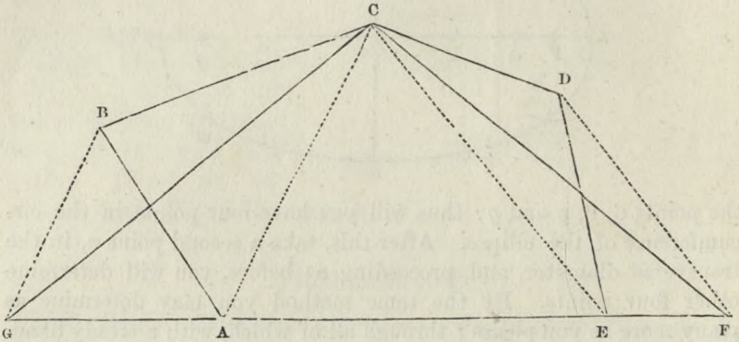
Draw the diagonal DB, and parallel to it draw CE, meeting AB produced in E. Draw DE; the triangle ADE is equal to the trapezium ABCD.

NOTE.—This and the following problem may be applied in finding the areas of trapeziums and irregular polygons, by first reducing them to triangles.

PROBLEM XVII.

To reduce an Irregular Polygon ABCDE, *of Five Sides, to a Triangle of equal area.*

Extend the side AE both ways at pleasure, and draw the diagonals CE and CA. Parallel to these diagonals draw the lines DF



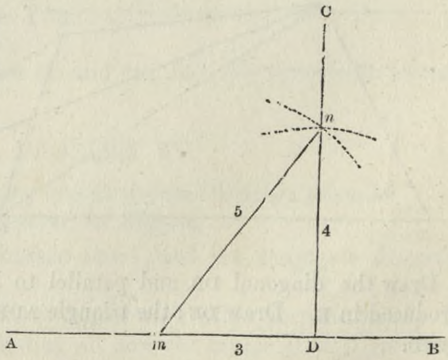
and BG; then draw CF and CG; and GCF will be the triangle required.

NOTE.—Any irregular polygon of more than five sides may be brought to a triangle of equal area by reducing it successively to a figure with one side less, until you bring it to a figure of three sides. Thus the trapezium ABCF or GCDE is equal to the polygon ABCDE as well as the triangle GCF.

PROBLEM XVIII.

To raise a Perpendicular from any Point D, *in a given Line* AB, *by a scale of equal parts.*

Make Dm = 3; and from the points D and m with the distances 4 and 5, describe arcs intersecting each other in n. From D, through the point n, draw the line DC, and it will be the perpendicular required.

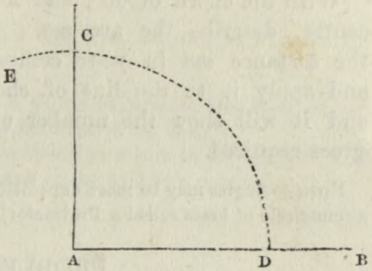


NOTE.—This problem may be performed by any other numbers in the same proportion; but 3, 4, and 5 are the least whole numbers that will make a right-angled triangle.

PROBLEM XIX.

To make a Right Angle by the Line of Chords on the Plane Scale.

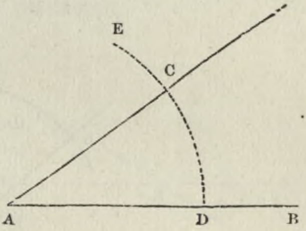
Draw the unlimited line AB; then take in your compasses 60° from the line of chords, and with A as a centre describe the arc ED. Take 90° from the same scale, and set off that extent from D to C. Draw the line AC, and CAD will be the angle required.



PROBLEM XX.

To make an Acute Angle equal to any Number of Degrees, suppose 33° 30′.

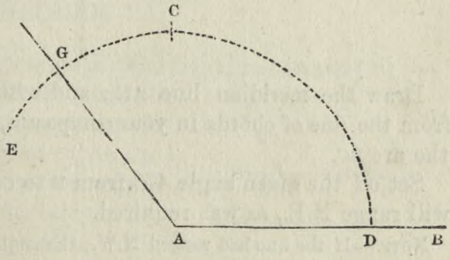
Draw the unlimited line AB; then take 60° in your compasses, and with A as a centre describe the arc ED. Then set off the angle 33° 30′ from D to C. Draw the line AC; and CAD will be the angle required.



PROBLEM XXI.

To make an Obtuse Angle equal to any Number of Degrees, suppose 125° 30′.

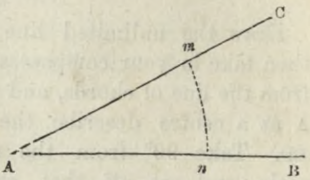
Draw the unlimited line AB; then take 60° in your compasses, and with A as a centre describe the arc ED. Then set off 90° from D to C; and from C to G set off the excess above 90°, which is 35° 30′. Draw the line AG; and GAD will be the angle required.



PROBLEM XXII.

To find the Number of Degrees contained in any given Angle BAC.

With the chord of 60°, and A as a centre, describe the arc *mn*. Take the distance *mn* in your compasses, and apply it to the line of chords; and it will show the number of degrees required.



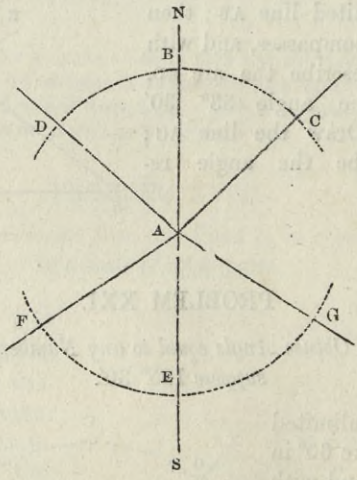
NOTE.—Angles may be more expeditiously laid down or measured by means of a semicircle of brass called a Protractor, the arc of which is divided into 180°.

PROBLEM XXIII.

To lay down a Line making a given Angle with the Meridian, or North and South Line.

EXAMPLES.

1. Let it be required to lay down a line that ranges N.E., making an angle of 45° with the meridian line. (*See* The Compass, Part II.)



Draw the meridian line AN; and with the sweep of 60°, taken from the line of chords in your compasses, and A as a centre, describe the arc BC.

Set off the given angle 45° from B to C; draw the line AC, and it will range N.E., as was required.

NOTE.—If the line had ranged N.W., the angle must have been set off on the other side of the meridian AN; and AD would have been the direction of the line.

2. Lay down a line that ranges S.W. b. W., making an angle of 26° 15′ with the meridian line.

Draw the meridian line AS; and with the chord of 60° describe the arc EF.

Set off 56° 15′ from E to F; draw the line AF, and it will range S.W. b. W., as was required.

NOTE 1.—If the line had ranged S.E. b. E. the angle must have been set off from E to G; and AG would have been the direction of the line.

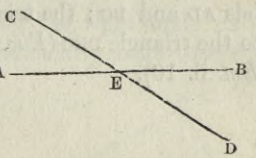
2. This problem will be found useful to young surveyors in laying down the first line, the range of which should be taken in the field by a compass.

GEOMETRICAL THEOREMS,

THE DEMONSTRATIONS OF WHICH MAY BE SEEN IN THE WORKS OF EUCLID, SIMPSON, AND EMERSON.

THEOREM I.

If two straight lines AB and CD cut each other in the point E, the angle AEC will be equal to the angle DEB, and CEB to AED (*Euclid*, i. 15; *Simpson*, i. 3; *Emerson*, i. 2).

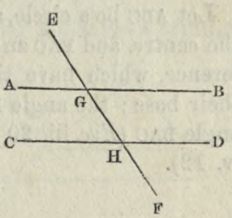


THEOREM II.

The greatest side of every triangle is opposite to the greatest angle (*Euc.* i. 18; *Simp.* i. 13; *Em.* ii. 4).

THEOREM III.

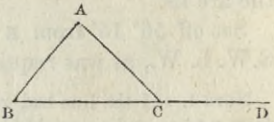
Let the right line EF fall upon the parallel right lines AB and CD; the alternate angles AGH and GHD are equal to each other; and the exterior angle EGB is equal to the interior and opposite upon the same side GHD; and the two interior angles BGH and GHD upon the same side are together equal to two right angles (*Euc.* i. 29; *Simp.* i. 7; *Em.* i. 4).



B

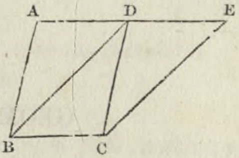
THEOREM IV.

Let ABC be a triangle, and let one of its sides BC be produced to D; the exterior angle ACD is equal to the two interior and opposite angles CAB and ABC; also the three interior angles of every triangle are together equal to two right angles (*Euc.* i. 32; *Simp.* i. 9, 10; *Em.* ii. 1, 2).



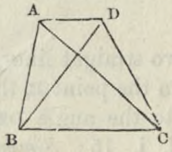
THEOREM V.

Let the parallelograms ABCD and DBCE be upon the same base BC, and between the same parallels AE and BC; the parallelogram ABCD is equal to the parallelogram DBCE (*Euc.* i. 35; *Simp.* ii. 2; *Em.* iii. 6).



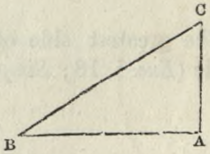
THEOREM VI.

Let the triangles ABC and DBC be upon the same base, BC, and between the same parallels AD and BC; the triangle ABC is equal to the triangle DBC (*Euc.* i. 37; *Simp.* ii. 2; *Em.* ii. 10).



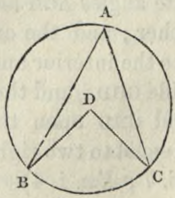
THEOREM VII.

Let ABC be a right-angled triangle, having the right angle BAC; the square of the side BC is equal to the sum of the squares of the sides AB and AC (*Euc.* i. 47; *Simp.* ii. 8; *Em.* ii. 21).

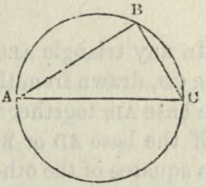


THEOREM VIII.

Let ABC be a circle, and BDC an angle at the centre, and BAC an angle at the circumference, which have the same arc BC for their base; the angle BDC is double of the angle BAC (*Euc.* iii. 20; *Simp.* iii. 10; *Em.* iv. 12).

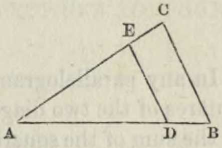


THEOREM IX.



Let ABC be a semicircle; then the angle ABC in that semicircle is a right angle (*Euc.* iii. 31; *Simp.* iii. 13; *Em.* vi. 14).

THEOREM X.



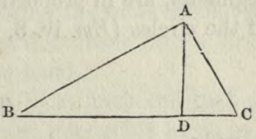
Let DE be drawn parallel to BC, one of the sides of the triangle ABC; then BD is to DA as CE to EA (*Euc.* vi. 2; *Simp.* iv. 12; *Em.* ii. 12).

THEOREM XI.

In the preceding figure, DE being parallel to BC, the triangles ABC and ADE are similar; therefore AB is to BC as AD to DE; and AB is to AC as AD to AE (*Euc.* vi. 4; *Simp.* iv. 12; *Em.* ii. 13).

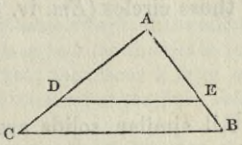
THEOREM XII.

Let ABC be a right-angled triangle, having the right angle BAC; and from the point A let AD be drawn perpendicularly to the base BC; the triangles ABD and ADC are similar to the whole triangle ABC, and to each other. Also the perpendicular AD is a mean proportional between the segments of the base; and each of the sides is a mean proportional between the base and its segment adjacent to that side; therefore BD is to DA as DA to DC; BC is to BA as BA to BD; and BC is to CA as CA to CD (*Euc.* vi. 8; *Simp.* iv. 19; *Em.* vi. 17).



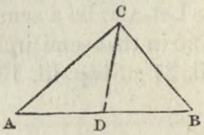
THEOREM XIII.

Let ABC and ADE be similar triangles, having the angle A common to both; then the triangle ABC is to the triangle ADE as the square of BC to the square of DE. That is, similar triangles are to one another in the duplicate ratio of their homologous sides (*Euc.* vi. 19; *Simp.* iv. 24; *Em.* ii. 18).



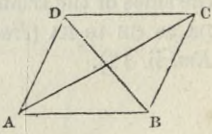
THEOREM XIV.

In any triangle ABC, double the square of a line CD, drawn from the vertex to the middle of the base AB, together with double the square of half the base AD or BD, is equal to the sum of the squares of the other sides AC and BC (*Simp.* ii. 11; *Em.* ii. 28).



THEOREM XV.

In any parallelogram ABCD, the sum of the squares of the two diagonals AC and BD is equal to the sum of the squares of all the four sides of the parallelogram (*Simp.* ii. 12; *Em.* iii. 9).



THEOREM XVI.

All similar figures are in proportion to each other as the squares of their homologous sides (*Simp.* iv. 26; *Em.* iii. 20).

THEOREM XVII.

The circumferences of circles, and the arcs and chords of similar segments, are in proportion to each other as the radii, or diameters, of the circles (*Em.* iv. 8, 9).

THEOREM XVIII.

Circles are to each other as the squares of their radii, diameters, or circumferences (*Em.* iv. 35).

THEOREM XIX.

Similar polygons described in circles are to each other as the circles in which they are inscribed, or as the squares of the diameters of those circles (*Em.* iv. 36).

THEOREM XX.

All similar solids are to each other as the cubes of their like dimensions (*Em.* vi. 24).

PART II.

A DESCRIPTION OF THE CHAIN, CROSS-STAFF, OFF-SET STAFF, COMPASS, AND FIELD-BOOK; ALSO DIRECTIONS TO YOUNG SURVEYORS WHEN IN THE FIELD; &c.

THE CHAIN.

LAND is commonly measured with a Chain, invented by Mr. Gunter, known by the name of 'Gunter's chain.'

It is 4 poles, 22 yards, or 66 feet in length, and divided into 100 equal parts, called links; each link being 7·92 inches. At every tenth link from each end is fixed a piece of brass with notches or points; that at 10 links having one notch or point; at 20, two; at 30, three; and at 40, four points. At 50, or the middle, is a large round plain piece of brass.

The chain being thus marked, the links may be easily counted from either end; the mark at 90, 80, &c., being the same as that at 10, 20, &c. Part of the first link, at each end, is made into a large ring or bow, for the ease of holding it in the hand.

The chain should always exceed 22 yards, by an inch and half or two inches; because, in surveying, it is almost impossible to go in a direct line, or to keep the chain perfectly stretched. Long arrows likewise keep the ends of the chains a considerable distance from the ground; the lines, consequently, will be made longer than they are in reality.

Chains, when new, are seldom a proper length; they ought always, therefore, to be examined; as should those likewise which are stretched by frequent use.

NOTE 1. In folding up the chain, it is most expeditious to begin at the middle, and fold it up double. When you wish to unfold it, take both the handles in your left hand, and the other part of the chain in your right; then throw it from you, taking care to keep hold of the handles. You must then adjust the links before you proceed to measure.

2. Chains which have three rings between each link are much better than those which have only two, as they are not so apt to twist.

THE CROSS-STAFF.

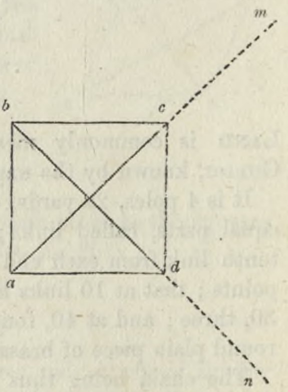
The Cross-Staff is an instrument used in the field by surveyors, to erect perpendiculars, and may very easily be made in the following manner.

Procure a piece of board about 6 inches square, either of sycamore, box, or mahogany.

Draw the two diagonals, and at their extremities fix four small studs or pins, which will serve as sights to direct to any object or angle.

Or, instead of studs or pins, you may saw two fine grooves at right-angles, about a quarter of an inch deep, in the board.

This being fixed upon a staff, of a convenient length for use, pointed with iron at the bottom to enter the ground readily, the instrument is called a cross-staff.



Suppose *abcd*, to represent a cross, and the groove *ac* to be directed to an object at *m*; then will the groove *bd* point to another at *n*.

Reverse the direction of the grooves, so that *bd* may be in the direction of *m*, then if *ac* be in the direction of *n* the instrument is correct.

NOTE 1.—The cross must be fixed upon the top of the staff, in such a manner that it may be easily turned without moving the staff itself.

2. The cross may be made of a circular piece of board; you must then draw two diameters crossing each other at right angles. The fourth part of a circle will answer the purpose equally well.

3. Great care ought to be taken in making this instrument, as its accuracy depends on the sights, or grooves, being at right angles with each other.

THE OFFSET-STAFF.

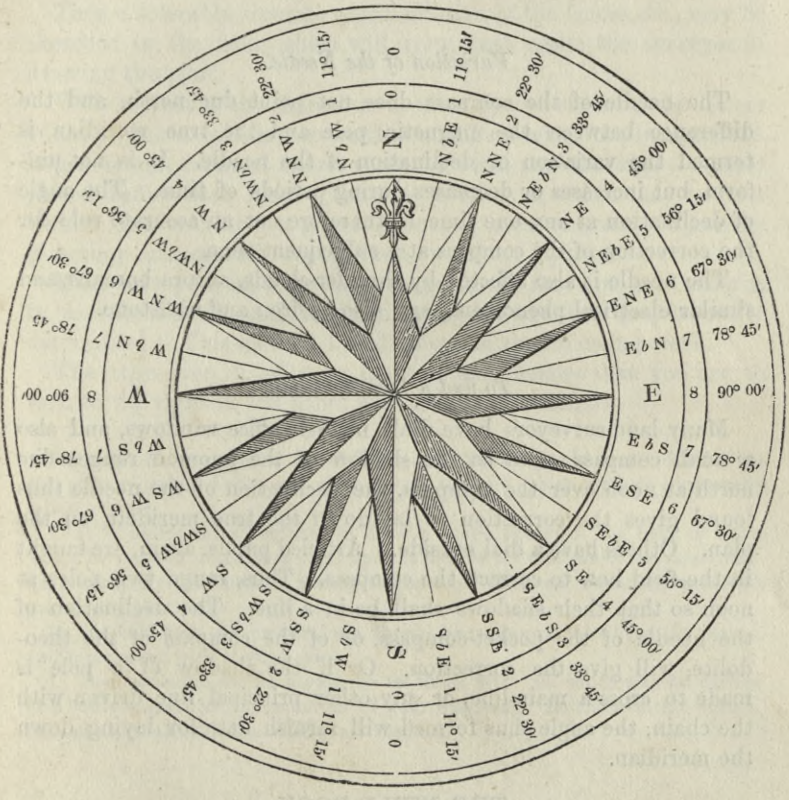
The Offset-Staff is an instrument used to measure short distances; and may be in length 10, 12, or 15 links. It would be advisable to number the links from each end, on opposite sides, with the figures 1, 2, 3, &c., as the staff thus marked will be more convenient for use.

NOTE.—As the cross-staff is sometimes thought incommodious, a small pocket-cross may be so contrived as to be readily fixed, upon occasion, to the offset-staff. This may be most expeditiously accomplished by means of a hole made through the cross, admitting the top of the staff, to the eighth link, counting from the bottom or piked end; at which place there must be attached a small shoulder, upon which the cross will rest.

THE COMPASS.

The Compass is an instrument used by surveyors, to point out the range or direction of lines, and also to show the bearings of objects. The circumference of the card of the compass contains 360°, and is divided into thirty-two equal parts, called Points, each containing 11° 15′.

Of these, the four principal (namely, East, West, North, and



South) are called Cardinal Points; from which the names of the others are derived.

To the under-side of the card, and in the direction of its north and south lines, is attached a magnetic bar of hardened steel, called the Needle, by which the north-point is directed towards the northern part of the horizon; and the other points, consequently, to their corresponding ones in the heavens.

The card and needle are suspended on an upright pin, called the Supporter, which is fixed in the bottom of a brass or wooden box; and the whole is covered with a plate of glass to prevent the action of the wind upon the card.

Although the compass is divided into thirty-two points, yet surveyors reduce them to eight, namely, the four cardinal or chief points, and the four midway between them, viz., the north-east, north-west, south-east, and south-west, which may be expressed by their initial letters, as E., W., N., S.; N.E., N.W., S.E., S.W.

Variation of the Needle.

The needle of the compass does not point due north, and the difference between the magnetic pole and the true meridian is termed the variation or declination of the needle. It is not uniform, but increases or decreases during periods of time. The angle of declination at any one time is therefore not an accurate rule for the correction of the compass at a subsequent time.

The needle is also affected by thunder-clouds, aurora borealis, and similar electrical phenomena, and also by iron and ironstone.

To find a true Meridian.

Many land-surveyors have dials in their office windows, and also a small compass; and as the shadow of the gnomon ranges due north at noon over the compass, the declination of the needle thus found gives the correction to lay down the true meridian on the plan. Others have a dial outside. Articled pupils, again, are taught in the field how to correct the compass. Thus, range two poles at noon so that their shadows shall be in a line. The declination of the needle of the pocket-compass, or of the compass of the theodolite, will give the correction. Or if the shadow of a pole is made to cross a main-line, or any other principal line driven with the chain, the angle thus formed will furnish data for laying down the meridian.

THE FIELD-BOOK.

Scarcely any two surveyors set down their field-notes exactly in the same manner. The method, however, now generally adopted, is to begin at the bottom of the page and write upward.

Each page of the book must be divided into three columns. In the middle column must be set down the distances on the chain-line at which any mark, offset, or other observation is made; and in the right and left-hand columns respectively those marks, offsets, and observations must be entered.

The crossings of fences, rivers, &c., may be denoted by lines drawn across the middle column, or part of the right and left-hand columns, opposite the distances on the chain-line at which they are crossed; and the corners of fields, and other remarkable turns in the fences, to which offsets are taken, may be denoted by lines joining or lying in the same relation to the middle column as the fences, &c., do to the chain line.

Thus a tolerably accurate representation of the fences, &c., may be sketched in the field, which will very much assist the surveyor in drawing the plan.

With respect to the characters used to denote stations, the letters of the alphabet will do very well in small surveys; but in those of a larger extent, numeral figures must be used, and the sign + (plus) placed before each figure; thus, + 1, or + 2, which may be read station first, or cross first; station one, or cross one, &c. Upon the plan they are generally represented by this ($\odot$) mark.

Most surveyors take the exact range of the first line, and enter it in their field-book; and from it the range of any other may be easily determined. This method I shall adopt in the following work.

The expression, R. off B, or L. off B, &c., denotes that you are to turn to the right or left hand, and measure from B, &c.

NOTE 1.—Many surveyors not only begin at the bottom of the field-book, but also at its right-hand side, and write toward the left, which method I generally follow myself.

2. It is useful for a beginner to draw a rough sketch of the field, or estate which he is about to measure; and upon it to note the stations in the same manner as they are put down in taking the survey. This will materially assist his memory in planning.

3. The field-book, for practical use, should be made convenient for the pocket, and interleaved with blotting-paper.

4. The field-notes should always be set down with ink, which may be carried in a bottle suspended from a button of your waistcoat. Double fountain-bottles such as are used by excise officers, are the best.

DIRECTIONS TO YOUNG SURVEYORS WHEN IN THE FIELD, &c.

In addition to the instruments already described, you must provide ten arrows, each about a foot in length, made of strong wire, and pointed at the bottom. These should be bent in a circular form at the top, for the convenience of holding them, and a piece of red cloth should be attached to each, that they may be more conspicuous among long grass, &c.

Poles, likewise, generally called Ranging-poles, or Station-staves, will be wanted as marks, or objects of direction, each about ten feet in length, piked with iron at the bottom, and having a red or white flag at the top, that they may be better seen at a distance. Thus equipped, and having entered the field or estate which you are about to survey, first, make yourself acquainted with its form; and then consider in what manner you must run your lines, according to the directions hereafter given in Parts III., IV., V.; after which you must proceed in the following manner.

Let your assistant or chain-leader take nine arrows in his left-hand, and one end of the chain with one arrow in his right; then, advancing toward the place directed, at the end of the chain, let him put down the arrow which he holds in his right hand. This the follower must take up with his chain-hand when he comes to it, the leader, at the same time, putting down another at the other end of the chain. In this manner he must proceed until he has put down his tenth arrow; then, advancing a chain farther, he must set his foot upon the end of the chain, and call out "change." The surveyor, or chain-follower, must then come up to him, if he have no offsets to take, and carefully count to him the arrows; and one being put down at the end of the chain, proceed as before, until the whole line be measured.

Each change ought to be entered in the field-book, or a mistake of 10 chains may happen, when the line is very long. The chain-follower ought to be careful that the leader always puts down his arrow perpendicularly, and in a right-line with the object of direction; otherwise the line will be made longer than it is in reality. The follower may direct the leader by the motion of his left-hand; moving it to the right or left, as circumstances require, and always placing his eye and chain-hand directly over the arrow which is stuck in the ground. The leader likewise, as soon as he has put down his arrow, ought to fix his eye upon the object of direction, and go directly toward it. This he may easily effect by finding a tree or a bush beyond the station to which he is going, and in a straight line with it and himself.

In hilly ground, if the follower lose sight of the mark toward which he is going, he must stand over his arrow; and the leader must move to the right or left, till he sees the follower in a direct line between himself and the mark from which they last departed.

The surveyor ought to put down at each station a small stake, called a station-stake, with the number of the station upon it; so that any of the stations may be readily found, if there be occasion to measure the distance between two of them, as a tie or proof-line, &c.

In large surveys, there must be a cross cut in the ground, at each station, making right-angles with the chain-line; so that if the stake should be pulled up, the cross may still remain, and serve as a director.

When a survey is taken with an intent to draw a finished plan, all remarkable objects should be noted down in the field-book; as roads, stiles, gates, trees, &c.

If the surveyor can conveniently procure two assistants, the one to lead the chain and the other to follow it, it will be much to his advantage; as he will thus be left at liberty to take offsets, note down dimensions, &c., without loss of time.

He ought always to observe to whom the boundaries belong.

If the ditch be in the field he is about to measure, both it and the hedge usually belong to the adjoining field. This, however, is not always the case; as it sometimes happens that the hedge is on the reverse side of the ditch. It is advisible, therefore, to inquire of some person resident on the spot concerning the hedges, &c.

In some places 3 feet from the roots of the quickwood are allowed for the breadth of the ditches; in some 4, in some 5, and in some 6; but 4 feet, or 6 links, are commonly allowed for ditches between neighbouring estates, and 7 links for ditches adjoining roads, commons, waste lands, &c.

The ditches and fences must always be measured with the fields to which they belong, when the whole quantity of land is required; but in measuring crops of corn, turnips, &c., only so much must be measured as is or has been occupied by the corn, &c.

Upon the surveyor depends all the care of measuring, remarking, noting down, &c. It absolutely behoves him, therefore, to be not only particularly careful in his entries, and correct in his dimensions, but also extremely accurate in his constructions and calculations.

NOTE.—The line in which you have the misfortune to lose an arrow must be remeasured.

DIRECTIONS CONCERNING SCALES, LAYING DOWN FIGURES, &c.

Any scale of equal parts may be used in planning, or laying down figures; but that which is most convenient for use is the ivory plotting-scale, so divided on its edges that you may prick off distances by laying it upon the line.

In laying down an offset by the plotting-scale, it is best, first, to prick off the base-line, and then upon it make a small pencil dot at every place where a perpendicular must be erected.

This being done, lay the scale across the base, so that the perpendicular line which goes across the scale, marked with *oo*, may coincide with the base, the edge of the scale at the same time touching one of the dots. From the dot, by the edge of the scale, draw a line (which will be perpendicular to the base), and upon it prick off the offset; or it may be pricked off without drawing a line.

Proceed thus till all the perpendiculars are erected, and then draw the fence through each of their extremities. If the fence be curved, it must be drawn by a steady hand, in the same manner as the circumference of an eclipse. (See p. 13.)

In planning, or laying down figures relating to surveying, the upper part of the paper or book used should always, if possible, represent the north. All the fences and chain-lines should first be pencilled: the first should then be drawn, and the latter dotted with ink. Great accuracy is required in the construction of figures, when the perpendiculars, &c., are to be measured by the scale. The lines should be very fine, the dots at the stations very small, and the points of the compass very sharp, in order that distances may be taken from the scale with the utmost correctness. The scale should never be smaller than two chains to an inch; for when its divisions are large, figures may be constructed with much more accuracy, and their perpendiculars, &c., measured with much greater exactness.

After having found the area of any field or estate, you may, however, lay it down by any scale that will reduce it to a more convenient size. Or you may divide the dimensions by 2, 3, 4, &c., in order to make them of a proper size to be laid down by a scale of 2, 3, or 4 chains to an inch.

NOTE 1.—A plotting-scale divided into two chains to an inch on one of its edges, and four on the other, is perhaps most useful for a school-boy; but practical surveyors prefer those which have both their edges divided in the same manner, because they are more convenient in planning; and a mistake cannot be made by using one edge instead of the other.

2. Many plotting-scales are now made of box, without the perpendicular line across them; and a small scale, called an 'offset-scale,' is used in laying down the offsets. In using these scales, the first line or division, on the plotting-scale, is made to coincide with the commencement of the base-line; and the end of the offset-scale is then brought in contact with the edge of the plotting-scale; and thus the offsets may be pricked off, without drawing the offset lines perpendicular to the base-line.

3. An instrument called a Pricker, which may be made by putting a fine needle into a wooden haft, is used by some persons, in pricking off distances from the plotting-scale; but a hard black-lead pencil, finely pointed, is preferable, because it does not injure the paper.

PART III.

TO SURVEY WITH THE CHAIN AND CROSS; ALSO TO MEASURE MERES, WOODS, AND LINES UPON WHICH THERE ARE IMPEDIMENTS.

CONFORMABLY to a statute of 34 Henry VIII., an acre of land is equal to 10 square chains; that is, 10 chains in length and 1 in breadth; or $220 \times 22 = 4840$ square yards; or $40 \times 4 = 160$ square rods, poles, or perches.

A statute pole or perch is $16\frac{1}{2}$ feet in length; but in different parts of the kingdom there were, by custom, poles of different lengths; as 15, 18, 21 feet, &c.

The various dimensions of a piece of land are taken in lineal measure, from which its area or content is calculated in superficial measure.

NOTE 1.—The method of reducing statute measure to customary, and the contrary, may be seen in Part VI.

2. By the Act 5 Geo. IV. Chap. 74, it is enacted that our present yard shall be called 'The Imperial Standard Yard;' that one-third part thereof shall be a foot, and the twelfth part of such foot shall be an inch; and that the rod, pole, or perch, in length, shall be $5\frac{1}{2}$ yards, the furlong 220 yards, and the mile 1760 yards.

3. By the same statute it is enacted that all superficial measures shall be computed by the 'Standard Yard,' or by certain parts or portions thereof; that the rood of land shall be 1210 square yards, and the acre of land 4840 square yards.

4. By the Act 5 & 6 Will. IV. Chap. 63, all *local* or *customary* weights and measures are abolished; but still it may be necessary to give the methods of reducing *local* or *customary* measures of land to statute measure, as in many *old plans* the contents are given in *local* or *customary* MEASURES.

A TABLE OF LINEAL MEASURES.

Inches	Link	Foot	Yard	Stat. Perch	Chain	Fur-long	Mile
7·92 =	1						
12	1·5151 =	1					
36	4·5454	3 =	1				
198	25	16·5	5·5 =	1			
792	100	66	22	4 =	1		
7920	1000	660	220	40	10 =	1	
63360	8000	5280	1760	320	80	8 =	1

A TABLE OF SQUARE MEASURES.

Square Inches	Square Link	Square Foot	Square Yard	Square Perch	Square Chain	Square Rood	Square Acre	Square Mile
62·7264 =	1							
144	2·2956 =	1						
1296	20·6611	9 =	1					
39204	625	272·25	30·25 =	1				
627264	10000	4356	484	16 =	1			
1568160	25000	10890	1210	40	2·5 =	1		
6272640	100000	43560	4840	160	10	4 =	1	
4014489600	64000000	27878400	3097600	102400	6400	2560	640 =	1

PROBLEM I.

Square Fields.

When you enter a field, which has the appearance of a square (for few are accurately such), fix your cross-staff in a corner of it, and if the two sides be at right angles, measure one of them, and enter its dimensions in your field-book. Proceed in like manner with each angle and side; and if you find all the angles right angles, and all the sides equal, the figure is a square.

To compute the Content.

RULE.—Multiply the side into itself, and the product will be the area, in square links. Cut off five places, as decimals, toward the right hand of the product, and those on the left will express the number of acres.

Reduce these decimals into roods and perches, by multiplying them successively by 4 and by 40, and cutting off five figures on the right as before, in each product.

If the dimensions be in yards, divide the square of the side by 4840, and the quotient will be acres.

Reduce the remainder, if any, into roods and perches, by multiplying it successively by 4 and by 40, as before.

NOTE 1.—Any person who is not in possession of a chain may take the dimensions in yards; or, where accuracy is not required, by pacing.

2. In measuring with the chain, it is best to set down the number of links, as 956: where, instead of reading 956 links, read 9 chains and 56 links.

3. The dimensions of small parcels of land, sold by the square yard, for building, &c., should be taken in feet and inches with a measuring-tape. Paving, digging, &c., should be measured in the same manner.

4. In computing the contents of fields, it is customary, among practical surveyors, to call the remainder a perch, if it exceeds half a one; but if it be less than half a perch, it is considered as nothing.

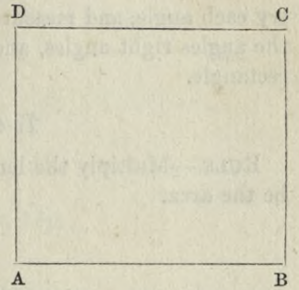
5. The learner should carefully *work over* and *put down* all the solutions given in this book, in order that he may better understand the different methods of calculation.

Examples.

1. What is the area in acres of the square ABCD, whose side is 956 links?

$$\begin{array}{r}
956 \\
956 \\
\hline
5736 \\
4780 \\
8604 \\
\hline
9{\cdot}13936 \\
4 \\
\hline
{\cdot}55744 \\
40 \\
\hline
22{\cdot}29760
\end{array}$$

22·29760 *Area 9a. 0r. 22p.*



2. Required the area in acres of the square whose side is 264 yards.

```
             264
             264
            ----
            1056
           1584
           528
          ------
    4840)69696(14a.
          4840
          -----
          21296
          19360
          -----
          ·1936
              4
          -----
    4840)7744(1r.
          4840
          ----
          2904
            40
          -----
  484,0)11616,0(24p.
          968
          ----
         ·1936
          1936 Area 14a. 1r. 24p.
          ====
```

3. If the side of a square be 1567 links, what is its area in acres?

Ans. 24*a.* 2*r.* 9*p.*

4. If the side of a square be 263 yards, what is its area in acres?

Ans. 14*a.* 1*r.* 6*p.*

PROBLEM II.

Rectangular Fields.

When you enter a field which has the appearance of a rectangle, try each angle, and measure each side, as before; and if you find all the angles right angles, and the opposite sides equal, the figure is a rectangle.

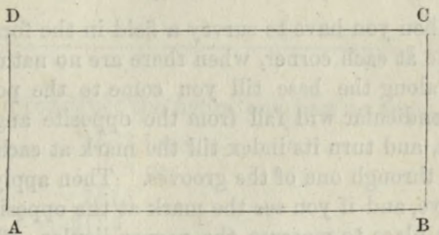
To compute the Content.

RULE.—Multiply the length by the breadth, and the product will be the area.

Examples.

1. What is the area of the rectangle ABCD, whose length AB is 1235 links, and breadth AD, 557 links?

```
  1235
   557
  ----
  8645
 6175
6175
-------
6·87895
      4
-------
3·51580
     40
-------
20·63200 Area 6a. 3r. 21p.
```



2. Required the area of a rectangle, whose length is 235 and breadth 162 yards.

```
          235
          162
          ---
          470
         1410
         235
         -----
484,0)3807,0(7a
      3388
      ----
       419
         4
      ----
  484)1676(3r.
      1452
      ----
       224
        40
      ----
  484)8960(18p.
      484
      ----
      4120
      3872
      ----
       248 Ans. 7a. 3r. 18p.
```

3. The length of a rectangular field is 1225 links, and its breadth 613 links; required the plan and area. *Area* 7*a.* 2*r.* 1*p.*

C

4. If the length of a rectangle be 135, and breadth 50 yards, what is its area? *Ans.* 1*a.* 1*r.* 23*p.*

NOTE.—As squares and rectangles seldom occur in surveying, it is more advisable to treat every field of four sides as a trapezium. (See Problem IV.)

PROBLEM III.

Triangular Fields.

When you have to survey a field in the form of a triangle, set up a pole at each corner, when there are no natural marks. Then measure along the base till you come to the point where you think a perpendicular will fall from the opposite angle. There plant your cross, and turn its index till the mark at each end of the base can be seen through one of the grooves. Then apply your eye to the other groove, and if you see the mark at the opposite angle, you are in the right place to measure the perpendicular; if not, move the instrument backward or forward along the line, till you can see the three marks as above directed. Enter in your field-book the distance from the end of the base to the cross, and the length of the perpendicular. Then measure the remainder of the base.

NOTE. 1—Be especially careful that, in measuring the two parts of the base and the perpendicular, no confusion of arrows takes place.

2. In ranging perpendiculars by the cross, you must always proceed as above directed.

Construction.

Having the place of the perpendicular, the figure may be easily constructed, as follows. From any scale of equal parts, lay off the base; erect the perpendicular at its proper point; draw a line from each end of the base to the end of the perpendicular, and the figure will be completed.

NOTE.—Having the diagonal, the two perpendiculars, and the place of each perpendicular given, you may construct any trapezium in the same manner.

To compute the Content.

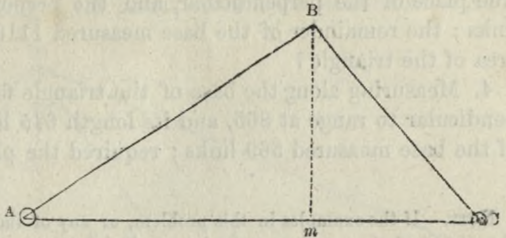
RULE.—Multiply the base and perpendicular together, divide the product by 2, and the quotient will be the area.

Or multiply half the base by the whole perpendicular, or the whole base by half the perpendicular, and the product will be the area.

Examples.

1. It is required to survey the triangular field ABC, and to find its area.

Measure from A toward C, and when you come to *m*, for instance at 935 links, try with your cross; and if this be the point for the perpendicular, measure *m*B = 625 links. Return and measure *m*C = 628 links, making the whole base = 1563 links; then construct the figure, and find its area.



1563 base.
625 per.
7815
3126
9378
2)976875
4·88437
4
3·53748
40
21·49920 *Area* 4*a*. 3*r*. 21*p*.

2. The distance between the beginning of the base and the place of the perpendicular is 125, the perpendicular 82, and the whole base 318 yards; what is the area of the triangle?

318 base.
82 per.
636
2544
2)26076
4840)13038(2*a*.
9680
3358
4
4840)13432(2*r*.
9680
3752
40
484,0)15008,0(31*p*.
1452
488
484
4 *Ans.* 2*a*. 2*r*. 31*p*.

3. Measuring along the base of a triangle 862 links, I found the true place of the perpendicular, and the perpendicular itself = 995 links; the remainder of the base measured 1110 links; what is the area of the triangle? *Ans.* 9*a.* 3*r.* 10*p.*

4. Measuring along the base of the triangle field, I found the perpendicular to range at 865, and its length 645 links; the remainder of the base measured 569 links; required the plan and area.

Area 4*a.* 2*r.* 20*p.*

NOTE.—If the examples in this problem, or any of the following problems be thought too few, more may easily be supplied by the teacher sketching fields, at pleasure, with his pen, which the learner may measure by a scale. This method will be found very advantageous; as it will give the learner a good idea in what manner he must run his lines, take his dimensions, and enter his notes, when he commences field-practice.

PROBLEM IV.

Fields in the form of a Trapezium.

A quadrilateral field, having unequal sides, may be surveyed by measuring a diagonal. This divides it into two triangles, to each of which it serves as a base.

To compute the Content.

RULE.—Multiply the sum of the two perpendiculars by the diagonal, divide the product by 2, and the quotient will be the area.

NOTE 1.—Always make choice of the longer diagonal, because the longer the base line of a triangle, the more obtuse is its subtending angle; and, consequently, there is the less chance to mistake, as the perpendicular will be shorter, and its place more easily and more accurately determined. After finishing the surveying, if you choose, measure the other diagonal, which will enable you to prove your work. (See Problems I. and II., Part IV.)

2. If a field be very long, or elevated in the middle, so that you cannot see from one end to the other, it may be divided into two or more trapeziums; or you may range your lines over the hill, as directed in Part V.

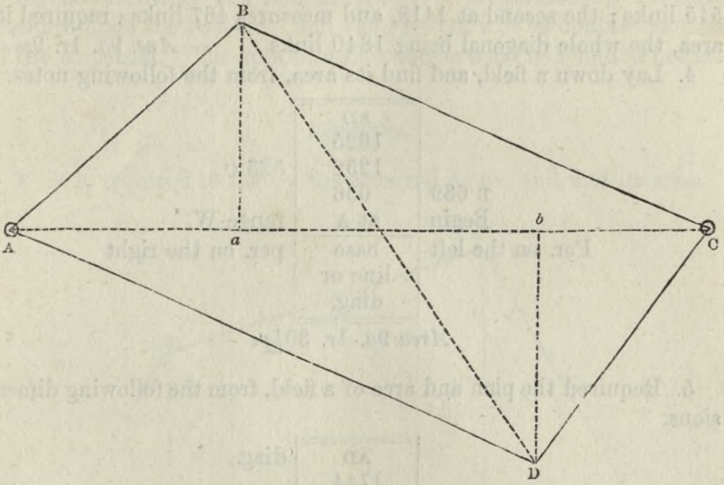
3. When two perpendiculars cannot be taken upon either of the diagonals, such fields must be divided into two triangles by measuring a diagonal from the base of one triangle, and one side of the field for the base of the other. (See example 6.)

4. Sometimes surveyors affect to reduce trapeziums into squares, or rectangles, by measuring all the sides, adding each two opposite sides together, and taking half their sum respectively for a mean length and breadth; but this method leads to very erroneous results. (See Part IV., Prob. II.)

Examples.

1. It is required to survey the trapezium ABCD, and find its area.

Measure from A toward C. Finding the perpendicular aB to rise at 473, and its length 437 links; return, and continue toward C,



till you come to the place where the second perpendicular bD rises. There note down its distance from A, 1128 links; measure bD = 508 links; then complete the measuring of the diagonal to C, and let the whole be 1490 links.

After this, measure the diagonal BD, for a proof-line, which you will find 1152 links.

$$
\begin{array}{r l}
\left.\begin{array}{r}437\\508\end{array}\right\} & \text{per.}\\
\hline
945 & \text{sum}\\
1490 & \text{diag.}\\
\hline
85050 & \\
3780 & \\
945 & \\
\hline
2)1408050 & \\
\hline
7{\cdot}04025 & \\
4 & \\
\hline
0{\cdot}16100 & \\
40 & \\
\hline
6{\cdot}44000 & \textit{Area } 7a.\ 0r.\ 6p.
\end{array}
$$

2. In taking the dimensions of a trapezium, I found the first perpendicular to rise at 539, and to measure 725 links; the second at

1890, and to measure 832 links; the whole diagonal measured 2456 links; required the area of the trapezium. *Ans.* 19*a.* 0*r.* 19*p.*

3. The first perpendicular of a trapezium rises at 467, and measures 545 links; the second at 1418, and measures 467 links; required its area, the whole diagonal being 1840 links. *Ans.* 9*a.* 1*r.* 9*p.*

4. Lay down a field, and find its area, from the following notes.

	AD	
	1625	
	1252	523 C
B 639	636	
Begin	at A	range W.
Per. on the left	base line or diag.	per. on the right

Area 9*a.* 1*r.* 30½*p.*

5. Required the plan and area of a field, from the following dimensions.

	AD	diag.
	1744	
C 545	1365	
	546	652 B
Begin	at A	range E.

Area 10*a.* 1*r.* 30*p.*

6. Lay down a field, and find its area, from the following notes.

	DB	diag.
	1095	
	488	298 C
	L. off D	
	AD	side
	1358	
B 532	410	
Begin	at A	range E.

Answer.
Double areas.
722456 triangle ABD
326310 triangle BCD
Whole area 5*a.* 0*r.* 39*p.*

ANOTHER METHOD.

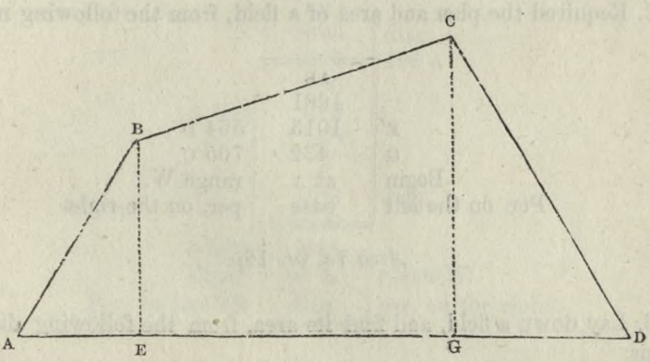
A field of four sides may sometimes be surveyed by dividing it into two right-angled triangles, and a trapezoid.

To compute the Content.

RULE.—Multiply the sum of the two perpendiculars by their distance upon the base-line, and the product will be double the area of the trapezoid. The area of each triangle must be found as before.

Examples.

1. It is required to survey the annexed figure, and find its area.



Measure the base AD, and enter in your field-book where the two perpendiculars rise, &c., as in the following notes.

	AD = 1326
GC = 645	AG = 952
EB = 422	AE = 265
Per.	Base

Triangle ABE
422 per.
265 base
2110
2532
844
111830

Triangle GCD
645 per.
374 base
2580
4515
1935
241230

Trapezoid EBCG
422 } per.
645 }
1067 sum
687 base
7469
8536
6402
733029

733029	double areas collected
241230	
111830	
2)1086089	
5·43044	
4	
1·72176	
40	
28·87040	*Area* 5*a*. 1*r*. 29*p*.

2. Required the plan and area of a field, from the following notes.

	AB	
	1681	
E	1015	564 D
G	432	705 C
Begin	at A	range W.
Per. on the left	base	per. on the right

Area 7*a*. 0*r*. 16*p*.

3. Lay down a field, and find its area, from the following dimensions.

	AB	
	1546	
D 625	1146	E
C 883	564	G
Begin	at A	range E.

Area 8*a*. 0*r*. 20½*p*.

PROBLEM V.

Fields comprehended under more than Four Straight Sides.

Fields having more than four sides may be surveyed by reducing them into triangles and trapeziums.

Thus, a field of five sides may be reduced into a triangle and a trapezium; of six, into two trapeziums; of seven, into two trapeziums and a triangle; of eight, into three trapeziums; &c.

The propriety of dividing fields in this manner depends entirely on the relation which the angles have to one another; it is, therefore, sometimes more accurate to divide them into triangles.

To compute the Content.

RULE.—By the rules given in the last two problems, find the double area of each triangle and trapezium contained in the figure.

Collect all the double areas into one sum, which divide by 2, and the quotient will be the whole area.

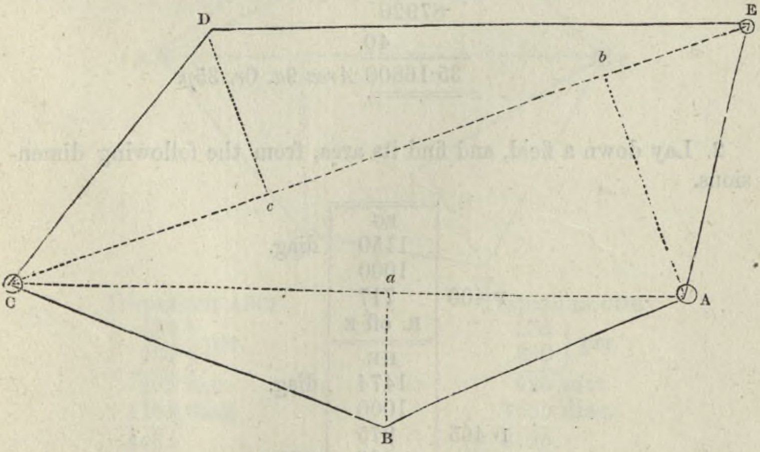
Examples.

1. Lay down a field, and find its area, from the following notes.

	CE	
	1666	diag.
	1326	496 A
	1000	
D 376	573	
	R off C	
	AC	
	1433	diag.
	1000	
B 273	643	
Begin	at A	range W.
Per. on the left	diag.	per. on the right

Construction.

From the notes, the figure obviously consists of five sides, and is divided into a triangle and a trapezium. Draw the base AC, which



make = 1433 links; at 643 links, let fall the perpendicular aB, upon which lay off 273 links; join AB and CB, and the triangle is completed. Then, with A as a centre, and 496 links in your compasses as a radius, describe an arc; and with C as a centre, and 1326 as a radius, describe another arc, intersecting the former in *b*.—Through *b* draw the diagonal CE = 1666 links; upon which, at 573 links, erect the perpendicular CD = 376 links. Join CD, DE, and EA, and the figure will be completed.

NOTE.—If the learner fully comprehend the above construction, he will not find it difficult to lay down the figures belonging to the following examples; as the same process will succeed in them and all similar cases.

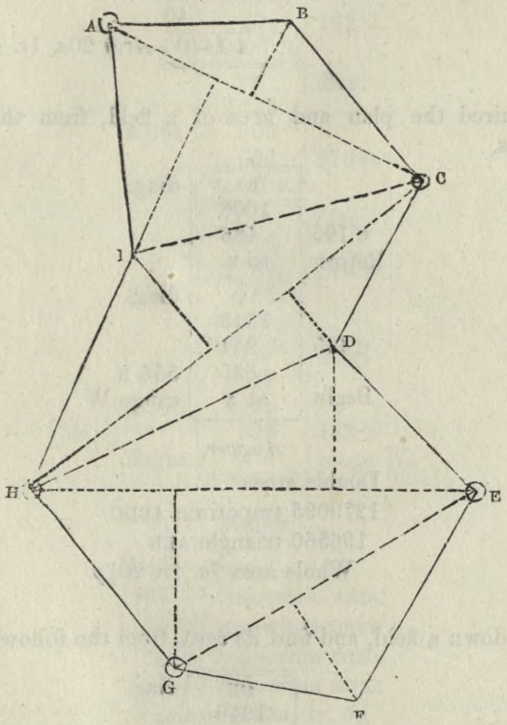
Triangle ABC	Trapezium ACDE
1433 base	376 } per.
273 per.	496 }
4299	872 sum
10031	1666 diag.
2866	5232
391209	5232
	5232
	872
	1452752

391209 }	double areas
1452752 }	collected
2)1843961	
9·21980	
4	
·87920	
40	
35·16800	*Area* 9*a.* 0*r.* 35*p.*

2. Lay down a field, and find its area, from the following dimensions.

	EG	
	1150	diag.
	1000	
F 400	717	
	R. off E	
	HE	
	1474	diag.
	1000	
D 465	975	
	465	575 G
	L. off H	

	CH	
	1635	diag.
	1000	
	910	390 I
D 235	575	
	R. off C	
	AC	
	1165	diag.
	1000	
B 263	530	
	400	630 I
Begin	at A	range E.S.E.



Trapezium ABCI

630 } per.
263 }
893 sum
1165 diag.
4465
5358
893
893
1040345

Trapezium CDHI

235 } per.
390 }
625 sum
1635 diag.
3125
1875
3750
625
1021875

Trapezium DEGH		Triangle EFG
575 } per.		1150 base
465		400 per.
1040 sum		460000
1474 diag.	1040345 } double areas collected	
4160	1021875	
7280	1532960	
4160	460000	
1040	2)4055180	
1532960	20·27590	
	4	
	1·10360	
	40	
	4·14400 *Area* 20*a.* 1*r.* 4*p.*	

3. Required the plan and area of a field, from the following dimensions.

	BA	diag.
	1008	
E 195	466	
Return	to B	
	AD	diag.
	1345	
C 415	944	
	855	536 B
Begin	at A	range W.

Answer.

Double areas
1279095 trapezium ABDC
196560 triangle AEB
Whole area 7*a.* 1*r.* 20½*p.*

4. Lay down a field, and find its area, from the following notes.

	DF	diag.
	1940	
	1040	362 B
E 581	825	
	R. off D	
	AD	diag.
	1488	
C 322	772	
	606	665 B
Begin	at A	range W.

Answer.

Double areas
1468656 trapezium ABDC
1829420 trapezium DEFB
Whole area 16*a.* 1*r.* 38*p.*

5. Draw the plan of a field, and find its area, from the following dimensions.

	HK	diag.
	1285	
I 382	740	
	600	162 G
	R. off H	
	FH	diag.
	1223	
E 661	803	
	666	276 G
	L. off F	
	DF	diag.
	1716	
	1080	246 B
E 409	761	
	R. off D	
	AD	diag.
	1547	
C 603	1023	
	525	488 B
Begin	at A	range W.

Answer.

Double areas.
1687777 trapezium ABDC
1123980 trapezium DEFB
1145951 trapezium FGHE
699040 trapezium HGKI
Whole area 23*a.* 1*r.* 5*p.*

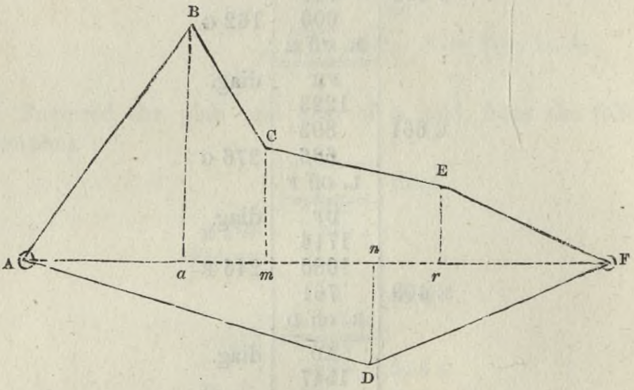
ANOTHER METHOD.

A field consisting of five, six, seven, or more sides may sometimes be surveyed by measuring one diagonal, and upon it erecting perpendiculars to all the opposite angles, on each side. This process will divide the whole field into right-angled triangles, and trapezoids, the areas of which must be found as before.

Examples.

1. Lay down a field, and find its area, from the following notes.

	AF	
	1896	diag.
E 259	1342	
	1132	325 D
	1000	
C 367	763	
B 756	522	
Begin	at A	range E.



Triangle AB*a*
756 per.
522 base
1512
1512
3780
394632

Trapezoid *a*BC*m*
756 } per.
367 }
1123 sum
241 base
1123
4492
2246
270643

Trapezoid *m*CE*r*
367 } per.
259 }
626 sum
579 base
5634
4382
3130
362454

Triangle *r*EF
554 base
259 per.
4986
2770
1108
143486

Triangle ADF		394632	double areas collected.
1896 base		270643	
325 per.		362454	
9480		143486	
3792		616200	
5688		2)1787415	
616200		8·93707	
		4	
		3·74828	
		40	
		29·93120	

Area 8*a*. 3*r*. 30*p*.

2. Lay down a field, and find its area, from the following dimensions.

	AK	diag.
0	1700	0
I 290	1465	*d*
w	1368	365 H
r	1055	381 G
F 144	986	*n*
m	794	218 E
e	515	350 D
C 250	444	*c*
a	150	275 B
0	000	0
Begin	at A	range W.

Answer.

Triangles and Trapezoids on the Right.

Double areas
41250 triangle AB*a*
228125 trapezoid *a*BD*e*
158472 trapezoid *e*DE*m*
156339 trapezoid *m*EG*r*
233498 trapezoid *r*GH*w*
121180 triangle *w*HK
938864 sum

Triangles and Trapezoids on the Left.

Double areas	
111000	triangle AcC
213548	trapezoid cCFn
207886	trapezoid nFId
68150	triangle dIK
600584	sum
938864	sum brought forward
1539448	sum total

Whole area 7*a.* 2*r.* 31½*p.*

3. It is required to lay down a field, and find its area, from the following notes.

	AL	diag.
M 460	2150	0
d	1670	295 K
I 395	1530	w
r	1345	160 H
G 670	1275	n
F 400	880	m
e	780	270 E
c	465	150 D
C 405	305	a
0	000	300 B
Begin	at A	range E.

Answer.

Triangles and Trapezoids on the Right.

Double areas	
209250	trapezoid ABDc
132300	trapezoid cDEe
242950	trapezoid eEHr
147875	trapezoid rHKd
141600	triangle dKL
873975	sum

Triangles and Trapezoids on the Left.

Double areas	
123525	triangle AaC
462875	trapezoid aCFm
422650	trapezoid mFGn
271575	trapezoid nGIw
530100	trapezoid wIML
1810725	sum
873975	sum brought down
2684700	sum total

Whole area 13*a.* 1*r.* 27¾*p.*

PROBLEM VI.

Fields comprehended under any Number of Crooked or Curved Sides.

When a field is bounded by crooked fences, measure a line as near to each as the angles or curves will permit; in doing which, take an offset to each corner or angle in the fence. Where the fences are curved, those offsets must be so taken that a right line drawn from the end of any one perpendicular to the end of the next, on each side, would neither exclude any part of the land to be measured, nor include any of that which is adjacent. Perpendiculars thus erected will divide the whole offset into right-angled triangles and trapezoids, the areas of which are found as before.

NOTE 1.—If the curves be so large that many of the offsets would be 2, 3, 4, or 5 chains long, it will be more expeditious and accurate to measure the base without taking any offsets, except such as are short, leaving stations in proper places along the base, to which, when you have obtained its length, you may return, and from them run fresh station-lines to some convenient point, or points, in the curved fence. Upon these lines take offsets as before. (See Example 3.)

2. If any of the fences be curved inward, it is frequently most convenient to measure a line on the outside of the field, and upon it erect perpendiculars to the curved fence, which, in this case, are called insets; and the area thus included must be subtracted from the area of the whole figure. (See Example 4.)

3. When the fences and ditches are to be measured with the field to which they belong, it is generally most practicable to fix the stations within the fences, at a little distance from the corners, and then to measure to the roots of the quick-wood, adding or subtracting 5 or 6 links, according to the custom of the place, for the breadth of the ditch. (See Example 5.)

4. When the offsets are small, their places on the base line may be determined by laying the offset staff at right angles upon the chain; but when large, and accuracy is required, they must be found by the cross, and measured by the chain.

5. The base of each triangle and trapezoid, forming an offset, may be found by subtracting the distances on the chain line from each other.

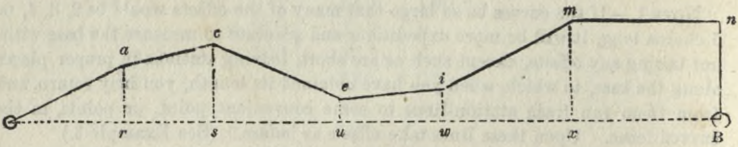
6. The following three methods of finding the area of offsets are erroneous. Some divide the sum of the offsets by their number, for a mean breadth; others divide that sum by one more than their number, for a mean breadth; and both multiply the whole base by the mean breadth, thus supposed to be found, for the area of the whole offset. The first of these methods generally gives the area too much; and the second sometimes too much and sometimes too little. A third method, which is usually more accurate than either of the preceding ones, is to set down each offset twice (accountiug that one where the boundary meets the station-line) except the first and last, which are only entered once. The sum of these offsets is then multiplied by the base, the product divided by the number of offsets set down, and the quotient given as the area required.

7. Directions for laying down offsets by a plotting-scale may be seen in Part II.

Examples.

1. Lay down the figure of a right-line offset, and find its area, from the following notes :—

	AB	
n 200	1569	
m 210	1249	
	1000	
i 70	952	
e 50	745	
c 159	450	
a 120	265	
0	000	
Begin at	A	range E.



BY THE TRUE METHOD.

Triangle A*ra*

265 base.
120 per.
5300
265
31800

Trapezoid *racs*

120 }
159 } per.
279 sum
185 base
1395
2232
279
51615

Trapezoid *sceu*

159 }
50 } per.
209 sum
295 base
1045
1881
418
61655

Trapezoid *ueiw*

50 }
70 } per.
120 sum
207 base
840
240
24840

Trapezoid *wimx*

70 } per.
210

280 sum
297 base
1960
2520
560
83160

Trapezoid *xmn*B

210 } per.
200

410 sum
320 base
8200
123
131200

31800
51615
61655
24840
83160
131200 } double areas collected

2)384270
1·92135
4
3·68540
40
27·41600

Hence the true area is 1*a.* 3*r* 27*p.*

BY THE FIRST FALSE METHOD.

120
159
50
70
210
200
6)809
134·8 mean breadth

1569 length
134·8 breadth
12552
6276
4707
1569
2·115012
4
·460048
40
18·401920

Here the area appears to be 2*a.* 0*r.* 18*p.*, which is too much by 31*p.*

BY THE SECOND FALSE METHOD.

```
   120                          1569 length
   159                          115·5 breadth
    50                          ------
    70                           7845
   210                          7845
   200                        1569
  ----                       1569
 7)809                       --------
 -----                       1·812195
 115·5 mean breadth                 4
 =====                       --------
                             3·248780
                                   40
                             --------
                             9·951200
                             ========
```

Here the area appears to be 1*a.* 3*r.* 1*p.*, which is too little by 17*p.*

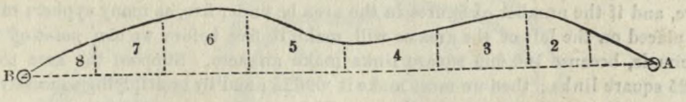
BY THE THIRD FALSE METHOD.

```
     0                          1569 length
   120                          1418 sum
   120                         ------
   159                         12552
   159                         1569
    50                        6276
    50                       1569
    70                   12)22·24842
    70                      --------
   210                       1·85403
   210                             4
   200                      --------
  ----                       3·41612
  1418 sum                        40
  ====                      --------
                            16·64480
                            ========
```

Here the area appears to be 1*a.* 3*r.* 16*p.*, which is too little by 11*p.*

2. Lay down a curve-line offset, and find its area, from the following notes.

	AB	
	1012	0
	892	53
	786	80
	645	95
	500	45
	350	63
	200	84
	100	52
	000	0
Begin at	A	range W.



52
100
5200 No. 1

52
84
136
100
13600 No. 2

63
45
108
150
5400
108
16200 No. 4

95
80
175
141
175
700
175
24675 No. 6

84
63
147
150
7350
147
22050 No. 3

45
95
140
145
700
56
14
20300 No. 5

80
53
133
106
798
133
14098 No. 7

53
120
1060
53
6360 No. 8

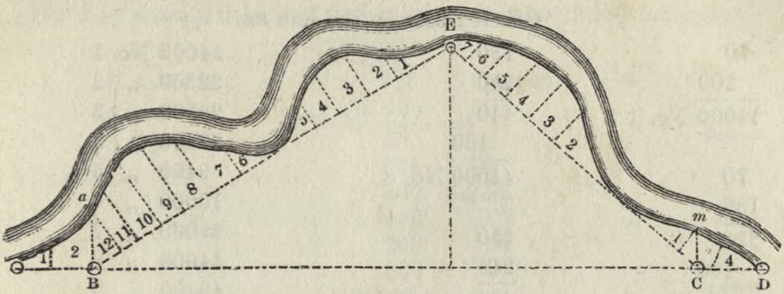
5200
13600
22050
16200
20300
24675
14098
6360
double areas collected

2)122483
·61241
4
2·44964
40
17·98560 *Area* 2*r.* 18*p.*

NOTE.—When the content of a small piece of land does not amount to an acre, and if the number of figures in the area be under five, as many cyphers must be placed on the left of the area as will make it five before we can *point-off* the decimals, because 100,000 square links make an acre. Suppose the area to be 9625 square links; then we must make it ·09625; and by multiplying successively by 4 and 40, we shall obtain for the area 15·4 square perches.

3. Lay down the figure of a piece of land adjoining a river, and find its area from the following notes.

	EB		
	1350		
	1265	140 *a*	
	1200	170	
	1100	244	
	1000	250	
	900	190	
	800	100	
	700	0	
	600	0	River
	500	94	
	400	142	
	300	153	
	200	70	
	000	0	
Left off E	and go	SW. to B	
	CE		
	1060	0	River 100 links in breadth
	1000	50	
	900	110	
	800	154	
	700	173	
	600	142	
	400	0	
	200	0	
	70	82 *m*	
	000	0	
Return	to C, and go	NW. to E	
	AD		
0	2463		
70	2300		
m 110	2200	C	
	2000		
River E 698	1410		
	1000		
a 165	250	B	
60	100		
0	000		
Begin	at A	range E.	



Triangle BCE

1950 base
698 per.
15600
1755
1170
1361100

Offsets taken on the line AD

60
100
6000 No. 1

60
165
225
150
11250
225
33750 No. 2

110
70
180
100
18000 No. 3

163
70
11410 No. 4

6000 No. 1
33750 „ 2
18000 „ 3
11410 „ 4
69160 sum

Offsets taken on the line CE

82
200
16400 No. 1

142
200
28400 No. 2

142
173
315
100
31500 No. 3

173
154
327
100
32700 No. 4

154
110
264
100
26400 No. 5

110
50
160
100
16000 No. 6

60
50
3000 No. 7

16400 No. 1
28400 „ 2
31500 „ 3
32700 „ 4
26400 „ 5
16000 „ 6
3000 „ 7
154400 sum

Offsets taken on the line EB

70
200
14000 No. 1

70
153
223
100
22300 No. 2

153
142
295
100
29500 No. 3

142
94
236
100
23600 No. 4

94
100
9400 No. 5

100
100
10000 No. 6

100
190
290
100
29000 No. 7

190
250
440
100
44000 No. 8

250
244
494
100
49400 No. 9

244
170
414
100
41400 No. 10

170
140
310
65
1550
186
20150 No. 11

85
140
3400
85
11900 No. 12

14000	No. 1
22300	„ 2
29500	„ 3
23600	„ 4
9400	„ 5
10000	„ 6
29000	„ 7
44000	„ 8
49400	„ 9
41400	„ 10
20150	„ 11
11900	„ 12
304650	sum

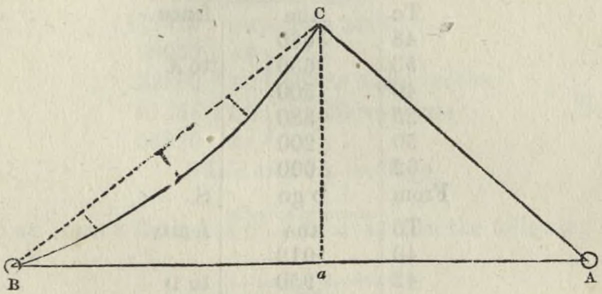
1361100
69160
154400
304650
} whole double areas collected

2)1889310
9·44655
4
1·78620
40
31·44800

Area 9*a.* 1*r.* 31*p.*

4. Lay down a field, and find its area, from the following notes.

	Ca	
	525	perpen.
	R. off C	
	BC	
	846	0
	600	64
	400	85
	200	50
	000	0
	R. off B	
	AB	
	1253	
	1000	
	586	*a*, the place of the perpen.
Begin at	A	range W.



Triangle ABC

```
   1253 base
    525 per.
   6265
  2506
 6265
 657825
```

Insets taken on the line BC

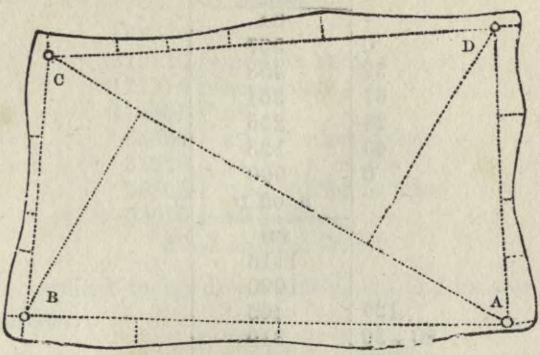
```
   50            85         10000 ⎫ double
  200            64         27000 ⎪ areas
10000           149         29800 ⎬ collected
                 200        15744 ⎭
               29800        82544

   50            246
   85             64
  135            984
  200           1476
27000          15744
```

```
 657825 triangle ABC
  82544 insets
2)575281 difference
 2·87640
       4
 3·50560
      40
20·22400 Area 2a. 3r. 20p.
```

5. Draw a plan of a field, and find its area, from the following notes.

	AC	diag.
	1155	
	1000	
495	915	
	360	520
From	A go	N.W.
To	the	fence
48	660	
53	630	to A
40	500	
25	380	
50	200	
62	000	
From	D go	S.
To	the	fence
40	1012	
42	950	to D
54	600	
33	400	
12	260	
30	150	
65	000	
From	C go	E.
To	the	fence
25	615	
35	550	to C
30	400	
10	240	
22	150	
45	000	
From	B go	N.
To	the	fence
33	1090	
45	1045	to B
56	800	
40	500	
48	300	
30	000	
Begin at	A and	go W.



Answer.

Double areas

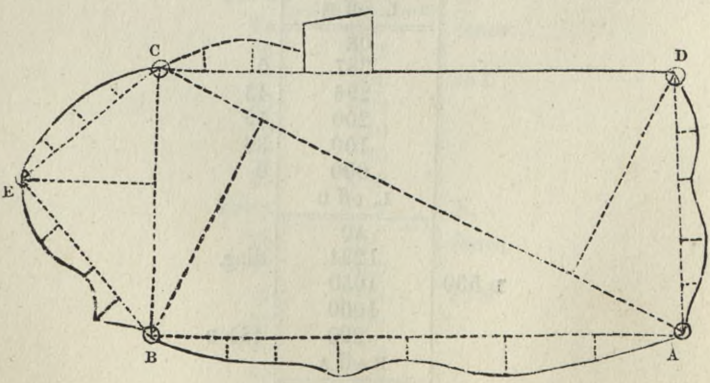
1172325	trapezium ABCD	
98055	AB	offsets taken on the different lines
32980	BC	
81254	CD	
58820	DA	

Whole area 7*a*. 0*r*. $34\frac{3}{4}$*p*.

6. Lay down a field, and find its area, from the following dimensions.

	EB	
	442	0
	345	74
	256	25
	115	56
	000	0
	L. off E	
	CE	
	387	0
	296	43
	200	59
	100	36
	000	0
	L. off C	
	AC	
	1294	diag.
B 530	1050	
	1000	
	290	485 D
	R off A	

	DA	
0	567	
32	458	
67	364	
24	235	
43	123	
0	000	
	R. off D	
	CD	
	1116	
	1000	
129	465	
80 + 30	310	
65	200	
42	100	
0	000	
	R. off C	
	BC	
	584	diag.
E 293	328	
	R. off B	
	AB	
0	1173	
37	1000	
44	900	
78	750	
46	600	
85	400	
42	200	
0	000	
Begin at	A	range W.



Answer.

Double areas		
1313410	trapezium ABCD	
171112	triangle BEC	
111401	AB	offsets taken on the different lines
62395	CD	
37326	DA	
26805	CE	
33850	EB	

Whole area 8*a*. 3*r*. 5*p*.

7. It is required to lay down a field, and find its area, from the following notes.

To	the	fence
80	1121	
100	1025	to A
83	930	
70	800	
100	700	
130	600	
190	500	
140	400	
70	300	
50	200	
40	100	
0	000	
	R. off D	
	BD	diag.
	1900	
	1300	823 A
	1000	
C 780	440	
	R. off B	
	AB	
0	1530	
66	1350	
73	1200	
85	1100	
94	1000	
73	900	
40	780	
70	630	
110	467	
76	300	
70	200	
69	100	
96	000	
Begin	at A	and go W.

Answer.

Double areas
3045700 trapezium ABCD
218592 offsets taken on the line AB
205555 ditto on the line DA
Whole area 17*a.* 1*r.* 15*p.*

8. Required the area and plan of a field, from the following dimensions.

	AD	
	1080	0
	1000	40
	950	60
	890	54
	820	38
	740	12
	650	10
	580	35
	535	60
	480	75
	420	65
	300	35
	220	30
	100	60
	50	70
	000	50
	L. off A	
	CA	
	1170	diag.
	920	540 B
D 525	225	
	R. off C	
	BC	
0	1065	
25	1005	
35	946	
50	870	
40	830	
90	780	
115	715	
110	650	
80	625	
75	510	
55	440	
70	330	
65	250	
35	215	
48	150	
40	100	
60	50	
55	000	
	R. off B	

To	the	fence
0	700	
40	645	to B
55	570	
72	500	
68	450	
49	375	
42	300	
37	225	
53	170	
40	130	
50	50	to A
52	000	
From the	fence	go N.

Answer.

Double areas
1246050 trapezium ABCD
67710 offsets taken on the line AB
129820 ditto on the line BC
91210 ditto on the line AD
Whole area 7*a*. 2*r*. 27¾*p*.

9. Lay down a field, and find its area, from the following notes.

	EA	
0	500	
18	450	
40	400	
74	350	
108	300	
104	250	
80	200	
35	100	
20	50	
0	000	
	R. off E	
	DE	
0	590	
20	550	
50	500	
130	450	
152	400	
110	350	
94	300	
110	250	
105	200	
80	150	
50	100	
25	50	
0	000	
Return	to	D

	AC	
	1220	diag.
B 550	915	
	700	524 D
	R. off A	
	DA	
	875	diag.
E 322	490	
	R. off D	
	CD	
0	750	
34	700	
55	650	
70	600	
84	550	
90	500	
95	450	
100	400	
75	350	
38	320	
44	200	
40	100	
30	50	
0	000	
	R. off C	
	BC	
0	640	
18	600	
25	550	
30	500	
25	450	
35	430	
65	350	
60	320	
40	250	
10	140	
20	100	
40	000	
	R. off B	
To	the	fence
65	1115	
70	1075	to B
60	1000	
55	920	
60	868	
40	800	
20	750	
28	700	
65	600	
80	550	

73	400	
70	300	
34	200	
40	134	
23	100	
0	000	
Begin	at A	range W.

Answer.

Double areas
1310280 trapezium ABCD
281750 triangle AED

116056	AB	offsets taken on the different lines
41020	BC	
83180	CD	
92400	DE	
53650	EA	

Whole area 9*a.* 3*r.* 22½*p.*

PROBLEM VII.

Narrow Pieces of Land.

A method frequently adopted, in surveying narrow pieces of land, is to take breadths in different places, add all these breadths together, and divide their sum by their number, for a mean breadth; and this supposed mean breadth is multiplied by the length, for the area : but, as the method of finding the mean breadth is liable to mistake, this process may lead to very erroneous results.

If, for example, a piece of land taper regularly from one end to the other, you may take its breadth at each end; half the sum of these breadths will then be its mean breadth; so that you may multiply this mean breadth by the length, for the area. But if it be irregular, you must take breadths in the widest and narrowest places, or at every particular curve, noting the place of each breadth upon the chain-line. These breadths will divide it into triangles and trapezoids, which compute as before.

NOTE. 1.—The breadths must be taken directly across the land to be measured, and therefore, if considerable, will require the use of the cross.

2. If a piece of land be curved, or longer on one side than on the other, by measuring along the middle you will obtain the true or mean length.

3. When several pieces of land, of various lengths, are contiguous to each other, it will generally be most expeditious to measure only one base-line, noting the point where each piece begins and ends, perpendicularly to the line. In this case

be especially careful that no confusion take place in noting down the breadths of the respective pieces.

4. Paring, reaping, &c., both in this and the foregoing problems, should be surveyed with a slack chain, in order to obtain the measurement of the surface.

5. It is best to take the first and last breadths of lands or ridges about half a chain from each end, and account them as the end-breadths; because, in turning, the plough usually makes some of them appear either broader or narrower than they are in reality. It may also be observed, that it is frequently necessary to take the breadths to half a link; for when the length is great, half a link in the breadth is too considerable to be neglected.

6. If a narrow piece of land be very irregular, you may obtain its area most accurately by measuring a base-line, in a convenient position; and upon it erecting perpendiculars to the boundaries, on each side.

7. In surveying with the chain and cross, when the area only of any field or piece of ground is required, it is unnecessary to lay down the figure.

Examples.

1. Find the area of a tapering piece of land, whose length is 2562 links, and breadth at one end 126, and at the other 232 links.

$$\left.\begin{array}{r}126\\232\end{array}\right\}\text{ breadths}$$

$$\begin{array}{rl}
2)\overline{358} & \text{sum} \\
\hline
179 & \text{mean} \\
2562 & \text{length} \\
\hline
358 & \\
1074 & \\
895 & \\
358 & \\
\hline
4{\cdot}58598 & \\
4 & \\
\hline
2{\cdot}34392 & \\
40 & \\
\hline
13{\cdot}75680 & \textit{Area } 4a.\ 2r.\ 14p.
\end{array}$$

2. Find the area of a piece of land, which is broadest towards the middle, from the following dimensions.

BY THE TRUE METHOD.

2322	169
2000	
1056	215
1000	
000	125
base.	per.

$$\left.\begin{array}{r}125\\215\end{array}\right\}\text{ per.}$$

$$\begin{array}{rl}
340 & \text{sum} \\
1056 & \text{base} \\
\hline
2040 & \\
170 & \\
34 & \\
\hline
359040 &
\end{array}$$

$$\left.\begin{array}{r}215\\169\end{array}\right\}\text{ per.}$$

$$\begin{array}{rl}
384 & \text{sum} \\
1266 & \text{base} \\
\hline
2304 & \\
2304 & \\
768 & \\
384 & \\
\hline
486144 &
\end{array}$$

```
  359040 } double areas
  486144 }  collected
2)845184
  4·22592
        4
  ·90368
       40
 36·14720 Area, 4a. 0r. 36p.
```

BY THE FALSE METHOD.

```
 125 }                         2322 length
 215 } breadths               169·6 breadth
 169 }                         13932
3)509                         20898
 169·6 mean                  13932
                             2322
                             3·938112
                                    4
                             3·752448
                                    40
                             30·097920
```

Here the area appears to be 3*a.* 3*r.* 30*p.*, which is too little by 1*r.* 6*p.*

Again, the dimensions remaining as before, suppose the piece to be narrowest towards the middle; the area by the false method will be the same as already found.

BY THE TRUE METHOD.

2322	169
2000	
1056	125
1000	
000	215
base.	per.

```
  215 }               125 }
  125 } per.          169 } per.
  340 sum             294 sum
 1056  base          1266 base
  2040               1764
 170                1764
 34                 588
 359040            294
                   372204
```

```
  359040 } double areas
  372204 }   collected
2)731244
  3·65622
        4
  2·62488
       40
 24·99520
```

The true area is $3a.$ $2r.$ $25p.$; hence the false area is too much by $1r.$ $5p.$

Lastly, the dimensions still continuing, suppose the breadth towards the middle to be greater than that at one end, and less than at the other; the false area will still be the same.

BY THE TRUE METHOD.

2322	125
2000	
1056	169
1000	
000	215
base.	per.

```
  215 } per.
  169 }
  384 sum
 1056 base
 2304
1920
384
405504
```

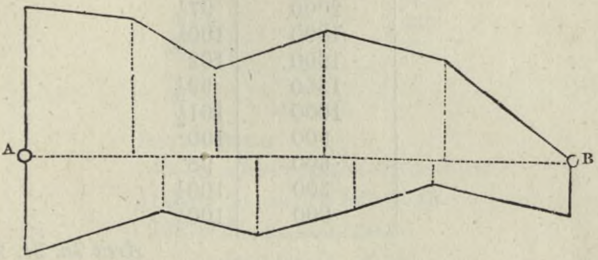
```
  169 } per.
  125 }
  294 sum
 1266 base
 1764
1764
588
294
372204
```

```
  405504 } double areas
  372204 }   collected
2)777708
  3·88854
        4
  3·55416
       40
 22·16640
```

The true area is $3a.$ $3r.$ $22p.$; hence the false area is too much by $8p.$

3. Draw a plan of an irregular piece of land, and find its area from the following dimensions.

	AB	
0	1325	136
246	1015	
	987	58
	790	134
318	718	
	560	162
223	465	
	345	125
346	266	
372	000	246
From	A go	E.



Answer.

Double areas
361176 offsets on the right
684860 ditto on the left
2)1046036 sum
5·23018 = 5*a*. 0*r*. 36$\frac{3}{4}$*p*. the *area* required.

4. Find the area of five lands, from the following dimensions.

2378	185
2000	190
1700	194
1400	198
1000	200
700	195
400	189
000	185

Area 4*a*. 2*r*. 13$\frac{1}{2}$*p*.

5. Required the area of six lands, from the following notes.

3422	189
3000	
2500	204
2000	
1800	226
1000	
800	191
000	165

Area 6*a.* 3*r.* 12*p.*

6. Find the area of seven lands, from the following dimensions.

NOTE.—In calculating the area, the half-links must be treated as decimals.

2900	99½
2600	98½
2300	101
2000	97½
1900	100½
1600	102
1300	99½
1000	101½
900	100
600	98
300	100½
000	100½

Area 2*a.* 3*r.* 23¾*p.*

7. It is required to lay down a narrow piece of land, and find its area, from the following dimensions.

	AB	
250	1230	460
	1050	250
60	1000	
	918	300
25	800	
	690	235
0	500	
	440	108
0	300	
	100	216
153	000	130
From	A go	N.

Answer.

Double areas
552890 offsets on the right
140800 ditto on the left
Whole area 3*a.* 1*r.* 35*p.*

8. Lay down a field, and find its area, from the following notes.

	BC	
0	521	
70	400	
97	300	
99	200	
78	100	
0	000	
	R. off B	
	AB	
0	1235	521 to C
57	1100	430
114	900	323
177	650	245
207	500	219
232	350	240
252	200	275
268	000	360
From	A go	N.

Answer.

Double areas
763685 offsets on the right of AB
414695 do. on the left of AB
70270 do. on the line BC
Area 6*a.* 0*r.* 38$\frac{3}{4}$*p.*

PROBLEM VIII.

Meres and Woods.

When you have a mere or wood to survey, by the help of your cross, fix four marks on its outside, in such a manner as to form a rectangle or square. Then measure each side of the rectangle or square, taking insets to the edge of the mere or wood, the area of which insets must be treated as directed in Note 2, Problem VI.

If the opposite sides be not found equal, or very nearly so, your marks do not form four right angles; in which case you must rectify your error.

NOTE 1.—It is a more expeditious method to measure the four sides of a quadrilateral figure, having one right angle, paying no regard to the length of the sides. Then construct the figure by Part I. Problem XIV., and draw the longer diagonal, upon which let fall a perpendicular from each of the opposite angles. This diagonal and these perpendiculars you must measure by the scale used in plotting.

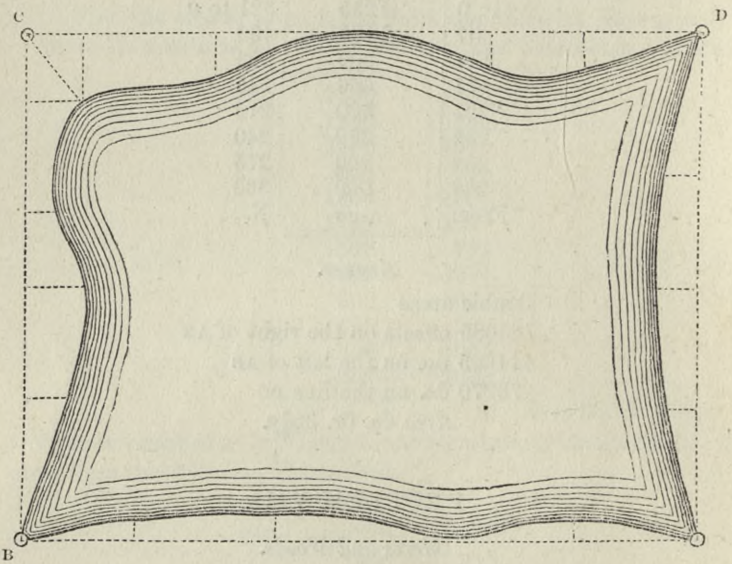
2. It sometimes happens that the mere or wood is of a triangular form; in this

case, the work may be very readily done by measuring the three sides of a triangle, taking insets as before. After which construct the triangle, and from the opposite angle let fall a perpendicular upon the base, and proceed as before.

3. By this problem you may measure fields into which you are not permitted to enter, or which contain obstructions.

Examples.

1. Let the following figure represent a mere; its area is required.



Having fixed four marks, A, B, C, and D, forming a rectangle; begin at A, and measure the line AB, taking the necessary insets, and entering them in your field-book. In the same manner proceed with the other three sides; and you will find noted the following dimensions.

DA	
1100	0
1000	
800	50
550	70
300	60
000	0
R. off D	

	CD	
	1450	0
	1200	70
	1000	80
	820	0
	600	0
	400	95
	110	150
	000	0
	R. off C	
	BC	
	1100	0
	950	110
	700	70
	550	142
	300	100
	000	0
	R. off B	
	AB	
	1450	0
	1200	55
	1000	
	900	75
	550	32
	250	65
	000	0
Begin at	A	range W.

Answer.

1595000 area of the rectangle ABCD

135550 AB
183800
168450 CD
95500 DA
(double areas of the insets taken on the different lines)

2)583300

291650 area of the whole insets

13·03350 ditto of the mere

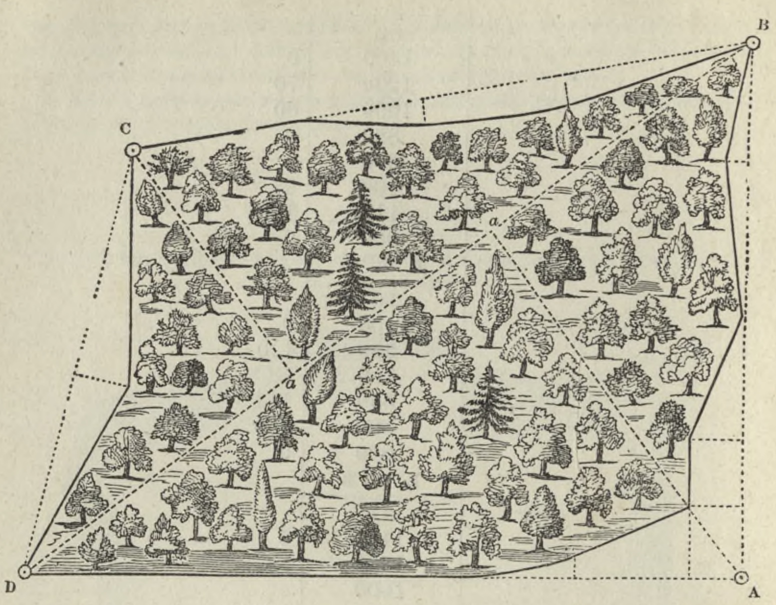
4

·13400

40

5·36000 *Area* 13*a.* 0*r.* 5*p.*

2. Let the following figure represent a wood; its area is required.



Set up your cross at A, and let your assistant fix the marks B and D, so that the angle at A may be a right angle; and measure the line AB, taking insets as before. Then fix the mark C, as most convenient; measure the other three lines, and you will find in your field-book the following notes.

	DA
0	1550
160	1440
50	1200
	1000
0	900
	L. off D
	CD
0	950
120	500
0	000
	L. off C
	BC
	1340
0	1050
	1000
50	700
60	400
0	000
	L. off B

	AB	
0	1150	
	1000	
50	900	
0	550	
100	300	
110	160	
0	000	
Begin at	A	range N.

Answer.

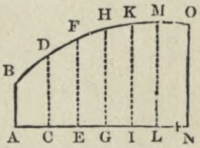
Having constructed the figure, you will find the diagonal BD to measure 1930, the perpendicular A *a* 923, and C *a* 605 links.

Double areas		
2949040	trapezium ABCD	
102000	AB	insets taken on the different lines
74500	BC	
114000	CD	
83000	DA	
373500	whole insets	
2575540	wood.	*Area* 12*a*. 3*r*. 20*p*.

PROBLEM IX.

To find the Area of a Segment of a Circle, or any other Curvilineal Figure, by means of Equidistant Ordinates.

RULE.—If a right line AN be divided into any even number of equal parts AC, CE, EG, &c.; and at the points of division be erected perpendicular ordinates AB, CD, EF, &c., terminated by any curve BDF, &c.; and if A be put for the sum of the extreme or first and last ordinates, AB, NO; B for the sum of the even ordinates, CD, GH, LM, &c., viz., the second, fourth, sixth, &c.; and C for the sum of all the rest, EF, IK, &c., viz., the third, fifth, &c., or the odd ordinates, wanting the first and last; then the common distance AC, or CE, &c., of the ordinates, being multiplied by the sum arising from the addition of A, four times B, and two times C; one-third of the product will be the area ABON, very nearly; that is, $\frac{A+4B+2C}{3}\times D$ = the area, putting D = AC, the common distance of the ordinates.



NOTE.—The foregoing rule being expressed in an algebraic form, is seldom properly understood by learners; but the following one may be easily comprehended and committed to memory.

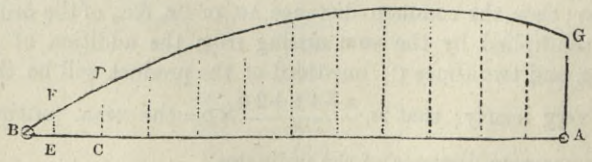
RULE.—To the sum of the first and last ordinates, add four times the sum of all the even ordinates, and twice the sum of all the odd ordinates, not including the first and last; multiply this sum by the common distance of the ordinates, divide the product by 3, and the quotient will be the area required.

NOTE.—The length of the base must be ascertained before you begin to take the ordinates, in order that you may divide it into an even number of equal parts; or you may take the dimensions without doing this, and find the areas of the pieces at the end, by the rules for triangles and trapezoids, which being added to that part of the figure computed by equidistant ordinates, will give the whole area. See the following examples.

Examples.

1. Required the plan and area of a piece of land, measured by equidistant ordinates, from the following notes.

	AB	
	1167	0
	1100	44 EF
	1000	97 CD
	900	139
	800	175
	700	206
	600	230
	500	248
	400	260
	300	264
	200	268
	100	262
	000	252
Begin	at A and	go W.



The first and last ordinates
252 the first ordinate AG
97 the last ordinate CD
349 sum

The even ordinates
262 second
264 fourth
248 sixth
206 eighth
139 tenth
1119 sum
4
4476 four times the sum

The odd ordinates
268 third
260 fifth
230 seventh
175 ninth
933 sum
2
1866 twice the sum

349 the first and last ordinates
4476 four times the sum, &c.
1866 twice the sum, &c.
6691 sum total
100 the common distance
3)669100
223033 the area of the figure ACDG

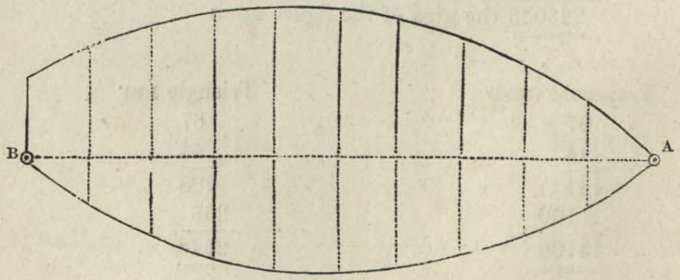
Trapezoid CEFD	Triangle EBF
97	67
44	44
141	268
100	268
14100	2948

Double areas
14100 trapezoid CEFD
2948 triangle EBF
2)17048 sum
8524 the area of the figure CBD
223033 ditto of the figure ACDG
2·31557 sum
4
1·26228
40
10·49120

Hence the area of the whole figure ACEBFDGA is $2a.\ 1r.\ 10\frac{1}{2}p.$ nearly.

2. Lay down a piece of ground, and find its area from the following equidistant ordinates.

	AB	
0	1000	0
85	900	75
150	800	125
200	700	160
230	600	180
247	500	185
250	400	177
240	300	157
216	200	125
180	100	73
130	000	0
Begin	at A and	go E.



The first and last ordinates
130 the first ordinate
000 the last ditto
130 sum

The even ordinates
$180 + 73 = 253$ second
$240 + 157 = 397$ fourth
$247 + 185 = 432$ sixth
$200 + 160 = 360$ eighth
$85 + 75 = 160$ tenth
1602 sum
4
6408 four times the sum

The odd ordinates
$216 + 125 = 341$ third
$250 + 177 = 427$ fifth
$230 + 180 = 410$ seventh
$150 + 125 = 275$ ninth
1453 sum
2
2906 twice the sum

130 the first and last ordinates
6408 four times the sum, &c.
2906 twice the sum, &c.
9444 sum total
100 the common distance
3)944400
3·14800 area in square links
4
·59200
40
23·68000 *Area* 3*a.* 0*r.* $23\frac{1}{2}$*p.*

3. Required the plan and area of a piece of ground, from the following equidistant ordinates.

	AB	
220	1200	
234	1100	
245	1000	
250	900	
246	800	
235	700	
221	600	
200	500	
176	400	
140	300	
100	200	
55	100	
0	000	
Begin	at A	range E.

Answer.

220 the first and last ordinates
4456 four times the sum, &c.
1976 twice the sum, &c.

6652 sum total
100 the common distance

3)665200

2·21733 area in square links
4

·86932
40

34·77280 *Area* 2*a.* 0*r.* 34¾*p.*

4. Find the area of 4 lands, measured by equidistant ordinates, from the following notes.

182	1290	70
183	1200	101
178	1100	115
189	1000	112
190	900	96
187	800	98
179	700	95
150	600	100
182	500	120
185	400	110
180	300	131
160	200	133
170	100	137
188	000	130

Answer.

602 the first and last ordinates
7092 four times the sum, &c.
2848 twice the sum, &c.
10542 sum total
100 the common distance
3)1054200
351400
24120 trapezoid at the end
3·75520 area in square links
4
3·02080
40
0·83200 *Area* 3*a.* 3*r.* $0\frac{3}{4}$*p.*

5. Find the area of 6 lands, by equidistant ordinates, from the following dimensions.

147	3090
153	3000
152	2700
150	2400
149	2100
147	1800
144	1500
142	1200
143	900
148	600
146	300
145	000

Answer.

298 the first and last ordinates
2936 four times the sum, &c.
1174 twice the sum, &c.
4408 sum total
300 the common distance
3)1322400
4·40800
13500 trapezoid at the end
4·54300 area in square links
4
2·17200
40
6·88000 *Area* 4*a.* 2*r.* $6\frac{3}{4}$*p.*

6. Required the plan and area of a piece of ground from the following equidistant ordinates.

	AB	
236	1200	180
170	1100	220
126	1000	246
90	900	265
67	800	270
55	700	269
57	600	260
66	500	243
87	400	215
120	300	180
170	200	134
232	100	65
327	000	0
Begin	at A and	go W.

Answer.

743 the first and last ordinates
7900 four times the sum, &c.
3264 twice the sum, &c.
11907 sum total
100 the common distance
3)1190700
3·96900 area in square links
4
3·87600
40
35·04000 *Area 3a. 3r. 35p.*

7. Required the plan and area of a field from the following equidistant ordinates.

	AB	
217	1096	202
187	1000	150
150	900	112
125	800	84
107	700	66
98	600	58
95	500	57
100	400	65
114	300	80
130	200	110
167	100	148
190	000	200
Begin	at A and	go N.

Answer.

727 the first and last ordinates
4384 four times the sum, &c.
1540 twice the sum, &c.
6651 sum total
100 the common distance
3)665100
221700
36288 trapezoid at the end
2·57988 area in square links
4
2·31952
40
12·78080 *Area 2a. 2r.* $12\frac{3}{4}p.$

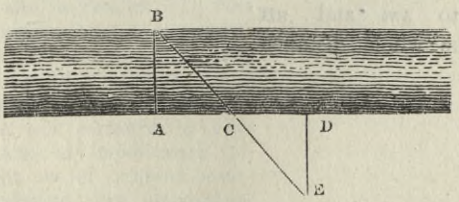
NOTE.—Whenever the rule given in this Problem can be applied, it will be found more easy, expeditious, and accurate, in finding the areas of offsets and of narrow pieces of land, than the rules for triangles and trapezoids.

PROBLEM X.

To find the Breadth of a River.

EXAMPLE.

Let the following figure represent a river, the breadth of which is required.



Fix upon any object B, close by the edge of the river on the side opposite to which you stand. By the help of your cross, make AD perpendicular to AB; also make AC = CD, and erect the perpendicular DE; and when you have arrived at the point E, in a direct line with CB, the distance DE will be = AB, the breadth of the river; for by Theo. 1, Part I., the angle ACB = DCE, and as AC = CD, and the angles A and D are right angles, it is evident that the triangles ABC and CDE are not only similar but equal.

NOTE 1.—The distance between A and the edge of the river must be deduced from DE, when it is not convenient to fix A close by the river's edge.

2. This Problem may also be well applied in measuring the distance of an inaccessible object; for let AC equal 8, CD equal 2, and DE equal 10 chains; then, by similar triangles, as CD : DE : : AC : AB; that is, as 2 : 10 : : 8 : 40 chains = AB. (See Theo. 11, Part I.)

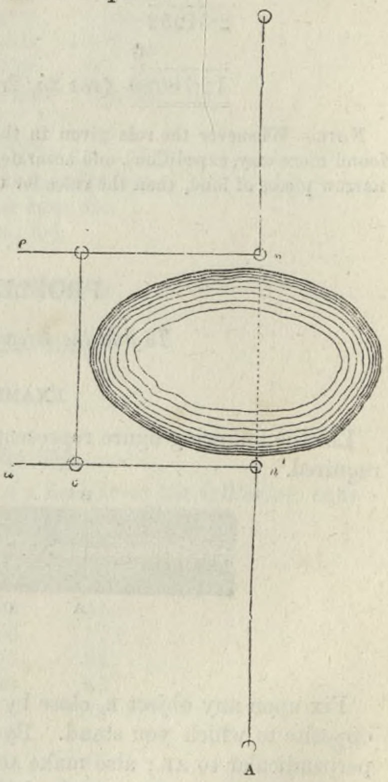
PROBLEM XI.

Lines upon which there are Impediments not obstructing Sight.

Suppose *mn* to represent a deep pit or water, and A and B two objects, the direct distance of which is required.

At the verge of the impediment, having fixed the mark *m*, in a right line with A and B, measure from A to *m;* and at *m*, by the help of your cross, erect the perpendicular *ma*, which measure to the outside of the interposed obstruction, as at *c*. Then on the other side, as at *n*, in a line with A and B, erect the perpendicular *ne;* and make *nb* equal to *mc*.

Measure *bc*, which will be equal to *mn;* and from *n* measure the distance *n*B; then *bc* added to A*m* and *n*B will give the whole distance AB.

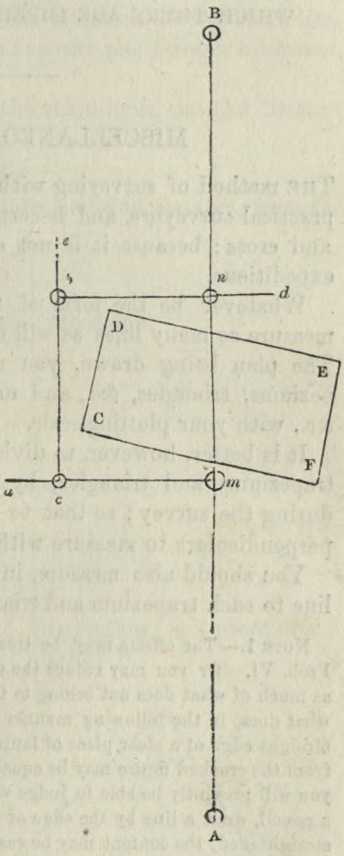


PROBLEM XII.

Lines upon which there are Impediments obstructing Sight.

EXAMPLE.

Suppose CDEF to represent the base of a building which obstructs the sight, and through which it is necessary that a straight line should pass from an object at A to B, the direction A*m*B on the side A being given.



Measure from A to *m*; at *m* erect the perpendicular *ma*, which measure until you are clear of the impediment, as at *c*.

Erect the perpendicular *ce*, which measure until you are beyond the building, as at *b*. Erect the perpendicular *bd*; and make *bn* equal to *mc*, at which point you will be in a direct line with *m*A. Erect the perpendicular *n*B, which measure; then *bc*, added to A*m* and *n*B, will give the whole distance AB.

NOTE.—In the tenth and previous editions of the work the range A*m* was evidently understood in this problem *as given*, and also in Problem VII. Part IV. p. 139, although not so expressed. This involves an omission; for the range or direction of a line must always be known before it can be measured. The error is now corrected in both examples. And as impediments obstructing sight are of frequent occurrence in surveying, two propositions have been added to find the line when the range A*m* is not given—the one with the chain, p. 89, and the other with the theodolite and chain, Part IX.

PART IV.

THE METHOD OF SURVEYING WITH THE CHAIN ONLY; AND OF MEASURING MERES, WOODS, DISTANCES, LINES, UPON WHICH THERE ARE IMPEDIMENTS, AND HILLY GROUND.

MISCELLANEOUS INSTRUCTIONS.

THE method of surveying with the chain only is adopted by most practical surveyors, and is certainly preferable to that by the chain and cross; because it is not only as accurate, but generally more expeditious.

Whatever be the form of the field or ground to be surveyed, measure as many lines as will enable you to plot it with accuracy. The plan being drawn, you may then divide the figure into trapeziums, triangles, &c., and measure the diagonals, perpendiculars, &c., with your plotting-scale.

It is better, however, to divide small pieces and single fields into trapeziums and triangles, by measuring the diagonals and bases during the survey; so that to find the area, you will have only the perpendiculars to measure with the scale.

You should also measure, in some convenient direction, a proof-line to each trapezium and triangle.

NOTE 1.—The offsets must be treated according to the directions in Part III. Prob. VI. Or you may reduce the crooked sides to straight ones, by including as much of what does not belong to the field under your survey as you exclude of what does, in the following manner:—Apply to the crooked line in question the straight edge of a clear piece of lantern horn, so that the small parts cut off by it from the crooked figure may be equal to those which are taken in (of this equality you will presently be able to judge very correctly by a little practice); then, with a pencil, draw a line by the edge of the horn. The sides being thus successively straightened, the content may be easily found.

2. A slender bow of cane or whalebone, strung with a silk thread, may be substituted for the horn. The thread must be applied to the crooked fence, and two marks made by which to draw a straight line.

3. The sides may also be straightened by a parallel ruler; but the operation is generally tedious, and must be performed with the greatest care, or it will not be more correct than the foregoing method.

4. When the three sides of a triangle are given, the area may be found as fol-

lows: From half the sum of the three sides subtract each side severally; multiply the half sum and the three remainders continually together, and the square root of the last product will be the area required. This method is too prolix, except in particular cases; the operation may, however, be considerably simplified by performing the multiplication and evolution by logarithms.

PROBLEM I.

Triangular Fields.

When you have a triangular field to survey, begin at the most convenient corner and measure each side; and while measuring any one of the sides, leave a mark in some situation on the chain-line, that the distance between it and the opposite angle being measured, may be a proof-line.

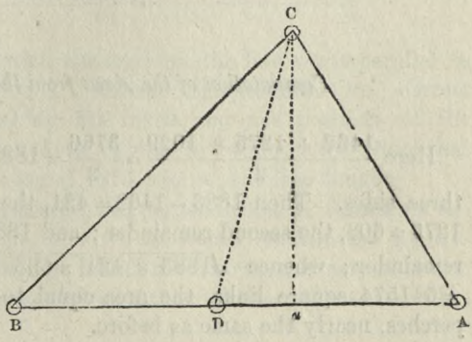
Or, leave marks upon any two of the chain-lines, and the distance between them will prove your work.

Examples.

1. It is required to construct a figure, and find its area, from the following notes.

	DC	
	913	
	return to D	proof-line
	CA	
	1029	
	R. off C	
	BC	
	1275	
	1000	
	R. off B	
	AB	
	1462	
	1000	
	800	D, station for a proof-line
Begin at	A	range W.

Having constructed the figure, you will find the line DC to measure 913 links, as in the field-book; hence, you may conclude there is no error committed in taking or setting down the dimensions.



NOTE 1.—If your proof-line upon the plan does not agree, or nearly so, with that taken in the field, you may be assured that some error has been committed; you must, therefore, repeat the survey in order to discover it.

2. When land is level, and the lines are well driven, and not very long, you will generally find them to meet correctly.

To find the Perpendicular.

(See Part I. Prob. VI.)

Or, if you make use of a plotting-scale, lay it across the base in such a manner that any of the lines which are opposite to each other on the edges of the scale, may coincide with the base, the edge of the scale at the same time touching the opposite angle; by that edge draw a line from the base to the opposite angle; this line, or perpendicular Ca, in the present case, you will find to be 878 links.

$$
\begin{array}{rl}
1462 & \text{base} \\
\underline{878} & \text{per.} \\
11696 & \\
10234 & \\
\underline{11696} & \\
2)\underline{1283636} & \\
6{\cdot}41818 & \\
\underline{4} & \\
1{\cdot}67272 & \\
\underline{40} & \\
\underline{\underline{26{\cdot}90880}} & \textit{Area } 6a.\ 1r.\ 27p.
\end{array}
$$

Computation of the Area from the Three Sides.

Here $\frac{1462 + 1275 + 1029}{2} = \frac{3766}{2} = 1883$, half the sum of the three sides. Then $1883 - 1462 = 421$, the first remainder; $1883 - 1275 = 608$, the second remainder; and $1883 - 1029 = 854$, the third remainder; whence $\sqrt{1883 \times 421 \times 608 \times 854} = \sqrt{411617533376}$ $= 641574$ square links, the area equal to 6 acres, 1 rood, and $26\frac{1}{2}$ perches, nearly the same as before.

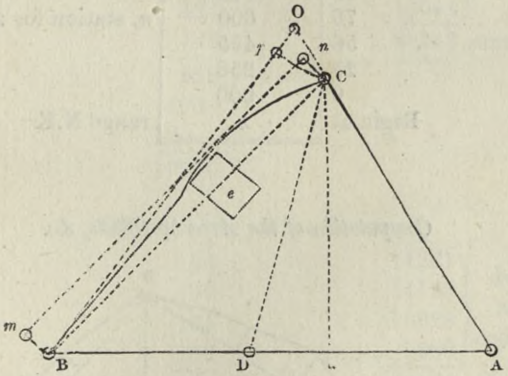
The same by Logarithms.

The log of	1883	. . .	$= 3{\cdot}2748503$
"	421	. . .	$= 2{\cdot}6242821$
"	608	. . .	$= 2{\cdot}7839036$
"	854	. . .	$= 2{\cdot}9314579$
Divide by the index of the root		. . . .	2)11.6144939
The quotient is the log of 641574, the area		. .	5·8072469

To find a Line between two Stations when the side of the field is crooked, and when the Sight on the Line sought is obstructed by a Farm Homestead or other Building.

Let ABC be the field, B and C the two stations, *e* the building on the crooked side, and BC the line sought.

From stations B and C range a parallel to the line sought, as in Problem VII., Part IV., placing a pole and making the angle *m* a right angle. Then take the inset *m*B, and measure from B to A, setting the proof-pole at D as in the preceding example; drive AC; next make the inset *n*C = *m*B, and the angle at *n* a right angle. If



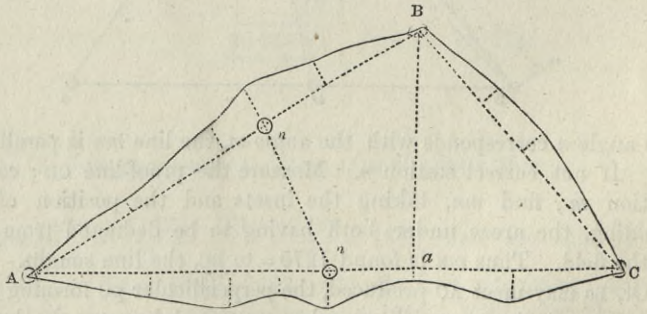
the angle *n* corresponds with the angle *m*, the line *mn* is parallel to BC. If not, correct station *n*. Measure the proof-line CD; correct station *m*; find *mn*, taking the insets and the position of the building, the areas under both having to be deducted from that of the field. Thus *mn* is found 1275 = to BC, the line sought.

Or, B*o* may meet AC produced, the perpendicular *p*C forming a tie-line so as to dispense with CD, when BC would become the hypotenuse to B*p* and *p*C, and thus be found, the base B*p* and perpendicular *p*C having been first measured with the chain. (Theorem VII., Part I.)

2. It is required to construct a figure, and find its area, from the following notes.

	CA	
0	1252	
37	1000	
69	824	
45	716	
72	610	*m*, station for a proof-line, which goes to *n*, and measures 352 links.
15	424	
55	212	
0	000	
	R. off C	
	BC	
0	683	
40	536	
64	354	
49	229	
0	000	
	R. off B	
	AB	
0	973	
48	745	
76	600	*n*, station for a proof-line
56	495	
25	256	
0	000	
Begin at	A	range N.E.

Computation of the Area by Offsets, &c.



Having constructed the figure, you will find the line *mn* to measure 352 links, as in the field-book. You also will find the perpendicular B*a* to be 528 links.

Triangle ABC

1252 base
528 per.
10016
2504
6260
661056

Offsets taken on the line AB.

256
25
1280
512
6400

25
56
81
239
729
243
162
19359

56
76
132
105
660
132
13860

76
48
124
145
620
496
124
17980

228
48
1824
912
10944

6400
19359
13860
17980
10944
(double areas collected)
68543 sum

Offsets taken on the line BC.

229
49
2061
916
11221

49
64
113
125
565
226
113
14125

64
40
104
182
208
832
104
18928

147
40
5880

11221
14125
18928
5880
(double areas collected)
50154 sum

Offsets taken on the line CA.

212
55
1060
1060
11660

55
15
70
212
14840

15
72
87
186
522
696
87
16182

72
45
117
106
702
117
12402

45
69
114
108
912
114
12312

69
37
106
176
636
742
106
18656

252
37
1764
756
9324

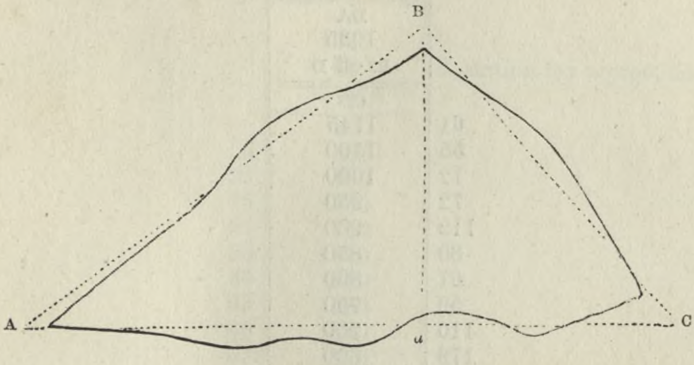
11660
14840
16182
12402
12312
18656
9324
double areas collected

95376 sum

661056
68543
50154
95376
whole double areas collected

2)875129
4·37564
4
1·50256
40
20·10240 *Area* 4*a*. 1*r*. 20*p*.

Computation of the Area by reducing the Crooked Sides to Straight Ones: generally called "Casting."



Having constructed the figure as before, and taken out the chain-lines, draw the three dotted lines AB, BC, and CA, in such a manner that the parts included may be equal to those excluded, as nearly as your eye can judge. Then the base AC being measured, will be found = 1390 links; and the perpendicular Ba = 630 links.

1390	base
630	per.
41700	
8340	
2)875700	
4·37850	
4	
1·51400	
40	
20·56000	*Area* 4*a.* 1*r.* 20*p.*

NOTE.—Although the method of finding the area by casting (which depends entirely upon the accuracy of the eye) is occasionally adopted, it is certainly less correct than that by offsets, &c. A learner, therefore, ought to practise both until he can habitually come very near to the truth by the former.

3. Lay down a field, and find its area, from the following notes.

	BD	
	760	proof-line
	return to B	
	DA	
	1035	
	R. off D	
	CD	
61	1145	
55	1100	
12	1000	
72	950	
119	900	
80	850	
61	800	
59	750	
110	700	
179	600	
210	550	
215	500	
212	450	
180	400	
159	350	
142	300	
165	250	
173	200	
161	150	
126	100	
65	50	
0	000	
	R. off C	
	AC	
	1590	
B	890	
Begin at	A, and	range W.

Answer.

Having constructed the figure, you will find the perpendicular Da, upon the base AC, to measure 740 links.

Double areas

1176600 triangle ACD

275770 offsets taken on the line CD

Area 7*a.* 1*r.* $1\frac{3}{4}p.$

4. Lay down a field, and find its area, from the following dimensions.

	DB 575 return to D	proof-line
	CA 1320 600 R. off C	D, station for a proof-line
	BC	
0	880	
31	800	
73	750	
95	700	
58	600	
60	550	
95	500	
60	380	
63	250	
60	200	
45	100	
55	000	
	R. off B	
To	the	fence
30	930	
17	875	to B
48	800	
65	700	
74	600	
65	500	
58	400	
55	300	
30	200	
17	100	
0	000	
Begin	at A and	range N.E.

Answer.

Having constructed the figure, you will find the perpendicular B*a*, upon the base AC, to measure 573 links.

Double areas
756360 triangle ABC
85060 offsets taken on the line AB
106270 ditto on the line BC
Area 4*a*. 2*r*. 38*p*.

5. Lay down a field, and find its area, from the following notes.

	DB	
	596	proof-line
	return to D	
	CA	
0	1080	
6	1000	
50	900	
110	800	
130	700	D, station for a proof-line
145	620	
106	550	
65	500	
30	450	
16	410	
36	350	
54	300	
70	250	
74	200	
86	150	
70	100	
46	50	
0	000	
	R. off C	
	BC	
0	848	
30	800	
60	750	
80	700	
70	650	
48	600	
20	520	
46	450	
90	380	
00	330	
10	270	
70	200	
40	150	
50	100	
45	000	
	R. off B.	

	To	the fence	
	50	800	
	45	755	to B
	30	700	
	40	650	
	75	600	
	130	550	
	170	500	
	156	450	
Brook 50 links in breadth	135	400	
	50	350	
	24	300	
	66	250	
	80	200	
	40	150	
	20	100	
	23	50	
	0	000	
	From	A	range N.E.

Answer.

Having laid down the figure, you will find the perpendicular B*a*, upon the base AC, to measure 587 links.

Double areas
633960 triangle ABC
110800 offsets taken on the line AB
100120 ditto on the line BC
149310 ditto on the line CA
Area 4*a*. 3*r*. $35\frac{1}{4}$*p*.

PROBLEM II.

Fields in the form of a Trapezium.

When you have a trapezium to survey, measure each side and both the diagonals, one of which will enable you to construct the figure, and the other will serve as a proof-line ; or, you may measure the longer diagonal, and a proof-line in any other direction most convenient.

NOTE 1.—From various obstructions it is sometimes impossible to take either of the diagonals; in such cases measure tie lines across the angles of the field at any convenient distance (not less than two chains) from the corners. These you will find sufficient for constructing the figure, and for proofs. Or, you may take an external angle, or angles, as directed in Problem IV.

2. When the lines, including the angle you intend to take with the chain, are of a considerable length, it will be necessary to measure more than two chains from the angular point before you take the chord-line, because a small inaccuracy in constructing the figure, when the angular distance is short, will throw the lines, when far produced, considerably out of their true position. It sometimes happens, however, in consequence of obstructions, that it is impossible to measure the chord-line at a greater distance from the angular point than one or two chains. In such cases multiply both the chord-line and angular distance by 2, 3, 4, or any larger number, as circumstances may require, and use the products resulting in laying down the figure.

3. When the measurement of the surface is required, for reaping, &c., you must let the chain touch the sides of the lands in all places where you measure across them. If you do not measure across the lands, but along the headland, then you must add as many links to the length of the chain-line as will make it equal to one measured across the lands, parallel to and near the headland.

You may easily ascertain what number of links you ought to add by stretching the chain across the lands, and putting down an arrow at each end, after which leave hold of one of the ends, and you will observe it recede from the arrow. The number of links by which it falls short of its former position add to each chain. Some lands you will find so low that nothing need be added to the chain-line, and some will require a link to four, three, two, or even (where the lands are very high) a link or more to one chain.

When the lands are high, if the lines measured along the headlands be not lengthened, the perpendiculars will obviously measure less than they ought to do ; consequently, the horizontal measure will be returned instead of the measure of the surface.

When the diagonal is measured with a slack chain it will give the measure of the surface; but in this case the perpendiculars will evidently be shorter than they would have been, if the diagonal had been measured with a tense chain; consequently, the measurement will be the same, or very nearly the same, whether the diagonal be measured, with a tense or slack chain, unless the headland lines be lengthened.

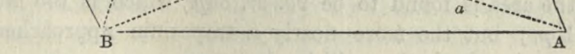
4. If two or three persons measure the same piece of land separately, or even if one person measure the same piece twice over, there will generally be a difference between the measurements; this difference, however, in small pieces, should scarcely ever exceed four or five perches.

5. When land, crops of corn, &c., are bought and sold, the buyer and seller commonly choose each a surveyor, and in their measurements it occasionally happens that there exists a considerable difference. In this case the best method, perhaps, of adjusting a dispute is, that the two surveyors meet and jointly remeasure the land. If this fail, they or their employers, the buyer and seller jointly choose an experienced surveyor, as an umpire, and abide by his decision.

Examples.

1. It is required to construct a figure, and find its area from the following notes.

	BD	
	1400	diag.
	1000	
	return to B	
	AC	
	1916	diag.
	1000	
	R. off A	
	DA	
	558	
	R. off D	
	CD	
	1626	
	1000	
	R. off C	
	BC	
	689	
	R. off B	
	AB	
	1492	
	1000	
Begin at	A	range W.



Having constructed the figure, lay your scale from B to D; and if you find it exactly 1400 links, as in the field-book, you may then measure the perpendicular $Ba = 468$ links, and the perpendicular $Da = 432$ links; from which you will readily compute the area required.

```
          468 ⎫
          432 ⎭ per.
          ───
          900 sum
        1916  diag.
       ───────
      2)1724400
       ───────
       8·62200
             4
       ───────
       2·48800
            40
       ───────
      19·52000 Area 8a. 2r. 19p.
```

BY THE FALSE METHOD.

Referred to in Note 4, Part III., Prob. IV.

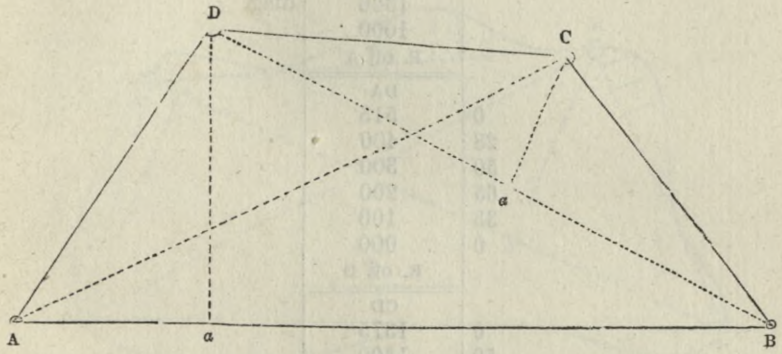
```
 1492 = AB                          1559
 1626 = CD                           623
2)3118                              4677
 1559 mean length                  3118
                                  9354
  689 = BC                       9·71257
  558 = DA                             4
2)1247                           2·85028
  623 mean breadth                    40
                                34·01120
```

Here the area is found to be *9a. 2r. 34p.*, which is too much by 1*a.* 0*r.* 15*p.*; but the more nearly a trapezium approaches to a square or rectangle the less will be the error.

2. Required the area of a field from the following notes.

DB 1365 1000 return to D	diag.
AC 1288 1000 L. of A	diag.

	DA 750 L. off D	
	CD 765 L. off C	
	BC 720 L. off B	
	AB 1600 1000	
Begin at	A	range E.



Having constructed the figure, you will find that, in consequence of the angle ADB being obtuse, a perpendicular from the angle A to the diagonal DB cannot be taken; you must, therefore, let fall the perpendicular Da from the angle D to the side AB, which you will find = 638 links. The perpendicular Ca will be found = 294 links.

Triangle ABD		Triangle BCD	
1600	base	1365	base
638	per.	294	per.
12800		5460	
48		12285	
96		2730	
1020800		401310	

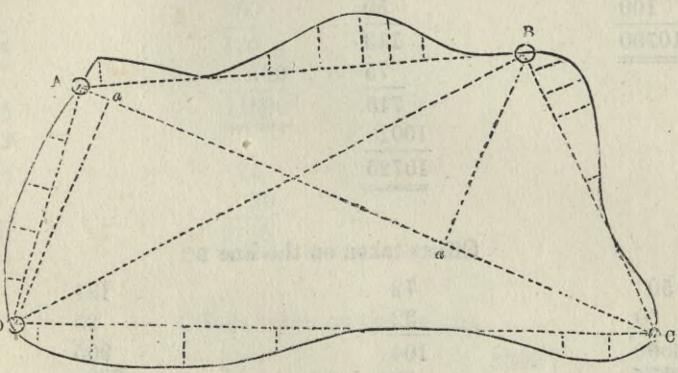
$$\begin{array}{r l}
1020800 & \Big\} \text{ double areas} \\
401310 & \quad \text{collected} \\
\hline
2)\overline{1422110} & \\
\hline
7{\cdot}11055 & \\
4 & \\
\hline
{\cdot}44220 & \\
40 & \\
\hline
17{\cdot}68800 & \textit{Area } 7a.\ 0r.\ 18p.
\end{array}$$

3. It is required to find the area of a field from the following notes.

	BD	
	1236	diag.
	1000	
	return to B	
	AC	
	1326	diag.
	1000	
	R. off A	
	DA	
0	515	
28	400	
50	300	
65	200	
33	100	
0	000	
	R. off D	
	CD	
0	1375	
50	1300	
75	1200	
84	1000	
52	800	
0	652	
0	356	
44	200	
50	100	
0	000	
	R. off C	
	BC	
0	664	
25	570	
0	483	
0	378	
32	300	
72	150	
85	100	
60	50	
0	000	
	R. off B	

	AB	
	946	
0	784	
50	725	
93	650	
106	600	
75	500	
32	400	
0	335	
0	242	
50	40	
0	000	
Begin at	A	range E.

BY OFFSETS, ETC.



Having constructed the figure, you will find the perpendicular Da = 512, and the perpendicular Ba = 446 links

Trapezium ABCD

$$\left.\begin{array}{r}512\\446\end{array}\right\}\text{per.}$$

958 sum
1326 diag.
5748
1916
2874
958
1270308

Offsets taken on the line AB

242
50
12100

65
32
130
195
2080

32
75
107
100
10700

75
106
181
100
18100

106
93
199
50
9950

93
50
143
75
715
1001
10725

59
50
2950

12100
2080
10700
18100
9950
10725
2950
double areas collected
66605 sum

Offsets taken on the line BC

50
60
3000

60
85
145
50
7250

85
72
157
50
7850

72
32
104
150
5200
104
15600

78
32
156
234
2496

181
25
905
362
4525

3000
7250
7850
15600
2496
4525
double areas collected
40721 sum

Offsets taken on the line CD

100
50
5000

50
44
94
100
9400

156
44
624
624
6864

148
52
296
740
7696

52
84
136
200
27200

84
75
159
200
31800

75
50
125
100
12500

75
50
3750

5000
9400
6864
7696
27200
31800
12500
3750
104210 sum

double areas collected

Offsets taken on the line DA

33
100
3300

33
65
98
100
9800

65
50
115
100
11500

50
28
78
100
7800

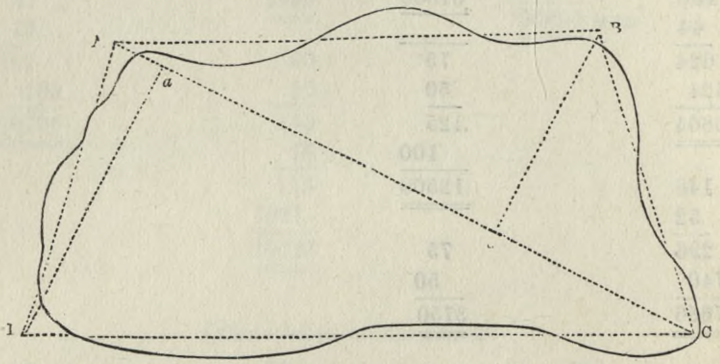
115
28
920
230
3220

3300
9800
11500
7800
3220
35620 sum

double areas collected

```
 1270308 ⎫
   66605 ⎪   whole
   40721 ⎬ double areas
  104210 ⎪   collected
   35620 ⎭
2)1517464
 7·58732
       4
 2·34928
      40
13·97120  Area 7a. 2r. 14p.
```

BY CASTING.

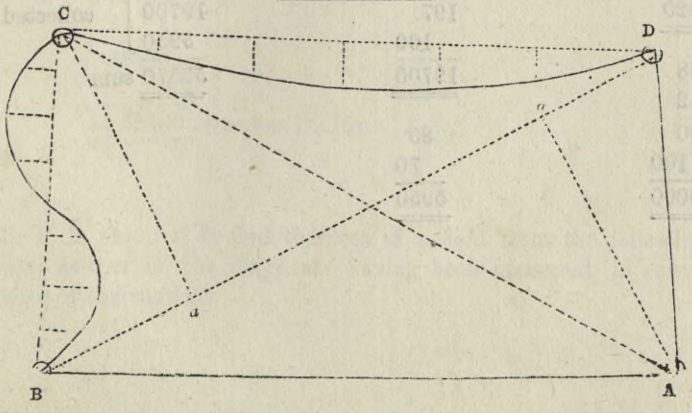


Having constructed the figure, draw the four dotted lines AB, BC, CD, and DA, in such a manner that the parts included may be equal to those excluded; then the diagonal AC will be found = 1364, and the perpendiculars Da = 636, and Ba = 476 links.

```
     636 ⎫
     476 ⎭ per.
    1112 sum
    1364 diag.
    4448
   6672
  3336
 1112
2)1516768
 7·58384
       4
 2·33536
      40
13·41440  Area 7a. 2r. 13p.
```

4. It is required to find the area of a field from the following notes.

	BD	
	1460	diag.
	1000	
	return to B	
	AC	
	1480	diag.
	1000	
	R. off A	
	DA	
	672	
	R. off D	
	CD	
	1244	0
	1000	47
	800	70
	600	85
	400	68
	200	30
	000	0
	R. off C	
	BC	
0	720	
85	650	
112	550	
88	450	
	360	cross the fence
	300	83
	200	130
	100	100
	000	0
	R. off B	
	AB	
	1350	
	1000	
Begin at	A	range W.



Having constructed the figure, you will find the perpendicular $Ca = 613$, and the perpendicular $Aa = 618$ links.

Trapezium ABCD

613 } per.
618 }
1231 sum
1460 diag.
73860
4924
1231
1797260

Insets taken on the line BC

100
100
10000

100
130
230
100
23000

130
83
213
100
21300

83
60
4980

10000 } double areas collected
23000 }
21300 }
4980 }
59280 sum

Offsets taken on the line BC

88
90
7920

88
112
200
100
20000

112
85
197
100
19700

85
70
5950

7920 } double areas collected
20000 }
19700 }
5950 }
53570 sum

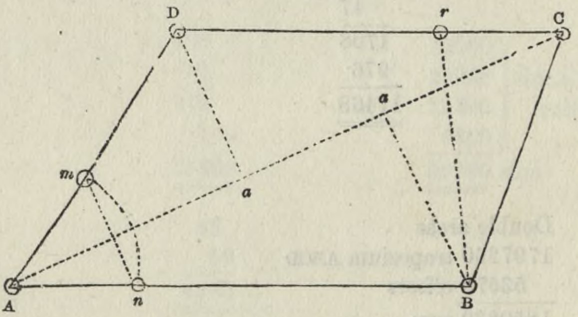
Insets taken on the line CD

```
200            85            6000 \
 30            70           19600 |  double
----          ----          30600 |  areas
6000          155           31000 |  collected
====           200          23400 |
              -----         11468 /
 30           31000         ------
 68           =====         122068 sum
----                        ======
 98            70
 200           47
-----         ----
19600         117
=====          200
              -----
 68           23400
 85           =====
----
153            244
 200            47
-----         -----
30600         1708
=====          976
              -----
              11468
              =====
```

```
Double areas
 1797260 trapezium ABCD
   53570 offsets
 -------
 1850830 sum
   59280 \
  122068 / insets
 -------
  181348 sum to be deducted for the above offsets
 -------
2)1669482 whole field
 -------
 8·34741
       4
 -------
 1·38964
      40
 -------
15·58560 Area 8a. 1r. 16p.
 =======
```

5. It is required to find the area of a field from the following notes; neither of the diagonals having been measured in consequence of obstructions.

	DA 476 L. off D	
	CD 618 200 L. off C	 *r*, proof-line, 17 to B
	BC 443 L. off B	
	AB 723	
	192 200 200	from *m* to *n* from A to *m*, on the line AD from A to *n*, on the line AB
Begin at	the angle A	range E.



Having constructed the figure, you will find the diagonal AC = 963, and the perpendiculars D*a* = 257, and B*a* = 316 links.

257 }
316 } per.
573 sum
963 diag.
1719
3438
5157
2)551799
2·75889
4
3·03596
40
1·43840 *Area* 2*a*. 3*r*. 1*p*.

6. Required the plan and area of a field from the following notes.

	BD	
	1437	diag.
	1000	
	return to B	
	AC	
	939	diag.
	L. off A	
	DA	
	567	
	L. off D	
	CD	
	712	
	L. off C	
	BC	
	765	
	L. off B	
	AB	
	1457	
	1000	
Begin at	A	range E.

Answer.

Having constructed the figure, you will find one of the perpendiculars = 560, and the other = 166 links; hence the area is = 5*a*. 0*r*. 34*p*.

7. It is required to lay down a field, and find its area, from the following notes.

	BD	
	662	diag.
	return to B	
	AC	
	533	diag.
	L. off A	
	DA	
	168	
	L. off D	
	CD	
	439	
	L. off C	
	BC	
	356	
	L. off B	
	AB	
	667	0
	400	62
	200	58
	000	0
Begin at	A	range E.

Answer.

Having constructed the figure, you find one of the perpendiculars $= 168$, and the other $= 218$ links; hence the area is $= 1a.\ 2r.\ 6p.$

8. It is required to lay down a field, and find its area, from the following notes; neither of the diagonals having been measured in consequence of obstructions.

	CE 1200 1000 L. off C	
	AC 1094 L. off A	
	EA 1800 1000 L. off E	
	DE 837 L. off D	
	BD 1528 1000 860 L. off B	C
Begin at	AB 621 A	range N.

Answer.

Having constructed the figure, you will find the diagonal BE $= 1927$, and the perpendiculars $= 580$ and 637 links respectively; hence the area is $= 11a.\ 2r.\ 36p.$

9. The plan and area of a field are required from the following dimensions.

DB 1730 return to D	diag.
AC 1660 R. off A	diag.

To	the	fence
99	1580	
110	1500	to A
100	1450	
116	1400	
132	1300	
115	1200	
65	1100	
33	1000	
25	950	
40	900	
150	850	
210	800	
250	700	
255	630	
240	550	
218	500	
117	400	
41	300	
18	250	
15	200	
100	150	
140	100	
157	50	
165	000	
	R. off D	
To	the	fence
60	1085	
82	920	to D
80	850	
42	750	
40	700	
121	600	
140	550	
136	500	
70	300	
25	450	
17	300	
14	250	
30	200	
70	150	
92	100	
100	000	
	R. off C	

To	the	fence
52	1440	
70	1340	to C
60	1250	
37	1200	
33	1150	
45	1100	
83	1000	
70	900	
25	800	
12	750	
20	700	
40	650	
48	600	
54	500	
59	450	
60	400	
72	350	
84	300	
70	200	
86	150	
80	100	
75	000	
	R. off B	
To	the	fence
67	1005	
78	930	to B
86	850	
90	750	
75	700	
40	650	
27	600	
36	550	
57	500	
85	450	
78	400	
58	300	
62	200	
79	100	
83	50	
80	000	
Begin	at A and	go N.

Answer.

Having constructed the figure, you will find the perpendiculars Aa = 810, and Ca = 708 links.

Double areas

2626140	trapezium ABCD
137945	offsets taken on the line AB
167800	ditto on the line BC
157520	ditto on the line CD
395420	ditto on the line DA
3484825	sum

Area 17*a.* 1*r.* 27¾*p.*

PROBLEM III.

Fields of more than Four Sides.

When a field consists of more than four sides, divide it into triangles and trapeziums, agreeably to the directions given in Part III., Prob. V. Then take the dimensions of each, as directed in the last two problems.

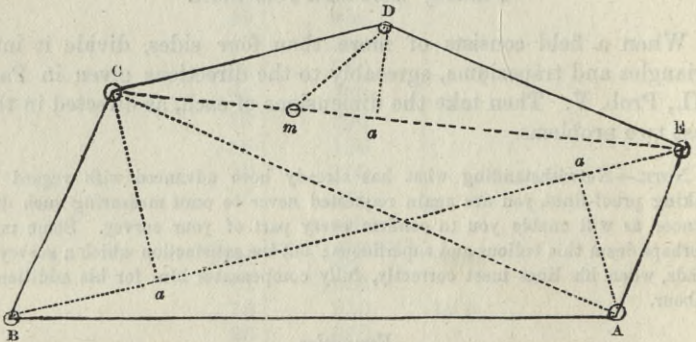
NOTE.—Notwithstanding what has already been advanced with regard to taking proof-lines, you are again requested never to omit measuring such distances as will enable you to confirm every part of your survey. Some may perhaps deem this tedious and superfluous; but the satisfaction which a surveyor finds, when his lines meet correctly, fully compensates him for his additional labour.

Examples.

1. It is required to find the area of a field from the following notes.

EB	
1510	diag.
1000	
R. off C	
CE	
1238	diag.
1000	
400	*m*, proof-line, goes to D,
R. off C	and measures 285
AC	
1200	diag.
1000	
R. off A	
EA	
383	
R. off E	

	DE 692 R. off D	
	CD 620 R. off C	
	BC 535 R. off B	
Begin at	AB 1334 1000 A	range W.



Having constructed the figure, you will find the perpendiculars $Ca = 410$, $Aa = 330$, and $Da = 215$ links.

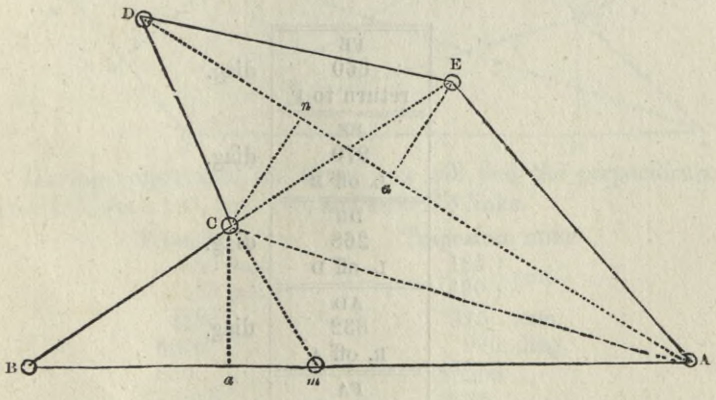
Trapezium ABCE	Triangle CDE
410 } per. 330 } 740 sum 1510 diag. 7400 370 74 1117400	1238 base 215 per. 6190 1238 2476 266170

1117400 } double areas
266170 } collected

2)1383570
6·91785
4
3·67140
40
26·85600 *Area* 6*a.* 3*r.* 27*p.*

2. It is required to find the area of a field from the following notes.

	DA 1042 return to D	diag.
	CE 420 R. off C	diag.
	AC 768 R. off A	diag.
	EA 585 R. off E	
	DE 518 R. off D	
	CD 365 L. off C	
	BC 390 R. off B	
Begin at	AB 1054 600 A range W.	*m*, proof-line, goes to C and measures 260



Having constructed the figure, you will find the perpendiculars $Ca = 223$, $Cn = 200$, and $Ea = 176$ links.

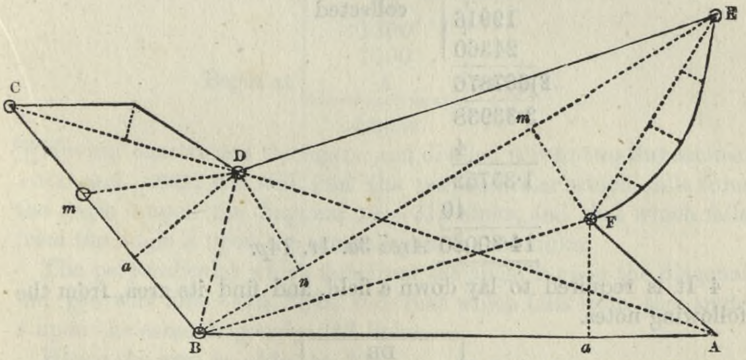
Triangle ABC		Trapezium ACDE	
1054	base	200	per
223	per	176	per
3162		376	sum
2108		1042	diag.
2108		752	
235042		1504	
		376	
		391792	

$$\begin{array}{r l}
235042 & \text{double areas} \\
391792 & \text{collected} \\
\hline
2)\,626834 & \\
\hline
3{\cdot}13417 & \\
4 & \\
\hline
{\cdot}53668 & \\
40 & \\
\hline
21{\cdot}46720 & \textit{Area } 3a.\ 0r.\ 21p.
\end{array}$$

3. It is required to find the area of a field from the following notes.

FB 660 return to F	diag.
BE 970 L. off B	diag.
DB 268 L. off D	diag.
AD 832 R. off A	diag.
FA 285 L. off F	

	EF	
0	384	
40	300	
53	200	
32	100	
0	000	
	R. off E	
	DE	
	804	
	L. off D	
	CD	
0	383	
52	200	
0	000	
	R. off C	
	BC	
	475	
	300	*m*, proof-line, goes to D,
	R. off B	and measures 255
	AB	
	850	
Begin at	A	range W.



Having constructed the figure, you will find the perpendiculars Fa = 185, Fm = 185, Dn = 190, and Da = 216 links.

Triangle ABF	Trapezium BDEF
850 base	185 } per.
185 per.	190 }
4250	375 sum
6800	970 diag.
850	26250
157250	3375
	363750

Triangle BCD	Offsets taken on the line CD
475 base	383
216 per.	52
2850	766
475	1915
950	19916
102600	

Offsets taken on the line EF

32	53	3200
100	40	8500
3200	93	9300
	100	3360
32	9300	24360 sum
53		
85	84	
100	40	
8500	3360	

$$\left.\begin{array}{r}3200\\8500\\9300\\3360\end{array}\right\}\text{double areas collected}$$

24360 sum

$$\left.\begin{array}{r}157250\\363750\\102600\\19916\\24360\end{array}\right\}\text{double areas collected}$$

2)667876
3·33938
4
1·35752
40
14·30080 *Area 3a. 1r. 14p.*

4 It is required to lay down a field, and find its area, from the following notes.

DB 1440 1000 return to D	diag.
AC 1078 L. off A	diag.
EA 1324 1000 return to E	diag.

	DF	
	712	diag.
	L. off D.	
	AD	
	818	diag.
	L. off A	
	FA	
	755	
	L. off F	
	EF	
	692	
	L. off E	
	DE	
	754	
	R. off D	
	CD	
	540	
	L. off C	
	BC	
	1048	
	L. off B	
	AB	
	1360	
	1000	
Begin at	A	range E.

Answer.

Having constructed the figure, and divided it into two trapeziums, ABCD and ADEF, you will find the perpendicular which falls from the angle C upon the diagonal DB = 315 links, and that which falls from the angle A upon the same diagonal = 758 links.

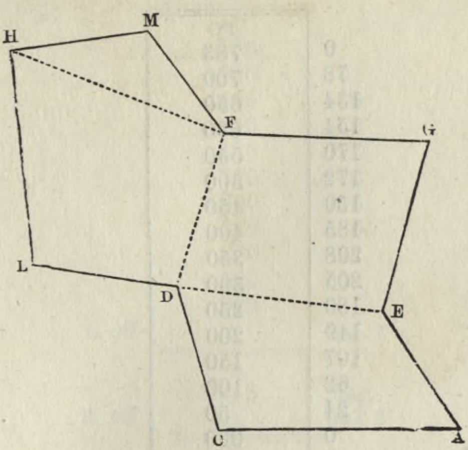
The perpendicular which falls from the angle D upon the diagonal EA, you will find = 425 links, and that which falls from the angle F upon the same diagonal = 287 links.

Hence the area is = 12*a.* 1*r.* 30*p.*

5. Required the plan and area of a field from the following notes.

	EB	
	424	proof-line
	R. off E	
	FE	
	750	diag
	L. off F	
	MF	
	400	
	R. off M	

	HM 460 R. off H	227 to K, proof-line
	LH 700 R. off L	
	DL 430 return to D	
	FL 740 R. off F	diag.
	HF 730 400 R. off H	diag. K
	DH 920 R. off D	diag.
	GD 950 return to G	diag.
	GE 580 R. off G	
	FG 630 R. off F	
	DF 540 R. off D	diag.
	AD 1050 R. off A	diag.
	EA 450 R. off E	
	DE 670 R. off D	diag.
	CD 500 R. off C	
	AC 780 500	 B
Begin at	A	range W.



Answer.

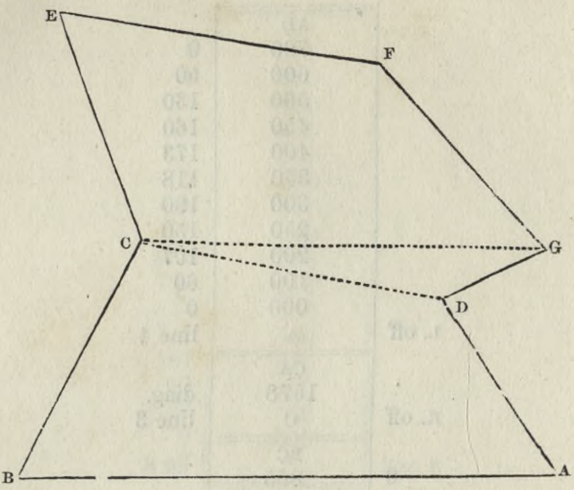
Having constructed the figure, you will find the perpendiculars of the trapezium ACDE = 354 and 195; of DFGE = 404 and 340; of DLHF = 426 and 316; and the perpendicular of the triangle FHM = 227 links. Hence the area of the field is = 10*a*. 2*r*. 25*p*.

6. Required the plan and area of a field from the following dimensions.

	BD	
	1480	diag.
Return	to B	line 14
	GE	
	1725	diag.
L. off	G	line 13
	CG	
	1295	diag.
L. off	C	line 12
	FC	
	935	diag.
L. off	F	line 11
	DF	
	793	diag.
R. off	D	line 10
	GD	
0	358	
37	300	
49	250	
60	200	
66	150	
62	100	
30	50	
0	000	
R. off	G	line 9

	FG	
0	783	
78	700	
134	650	
154	600	
170	550	
172	500	
150	450	
185	400	
208	350	
205	300	
180	250	
149	200	
107	150	
62	100	
24	50	
0	000	
R off	F	line 8
	EF	
0	1043	
36	1000	
67	900	
85	800	
100	700	
140	600	
152	550	
143	500	
135	450	
110	400	
65	350	
50	300	
40	200	
25	100	
0	000	
R. off	E	line 7
	CE	
0	743	
70	600	
135	500	
160	450	
185	400	
190	350	
179	300	
150	250	
95	200	
60	100	
0	000	
R. off	C	line 6
	DC	diag.
	1000	
L. off	D	line 5

	AD	
	700	0
	600	50
	500	130
	450	160
	400	173
	350	118
	300	190
	250	150
	200	107
	100	60
	000	0
L. off	A	line 4
	CA	
	1578	diag.
R. off	C	line 3
	BC	
0	865	
60	750	
104	650	
72	600	
88	500	
100	400	
86	350	
75	300	
95	200	
80	100	
70	000	
R. off	B	line 2
To	the	fence
80	1820	
73	1750	
60	1600	
56	1500	
70	1400	
95	1300	
120	1200	
105	1100	
110	1000	
140	900	
186	800	
184	700	
125	600	
114	500	
93	400	
86	300	
75	200	
70	100	
0	000	
Begin at	A	range W, line 1



NOTE.—The preceding figure represents the chain-line, forming the two trapeziums and the triangle, in the 6th Example.—When the learner constructs the figure, he must of course lay down the offsets from the notes, and dot all the chain-lines as before directed.

Answer.

Having constructed the figure, you will find the perpendiculars of the trapezium ABCD, falling upon the diagonal CA, to measure 862 and 314 links; the perpendicular of the triangle DGC, falling upon the diagonal CG, to measure 184 links; and the perpendiculars of the trapezium CEFG, falling upon the diagonal GE, to measure 513 and 300 links.

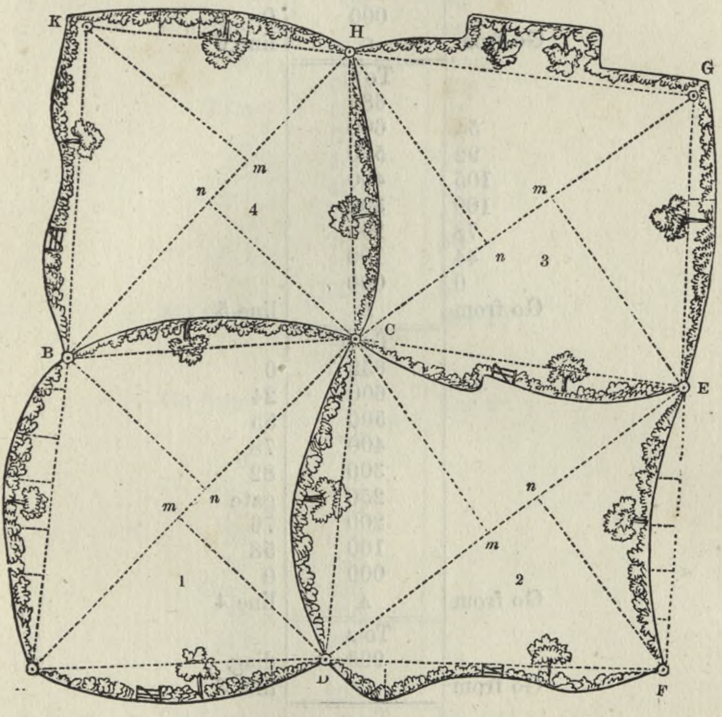
Double areas

1855728	trapezium ABCD
238280	triangle DGC
1402425	trapezium CEFG
362460	offsets taken on the line BC
133750	ditto on the line BC
143250	ditto on the line AD
149010	ditto on the line CE
157548	ditto on the line EF
200374	ditto on the line FG
30696	ditto on the line GD
4673521	sum

Area 23*a.* 1*r.* $18\frac{3}{4}p.$

7. Draw a plan of a small estate, consisting of four fields, and find the respective areas of the different inclosures, and the contents of the whole, from the following dimensions.

NOTE.—The field-notes in this Example are entered from the left towards the right; but in all the foregoing Examples, they are entered from the right towards the left. Both methods are consequently exhibited; and, of course, the learner is at liberty, when he commences field-practice, to follow that of which he most approves.



Scale 4 Chains to an inch.

	To B	
0	662	
53	600	
88	500	
96	400	
92	300	
70	200	
35	100	
0	000	
Begin at	A	and go N. line 1

	To E	
	712	0
	600	43
	500	52
	400	45
	340	gate
	274	28 + 52
	200	76
	100	42
	000	0
Go from	C	line 6
	To C	
	685	
54	600	
92	500	
105	400	
100	300	
78	200	
44	100	
0	000	
Go from	D	line 5
	To D	
	632	0
	600	24
	500	55
	400	78
	300	82
	250	gate
	200	76
	100	58
	000	0
Go from	A	line 4
	To A	
	995	diag.
Go from	C	line 3
	To C	
0	615	
46	500	
53	400	
62	300	
60	200	
50	100	
0	000	
Go from	B	line 2

	To H	
	750	0
	700	18
	600	38
	490	60 + 35
	350	104
	220	15 + 80
	100	25
	000	30
Go from	G	line 11
To	the	fence
	668	22
To G	638	24
	600	26
	500	42
	400	48
	300	45
	200	30
	100	18
	000	0
Go from	E	line 10
	To E	
	965	diag.
Go from	D	line 9
	To D	
0	735	
32	700	
75	600	
55	500	
20	430	
Gate	380	
28	300	
42	200	
40	100	
0	000	
Go from	F	line 8
	To F	
	593	0
	500	30
	400	52
	300	57
	200	48
	100	32
	000	0
Go from	E	line 7

	Finis	
	To H	
	895	diag.
Go from	B	line 16
	To B	
	720	0
	600	40
	500	48
	460	gate
	400	32
	350	18
	300	28
	200	68
	100	55
	000	34
Go from	K	line 15
To	the	fence
	580	28
To K	546	30
	500	32
	400	46
	300	45
	200	40
	100	34
	000	0
Go from	H	line 14
	To G	
	908	diag.
Go from	C	line 13
	To C	
0	624	
54	500	
65	400	
63	300	
52	200	
30	100	
0	000	
Go from	H	line 12

Answer.

Having drawn the plan, you will find the perpendiculars of the different trapeziums to measure as follow, viz :—

$Dm = 426$, and $Bn = 400$, in No. 1 ;
$Cm = 503$, „ $Fn = 448$, „ No. 2 ;
$Hn = 515$, „ $Em = 498$, „ No. 3 ; and
$Cn = 428$, „ $Km = 439$, „ No. 4.

AREA OF No. 1.

Double areas
821870 trapezium ABCD
84786 offsets on AB
54890 ditto on BC
72968 ditto on AD

1034514 sum
93790 insets on CD

2)940724 difference

4·70362 *Area* in square links
4

2·81448
40

32·57920 *Area* 4*a*. 2*r*. $32\frac{1}{2}$*p*.

AREA OF No. 2.

Double areas
917715 trapezium DCEF
93790 offsets on CD
55510 ditto on DF

1067015 sum
60758 insets on CE
43590 ditto on EF

104348 sum

962667 difference

Area 4*a*. 3*r*. 10*p*.

AREA OF No. 3.

Double areas
919804 trapezium CEGH
60758 offsets on CE
42480 ditto on EG
81310 ditto on GH

1104352 sum
54096 insets on CH

1050256 difference

Area 5*a*. 1*r*. 0*p*.

Area of No. 4.

Double areas

775965	trapezium BCHK
54096	offsets on CH
41024	ditto on HK
57200	ditto on KB
928285	sum
54890	insets on BC
873395	difference

Area 4*a.* 1*r.* 18½*p.*

Content.

No.	A.	R.	P.
1. . . .	4	2	32½
2. . . .	4	3	10
3. . . .	5	1	0
4. . . .	4	1	18½
Sum	19	0	21

Note 1. In the last example every field is measured separately; but they are so connected by the chain-lines that no difficulty can arise to the learner in planning them. It may also be observed that no proof-lines were measured; but they should never be omitted in practice: if they be, the surveyor cannot depend upon the accuracy of his work.

2. If the foregoing estate be laid down upon a sheet of drawing paper, by a scale of one chain, or of two chains to an inch, a finished plan may then be made and ornamented with Indian ink, in a similar manner to Plates IX. and XI. Or the quickwood hedges may be made by a pen and Indian ink; or they may be represented by running narrow shades of colouring along the lines which form the boundaries of the fields; and each field may then be washed over with a different colour, mixed up thinly with water, and laid on with a small brush or camel's hair pencil. (See Part V. for the method of transferring a rough plan to a clean sheet of paper, in order to make a finished plan, with proper embellishments.)

3. In drawing the finished plan, all the out-boundaries may be considered as belonging to the fields which they respectively adjoin; that fence from B to C may be made as belonging to No. 1, that from C to D as belonging to No. 2, that from C to E as belonging to No. 3, and that from C to H as belonging to No. 4. (See a remark on the 27th page relating to fences.)

4. The title of the finished plan of the foregoing estate may run thus:—Plan of an Estate lying in the Parish of Bradford, in the West-Riding of the County of York.

PROBLEM IV.

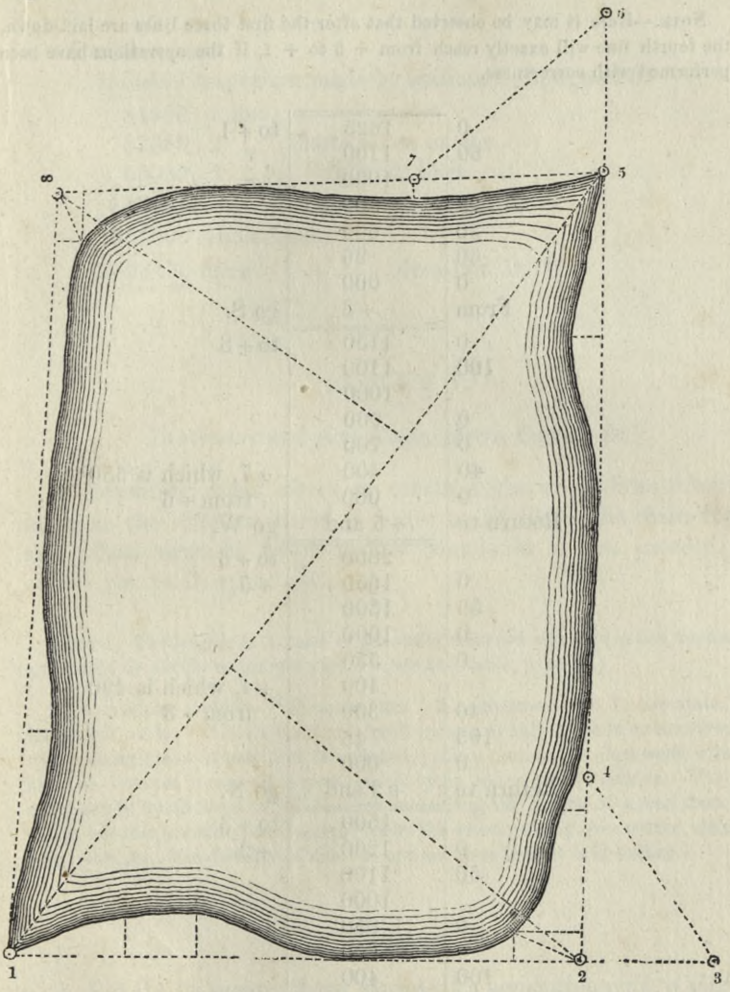
Meres and Woods.

The method of measuring Meres and Woods by the Chain and Cross has already been shown in Part III. It is here proposed to survey them by the Chain only.

In this case, you must not only measure on the outside of the mere, or wood, and take insets as before directed; but also take such external angles, or tie-lines, as will enable you to lay down the figure.

Example.

Let the following figure represent a mere, the area of which is required :—



Begin at + 1, and measure eastward as far as + 2, taking insets as you proceed; then produce the line to + 3. Return to + 2, and measure northward as far as + 4; thence run a line backward to

+ 3, which will tie the first and second lines. Return to + 4, continue the line to + 5, and produce it to + 6. Return to + 5, and proceed westward to + 7, the distance between which and × 6, being measured, will tie the second and third lines. Return to + 7, and continue the line to + 8. From + 8 proceed to + 1, and you will have obtained the following dimensions.

NOTE.—Here it may be observed that after the first three lines are laid down, the fourth line will exactly reach from + 8 to + 1, if the operations have been performed with correctness.

0	1625	to + 1
60	1100	
	1000	
23	800	
30	600	
60	96	
0	000	
From	+ 8	go S.
0	1150	to + 8
100	1100	
	1000	
0	900	
0	700	
40	400	+ 7, which is 550
0	000	from + 6
Return to	+ 5 and	go W.
	2000	to + 6
0	1650	+ 5
56	1300	
0	1000	
0	550	
	400	+ 4, which is 490
40	300	from + 3
103	48	
0	000	
Return to	+ 2 and	go N.
	1500	to + 3
0	1200	+ 2
50	1100	
	1000	
0	850	
0	600	
100	400	
80	250	
0	000	
Begin at	+ 1	range E.

Answer.

Having constructed the figure, you will find the diagonal, drawn from + 1 to + 5 = 2085, the perpendicular from + 2 upon the diagonal = 950, and that from + 8 = 890 links.

Double areas

3836400 trapezium made by stations 1, 2, 5, and 8.

84500	1 line	insets taken on the different lines
87380	2 „	
53000	3 „	
118120	4 „	

343000 whole insets

3493400 mere — *Area* 17*a.* 1*r.* 35*p.*

PROBLEM V.

To measure and plan Roads, Rivers, Canals, &c.

In measuring roads, rivers, or canals, angles or tie-lines must be taken at the different turns, in order to lay down the chain-lines; and offsets must be taken to the boundaries as you proceed, to enable you to draw the plan.

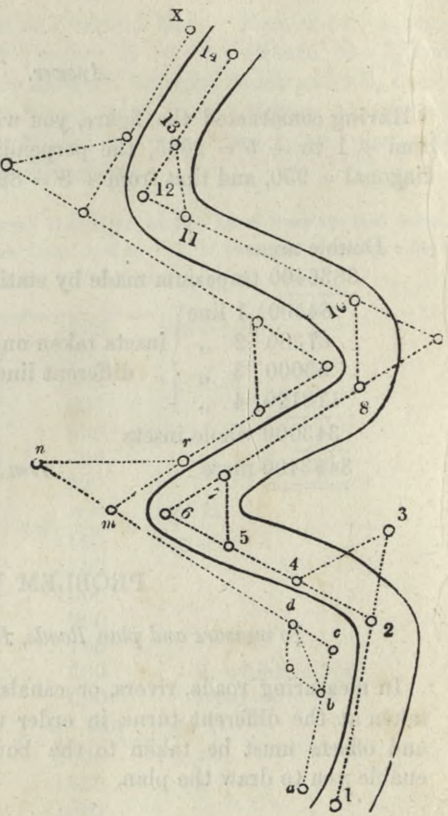
NOTE 1. The length of a road is generally returned either in miles, furlongs, and poles, or else in miles and yards. (See the Table, page 30.)

2. A machine called a "Perambulator" is sometimes used to ascertain the lengths of roads. It has a wheel of 8 feet 3 inches, or half a pole in circumference, which being made to pass over the ground, puts in motion the clockwork within; and the distance measured is pointed out by an index on the outside. This instrument is much more expeditious for measuring the length of a road than the chain, but it is certainly less correct; for by the wheel passing over stones, sinking into holes, &c., the distance is made to appear more than it is in reality.

Examples.

1. Let the following figure represent a serpentine road, a plan of which is required.

Begin at + 1, and measure to + 3, taking offsets on both sides, as you proceed. Return to + 2, and measure to + 4, from which run a line to + 3, which will tie the first and second lines. Return to + 4, and continue the line to + 6. From + 6, proceed as before, until you arrive at + 14; and you will have obtained the following dimensions, from which a plan may be drawn.



	To + 14	
58	350	60
68	200	44
	150	+ 13 is 184 from + 11
50	100	80
Go from	+ 12	line 5
	To + 12	
30	720	
70	650	
	600	+ 11
86	550	33
70	300	50
	200	+ 10 is 200 from + 8
120	135	cross-fence
Go from	+ 9	line 4

	To + 9	
	700	
	600	cross-fence
	500	+ 8
38	480	84
40	300	60
52	180	65
	150	+ 7 is 160 from + 5
50	100	
Go from	+ 6	line 3
	To + 6	
20	512	
50	450	
52	380	70
	350	+ 5
20	300	80
	200	+ 4 is 232 from + 3
18	100	93
Go from	+ 2	line 2
	To + 3	
	600	
	480	cross-fence
	400	+ 2
38	350	95
15	300	
28	200	80
55	000	70
Begin	at + 1	line 1

2. Let the foregoing figure represent a river, a plan of which is required.

Begin at *a*, and measure to *c*, taking offsets to the river's edge as you proceed. From *c* measure to *d*, and there take the tie or chord-line *db*, which will enable you to lay down the first and second lines. Continue the second line to *n*, and from *m* measure to *r*, at which place take the tie-line *rn;* and thus proceed until you come to the end of your survey at *x*.

The breadth of a river, if it be everywhere nearly the same, may be taken in different places by the next problem or by Problem X. Part III.; but if it be very irregular, dimensions must be taken on both sides, as above.

When the area is required, it must be found from the plan, by dividing the river into several parts, and taking the necessary dimensions by the scale.

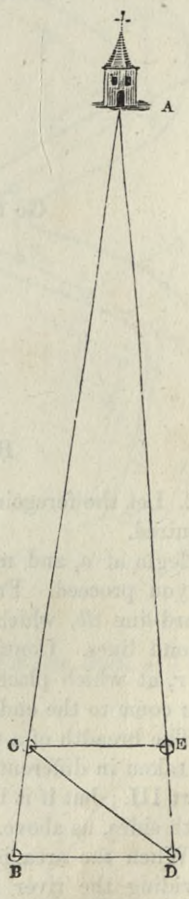
Note.—Any bog, marsh, mere, or wood, whatever may be its number of sides, may be measured by this problem.

PROBLEM VI.

Taking Distances by the Chain and Scale.

EXAMPLE.

Required the distance of an object at A, from B.



First, make a station at B; then, in a direct line with BA, set up a pole, suppose at C; measure the distance BC. Return to B, and measure in any direction, making an angle with BC, suppose to D; then set up a pole in a direct line with DA, as at E. Measure the lines DE and EC, and also the diagonal CD; these will enable you to construct the trapezium BCED.

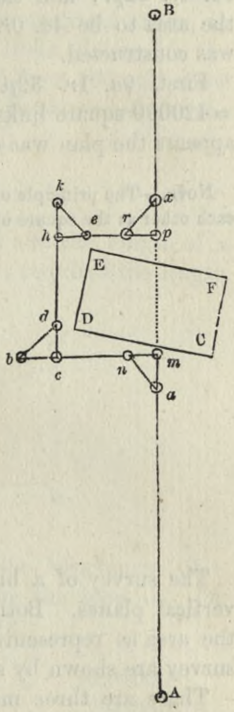
The lines BC and DE, produced, will evidently meet at A.

Measure the line BA with the same scale by which you have constructed the trapezium, and it will be the distance required.

PROBLEM VII.

To erect a Perpendicular by the Chain, or to measure Lines upon which there are Impediments.

EXAMPLE.



Suppose CDEF to represent the base of a building, through which it is necessary a line should pass from A, to an object at B, the direction A*m*B on the side A being given.

Measure from A to *m;* and from *m,* measure back to *a*, 40 links. Let one end of the chain be kept fast at *a*, and the eightieth link at *m;* take hold of the fiftieth link, and stretch the chain so that the two parts *an* and *mn* may be equally tight: then will *mn* be perpendicular to *am*.

For *mn* will be 30, *am* 40, and *an* 50 links; or the sides of the right-angled triangle *amn* will be in proportion to each other as 3, 4, and 5. (See Prob. XVIII. Part I.)

Measure from *m*, upon the line *mn* continued, until you are clear of the impediment, as at *c;* then continue the line 40 links farther, to *b*. Find by the above process the perpendicular *cd;* and proceed in that direction till you are beyond the building, as at *h*. Again erect the perpendicular *he*, upon which measure till you have made *hp* equal to *mc;* and you will then be in a direct line with *m*A. Erect the perpendicular *px*, which (if you have conducted the work with correctness) will be in a right line with B. Measure the distance *p*B; then A*m*, added to *ch* (= *mp*), and *p*B, will give the whole length of the line AB.

PROBLEM VIII.

Having the Plan of a Field, and its true Area, to find the Scale by which it has been constructed.

RULE.—By any scale whatever measure such lines as will give you the area of the figure: then say, as this area is to the square of the scale by which it was found, so is the true area to the square of the scale required.

Example.

Suppose the true area of a field, the plan of which is given, to be 9*a.* 1*r.* 32*p.*; and that by a scale of 2 chains to an inch, I find the area to be 4*a.* 0*r.* 32*p.*; required the scale by which the plan was constructed.

First, 9*a.* 1*r.* 32*p.* = 945000 square links; and 4*a.* 0*r.* 32*p.* = 420000 square links; then, as 420000 : 4 : : 945000 : 9. Hence it appears the plan was constructed by a scale of 3 chains to an inch.

NOTE.—The principle of this process is, that the areas of similar figures are to each other as the square of their homologous sides. (Theo. XVI. Part I.)

HILLY GROUND.

The survey of a hill is taken conjointly upon the horizontal and vertical planes. Both planes are generally required to determine the area as represented upon the plan. The details of the vertical survey are shown by sections.

There are three methods practised in finding the area of a hill, viz., the horizontal, the hypotenusal, and the superficial.

The *first way* is that which determines the horizontal area. It accords with trigonometry on the horizontal plane, and is that which is generally practised in Land-surveying.

The *second way* gives the hypotenusal area, and also the angle which the hypotenusal plane makes with the horizon. Surveys of this kind are frequently ordered for plantation and other grounds, and for whole estates when they lie on the sloping sides of large hills.

The *third way* returns the superficial area. It is generally adopted for paring and burning, trenching, ploughing, reaping, and other agricultural works.

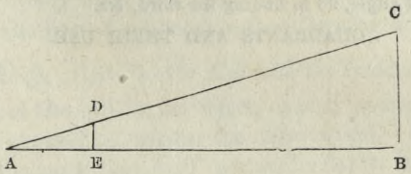
Methods for reducing Hypotenusal to Horizontal Lines.

METHOD I.

When the hill is of a regular slope, take its altitude with a theodolite, or with a quadrant; then, by a trigonometrical canon, in which the hypotenuse may be counted 100 links, determine the number of links in the base. These deducted from 100 will show the number of links by which each chain must be shortened, for the purpose of planning.

Example.

Suppose the altitude of a hill to be 16° 15′, and the length of a line measured upon its surface to be 2550 links; required the length of the line that must be used in planning.



In the right-angled triangle ABC are given the hypotenuse AC = 2550, and the angle BAC = 16° 15′, to determine the base AB. Or AD = 100, and the angle EAD = 16° 15′, to find AE.

As radius	10·00000
Is to the hypot. AD = 100 links . . .	2·00000
So is the co-sine of the angle EAD = 16° 15′ .	9·98229
To AE = 96 links	1·98229

Hence it appears that 4 links must be subtracted from each chain; consequently (25 × 4 + 2 =) 102 links must be taken from AC; hence AB = 2448 links, the line required.

PROOF.—As, 1 : 2550 :: ·96005 (the nat. co-sine of 16° 15′) : 2448·1275 links = AB.

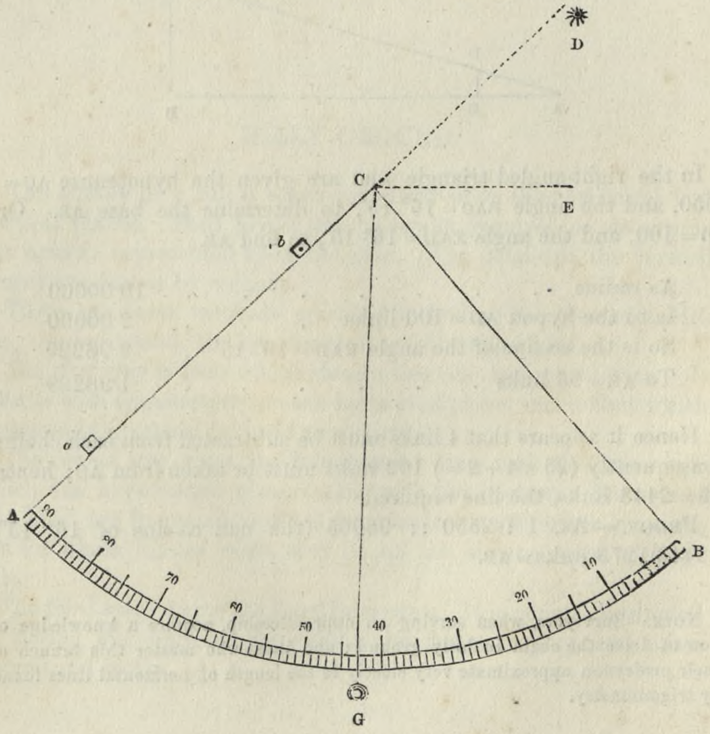
NOTE.—Surveyors, when serving an apprenticeship, acquire a knowledge of how to drive the chain in hilly ground; and those who master this branch of their profession approximate very closely to the length of horizontal lines found by trigonometry.

A Table for reducing Hypotenusal to Horizontal Lines.

Different Altitudes of Hills.		Links to be subtracted from each Chain measured upon the Surface.	Different Altitudes of Hills.		Links to be subtracted from each Chain measured upon the Surface.
Deg.	Min.	Links.	Deg.	Min.	Links.
5	44	½	29	32	13
8	6	1	30	42	14
11	28	2	31	47	15
14	4	3	32	52	16
16	16	4	33	54	17
18	12	5	34	55	18
19	57	6	35	54	19
21	34	7	36	52	20
23	4	8	37	49	21
24	30	9	38	44	22
25	51	10	39	39	23
27	8	11	40	32	24
28	21	12			

NOTE.—To construct the above table, suppose the base AB, in the preceding triangle to be=99·5, and the hypotenuse AC=100 ; then by Trig. as 100 : 1 : : 99·5 ·995, the nat. co-sine of the angle BAC=5° 44′. In the same manner, the rest of the angles are obtained, by different operations, accounting the base 99 in finding the second angle, 98 in finding the third, &c.

QUADRANTS AND THEIR USE.



Quadrants differ in construction, but are similar in principle. The instrument is for taking vertical angles. In surveying, it has been nearly superseded by the semicircle of the theodolite and spirit-level, and in astronomy by the mural circle.

1. The annexed diagram ABC represents a quadrant of the simplest mechanism. It is suspended from C either by the hand or top of a staff, the plumb-line CG being the index. The sights through which the object at D is viewed are two small holes drilled in the brass plates *a* and *b* on the radius AC.

2. Another example is similar to the last, a telescope being substituted for the brass sights on AC.

3. A third kind has a spirit-level on the radius BC, which is placed horizontally on CE; the quadrant or arc BA is towards D; and a telescope is affixed longitudinally on the revolving index CG, through which the object is viewed.

4. A fourth construction has a spirit level on AC, which is placed horizontally on CE; BC is towards the object; and the telescope is now fixed tangentially upon the revolving index at the circumference of the arc, the line of collimation thus forming a tangent.

To take the Altitude of a Hill with the Quadrant.

Upon the top of the hill fix an object, exactly as high as your eye will be from the ground, in taking the observation. At the bottom of the hill fix the quadrant staff perpendicularly to the horizon, which may be easily done by means of the plummet. Then with one eye at A, the other being closed, look through the sights, turning the quadrant until you perceive the object at D; so will the arc BG, cut off by the plumb-line CG, be the measure of the angle DCE, or the altitude of the hill, in degrees, above the horizon.

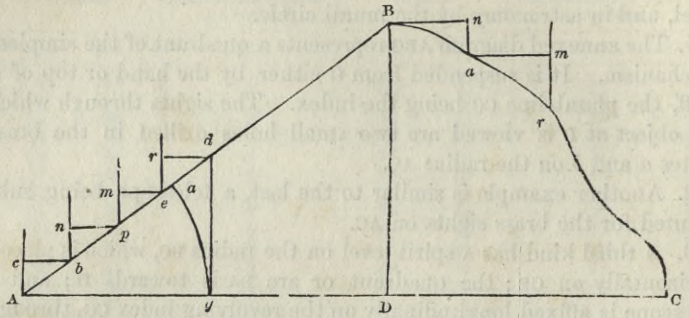
To take the Altitude of a Steeple, &c., with the Quadrant.

Screw the quadrant fast to its staff, so that the plummet may hang exactly at 45° when the staff is perpendicular to the horizon. Then move the staff backward or forward (always keeping it perpendicular), until you can see the top of the object through both the sights. Measure the distance between the bottom of the staff and that of the object, which being added to the height of your eye, will give the altitude required.

METHOD II.

As the foregoing method of reducing hypotenusal to horizontal lines can only be applied, with accuracy, when hills are of a regular slope, surveyors, in general, elevate the chain, as they ascend or descend a hill, in order to preserve the horizontal line.

Examples.



Suppose the lines AB and BC to represent the acclivity and declivity of an irregular hill, it is required to measure them, and to preserve the horizontal line AC.

From A stretch the chain toward B, and suppose it to reach to *a*; the same extent, upon the base, will evidently reach from A to *g*; and a perpendicular erected from *g* will intersect the line AB in *d*; hence the distance A*d*, upon the hypotenuse, will make one chain upon the base. At A, stick your offset-staff into the ground, perpendicularly to the horizon, and let your assistant hold the chain, suppose at the twenty-fifth link, close to the surface of the hill, as at *b*; at the same time you must elevate the end of the chain to *c* forming the horizontal line *cb*; then move forward to *b*, at which place fix your staff again, as before. Let your assistant hold the fiftieth link at *p*, while you elevate the twenty-fifth to *n*, forming the horizontal line *np*. Again, fixing your staff at *p*, elevate the fiftieth link to *m*, while your assistant holds the seventy-fifth at *e*. Lastly, put down the staff at *e*, and elevate the seventy-fifth link to *r*, while the hundredth is held by your assistant at *d*. There he must put down an arrow, and thus you must proceed until you arrive at B, where you will have obtained the horizontal line AD.

In descending from B to C, let your assistant hold one end of the chain at B, while you elevate, suppose, the fiftieth link to *n*, forming the horizontal line B*n*; then fix the staff at *a*, perpendicularly to the horizon, and touching the chain at *n*. Next, let your assistant hold the fiftieth link at *a*, while you elevate the hundredth to *m*, and put down the staff at *r*, as before. In this manner, having arrived at C, you will have obtained the horizontal line DC, which being added to AD, will give the base or horizontal line AC, as required.

NOTE 1. If you wish to obtain the hypotenusal, as well as the horizontal line, divide your field-book into four columns, in one of which you must enter the number of links between *a* and *d*, &c., which being added to the horizontal will give the hypotenusal line.

2. When the ascent or descent of a hill is great, you will not be able to elevate more than 10 or 15 links of the chain at one time; for, in such cases, if you attempt to elevate 20 or 30 links, you will find that the perpendiculars AC, *bn*, &c. will exceed your own height before you can form the horizontal lines *cb*, *np*, &c. (See the last figure.)

METHOD III.

Hypotenusal lines may likewise be reduced to horizontal ones during the survey by quadrants constructed for the purpose. Of these two examples are subjoined, viz., King's Quadrant and Nesbit's Quadrant.

KING'S QUADRANT.

Description.

'The quadrant is fitted to a wooden square, which slides upon an offset-staff, and may be fixed at any height by means of a screw, which draws in the diagonal of the staff, thus embracing the four sides, and keeping the limb of the square perpendicular to the staff. The staff should be pointed with iron to prevent wear. When the staff is fixed in the ground on the station-line, the square answers the purpose of a cross staff, and may, if desired, have sights fitted to it. The quadrant is three inches radius, of brass, is furnished with a spirit-level, and is fastened to a limb of the square by means of a screw.

'When the several lines on the limb of the quadrant have their first division coincident with their respective index-divisions, the axis of the level is parallel to the staff.

'The first line next the edge of the quadrant is numbered from right to left, and is divided into 100 parts, showing the number of links in the horizontal line which are completed in 100 links on the hypotenusal line, and in proportion for any smaller number.

'The second, or middlemost line, shows the number of links the chain is to be drawn forward, to render the hypotenusal measure the same as the horizontal.

'The third, or uppermost line, gives the perpendicular height, when the horizontal line is equal to 100.'

Use.

'Lay the staff along the chain-line on the ground, so that the plane of the quadrant may be upright; then move the quadrant till the bubble stands in the middle, and on the several lines you will have—1. The horizontal length gone forward in that chain; 2. The links to be drawn forward to complete the horizontal chain; 3. The perpendicular height or descent made in going forward one horizontal chain.

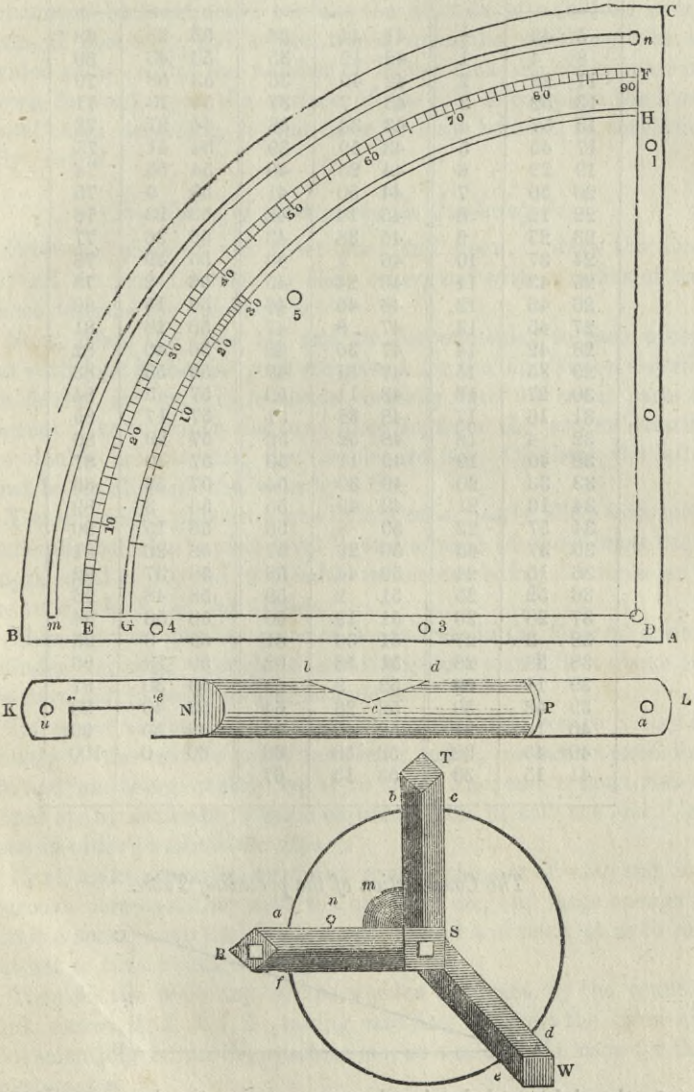
'The first two lines are of the utmost importance in surveying land, which cannot possibly be planned with any degree of accuracy without having the horizontal line; and this is not to be obtained by any instrument in use without much loss of time to the surveyor; whilst with this he has only to lay his staff along the ground, and set the quadrant till the bubble is in the middle of the space, which is very soon performed. And he saves by it more time in plotting his survey than he can lose in the field; for as he completes the horizontal chain as he goes forward, the offsets are always in their right places, and the field-book being kept by horizontal measure, his lines are sure to close.

'If the superficial content, by the hypotenusal measure, be required for any particular purpose, he has that likewise, by entering in the margin of the field-book the links drawn forward in each chain, having thus the hypotenusal and horizontal length of every line.

'The third line, which is the perpendicular height, may be used with success in finding the height of timber. Thus, measure with a tape of 100 feet the surface of the ground from the root of the tree, and find by the second line how much the tape is to be drawn forward to complete the distance of 100 horizontal feet; and the line of perpendiculars shows how many feet the foot of the tree is above or below the place where the 100 feet distance is completed.—Then, inverting the quadrant by means of sights fixed on the staff, place the staff in such a position as to point to that part of the tree whose height you want; and sliding the quadrant till the bubble stands level, you will have on the line of perpendiculars on the quadrant the height of that part of the tree above the level of the place where you are; to which add or substract the perpendicular height of the place from the foot of the tree, and you obtain the height required.'

NESBIT'S QUADRANT.

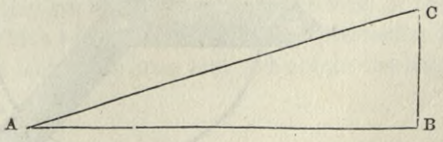
PLATE II.



The following table, by which the Quadrant may be constructed, shows the number of links to be drawn forward upon the surfaces of hills of different altitudes to complete the horizontal chain.

Deg.	Min.	Links.	Deg.	Min.	Links.	Deg.	Min.	Links.
5	43	½	41	44	34	53	28	68
8	4	1	42	12	35	53	43	69
11	22	2	42	40	36	53	58	70
13	52	3	43	7	37	54	13	71
15	57	4	43	34	38	54	27	72
17	45	5	43	59	39	54	41	73
19	22	6	44	25	40	54	55	74
20	50	7	44	50	41	55	9	75
22	12	8	45	14	42	55	23	76
23	27	9	45	38	43	55	36	77
24	37	10	46	1	44	55	49	78
25	43	11	46	24	45	56	2	79
26	46	12	46	46	46	56	15	80
27	45	13	47	8	47	56	28	81
28	42	14	47	30	48	56	40	82
29	35	15	47	51	49	56	53	83
30	27	16	48	11	50	57	5	84
31	16	17	48	32	51	57	17	85
32	4	18	48	52	52	57	29	86
32	49	19	49	11	53	57	40	87
33	33	20	49	30	54	57	52	88
34	16	21	49	49	55	58	3	89
34	57	22	50	8	56	58	15	90
35	37	23	50	26	57	58	26	91
36	15	24	50	44	58	58	37	92
36	52	25	51	2	59	58	48	93
37	28	26	51	19	60	58	58	94
38	3	27	51	36	61	59	9	95
38	38	28	51	53	62	59	19	96
39	11	29	52	9	63	59	30	97
39	43	30	52	26	64	59	40	98
40	14	31	52	42	65	59	50	99
40	45	32	52	58	66	60	0	100
41	15	33	53	13	67			

The Construction of the preceding Table.



In the right-angled triangle ABC, suppose the base AB to be 100, and the hypotenuse AC 100·5 ; then by trigonometry as 100·5 : 1 : : 100 : ·99502, the co-sine of the angle BAC = 5° 43′. In the same manner, the rest of the angles are obtained by different operations, accounting the hypotenuse 101 in finding the second angle, 102 in finding the third, &c.

Now from the preceding table it evidently appears that if an instrument be constructed to take the altitude of a hill at every chain, if necessary, and a line traced upon the instrument be so divided as to exhibit the number of links which the chain must be drawn forward upon the surface of the hill, to complete the horizontal chain according to the table, it may be used in surveying hilly ground.

The Method of Constructing the Quadrant, &c.

Procure a piece of soft sheet-brass, and upon it draw the lines AB and AC perpendicular to each other, and with a radius of five inches describe the quadrant BC.

Next, draw the lines DE and DF perpendicular to each other; and with four inches in your compasses for the first sweep describe the double arc EF, which divide correctly into 90 equal parts or degrees. At a proper distance, likewise, from the arc EF describe the double arc GH, and the double arc *mn*. Of these, the latter must be cut through the brass by a file.

You must also procure a small glass tube, nearly filled with spirit (generally called a "spirit level"), and a piece of sheet-brass KL, in length equal to AB, and in breadth rather exceeding the diameter of the tube, which call the "index."

Then procure another piece of sheet-brass in the form of a semi-cylinder NP, large enough to admit the tube; and in it make the aperture *bcd*, in order to see the bubble.

Its edges solder to the index KL, so that the centre *c* may be exactly in the middle point between *r* and *a*; *ra* rather exceeding DE, and *au* being exactly equal to D*m*. The end N must also be closed up by soldering a piece of brass upon it, and the end P left open in order to admit the tube.

Next, make a wooden quadrant, exactly the size of ABC, and in it a groove corresponding with the aperture *mn*, and large enough to admit a small screw-nail, with a square head and neck, so as to run, but not to turn round, in the groove *mn*.

Then fix the plate ABC to the wooden quadrant, by the counter-sunk screws, 1, 2, 3, 4, 5; taking care first to insert the screw-nail abovementioned into the aperture *mn*, at a small hole made for that purpose at *n*.

Next, fix the index KL upon the face of the quadrant by a screw-nail passing through it at *a*, which must enter the quadrant exactly at the centre D. The nail in the aperture *mn* must likewise pass through the hole at *u*, and upon the end of this nail must be screwed a small nut, by which the end K of the index may be made fast at any altitude.

Now, to divide the arc GH move the end K of the index toward C, until the line or edge *re*, which must be exactly in the centre of the index, cuts the arc EF at 8° 4′, as per table; and upon the arc GH mark the first division. In the same manner, move the index until it cuts of 11° 22′, and there mark the second; continuing these operations until you have made as many divisions as are necessary.—The divisions marked upon the arcs EF and GH, must then be properly cut and figured by an engraver.

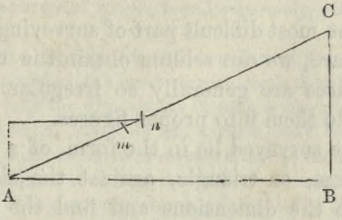
Next, procure a wooden cross, RTSW, the three limbs of which must each be in length equal to AB or AC, and must form with each other three right angles, RST, TSW, and WSR.

This cross must be made to slide upon an offset-staff by means of a square or rectangular aperture through the limb RS; and if a screw be fixed in the side of the limb at *n*, the cross may be fastened to the staff at any convenient height, by turning the screw against the side of the staff. As it will be somewhat difficult, however, on account of the limb RS being hollow, to make a joint at S sufficiently strong to keep the limbs at right angles with each other, they may be supported by means of the brackets *ab*, *cd*, and *ef*. The quadrant ABC must then be fixed upon the square RST, by means of two screws passing through the bracket *ab*, and one through the bracket *m*, so that the outside of the limb SR may coincide with AB, and the outside of the limb ST with AC.

To fix the tube or spirit-level correctly in the semi-cylinder NP, screw the index fast at no altitude, and place the edge AB of the quadrant upon a level table, which you may do by laying the tube upon it, and varying the position of the table until the bubble stands in the centre of the tube; then put the tube into the semi-cylinder NP, and fix it in such a manner that the bubble may be seen at *c*; after which close up the end P with brass or putty.

NOTE.—If the quadrant be made the same size as that in Plate II. instead of five inches radius, as before directed, it will save much trouble in dividing, as the engraver may then follow the divisions given in the plate.

The Method of Proving the Quadrant.



Let AC represent a strong plank, placed with one end against the perpendicular wall BC, and the other upon the horizontal plane AB. Lay on offset-staff, suppose of 12 links, upon AC, with one end at A and the other at *m;* then elevate the lower end, so that the staff A*n* may be parallel to AB. Measure the distance *mn*, which suppose to be 10·5 inches; then say, as 12 links is to 10·5 inches, so is 100 links to 87·5 inches, or 11 links.

Next, lay the edge AB of the quadrant upon the plank AC, and elevate the end K of the index until the bubble stands at *c;* and if the index cuts off 11 links, or nearly so, upon the arc GH, the quadrant is correct.

The Method of Using the Quadrant.

Lay the staff, with the quadrant fixed to it, along the chain-line, so that the edge AB of the quadrant may come in contact with the ground; then elevate the end K of the index until the bubble stands at *c;* and you will have the altitude of the hill upon the arc EF, and the number of links to be drawn forward to complete the horizontal chain upon the arc GH. If you fix the bottom of the staff into the ground upon the chain-line, the limbs ST and SW will serve as a cross by which perpendiculars may be erected.

NOTE 1. In using the quadrant, care should be taken to place it upon the even part of the surface of the hill.

2. In measuring and reducing a line upon a hill, if it happen that the end of the chain reaches exactly to the station at the end of the line, you must then deduct from the chain instead of drawing it forward. For example: if you find that the chain ought to be drawn forward 6 links, you must set down 94 instead of 100 links. Or, if the fiftieth link reach to the station, you must enter 47 instead of 50 links, &c.

3. If you determine, by elevating the chain, and also by the quadrant, the number of links to be drawn forward upon the surface of a hill, in order to complete the horizontal chain, you will seldom find them precisely the same; because it is almost impossible to prevent the chain from forming a curve line, or to keep the staff perpendicular to the horizon. In every case, however, the conclusions of an instrument constructed upon mathematical principles are to be preferred.

Methods for finding the Hypotenusal Measure of Hilly Ground.

This is by far the most difficult part of surveying; and, though we may approach toward, we can seldom obtain the true area of hills, because their surfaces are generally so irregular that it is almost impossible to divide them into proper figures.

If the land to be surveyed lie in the form of a square, rectangle, trapezoid, trapezium, or triangle, against the side of a hill of a regular slope, take the dimensions and find the area in the same manner as if the figure lay upon a plane. But should it be required to find the area of a field (suppose in the form of a trapezium) in which there is a hill so situated as to effect the diagonal only, if the sides and diagonal be measured, and the figure laid down according to those dimensions, the perpendiculars will obviously measure less than they would have done, had the diagonal been reduced to a horizontal line; consequently, we cannot obtain the hypotenusal measure of such a field by the common method of measuring trapeziums or triangles.

In such cases, it is perhaps best, first, to measure the hill only. For this purpose, surround its base by station-staves, dividing it into an irregular polygon, each side of which must be measured. Then fix upon a convenient place near the top of the hill for a station, and between it and each station at the bottom measure a line. Thus will the whole surface be divided into triangles, the areas of which must be found by laying down each triangle separately. Or, from the three sides, you may find the area of each triangle, as already directed.

Next, measure the remainder of the field, by dividing it into proper figures. Collect all the areas together, and their sum will be the area required.

When the land to be surveyed ascends a hill on one side, occupies a plane upon the top, and descends on the other side, you must divide it into such figures as will enable you to approach as nearly as possible to the true area.

The foregoing directions may, perhaps, be found useful to a learner, but, in practice, much will always depend upon the surveyor; he ought, therefore, to be very careful, whatever be the shape or size of the hill, to divide it into such squares, rectangles, trapezoids, trapeziums, or triangles, as are most likely to give him the hypotenusal measure.

NOTE 1. In surveying a triangular field, of which one side passes over a hill, the other two being upon a horizontal plane of the base, it will be necessary to divide it into two triangles, by measuring a line from some part of the fence passing over the hill to the opposite angle. Thus will two sides of each triangle be affected by the hill, the areas of which, found separately, will give the hypotenusal measure of the field.

2. After making some experiments and considering the subject very maturely, the author is of opinion that the most correct method of finding the surfaces of hills in general, is to take the dimensions in such a manner that the areas of the different figures into which the hills are divided, may be found from the lines measured in the field, without having recourse either to the scale or plan. Hence, if the figures be rectangles, their lengths and breadths must be measured in the field; and if they be triangles, trapeziums, or trapezoids, their bases and perpendiculars must be measured in the field.

Examples.

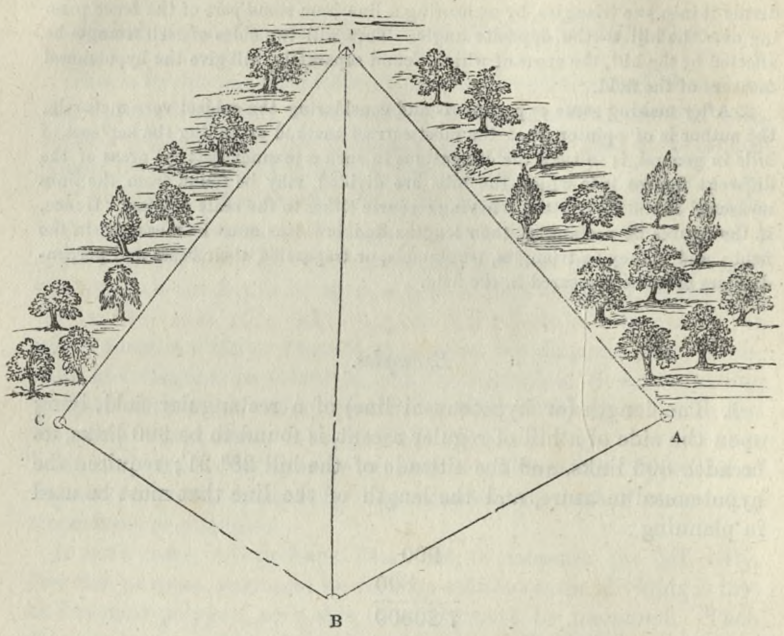
1. The length (or hypotenusal line) of a rectangular field, lying upon the side of a hill of regular ascent, is found to be 900 links, its breadth 800 links, and the altitude of the hill 28° 21′; required the hypotenusal measure, and the length of the line that must be used in planning:

$$\begin{array}{r}
900 \\
800 \\
\hline
7{\cdot}20000 \\
4 \\
\hline
{\cdot}80000 \\
40 \\
\hline
32{\cdot}00000 \\
\hline\hline
\end{array}\quad \textit{Area } 7a.\ 0r.\ 32p.$$

Now, by the table, page 142, we find that 12 links must be deducted from each chain; hence $9 \times 12 = 108$, which being taken from 900, leaves 792 links, the length of the line required.

NOTE.—If we multiply 792 by 800, we find the product 633600 square links equal to 6A. 1R. 14P. the horizontal measure, which is less than the hypotenuse by 3R. 18P.

2. Let ABCD represent a field in the form of a trapezium, lying upon the side of a hill of an irregular ascent, the sides AB and BC being upon the horizontal plane of the base; required the horizontal and hypotenusal measures from the following notes:



	110	BD	
	6	1154	
	12	1100	
	11	1000	
	12	900	
	10	800	
	11	700	
	10	600	
	8	500	
	9	400	
	7	300	
	8	200	
	6	100	
From	B	go N.	diag.
		AC	diag. reduced for
		1300	a proof-line.
		R. off A	

	78	DA	
	6	990	
	7	800	
	9	700	
	8	600	
	7	500	
	9	400	
	10	300	
	11	200	
	11	100	
		R. off D	
	96	CD	
	5	1044	
	10	1000	
	11	900	
	12	800	
	11	700	
	9	600	
	10	500	
	8	400	
	7	300	
	6	200	
	7	100	
		R. off C	
		BC	
		800	
		R. off B	
		AB	
		700	
Begin	at	A	range S.W.

The Operation of finding the Horizontal Measure.

First, 700 + 1154 + 990 = 2844, the sum of the three sides, which being divided by 2, gives 1422. From this number deduct severally each side, and we obtain 722, 268, and 432, for the three remainders. Then, by multiplying the half sum and the three remainders continually together, and extracting the square root of the product, we obtain 344768 square links, the horizontal measure of the triangle ABD.

In a similar manner, we find the horizontal measure of the triangle BCD = 405559 square links; which, added to 344768, gives 750327 square links, equal to 7*a.* 2*r.* the horizontal measure of the trapezium ABCD.

The Operation of finding the Hypotenusal Measure.

First, 1154 + 110 = 1264, the hypotenusal line BD; and 990 + 78 = 1068, the hypotenusal line DA. Then, 700 + 1264

+ 1068 = 3032, the sum of the three sides, which being divided by 2, gives 1516. From this number, deduct severally each side, and we obtain 816, 252, and 448, for the three remainders. Then, proceeding as before, we obtain 373709 square links, the hypotenusal measure of the triangle ABD.

In a similar manner we find the hypotenusal measure of the triangle BCD = 437917 square links, making jointly 821626 square links, equal to 8*a.* 0*r.* 34*p.* the hypotenusal measure of the trapezium ABCD, which exceeds the horizontal measure by 2*r.* 34*p.*

General Directions for the Vertical Survey of a Hill.

Let ABC, in the diagram page 144, represent the transverse section of a hill, AC its base, and B an object at the top of the hill or section, whose altitude is represented by the perpendicular BD. The line BD may also indicate the plane of a longitudinal section.

The line AB is assumed to show the acclivity of the hill lying in the hypotenusal plane: and the crooked line B*r*C the declivity on the opposite side where the surface diverges from the hypotenusal plane BC, this line not being drawn.

The line AB being the base-line of the survey on that side of the hill from which the bearings are taken, it requires to be carefully measured with the chain, the same as if it lay on the horizontal plane in horizontal surveying.

The lines AD and BD (or any other lines parallel to them that may be required) are determined by trigonometry in the following manner :—

The angle BAD is found by taking the angle of depression at B, the two angles being equal (Theo. III. Part I.), and their complement ABD should be taken at the same time as a check angle.

The angle of elevation taken at A does not give the correct angle BAD, because the perpendiculars at A and B represent radii, and therefore are not parallel : whereas a line drawn from A so as to make with AB the complement of BAD is parallel to BD.

The two lines BD and AD remaining undetermined may be found by the following formulæ, AB being taken as the common radius.

1. To find BD, R : sin A : : AB : BD
2. To find AD, R : cos A : : AB : AD
3. To find AD, R : sin B : : BA : AD
4. To find BD, R : cos B : : BA : BD

Any other points in AB as *d* may thus be determined, the triangle A*dg* being similar to the triangle ABD.

The details of the survey on the opposite side of the hill are more complicated, owing to the uneven surface or crooked line B*r*C involving oblique-angled triangles and impediments to sight between the station-poles at B and C. Thus, if the flag-staff at B cannot be seen from station C extend the base line, *if practicable*, as in Prob. V. Sect. V. Part VIII. until it becomes visible, at which place set a station-pole E, and take a second distance in the same direction to station F for a base line. Measure EF with the chain, and take the two adjacent angles at E and F with the quadrant or theodolite. Their sum deducted from 180° gives the other angle of the triangle EBF. BE is now found by Case I. Sect. III. Part VIII. Next take the angle of depression at B and its complement EBD, which will give the angle E sought, when BD and DE may be found by the above formulæ; measure CE with the chain and deduct it from DE, which will give CD.

The hypotenuse BC and the angles which it makes with BD and DC remain undetermined, and may be found from the following formulæ, BC being, in 1 and 2, the common secant sought, and the lines found, BD and DC, tangent and radius; but, in 3 and 4, it is radius and BD and DC sin and cos.

1. To find BC, R : sec B : : BD : BC
2. To find CB, R : sec C : : CD : CB
3. To find B, BC : CD : : R : sin B
4. To find C, CB : BD : : R : sin C

If the base line EF cannot be found in the horizontal plane AC, so that the station-poles E and F are seen from B, then an intermediate station-pole may be placed farther down the hill, as at *r*, making a triangle BC*r* within the vertical plane; *r*CE and *r*CF being without. Should this hypothesis fail, then a base line must be taken upon a different plane, say upon the opposite hill, and the hypotenusal line BC or *r*C found from the bearings taken thereon, as directed in Sect. VI. Part IX. Or the whole of the survey may be taken downwards from the top of the hill; and as the details of the operation are similar to those already given, with the exception that the offsets are taken from parallels by means of the levelling staff, they may be left for the student to work out as a simple but instructive exercise.

Of surveys involving less or more details of the above kind on the vertical plane, the following familiar examples may be enumerated.

1. In mountainous district roads ascend from the valleys, climbing zigzag along the sides of high hills, and not unfrequently pass over the tops of low ones. Their formation often involves a vast amount of levelling, and before such work can be undertaken, a vertical as well as a horizontal survey is necessary.

2. In many places a vertical section is necessary to show the true elevation, climate, and position of towns and other places.

3. When a survey is ordered with a view to determine the geological character, or the agricultural and mineral value, of an estate, a vertical section is required to show the depth of the soil, and the geological strata below. Geological surveys and maps furnish an illustration.

4. Large areas of peat-moss are to be found on hill-sides, and when the moss forms a 'live bog' a survey may be ordered on public grounds; the safety of the inhabitants farther down the hill being in danger.

5. Quarries of slate, marble, and stone for building are often situated towards the top of hills, and surveys are required to ascertain the depth of earth upon the rock (as in Example 3), the thickness of the strata, the elevation, and probable expense of opening and working.

6. On the Wealden and other clays, low hills often yield fine brick-earth; hence the nature of the survey.

7. Gravel is found drifted into low hills and flat table-lands in districts where it possesses a high value for road-making and building purposes; hence the survey.

8. Fortifications and other military works often involve a large amount of vertical surveying.

9. It has often been proposed to give a true plan of a country by means of miniature hills and valleys, and several exceptionary surveys of this kind have been made. The position of the hills and valleys determine the direction of the transverse and longitudinal sections.

10. Railways, embankments, rivers, canals, &c., give rise to a larger amount of vertical surveying than any of the preceding examples. Special directions for surveys of this kind will be found in subsequent parts of the work.

PART V.

THE METHOD OF SURVEYING AND PLANNING LARGE ESTATES.

VARIOUS METHODS are adopted in taking the dimensions of large estates or lordships, of which the following four examples may be given.

METHOD I.

Having made yourself acquainted with the form of the estate, select two suitable places, at the greatest convenient distance from each other, as grand stations ; and measure a principal base, or what is generally called a " main-line," from one to the other, noting every hedge, brook, or other remarkable object, as you cross or pass it ; taking offsets likewise to the bends or corners of the hedges that are near you.

Next, fix upon some other suitable place, towards the outside of the estate, as a third grand station ; to which, from each extremity of the diagonal or main-line, or from two convenient points in it, lines must also be run.

These three lines being laid down, will form one large triangle ; and in a similar manner, if necessary, on the other side of the diagonal or main-line a second triangle may be formed.

The survey must then be completed by forming smaller triangles on the sides of the former, and measuring such lines as will enable you to obtain the fences of each enclosure, the boundaries of rivers, roads, lakes, &c. &c., and prove the whole work.

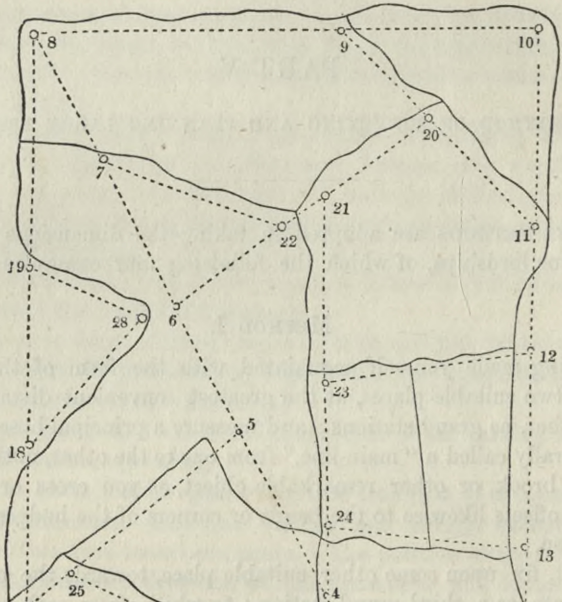
NOTE 1. If the estate be of a triangular form, three lines must be run in the most convenient manner, so as to form the largest triangle possible ; after which other lines must be measured, offsets taken, &c. &c. ; so that all the fences may be obtained, and the survey completed, as in Plate VIII.

2. When an estate is divided into two triangles, it is generally best to finish one of them before you measure any of the internal lines of the other, as in Plate III. Sometimes, however, it is more convenient and expeditious to run some lines in the second triangle before you have finished the first, as in Plate X.

3. Estates similar to those in Plates III. and VIII. are very easy to survey, as

they contain no impediments; but it is otherwise with estates like that in Plate X. where the windings of rivers, roads, and fences, make it necessary to run a great number of lines in order to obtain a correct plan of the whole estate, and the true area and position of every part.

PLAN. PLATE III.



Scale, 8 *Chains to an Inch.*

4. In extensive surveys, where two surveyors are employed, it is best to consider the estate as divided into two distinct parts by the main-line. Each surveyor may then take a part, and make use of the base-line between them, and

measure such other lines as are necessary to complete that part of the survey which he undertakes. By this means the lines of one surveyor do not become entangled with those of the other, while the work of both is more expeditiously and correctly performed than if the two were employed on the same side of the main-line.

5. It is sometimes advisable to divide a very large estate in the following manner : Measure a line across the estate as near to the middle as convenient ; and at right angles to this line measure another line through the middle of the estate. These two lines being tied together by a connecting line measured from one to the other, will divide the estate into four parts, which may be measured separately by dividing them into triangles, and taking such dimensions as are necessary to complete the survey. This method is more especially advisable where three or four surveyors are employed in measuring a large lordship ; for the first two lines being considered as out-boundaries, the estate may be measured in four separate but connected parts.

6. The method of surveying estates by dividing them into triangles is illustrated by Plates III., VIII., and X., the last two of which are actual surveys. The field-notes belonging to them are given in an engraven Field-Book ; and Plates IX. and XI. are the finished plans.

7. No notes are given to Plate III., but the directions of the lines may be ascertained by the following particulars : The first, or main-line, leads from + 1 to + 8, the second line from + 8 to + 10, and the third from + 10 to + 1, which three lines form the first large triangle. The fourth line extends from + 2 to + 15, and the fifth from + 15 to + 8, which two lines and part of the main-line form the second large triangle. The sixth line leads from + 9 to + 11 ; the seventh from + 20 to + 6 ; the eighth from + 7 to + 22 ; the ninth from + 21 to + 4 ; the tenth from + 24 to + 13 ; and the eleventh from + 12 to + 23 ; which complete the survey of the first triangle. The twelfth line extends from + 5 to + 17 ; the thirteenth from + 25 to the main-line south-ward of + 3 ; the fourteenth from + 1 to + 14 ; the fifteenth from + 14 to + 26 ; the sixteenth from + 27 to + 16 ; the seventeenth from + 18 to + 28 ; and the eighteenth from + 28 to + 19 ; which finish the whole survey.

8. The content of the estate may be found in the following manner : Measure the lines upon the plan, and take the necessary offsets, by a scale of 8 chains to an inch, and enter the dimensions in a Field-Book. From the dimensions thus obtained, draw a plan by a scale of 2 chains to an inch ; then straighten the fences, as directed in Part IV. or Part V., and measure diagonals, perpendiculars, &c., from which compute the content of each field. The diagonals, perpendiculars, and contents may be entered in a Book of Castings, similar to those belonging to Plates VIII. and X. Any other scale will do just the same for practice.

9. Taking the dimensions, &c., as directed in the last note, will be found of infinite service to the learner : as it will tend to make him very expert in entering the field-notes, laying down the lines, and casting the contents, which are no small acquisitions towards becoming a complete land-surveyor.

10. In the first edition, the notes were entered from the right towards the left, in the engraven Field-Book ; in this edition they are entered from the left towards the right.

11. Some surveyors represent the crossings of fences by lines drawn across the right and left hand columns of the Field-Book ; and others by lines crossing the middle column. The author prefers the latter method ; but every surveyor will, of course, follow that of which he most approves.

12. Many surveyors enter their notes in a book about four inches and a half in breadth, and fourteen or fifteen in length when opened; and others prefer a book about eight or nine inches long, and seven or eight inches broad when opened. (See the description of the Field-Book, Part II.; and also the engraven Field-Book belonging to Plates VIII., X., and XII.)

METHOD II.

Measure a main-line as nearly to one of the out-boundaries of the estate as the curves in the hedges will permit; noting the crossings of fences, and taking offsets as before directed.

At a convenient distance measure another main-line parallel, or nearly parallel, to the first line, so that a number of fences running in that direction may be obtained; and from any two stations in the first line measure lines to some station in the second main-line, thus forming a triangle; so will a station in the second main-line become determined or fixed.

From the first main-line to the second, or from the second to the first, measure lines in order to obtain all the fences which run in that direction. The remainder of the fences of the enclosures contained between the first and second main-lines being obtained by running lines in the most convenient manner, you will have completed the dimensions of a portion of the estate, which may then be laid down.

Parallel or nearly parallel to the second main-line, and at a proper distance from it, measure a third; and proceed with the internal lines as before, and you will obtain the dimensions of another portion of the estate, which may also be laid down.

Carry on the survey in a similar manner, until you finish it.

NOTE 1. This method is illustrated by Plate IV. The field-notes are not given; but the following particulars exhibit the directions of all the lines: The first main-line leads from + 1 to + 6; the second from + 6 to + 7; and the third line, or second main-line, from + 7 to + 16. The fourth line extends from + 16 to + 1; the fifth or tie line from + 16 to + 2; the sixth from + 2 to + 14; the seventh from + 17 to + 18; the eighth from + 12, through + 18 to + 3; the ninth from + 4 to + 10; and the tenth line leads from + 8 to + 5; thus all the fences between the first and second main-lines are obtained.

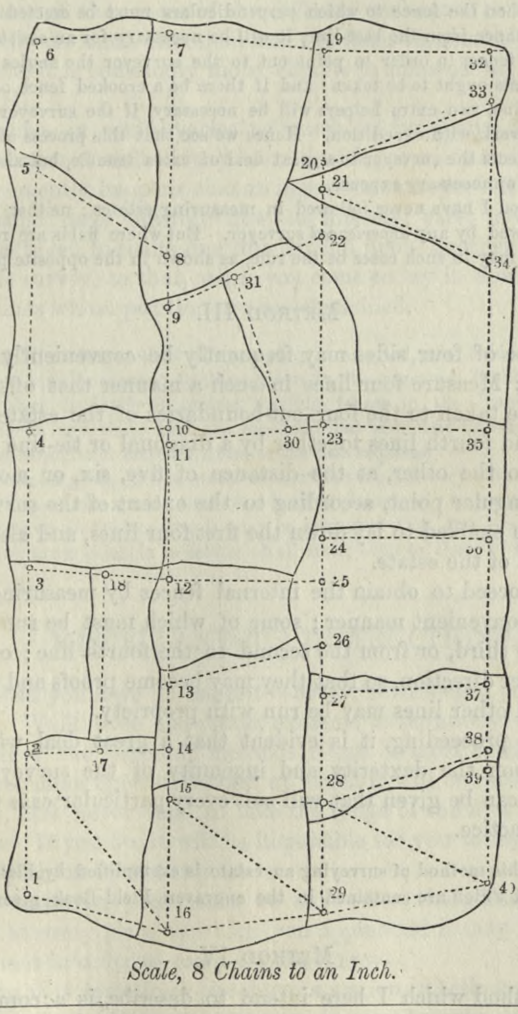
The eleventh line, or third main-line, leads from + 19 to + 29; the twelfth from + 29 to + 16; the thirteenth from + 29 to + 15; the fourteenth from + 15 to + 28; the fifteenth from + 26 to + 13; the sixteenth from + 12 to + 25; the seventeenth from + 23 to + 11; the eighteenth from + 30 to + 31: the nineteenth from + 9 through + 31 to + 22; and the twentieth from + 7 to + 19, which completes the survey between the second and third main-lines.

The twenty-first line, or fourth main-line, extends from + 32 to + 40; the twenty-second from + 40 to + 29; the twenty-third from + 40 to + 28; the twenty-fourth from + 28 to + 38; the twenty-fifth from + 37 to + 27; the twenty-sixth from + 24 to + 36; the twenty-seventh from + 35 to + 23; the twenty-eighth from + 34 to

+ 21; the twenty-ninth from + 20 to + 33; the thirtieth from + 32 to + 19, which finish the whole estate.

2. In order to practice the learner, a Field-Book may be formed, and the content of the estate found in the same manner as directed in Note 8, Method I.

PLAN. PLATE IV.



Scale, 8 Chains to an Inch.

3. Some writers on surveying instruct their pupils to measure main-lines through the estate to be surveyed; and upon these, by the help of a cross, to erect perpendiculars to the opposite angles and curved fences ; and upon these perpendiculars

again, if necessary, to erect other perpendiculars ; thus dividing the whole estate into right-angled triangles and trapezoids.

This method is extremely tedious, as many of the perpendiculars will be 12 or 15 chains in length, when the fields are large; and where the fences are much curved, it becomes almost impracticable, in consequence of the great number of offsets that must be taken, in order to obtain a correct plan of the estate.

Besides, when the fence to which perpendiculars must be erected is at a considerable distance from the base-line, it will be necessary for an assistant to walk along by the fence, in order to point out to the surveyor the angles and curves to which offsets ought to be taken : and if there be a crooked fence on each side of the base-line, two extra helpers will be necessary, if the surveyor intends to perform his work with expedition. Hence we see that this process of measuring not only subjects the surveyor to a great deal of extra trouble, but also to a very considerable unnecessary expense.

This method I have never followed in measuring estates ; neither have I ever seen it followed by any *experienced* surveyor. But where fields are rectangular, the contrary may in such cases be the rule, as shown in the opposite page 165.

METHOD III.

An estate of four sides may frequently be conveniently surveyed as follows : Measure four lines in such a manner that offsets or insets may be taken to the four out-boundaries of the estate ; and tie the first and fourth lines together by a diagonal or tie-line measured from one to the other, at the distance of five, six, or more chains from the angular point, according to the extent of the survey ; thus you will be enabled to lay down the first four lines, and also the out-boundaries of the estate.

Next proceed to obtain the internal fences by measuring lines in the most convenient manner ; some of which must be run from the first to the third, or from the second to the fourth line ; or in some other proper direction, so that they may become proofs and fast-lines, into which other lines may be run with propriety.

In thus proceeding, it is evident that a great deal will always depend upon the dexterity and ingenuity of the surveyor, as no directions can be given that will suit every particular case to be met with in practice.

NOTE.—This method of surveying an estate is exemplified by Plate XII., the field-notes of which are contained in the engraven Field-Book given with this work.

METHOD IV.

The method which I here intend to describe, is a compound of all the foregoing methods of surveying with the chain ; for as there are never two estates to be met with which are exactly alike, sometimes one method claims the preference, and sometimes another ;

but a skilful surveyor will always adopt that by which he can take his dimensions and proofs with the greatest accuracy by the fewest lines.

If an estate be in the form of an irregular polygon of five, six, or more sides, and the fences very crooked, such an estate may generally be most easily surveyed by dividing it into triangles, as in Method I.; but if many of the fences of the different enclosures run a considerable way in the same direction, and the fields in general pretty neat trapeziums, it is commonly more eligible to proceed as directed in Method II.

Sometimes an estate varies so much in its shape, that all the methods before described may be used with propriety and advantage; and it frequently happens that an ingenious surveyor adopts methods in particular cases entirely new to himself; care, however, must always be taken to make one line depend upon another throughout the whole survey, so that, when you come to lay it down, you may find no lines whose position are undetermined.

NOTE 1. Whatever method of surveying is adopted, the field-notes must be entered in a similar manner to those given in the engraven Field-Book. Some surveyors place the letter S against straight fences in the Field-Book, to distinguish them from those that are crooked; but they may be very well denoted by drawing straight or crooked lines, as the case requires.

2. The estates given in this work as examples are not very extensive, in consequence of the serious expense that attends large plates, and the great inconvenience of folding them in books; but it may be remarked that the foregoing methods of surveying are applicable to estates of all sizes, even to those of many thousand acres.

MISCELLANEOUS INSTRUCTIONS.

1. When you have an estate to survey, never begin your work too hastily. Walk over the estate, examine it minutely, and observe by which of the foregoing methods it can be most easily measured. Next determine upon that point at which it will be most convenient to begin, and never omit to take the range of the first line with a compass. If you do, it will be impossible for you to lay it down in its true position upon the plan.

2. In measuring your main or any other chain-line, put down stations at every place to which you apprehend it may be necessary to run lines in order to complete the survey.

3. You may sometimes put down a station, whether you see any particular use for it or not; because it may become serviceable in correcting an error should one be committed; and if it be not used, it will be immaterial.

4. In measuring your internal lines, it will give you the least trouble to run them from one station to another, if you can make it convenient; if not, you must run them from, and continue them to some chain-line, and measure the distance upon that line to the nearest station, which may be entered in the Field-Book thus: run upon the first line, 30 links S. of + 1, &c.

5. The place where you run upon or cross a chain-line, may be easily ascertained by setting up poles at two of the nearest stations in that line; the crossing will be at the place where you are in a direct line with these poles, which may be represented by marks cut in the ground, pointing out the directions of the lines.

6. In ranging the poles, there must be one fixed at the station from which you intend to depart, and another at the place toward which you direct your line, if there be no natural mark, as a tree, the corner of a house, &c. Then, in a straight line with these marks, put down poles at the distance of 4, 6, or 10 chains from each other, according as impediments may render them necessary.

7. When you are measuring a line across a valley, you must proceed forward until you are likely to lose sight of the station to which you are going; then, let your assistant take a pole to the other side of the valley, and direct him to place it exactly in the line which you are measuring, so as to be seen from the bottom of the valley; to this you may continue your line, and thence to the end.

8. When the stations between which you wish to run a line are so far distant that you cannot see from one of them to the other, or when your view is obstructed by an elevation between them, you must then, accompanied by your assistant, go to the place whence you can distinctly see both; and turning face to face, at a little distance, direct each other to the right or left, until you are both in a right line with the stations; then, one of you putting down a pole, the line will be correctly found. If the line, however, be so long that you cannot possibly find it by the above methods, it must be ranged at random; but, in this case, you should be extremely careful that your pole-ranger keeps one pole in a direct line with another, which he may accurately effect by always having, at least, two behind him.

9. In measuring a line which passes over a hill, you must attend to the directions given in Part IV. in the method of measuring Hilly Ground; but you will not always find your lines to meet correctly in surveying mountainous estates.

10. When a river runs through the estate it will be necessary to continue some of your lines across the river, in order to tie the whole survey together.

11. Rivers, large brooks, public roads, and common sewers, should not be included in the area, but only delineated upon the plan. If, however, their areas be required, they should be given separately.

12. Marshes, bogs, heaths, rocks, &c., belonging to the estate should be distinctly represented upon the plan; and their measurements separately returned.

13. You will generally have an opportunity of representing some part of each hedge in your Field-Book; and you may denote on which side of the ditch the fence stands by drawing a small bush, or by specifying it in writing.

14. In surveying estates, the crossings of fences must be taken at the outer extremities of the ditches, and not at the roots of the quickwood, because the ditch, and not the fence, is the division-line between adjoining fields; but in measuring enclosures which are separated by walls, the case is generally different, as the walls most commonly form the lines of division. It may also be observed that the ground upon which a wall stands must be measured with the field to which the fence belongs; and as walls are generally broader at the bottom than at the top, it is necessary to attend to this circumstance in taking the dimensions.

15. When the surveyor finds it convenient, he may put down stations at the outer extremities of the ditches; and in planning, the stations will, of course, fall upon the black lines, because they always represent the boundaries between adjoining fields. This accounts for several of the stations appearing on the black lines of the rough plans in Plates VIII., X., and XII.

16. In taking a survey, you must enter in your Field-Book the name of each field, or of its proprietor or occupier; or you may make such remarks as will enable you to distinguish the fields from each other, &c., and after the plan is drawn, acquire from persons acquainted with the estate every necessary additional information.

17. When hedges obstruct your sight in running the lines, it will be necessary to cut down part of their tops in order to see the poles.

18. If it should happen that you measure a line for which you have no particular use, it will serve as an additional proof; it is evident that you had better measure too many lines than too few.

19. In taking a survey, you ought to observe to whom the adjoining ground belongs, and specify the same upon the plan.

20. Some of our practical surveyors use only nine arrows. When the leader has advanced ten chains, the follower goes up to him, and places his foot or offset-staff at the end of the chain, instead

of the tenth arrow; but in this method I do not perceive any particular advantage.

GENERAL RULES FOR PLANNING LARGE SURVEYS.

The method of laying down a large survey from the Field-Book may easily be acquired by practice; but as the least appearance of difficulty generally discourages a learner, it is presumed that the following directions may be found acceptable.

Having provided a sheet of drawing-paper of proper size, trace with a pencil a meridian, or north and south line, in such a manner that your first station may be in some convenient point in this line. Then, from your first station draw your first or main-line, making its proper angle with the meridian-line, which you may then take out with indiarubber.

Next, take separately in your compasses your second and third lines, or any two more convenient ones, forming a triangle with the main-line; and placing one foot of your compasses in the proper centres respectively, describe arcs intersecting each other. Thus will you have three points from which to form a triangle.

In the same manner proceed with each triangle formed upon the main-line (or upon any other line), proving your work as you advance, until all the triangles are laid down; and if you find all your lines correctly meet, it will be an infallible proof of the accuracy of the work.

The chain-lines being thus laid down, next prick off the crossings of fences, and draw lines in their proper situations, from one crossing to another, to represent the straight fences.

The curved fence must be formed by laying down the offsets, as already directed.

When the whole survey is planned, all the fences must be drawn with Indian ink, the chain-lines and offsets dotted, and the stations, gates, stiles, &c., marked in their proper places; the sheet will then represent what is called a 'Rough Plan.'

NOTE 1. When a fence represents a chain-line it must not be dotted.

2. Practical surveyors seldom dot their chain-lines or offsets, but only mark their stations upon the plan; but it is more satisfactory to a learner to be able to see all his chain-lines at a single view.

3. In taking a very large survey, it is necessary that the work be laid down and proved every night; for if an error be committed, and the survey continued two or three days before it is discovered, the detection in the field will probably be attended with a great deal of trouble.

4. In laying down large surveys it sometimes happens that one sheet of paper will not contain the whole ; in this case, two or more must be pasted together.

5. When you have to lay down a line exceeding the length of your scale draw a line with your pencil in some convenient place upon the plan, and upon it, at two or more operations, prick off the distance in question, which you may then take in your compasses.

6. Beam Compasses are very useful in drawing large circles. They consist of a long straight beam or bar, carrying two brass cursors; one of them fixed at one end, the other sliding along the beam, with a screw to fasten it. To the cursors may be screwed points of any kind, as of steel, pencils, &c. To the fixed cursor is sometimes applied an adjusting or micrometer screw, by which an extent may be obtained to a very great nicety.

DIRECTIONS FOR PLANNING THE ESTATE IN PLATE VIII.

From the dimensions in the engraven Field-Book.

It appears by the first page of the Field-Book that the range of the first line is NNW. ; and by referring to the compass, Plate I. (page 23), we find that the angle which this line makes with the meridian line is 22° 30′.

By Prob. XXIII., Part I., lay down a line making an angle of 22° 30′ with the meridian line ; and by a scale of four chains to an inch prick off 2802 links from cross or station (+) 1 to + 3, and you will thus have the part of the first line.

Now, as the third line could not be run to + 1, in consequence of a large quickwood hedge intervening too far to be cut down, it was necessary to produce the first line 30 links southward, in order that the first three lines might form a triangle; consequently, the first line must be continued 30 links southward from + 1, in laying down the plan ; and this continuation completes the first line.

Take the second line, 3075 links, in your compasses, and with one foot in + 3, describe an arc; and with 3270 links, the third line in your compasses, and one foot in a point 30 links south of + 1, describe another arc, intersecting the former in + 6 ; join these three points by drawing lines from + 3 to + 6, and from + 6 to the above-named point, and you will thus form the triangle 1 3 6.

Next, prick off stations 2, 4, 5, 7, and 8 ; and lay your plotting-scale from + 2 to + 8, and if it measure 1046, as in the Field-Book, line six, you have good reason to conclude that your dimensions are thus far correctly taken and laid down.

Also, mark off + 10, try its distance from + 4 ; likewise examine the distance from + 5 to + 7 ; and if you find both these lines the same as in the Field-Book, your survey is evidently correct.

With the fourth line, 257, in your compasses, and one foot in + 1, describe an arc; and with the fifth line, 1004, as a radius, and + 2 as a centre, make another arc cutting the former in + 9; hence you have three points by which to form the triangle 1 9 2.

Lastly, complete the rough plan by pricking off, and drawing all the straight fences; laying down the off-sets; showing the gates; numbering the fields, &c. &c., as in the Plate.

DIRECTIONS FOR PLANNING THE ESTATE IN PLATE X.

From the dimensions in the engraven Field-Book.

We find from the fourth page of the Field-Book that the first line ranges W. b. N. $\frac{1}{2}$ W., making an angle with the meridian line 84° $22\frac{1}{2}'$.

By Prob. XXIII., Part I., draw a line, making an angle of 84° $22\frac{1}{2}'$ with the meridian line; and by a scale of four chains to an inch, prick off 5445 links from + 1 to + 12, and you will thus obtain the first line, upon which prick off stations 2, 3, 4, 5, 6, 7, 8, 9, 10, and 11.

With 900 (part of the third line) in your compasses, and + 1 as a centre, describe an arc; and with 625, the fourth line, as a radius, and one foot in + 2, intersect the former arc in + 22. From + 2 draw a line to + 22; and from + 1, through + 22, draw the third line, equal to 1360, and you will obtain + 23.

With the second line, 3790, in your compasses, and + 12 as a centre, describe an arc; and with 925, part of the fifth line, as a radius, and + 23 as a centre, describe another arc, cutting the former in + 21. From + 23, through + 21, draw a line equal to 2090, and you will thus obtain the fifth line, and also stations 24, 25, and 26.

Draw a line from + 12 to + 21, and you will have the second line, and also stations 13, 14, 15, 16, 17, 18, 19, and 20; and draw another from + 26 to a point in the first line, 295 W. of + 10, and you will obtain the sixth line, and likewise stations 27 and 28.

With 2325, the twenty-sixth line, in your compasses and + 3 as a centre, describe an arc; and with 1210, part of the twenty-seventh line, as a radius, and one foot on the first line, 150 W. of + 7, describe another arc, cutting the former in + 33. Draw a line from + 3 to + 33; and from + 33, through the intersection of the first line, draw the twenty-seventh line, equal to 2040, and you will obtain + 43.

Next, with 446, the seventh line, in your compasses, and + 12 as a centre, describe an arc; and with 2528, the eighth line, as a radius and + 33 as a centre, describe another arc, cutting the former in + 29. Draw lines from + 12 to + 29, and from + 29 to + 33, and you will obtain stations 30, 31, and 32; and also draw the ninth line from + 32, through + 8, and + 16, to + 27, and you will have stations 34, 35, 36, and 37.

Join stations 11 and 13, and you will obtain the tenth line; 28 and 30, and you will have the eleventh line; 37 and 18, and you will obtain the twelfth line; 19 and 25, and you will have the thirteenth line; 38 and 20, and you will obtain the fourteenth line; 20 and 39, and you will have the fifteenth line; 24 and 39, and you will obtain the sixteenth line; 39 and 19, and you will have the seventeenth line; 17 and 27, and you will obtain the eighteenth line.

From + 31, through + 9, draw the twentieth line, equal to 1175, and you will obtain stations 40 and 41; join 36 and 40, and you will have the nineteenth line; and from + 41, through + 34, draw a line to a point in the first line 72 E. of + 7, and you will obtain the twenty-first line.

Draw a line from + 35 to + 43, and you will have + 42, and the twenty-third line; and join + 42 and + 36, and you will obtain the twenty-second line. Draw a line from + 23 to the first line, 115 E. of + 5, and you will obtain + 45, and the twenty-ninth line; from + 43 to + 45, and you will have + 44, and the twenty-fourth line; from + 3 to + 45, and you will obtain the twenty-fifth line.

Lastly, complete the rough plan, by pricking off, and drawing all the straight fènces; laying down the offsets; making the gates; forming the bases of buildings; shading the river; numbering the fields, &c. &c., as in the Plate.

NOTE 1. Hot-pressed drawing-paper is best for plans. Parchment and vellum are more durable, and generally used when gentlemen are desirous that the plans may be handed down to their posterity. Vellum exceeds parchment in durability; and it may be necessary to remark, that when either of them is used for planning, it must first be rubbed with clean flannel dipped in the best Paris whiting. This operation clears its surface from grease, and facilitates the movements of the pen.

2. In damp weather paper expands, in dry weather it contracts; consequently, if a plan be drawn when the paper is in a moist state, and the content be not found till after it has become perfectly dry, the diagonals and perpendiculars will measure too little, and will of course give the area too little also, but if the plan be drawn when the paper is dry, and the area be found after it has expanded by a change in the atmosphere, the diagonals and perpendiculars will measure too much, and will consequently give the area too much likewise. Hence the necessity of having the paper in the same state of dryness when you find the area that it was in when you laid down the chain-lines, offsets, &c.

3. The most expeditious method of laying down crooked fences is by means of an offset scale, which must be used with the plotting-scale in the following manner: Lay one edge of the plotting scale close by the base-lines, and bring the end of the offset-scale in contact with the edge of the plotting-scale, so that the edges of the scales may form a right-angle ; then by the edge of the offset-scale prick off, in its proper situation, the first offset, with a pencil finely pointed. Keep the plotting-scale firm, and slide the offset scale to the place of the next perpendicular, which prick off as before ; and thus proceed until all the offsets are finished.

4. Properly prepared lead pencils, of different degrees of hardness, for the use of engineers, architects, land-surveyors, and artists, are always in high repute among draftsmen. For land-surveyors they should bear pointing well, so as to produce fine lines.

TO COMPUTE THE CONTENTS.

After the whole survey is laid down, practical surveyors straighten the crooked fences of each field, as directed in Part IV. ; and then divide the fields into trapeziums and triangles, and take such dimensions by the scale as are necessary to find the separate area of each field. They then collect all the areas into one sum ; afterward find the area of the whole survey, as if it were a single field, and if it appears to be equal, or nearly equal, to the sum of the separate areas, previously found, they justly infer that their survey is correct.

NOTE 1. Those who do not approve of finding the area by the method of casting, may make use of the offsets taken in the survey, where convenient ; and if more be wanted, they may be measured by the scale ; for in measuring a number of small parts by it, some will probably be taken a little too large and others a little too small, so that, in the end, they will nearly counterbalance each other.

2. Practical surveyors generally lay down their lines by a scale of 4 chains to an inch, when their surveys are very large ; and in computing the contents, they measure the bases and diagonals by the same scale, but the perpendiculars by a scale of 2 chains to an inch ; consequently, the product of the base and perpendicular of a triangle will be its area. To treat small surveys, in a similar manner, by a scale of 2 chains and of 1 chain to an inch, must, of course, be correct.

3. When the survey is not very large, the content of each field may be set down in some convenient place upon the plan. In other cases, it may be entered within the field itself. Some gentlemen, however, prefer having the areas of their estates given in a book of particulars, containing numbers, or letters of reference, corresponding to those upon the plan.

4. As some surveyors prefer a parallel ruler to a lantern ruler, or a bow of whalebone and silk, for reducing crooked fences to straight ones, I have in the following problems given the method of using that instrument.

5. When there are no dimensions given in the following problems, the figures may be measured by a scale, and then laid down in the learner's book ; after which

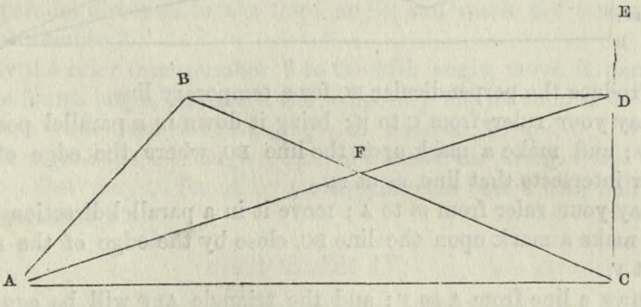
the operations by the parallel ruler may be performed. Or, for practice, figures may be made at pleasure; and the necessary equalising lines drawn, according to the subsequent directions.

THE USE OF THE PARALLEL RULER

IN REDUCING CROOKED FENCES TO STRAIGHT ONES, IN ORDER TO FIND THE AREAS OF FIELDS BY THE METHOD OF CASTING.

PROBLEM I.

To draw a right line AD *from the point* A *through the line* BC, *so that the quantities on each side of the line* AD *may be equal.*



Draw with your pencil a temporary line CE, at pleasure; then, your ruler being closed, lay it from C to A; hold the side that is next to you fast; open the other to B; make a mark with your pencil upon the temporary line CE, where the edge of the ruler cuts that line, as at D; draw a line from A to D, and the quantities on each side of this line will be equal; that is, the triangle ABF will be equal to the triangle CDF.

Demonstration.

Draw the line AC, and also the line BD, which is evidently parallel to AC; then by Theo. VI., Part I., the triangle ABC is equal to the triangle ACD; take away the triangle ACF, which is common to both, and there remains the triangle ABF equal to the triangle CDF.

NOTE 1. The solutions of all the following problems are founded upon the foregoing demonstration.

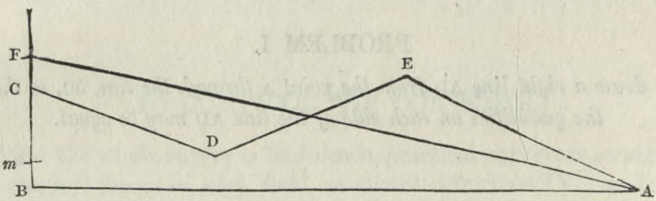
2. If it had been required to draw the equalising line from the angle C, through the line AB, the temporary line must have been made from the angle A.

3. All the operations must be performed with the utmost care and accuracy; and if, at any time, the ruler be suffered to slip, the work must be repeated, or it will not be correct.

4. When an error has been committed, it may be frequently discovered by the eye, after the equalising line is drawn.

PROBLEM II.

Let the irregular figure ABCDEA *represent an offset taken in surveying a field; it is required to draw a right line from the angle* A, *so as to reduce the figure to a right-angled triangle.*



Produce the perpendicular BC for a temporary line.

Lay your ruler from C to E; bring it down in a parallel position to D; and make a mark upon the line BC, where the edge of the ruler intersects that line, as at *m*.

Lay your ruler from *m* to A; move it in a parallel direction to E, and make a mark upon the line BC, close by the edge of the ruler, as at F.

Draw a line from A to F; and the triangle ABF will be equal to the irregular figure ABCDEA; hence the area may be found by multiplying the base AB by half the perpendicular BF.

NOTE 1. In practical operations, the equalising and temporary lines must be made with a pencil finely pointed, and effaced with india-rubber, after the area is found.

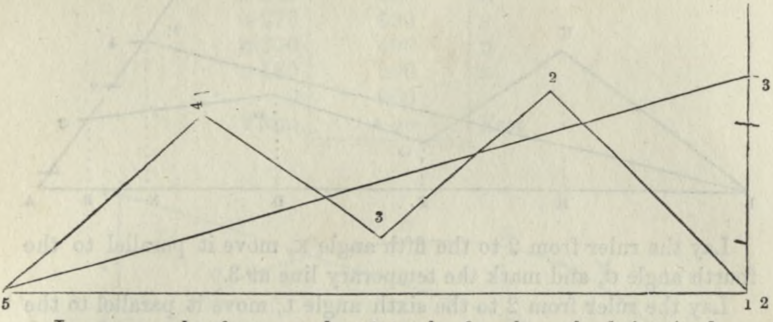
2. If perpendiculars be let fall from the angles E and D, upon the base AB, the necessary dimensions taken by a scale, and the area of the irregular figure ABCDEA obtained by the rules for triangles and trapezoids, it will be found equal to the area of the right-angled triangle ABF: great care, however, must be used to make the lines very fine, and to take the dimensions of all the figures with the utmost accuracy.

PROBLEM III.

It is required to reduce the offset 1 2 3 4 5 *to a right-angled triangle by drawing an equalising line from the fifth angle through the irregular fences.*

Perpendicularly to the base, and from the first angle, draw a temporary line.

Lay your ruler from the first to the third angle, move it in a parallel position to the second angle, and mark the temporary line at number 1.



Lay your ruler from number 1 to the fourth angle, bring it down in a parallel direction to the third angle, and mark the temporary line at number 2.

Lay the ruler from number 2 to the fifth angle, move it parallel to the fourth angle, and mark the temporary line at number 3.

Draw a line from the fifth angle to number 3; and 5 1 3 will be the right-angled triangle required; hence the area of the irregular offset may be found by multiplying the base 1 5 by half the perpendicular 1 3.

PROBLEM IV.

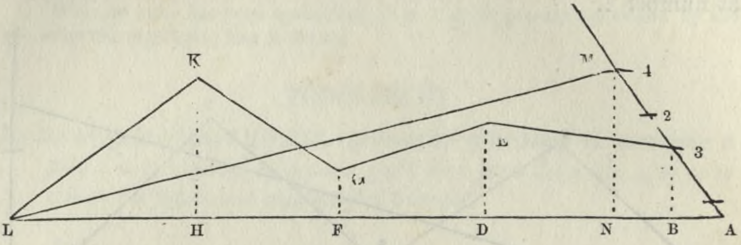
It is required to lay down a right-line offset from the following dimensions; to reduce it to a scalene triangle by the parallel ruler; and to find its area both by the method of offsets and casting.

	AL	
	1500	0
H	1100	300 K
F	800	100 G
D	500	200 E
B	100	150 C
	000	0
From	A go	West

Having laid down the figure, produce the side AC, at pleasure, for a temporary line.

Lay the ruler from the first angle A to the third angle E, move it parallel to the second angle C, and mark the temporary line at 1, which, in this case, is at the second angle, because AC produced is the temporary line.

Lay the ruler from 1 to the fourth angle G, move it parallel to the third angle E, and mark the temporary line at 2.



Lay the ruler from 2 to the fifth angle K, move it parallel to the fourth angle G, and mark the temporary line at 3.

Lay the ruler from 2 to the sixth angle L, move it parallel to the fifth angle K, and mark the temporary line at 4.

Draw a line from the sixth angle L to number 4 (M), and ALM will be the scalene triangle required.

Computation of the Area by Offsets.

Here $150 \times 100 = 15000$, twice the area of the triangle ABC; $150 + 200 \times 400 = 350 \times 400 = 140000$, twice the area of the trapezoid BDEC; $200 + 100 \times 300 = 300 \times 300 = 90000$, twice the area of the trapezoid DFGE; $100 + 300 \times 300 = 400 \times 300 = 120000$, twice the area of the trapezoid FHKG; and $400 \times 300 = 120000$, twice the area of the triangle HLK; then $15000 \times 140000 \times 90000 \times 120000 \times 120000 = 485000$, twice the area of the whole offset; and $485000 \div 2 = 242500$ square links = 2A. 1R. 28P. the area required.

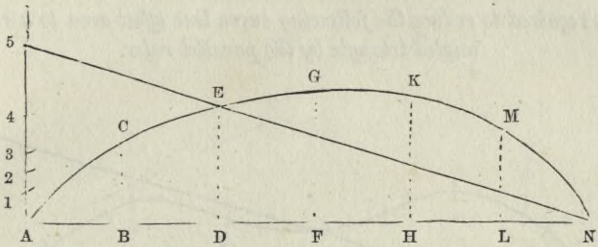
Computation of the Area by Casting.

From the angle M let fall the perpendicular MN, which you will find to measure 323 links; then $\frac{323 \times 1500}{2} = \frac{484500}{2} = 242250$ sq. links = 2A. 1R. 27·6P. the area required. It differs only four-tenths of a perch from the area found by offsets.

PROBLEM V.

Lay down a curve line offset from the following dimensions: reduce it to a right-angled triangle by the parallel ruler; and find its area both by equidistant ordinates and casting.

	AN	
0	1200	
M 190	1000	L
K 260	800	H
G 270	600	F
E 250	400	D
C 180	200	B
0	000	
From	A go	East



Having laid down the figure, erect the perpendicular AP for a temporary line.

Lay the ruler from A to E, move it parallel to C, and mark the temporary line at 1.

Lay the ruler from 1 to G, move it parallel to E, and mark the temporary line at 2.

Lay the ruler from 2 to K, move it parallel to G, and mark the temporary line at 3.

Lay the ruler from 3 to M, move it parallel to K, and mark the temporary line at 4.

Lay the ruler from 4 to N, move it parallel to M, and mark the temporary line at 5.

Draw a line from N to 5 (P), and NAP will be the right-angled triangle required.

Computation of the Area by equidistant Ordinates.

See Prob. IX. Part III.

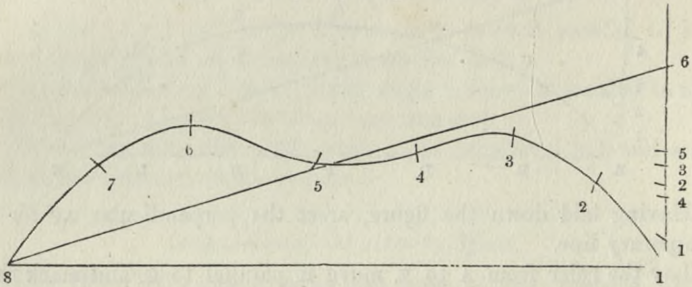
Here the sum of the first and last ordinates is nothing; $(180 + 270 + 190) \times 4 = 640 \times 4 = 2560$, four times the sum of the even ordinates; and $(250 + 260) \times 2 = 510 \times 2 = 7020$ twice the sum of the odd ordinates; then $\frac{2560 + 1020 + 200}{3}$ $= \frac{3580 \times 200}{3} = \frac{71600}{3} = 238666$ square links = 2A. 1R. 21·8P. the area required.

Computation of the Area by Casting.

Measure the perpendicular AP, which you will find to be 398 links; then $\frac{398 \times 1200}{2} = \frac{477600}{2} = 238800$ square links = 2A. 1R. 22P. the area required; which differs only two-tenths of a perch from that found by equidistant ordinates.

PROBLEM VI.

It is required to reduce the following curve line offset area to a right-angled triangle by the parallel ruler.



Erect a perpendicular at one end of the base for a temporary line, and assume a competent number of points in the curve to denote angles.

Lay the ruler from 1 to 3, move it parallel to 2, and mark the temporary line at 1.

Lay the ruler from 1 to 4, move it parallel to 3, and mark the temporary line at 2.

Lay the ruler from 2 to 5, move it parallel to 4, and mark the temporary line at 3.

Lay the ruler from 3 to 6, move it parallel to 5, and mark the temporary line at 4.

Lay the ruler from 4 to 7, move it parallel to 6, and mark the temporary line at 5.

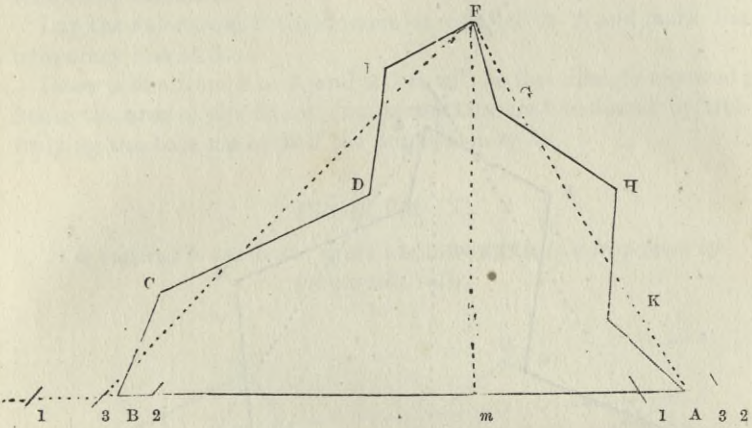
Lay the ruler from 5 to 8, move it parallel to 7, and mark the temporary line at 6.

Draw a line from 8 to 6, and 8 1 6 is the triangle required; hence the area of the irregular offset may be found by multiplying the base 1 8 by half the perpendicular 1 6.

NOTE.—When a curve-line offset area has to be reduced to a triangle by the parallel ruler, a competent number of points must be assumed in the curve to denote angles. These points must be taken at such distances from each other that a right line drawn between any two adjacent points would *nearly* coincide with the curve.

PROBLEM VII.

It is required to reduce the irregular figure ABCDEFGHK *to a triangle by the parallel ruler.*



Produce the base AB both ways, at pleasure, for a temporary line.

Lay the ruler from A to H, move it parallel to K, and mark the temporary line at 1.

Lay the ruler from 1 to G, move it parallel to H, and mark the temporary line at 2.

Lay the ruler from 2 to F, move it parallel to G, and mark the temporary line at 3.

Draw a line from F to 3, and it will be a side of the required triangle.

Again, lay the ruler from B to D, move it parallel to C, and mark the temporary line at 1.

Lay the ruler from 1 to E, move it parallel to D, and mark the temporary line at 2.

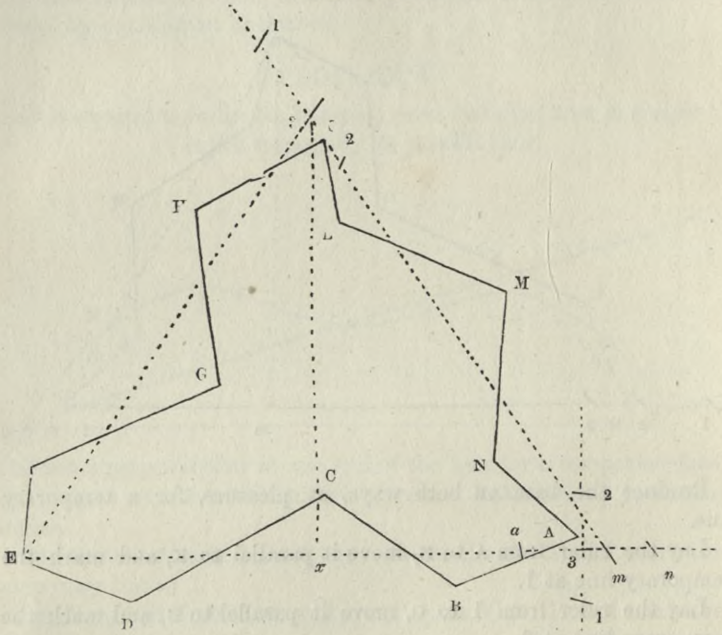
Lay the ruler from 2 to F, move it parallel to E, and mark the temporary line at 3.

Draw a line from F to 3, and 3 F 3 will be the triangle required; hence the area of the irregular figure ABCDEFGHK may be found by multiplying the base 3 3 by half the perpendicular F*m*.

NOTE.—The method of reducing fields of four or five sides to triangles of equal areas may be seen in Problems XVI. and XVII. Part I.

PROBLEM VIII.

It is required to reduce the figure ABCDEFGHKLMN *to a triangle by the parallel ruler.*



Draw the temporary line 1 2 to touch the angle A.

Lay the ruler from A to C, move it parallel to B, and mark the temporary line at 1.

Lay the ruler from 1 to D, move it parallel to C, and mark the temporary line at 2.

Lay the ruler from 2 to E, move it parallel to D, and mark the temporary line at 3.

Draw a line from E to 3, and produce it, at pleasure, for a temporary line.

Lay the ruler from 3 to M, move it parallel to N, and mark the temporary line at *a*.

Lay the ruler from *a* to L, move it parallel to M, and mark the temporary line at *n*.

Lay the ruler from *n* to K, move it parallel to L, and mark the temporary line at *m*.

Draw a line from *m* to K, and produce it, at pleasure, for a temporary line.

Lay the ruler from K to G, move it parallel to H, and mark the temporary line at 1.

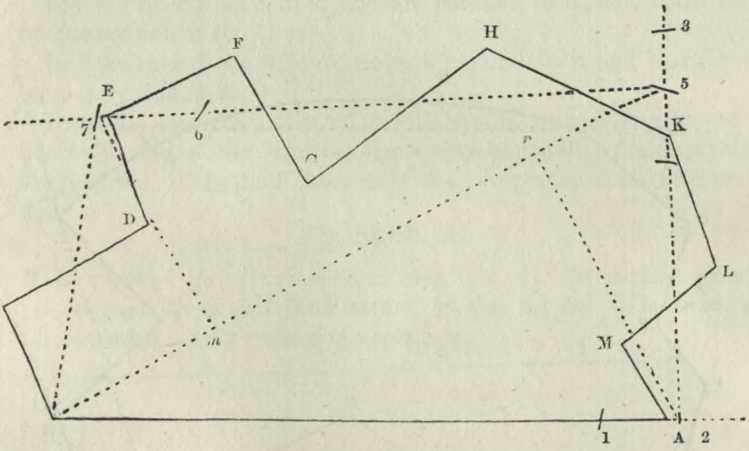
Lay the ruler from 1 to F, move it parallel to G, and mark the temporary line at 2.

Lay the ruler from 2 to E, move it parallel to F, and mark the temporary line at 3.

Draw a line from E to 3, and E 3 *m* will be the triangle required; hence the area of the figure ABCDEFGHKLMN may be found by multiplying the base E*m* by half the perpendicular 3*x*.

PROBLEM IX.

It is required to reduce the figure ABCDEFGHKLM *to a trapezium by the parallel ruler.*



Produce the line AB, at pleasure, for a temporary line.

Lay the ruler from A to L, move it parallel to M, and mark the temporary line at 1.

Lay the ruler from 1 to K, move it parallel to L, and mark the temporary line at 2.

Draw a line from 2 to K, and produce it, at pleasure, for a temporary line.

Lay the ruler from K to G, move it parallel to H, and mark the temporary line at 3.

Lay the ruler from 3 to F, move it parallel to G, and mark the temporary line at 4.

Lay the ruler from 4 to E, move it parallel to F, and mark the temporary line at 5.

Draw a line from 5 to E, and produce it, at pleasure, for a temporary line.

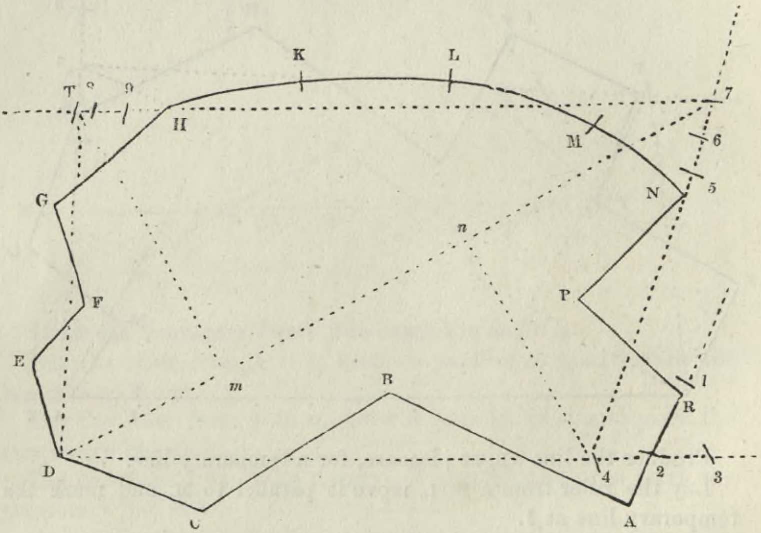
Lay the ruler from E to C, move it parallel to D, and mark the temporary line at 6.

Lay the ruler from 6 to B, move it parallel to C, and mark the temporary line at 7.

Draw a line from 7 to B, and B 7 5 2 will be the trapezium required; hence the area of the figure ABCDEFGHKLM may be found by multiplying the diagonal B 5 by half the sum of the two perpendiculars $7m$ and $2n$.

PROBLEM X.

It is required to reduce the figure ABCDEFGHKLMNPR *to a trapezium by the parallel ruler.*



Continue AR for a temporary line.

Lay the ruler from A to C, move it parallel to B, and mark the temporary line at 1.

Lay the ruler from 1 to D, move it parallel to C, and mark the temporary line at 2.

Draw a line from D to 2, and produce it, at pleasure, for a temporary line.

Lay the ruler from 2 to P, move it parallel to R, and mark the temporary line at 3.

Lay the ruler from 3 to N, move it parallel to P, and mark the temporary line at 4.

Draw a line from 4 to N, and produce it, at pleasure, for a temporary line.

Lay the ruler from N to L, move it parallel to M, and mark the temporary line at 5.

Lay the ruler from 5 to K, move it parallel to L, and mark the temporary line at 6.

Lay the ruler from 6 to H, move it parallel to K, and mark the temporary line at 7.

Draw a line from 7 to H, and produce it, at pleasure, for a temporary line.

Lay the ruler from H to G, move it parallel to F, and mark the temporary line at 8.

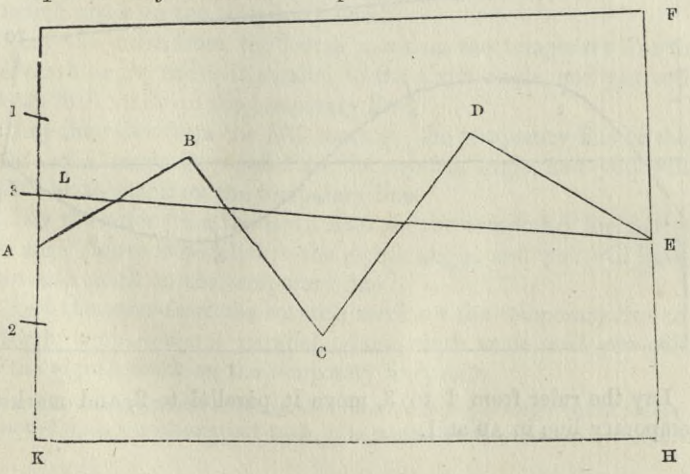
Lay the ruler from 8 to E, move it parallel to F, and mark the temporary line at 9.

Lay the ruler from 9 to D, move it parallel to E, and mark the temporary line at T.

Draw the line DT, and D T 7 4 will be the trapezium required; hence the area of the irregular figure may be found by multiplying the diagonal D 7 by half the sum of the two perpendiculars Tm and 4 n.

PROBLEM XI.

It is required to draw an equalising line, by the parallel ruler, through the crooked fence ABCDE, *so that the two fields which it separates may be reduced to trapeziums.*



Lay the ruler from A to C, move it parallel to B, and mark the temporary line in KG at 1.

Lay the ruler from 1 to D, move it parallel to C, and mark the temporary line at 2.

Lay the ruler from 2 to E, move it parallel to D, and mark the temporary line at 3.

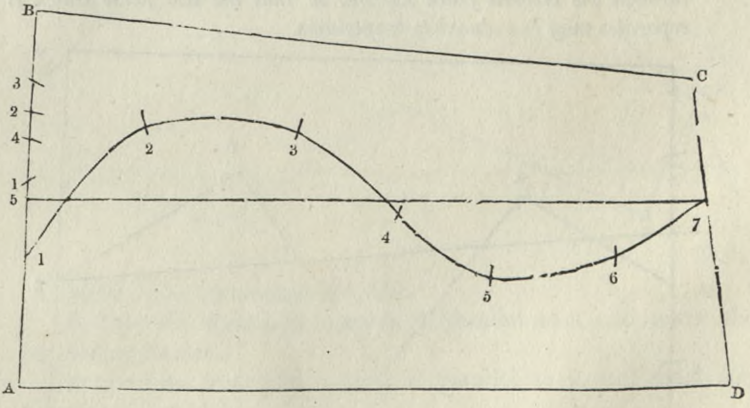
Draw a line from E to 3 (L), and the irregular figure ABCDEFG will be reduced to the trapezium LEFG, and the irregular figure ABCDEHK to the trapezium LEHK; hence their respective areas may be obtained by measuring diagonals and perpendiculars.

NOTE 1.—Sometimes the proprietors of adjoining estates agree to straighten crooked fences or brooks by *giving* and *taking* equal quantities of land. When this is the case, you must first measure and plan the ground, then draw the equalising line as directed in the last problem, and take the distance from A to L very correctly by the scale. Measure this distance in the field from the angle A; range the division line E L, and stake it out; and the work will be completed.

2. It will be advisable to measure, both on the plan and in the field, the parts cuts off on each side by the division-line, in order to prove the work; for an error committed in dividing land is of serious consequence, if it be not discovered and rectified before the new fence is made.

PROBLEM XII.

It is required to draw an equalising line by the parallel ruler, so that the curved fence which separates the two fields in the following figure may be reduced to a straight fence.



Lay the ruler from 1 to 3, move it parallel to 2, and mark the temporary line in AB at 1.

Lay the ruler from 1 to 4, move it parallel to 3, and mark the temporary line at 2.

Lay the ruler from 2 to 5, move it parallel to 4, and mark the temporary line at 3.

Lay the ruler from 3 to 6, move it parallel to 5, and mark the temporary line at 4.

Lay the ruler from 4 to 7, move it parallel to 6, and mark the temporary line at 5.

Draw a line from 7 to 5, and it will reduce the figure ABCD to two trapeziums; hence their respective areas may be found by measuring diagonals and perpendiculars.

The following general rule for the parallel ruler will be found of considerable service to learners, and may be easily committed to memory.

GENERAL RULE.

1. Lay the ruler from the first to the third angle, move it parallel to the second angle, and you will have the first mark on the temporary line.

2. Lay the ruler from the first mark on the temporary line to the fourth angle, move it parallel to the third angle, and you will have the second mark on the temporary line.

3. Lay the ruler from the second mark on the temporary line to the fifth angle, move it parallel to the fourth angle, and you will have the third mark on the temporary line.

4. Lay the ruler from the third mark on the temporary line to the sixth angle, move it parallel to the fifth angle, and you will have the fourth mark on the temporary line.

5. Lay the ruler from the fourth mark on the temporary line to the seventh angle, move it parallel to the sixth angle, and you will have the fifth mark on the temporary line.

6. Lay the ruler from the fifth mark on the temporary line to the eighth angle, move it parallel to the seventh angle, and you will have the sixth mark on the temporary line.

7. Lay the ruler from the sixth mark on the temporary line to the ninth angle, move it parallel to the eighth angle, and you will have the seventh mark on the temporary line.

8. Lay the ruler from the seventh mark on the temporary line to the tenth angle, move it parallel to the ninth angle, and you will have the eighth mark on the temporary line, &c.

NOTE.—As the operations of the parallel ruler, in straightening crooked fences, are founded upon a mathematical truth, it is preferable to a lantern horn: but the latter may be applied with much more expedition than the former ; and if it be used by a skilful hand, its results will be sufficiently correct for general practice. (See Problems I. and II. Part IV.)

A BOOK OF DIMENSIONS, CASTINGS, AND AREAS

BELONGING TO PLATE VIII.

Names of Propietors.	No. on the plan.	Diagonals.	First Perpend.	Second Perpend.	Sum of Perpend.	Quantity in A dec.	Quantity in A.	R.	P.
Mr Dalton's Close	1	625	180	—	180	1·12500	1	0	20
Mr Cayley's Close	2	916	52	247	299	2·73884	2	2	38
Mr Whisker's Close	3	742	123	155	278	2·06276	2	0	10
Mr Straker's Close	4	2094	160	481	641	13·42254	13	1	28
Mr Straker's Close	5	1855	328	268	596	11·05580	11	0	9
Mr Ellard's Close	6	2197	574	—	574	12·61078	12	2	18
					Whole quantity	43·01572	43	0	3

NOTE 1. In the first edition of this work, the content of the estate in Plate VIII. was found from a plan of 2 chains to an inch. The bases, diagonals, and perpendiculars were measured by the scale used in planning; the offsets taken in the field were used where convenient, and when those were insufficient, more were measured by the scale. In this edition, the crooked fences have been straightened by the parallel ruler, the bases and diagonals measured by a scale of 2 chains to an inch, and the perpendiculars by a scale of 1 chain to an inch ; hence the area of each triangle was found by multiplying the base by the perpendicular, and the area of each trapezium by multiplying the diagonal by the sum of the two perpendiculars.

The diagonals, perpendiculars, and areas are entered in the foregoing Book of Castings; and it may also be observed that the bases of triangles are put down in the column of diagonals, and their perpendiculars in the first column of perpendiculars.

2. In straightening crooked fences by the parallel ruler, it frequently happens that one equalising line will serve for two adjoining fields; and almost every irregular figure may be reduced either to a triangle or a trapezium. (See Problems XVI. and XVII. Part I.; and also the use of the Parallel Ruler, Part V.)

A BOOK OF DIMENSIONS, CASTINGS, AND AREAS

BELONGING TO PLATE X.

Names of the Fields.	No. on the Plan.	Diagonals.	First Perpend.	Second Perpend.	Sum of Perpend.	Quantity in A dec.	Quantity in A.	R.	P.
Grime Garth .	1	1166	265	95	360	4·19760	4	0	32
House Ing. .	2	1581	440	—	440	6·95640	6	3	33
Sandy Field .	3	1098	196	97	293	3·21714	3	0	35
Low Holme .	4	861	259	88	347	2·98767	2	3	38
Brooke Close .	5	1130	215	—	215	2·42950	2	1	29
Low Close .	6	969	243	—	243	2·35467	2	1	17
Marsh Close .	7	1209	336	—	336	4·06224	4	0	10
Green Meadow	8	846	153	200	353	2·98638	2	3	38
Horse Pasture	9	741	161	170	331	2·45271	2	1	32
Cow Pasture .	10	962	150	104	254	2·44348	2	1	31
Calf Garth .	11	725	180	133	313	2·26925	2	1	3
Long Meadow	12	1781	226	224	450	8·01450	8	0	2
River Close .	13	810	178	174	352	2·85120	2	3	16
Primrose Close	14	1046	183	210	393	4·11078	4	0	18
Bridge Ing. .	15	1733	287	153	440	7·62520	7	2	20
Shady Ing. .	16	875	159	187	346	3·02750	3	0	4
Hare Park .	17	880	147	167	314	3·76320	2	3	2
Long Tongue .	18	1092	284	—	284	3·10128	3	0	16
Whole quantity,						67·85070	67	3	16

NOTE.—In the first edition of this work, the content of the estate in Plate X. was found from a plan of 2 chains to an inch, by making the crooked fences straight by a lantern horn, as directed in Part IV. In this edition, all the crooked fences have been straightened by the parallel ruler, the bases and diagonals measured by a scale of 2 chains to an inch, and the perpendiculars by a scale of 1 chain to an inch; and hence the foregoing Book of Dimensions, Castings, and Areas was formed.

TO TRANSFER A ROUGH PLAN

TO A CLEAN SHEET OF PAPER, OR TO A SKIN OF PARCHMENT OR VELLUM, IN ORDER TO MAKE A FINISHED PLAN; ALSO TO ENLARGE OR REDUCE PLANS, ETC.

METHOD I.

y Points.

Having laid the fresh sheet upon a smooth table, lay the rough plan upon it, and with four small nails (or weights or books) fasten the corners of both to the table. Then, with your pricker, pierce the extremities of straight lines, and as much of the curved ones as will enable you to draw them on the new plan. Next separate the papers, and trace the outlines and fences with a black-lead pencil, after which draw them with a fine pen and good Indian ink.

NOTE.—Common ink ought never to be used in planning, because it not only sinks too deep into the paper, but generally, in process of time, becomes discoloured.

METHOD II.

By Tracing-Paper.

Take a sheet of writing-paper, of the same size as the rough plan, and rub one side of it with black-lead powder; then lay it upon the sheet which you intend for your new plan, with the black side downward; upon both lay the rough plan, and fasten them all to the table, as before directed. Next run your tracer gently over all the lines upon the plan, so that the black-lead under them may be transferred to the fresh paper. They must then be drawn with Indian ink, as before directed.

NOTE.—This method of transferring is preferable to the former, because it does not injure the plans.

METHOD III.

By a Copying-Glass.

A copying-glass is a large square or rectangular piece of the best window-glass, fixed in a frame of wood, which can be raised to any angle, like a desk, the lower side resting upon a table; and a screen of blue paper may be fitted to the upper edge, and stand at right angles to it.

Place this frame at a convenient angle, against a strong light; fix the old plan and clean paper firmly together by pins, the clean paper uppermost, and on the face of the plan to be copied; lay them with the back of the old plan next the glass, namely, that part which you intend to copy first.

The light through the glass will enable you to perceive distinctly every line of the plan upon the clean paper, and you can easily trace over them with a pencil; and having finished that part which covers the glass, slide another part over it, and copy this, and thus continue till the whole be copied.

NOTE.—Those who have not a copying-glass may use a rectangular piece of window glass fixed in a common frame; and when copying it may be placed in an inclining position, with its top against a window, and its bottom upon the window-seat, if it be nearly level with the bottom of the window. A pane in a window is not unfrequently used for copying small drawings.

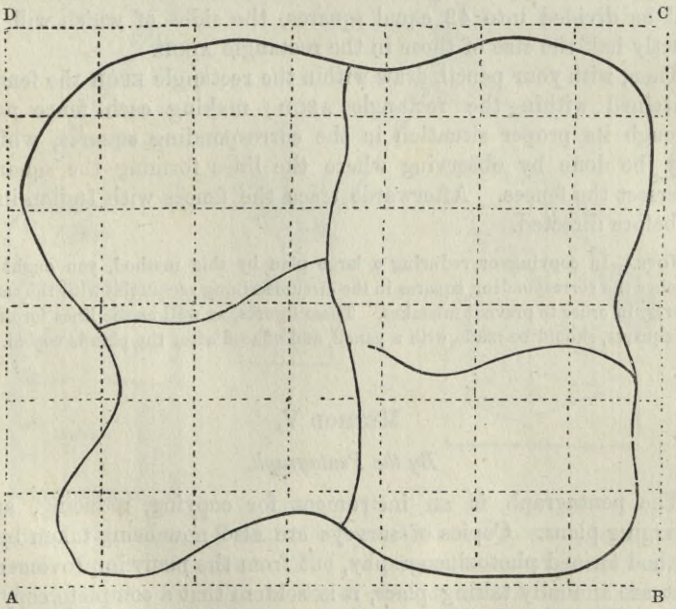
METHOD IV.

By Similar Squares.

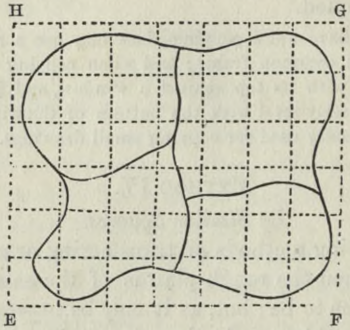
The three foregoing methods of transferring or copying plans can only be applied when the rough plan is of the same size which you wish the finished one to be; but, as it may be necessary to reduce the size of the original, this may be done by similar squares.

Example.

Suppose the following inclosures to have been laid down by a scale of 2 chains to an inch; it is required to reduce them to one of 4 chains to an inch.



By a scale of 2 chains to an inch, draw the line AB = 7 chains. At A and B erect the perpendiculars AD and BC, each of which make = 6 chains, and join DC. Divide the lines AB and DC each into 7 equal parts, and the lines AD and BC each into 6 equal parts; join the opposite points of division, and the rectangle ABCD will be divided into 42 equal squares, the side of each being one chain.



Next, by a scale of 4 chains to an inch, draw the line EF = 7 chains. At E and F erect the perpendiculars EH and FG, each of which make = 6 chains, and join HG. Divide the lines EF and HG each into 7 equal parts, and the lines EH and FG each into 6 equal parts, join the opposite points of division, and the rectangle EFGH will be divided into 42 equal squares, the sides of which will be exactly half the size of those in the rectangle ABCD.

Then, with your pencil, draw within the rectangle EFGH the fences contained within the rectangle ABCD; making each fence pass through its proper situation in the corresponding squares, which may be done by observing where the lines forming the squares intersect the fences. Afterwards trace the fences with Indian ink, as before directed.

NOTE.—In copying or reducing a large plan by this method, you ought to number the corresponding squares in the circumscribing rectangles with the same figures, in order to prevent mistakes. These figures, as well as the lines forming the squares, should be made with a pencil, and effaced after the plan is copied.

METHOD V.

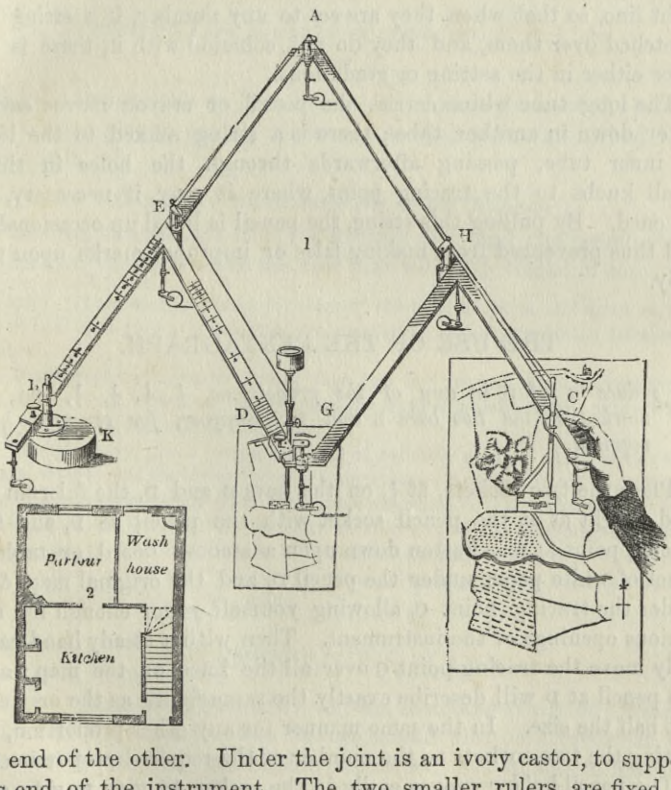
By the Pentagraph.

The pentagraph is an instrument for copying, reducing, and enlarging plans. Copies of surveys are also now being taken by a method termed photozincography, but from the many improvements that are annually taking place, it is seldom that a complete copy is called for, all the changes made in roads, fences, &c., requiring to be entered in the new plan. Surveyors will therefore have chiefly to rely upon their plotting scales, proportional compasses, &c. At the same time, much may often be done by the pentagraph, so that its use should be learned.

DESCRIPTION OF THE PENTAGRAPH.

The pentagraph is generally made of wood or brass, from 12 inches to two feet in length, and consists of four flat bars or rulers, two of them long, and two short, as illustrated in the plate below. The two longer are joined at the end A by a double pivot, which is fixed to one of the rulers, and works in two small holes placed at

PLATE V.



the end of the other. Under the joint is an ivory castor, to support this end of the instrument. The two smaller rulers are fixed by pivots at E and H near the middle of the larger rulers, and are also joined together at the other end G.

By the construction of this instrument, the four rulers always form a parallelogram. There is a sliding box on the longer arm, and another on the shorter arm. These boxes may be fixed at any part of the rulers, by means of their milled screws; and each of these boxes is furnished with a cylindric tube, to carry either the tracing point, pencil, or fulcrum.

The fulcrum or support K is a leaden weight; on this the whole instrument moves when in use.

To the longer instruments are sometimes placed two moveable rollers, to support the pentagraph and facilitate its motions. Their situation may be varied as occasion requires.

The graduations are placed on two of the rulers, B and D, with the proportions of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c. to $\frac{1}{12}$, marked on them.

The pencil-holder, tracer, and fulcrum must in all cases be in a right line, so that when they are set to any number, if a string be stretched over them, and they do not coincide with it, there is an error either in the setting or gradations.

The long tube which carries the pencil or crayon moves easily up or down in another tube; there is a string affixed to the long or inner tube, passing afterwards through the holes in three small knobs to the tracing point, where it may, if necessary, be fastened. By pulling this string, the pencil is lifted up occasionally, and thus prevented from making false or improper marks upon the copy.

THE USE OF THE PENTAGRAPH.

To reduce a plan in any of the proportions, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, &c., as marked on the two bars B *and* D. *Suppose, for example, $\frac{1}{2}$ is required.*

Place the two sockets, at $\frac{1}{2}$, on the bars B and D, the fulcrum or lead weight at B, the pencil socket with the pencil at D, and the tracing point at C. Fasten down upon a smooth board or table a sheet of white paper under the pencil D, and the original map, &c., under the tracing point C, allowing yourself room enough for the various openings of the instrument. Then with a steady hand carefully move the tracing point C over all the lines on the map; and the pencil at D will describe exactly the same figure as the original, but half the size. In the same manner for any other proportion, by setting the two sockets to the number of the required proportion.

The pencil-holder moves easily in the socket, to give way to any irregularity in the paper. There is a cup at the top for receiving an additional weight, either to keep down the pencil to the paper, or to increase the strength of its mark.

A silken string is fastened to the pencil-holder, in order that the pencil may be drawn up off the paper, to prevent false marks when crossing the original plan, in the operation.

If the original should be so large that the instrument will not extend over it at one operation, two or three points must be marked on the original, to correspond with the same upon the copy. The fulcrum and copy may then be removed into such situations as to admit the copying of the remaining part of the original; first observing, that when the tracing point is applied to the three points marked on the original, the pencil falls on the three corresponding points upon the copy. In this manner by repeated shiftings a pentagraph may be made to copy an original of ever so large dimensions.

To enlarge a Plan in any of the proportions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c.
Suppose $\frac{1}{2}$.

Set the two sockets at $\frac{1}{2}$, as before, and change places of the pencil and tracing point; namely, place the tracing point at D, and the pencil at C.

To copy a Plan the same size as the Original.

Place the two sockets at $\frac{1}{2}$, the fulcrum at D, and the pencil at B. In this case the lines upon the new plan will be reversed in copying.

NOTE 1. There are sometimes divisions of 100 unequal parts laid down on the bars B and D, to give any intermediate proportion, not shown by the fractional numbers.

2. Pentagraphs of a greater length than two feet are best made of hard wood mounted in brass, with steel centres, upon the truth of which depends entirely the equable action of this useful instrument.

3. Though I have given various methods of reducing plans, I would advise the learner, after he has found the contents from a plan of 2 chains, or of 1 chain to an inch, to draw another rough plan, of the same size which he intends his finished one to be; and then to transfer it to a clean sheet by any of the foregoing methods. This may appear a little tedious, but it will make the learner very expert in laying down his lines, which will be found of great advantage to him when he enters upon the Practical Part of Surveying.

TO EMBELLISH A PLAN.

In order to make a neat, finished plan, some knowledge of drawing is absolutely necessary. The learner should also be a proficient in plain and ornamental penmanship; or he will not be able to finish a plan either with beauty or elegance. Every person who would excel in this art should devote all his leisure hours to copying and making out drawings, either from plans or copper-plates well executed, as nothing but practice will make a good draughtsman.

Method I.

Plans neatly finished with Indian Ink and Colours.

Having transferred the plan to a clean sheet of drawing-paper, or to a skin of parchment or vellum, by any of the foregoing methods, draw all the straight lines very finely, by the edge of a ruler, with a drawing-pen and Indian ink; but the curved lines must be drawn by a steady hand.

Proceed next to make the representation of hedges, bushes, trees, woods, gates, stiles, bridges, the bases of buildings, &c., &c., in their proper places; running a single dotted line, in an open field, for a footpath, and a double one for a carriage-road.

Hills may be shaded with a brush or hair-pencil and Indian ink. The first wash should be weak, and the edges of the shade, particularly at the top and bottom of the hill, must be softened off with clear water, and a clean brush, kept for that purpose, at one end of the pencil-handle; the other end being occupied by the Indian ink brush.

When the hills are very steep, and rise one above another, as those in Wales, Derbyshire, Yorkshire, Westmoreland, Cumberland, Northumberland, and Scotland, they must all be shaded according to their various inclinations; always letting one wash dry before another is laid on, and never neglecting to soften off the edges of each shade with water.

If some parts of the hills be rocky, tint them with a colour resembling stone, after they have been shaded with Indian ink and a hair-pencil in the manner exhibited in No. 2, Plate VII.

It may also be observed, that when the inclination of a hill is inconsiderable, it is never noticed by surveyors in shading or finishing their plans, and if hills be flat at the top, they are left nearly white.

The method of shading high moorish ground and hilly fields may be seen in Plates VI. and VII.; except they must not be done with lines in imitation of engraving, but with repeated washes of Indian ink.

After hills have been properly shaded with Indian ink, they may then be coloured in the manner hereafter directed for meadow, pasture, and arable land.

Lakes, rivers, brooks, &c., may also be shaded with a brush and Indian ink, pretty strongly at the edges, and softened off towards the middle; and when they are dry, they may be washed over with a light tint of Prussian blue. The shape of arrows should also be made in brooks and rivers, to show in what direction the streams run.

Meadow and pasture ground should be coloured with a transparent green, the pasture rather lighter than the meadow; arable land with various shades of fine brown, so that too many fields may not appear exactly alike; and some surveyors use both red, blue, lake, and yellow in colouring plans.

If the quickwood hedges be not made with a pen and Indian ink, in imitation of bushes, they may be represented by running narrow shades of colouring along the black lines which form the boundaries of the different enclosures.

Roads should be washed with a brownish tint, and the bases of buildings with a red one, or with Indian ink, laid on with a brush of a convenient size, as it is difficult to manage large brushes in shading small spaces.

Sands upon the sea-shore may be washed over with a mixture of brown, lake, and gamboge.

Greens of various shades may be composed of blue and yellow; a pleasing variety of brownish tints may be produced by mixing lake, red, or yellow, with a little brown; and a shade for water may be formed of Indian ink and Prussian blue.

All the washes should be made thin, and laid on in a very neat manner, as nothing disfigures a plan or map so much as daubing on the colours too thickly.

If the estate be small, the area of each enclosure may be put down in some vacant part of the plan; but if it be large, the areas must either be entered within the fields themselves, or in a book of particulars, which may also contain any remarks that the surveyor may think necessary to make to his employer concerning the estate.

In some convenient parts of the plan, write, in various hands, with Indian ink, the title of the estate, ornamented with a compartment or device. In another vacancy introduce the scale by which the plan has been laid down, and also meridian-line, with the compass or flower-de-luce pointing to the north.

The whole may then be bordered with black lines, at a convenient distance from each other, and the space between them shaded with a hair-pencil and Indian ink. See Plates IX. and XI.

NOTE 1. If the learner examine a well-finished coloured map of England, or any other country, he will fully comprehend what has been said on the subject of embellishing plans.

2. Indian ink must always be used in planning; and as it is frequently of a very bad quality, it is advisable to try it before you purchase, by wetting one end of the cake, and rubbing it upon white paper. The blackest and freest is considered the best.

3. The most convenient colours are those ready prepared in cakes, which must be used in the following manner: Dip one end of the cake in clear water, and rub

a little of it upon a clean wedgewood or earthen plate ; then mix it with water by your hair-pencil, until you have brought it to any consistency you please. Indian ink must be prepared for use in the same way.

4. The following water-colours may be enumerated; viz. :—

Vandyke Brown.	Yellow Ochre.	Vermilion.
Raw Sienna.	Indian Yellow.	Prussian Blue.
Burnt Sienna.	Light Red.	Prussian Green.
Gamboge.	Lake.	Sap Green.

By means of these colours a great variety of tints may be formed ; and a little practice will soon enable the learner to produce any shade that may be wanted for plans or maps.

5. When the price for measuring and planning is *very small*, surveyors generally finish their plans neatly, but without either colours, compartments, or embellishments of any kind.

6. Professional surveyors always enter in their field-books the day of the month and date of the year when they begin to survey an estate ; and in finishing their plan, they date them accordingly, and also insert their own names, in order that gentlemen may know when and by whom their estates were surveyed.

Method II.

Plans highly finished with Indian Ink and Colours.

The foregoing method of finishing plans is very expeditious, and may suffice when the price allowed for surveying will not admit of much time being spent in making embellishments; but when a highly-finished plan is wanted, the following method must be adopted.

Meadows.

With a pen, or a very fine-pointed hair-pencil, and light Indian ink, make perpendicular and inclining strokes over the whole meadow, as represented in No. 1, Plate VI., and then wash it with a fine transparent green. The strokes must be of various lengths, but none of them should exceed the 10th part of an inch.

Pasture Grounds.

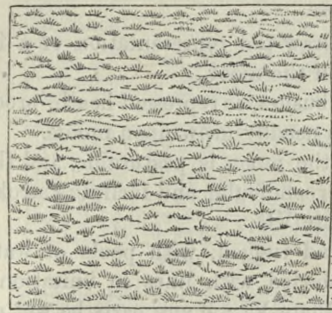
Pastures may be shaded with upright and sloping strokes, of various lengths, as represented in No. 2, Plate VI., and then washed over with a green somewhat inclining to yellow. None of the strokes should exceed the 20th part of an inch in length.

Corn Fields.

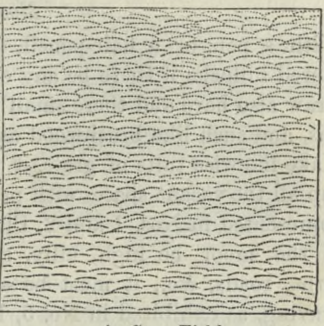
By the edge of a ruler, or by the hand, draw (in short dashes) fine parallel lines, at equal distances from each other, so as to give the fields the appearance of being divided into ridges and furrows, as represented in Nos. 3 and 4, Plate VI. ; and then wash each field over with a different tint of brown, inclining to yellow.

PLATE VI.

1. *Meadow.*

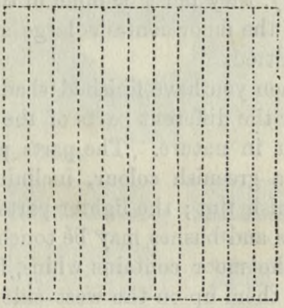


2. *Pasture.*



3. *Corn Field.*

4. *Corn Field.*



5. *High Moorish Ground.*

Corn fields neatly finished in this manner give a plan a very fine appearance.

Moors.

With a pen, or a hair-pencil, draw the representation of a few scattered hillocks if there be any on the moor.

Draw also here and there small bushes, to represent heath, broom, whins, and such like brushwood as usually grow upon moors.

Make likewise tufts of grass, if the moor is pasturable, and then fill up all the vacant spaces with perpendicular and inclining strokes, as represented in Nos. 1 and 2, Plate VI.

If the moor be high, hilly, and rugged, with pools of water, caverns, roads, &c., it must be shaded with lines in imitation of engraving, as exhibited in No. 5, Plate VI.; if any parts be wet and marshy, they must be done in the same manner as marshy ground; and if the moor contains large stones, rocks, or trees, they must not be omitted.

When you have finished shading with Indian ink, you must then colour the different parts of the moor in the same manner as they appear in nature. The parts producing herbage must be washed with a greenish colour, inclining to blue; the dark parts with a brownish tint; the lighter parts with a yellowish one, &c.; and the shrubs and bushes may be touched up with a fine lightish green.

If the moor contains whins, first wash them with green, and then touch them up on the west side with yellow, which will give them the appearance of being in blossom.

By proceeding as above directed, a variety of pleasing effects and shades will be produced, and you will be able to give your plan a very fine appearance, and make it resemble even nature itself.

Marshy Ground.

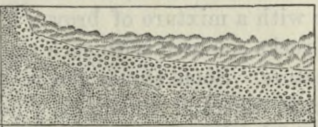
With a pen, or a fine-pointed hair-pencil, and palish Indian ink, draw, by the hand, shortish horizontal strokes of various lengths, pretty closely to each other. Make also the representation of reeds, rushes, sedges, and strong herbage, as exhibited in No. 1, Plate VII.; wash the whole over with a palish green, inclining to blue; and then touch up the reeds, rushes, sedges, &c., with a stronger green, which soften off either towards the right or left with a lighter one, or with clear water. (See the Method of Shading Trees.)

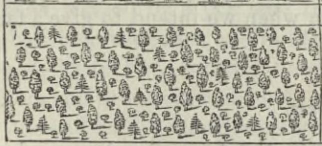
Sands and Rocks.

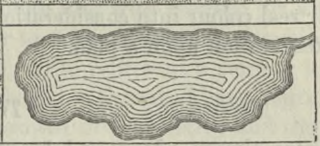
Sands on the sea-shore, &c., must be represented by small dots with a pen and Indian ink; loose stones by figures resembling small circles and ovals, but more irregular; and rocks must be made to

PLATE VII.

1. *Marshy Ground.* 2. *Sands and Rocks.*

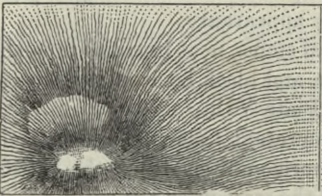


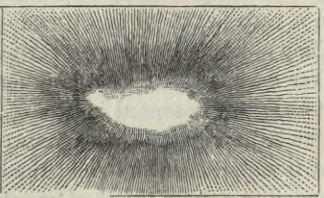




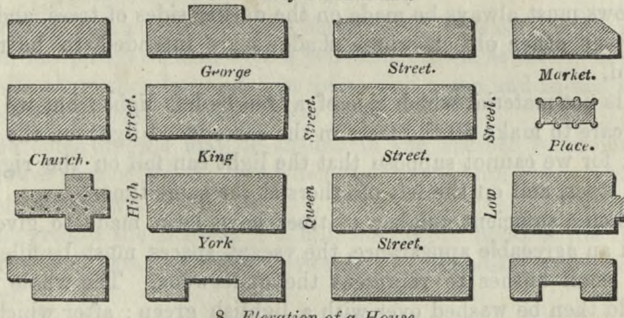
3. *Trees.* 4. *Water.*

5. *Hilly Fields.* 6

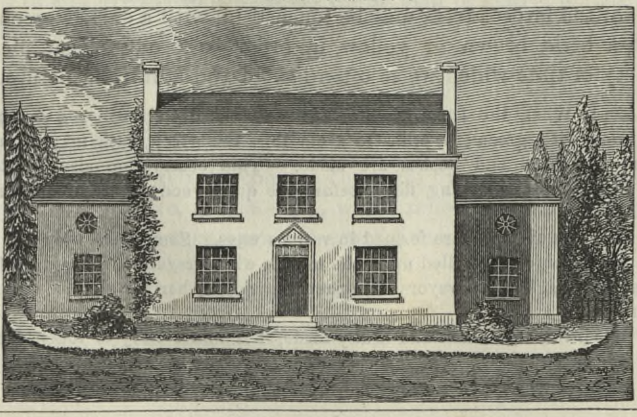




7. *Plan of a New Town.*



8 *Elevation of a House.*



appear rugged and rough, and to rise in succession, one above another, as exhibited in No. 2, Plate VII. The sands may then be washed over with a mixture of brown, lake, and gamboge; and the stones and rocks coloured with such tints as will give them the appearance of nature. Some stones and rocks are whitish, some yellowish, some greyish, others brownish, &c.; hence the propriety of always taking their real colour into consideration when we intend to give a faithful representation upon a plan.

Trees.

Trees always adorn and beautify the face of nature, and when they are neatly drawn, with a fine pen and Indian ink, they give a plan a very beautiful and pleasing appearance.

They must be made with vertical stems, neat broadish tops, shaded darker on one side than the other, and black horizontal shades at the bottom, as represented in No. 3, Plate VII.

The lighter parts of the trees represent that side upon which the light is supposed to fall, and the horizontal shades at the bottom are intended to denote the shadows of the trees upon the ground. These shadows must always be made on the darker sides of trees, and also of every other object, where shadows are intended to be represented.

It is not material which side of a tree be left light; but we must take care to make all the trees in the same wood light on the same side; for we cannot suppose that the light can fall on the right of some trees, and on the left of others at the same time.

When a sufficient number of trees have been made to give the wood an agreeable appearance, the vacant spaces must be filled up with small bushes to represent the underwood. The whole wood should then be washed over with a lightish green; after which the tops of the largest trees may be touched up with a darker green, and with a little brown or yellow, in order to produce that pleasing variety of tints which we so often behold and admire in nature.

NOTE 1. When the Indian ink is not perfectly dry, it will run in washing the wood with green; in order to avoid this, the green wash may be laid on before the trees and bushes are made. — This observation also points out the propriety of colouring fields before the quickwood fences are made with Indian ink.

2. The tops of trees are formed in various ways. Sometimes they are made with jagged edges, and filled up in the middle with irregular strokes, in different directions, and some surveyors form them entirely by horizontal lines of various lengths.

3. When trees are small and neatly made, it is unnecessary to touch them up with any colour.

4. Quickwood hedges must be made with a pen and Indian ink, in imitation of bushes, and when trees are properly introduced, they have a very good effect in the hedgerows. (See Plates IX. and XI.)

Lakes, Rivers, and the Sea-Shore.

Water must first be coloured with a fine tint of Prussian blue, and then shaded by a pen and Indian ink, with crooked or waved lines, bold near the edges, and fainter towards the middle, as exhibited in No. 4, Plate VII., which is intended to represent a mere or lake.

Rivers and brooks must also be shaded with waved lines, continued from one end to the other, as represented in Plate XI.; and the sea-shore in a similar manner, but much stronger and bolder than either lakes or rivers.

NOTE 1. Some draughtsmen do not wash with Prussian blue until they have finished shading with Indian ink, but it is much better to colour the water before it is shaded, as the ink frequently runs when a wash is laid upon it.

2. Here it may not be improper to observe that in colouring lakes, rivers, &c., with Prussian blue, the wash should be pretty strong at the edges, and softened off with water towards the middle.

Hilly Ground.

Meadow and pasture ground should first be washed with a fine green, and ploughed land with a yellowish brown, as before directed; the hills must then be shaded in lines, with a pen and Indian ink as represented in Nos. 5 and 6, Plate VII.

The sides of hills may be shaded in the manner represented in the lower part of No. 6; and when the top of a hill is level, it must be left almost without shade.

The greater the altitude of a hill, the deeper must be the shade; but the level part of a valley between two hills must be very faintly shaded.

It will add greatly to the beauty of the plan or map if all the hills be introduced in their proper places. When this is the case, and the hills are properly shaded, they form what is called a *bird's eye view;* it being supposed that the eye of the observer is elevated to some distance from the ground.

What has been said on this subject will be fully comprehended by the learner, if he carefully examine the plate to which I have already referred, and also No. 5, Plate VI., which represents a high moorish district shaded in a very neat and expressive manner.

Pleasure Grounds.

In order to draw a true plan of pleasure grounds, it is necessary to measure such lines, in taking the survey, as will enable you to lay down correctly the shrubberies, grass-plots and fish-ponds, the bases of summer-houses and alcoves, and the turnings and windings of all the gravel-walks, &c.

The trees, bushes, bases of buildings, &c., must then be neatly made; the fish-ponds and grass-plots properly coloured and shaded, as before directed, for lakes and meadows: and the gravel-walks washed with a fine brown inclining to yellow.

NOTE 1. If the mansion-house, stables, gardens, &c., be situated within the pleasure grounds, the greatest care should be taken to lay them down correctly, so that the smallest inaccuracy in the plan shall not be perceptible.

2. When pleasure grounds are surveyed and planned with adjoining estates, the same scale must, of course, be used for the whole; but when the former are measured separately, a large scale should be chosen, in order to allow sufficient room to plan every object distinctly.

Gardens.

Gardens should be correctly and neatly planned, and finished in a tasteful and elegant manner.

The hothouses, greenhouses, grass-plots, gravel-walks, beds, &c., should all be drawn and laid out as they appear in the garden itself.

The divisions between the different beds may be made with short dashes, as represented in Nos. 3 and 4, Plate VI.; the beds should then be lightly shaded with a pen and Indian ink; rows of bushes inserted along the sides of the walks, and at the divisions of the various beds; and trees should be shown as before directed, if there be any in the garden.

The gravel-walks must then be washed with a yellowish brown; the grass-plots with green; and the different beds with a light tint of yellow, red, lake, blue, green, or any other colours, so as to produce a pleasing variety: and the trees may be touched up with a little dark green, and occasionally a brownish or yellowish tint may be used, to give them an autumnal appearance.

NOTE. When plans are to be finished with colours, it is not necessary to shade them so much with Indian ink as when they are finished with Indian ink only.

The Bases of Buildings.

The outlines of the bases of buildings must be made with a drawing-pen and Indian ink, bold and black on the south and east sides, or on the north and west sides; and the spaces in the middle filled up with oblique lines, as represented in No. 7, Plate VII., which is given expressly for the purpose of making the learner fully acquainted with the method of shading the bases of buildings, drawing the plans of villages, towns, &c.

NOTE 1. When a proprietor wishes to have a plan of his buildings, offices, yards, &c., upon a large scale, the dimensions should be taken in feet and inches, or in feet and tenths, which is preferable ; because the chains and tenths of a chain, upon the plotting-scale, may then be considered as feet and tenths, and used accordingly in planning ; or, when it is more convenient, each chain may be called ten feet ; consequently, each division will then become one foot. (See Note 3, Prob. I., Part III.)

2. When it is intended to lay down buildings by a large scale, the thickness of the walls, the lengths and breadths of rooms and passages, the widths of doors and windows, the projections of fireplaces, and other necessary dimensions, should be taken, in order to produce a correct plan.

3. After the base of a wall has been formed by parallel lines, drawn at such a distance from each other as to exhibit the wall's thickness, the space between these lines may then be shaded by oblique lines as before directed. The doorways should be left open ; the window-bottoms represented by omitting to shade them with oblique lines ; the chimney-bottoms or fireplaces exhibited by making the inside of the wall to project into the room at right angles ; and the steps of the stairs denoted by parallel lines, drawn at proper distances from each other. The insides of the rooms may either be left white or coloured at the option of the draughtsman ; and if it be thought tedious to shade the bases of the walls with oblique lines, they may be done with a brush and Indian ink.

4. The name of every room, office, yard, &c., must be given, either within the rooms themselves, or in the margin of the plan ; and when the premises are extensive, the names of the rooms, out-offices, yards, &c., will be numerous ; there will probably be the kitchen, back-kitchen, parlour, hall, breakfast-room, dining-room, drawing-room, dairy, pantry, stairs, brew-house, wash-house, coal-house, carriage-house, stables, cow-house, calf-house, hog-sty, soil-hole, barn, stable-yard, court-yard, orchard, garden, &c. What has been said on this subject will be easily comprehended by inspecting No. 2, Plate V., which is the ground-plan of a small house, laid down by a large scale, in order to show the learner how he must proceed with plans of a similar nature.

5. When premises are to be sold, every convenience should be pointed out on the plan, in order to promote the sale, and it will be found very advantageous to have plans of the cellars and the upper stories, and even the elevations ; but this is more properly the business of an architect than that of a land surveyor. Some persons, however, will find it of considerable advantage to obtain a knowledge of both these sciences, as gentlemen frequently want not only plans of their estates, but also architectural drawings of their buildings.

The Elevations of Buildings.

In order to give a *perspective* view of the elevation of a building, it is necessary to be acquainted with the art of drawing in perspective ; but an *architectural* view may be produced by taking the dimensions of the building, and laying them down by a scale of equal parts.

When it is intended to give the elevation of any buildings belonging to a farm, or the elevation of a mansion-house and offices belonging to a gentleman's estate, the length from end to end, the perpendicular height from the ground to the eaves, the height of the gable ends, the height and breadth of the chimney-tops, the height and width of the doors and windows, their situations in the walls, and every other necessary dimension, must be measured; then these dimensions, being correctly laid down by a scale, will give an architectural view of the elevation of the building in question.

What has been advanced on this subject will be farther illustrated by referring to No. 8, Plate VII., which is an architectural view of a gentleman's house, given for the inspection and improvement of the learner. The house itself is built with gable ends, but the roofs of both the wings are hipped at one end, which make a pleasing contrast in the elevation.

NOTE 1. After the outlines of an elevation are drawn, the common method of shading is by a brush and Indian ink, as it is generally thought too tedious to shade with strokes in imitation of engraving. The roof should be shaded pretty strongly at the ridge, and softened off towards the middle with water. It may then be washed with Prussian blue; and if the washes, both of Indian ink and colour, be light and often repeated, a more agreeable softness will be produced than by laying on only two or three strong washes. When the roof has been thus shaded, lines may be drawn parallel to the eaves, decreasing gradually in their distance from each other towards the ridge, to represent the edges of the slates. If the house be covered with tiles, the lines must be at equal distances from each other; because tiles of different sizes are never laid upon the same house.

2. If the front of a building project beyond the wings, it must be denoted by making its shadow fall upon one of the wings; but if the wings project beyond the front, the shade of one of them must be made to fall upon the front. (See No. 8, Plate VII., where the shade of the front falls upon the right wing; if the wings had projected, the shade of the left wing would have fallen upon the front.)

3. If a house be built of brick, it may be coloured red; if of stone, a colour may be chosen to resemble it; and when a roof is covered with grey slates, blue slates, or red tiles, it may be coloured accordingly. Sometimes the front of a building is shaded with Indian ink, the roof tinted with blue, and the stone door-post, window-jambs, string-courses, chimney-tops, &c., coloured so as to resemble stone. Indian ink, however, is generally used for fronts in preference to any colour, as it is considered to give buildings a much richer appearance.

4. If there be trees about the buildings, they may be *etched* with a pen and Indian ink in imitation of engraving; the ground in front should be properly shaded; the gravel-walks coloured with a light brown; and if the elevation be bordered with black lines, as in No. 8, Plate VII., the sky may be coloured with a fine blue, or shaded with Indian ink.

5. The elevations of buildings belonging to estates that have been surveyed should be given on vacant parts of the plan as embellishments: it is very seldom indeed that they are drawn in their true situations, because they would intercept the view of the ground-plot; and besides, they are generally laid down by a much larger scale. The mansion-house of a nobleman, well executed, on a vacant part of the plan of his estate, has a very pleasing effect, and will never fail to gratify the proprietor.

6. It is almost superfluous to remind the young draughtsman that he should always keep his hands perfectly clean, and off his plans, so as to preserve them

from being in the least soiled in drawing them, as nothing exhibits the carelessness of a draughtsman in a more conspicuous light than seeing his work besmeared with dust, ink, or colours.

METHOD III.

Plans highly finished with Indian Ink.

A plan highly finished with Indian ink only has a very elegant appearance, and is considered by most persons to excel those done in colours; but the process is very tedious, and requires much time to do it neatly; however, if the surveyor be well paid for his time, he ought to finish his plans in that manner which is most likely to give satisfaction to his employers.

Many surveyors keep plans by them, finished in various ways, as specimens, in order that gentlemen may have an opportunity of choosing in what manner they will have the plans of their estates executed.

Shading with the Pen.

In finishing a plan with Indian ink, a fine pen ought to be used; and the fields should be shaded in a great variety of forms, in imitation of engraving, as exhibited in Plates VI., IX., and XI.

Some fields should be done lighter, and others darker, so as to produce a pleasing contrast of light and shade. Some may be executed in such a manner as to resemble corn-fields, as in Nos. 1 and 6, Plate IX., and 13 and 16, Plate XI., and others may be shaded like meadow and pasture, as exhibited in Nos. 1 and 2, Plate VI.

High moorish ground should be shaded as represented in No. 5, Plate VI.; and marshy grounds, sands, loose stones, rocks, trees, water, hilly fields, and the bases of buildings, as denoted in Plate VII.; and even the elevations of buildings look very elegant when they are finely shaded with lines, as No. 8 in the Plate to which we last referred.

NOTE. In finishing a plan with Indian ink only, it is necessary to shade it much closer and deeper than in finishing with Indian ink and colours.

Penmanship.

In making finished plans, no ornaments or embellishments will compensate for bad penmanship.

Writing, German-text, printing, and figures, are all essentially necessary for a draughtsman; and whoever would excel in the art of planning should use his utmost endeavours to become a complete and elegant penman.

He should practise the various hands, either by copies well written or by good copper-plates, until he can make all the letters and figures correctly and with true taste; and it will save him much trouble in making compartments and devices if he can acquire the art of flourishing and ornamenting neatly and elegantly with the pen.

Ornaments.

Any compartment or device may be chosen to fill up the vacant corners of a plan, such as the compass, scrolls of paper, wreaths or festoons of leaves and flowers, branches or sprigs of oak, palm-tree, weeping-willow, myrtle, laurel, olive, &c.; also shields, coats of arms, columns supporting vases or urns, mathematical instruments, cattle, sheep, or whatever else may please the fancy of the draughtsman.

Ornaments on Plate IX.

In the N.W. corner is a device formed of an oak branch, leaves and acorns on the left side; and on the right side is a branch of large pointed leaves resembling sedges or sweet flags, intertwined with a string of small leaves; and both branches are united at the bottom by a bunch of riband.

In the S.W. corner is a scroll of paper, supported by a fluted column; by the side of which are some ears of corn, and at the bottom a few blades of grass and herbage.

In the N.E. corner is the sun in his meridian splendour, with a fancy device resembling an ogee cornice, fronted with reeds, and from each end of the cornice is suspended a festoon of small leaves.

In the S.E. corner is a plotting-scale, a pair of compasses, two drawing-pens, and a writing-pen, interwoven with a garland of small leaves and berries, resembling those of the myrtle.

Ornaments on Plate XI.

In the N.W. corner is a fancy device in the form of an oval, and in the N.E. corner is a rectangular device, with the exception of the arch at the top. This device is ornamented with a bunch of riband, and two festoons of small leaves and berries, hanging upon two scutcheons or shields.

In the S.W. corner is a column, at the top of which is a vase encircled with leaves and flowers. On the west of the column Britannia is seated, leaning on her shield, holding a spear in her right hand, and with her left hand pointing out the science of surveying. To the east of the column are two sheep, emblems of agriculture.

The plotting-scale, drawing-pens, &c., are nearly similar to those in the last plate.

In the S.E. corner is a parallel ruler, a plane table, a terrestrial globe, a crowing cock, and a youth seated upon a beehive, with a pair of compasses in his hand, at work upon Plate XII.

The cock is an emblem of early rising, and the beehive may be considered as an emblem of industry; and it may here be remarked, that it is impossible to attain eminence in the art of surveying without early rising, industry, and perseverance.

MISCELLANEOUS INSTRUCTIONS

Relating to Surveying, Planning, Casting, Valuing, &c.

1. The title of a plan should set forth the name of the proprietor, and also the name of the township, hamlet, parish, and county in which the estate is situated.

2. The names of the adjoining lordships, or the names of the proprietors of the adjoining lands, should be given on the plan, in order to point out clearly the situation of the estate and corroborate the title.

3. All principal roads passing through the estate, from one highway to another, should be laid down, and the places to which they lead specified.

4. All footpaths and bridle-roads should be pointed out, in order to determine the public right, and guard against encroachments.

5. All occupation and privileged roads through adjoining estates should be noticed either on the plan or in the reference book.

6. All ancient highways leading through the estate, although not now in use, should be particularly specified, and the names of the proprietors given, to show in whom the privilege of reopening them, if necessary, is vested.

7. The ancient and proper names of fields should be preserved, as it generally creates confusion and mistakes when new ones are assigned without sufficient authority.

8. It has already been observed, that the extremities of the ditches are generally the boundaries between adjoining fields; this, however, is not always the case, as the stem of the quickwood sometimes forms the boundary; hence the necessity of obtaining an assistant who is well acquainted with all the local customs of the place.

9. The greatest care must be taken to find the area of each field correctly; and particularly if the survey be taken for an enclosure, or to make a valuation for the land-tax, poor-rates, county-rates, and other assessments; for it is evident that if the survey be incorrect,

the valuation can never be equitable, and will consequently produce nothing but disputes and dissatisfaction among the proprietors and occupiers, instead of peace, harmony, and friendship.

10. In valuing for an assessment, great care should be taken not to over-rate the land that is of a poor quality and lies far from the means of improvement; for bad land costs the occupier as much in labour and seed as good land, and is far less productive. (See more observations on valuing land in Part VI.)

11. In reducing a plan for portable use, care should be taken to choose a scale sufficiently large to exhibit all the irregularities in the fences, buildings, &c.

12. Several small farms, or detached pieces of land, belonging to one proprietor, may be laid down upon the same sheet. They ought not, however, to be joined together, but planned as separate estates.

13. When one sheet of drawing-paper is too small to contain the survey, two or more must be neatly pasted together; and when those parts that have been wet with the paste are nearly dry, they may be made smooth by a warm *iron*. The edge of one of the sheets should be cut even, and laid nearly half-an-inch over the edge of the other sheet; and a piece of clean paper should be laid under the *iron*, to prevent it from soiling the plan.

14. It has already been observed that the surveying-chain should frequently be measured. The readiest method of doing this is to drive two stakes or pins into the ground, exactly at the distance of 22 yards from each other. Professional surveyors measure their chains in this manner every morning, when they are engaged in extensive measurements. When the chain has become too long, it is better to cut a little from several of the links than to take off the rings; care, however, must be taken to keep each 10 links of an equal length, or the dimensions will be incorrect.

15. The book of particulars before mentioned is generally called 'A Terrier of the Survey,' and should contain references corresponding to those upon the plan; also the name of each field, or the name of the proprietor, or of the occupier, and the area of each field in acres, roods, and perches. If the surveyor value the estate, the terrier ought to contain the value per acre to let or for sale; the annual value of each field to let, or the total value for sale; and also the cultivation of each field: thus will the proprietor be furnished with every necessary particular relating to his estate.

16. The terrier may likewise contain remarks and observations on the quality of the soil; and point out the method of improving wet marshy grounds, by draining them; commons and waste lands, by enclosing them; large fields, by dividing them, &c.

17. Some surveyors return three measurements of each field in the terrier; viz., the land in cultivation, the hedges and waste land, and the total quantity or sum of both.

18. Arable lands should be distinguished from permanent meadows, and those which the tenant is prohibited from breaking up should be particularly noticed.

19. In writing out a valuation book for the purpose of making assessments, all the lands and tenements in the occupation of the same tenant should be collected together, and put down on the left-hand page of the book. At the top of the page must appear the name of the tenant; and in the first and second columns respectively, the names of the proprietors and the numbers on the plan. The third fourth, fifth, and sixth columns must contain the name, measurement, value per acre, and total value of each field respectively. The right-hand page may be left blank for incidental remarks, when a change of occupation takes place, or when any circumstance occurs that affects the arrangement of the book.

20. When the valuation is high, it is frequently thought prudent to calculate the assessments from one-fourth, one-half, or three-fourths of the amount; this, however, is more properly the consideration of the occupiers than that of the land-surveyor. Sometimes the assessments are calculated from one-half or three-fourths of the valuation of the land, and from one-fourth of the valuation of the buildings.

A Terrier of the Survey in Plate IX.

Nos. on the Plan.	Names of the Fields.	Cultivation of the Ground.	Area in A.	R.	P.	Value per Acre to rent. £	s.	d.	Total Value per Annum to rent. £	s.	d.
1	Calf Garth .	Pasture . .	1	0	20	2	12	6	2	19	$0\frac{3}{4}$
2	Lane Close .	Arable . .	2	2	38	1	16	0	4	18	$6\frac{1}{2}$
3	Low Close .	Permanent Meadow	2	0	10	2	2	6	4	7	$7\frac{3}{4}$
4	Turnpike Close	Arable . .	13	1	28	1	14	0	22	16	$5\frac{1}{4}$
5	Daisy Field .	Meadow .	11	0	9	1	15	6	19	12	$5\frac{3}{4}$
6	Triangle . .	Pasture . .	12	2	18	2	3	6	27	8	$7\frac{1}{2}$
		Sum total .	43	0	3	……			82	2	$9\frac{1}{2}$

NOTE 1. The annual value of each field may be found from the area, and the value per acre by the Rule of Three.

2. If one tenant occupy all the foregoing estate, his rent will be 82*l.* 2*s.* 9½*d.* per annum; and if the assessments be made from three-fourths of the annual value, he will be assessed at 62*l.* 12*s.* 1*d.*

3. The terrier may be divided into any number of columns, to suit the purpose of the surveyor; and when the observations, remarks, &c., are too numerous to be contained in the columns of one page, each two opposite pages may be divided into columns, in which may be entered every necessary information relating to the estate.

4. In extensive surveys and valuations, an alphabetical index should be annexed to the terrier or valuation-book, in order that the name of any particular proprietor or occupier may be more readily found.

PART VI.

RULES AND DIRECTIONS FOR APPORTIONING LAND; FOR LAYING OUT ANY GIVEN QUANTITY OF LAND, IN ANY PROPOSED FIGURE; FOR PARTING FROM ANY FIELD OR FIGURE ANY QUANTITY OF LAND REQUIRED; AND FOR DIVIDING A PIECE OF LAND AMONG SUNDRY CLAIMANTS IN THE PROPORTION OF THEIR RESPECTIVE CLAIMS, OR A COMMON, ETC., OF VARIABLE VALUE AMONG ANY NUMBER OF PROPRIETORS, IN THE PROPORTION OF THEIR RESPECTIVE INTERESTS.—LANDS INTERSECTED BY A RAILWAY.—VERTICAL SURVEY OF EMBANKMENTS, RIVERS, CANALS, AND LANDS RECLAIMED FROM THE SEA.—ALSO, THE METHOD OF REDUCING STATUTE MEASURE TO CUSTOMARY, AND *VICE VERSA*.

SECTION I.

Rules and directions for laying out any given quantity of land in any proposed figure, and for parting from any field or figure any quantity of land required.

When the land to be laid out or parted off is given in acres, roods, and perches, it must first be reduced into square links; in which process the following table will be found extremely useful.

When it is required to part off from any field or figure any quantity of land, it is generally necessary, first, to measure the whole, if the dimensions be not given.

A Table for Reducing Acres, Roods, and Perches into Square Links.

Acres.	Sq. Links.	Perches.	Sq. Links.	Perches.	Sq. Links.
1	100000	1	625	21	13125
2	200000	2	1250	22	13750
3	300000	3	1875	23	14375
4	400000	4	2500	24	15000
5	500000	5	3125	25	15625
6	600000	6	3750	26	16250
7	700000	7	4375	27	16875
8	800000	8	5000	28	17500
9	900000	9	5625	29	18125
10	1000000	10	6250	30	18750
20	2000000	11	6875	31	19375
30	3000000	12	7500	32	20000
40	4000000	13	8125	33	20625
50	5000000	14	8750	34	21250
60	6000000	15	9375	35	21875
70	7000000	16	10000	36	22500
80	8000000	17	10625	37	23125
90	9000000	18	11250	38	23750
100	10000000	19	11875	39	24375
Roods.	Sq. Links.	20	12500		
1	25000				
2	50000				
3	75000				

PROBLEM I.

To reduce any number of acres, roods, and perches, into square links.

RULE.—Reduce the given quantity of land into perches, which multiply by 625, the number of square links in one perch, and the product will be the square links required. Or, find the equivalents of the acres, roods, and perches respectively in the foregoing table.

Examples.

1. Reduce 6A. 3R. 25P. into square links.

By the rule.

```
 A.  R.  P.
  6   3  25
  4
 27
 40
1105
 625
5525
2210
6630
690625 Ans.
```

By the table.

```
        sq. links.
 6A. = 600000
 3R. =  75000
25P. =  15625
        690625 Ans.
```

2. Required the number of square links in 96A. 2R. 36P.

Ans. 9672500.

PROBLEM II.

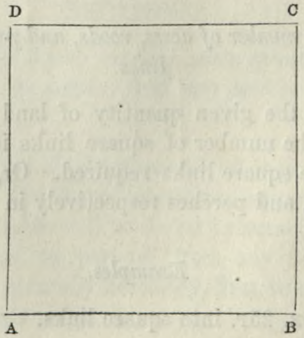
To lay out, in a square, any quantity of land proposed.

RULE.—Extract the square root of the proposed area, and it will be the side of the square required.

Examples.

1. Lay out in a square 7 acres, 1 rood, and 24 perches.

```
          sq. links.
  7A. = 700000
  1R. =  25000
 24P. =  15000
         740000(860·2 links, the side of the square.
         64
    166)1000
         996
     17202)40000
           34404
           .5596
```



In laying out the square in the field, let AB represent one of its sides, which make = 860·2 links. At A erect the perpendicular AD, which make = AB; and at B erect the perpendicular BC, which make also = AB. Then measure the line CD, and if you find it = 860·2 links, the work is right.

2. Required the side of a square which shall contain 15A. 2R. 32P. *Ans.* 1253 *links.*

PROBLEM III.

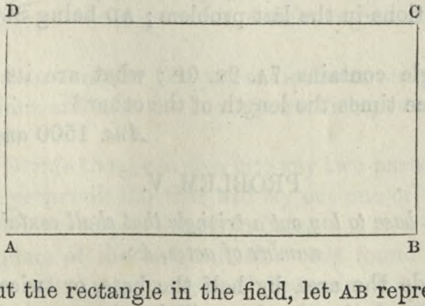
Upon a given line, to make a rectangle that shall contain any proposed quantity of land.

RULE.—Divide the proposed area by the given side, and the quotient will be the other side of the rectangle.

Examples.

1. Lay out 3A. 3R. 26P. in the form of a rectangle, one side of which must be 850 links.

```
         sq. links.
  3A. = 300000
  3R. =  75000
 26P. =  16250
85,0)39125,0(460·3 links, the other side.
     340
     ---
     .512
      510
     ----
     . . 250
         255
```



In laying out the rectangle in the field, let AB represent the given side. At A, erect the perpendicular AD, which make = 460·3 links; and at B, erect the perpendicular BC, which make = AD. Then measure the line CD, and if you find it = AB, the work is right.

2. If one side of a rectangle be 525 links, required the other side, so that the figure may contain 6A. 2R. 23P. *Ans.* 1265·5 *links.*

PROBLEM IV.

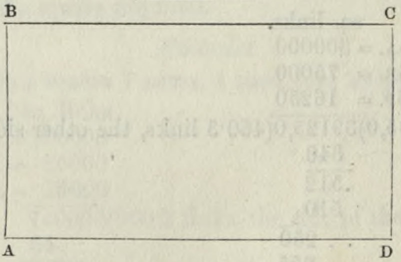
To lay out any given quantity of land in a rectangle, so that one of its sides shall be two, three, four, or any number of times as long as the other.

RULE.—Divide the given area by the given number, and the square root of the quotient will be the shorter side, which multiply by the given number, and the product will be the longer side.

Examples.

1. Lay out 3A. 0R. 32P. in the form of a rectangle, one of the sides of which shall be twice as long as the other.

sq. links.
3A. = 300000
32P. = 20000
2)320000
160000(400 links, the shorter side.
16   2
. .0000 800 links, the longer ditto.



Let ABCD represent the rectangle, which you must lay out according to the directions in the last problem; AD being 800 and AB 400 links.

2. A rectangle contains 7A. 2R. 0P.; what are its sides, one of them being three times the length of the other?

Ans. 1500 *and* 500 *links.*

PROBLEM V.

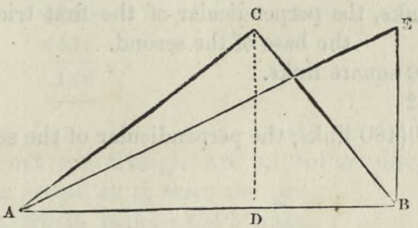
Upon a given base to lay out a triangle that shall contain any given number of acres, &c.

RULE.—Divide the area by half the base, or twice the area by the whole base, and the quotient will be the perpendicular of the triangle.

Examples.

1. Lay out 3A. 2R. 16P. in the form of a triangle, the base of which must be 1200 links.

$$\begin{array}{rl} & \text{sq. links.} \\ 3\text{A.} = & 300000 \\ 2\text{R.} = & 50000 \\ 16\text{P.} = & 10000 \\ \hline 6{,}00) & 3600{,}00 \\ \hline & 600 \text{ links, the perpendicular.} \end{array}$$



Upon any part of the given base AB, suppose at D, erect the perpendicular DC, which make = 600 links; then stake out the lines AC and BC; so will ABC be the required triangle. But if the perpendicular be erected at either end of the base, as at B, then the line AE must be staked out; and ABE will be the triangle required.

2. Required the perpendicular of a triangle, which contains 6A. 2R. 37P., its base being 1556 links. *Ans.* 865·2 *links.*

PROBLEM VI.

To lay out a trapezium that shall contain any number of acres, &c., having one of its sides or a base line given.

RULE 1.—Divide the given area into two parts, either equal or unequal; and then, by the last problem, find the perpendicular that will lay out one of these parts in a right-angled triangle upon the given base.

You must then consider this perpendicular as one of the diagonals of the trapezium, and also the base upon which you must lay out the other triangle.

RULE 2.—Divide the given area into any two parts as before; and then find the perpendicular that will lay out one of these parts in a right-angled triangle upon the given base.

Add the square of the perpendicular thus found to the square of the given base, and the square root of the sum will be the hypotenuse. Consider this hypotenuse as one of the diagonals of the trapezium, and also the base upon which the other triangle must be laid out.

Examples.

1. Lay out 8 acres in a trapezium, upon a given side of 800 links.

BY THE FIRST RULE.

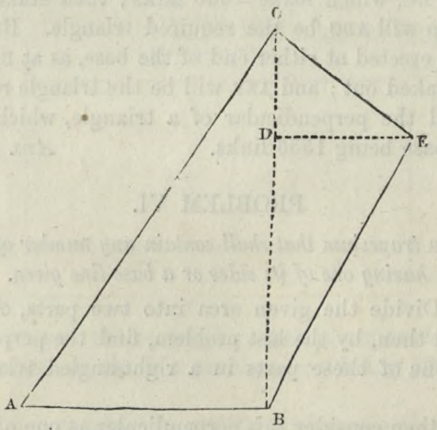
Divide the given area into 5 and 3 acres, and let the triangle upon the given side contain the greater part.

5A. = 500000 square links.

$$\begin{array}{l} \quad\quad\quad 2 \\ 8,00)10000,00 \\ \quad\quad 1250 \text{ links, the perpendicular of the first triangle, and also the base of the second.} \end{array}$$

3A. = 300000 square links.

$$\begin{array}{l} \quad\quad\quad 2 \\ 125,0)60000,0(480 \text{ links, the perpendicular of the second triangle.} \\ \quad\quad 500 \\ \quad\quad 1000 \\ \quad\quad 1000 \\ \quad\quad \ldots\ldots 0 \end{array}$$



In laying out the trapezium in the field, let AB represent the given side. At B, erect the perpendicular BC, which make = 1250 links. Then, upon any part of the line BC, as at D, erect the perpendicular DE, which make = 480 links. The four outlines being properly staked out, the work will be completed.

BY THE SECOND RULE.

5A. = 500000 square links.

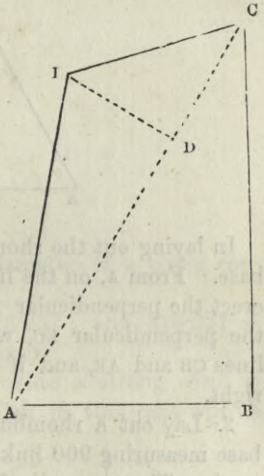
$$\begin{array}{l} \quad\quad\quad 2 \\ 8,00)10000,00 \\ \quad\quad 1250 \text{ links, the perpendicular of the first triangle.} \end{array}$$

Then, $\sqrt{1250^2 + 800^2} = \sqrt{1562500 + 640000} = \sqrt{2202500}$ = 1484 links, the hypotenuse of the first, and also the base of the second triangle.

3A. = 300000 square links.
2

1484)600000(404·3 links, the perpendicular of the second triangle.
5936
. .6400
5936
·4640
4452
.188

Having laid out the triangle ABC as before directed, upon any part of the line AC, as at D, erect the perpendicular DE, which make = 404·3 links. Stake all the outlines, and the work will be completed.



2. Lay out 12A. in a trapezium, upon a given side of 1400 links.

Ans. Supposing the given area divided into 7 and 5 acres : then, by the first Rule, the perpendicular of the first triangle is found to be 1000 links, and that of the second the same.

By the second Rule, the perpendicular of the first triangle is found to be 1000 links, the base of the second 1720·5, and its perpendicular 581·2 links.

PROBLEM VII.

Upon a given base, to lay out a rhombus of any content less than the square of the base.

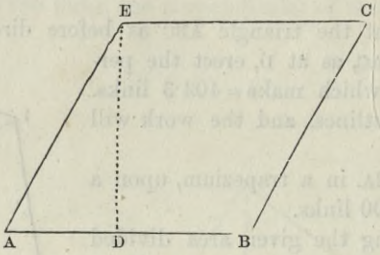
RULE.—Divide the content by the base, and the quotient will be the perpendicular. Then from the square of the base subtract the square of the perpendicular, and find the square root of the remainder. Upon the base, from one of its extremities, measure a line equal to this root, and at this point erect a perpendicular.

Examples.

1. Lay out in a rhombus 5A. 2R. 16P. its base being 800 links.

$$\begin{array}{rr} & \text{sq. links.} \\ 5\text{A.} = & 500000 \\ 2\text{R.} = & 50000 \\ 16\text{P.} = & \underline{10000} \\ 8{,}00) & \underline{5600{,}00} \\ & 700 \text{ links, the perpendicular.} \end{array}$$

Then, $\sqrt{800^2 - 700^2} = \sqrt{640000 - 490000} = \sqrt{150000} = 387{\cdot}3$ links, at which distance from one of the extremities of the base the perpendicular must be erected.



In laying out the rhombus in the field, let AB represent the given base. From A, on the line AB, measure 387·3 links to D; and at D erect the perpendicular DE, which make = 700 links. At E, erect the perpendicular EC, which make = the base AB. Measure the lines CB and AE, and if you find each of them = AB, the work is right.

2. Lay out a rhombus, which shall contain 6A. 1R. 8P. upon a base measuring 900 links.

Ans. The perpendicular is found to be 700 links, and the distance at which it must be erected from one of the extremities of the base 565·7 links.

PROBLEM VIII.

To lay out any given quantity of land in a circle.

RULE 1.—If we multiply the square of the diameter of any circle by ·7854, the product will be the area; consequently, if we divide the area by ·7854, the quotient will be the square of the diameter.

2. Multiply the square root of the area by 1·12837, and the product will be the diameter.

Examples.

1. Lay out one acre of land in a circle.

sq. links.

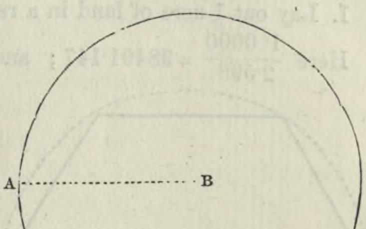
·7854)100000·000000(127323·65 links, the square of the diameter.

7854
21460
15708
.57520
54978
.25420
23562
.18580
15708
.28720
23562
.51580
47124
.44560
39270
.5290

Then, $\sqrt{127323 \cdot 65} = 356 \cdot 82$ links, the diameter of a circle containing one acre of land.

Or, by the second rule, $\sqrt{100000} = 316 \cdot 227$; and $316 \cdot 227 \times 1 \cdot 12837 = 356 \cdot 82$ links, the diameter, as before.

In laying out the circle in the field, provide a strong cord, in length equal to the radius of the circle, which in this case will be 178·4 links; and fixing one of its ends at B, as a centre, make the other fast to your offset-staff near its lower extremity. Then stretch the cord to A, and with the line AB describe the circle, keeping the staff perpendicular, and making a mark on the ground with its spike, by which you must stake out the circumference.



Or at proper intervals stretch the radius AB, and put down stakes in such a manner as to form the circumference.

2. Required the diameter of a circle, which will contain half an acre of land. *Ans.* 252·3 *links.*

PROBLEM IX.

To lay out any given quantity of land in a regular polygon.

RULE.—Divide the area of the required polygon by the area standing opposite to its name in the subjoined table, and the square root of the quotient will be the length of the side. Multiply the side thus found by the polygon's number in the column of radii, and the product will be the radius of the circle circumscribing the required polygon.

A table of regular polygons, with their areas, and the radii of their circumscribing circles, when the side of the polygon is 1.

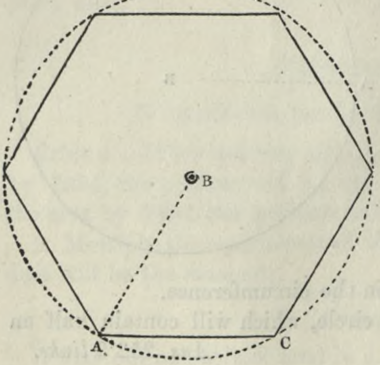
No. Sides.	Names.	Areas.	Radii.
3	Triangle . .	0·433	0·577
4	Square . . .	1·	0·707
5	Pentagon . .	1·72	0·851
6	Hexagon . .	2·598	1·
7	Heptagon . .	3·634	1·152
8	Octagon . .	4·828	1·306
9	Nonagon . .	6·182	1·462
10	Decagon . .	7·694	1·619
11	Undecagon . .	9·365	1·775
12	Duodecagon . .	11.196	1·932

NOTE. If the square of the side of any polygon be multiplied by the area standing opposite to its name in the preceding table, the product will be the area of the polygon.

Examples.

1. Lay out 1 acre of land in a regular hexagon.

Here $\frac{100000}{2\cdot598} = 38491\cdot147$; and $\sqrt{38491\cdot147} = 196\cdot191$ links, the side of the required polygon, and also the radius of the circumscribing circle, because the side of a regular hexagon and the radius of its circumscribing circle are always equal to each other; hence the multiplier in the table is 1.



To lay out the hexagon in the field, draw the circumscribing circle as directed in the last problem. Then the radius AB, which is equal to the side of the hexagon, being applied six times, will just go round the circumference, and form the polygon required.

2. Lay out half an acre of land in a regular octagon.

Ans. The side of the required octagon is 101·76, and the radius of its circumscribing circle 132·898 links.

PROBLEM X.

To lay out any given quantity of land in an ellipsis, having one of the diameters given.

RULE.—If we multiply the rectangle of the two diameters of an ellipsis by ·7854, the product will be the area; consequently, if we divide the area by ·7854, and that quotient by the given diameter, the latter quotient will be the diameter required.

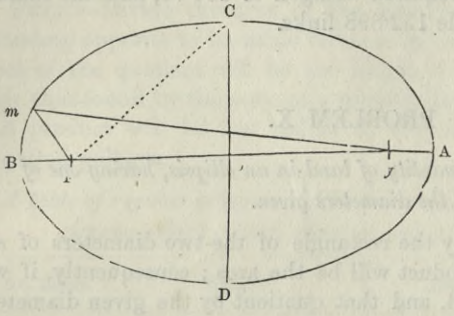
Examples.

1. Lay out an ellipsis, which shall contain one acre, with a transverse diameter of 450 links.

```
         sq. links.
·7854)100000·00000(127323·6 quotient.
       7854
       21460
       15708
       .57520
        54978
        .25420
         23562
         .18580
          15708
          .28720
           23562
           .51580
            47124
            .4456
```

Then, $\frac{127323\cdot6}{450} = 283$ links, the conjugate diameter.

By Prob. XV., Part I., construct the ellipse ABCD; then by a property of the ellipse, the square of the distance of the focus from the centre is equal to the difference of the squares of the semi-diameters: hence, we have $\sqrt{225^2 - 141{\cdot}5^2} = \sqrt{30602{\cdot}75} = 175$ links, equal Fo, or fo: and $225 - 175 = 50$ links, equal AF, or Bf.



Again, by another property of the ellipse, the sum of the two lines drawn from the foci, and meeting in any point in the circumference, is equal to the transverse diameter; that is, $Fm + fm =$ AB.

Procure, therefore, a cord, and upon it make two loops, so that the distance between them may be equal to the transverse diameter; then measure in the field the diameter AB, putting down a stake at each focus, and one at the centre o. At o erect the perpendiculars oC and oD, making each $= 141{\cdot}5$ links.

Put the two loops over the stakes at F, f, and stretch the cord so that the two parts Fm, fm, may be equally tight; at m put down a stake as one point in the circumference of the ellipse, and in the same manner determine as many others as you please.

But if the ellipse be very large, so that you cannot conveniently procure a cord as long as the transverse diameter, you must then erect perpendiculars, called ordinates, at every 50 links, or at every chain's length, &c., upon that diameter, and measure the lengths of these perpendiculars by the scale.

Then measure in the field the transverse and conjugate diameters, and erect the perpendiculars in their proper places, always remembering to put down a stake at the end of each perpendicular.

2. Lay out an ellipse which shall contain 8A. 3R. 8P., one of the diameters being given equal to 800 links.

Ans. The other diameter is = 1400 links.

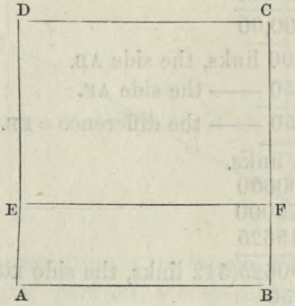
PROBLEM XI.

To part from a square or rectangle any proposed quantity of land by a line parallel to one of its sides.

RULE.—Divide the proposed area by the side upon which it is to be parted off, and the quotient will be the length of the other side of the figure required.

Examples.

1. From the square ABCD, containing 6A. 1R. 26P., part off 3A. by a line parallel to AB.

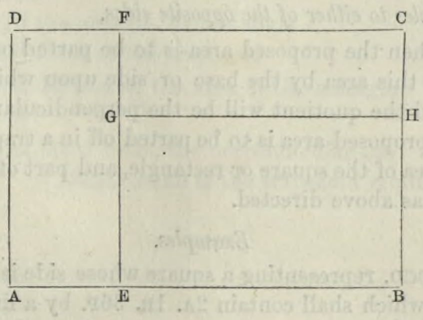


sq. links.
6A. = 600000
1R. = 25000
24P. = 15000

640000(800 links, the side of the square.
64
. . 0000

Then, $\frac{300000}{800} = 375$ links, the side AE or BF required.

2. From the rectangle ABCD, containing 8A. 1R. 24P., part off 2A. 1R. 32P. by a line parallel to AD = 700 links. Then, from the remainder of the rectangle, part off 2A. 3R. 25P. by a line parallel to AB.



sq. links.
2A. = 200000
1R. = 25000
32P. = 20000

7,00)2450,00

.350 links, the side AE or DF.

$$\begin{array}{rl} & \text{sq. links.} \\ 8\text{A.} = & 800000 \\ 1\text{R.} = & 25000 \\ 24\text{P.} = & \underline{15000} \\ 7{,}00) & \underline{8400{,}00} \\ & 1200 \text{ links, the side AB.} \\ & 350 \text{ —— the side AE.} \\ & \overline{850} \text{ —— the difference = EB.} \end{array}$$

$$\begin{array}{rl} & \text{sq. links.} \\ 2\text{A.} = & 200000 \\ 3\text{R.} = & 75000 \\ 25\text{P.} = & \underline{15625} \\ 850) & 290625(342 \text{ links, the side EG or BH.} \\ & \underline{2550} \\ & .3562 \\ & \underline{3400} \\ & .1625 \\ & \underline{1700} \end{array}$$

3. Part off 6A. 3R. 12P. from a rectangle containing 15A. by a line parallel to the longer side, the shorter being 1000 links.

Ans. The longer side of the given rectangle is 1500, and the shorter side of the rectangle required is 455 links.

PROBLEM XII.

To part from a square or rectangle any proposed quantity of land, either in a right-angled triangle or trapezoid, by a line drawn from any of the angles to either of the opposite sides.

RULE.—When the proposed area is to be parted off in a triangle, divide double this area by the base or side upon which it is to be parted off, and the quotient will be the perpendicular.

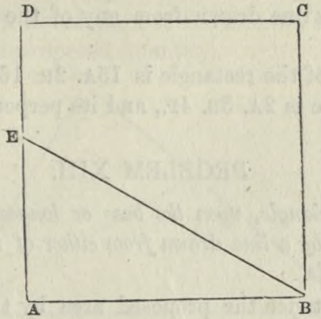
When the proposed area is to be parted off in a trapezoid, subtract it from the area of the square or rectangle, and part off the remainder in a triangle, as above directed.

Examples.

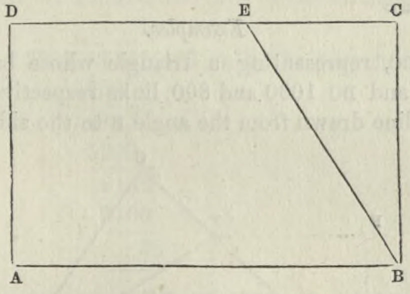
1. From ABCD, representing a square whose side is 900 links, part off a triangle which shall contain 2A. 1R. 36P. by a line drawn from the angle B to the side AD.

$$\begin{array}{rl} 2\text{A. } 1\text{R. } 36\text{P.} = & 247500 \text{ square links.} \\ & \underline{\qquad\qquad 2} \\ 9{,}00) & \underline{4950{,}00} \\ & 550 \text{ links, the perpendicular AE.} \end{array}$$

Hence ABE is the triangle required.



2. From ABCD, representing a rectangle whose length is 1265 and breadth 758 links, part off a trapezoid which shall contain 7A. 3R. 24P. by a line drawn from the angle B to the side CD.




```
sq. links
958870 the area of the rectangle
790000 ditto of the trapezoid
------
168870 difference, the area of the triangle
     2
------
758)337740(445·5 links, the perpendicular CE.
    3032       Hence ABED is the trapezoid required
    ----
    .3454
    3032
    ----
    .4220
    3790
    ----
     ·4300
     3790
     ----
     .510
```

3. From a rectangular field, whose length is 1560 and breadth 1000 links, it is required to part off a trapezoid which shall contain

12A. 3R. 12P. by a line drawn from any of the angles to the longer opposite side.

Ans. The area of the rectangle is 15A. 2R. 16P.; consequently the area of the triangle is 2A. 3R. 4P., and its perpendicular 555 links.

PROBLEM XIII.

To part from a triangle, upon the base or longest side, any proposed quantity of land by a line drawn from either of the angles at the base to the opposite side.

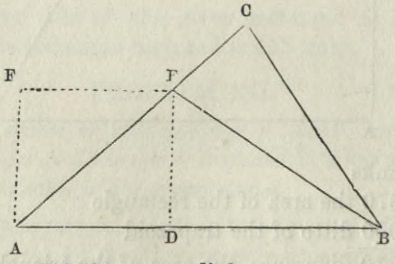
RULE.—Divide twice the proposed area by the base upon which it is to be parted off, and the quotient will be the perpendicular.

Or, if the proposed area be divided by half the base, the quotient will be the perpendicular.

NOTE.—A parallel ruler may be used with advantage in this and several of the following problems.

Examples.

1. From ABC, representing a triangle whose base AB is 1200, and sides AC and BC 1000 and 800 links respectively, part off 2A. 2R. 24P. by a line drawn from the angle B to the side AC.



2A. 2R. 24P. = 265000 square links

$$\begin{array}{r} 265000 \\ 2 \\ \hline 12{,}00)5300{,}00 \\ \hline 441{\cdot}6 \end{array}$$

441·6 links the perpendicular DE.

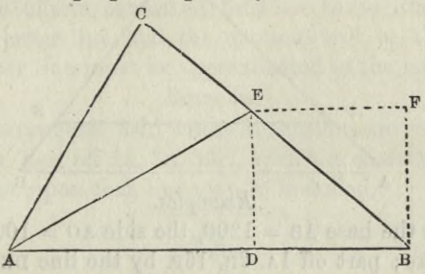
At A erect the perpendicular AF, which make = 441·6 links; then draw FE parallel to AB, and it will intersect the side AC in the point to which the division-fence BE must be made.

Or, by the plotting scale, erect the perpendicular DE = 441·6 links, which will determine the point E.

By the scale you will find AE = 664 links; measure, therefore, in the field, 664 links from A to E; stake out the line BE, and ABE will be the triangle required.

2. From ABC, representing a triangle whose base AB is 1300, and sides BC and AC 1100 and 900 links respectively, part off 1A. 3R. 36P.

by a line drawn from the angle A to the side BC, so that the triangle AEC may contain the proposed quantity.



From the three sides, by Note 4, Part IV., the area of the given
triangle is found = 488076 square links
and 1A. 0R. 36P. = 197500 square links
The difference = 290576, the area of the triangle ABE

$$\begin{array}{r}
290576 \\
2 \\
\hline
1300)581152(447 \text{ links, the perpendicular DE.} \\
5200 \\
\hline
.6115 \\
5200 \\
\hline
.9152 \\
9100 \\
\hline
..52 \\
\hline
\end{array}$$

By the mode described in the last example, determine the point E, which you will find at the distance of 658 links from the angle B; measure this distance in the field from B to E, and proceed as before.

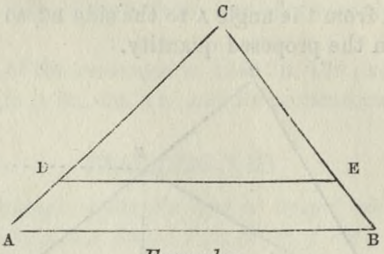
3. From a triangular field whose sides are 1500, 1200, and 1000 links respectively, part off 3A. 2R. 16P. by a fence made from the greater angle at the base to the opposite side.

Ans. The perpendicular of the triangle required is found to be 480 links; and it rises upon the base at the distance of 537 links from the less angle.

PROBLEM XIV.

To part from a triangle any proposed quantity of land by a line parallel to any one of its sides.

RULE.—The areas of similar triangles are to one another in the duplicate ratio of their homologous sides; hence, as the area of the triangle ABC is to the square of the side AC, or BC, so is the area of the triangle DEC to the square of the side DC, or EC. (See Theo. XIII. Part I.)



Examples.

1. Suppose the base AB = 1200, the side AC = 1000, and the side BC = 800 links; part off 1A. 2R. 16P. by the line DE parallel to AB.

From the three sides, by Note 4, Part IV., we find the area of the triangle.

ABC = 396863 square links
and 1A. 2R. 16P. = 160000 square links
The difference = 236863, the area of the triangle DEC.

Then, as 396863 : 1000 × 1000 : : 236863 : 596838·20; and $\sqrt{596838{\cdot}20}$ = 772·5 links = DC; hence 1000 − 772·5 = 227·5 links = AD. Again, as 396863 : 800 × 800 : : 236863 : 381976·45; and $\sqrt{381976{\cdot}45}$ = 618 links = EC; then 800 − 618 = 182 links = BE.

Measure therefore in the field 227·5 links from A to D; and from B to E measure 182 links; stake out the line DE, and the work will be completed.

2. From a triangular field, whose sides are 1800, 1500, and 1200 links respectively, part off 3A. 2R. 32P. by a line parallel to the shortest side.

Ans. The area of the given triangle is 892941 square links; the area of the triangle made by the line of division is 522941 square links; and one of its sides from the angle opposite the line of division to the commencement of that line, is 1147·9, and the other 1377·4 links.

PROBLEM XV.

To part from a rectangle or triangle any proposed quantity of land upon a line on which there are offsets, when the area of those offsets is to be considered as part of the portion to be parted off.

RULE.—Find the area of the offsets, which subtract from the portion to be parted off, and then proceed with the remainder as directed in the preceding problems.

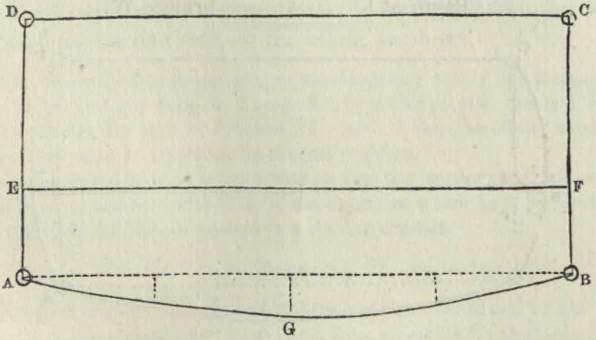
But, in a rectangle, when there are offsets on one or both of the lines, adjoining that upon which the given quantity is to be parted off, reject these offsets, and proceed as before directed.

Then, having found the distance at which the line of division must be from that upon which the given quantity is to be parted off, find the area of the offsets contained between those lines, which area divide by the latter line, and the quotient will be the distance by which the former line must be approximated to the latter.

Examples.

1. From a rectangular field whose dimensions are contained in the following notes, part off 2A. 3R. 32P., upon the chain-line AB, so that the offsets taken upon that line may be included.

	DA	
	560	
	L. off D	
	CD	
	1200	
	1000	
	L. off C	
	BC	
	560	
	L. off B	
	AB	
	1200	0
	1000	
	900	60
	600	80
	300	50
	000	0
Begin at	A	range E

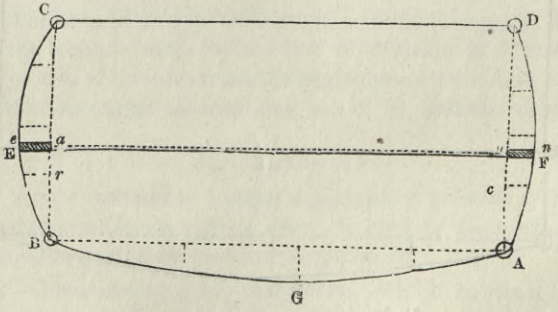


sq. links
2A. 3R. 32P. = 295000
57000 the area of the offsets
12,00)2380,00 the difference
198·4 links = AE, or BF.

Hence the irregular figure AGBFE contains 2A. 3R. 32P.

2. From a rectangular field whose dimensions are contained in the following notes, part off 2A. 2R. 8P. by a line parallel to the chain-line AB, so that the offsets taken upon this line, and also those upon the two adjoining lines contained between the chain-line AB and the line of division may be included.

	DA	
0	500	
40	350	
55	250	
45	150	
0	000	
	R. off D	
	CD	
	1000	
	R. off C	
	BC	
0	500	
40	400	
60	250	
45	150	
0	000	
	R. off B	
	AB	
0	1000	
50	700	
70	450	
40	200	
0	000	
Begin at	A	range W.



sq. links
2A. 2R. 8P. = 255000
40250 the area of the offsets taken on AB
1,000)214,750 the difference
214·750 links = Ba or Am, which we may call 215 links. Now, 215 − 150 = 65 = ra = cm; and, by the scale, ae

is found to measure 58, and mn 53 links; hence the area of the offset Bae + the area of the offset $Amn = 13282$, which, divided by 1000, gives 13 links, the distance by which the line en must be approximated to AB. Consequently, EF is the true line of division; and the regular figure AGBEF contains 2A. 2R. 8P. minus the two shaded offsets.

PROBLEM XVI.

To part from a trapezium, or any irregular polygon whatever, any proposed quantity of land by a line drawn parallel to any of the sides, or by a line drawn from any of the angles, or from any assigned point in one of the sides to any of the opposite sides.

RULE 1. Having laid down the whole figure, draw a guess-line in the direction required, parting off, as nearly as can be judged, the proposed quantity; after which, by the scale, measure, with the greatest accuracy, the guess-line, and also the quantity thus parted off.

Then, if the guess-line or line of division be drawn from an angle, or from any assigned point in a side, divide the difference between the proposed quantity and the quantity parted off, by half the guess-line, and the quotient will be the perpendicular to be set off, on one side, or the other, of the guess-line, accordingly as the quantity parted off is more or less than the quantity proposed. To the end of this perpendicular, from the point assigned, draw a new line of division, and it will part off the quantity required.

2. But if the guess-line be drawn parallel to any of the sides, divide the difference before mentioned by the whole guess-line, and the quotient will be the perpendicular to be set off from each end of the guess-line, on one side, or the other, as above.

NOTE 1. When from a trapezium, approaching very nearly to a rectangle, it is required to part off any number of acres, &c., by a line parallel to one of its sides, it may be done as directed in Problem XI.; and if there be offsets upon any of the lines, they must be treated as in the last problem.

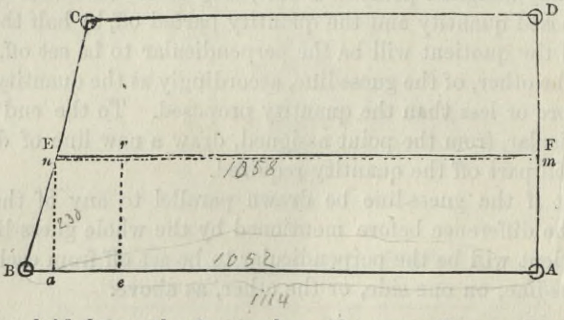
2. In using guess-lines, it is not necessary that the learner should draw them so as to coincide in measure with those of the examples which he is performing. It will be sufficient for him to proceed in a similar manner.

Examples.

1. From a trapezium whose dimensions are contained in the following notes, part off 2A. 2R. 24P. by a line parallel to the side AB.

BD	
1249	diag.
1000	
return to B	

	AC	
	1112	diag.
	1000	
	R. off A	
	DA	
	550	
	R. off D	
	CD	
	979	
	R. off C	
	BC	
	557	
	R. off B	
	AB	
	1114	
	1000	
Begin at	A	range W.



Having laid down the figure, draw the guess-line *mn* parallel to AB; and from *n* let fall the perpendicular *an*; then, suppose *mn* = 1058 links, *an* will be = 230, and *a*A = 1052 links; therefore, B*a* = 1114 − 1052 = 62 links.

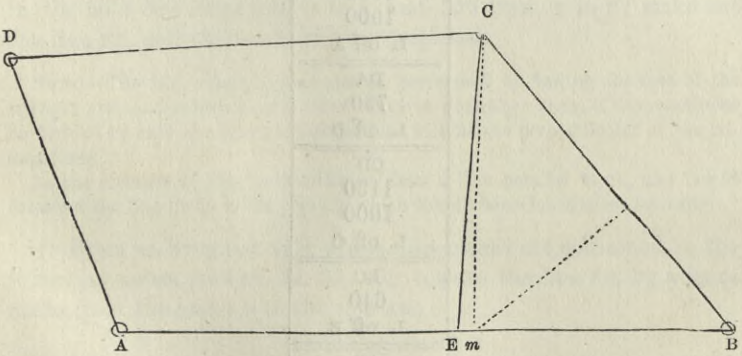
	sq. links	
Then, 1055 × 230 =	242650	the area of the trapezoid A*anm*
and 230 × 31 =	7130	the area of the triangle B*an*
The sum =	249780	the area of the trapezium AB*nm*
2A. 2R. 24P. =	265000	
	15220	the difference between the quantity

proposed and the quantity parted off by the guess-line; which, divided by 1058, gives 14·4 links to be set off perpendicularly from *m* and *n* toward D and C. Hence, EF is the true line of division and the trapezium ABEF contains 2A. 2R. 24P.

As A is very nearly a right angle, measure in the field 230 + 14·4 = 244·4 links from A to F. Then, upon any part of the line AB (toward B), as at *e*, erect the perpendicular *er*, which make = 244·4 links; stake out the line E*r*F, and the work will be completed.

2. From a trapezium whose dimensions are contained in the following notes, part off, in a triangle, 1A. 3R. 12P. by a line drawn from the angle C to the side AB.

	BD	
	1634	diag.
	1000	
	return to B	
	AC	
	1010	diag.
	L. off A	
	DA	
	620	
	L. off D	
	CD	
	1010	
	L. off C	
	BC	
	820	
	L. off B	
	AB	
	1300	
	1000	
Begin at	A	range E.



Having laid down the figure, draw the guess-line C*m*, which suppose = 638 links. From *m* let fall the perpendicular *ma*, which will be = 417 links.

sq. links

Then, $410 \times 417 = 170970$ the area of the triangle BCm

1A. 3R. 12P. = 182500

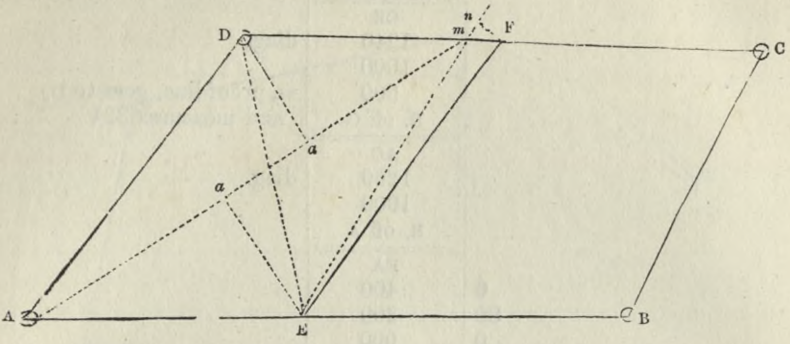
11530 the difference between the quantity proposed and the quantity parted off by the guess-line, which divided by 319 (half the guess-line) gives 36 links, to be set off from m towards A. Hence EC is the true line of division; and the triangle BCE contains 1A. 3R. 12P.

Also, AE is found = 731 links: measure, therefore, in the field 731 links from A to E; stake out the line EC, and the work will be completed.

NOTE.—The rules given in this problem for parting off land from irregular figures are generally adopted by practical surveyors, because they may be applied to any irregular figure whatever. Land, however, may sometimes be parted off more directly: for instance, the foregoing example may be performed by the mode followed in Problem XIII., *i.e.*, if the given quantity, in square links, be divided by half the line BC, the quotient will be the perpendicular of the triangle BCE; then, at the distance of this perpendicular, a line drawn parallel to BC will intersect the line AB in E, the point to which the division-fence must be made.

3. From a field whose dimensions are contained in the following notes, part off 3A. 2R. 16P. toward AD, by a fence made from the side AB to the side CD, so that the fence may commence at the distance of 600 links from A.

	BD 1050 return to B	diag.
	AC 1708 1000 L. off A	diag.
	DA 790 L. off D	
	CD 1130 1000 L. off C	
	BC 640 L. off B	
Begin at	AB 1320 1000 A	range E.



Having constructed the figure, set off 600 links from A to E, and draw the guess-line Em, which suppose = 702 links; the diagonal Am will be = 1132, the perpendicular Da = 278, and the perpendicular Ea = 318 links. Hence, the area of trapezium ADmE is found = 337336 square links; but the quantity proposed (360000 square links) exceeds the quantity parted off by 22664 square links; this divided by 351 (half the guess-line) gives 64·5 links, to be set off from the line Em, perpendicularly towards BC.

Now, continue the line Em, and upon it erect the perpendicular nF = 64.5 links. The line FE will be the true line of division; and the trapezium ADFE contains 3A. 2R. 16P.

If it had been required to set off the perpendicular on the other side of the line Em, you must still have erected it so that its end might have touched the line CD.

Now, by the scale DF is found = 553 links. Measure, therefore, in the field 600 links from A to E, and 553 from D to F; stake out the line FE, and the work will be completed.

NOTE.—The last example may also be performed by finding the area of the triangle ADE, and subtracting it from the given quantity; then, if the remainder be divided by half the line DE, the quotient will be the perpendicular of the triangle DEF.

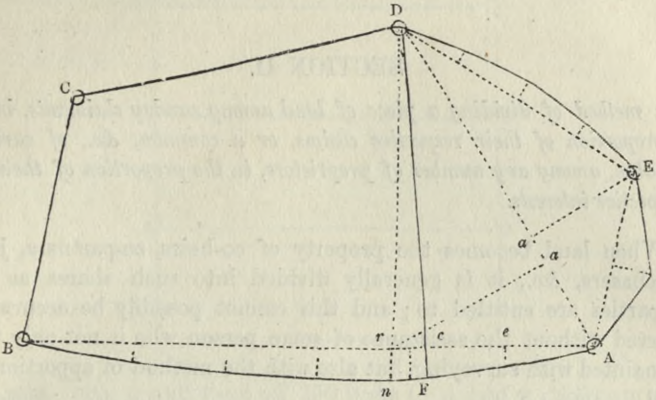
At the distance of this perpendicular, draw a line parallel to DE, and it will intersect the line CD in F, the point to which the division fence must be made.

4. From an irregular field whose dimensions are contained in the following notes, part off 2A. 3R. 20P. toward the line AE, by a fence made from the angle D to the side AB.

EB	
1398	diag.
1000	
R. off E	

	CE	
	1240	diag.
	1000	
	500	*m*, proof-line, goes to D, and measures 324
	R. off C	
	AC	
	1260	diag.
	1000	
	R. off A	
	EA	
0	400	
80	200	
0	000	
	R. off E	
	DE	
0	600	
25	450	
35	300	
20	150	
0	000	
	R. off D	
	CD	
	740	
	R. off C	
	BC	
	550	
	R. off B	
	AB	
0	1250	
35	1000	
50	800	
60	600	
50	400	
30	200	
0	000	
Begin at	A	range W.

Having laid down the figure, draw the guess-line D*n*, which suppose = 766 links; then the diagonal AD will be = 824, the perpendicular E*a* = 278, and the perpendicular *ra* = 372 links; *re* also will be = 228, and *rn* = 52 links.



sq. links
267800 the area of the trapezium ArDE

12000	DE	the area of the offsets taken on the different lines
16000	EA	
12348	Ar	

308148 the area of the irregular figure AnDE

2A. 3R. 20P. = 287500

20648 the difference between the quantity proposed and the quantity parted off by the guess-line, which divided by 383 (half the guess-line) gives 54 links, to be set off from *n* toward A. Hence DF is the true line of division; and the irregular figure AFDE contains 2A. 3R. 20P.

Now, by the scale, A*c* is found = 377 links. Measure, therefore, in the field 377 links from A to *c*; stake out the line D*c*F and the work will be completed.

NOTE 1. If the area of the irregular figure ADE be subtracted from the given quantity, and the remainder divided by half the line AD, the quotient will be the perpendicular of the triangle ADF; the side AB being nearly straight from A to F.

Now, at the distance of this perpendicular, draw a line parallel to AD, and it will intersect the side AB in F, the point to which the division-fence must be made.

2. It is not absolutely necessary to survey and plan a whole field in order to part a portion from it, as the guess-line and portion parted off may be measured in the field; but, in my opinion, the former, in general, is a more eligible method than the latter, as you have a better opportunity of proving your work.

SECTION II.

The method of dividing a piece of land among sundry claimants, in the proportion of their respective claims, or a common, &c., of variable value, among any number of proprietors, in the proportion of their respective interests.

When land becomes the property of co-heirs, co-partners, joint purchasers, &c., it is generally divided into such shares as the co-parties are entitled to; and this cannot possibly be accurately effected without the assistance of some person who is not only well acquainted with surveying, but also with the method of apportioning land.

In this process an error is evidently much more material than one committed in surveying.—When a field, &c., is to be divided into any number of parts, equal or unequal, it is necessary, first, to ascertain its dimensions, and next to inquire of the parties concerned in what part of the property in question they wish their respective shares to lie.

PROBLEM I.

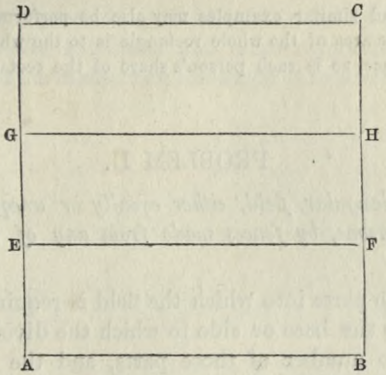
To divide a square or rectangle, either equally or unequally, among any number of persons, by lines parallel to one of its sides.

RULE.—If the parts into which the field is required to be divided be equal, divide the side which will be cut by the division-fences by the number of those parts, and the quotient will be the distance at which the division-fences must be placed from each other, and from the outsides to which they are parallel. But, if the parts be unequal, you must then part off each person's share as directed in Section I. Problem XI.

Examples.

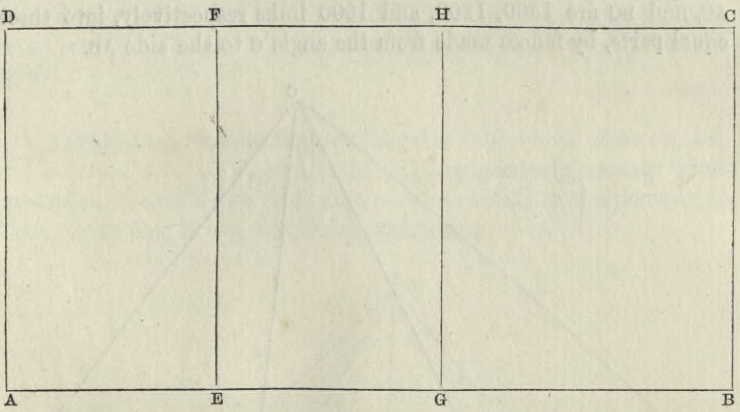
1. Divide the square ABCD containing 5A. 2R. 20P. into three equal parts by fences parallel to the side AB.

Here 5A. 2R. 20P. $= 562500$ square links; and $\sqrt{562500} = 750$ links, the side of the square. This, divided by 3, the number of parts, gives 250 links, the distance at which the first division-fence must be placed from AB, &c. From A and B, therefore, set off 250 links to E and F; join EF, and the rectangle ABFE will be one of the parts required.



Again, from E and F set off 250 links to G and H; join GH, and the rectangles EFGH and GHCD will be the two other parts required.

2. Divide ABCD, representing a rectangular field whose length is 1500 and breadth 800 links, among three men, A, B, and C, by fences parallel to the side AD, so that A may have 3A., B 4A., and C the remainder.



Here 3A. = 300000 square links, which divided by 800 gives 375 links = AE or DF; hence the rectangle AEFD contains A's share.

Again, 4A. = 400000 square links, which divided by 800 gives 500 links = EG or FH; hence the rectangle EGHF contains B's share.

Now, the rectangle ABCD is found to contain 12A., consequently, the rectangle GBCH, containing 5A., is C's share.

NOTE.—This and similar examples may also be performed by the following proportion: As the area of the whole rectangle is to the whole base or side cut by the division-fence, so is each person's share of the rectangle to his share of the base.

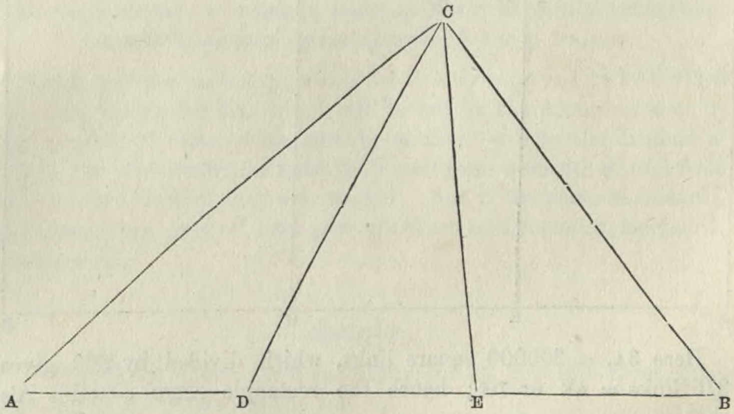
PROBLEM II.

To divide a triangular field, either equally or unequally, among any number of persons, by fences made from any of its angles to the opposite side.

RULE.—If the parts into which the field is required to be divided be equal, divide the base or side to which the division-fences are to be made by the number of those parts, and the quotient will be each person's share of the base. But, if the parts be unequal, say, as the area of the whole triangle is to the whole base, so is each person's share of the triangle to his share of the base. (See Simpson's Geom. IV. 7, and Euclid VI. 1.)

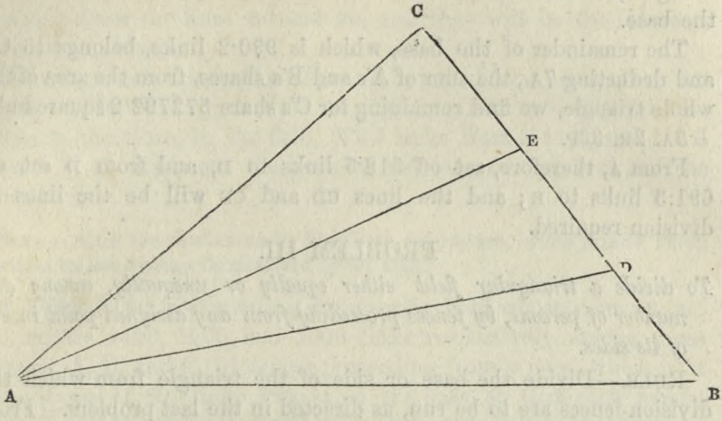
Examples.

1. Divide ABC, representing a triangular field, whose sides AB, AC, and BC are 1500, 1200, and 1000 links respectively, into three equal parts, by fences made from the angle C to the side AB.



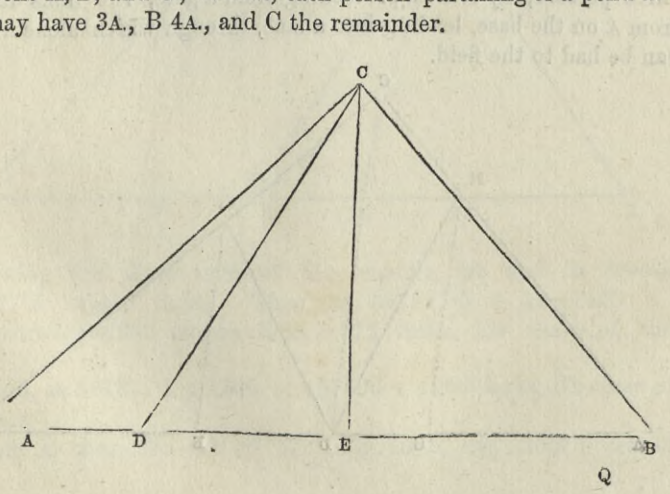
Here AB = 1500 links, which divided by 3 (the number of parts) gives 500 links, each person's share of the base. From A, therefore, set off 500 links to D, and from D 500 links to E; draw the lines CD and CE, to represent the division-fences; and the triangles ADC, DEC, and EBC, are the three equal parts required.

2. Divide ABC, representing a triangular field whose sides AB, AC, and BC are 1450, 1150, and 960 links respectively, into three equal parts, by fences made from the angle A to the side BC.



Here BC = 960 links, which divided by 3 (the number of parts) gives 320 links, each person's share of the side BC. From B, therefore, set off 320 links to D, and from D set off the same distance to E; and the lines AD and AE will be the lines of division required.

3. Divide ABC, representing a triangular field whose sides AB, AC, and BC are 2200, 1700, and 1500 links respectively, among three persons A, B, and C; so that, each person partaking of a pond at C, A may have 3A., B 4A., and C the remainder.



Having the three sides of the triangle, we find its area = 1272792·2 square links; then, as 1272792·2 : AB = 2200 : : A's share = 300000 square links : 518·5 links, A's share of the base.

Again, as 1272792·2 : 2200 : : 400000 : 691·3 links B's share of the base.

The remainder of the base, which is 990·2 links, belongs to C; and deducting 7A., the sum of A's and B's shares, from the area of the whole triangle, we find remaining for C's share 572792·2 square links = 5A. 2R. 36P.

From A, therefore, set off 518·5 links to D, and from D set off 691·3 links to E; and the lines CD and CE will be the lines of division required.

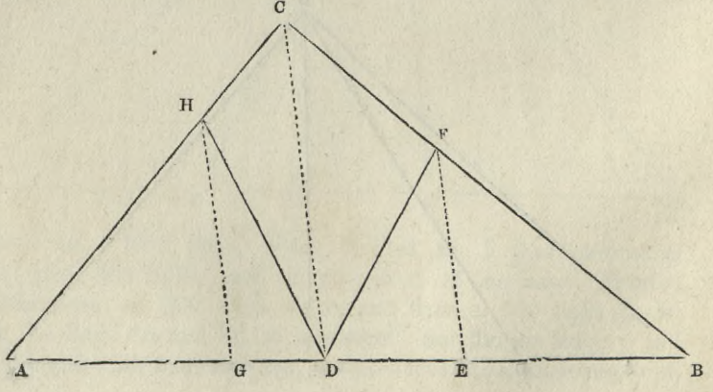
PROBLEM III.

To divide a triangular field either equally or unequally, among any number of persons, by fences proceeding from any assigned point in one of its sides.

RULE.—Divide the base or side of the triangle from which the division-fences are to be run, as directed in the last problem. From the assigned point, draw a line to the opposite angle; and parallel to this line draw a line from each point of division on the base, until it intersects the opposite side. From these points of intersection draw lines to the point assigned, and they will be the lines of division required. (See Dr Hutton's Course of Mathematics, vol. iii. chap. 7, prob. 2.)

Examples.

1. Divide ABC, representing a triangular field, whose sides AB, BC, and AC are 1500, 1150, and 950 links respectively, equally among three persons, by fences proceeding from a gate, 700 links distant from A on the base, leading into a lane, through which alone a road can be had to the field.

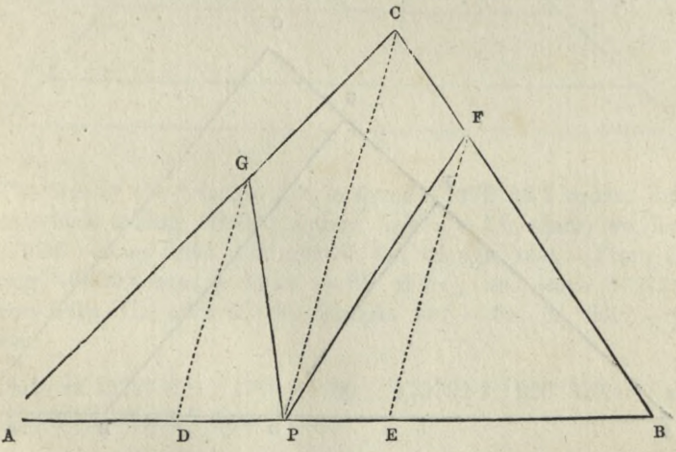


By the question AB = 1500 links, divided by 3, gives 500 links, each person's share of the base. From A, therefore, set off 500 links to D, and from D set off the same extent to E. Then from the gate G draw the line GC; and, parallel to it, the lines DH and EF. From H and F draw the lines HG and FG, and they will be the fences of division required.

Now, by similar triangles (Theo. XI., part I.), as AG : AC :: AD : AH = 678·5 links; and as BG : BC :: BE : BF = 718·7 links. Measure, therefore, in the field, 678·5 links from A to H, and 718·7 from B to F; stake out the lines HG and FG, and the work will be completed.

NOTE.—After the points H and F have been determined, by the parallel ruler, the lines AH and BF may be measured by the scale.

2. Divide ABC, representing a triangular field whose sides AB, AC, and BC are 1400, 1200, and 1000 links respectively, among three persons, A, B, and C, by fences proceeding from a pond which is at the distance of 600 links from A on the base, so that each person partaking of the pond, A may have 1A. 2R. 10P., B 1A. 3R. 20P., and C the remainder.



Having the three sides of the triangle, we find its area = 587877·5 square links. Then, as 587877·5 : AB = 1400 : A's share = 156250 square links : 372 links, his share of the base.

Again, as 587877·5 : 1400 :: 187500 : 446·5 links, B's share of the base.

From A, therefore, set off 372 links to D, and from D set off

446·5 to E. Then from the pond P draw the line PC, and parallel to it the lines DG and EF. From G and F draw the lines GP and FP; and the triangle APG will contain A's share; the trapezium PGCF, B's share, and the triangle BPF, C's share = 244127·5 square links = 2A. 1R. 30P.

By similar triangles we find AG = 744, and BF = 726·8 links; proceed, therefore, in the field, as directed in the last example.

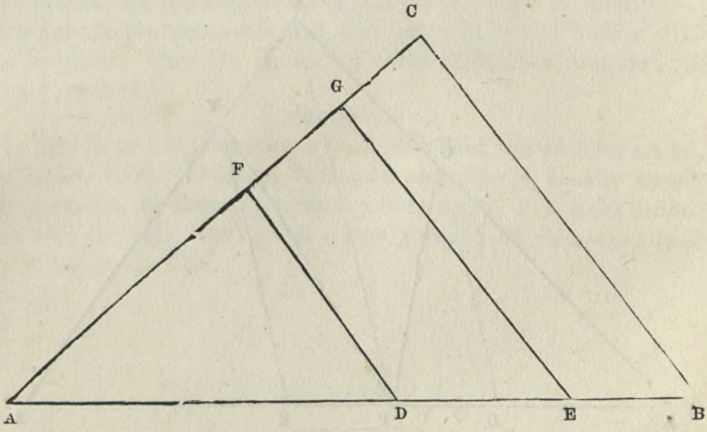
PROBLEM IV.

To divide a triangular field, either equally or unequally, among any number of persons, by fences made parallel to one of its sides.

RULE.—By the rule given in Section I., Problem XIV., part off the first person's share; proceed with the remainder of the triangle, and the next person's share in the same manner, and thus continue till the whole triangle is divided.

Examples.

1. Divide ABC, representing a triangular field whose sides AB, AC, and BC, are 1500, 1200, and 1000 links respectively, into three equal parts by fences made parallel to the side BC.



The area of the given triangle is found = 598116·9 square links, which divided by 3 gives 199372·3 square links for each person's share; consequently, 398744·6 square links is the area of the triangle AEG.

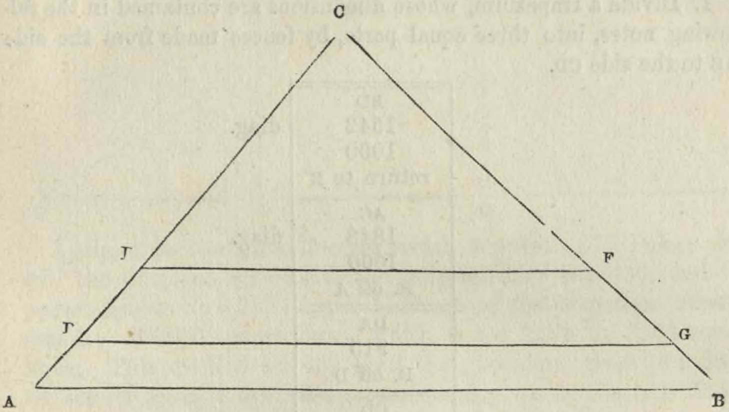
Then, as 598116·9 : 1500 × 1500 : : 398744·6 : 1500000; and $\sqrt{1500000} = 1224{\cdot}7$ links = AE.

And as 598116·9 : 1200 × 1200 : : 398744·6 : 960000; and $\sqrt{960000} = 979{\cdot}8$ links = AG.

Again, as 398744·6 : 1224·7 × 1224·7 :: 199372·3 : 749945·04; and $\sqrt{749945{\cdot}04}$ = 865·9 links = AD.

And as 398744·6 : 979·8 × 979·8 :: 199372·3 : 480004·02; and $\sqrt{480004{\cdot}02}$ = 692·8 links = AF. Hence the triangle ABC is divided into three equal parts, as required.

2. Divide ABC, representing a triangular field whose sides AB, BC, and AC are 2200, 1700, and 1500 links respectively, among three persons, A, B, and C, by fences made parallel to the base AB, so that A may have 3A., B 4A., and C the remainder.



The area of the triangle ABC is found = 1272792·2 square links, from which taking 300000 square links (= A's share) we leave 972792·2 square links, the area of the triangle DGC. From this taking 400000 square links (= B's share), we leave 572792·2 square links, the area of the triangle EFC = 5A. 2R. 36P. = C's share.

Then, as 1272792·2 : 1700 × 1700 :: 972792·2 : 2208820·46; and $\sqrt{2208820{\cdot}4}$ = 1486·2 links = CG.

And, as 1272792·2 : 1500 + 1500 :: 972792·2 : 1719669·91; and $\sqrt{1719669{\cdot}91}$ = 1311·3 links = CD.

Again, as 972792·2 : 1486·2 × 1486·2 :: 572792·2 : 1300563·40; and $\sqrt{1300563{\cdot}40}$ = 1140·4 links = CF.

And, as 972792 : 1311·3 + 1311·3 :: 572792·2 : 1012467·60; and $\sqrt{1012467{\cdot}60}$ = 1006·2 links, = CE. Hence the triangle ABC is divided into three parts, as required.

PROBLEM V.

To divide a trapezium, or an irregular polygon, equally or unequally, among any number of persons, by fences made in a given direction.

RULE.—By the rules given in Section I. Problem XVI. part off the first person's share; proceed with the remainder of the figure, and the second person's share in the same manner; and thus continue till the whole figure is divided.

Examples.

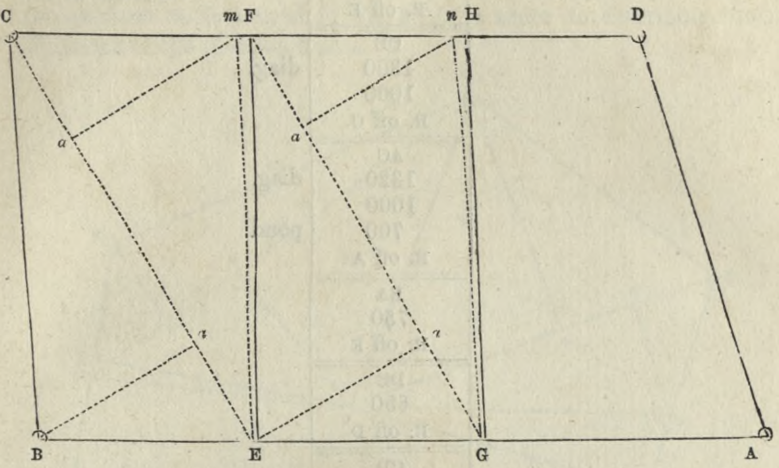
1. Divide a trapezium, whose dimensions are contained in the following notes, into three equal parts, by fences made from the side AB to the side CD.

	BD	
	1542	diag.
	1000	
	return to B	
	AC	
	1848	diag.
	1000	
	R. off A	
	DA	
	915	
	R. off D	
	CD	
	1347	
	1000	
	R. off C	
	BC	
	885	
	R. off B	
	AB	
	1547	
	1000	
Begin at	A	range W.

The area of the triangle ABC is found = 681942, and the area of the triangle CDA = 585949 square links; consequently the area of the trapezium ABCD = 1267891 square links, which divided by 3 gives 422630 square links for each person's share.

Now draw the guess-line E*m*, which suppose = 880 links; then the diagonal EC will be found = 1028, the perpendicular B*a* = 387 and the perpendicular *ma* = 424 links; hence the area of the trapezium BC*m*E is found = 416854 square links, which is too little by

5776 square links. This divided by 440 (half the guess-line) gives 13 links, to be set off from m toward D; consequently EF is the true line of division.



Again, draw the guess-line Gn, which suppose $= 878$ links; then will the diagonal GF $= 1017$, the perpendicular E$a = 430$, and the perpendicular N$a = 385$: hence the area of the trapezium EFnG is found $= 414427$ square links, which is too little by 3203 square links. This divided by 439 (half the guess-line) gives 19 links to be set off from n toward D; consequently GH is the true line of division; and the trapezium ABCD is divided into three equal parts, as required.

Now, by the scale, we find BE $= 450$, EG $= 500$, CF $= 508$, and FH $= 468$ links, which distances must be measured in the field, in order to determine the situation of the division-fences.

NOTE.—If we subtract the area of the triangle BCE from the quantity to which each person is entitled, and divide the remainder by half the line CE, the quotient will be the perpendicular of the triangle CEF. By drawing a line parallel to CE, at the distance of this perpendicular, the point F may be determined.

In a similar manner may be parted off the trapezium EFHG.

2. Divide a field, whose dimensions are contained in the following notes, among three persons, A, B, and C, so that each partaking of a pond at P, A may have 3A., B 4A., and C the remainder.

BD	
1447	diag.
1000	
R. off B	

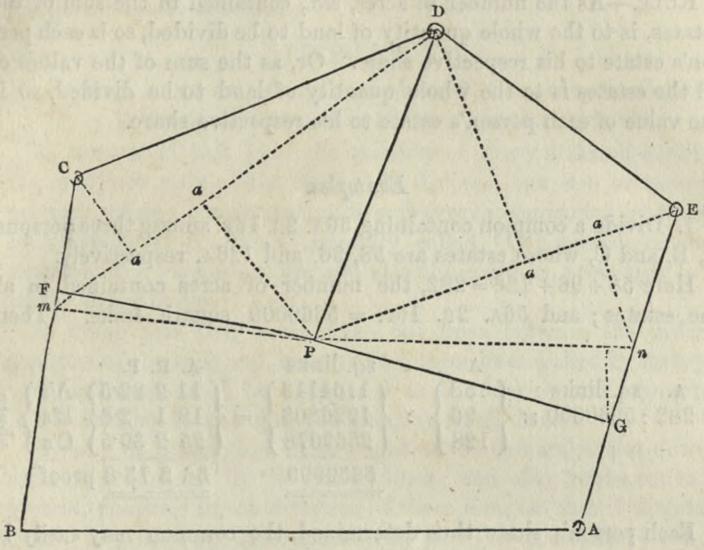
	EB	
	1603	diag.
	1000	
	R. off E	
	CE	
	1300	diag.
	1000	
	R. off C	
	AC	
	1320	diag.
	1000	
	700	pond
	R. off A	
	EA	
	750	
	R. off E	
	DE	
	650	
	R. off D	
	CD	
	850	
	R. off C	
	BC	
	800	
	R. off B	
	AB	
	1200	
	1000	
Begin at	A	range W.

From the pond P draw the line PD and also the guess-line P*m*, which suppose = 558 links; then will the diagonal D*m* be = 1025, the perpendicular P*a* = 400, and C*a* = 195 links; hence the area of the trapezium P*m*CD is found = 304937 square links, which exceeds A's share by 4937 square links. This divided by 279 (half the guess-line) gives 17·7 links, to be set off from *m* toward C; consequently PF is the true line of division, and the trapezium PFCD contains A's share.

Again, draw the guess-line P*n*, which suppose = 696 links, then the diagonal PE will be = 848, the perpendicular *na* = 247 and D*a* = 552 links; hence the area of the trapezium P*n*ED is found = 338776 square links, which is less than B's share by 61224 square links. This divided by 348 (half the guess-line) gives 176 links to be set off from the line P*n* perpendicularly toward A; consequently, PG is the true line of division, and the trapezium PGED ontains B's share.

Now the irregular polygon ABFPG contains C's share, which will be found = 4A. 3R. 19P.

By the scale we find AG = 252, and BF = 545 links, which distances must be measured in the field, in order to determine the situations of the division-fences.



NOTE 1. The foregoing example may also be performed by subtracting the area of the triangle CDP from A's share, and then laying out the remainder in the triangle CFP, as before directed.

In a similar manner may be parted off B's share.

2. The division of the last, or any other figure, may be proved by finding the area of the whole figure, which, if equal, or nearly equal, to the sum of the areas of the parts into which it has been divided, demonstrates the work to be right.

PROBLEM VI.

To divide a common, or any quantity of land, of uniform value, among any number of proprietors, in the proportion of their respective interests.

In this case the land to be divided must first be surveyed, and next the estate of each proprietor, if its quantity be unknown. Then, if it be required to make the division according to the value of each person's estate, there must be proper persons appointed to value them, which, in this problem, we will suppose, may be done at so much per acre, uniformly, throughout each estate.

NOTE 1. When the land to be divided is of uniform value nothing more is wanted than its quantity.

2. It is immaterial whether the land be valued at 5*s.* or 5*l.* per acre, if the same proportion, according to the quality of the land, &c., be observed in valuing each person's estate.

To determine each person's share.

RULE.—As the number of acres, &c., contained in the sum of the estates, is to the whole quantity of land to be divided, so is each person's estate to his respective share. Or, as the sum of the values of all the estates is to the whole quantity of land to be divided, so is the value of each person's estate to his respective share.

Examples.

1. Divide a common containing 56A. 2R. 16P. among three persons, A, B, and C, whose estates are 58, 96, and 128A. respectively.

Here $58 + 96 + 128 = 282$, the number of acres contained in all the estates; and 56A. 2R. 16P. = 5660000 square links. Then,

$$\begin{array}{l} \text{A.} \quad \text{sq. links} \\ \text{as } 282 : 5660000 :: \end{array} \left\{\begin{array}{r} \text{A} \\ 58 \\ 96 \\ 128 \end{array}\right\} : \left\{\begin{array}{r} \text{sq. links} \\ 1164113 \\ 1926808 \\ 2569078 \\ \hline 5659999 \end{array}\right\} = \left\{\begin{array}{rrr} \text{A.} & \text{R.} & \text{P.} \\ 11 & 2 & 22{\cdot}5 \\ 19 & 1 & 2{\cdot}8 \\ 25 & 2 & 30{\cdot}5 \\ \hline 56 & 2 & 15{\cdot}8 \end{array}\right\} \begin{array}{l} \\ \text{A's} \\ \text{B's} \\ \text{C's} \\ \text{proof} \end{array} \text{share}$$

Each person's share thus determined, the common may easily be divided by the methods already described.

2. Three gentlemen, A, B, and C, have each an estate consisting of 300A.; divide among them, according to the values of their estates, 75A. 3R. 32P.; A's estate being valued at 25*s.*, B's at 32*s.*, and C's at 40*s.* per acre per annum.

$$\text{Here } 300 \times \left\{\begin{array}{r} s \\ 25 \\ 32 \\ 40 \end{array}\right\} = \left\{\begin{array}{rl} s & \\ 7500 & \text{the value of A's estate} \\ 9600 & \text{ditto of B's} \\ 12000 & \text{ditto of C's} \\ \hline 29100 & \text{sum} \end{array}\right.$$

And 75A. 3R. 32P. = 7595000 square links. Then

$$\begin{array}{l} s. \quad \text{sq. links} \\ \text{as } 29100 : 7595000 :: \end{array} \left\{\begin{array}{r} s \\ 7500 \\ 9600 \\ 12000 \end{array}\right\} : \left\{\begin{array}{r} \text{sq. links} \\ 1957474 \\ 2505567 \\ 3131958 \\ \hline 7594999 \end{array}\right\} = \left\{\begin{array}{rrr} \text{A.} & \text{R.} & \text{P.} \\ 19 & 2 & 12 \\ 25 & 0 & 9 \\ 31 & 1 & 11 \\ \hline 75 & 3 & 32 \end{array}\right\} \begin{array}{l} \\ \text{A's} \\ \text{B's} \\ \text{C's} \\ \text{proof} \end{array} \text{share}$$

NOTE.—It sometimes happens that two, three, or more persons join in taking a common pasture, and agree to pay in proportion to the number of cattle with which each person depastures. In such cases, when the whole of the cattle graze an equal time, you must make use of the rule of Single Fellowship, by saying, as the whole of the cattle is to the rent of the whole pasture, so is each person's cattle to his share of the rent. But when the cattle graze an unequal time, you must then have recourse to the rule of Double Fellowship, by saying, as the sum of the products of each person's cattle and time is to the whole rent, so is each person's product to his share of the rent.

PROBLEM VII.

To divide a common, &c., of variable value, among any number of proprietors, in proportion to their respective interests.

In a work of this kind the quantity of every different quality is required, not only of the land to be divided, but also of each proprietor's estate; consequently the surveyor, accompanied by the persons appointed to value, generally called 'Commissioners,' must examine each person's estate, and also the common, previously to the survey being taken.

In doing this they must stake out lines between the different qualities of the soil; and, in surveying, these lines (called by surveyors 'quality-lines') must be considered as boundaries, and represented in the field-book, and upon the plan, by small dashes.

By way of distinction there ought to be two stakes put down at each angle formed by the quality-lines; and also marks cut in the ground, pointing in the direction of these lines, so that if the stakes should be pulled up, these marks may serve as directors.

When the survey is finished and laid down, every different quality represented upon the plan must be successively numbered, 1, 2, 3, &c. The surveyor must then require the commissioners to put the different valuations upon the land; and, in doing this, he must accompany them with the plan, in order that both he and they may know the ground corresponding with each number. Surveyors generally use letters to represent the different values of land:

Thus *a*	may	denote	1 shilling
b	,,	,,	2
c	,,	,,	3
d	,,	,,	4
e	,,	,,	5
f	,,	,,	6
g	,,	,,	7
h	,,	,,	8
i	,,	,,	9
o	,,	,,	10
s	,,	,,	20
and *x*	,,	,,	30 shillings

By putting three of these letters together, and adding their separate values, the value per acre per annum may be set down as high as sixty shillings; and, by adding more letters, it may be carried to any height required. By the use of these letters the confusion arising from a multiplicity of figures is avoided.

The land being valued, you must then proceed to find the quantity contained under each number on the plan, and also its value.

In doing this, it is unnecessary to bring the decimals into roods and perches, or to retain more of them than the three next the acres, as the operation is thus considerably simplified.

If the fourth figure in the decimals be 5, or greater, add 1 to the third: that is, if the content be 3·54585, set down 3·546.

When the content does not amount to an acre, and the number of figures is under five, add as many ciphers to the left as will complete that number: that is, if the content be ·8626, set down ·086. Then, multiply the acres and decimals, contained in each number, by the valuation per acre put upon the respective numbers, and the product will be the value in shillings and decimals. (See the note on page 54.)

MISCELLANEOUS OBSERVATIONS ON VALUING LAND.

1. Proprietors require to appoint judicious commissioners to value for an enclosure. They should not only be well acquainted with the quality of the soil, but should also be able to judge how far every part of the common is capable of being improved after it has been inclosed, or they will not be able to put a just valuation upon it.

2. In valuing, not only the quality of the land, but also its situation, must be attended to; for, if one part of the land to be divided lies in a valley (not subject to be flooded), near a proprietor's messuage, and another part upon a hill, at the distance of two or three miles, it is evident, allowing the land to be all of the same quality, that the former situation is much more desirable than the latter; because it is nearer the homestead, and consequently better situated for agricultural purposes.

3. The manner in which the climate and seasons may operate upon the produce of the ground, in consequence of its local situation, should always be taken into consideration. If one field lies towards the south, and another towards the north, and both be of the same quality, the field that faces the south is more valuable than the other, as the crops on the former will not only be brought to a

greater degree of perfection by the influence of the sun, but will also be ready for harvesting sooner, and consequently may be brought to an earlier and frequently to a better market.

4. In valuing a common for an inclosure, the improvements that may be made by fencing, draining, and cultivation, should never be overlooked. If one person should have an allotment awarded to him in the best part of the common, but where no improvement can be made, and another person's allotment, of equal value, be laid out in the worst part of the common, but where much improvement may easily be made by cultivation, it is manifest that the latter allotment will, in a few years, be more valuable than the former. Besides, as quantity is always given to compensate for any deficiency in quality, the proprietor who has his common-right laid out in the worst part of the ground, will not only receive more land than the other, but will soon be able, by a trifling expense in cultivation, to make it worth more per acre.

5. In valuing either old inclosed lands or commons, the distance of the ground from good springs of water should be regarded. In many parts of England, and particularly upon the Wolds in Yorkshire, the occupiers of land frequently suffer great inconvenience in driving their cattle a considerable distance to watering-places; and the cattle themselves are sometimes much injured in droughty summers, for want of a regular supply of wholesome water. Hence a farm that is well watered is worth more to rent than another farm of equal quantity and quality, but destitute of water.

6. The distance of farms, common-rights, &c., from market-towns is also of considerable importance; because land always increases in value as it approaches the vicinity of large towns. Besides, as the means of tillage abounds in such places, improvements may be made at less expense than where the land lies at the distance of many miles. It may also be remarked that the occupiers of the former can always find a ready market for the produce of their land, while the occupiers of the latter are under the necessity of being at a considerable expense in transporting their goods to market, and in procuring the various articles that are indispensably necessary for the use of their families.

Appellations given to certain lands.

1. Moors are large, uncultivated tracts of ground, generally overgrown with furze, broom, heath, and other small shrubs, as Rumblesmoor in Yorkshire, and Blackstone-edge, partly in Yorkshire and partly in Lancashire.

2. A Fell is a large open portion of land, generally less overrun with shrubs than a moor, as Gateshead Fell in the county of Durham.

3. A Heath is an open ground, abounding with the plant called heath, or any other shrubs, as Hounslow Heath in Middlesex.

4. Wolds are high open grounds, as the Wolds in Yorkshire and Lincolnshire.

5. Downs are fine open pasture grounds, as the Downs in Kent, Sussex, and Surrey.

6. Fens are low wet tracts of ground, as the Fens in Lincolnshire.

7 Marshes are low swampy grounds, and when adjoining the sea or the sides of rivers, they are mostly excellent pastures; as the Marshes in the counties of Durham and York, contiguous to the river Tees; those in the counties of York and Lincoln, contiguous to the river Humber; and the rich marsh of Romney, in the county of Kent, adjoining the straits of Dover.

8. Mosses are black, turfy, boggy moors, as Ashton Moss, and many others in Lancashire.

9. Forests are wild uncultivated tracts of ground, generally abounding with trees, as Sherwood Forest, in Nottinghamshire, and the New Forest, and that of East Bere, in Hampshire.

10. Ings are large open meadows, generally situated on low, level grounds. Fields and tracts of land known by the local name of "The Ings," abound in almost every county of England.

11. Holmes are hilly, fenny, or level grounds, adjoining to, or encompassed by rivulets or brooks. Many rich and fertile pasture grounds in this country are known under the local appellation of "the Holmes."

12. Open fields are unenclosed lands, generally divided into furlongs by mere forms, and occupied by different tenants.

Some furlongs are usually in corn, some in meadow, and others in pasture; and the cattle and sheep which depasture are tended by shepherds. Large tracts of land upon the Wolds in Yorkshire are cultivated in this manner.

13. A Furlong of land is used in some old books to express the eighth part of an acre; hence 20 perches, or 605 square yards, make a furlong.

The term is also used to denote any number of lands adjoining each other in open fields, and running in the same direction from one head-land to another, and known by some particular name, in order to distinguish the different parts of the field from each other.

14. Mereforms are narrow pieces of swarth, dividing lands or furlongs in open fields from each other.

15. An Ox-gang or *Ax-gate* of land is usually taken for 15 acres, being as much land as it is supposed one ox can plough in a year.

In Scotland 13 acres are denominated an Ox-gang, and in some places the term is used to denote as much land as will summer one ox.

This word is corruptly called *Osken* in Lincolnshire and some other counties.

16. A Hide of land, sometimes met with in old books, was such a quantity as might be cultivated, in the compass of a year, with one plough, having meadow and pasture sufficient to feed the cattle belonging thereto. The term was also frequently used to denote as much land as would maintain a family.

Some writers make the hide to contain 60, some 80, some 100, and others 120 acres.

Sir William Dugdale, the antiquarian, says, that a Barony, in former ages, was a certain portion of land held immediately of the king, and contained not less than 40 hides, or 3840 acres—a statement that gives 96 acres to a hide.

Directions for setting out new roads, sand-pits, quarries, watering-places, &c., and for dividing commons and waste lands into allotments.

1. Before commons and waste lands are divided and allotted, new roads must be set out upon them, in the most convenient and advantageous manner. They should, whenever it is practicable, be set out in such directions as to form right angles, or as nearly right angles as possible, at the places where they meet or intersect each other, or come in contact with ancient highways. They should not be less than thirty feet in breadth, and set out in right lines; because straight roads not only look better than crooked ones, but also occupy less ground.

2. All old roads leading over commons or waste lands about to be inclosed, may be stopped or diverted at the discretion of the commissioners; and such old roads must be surveyed and allotted as part of the common or waste lands.

3. Certain portions of commons should always be set out for sand or gravel pits, and for quarries, if the commons contain either sand, gravel, or stone. The portions of ground thus set out are considered

as public property, from which every person who receives a common-right may take materials for building houses, making fences, and repairing roads.

4. If there be any good springs of water on commons, they must either be left uninclosed for the public watering-places, or the water must be conveyed to more convenient situations, by means of drains, or channels, and troughs or reservoirs made for its reception.

5. In some places the lord of the manor claims one-twelfth, in some one-sixteenth, in others only one-twentieth, of all commons and waste lands; whatever be his claim, however, it must be set out before any other allotment, after its value has been ascertained from the quantity and value of the whole common. Besides this allotment, the lord of the manor will, of course, be entitled to his proportional share of the remainder of the waste lands, in the same manner as any other proprietor.

6. When it can be done, it is very desirable to ascertain the value of any other claim, and to set out, for the proprietor of the same, an allotment of equivalent value; thus will the whole place become free, and the occupiers of lands be exempt from what they may generally deem an unpleasant tax upon their industry, but which may, nevertheless, be as justly due to the claimants as the rent of a farm is to the landlord.

7. If the clerk's salary arise from the lands, which is the case in some places, a common-right may also be set out in lieu of it; and if another can be obtained as a small endowment for a town's school, the inhabitants will not have cause to repent, if they be judicious in the choice of a master.

8. After the roads, sand-pits, quarries, watering-places, manorial rights, &c., have been set out, the remainder of the common or waste lands must be equitably divided (quantity, quality, and situation of place being regarded), among the owners and proprietors of messuages, cottages, lands, tenements, and hereditaments situated in the township or place where the inclosure is to be made and executed.

NOTE—The first step towards inclosing wet, marshy grounds, is to have them well drained, for without this be done, every attempt at improvement will be vain.

To determine the value of each proprietor's allotment, or claim upon the common.

In doing this the value only can be used; for, if we make use of the quantity, in allotting land of different qualities, the proprietor who has his allotment in land of the best quality will obviously receive more than his just right, while those whose allotments fall in land of inferior quality lose part of their property. Hence, you must say, as the value of the whole estates is to the value of the common, or land to be divided, so is the value of each person's estate to the value of his allotment, or claim upon the common.

To set off, upon the plan, each proprietor's allotment or share of the common.

When you find that a proprietor's allotment falls in that part of the common which is of uniform quality, you may easily determine the quantity to which he is entitled, by saying, as the value put upon the number in which his allotment falls, is to 1 acre, so is the value of his claim to the quantity of land which his allotment must contain. Then set off the allotment upon the plan by some of the methods already described.

But it commonly happens that a proprietor's allotment falls in different numbers. In such a case you must draw a guess-line or lines, and measure separately, by the scale, the pieces cut off belonging to the different numbers: then multiply the different quantities by their respective values; and if the sum of the products be equal to the value of the claim in question, the guess-line or lines are right; if not, they must be altered, until they part off the exact portion. After each proprietor's allotment is set off upon the plan, if you find the quantity and value of all the allotments equal to the quantity and value of the whole common, the division is right.

Example.

Lay down a plan from the engraven field-book belonging to Plate XII., and divide the common among the three proprietors, A, B, and C, according to the different qualities of their estates and of the common.

NOTE 1.—The learner should lay down the plan, from the field-notes, by a scale of two chains to an inch, and find the areas of all the fields from his own dimensions, as directed in Part V. The diagonals and perpendiculars from which the above areas were found, are not given, as this would have rendered the work too easy to exercise the genius of the student; he may, however, retain his own dimensions, and enter them in "A Book of Dimensions, Castings, Quantities, Qualities, and Values, adapted to Plate XII." (See page 187.)

A Book of Quantities, Qualities, Values, &c.

Belonging to Plate XII.

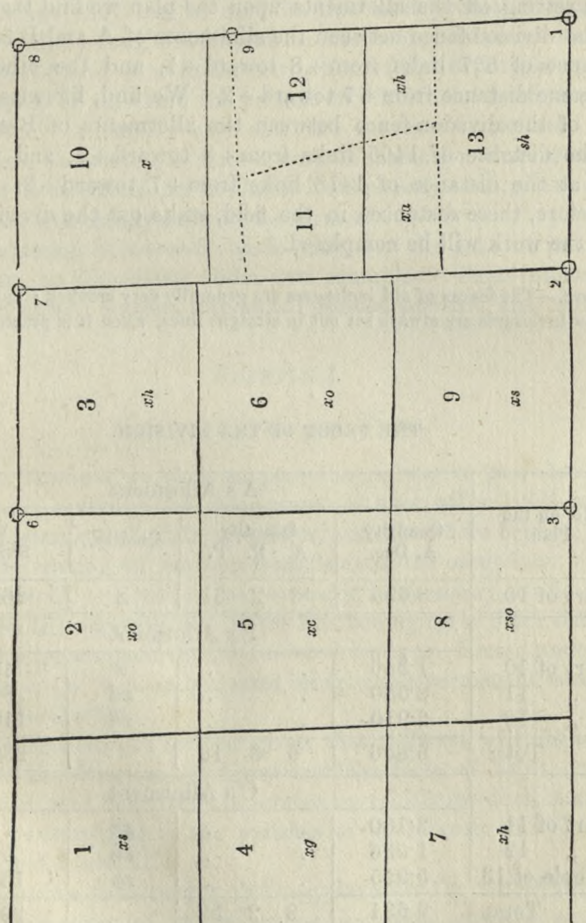
No. on the Plan.	A's Estate.		
	Quantity. A. Dec.	Quality.	Value Shil. Dec.
1	7·565	*xs*	378·250
2	7·609	*xo*	304·360
3	7·301	*xh*	277·438
Total	22·475		960·048
	B's Estate.		
4	7·858	*xg*	290·746
5	7·892	*xc*	260·436
6	8·223	*xo*	328·920
Total	23·973		880·102
	C's Estate.		
7	7·819	*xh*	297·122
8	7·078	*xso*	424·680
9	7·431	*xs*	374·050
Total	22·378		1095·852
	The value of the whole Estates		2936·002
	The Common.		
10	10·061	*x*	301·830
11	4·680	*xd*	159·120
12	4·446	*xh*	168·948
13	5·995	*sh*	167·860
Total	25·182		797·758

2. If the learner should not be able to find such dimensions as will make his areas agree exactly with those given in the foregoing Book of Quantities, it will be a matter of no consequence, provided the difference be not too considerable; and as any difference in the areas will also produce a difference in the values, all the numbers in his book will differ from the given numbers. This, however, will tend much to his improvement, as he will be under the necessity of making all his own calculations, both in finding the areas and values of the different fields, and also in dividing the common and proving the division.

The operation of finding the value of each proprietor's share of the common; and directions for setting out the allotments in the field.

$$\text{As } \overset{s.}{2936} : \overset{s.}{797{\cdot}758} :: \left\{\begin{array}{r} \overset{s.}{960{\cdot}048} \\ 880{\cdot}102 \\ 1095{\cdot}852 \end{array}\right\} : \left\{\begin{array}{r} \overset{s.}{260{\cdot}860} \\ 239{\cdot}137 \\ \underline{297{\cdot}761} \\ \underline{\underline{797{\cdot}758}} \end{array}\right. \left.\begin{array}{l} \text{A's} \\ \text{B's} \\ \text{C's} \end{array}\right\} \text{value} \quad \text{proof.}$$

PLATE XII.



A PLAN FROM THE ENGRAVEN FIELD BOOK.

Scale, 8 chains to an inch.

As the whole of A's allotment will fall in No. 10, say, as 30 : 1 : : 260·860 : 8·695 acres, the quantity of land which A's allotment must contain.

From 10·061 take 8·695, and we have 1·366, the remainder of No. 10, in value = 40·980, which will form part of B's allotment. Then, from 239·137 take 40·980, and there remains 198·157; consequently, B must have land equivalent to this value from Nos. 11 and 12.

The remainder of these Nos. and the whole of No. 13 will be C's allotment, which you must measure, &c., as a proof.

In setting off the allotments upon the plan we find that one end of the division-fence between the allotments of A and B falls at the distance of 827 links from + 8 toward + 1, and the other end at the same distance from + 7 toward + 2. We find, likewise, that one end of the division-fence between the allotments of B and C fall at the distance of 1465 links from + 8 toward + 1, and the other end at the distance of 1478 links from + 7 toward + 2. Measure, therefore, these distances in the field, stake out the division-fences, and the work will be completed.

NOTE.—The fences of old inclosures are generally very crooked; but the fences of new inclosures are always set out in straight lines, when it is practicable.

THE PROOF OF THE DIVISION.

No. on the Plan.	Quantity A. Dec.	Quantity A. R. P.	Quality.	Value Shil. Dec.
	A's Allotment			
Part of 10	8·695	8 2 31	x	260·850
	B's Allotment			
Part of 10	1·366	. .	x	40·980
„ 11	2·580	. .	xd	87·720
„ 12	2·910	. .	xh	110·580
Total	6·856	6 3 16		239·280
	C's Allotment			
Part of 11	2·100	. .	xd	71·400
„ 12	1·536	. .	xh	58·368
Whole of 13	5·995	. .	sh	167·860
Total	9·631	9 2 52		297·628
Sum total	25·182	25 0 29		797·758

GENERAL DIRECTIONS FOR APPORTIONING LAND INTERSECTED BY A RAILWAY.

On referring to Plate XIII. at the end of the work, it will be seen how railways, by intersecting fields, necessitate new lines of fences. Thus, in the parishes of Westbrook and Winston, there is not a single field through which the line passes but what requires to be altered, either by land being added to or taken from it.

Estates, parishes, and counties are severed in a similar way to farms, lands being taken from one that require to be added to another before they can be used in the most profitable manner: while the railway itself has become an intervening estate, containing a large area of land, now in the occupation of a new proprietary.

In a survey of this kind the present and future value of the land is that which principally engrosses the surveyor's attention. In other words, the survey is more a work of valuation than measurement, and as directions for the latter have already been given, those now offered will chiefly refer to the former.

The valuation of severed lands may, for the sake of practical instruction, be illustrated under two examples. First, intersected parks and farms; second, intersected estates, parishes, and counties.

EXAMPLE I.

Intersected Parks and Farms.

In this example we shall suppose the railway to pass through a large estate, severing the game-preserves and other lands in the natural possession of their proprietor, and also those held on lease by tenants, cutting off portions from fields in the occupation of one farmer so small, or detached, as to render it necessary to add them to fields on the opposite side of the line belonging to other tenants. It may also be assumed that the severed lands are farmed under different systems of husbandry, some being under permanent meadow, others under aration.

As a general rule, it may be given that railways will ultimately determine the direction of fences and the form of fields, just as roads, rivers, and boundaries of estates have hitherto done, similarly to what is exemplified in the parishes of Westbrook and Winston, as illustrated Plate XIII.

The doctrine thus taught obviously involves the resubdivision of land on both sides of the railway as the most profitable practice. To this rule there are but few exceptions. It implies that if the lands are allowed to remain as they are, in a severed state, a permanent loss will be sustained; but, as in the parallel example of a road, present losses will be more than compensated by future gains, provided a proper subdivision is effected; consequently, as the proper subdivision of the land may be presumed to be the object of the survey, the two alternatives call for special consideration at its commencement.

1. If lands intersected by a railway are allowed to remain in a severed state, or if they are improperly laid out into fields, their value per acre for agricultural purposes will be permanently decreased.

2. If lands intersected by a railway are properly laid out into rectangular fields of the required size, their value per acre will be permanently increased.

3. Capital invested according to the first rule will not return interest. The investment is a dead loss, and ought, therefore, to be borne exclusively by the landlord, because the tenant, on his part, under such circumstances, has annually to bear an increase of expense and decrease of profit, as a permanent loss per acre.

4. Capital invested in accordance with the second rule will return interest, and if the investment is judiciously made, the amount of interest realised will, in the vast majority of cases, be highly remunerating to the landlord, while the improvements effected ought at the same time to afford equal encouragement to tenant's capital.

The agricultural losses sustained under the first rule may be accounted for by reference to Plate XIII. Thus, if we suppose, what is too commonly the practice, that the triangular portion of field No. 5, lying between the railway and the open drain in the parish of Westbrook, is added to field No. 6, by the removal of the fence that is cut by the line, and that the triangular portion of No. 6, lying between the railway and the road, is added to No. 5, then, under such data, a permanent loss per acre will be sustained of a twofold character, apart from the dead loss borne in removing old fences and making new ones; and also apart from the greater area of land under fences.

The twofold loss thus sustained is occasioned by an increase of expense and a decrease of produce, as formerly stated, the first arising from working acute-angled and shapeless fields, which will be greater under steam-culture than it has hitherto been under horse-culture; and the second from the greater length of headlands.

Another source of loss arises from roads crossing the line, as in the case of the farm-road leading up to Hill Farm, which is raised eleven inches; but in many instances the inclination amounts to several feet, so that in hay and corn harvest two horses are required to do what formerly was done with one, while a boy has to attend to the gates at these two busy and expensive periods of labour.

Permanent losses of this kind are first experienced by tenants, but ultimately by landowners. In the case of the home-farm the latter bears the whole loss.

There are other losses which have to be valued that are not of a permanent character, and which, therefore, may have to be solely borne by the tenants whose lands are severed. Thus, an inferior quality of soil, in a high state of fertility by artificial means, taken from a tenant at the commencement of his lease, may be worth more annually for a short term to his neighbour who gets it than a better quality of land in an exhausted state from mismanagement received in exchange. Instances might be quoted where differences amount to more than the rent of the land during the remainder of a lease; and with the progress of science and an increasing amount of capital invested in farming, such differences are of greater importance than they were formerly.

For agricultural purposes a rectangle is the most profitable form of a field. This may be given as an established *formula*—that which is practically embodied in the second and fourth rules above.

Any deviation from the rectangle, occasioned by the inclination of the land or other cause, will be attended with a corresponding permanent loss; and the amount of this loss will be greater under steam-culture, as now performed by a wire rope, than it has hitherto been under horses. In other words, the best system of cultivation is that for which provision should be made by the surveyor.

As the railway determines one boundary fence, the direction of the parallel and perpendicular fences may also be considered as ultimately fixed. Thus, the two *limit of deviation* lines may be taken to represent the first two parallels, one on either side of the railway, as shown in Plate XIII.

In the case of the park, woods, game-preserves, pleasure-grounds, and home-farm, in the natural possession of their owner, present losses are generally greater, and future profits more distant, owing to the larger amount of ornamental work destroyed by the railway, and the corresponding increase of capital required for new improvements of this description. But, granting this, a railway may be made far less objectionable in a park than a common road, while in very many parks and demesnes it may be the *base-line* of much-needed improvements, alike for ornament, pleasure, game, and profit.

When an old plan of the estate is furnished, as is generally the case, a copy of the park may be taken by tracing-paper, and the railway laid down upon it. This will enable the surveyor to determine the *new design*, and apportion the grounds.

In a work of this kind every case must be its own rule. When the line crosses a river, road, or valley, at a distance from the mansion, it may have to be left open for ornament, but generally it will require to be covered. This may be done by planting forest timber, ornamental trees, and shrubbery favourable to game, to compensate for other grounds taken into the home-farm to square off arable fields. In the case of permanent meadows and other grass lands, if angles and straight lines are avoided, the more shapeless the areas are the better, provided the outlines are natural. Although game require retirement, yet pure air, light and sunshine, with natural food, are essentially necessary to health and the fine normal flavour of their flesh, and if these are secured for them, they will soon become familiar to the most noisy traffic on the line, and even prefer it to the interior of close, dark, and unhealthy forests and game-preserves.

Lands thus laid out for timber and game-preserves on both sides of a railway should communicate with each other by bridges under or over the line, as shown in the section Plate XIII. at No. 1 and No. 2; and where a choice exists, the latter, generally speaking, will give more satisfaction than the former. When a railway crosses a river, a brook, or even a drain, a footpath on one side will form a very inviting communication between game preserves.

Example II.

Intersected Estates, Parishes, and Counties.

A railway may be considered a permanent landed estate, in the possession of a separate and distinct proprietary, and, therefore, a suitable boundary between farms, estates, parishes, and counties.

The practical rule involved in this can be profitably carried out in all cases where a railway intervenes between farms, estates, parishes, and counties, in such a manner that the land taken from one can be equitably given in exchange for what has been cut off from another, the work of apportioning being made in accordance with the old familiar maxim of "take and give," as shown in Prob. XI. and XII., Part V.

It may also be profitably carried out in cases where the small differences of value between portions of severed land can be settled, either by money or by an exchange of land in some other place. Thus, A may receive from C land to compensate for what B has received from A, while C may receive from B what will balance the triune account.

Exchanges of this kind enhance the value of land in several ways. It does so to railway companies and the public as follows. Thus, if timely notice were given the former that the lands intersected by their line would be exchanged, and that their estate would be made a permanent boundary, they would often be enabled to obviate bridges under and over the line, and crossings on a level with the rails; consequently, capital would be economised in its formation, and expenses in working it afterwards, while in the outset they could afford to give a higher price for the land.* The safety of the public, again, would be better cared for, more especially at the crossings to fields and farms on a level with the rails, cattle being very liable to get upon the line at such places, gates to be left open, and things to be dropped from carts and left upon the rails.

In exchanges between landowners of portions of their estates present and future values require to be considered under the following three heads :—1. The agricultural value of the land; 2. The mineral resources of the land; 3. The adoption of land for building purposes, and others of a kindred character.

1. In examples of land exclusively adapted for agricultural purposes *to present values must be added facilities for future improvement.* Thus, according to the old adage, "Lay clay on sand and you buy land." If, therefore, an estate possesses within itself facilities for improvement by the mixing of soils, and which may not have existed prior to the formation of the railway, the materials having to be conveyed along the line or part of them, such must be carefully taken into account. On the other hand, some sandy, clayey, or mossy lands may be worth more to another estate than to the one to which they belong, because the former possesses such facilities while the latter does not.

2. Railways have considerably affected the mineral value of land. Generally an advance has been realised, but there are examples to the contrary, so that every individual case must furnish its own rule for valuation.

3. In the immediate vicinity of towns, or railway stations, or places on the line where a new railway station might advantageously be made, lands adapted for building purposes always form a prominent topic in a survey accompanied with a valuation and plan of exchange.

When the growth of the population of a place is rapid under the prospect of a continued state of prosperity, lands for building may increase in value.

Again, when a town is overcrowded by narrow streets and antiquated houses, there is every prospect of suburban lands increasing in value, if the trade of the place is flourishing.

The extension of railway accommodation is also thinning out many of our large towns, and as the inhabitants are being spread over a large area, market-gardens, nursery-grounds, dairies, and various manufactories are being removed farther inland. In all such cases there is an increasing demand for land a short distance from town, with a corresponding increase of value.

Rectangular areas of land are of more value for building purposes than triangular and shapeless pieces. This is of growing importance to landowners and tenants for long terms, more so than it was some time past. The increase of value is partly owing to the architectural reformation that old streets and buildings are now undergoing; but principally to the general progress of science demanding a greater economy of time and labour. Thus, when a street crosses another at right angles, the traffic in the one has only to wait until that of the other moves across the breadth of the street in which the stoppage takes place. But when streets cross each other diagonally the crossings increase in length, so that the expense of time and labour may be twice and even thrice that of the rectangular system. And this, too, is not the worst feature of diagonal crossings, for horses cannot turn round the acute angles without stopping both the up and down traffic, while in turning at obtuse angles the opposite error is experienced, viz., too great a velocity, and tear and wear of streets and vehicles. To these practical objections must be added other three, viz., the greater area of land consumed both by streets and houses; the increase of expense in their formation and annual repair; and the reduction in the accommodation which they afford for business; and when the student sums up the objections on both sides, he will then be able to appreciate the difference between the rectangular and diagonal systems of laying off land for building purposes, and how much two proprietors of building lands would gain by exchanging triangular and shapeless pieces of land situated on opposite sides of a railway, so as to enable each to square off his property, and thus give it the highest value.

SECTION III.

GENERAL DIRECTIONS FOR TAKING SURVEYS OF RIVERS, CANALS, EMBANKMENTS, AND LANDS RECLAIMED FROM THE SEA.

The directions in this section will be given under the following six examples, viz. :—

EXAMPLE I.—Directions for the survey of lands to be reclaimed from the sea.

EXAMPLE II.—Directions for the survey of a tidal river.

EXAMPLE III.—Directions for the horizontal survey of lands reclaimed under Ex. I. and II.

EXAMPLE IV.—Directions for the survey of a river in a large valley from high water level upwards.

EXAMPLE V.—Directions for the survey of mountain rivers.

EXAMPLE VI.—Directions for the survey of canals, irrigation-works, and warp-lands.

EXAMPLE I.

DIRECTIONS FOR THE SURVEY OF LANDS TO BE RECLAIMED FROM THE SEA.

Before land is reclaimed from the estuary of a tidal river on one or both banks, by means of embankments, a survey of it is usually made. The object of the survey is to enable the owner or owners to ascertain the area that can be recovered, the dimensions of the embankments required, and whether the land, when reclaimed, will return sufficient interest to justify the investment of capital necessary to complete the work, and lay out the land for agricultural purposes. Such a survey necessarily includes a plan and section of the embankments and mouth of the river. The land to be reclaimed may be presumed to extend to both sides, but as the two areas are similar in character, directions for one will be sufficient.

The embankments are thus what are usually termed *the sea embankments*, in contradistinction from those of the next example that extend inland along both sides of the tidal part of the river.

Parallel examples are to be found at most of our large rivers, such as those that flow into the Wash, from which large areas of fen land have been reclaimed and added to the counties of Lincoln, Huntingdon, and Cambridge. Holland presents examples of still greater magnitude. In this country, in our colonies, and other parts of the world, a vast amount of land may yet be profitably reclaimed.

In some places surveys are easily taken both for plan and section. Thus, where the land is only under water at spring tides, it will be covered with herbage, so that a complete survey, horizontal and vertical, may be taken at first, as no difficulty stands in the way of determining the position of the embankments, fences, water-courses, roads, buildings, &c., required in laying out the land reclaimed for agricultural purposes. In others the work embraces an indefinite amount of sectional detail, owing to the nature of the subsoil, and the difficulty of determining the position and magnitude of the embankments. The latter will be the most instructive example to adopt. The more advisable course in examples where the position of the embankment cannot be determined at once, is to include two areas of land by means of two boundary lines, the one area greater than will probably be reclaimed, and the other less.

The boundary line of the largest area may be termed the *maximum-extreme*, or the exterior line, or outer line; and the boundary line of the lesser area the *minimum-extreme*, or the *interior line*, or *inner line*.

A preliminary horizontal survey for the plan will thus be necessary. It will embrace the details of the area between the outer and inner boundary lines, but only a general outline of the lands inside and outside.

The survey for the sections should precede that for the plan, as the position of the two boundary lines cannot be determined until the former work is concluded. This arises from the treacherous nature of the subsoil, which, in the vast majority of cases, cannot be judged as to quality from mere superficial appearances. The boundary lines have consequently to be carefully examined before the several stations are finally fixed upon for the maximum and minimum extremes between which the position of the embankment is presumed to run. In doing so, to avoid, if possible, going over the line a second time, the distances of the ground line below the surface level of the sea at high water should be taken and entered into the field-book, which will form part of the details of the sectional survey.

Directions for ranging station-poles.

The first station-poles that require to be set are those at the mouth of the river, two on each side, one for each boundary-line.

The outfall or mouth of the river is often the most difficult point in the survey to determine. In some cases, for example, it is more advisable to make a new outfall than to continue the old one; consequently this is the first point that requires to be settled, and the following general rules may be given for that purpose:—

1. When the channel of the river at low water contracts at its mouth, when the subsoil is of such a quality as to permit of an embankment being easily thrown across this narrow place from one side to the other, and when the old bed of the river above this expands into basins, it may then be advisable to change the outfall.

2. When the land to be reclaimed extends to a great distance on each side of the river, and when an embankment can be thrown across the channel, the river may be divided, and its waters discharged into the sea by two outfalls, one on each side. This may be done partly to intercept the water from tributary streams and drains, and partly to throw it into the sea at a higher level.

3. It may be advisable to change the outfall in order to economise capital invested in the formation of the embankment.

4. The outfall may be changed to afford increased facilities for inland navigation.

5. The outfall may be changed to facilitate the accumulation of mud, and the formation of new lands for subsequent reclamation from the ocean.

Having fixed upon the place for the station-poles at the mouth of the river, which we shall suppose is not to be changed, two on each side, a transverse section of the channel and embankments between them should be taken and entered into the field-book. The ranging of the other station-poles of the survey may then be proceeded with, at the different angles or points of the two boundary lines, together with those of the seaward and landward stations.

If the ground cannot be conveniently driven with a chain, which is often the case, the poles should be ranged purposely for the *three areas* of land, being ascertained by trigonometry; the first comprising all included by the survey beyond the outer boundary line; the second that within the inner line; and the third area, the land lying between the two boundary lines.

One reason for ranging two boundary lines, is to furnish the necessary data for obtaining an estimate of any line which may be determined upon between them for the embankment. So far as the work of the surveyor is concerned, no more difficulty is experienced in giving a section of the embankment at the exterior boundary than at the interior one. In point of fact the two lines are two sections, either of which may be adopted, or any third line between them. But were only one longitudinal section of the embankment given, it might be otherwise with those who have to carry out into practice the proposition of reclaiming the land from the ocean. But when they have the alternative left of turning to the right hand or left, between two extremes sufficiently far apart to meet all the exigencies of the case, the practical solution of the problem as to the position of the embankment can then be determined satisfactorily.

When the boundary lines are covered with a sufficient depth of water to float a flat-bottomed boat or coble, the station-poles may often be more conveniently ranged by such means, during the flow and ebb of the tide, than at low water. And when thus ranged the levels of both boundary lines may be taken at high water by the same means, and the ground, longitudinally and transversely, between them also examined by boring into the subsoil under the water over the edge of the boat. Between the seaward and landward stations the surveying staff can thus employ their time without interruption until the station-poles are ranged, the lines examined, and the levels are taken.

The station-poles in the two boundary lines are ranged purposely to obtain the levels of the transverse and longitudinal sections, as well as the boundary lines themselves, and care should be taken that the line of the intended embankment lies between them, so that when it is made, the exterior line shall fall without, upon the sea side, and the interior one shall fall within, upon the opposite or land side.

The area between the boundary lines should be carefully examined, so as to determine if a sound foundation for the embankment exists between them.

The station-poles in each boundary line should be numbered, and the two lines distinguished from each other by different letters of the alphabet, and this requires to be done both upon the plan and sections. Thus, the station-poles of the exterior line may be numbered 1*a*, 2*a*, 3*a*, &c., and those of the interior line 1*b*, 2*b*, 3*b*, &c. And as the survey commences at the mouth of the river, it will be more convenient for the consecutive numbers to run from that point, the two stations on each side; there being 1*a* and 1*b* on the left bank, and 1*c*, 1*d* on the right bank.

The number of station-poles in each boundary line require to be equal, and the corresponding numbers should be opposite each other. Thus 1*a* should be opposite 1*b*, 5*a* opposite 5*b*, and so on for the other numbers in each line.

The object of this order is to get the data for the transverse sections. Thus between 1*a* and 1*b* there is a vertical plane that must be surveyed and shown upon the plan by a section, the ground line being represented by the bottom line, and the surface or high-water level at the top of the station-poles being shown by the upper line.

The number of transverse sections will depend upon the nature of the ground. If the surface is a regular plane, one for each straight line will be sufficient, but if undulating, one may be required for every concave and convex part.

The transverse section has a twofold purpose to serve. *First,* It shows the slope or angle of depression seawards which is required both for the embankment and the drainage of the land. *Second,* The levels being thus given at the two extremes (the outer and inner boundary lines), and shown upon the transverse section, the height of the surface level at high water above any point in the ground line between them can be easily ascertained from the scale upon which it is drawn.

Two longitudinal sections are required, one for each boundary line. They show the top and bottom levels at each station-pole along both lines, and the numbers and letters of the alphabet that distinguish them from each other should be carefully attended to, so as to obviate error in copying from the working plan and section.

The levels are taken at high water by placing a mark upon the station-poles at the highest point to which the tide rises for the surface level, and by measuring its distance from the ground to obtain the bottom level; and the marks thus placed upon the station-poles should be carefully examined a second time at high water, in order to ascertain if the levels are correctly taken, before the base line is ranged and the theodolite placed for taking the bearings. This is usually termed "checking the levels." It should be done between flow and ebb, when the weather is calm and the surface of the water "smooth" and free from ripple, and when right they will all lie in one plane.

Directions for the Horizontal Survey.

As the horizontal plane intersects the vertical ones at the several pairs of station-poles, the points of intersection and lines which they form are consequently common to both surveys. In all cases, therefore, when the vertical lines cannot be measured with the chain, this part of the vertical survey may be profitably supplied from data taken under the horizontal survey. Such, according to hypothesis, is the case in the present example.

The base line should range parallel to the general direction of the boundary lines, or as nearly so as practicable. Level ground and easily gone over should be chosen, in order to secure the greatest accuracy in measuring with the chain. This is necessary, as the true length of the lines between the station-poles in the two boundary lines will depend upon that of the base line. In the field the significant practical rule, *the best of three*, is generally given, *i.e.*, measure the base line three times and take the average.

The bearings are taken as in ordinary surveys with the theodolite, and directions for such will be found in Sect. VI. Part IX. and p. 385.

The stations and lines which they form should be shown upon the plan, as represented in the plan of the Preliminary Survey for a projected Railway, Part X. Sect. I. And the two boundary lines may be represented in a similar way to what the two lines indicating the *Limits of Deviation* of the Parliamentary Survey are in Plate XIII.

If the Surveyor undertakes to lay down a line representing the centre of the embankment, it may be determined as in Sect. I. Part X., and shown between the exterior and interior boundary lines upon the plan, as the line of railway is represented in Plate XIII. between the two lines indicating the *Limits of Deviation*.

The object of thus showing the working details of the survey, both horizontal and vertical, is to enable engineers, contractors, or their draughtsmen to take off working plans more readily for the execution of the work of embanking, &c., and for this purpose doubles of the plans and sections are generally ordered when surveys are extensive.

Where several landowners are interested they may order an approximate estimate of their respective shares of the land to be reclaimed, which can easily be taken and shown upon the plan, by rules previously given in the work. But the final work of apportioning shares should be left to a more complete survey when the embankments are finished, and when the lands recovered from the sea and the river can be laid out into fields, and the lines accurately measured with the chain. (See Ex. III.)

The position of sluices, syphons, or pumping stations, and existing embankments, require to be taken and shown upon the plan, and also the position of watercourses necessary for the drainage of adjoining lands that lie above the level of the top of the embankments.

The general rule for the latter is, that no water should be allowed to descend from a higher level into the watercourses of the lands to be reclaimed that can otherwise be drained off to the sea or river.

Directions for Longitudinal and Transverse Sections for Embankments.

When the survey includes longitudinal and transverse sections of the embankment, it is more accurately and easily done to show them upon a separate plan by themselves, than to crowd them into the preliminary one, along with those previously given.

This part of the survey includes the ground from whence materials are to be obtained for the formation of the embankment; and the quantity of materials will depend upon the dimensions of the embankment, quality of soil, and exposure to the sea.

The exceptions are few where the foundation of an embankment can be safely laid upon the surface of the ground to be reclaimed. Even in those cases where it has to be run through a shallow arm of the sea, according to the plan pursued in many cases in Holland, and also in this country, a proper foundation should afterwards be dug out inside of sufficient breadth to contain a body of clay that will prevent the water outside from finding its way by any porous strata to the interior, and thus ultimately endangering the safety of the whole area of land reclaimed.

As a general rule, sea embankments should be formed of inorganic materials, as clay or clayey gravel, and the nearest ground containing such, if at command, is generally that included in the survey. With the means of railway conveyance now at command, contractors enjoy many facilities for getting an ample supply of such materials that were not within the reach of their predecessors, so that if the surveyor finds a choice, the best quality may be preferable, although situated at a greater distance.

The principal reasons why vegetable and other organic matter should be excluded from the interior of embankments are as follows :—

1. All organic matter of the kind in question is subject to decomposition. If, therefore, an embankment, or any portion of it extending from the outside to the inside, is formed of materials that contain a large percentage by bulk of such, this percentage may in the course of time be dissipated in the atmosphere, and the inorganic portion reduced to an open porous body approaching in character as if formed of pure sand or gravel, through which the sea-water will ooze, and eventually flow.

2. Embankments which contain a large percentage of vegetable matter are liable to destruction from vermin of a very multifarious and indefinable character, as rats, moles, mice, worms, and innumerable kinds of insects, some of which are so small as to be invisible

to the naked eye. This motley brood live upon each other and the decaying vegetable matter as their normal food: the organic portion of the embankment is thus literally consumed, while the inorganic part that remains is so burrowed as to resemble in some measure a sponge, through which the sea water will ooze, eventually affecting a breach, as in the previous case.

3. A third reason may consist of a combination of the first and second, and in practice this is perhaps what is generally experienced, the depredation of insects and other animals, together with the action of their excreta and putrefying remains, hastening the chemical changes that destroy the stability of the embankment.

When old embankments that have to remain are included in the survey, they should be carefully examined in order to have destructive causes of the above kinds, and the extent of injury done, faithfully delineated. For this purpose a good microscope is essentially necessary; and if it is discovered that the embankment is giving way, it may be advisable to recommend the employment of men to whose professional labours these branches of science belong. The amount or area of damage done will, however, require to be examined by the surveyor, and shown upon the plan and section of the old embankment.

EXAMPLE II.

DIRECTIONS FOR THE SURVEY OF A TIDAL RIVER.

The present survey may be presumed to contemplate improvements extending from the mouth of a tidal river upwards to the highest point where the tide flows. The field operations lie between Examples I. and IV., and are chiefly sectional in character, embracing longitudinal and transverse sections of the channel and its embankments.

Many of the directions given in the last example relative to the ranging of the station poles, the taking of levels at high water, and the survey of the horizontal and vertical planes for the preliminary plan and sections, are applicable to the present, and therefore need not be repeated.

The chief points requiring notice are as follows:—*First*, The breadth of the river as determined by its tributaries. *Second*, The breadth of the river as determined by tidal action. *Third*, The velocity of the river as determined by its tidal length. *Fourth*, New channel and embankments. *Fifth*, Methods for counteracting tidal action. *Sixth*, Influx of tributary rivers.

1. *The breadth of a river as determined by its tributaries.*

The increase required in the breadth of a river to contain the waters of tributary streams, so as to maintain its velocity at low water, and prevent damming back above the point of junction, is directly as the volumes of water which the tributaries discharge.

If, for example, a tributary river discharges a volume of water equal to that flowing in the tidal river, then the breadth of channel required by the latter below the point of junction will be double that above where the two rivers unite. Or the breadth of the united rivers, as it requires to be shown upon the plan and transverse section, is equal to the sum of the breadths of the two rivers before their junction, their velocities and volumes of water being equal.

Had the tidal river contained twice the volume of water it received from its tributary, then the increase of breadth required would have been one-third; and for any other increase of volume, greater or less, a corresponding increase in the breadth of the channel at low water is necessary, to preserve the natural velocity of the tidal river below the point of junction, and to prevent the damming back process above this point (allowing no increase of breadth for friction, which must always be determined on the spot).

In applying the above data either to the laying out of a new channel for the tidal part of a river, or in straightening or effecting any improvement of the old one, it must be borne in mind that an increase in the depth will not compensate for breadth. The rule is founded upon an established law that cannot be violated without consequential losses being sustained, for a greater velocity effected by an increased altitude of the surface-level of the water, involves the damming back process above the point where this superelevation of the surface-water takes place; and when the increase of depth is effected by digging a deeper channel, the practice is abortive, amounting to a standing pool below the natural inclination of the bed of the river, so that no increase of velocity is obtained to convey the extra volume of water received from the tributary.

2. *The breadth of a river as determined by tidal action.*

The action of a tidal river upon the sides or slopes of its channel and embankments, and also upon its bottom, is directly as its depth, the quality of the ground being equal or uniform.

This rule is based upon the established data at sea, viz., the deeper the ocean the higher the waves and the greater the force with which they strike the vessel or lash the shore, and the heavier the amount of damage done during a storm.

On a sandy, gravelly, clayey, or muddy sea-shore, the counteracting force of the return water down the inclined slope formed by the flowing and ebbing tide is a safer defence to the slope than the more solid materials of its structure.

Under differences of geological strata the coarser the quality of the gravel or sand, the more obtuse is the angle of the slope of the sea-beach, and *vice versâ*.

The same practical data will be found exemplified in beds of rivers that have been scooped out by the natural action of the water.

The above data may be seen instructively illustrated on a sea-beach during a storm, or channel of a river during heavy floods; and both examples should be examined on shores of different qualities of soil, to obtain the invaluable information they afford.

The length of the transverse section representing the breadth of a tidal river at its mouth will therefore be as the height of the high-water level line above the ground line or bottom of the channel; but inversely as the angle of depression which the ground line makes with the horizon or high-water level.

As the extra depth of the river above its mouth produced by tidal action gradually decreases until it ceases to exist, it consequently follows that the extra breadth required for tidal action will also decrease in a corresponding manner, and finally cease to exist at the point where the tide ceases to flow.

This gradual decrease of breadth requires to be carefully ascertained in the survey and delineated upon the plan, and also shown by the transverse sections of the river, at its mouth and at the junctions of the tributary streams and upper end of the survey where tidal action terminates.

The general doctrines taught in the preceding directions have reference to the uniform retarded and accelerated velocity of the river during the flow and ebb of the tide, so essentially necessary to the safety of its channel and embankments. Thus during the flow of the tide the velocity of the river will be retarded, and the direction of the current at its mouth and upwards gradually reversed, owing to the damming back process that takes place by the rising of the surface-level of the ocean above that of the water in the river. But when the ebb commences the river will again begin to flow to the ocean with an accelerated velocity. Now, if the above rules have been properly carried out, then the retarding and accelerating processes will be so uniform as hardly to be perceptible to the eye of the observer, the downward current of the river being gradually counteracted by the upward flow of the tide, while the elevated surface of the dammed-back water will during the ebb as gradually fall to the lower-water level.

If, however, the river is confined between high embankments at its mouth, and if the channel upwards is of unequal breadth, being narrow in some places, and spread out into basins of great width in others, so as to form inland seas or lakes during high water, then "races" (as they are sometimes termed on old plans and in old deeds) will be found in the narrow parts between them. The force of the stream in such places is not easily calculated, or embankments defended from the impetuosity of the current during spring-tides, when a more than ordinary rapid fall of the ocean in front is attended by a heavy inland flood behind, or *vice versâ*, when an extra high tide is accompanied with a low level of the river, so that there is little or no counterbalancing action to the upward current, by the damming process of the downward one.

3. *The velocity of the river as determined by its tidal length.*

The velocity of a tidal river at low water is inversely as the length of the tidal part; or, the shorter the distance between the mouth of a tidal river and the highest point to which the tide reaches, the greater the velocity of the river at low water; and, *vice versâ*, the greater the distance the less the velocity.

By straightening a meandering tidal river its velocity at low water will be increased, because its course will be shortened.

The practical data upon which the above two rules are founded may be given thus. If we assume the difference of the depth of the river at its mouth between low water and high water to be 12 feet, then this is the fall of the whole tidal portion of the river included in the survey. If we next assume that the length of the tidal part is twelve miles, that the ground is of equal quality and the inclination uniform throughout its length, it will give a fall of one foot per mile. But if the length is twenty-four miles, and the other conditions the same, then the fall per mile will only be six inches; consequently the velocity in the latter example will only be that due to six inches, or one-half the former.

The formation of bars in the beds of rivers is produced by a decrease of velocity, the sand and gravel being deposited when the force of the current is so reduced as not to be sufficient to carry them farther. Bars across the mouths of rivers are formed in the same way, or jointly by the action of the sea.

The extra depth of the tidal part of a river which gradually increases towards the sea, when the bottom soil is of uniform quality, is produced, partly by the accelerated velocity of the water during the ebbing of the tide; partly by the scooping-out process of opposing currents during the flow of the tide; and partly by the action of the waves towards the sea in stormy weather.

If the ground forming the bed of the river is not composed of materials of uniform quality, the inclination of the bottom will not be uniform throughout its length.

Bars formed in the bed of the river require to be shown both by longitudinal and transverse sections; inequalities in the bottom, produced by different gradients, only in the former, unless special circumstances call for a transverse representation. Bars beyond the mouth of the river seawards are usually shown upon the plan, the depth of water over them being represented by figures. All bars, both at sea and in the channel of the river, if dry at low water, require to be surveyed and shown upon the plan as dry land; where rock crops out in the bottom, it should be indicated both on plan and section, and also any special diversity of soil, as a change from clay to sand.

4. *New channel and embankments.*

A new channel for a river may be occasioned either by changing its outfall, or straightening the course of the old one, and both (it may be presumed) are included in the survey.

A new outfall.

In surveying ground for a new outfall, the inclination of the bottom of the sea and the quality of the ground require, in the vast majority of cases, to be carefully examined for a considerable distance out seawards, before the depth and breadth of the mouth of the new channel can be determined, according to the directions given under the second head of this example. The exposure of the new outfall also requires to be attended to; for if the one side of the channel is more exposed than the other, a greater slope must be given both to it and the embankment, so as to make the return action of the water down the inclined plane defend it, as formerly shown, during a heavy storm.

In a few exceptional cases the new outfall made be made over a rocky bottom at various levels between high and low water, in which the breadth of the river from its mouth upwards will be determined accordingly. Thus, if the new channel is made to intercept the water from the elevated grounds on one side of the low lands, and to discharge itself over a rocky bottom on a level with high water, then the new river will cease to be a tidal one and to be subject to tidal action; consequently its breadth will be determined by its inclination and volume of fresh water, as in the next example. If, however, the rock is on a level with low water, then the breadth of the river will depend upon the nature of the ground forming the sides of the channel, and upon the exposure of its mouth to the waves of the sea during storms; and for intermediate levels the rock will defend the channel from tidal action according to its height, tidal action being inversely as the height of the rocky bottom above low water level.

Straightening a river.

In the language of practice, straightening a river more frequently means reducing the number of curves and right lines than the formation of one straight channel.

In every survey, therefore, the direction of a new channel for a river has to be determined on the spot.

If the object of a new channel is to effect a more efficient state of drainage, then the nearer its direction approaches a right line, the shorter will be the whole length of its course, and the greater the fall gained at low water for any given distance.

By such means the length of the tidal portion of some rivers in the United Kingdom may be reduced to one-half—of others to one-third, and of one or two exceptions to one-fourth of their present length. In such cases the increase of fall gained, and also the area of land reclaimed from the river, may be easily calculated.

The directions for the capacity of the straight portion of a new channel are the same as those given for the survey of the longitudinal and transverse sections of the old.

If a new channel is to be opened between two bends of the old so as to cut off one intervening bend, the best line of direction will be that of tangents to the two curves.

In straightening a river according to this rule the radius of any curve may be easily increased, so as to change the angularity of the current when it strikes the concave bank at too acute an angle. This may be done, for example, by extending the tangential direction of the new channel outwards, the required distance from both the old bends, so that the two station poles at the two extremities of the straight portion of the new channel shall be in the two curves at the two points where the two radii touch the new line of channel, so as to form with it right angles.

In the field this is done by going backwards until the whole of the curve on one hand and new channel on the other shall appear on the river side of the station poles; and the centre of the circle at each curve is the point where two lines ranging from station poles at the two extremities of the curve intersect each other, the one line forming a right angle with the new channel, upwards or downwards as the case may be, and the other a right angle with the direction of the old channel, downwards or upwards as the case may be.

The less the radius of any curve, the less must be the inclination of the slope of the channel and embankment on the concave side, to defend it from the current, and, *vice versâ*, on the opposite or convex side to maintain the capacity of the river required by tidal action.

A transverse section of a curve of a river is thus different from that of the straight portion of the channel.

5. *Methods for counteracting tidal action.*

One method for obviating the destructive action of tides in tidal rivers has already been given, viz., by changing the outfall to a rocky bottom.

A second method is by a tide-lock, or several tide-locks, if the volume of water is large. Such may be constructed either at the mouth of the river, or in the first narrow place above it, if preferable.

A third method may consist of a tide-lock for shipping at some distant point communicating with the river by means of a canal, the channel of the river itself being defended by a series of waterfalls from the high to the low water level, constructed on the principle of a salmon ladder, and which might serve that purpose.

A fourth method consists of floodgates or sluices, in number according to the size of the river, across its mouth or some more convenient place above it.

These several methods are now in operation, and hence call for surveys.

The details of the practice are similar to those of canal-surveying, and will be given under Example VI.

6. *Influx of tributary rivers.*

The survey of a tidal river generally includes that of its tributaries to a distance as far up from their mouths as contemplated improvements extend.

Where lands have been reclaimed, or are proposed to be so, at the mouth of a tidal river, as in the example, the first two tributaries surveyed are those that previously discharged, or now discharge, the drainage waters of the more elevated grounds through the unreclaimed land into the sea, which require to be diverted from their old channels and turned into the river by two new courses—one on each side the main stream.

Where old embankments exist at the inland boundary of the newly-reclaimed land, they may probably be turned to profitable account by forming one of the embankments of the first tributary, if the upland drainage-water cannot be intercepted at a higher level.

In those cases where no land has previously been reclaimed, and where embankments do not exist, the high-water levels of spring tides often determine the course of the two first tributaries for intercepting the drainage-water of the elevated grounds, and discharging it into the main river by opposite channels.

These are questions that always require to be determined on the spot by taking the levels of high water at the necessary station poles, and the other station-levels on more elevated ground. They are seldom surrounded with much practical difficulty; and, as the formation of the two first intercepting tributaries form the initiatory work of the contractor in the reclaiming of the lands lying between them and the ocean, this division of the survey should be the first finished, as the plans or copies of them may be demanded before the others.

The waters of a tributary and main river should unite with equal velocities, so as not to form bars across either channel, or in any way disturb the uniform flow of the principal stream.

Where a tidal river runs through a narrow valley, tributaries very frequently enter at too great a velocity; but such can easily be reduced by means of a waterfall, or series of waterfalls, as in the case of mountain streams in Example V.

The longitudinal and transverse sections of tributary rivers are similar to those of the principal, so that the directions given relative to the latter are applicable to the former.

EXAMPLE III.

GENERAL DIRECTIONS FOR THE HORIZONTAL SURVEY OF LANDS RECLAIMED UNDER THE FIRST AND SECOND EXAMPLES.

When a large area of land has been recovered from the sea and a tidal river by embankments, it requires to be surveyed and laid out into fields and farms for agricultural purposes. And if it belongs to different proprietors, as is frequently, if not generally, the case, when the reclaimed lands extend to both sides of the river, the whole will have to be apportioned according to the several proprietary rights involved.

In its general character the survey resembles that of a common proposed to be apportioned under the Inclosure Act, for which directions have already been given; but in the following details it differs in many respects—viz., 1. Quality of soil; 2. Drainage; 3. Embankments; 4. Roads, fences, and farm buildings; and 5. Area outside embankments.

1. *Quality of soil.*

The agricultural value of the land cannot be separately investigated from its proprietorship. This arises from the difficulty of distinguishing at times between different qualities of soil, especially that reclaimed from the sea, before it has yielded crops of any kind, so as to afford practical evidence of its productive value per acre; consequently, surveys of this kind, when marches are not well defined, often call for the highest degree of professional talent to make the most of the land, and to do justice to the landowners and tenants interested. On the other hand, when the line of subdivision between properties is well defined, and when the land is of uniform quality, the problems of apportioning and laying it out into farms and fields are of easy solution.

2. *Drainage.*

In laying off the principal drains for the drainage of the land they should, as far as practicable, form boundary lines between estates and farms. They are usually divided into main and tributary. The former are the largest; they carry the waters of the latter directly to the sluices or pumping-stations at the embankments; and the latter, sometimes designated cross-drains from their position, discharge the water they collect from the fields and farm ditches into the former. When the land is comparatively level, the water in the small ditches between the headlands of fields very frequently flow to both sides from the middle—a difference in the depth being all the fall that can be given them. In such cases the *water-shed* should be shown upon the plan. In other cases the water-shed may be at one side or between two fields—the water drained from the one field running to the right, and that from the other flowing to the left. In other cases, again, the field ditches are cut to a more uniform depth for the purpose of bottom irrigation, as shown under Example VI.

Straight lines and rectangular fields form the rule, one that also applies to main drains, estates, and farms. The tributary drains, however, where they enter the main drain, should have a gentle curve in the direction of the main current, so as the waters of the former shall not retard the velocity of those of the latter in times of flood, or silt up the channel when the stream is low, and its force reduced to a minimum.

The laying off of the main and tributary drains necessarily involves a large amount of levelling, which is now done with instruments, either spirit-levels or draining-levels; and, as the inclination is often less than 1 in 5000, the work requires to be carefully performed, otherwise flood-water will prove its inaccuracy.

3. *Embankments.*

The lines for the main drains and other subdivisions having been determined, and station poles ranged for measuring them with the chain, the ranging of station poles along the top of the embankments from one extremity of the survey to the other, may be proceeded with as the next step in field operations. The base-line of the previous survey with the theodolite may be continued; but if the lines along the embankments form a series of right lines of considerable length, with but few curves to break their angular connexion, it may be advisable to adopt one of them for a base-line —or the whole series for a series of base-lines—as right lines along the top of embankments can always be driven with greater accuracy than those across broken and uneven ground. The true position or range of the right lines on both sides of a curve can be easily determined by special check-lines and the other bearings of the survey.

The preliminary survey was made exclusively with the theodolite, the measuring of the base-line with the chain excepted; the present is taken both with the chain and theodolite, the distance between the station poles now placed upon the top of the embankment, as well as at the base, being carefully measured with the chain, as also the length and angle of the slopes and total breadth of the base: all deviations from the former dimensions or position of the embankment made during its formation require to be measured, as also the inside drains running parallel to it, and junction of the main drains therewith. The true position of sluices, pumping stations, &c., require to be shown.

The directions for taking the bearings in this survey are similar to those given in the preceding, and also those for laying down the true position of the embankments and river upon the plan.

The lines and offsets to subdivisions, &c., are measured with the chain and offset-staff, and the measurements entered in the field-book as in ordinary surveys.

4. *Roads, Farms, and Farm-buildings.*

Public roads, and private ones leading from them to farm homesteads, require to be between fences, but farm roads from the homestead to the fields may run alongside fences, being only fenced on one side.

In laying off land for public roads in a level district of newly-reclaimed fen or marsh land, the two chief points for consideration are as follows :—*First*, that without increasing their length they shall serve as much as possible the private purposes of the individual farms through which they pass; *Second*, that they shall form the shortest or the nearest and most direct route to church, markets, railway-stations, &c.

Sometimes the direction of roads and fences is determined by dry grounds rising above the general level of lands recovered from the sea and tidal rivers, and offering inviting sites for farm-buildings. As such are in many cases invaluable, they may not only determine the ground for the homestead, and the direction of roads and fences, but also the size of farms; consequently such are preliminary questions that ought to be settled with landowners and tenants before boundary and subdivision lines are finally determined.

Live fences have sometimes an open ditch on each side, in other cases only a ditch on one side, while in a few exceptionary examples as yet the lands are thoroughly drained and cultivated close up to the bottom of the hedges, the hedgerows being regularly hoed and kept as clean and free from weeds as drilled green crops of beans. Such, therefore, are also preliminary topics that, like the above, should be determined before the lines are finally ranged for being driven with the chain, more especially if the plan is to be drawn to a large scale, and everything shown in a highly-finished style.

The directions for laying off ground for farm-buildings, including labourers' cottages, water for the same, and for stock in the fields, are similar to those given in a subsequent place, Part VII.

Lands reclaimed from the river are generally added to the adjoining farms, the fences running directly to the river. This may give rise to alterations in the cross-fences, and the rule for determining the position of both subdivisions will be the size of the farm and fields in every individual example.

5. *Area outside Embankments.*

The extent of the survey outside the embankments will depend much upon whether land is accumulating, or the contrary. If new deposit is being formed at every tide, landowners have an interest in ascertaining the direction of this new formation, in order to watch its progress and preserve the rights which they or their posterity may possess to reclaim it from the sea at some future period. A survey, therefore, may be presumed to determine the proprietary rights of those who employ the surveyor. It may extend to a considerable distance beyond what dry land appears above low water, and the depths of ground under low water level are taking by soundings, and shown upon the plan in their proper places. Where the nature of the ground will not permit of station poles being set, as is frequently the case, buoys require to be anchored for taking the bearings. Buoys may thus be in deep water and station poles on dry ground, and the bearings are taken partly with the theodolite from the top of the embankments, or any other land station, and partly with the compass—a mariner's compass being preferable in a boat at sea to the pocket one of the land-surveyor or the compass of the theodolite. The position of the buoys at sea, and land-stations from which the bearings of the former are taken, should be shown upon the plan, so as to enable landowners at any time to take the depth of the sea at low water at any desired point, in order to ascertain whether deposit was making progress or not.

EXAMPLE IV.

GENERAL DIRECTIONS FOR THE SURVEY OF A RIVER IN A LARGE VALLEY FROM HIGH-WATER LEVEL UPWARDS.

The survey under this example is presumed to contemplate improvements in straightening and embanking a large river immediately above high-water level, either in terms of 24 & 25 Vict. c. 133, or by the mutual agreement of the proprietors interested. It may be presumed either to commence at that point of the river where Example II. terminated, and to end where the next example begins, or to commence at the ocean. Field operations will comprise horizontal data, as shown upon the plan, and vertical data, as represented by longitudinal and transverse sections. But as rules for the former have already been given, the directions will chiefly apply to the latter, the vertical survey under the following eight heads:—1. Outfall levels of the river and its tributaries; 2. General fall of the river; 3. Volume of water; 4. Area of land required for embankments; 5. Subdivision and apportioning of land where the river is straightened; 6. Water power, irrigation works, bleaching grounds, &c.; 7. Provision for the drainage of adjoining lands;

8. New roads, bridges, fences, and drainage, filling and levelling of the old channel.

1. *Outfall levels of the river and its tributaries.*

The beds of rivers at high water outfalls may frequently be lowered so as to drain more efficiently at low water the lands lying on both banks upwards. The outfall-level is, therefore, the first that has to be surveyed.

There are four methods by which the surface level of the outfall of a river may be lowered; viz., *first*, by widening the channel; *second*, by deepening the channel; *third*, by straightening the channel; and *fourth*, by increasing the velocity of the current.

FIRST METHOD.

Rivers gradually increase in breadth from their sources to their mouths, according to the influx of their tributaries, the width being thus inversely as the depth at any section.

The natural breadth of a river is, by the above rule, determined by its tributaries and the quality of the soil of which the bed is formed; consequently, if the natural breadth of the outfall is doubled, the depth will be reduced one-half, and the surface level thereby lowered in the same ratio (making no allowance in breadth for extra friction). Thus, if the present depth is 6 feet and the width 300, then by increasing the width to 600 feet, the surface level of the outfall will be lowered three feet for drainage, the velocity in both cases being the same.

In practice, the rule will be found generally to apply thus: the river at the outfall, or towards this point, will be found to be narrowed by various obstructions, thereby damming back the water and raising its surface level; consequently, if such obstructions are removed, it follows that the river will be widened, and its surface level lowered.

The narrowing causes that require special notice are rock; the central piers of bridges; tide-locks and sluices; rocky channels; quays and harbourage for shipping; boats and barges moored or plying in the river; trees deposited; piles driven in for fencing and other purposes.

When obstructions of the above kind can be removed the increase of breadth gained requires to be shown upon the plan; and the same data applied to the outfalls of tributaries.

SECOND METHOD.

When the present breadth of the outfall is sufficient, but the bottom of the channel so elevated by rocks, bars, or the like, that it can be lowered, such would lower the surface level at low water, and thus improve the drainage during a large period of the ebb and flow of the tide. This would allow the tide to flow farther up the river, thereby subjecting it to tidal action, for which provision will require to be made as previously directed (Example II.), but if half the period of the ebb and flow can be gained for drainage, the advantage may do more than compensate for all expenses.

The formation of new bars across the outfall, in examples where old ones have been removed, can be prevented by reducing the current of the river to a uniform velocity. Thus, the cause of the deposit of sand and gravel being a greater velocity and force of current at some part in the river above the outfall than at the outfall itself, it follows that if the river is deepened at that part, and widened, if necessary, so as to bring the bottom and sides of the channel to a uniform inclination, then the formation of a new bar could not take place; for the velocity and force of the current at the latter section would be equal to its velocity and force at the former section; consequently, no depositing materials would exist to be thrown down, the scooping out and depositing causes being both obviated.

The details of the survey under this method lie chiefly on the vertical plane in the longitudinal direction of the river, but two transverse sections will be required, one at the outfall, and the other at that part of the river above proposed to be deepened.

THIRD METHOD.

The increase of fall gained by straightening a river has already been shown in Example II. Thus, if by straightening a river its length can be reduced to one-half, the fall per mile will be doubled, and the surface level lowered by such means directly as the velocity is increased. Hence the gain for drainage.

In practice, the rule may be carried out in two ways. *First*, By straightening the river and removing the outfall farther down the tidal channel, which would improve the drainage at low water. *Second*, The old tidal bed may be preserved, and two tributaries, one on each side, taken farther down by two straight channels to improve the drainage. An extra fall of several feet may frequently be gained by such means. Under the second way the surface level of the outfall in the old bed will be lowered directly as the amount of tributary water removed.

Under this method the details of the survey are chiefly on the horizontal plane.

FOURTH METHOD.

It is manifest that if the velocity of a river at its outfall can be doubled, the depth will be reduced one-half, and the surface level lowered in the same ratio, as the quantity of water discharged in both cases would be equal in equal times.

In practice, the rule is seldom applicable to examples of the kind under survey unless where stagnation of the water in the river takes place at the outfall, not from narrowing cases, as in the *first method*, but from the reverse, too great a breadth, the object of the survey being to obtain the necessary sectional data for confining the river within a proper breadth of channel at its outfall, and thus obtain a sufficient velocity to prevent deposit of every kind.

This extra breadth of outfall may be effected by one of two causes either by a rapid above having a sufficient force to scoop out a broad pool at the outfall, and thus form a bar on the other side; partly by the joint action of the tide; or else by too small a velocity, the growth of reed, and accumulation of muddy deposit. Both examples are more frequently met with when the outfall is at the sea, than when it is far inland at the point where the ordinary tide ceases to flow.

In the case of a rapid above the outfall and a bar below, the rule is to deepen the channel at both places, so as to bring the bottom to a uniform inclination, as under the second method, and to reduce the width of the outfall section to its proper dimensions.

In the case of a deposit of mud the narrowing of the channel at the outfall, and the confining of the river in its proper rectilinear direction, will increase the velocity, clear out the bed and lower the surface level of the outfall at low water; but if the growth of reed has been allowed to commence, and the plants to take deep root, they may require to be cut: but the execution of works of this kind do not belong to the survey: at the same time, the surveyor may be called upon to report as to their magnitude and how they are to be done.

In both these examples the details of the survey are similar to those of the second method.

The above methods of lowering the surface level of the outfall of the principal river are applicable to the outfall levels of its tributaries; but when the general inclination of the bed of the former is altered, that will also affect the outfall levels of the latter.

2. *General fall of the river.*

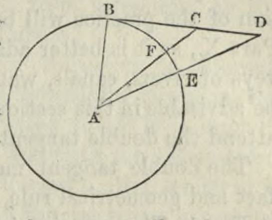
The position and level of the outfall section having been taken the next branch of field operations is to find the whole fall of the river between the lower and upper ends of the survey, in order to determine its general inclination, and how much additional depth can be gained for drainage and water-power purposes. Technically the operation is termed *levelling*, an illustrated description of which will be found in Sect. I. Part X. as applicable to railway surveying. The work is done by instruments of various kinds, but as levels thus taken are subject to errors that require to be corrected, it is usual in river surveying to check them by *water levels*, all our large valleys affording ample opportunity for taking "check levels" of this kind. In this and in several other respects, the survey differs widely from that of a railway.

The errors that require to be corrected are three in number: *first*, tangential errors; *second*, errors due to refraction; and *third*, hypotenusal errors.

Tangential Errors.

There are three errors of this kind that require notice. *First*, deviation from the arc as to curvature, or errors in curvature; *second*, deviation from the arc as to length, or errors in length; and *third*, deviation from the horizontal level, or errors in the practice of taking the horizontal levels.

First. If BE, in the annexed diagram, represents an arc of 60°, then AB is radius, BD tangent or the horizontal level, and AD secant. In the direction of the secant AD the extent of deviation is ED, which in this case is equal to radius; but if a perpendicular is dropped from BD to E, parallel to AB, the extent of error will then be equal to the versed sine, or half the radius AB, when the arc is 60°. The former is the extent of error usually adopted, viz., the difference between the radius and the secant of the arc; so that, taking the diameter of the earth at 7912 miles, the correction for 60° would be 3956 miles, *i.e.*, the semi-diameter.



If we suppose the length of the tangent one mile, then, according to Theor. VII. Part I., ED is equal to $\sqrt{(AB^2 + BD^2)} - AB$; the extent of error would, therefore, be $\sqrt{(3956^2 + 1^2)} - 3956 = {\cdot}000128 +$ of a mile, or 8 inches. For two miles $\sqrt{(3956^2 + 2^2)} - 3956 = {\cdot}000505 +$ of a mile or 32·028 inches.

Second. The next error consists in taking the length of the tangent for the length of the arc, or rather a rough guess for it, as the line measured in practice makes an angle of elevation with (BD) the horizontal level. Thus in the diagram if BF is an arc of 45°, then BC is tangent and equal to radius or 3956 miles; whereas BF is only 3107·04 miles, according to data subsequently given under Example VI. The error in length for 45° is, therefore, 848·9 miles. For 60° the tangent BD is 6301·6 miles; the arc BE 4142·7 miles, giving 2158·8 miles as the tangential error in length.

Third. There are two methods of taking horizontal levels, the one with a backsight from the station, at which the level is placed to the levelling staff; and the other with a backsight and a foresight to two levelling staves. The former practice may be termed the single tangent method, and the latter the double tangent method.

The single tangent method of taking levels is the correct one. It is subject to the two preceding tangential errors, and also to those arising from refraction and measurement with the chain. As the dead levels of canals will furnish a more instructive illustration of the practice than the surface levels of a river, it will be illustrated under Example VI. But general directions will be given under the longitudinal section of this example.

The double tangent method of levelling, although it has received the sanction of long use as being sufficiently correct for general purposes, is nevertheless erroneous; consequently the single tangent method has to be adopted when a greater degree of accuracy is required, or when obstructions prevent the level from being placed half-way between the two levelling staves. An illustrated description of the practice will be given under Railway Surveying, Sect. I. Part X., as it is better adapted for roads and railways than for surveys of rivers, canals, waterworks, and the like. It will, however, be advisable in this section to point out to the student the errors that attend the double tangent method of levelling.

The double tangent method is based on assumptions contrary to fact and geometrical rule, as the following three examples will show.

Ex. 1. The practice in the case of an acclivity or a declivity assumes that the two secants of the two arcs are equal, while it proves the contrary, the readings from the two levelling staves indicating a difference. Thus, as the two levelling staves represent two "normals," or portions of two radii, it follows that when they indicate a difference, as they always do on an inclined plane, the two radii, with their respective arcs, tangents, and secants, are also different, which is the contrary of the assumption that they are equal.

Ex. 2. If an exceptionary position of the level and levelling staves is assumed, viz., that the readings from the latter are equal, thereby proving the two tangents, secants, radii, and arcs equal, as when the ground is a horizontal level, or when the level is placed on the top of a hill, and the two levelling staves on the two declivities downwards—or when it is placed in the bottom of a concave surface or valley, and the two levelling staves on the two opposite acclivities upwards—then another assumption (which will be pointed out under the next example), that the back-secant is different from the fore-secant, and that the two thereby correct for curvature, is contrary to the previous one that they are equal.

Ex. 3. The practice assumes that the level is equidistant from the two levelling staves between which it is placed; (see diag. Sect. I. Part X.), that the latter represent two secants; the former a common radius; the two lines of sight two tangents; and that as the two tangential deviations are the one *minus* and the other *plus*, they, therefore, correct each other. Now these assumptions, passing over the errors already pointed out, can only, under the most favourable circumstances of the case, be received as very distant approximations, obtained contrary to geometrical rule, for the half-way position of the level cannot be determined until the whole distance is measured with the chain, until the horizontal levels are taken, and the true lengths of the two arcs and two tangents ascertained, which cannot be done by the practice itself. Hence the reason why surveyors are obliged to adopt the other method.

Errors due to refraction.

Directions for correcting errors arising from refraction on the horizontal plane will be found in Sect. I. Part X.

On the vertical plane the correction for refraction is different. Thus, when the telescope is levelled in a horizontal direction, the amount of refraction is 33′; but when elevated to 45° it is only 1′, or more correctly 57″, being three seconds less than the thirty-third part of what it is at the horizon; while directed to the zenith it is *nil*, a star or bird passing over the field of the telescope appearing then in its true position.

Hypotenusal Errors.

Errors of this kind arise from the undulating and uneven surface of the ground, which requires very great skill in driving the chain to take the distances between the level and levelling staff with that degree of accuracy which the nature of the survey demands. In horizontal surveying the correctness of the length of the lines measured with the chain is tested both by tie lines and the trigonometrical data obtained from the bearings taken with the theodolite; but in a vertical survey there are fewer opportunities of testing the geometrical truthfulness of the work.

The line measured with the chain in the vertical plane is the hypotenuse of a right-angled triangle, represented by *ci* in the diagram to Example VI., the triangle there being *ixc*, and the errors that require to be corrected are the deviations from that line. In practice the rule is to place the level and levelling staff in the same hypotenusal plane. This frequently gives rise to short backsight distances at one operation, when the greatest degree of accuracy is ordered. And when a concave or convex surface intervenes between the level and levelling staff, a second levelling staff, and a third if required, is, or are, placed between upon the highest and lowest grounds; the object of the intervening levelling staves being to take the offsets from the line of collimation or sight, so as to determine the curvature of the surface (the line measured), and the true length of the hypotenusal line sought. (Station poles will answer for intermediate staves.) If, in measuring the hypotenuse on the horizontal plane the leader was to place his arrows sometimes a few feet on one side of the line, and sometimes a few feet on the other, the driver would be aware that the measurement thus found would exceed the true length of the hypotenuse sought, just as the two sides of a parallelogram do that of the diagonal. And the same practical rule applies to similar deviations from the hypotenuse on the vertical plane.

Check-water levels.

In valleys check-water levels can frequently be taken by the water levels of canals and mills; or they may be taken by damming back the water in side drains, or even in the river itself under survey. Examples of this latter kind might be quoted where the dead level for several miles was ascertained in less time than it could otherwise have been. Thus, a dam was made in the evening, and next morning stakes were driven in to the water's edge, which finished the work. In extensive surveys, where the investment of large capitals is involved, check-water levelling should never be lost sight of when at command; more especially if the inclination of the river is small, and consequently every inch of fall gained for drainage invaluable.

Longitudinal section.

In comparatively level valleys the inclination of rivers is frequently so nearly uniform that a longitudinal section is not ordered, transverse sections at particular parts being considered sufficient. In other cases longitudinal sections may be required for those parts of the river that are straightened; but when extensive improvements throughout the survey are contemplated, a section of the whole length may be necessary.

If we suppose that the diagram, Prob. V. Part IV., represents a portion of a river that is proposed to be straightened by a new rectilinear channel; that a and x are two stations in the survey; and that the new channel is to be on the same side of these two stations as the old one, it will give the student an illustration of an example of the latter kind requiring a longitudinal section for the whole length.

The horizontal survey may be taken as there directed; a plan drawn according to the dimensions given in the field-book; the course of the new channel represented by two straight parallel lines, and the severed lands apportioned.

Under such conditions the dimensions sought are the lengths of the old and new channels, the fall between station x and station a, and the surface levels of the old river, when such are necessary.

If $+a$ is visible from $+x$, or if $+x$ and $+a$ are seen from a third station beyond $+n$, so as to form two straight lines xn and na, then the line ax sought may be found as follows:—

Measure an and nx with the chain, and take the included angle with the theodolite, which will give ax by Case II. Part VIII.

If $+a$ is visible from $+x$, the angle of depression would give the angles of the right-angled vertical triangle of which ax is hypotenuse, so that the fall may be got by trigonometry, the correction for curvature being the radius of the earth at x *minus* the versed sine of the arc between that radius and the radius of the earth at a, similar to what is shown in the diagram, Example VI., the corresponding right-angled triangle there being icx, and the correction $ix - xz = zi$, the fall between i and c. Or it may be got by various formulæ given under Example VI.

If the levelling staff when placed at $+a$ is sufficiently high to reach the line of collimation or horizontal level at $+x$, the fall will thus be determined at once, the correction for curvature being now secant *minus* radius.

But if the levelling staff has to be placed at $+n$ and $+a$, or any other number of intermediate stations, then the sum of the falls is the total fall, and the corrections for curvature and refraction have to be made at each fall, and not for the sum as a whole.

In a similar manner if three intermediate station-poles are ranged in a line between $+a$ and $+x$, one in cm, the second in mr, and the third in the line opposite, it will intersect these three lines (which belong to the horizontal survey) so as to form four triangles.

In each of these two sides and the included angle are determined by the horizontal survey, all the given angles being proved by tie lines. The four remaining sides or distances can therefore be got by trigonometry, when their sum will give the total distance *ax*.

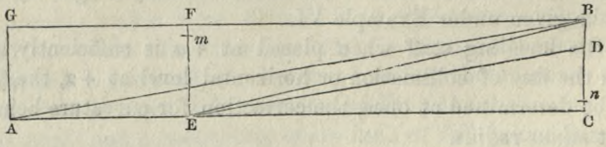
The length and fall of the new channel thus found will give its inclination, and also the fall of the old one, but not the length and inclination of the latter.

The length of the old channel is determined by the horizontal survey. It should be carefully measured with the chain, and will be found in the above example nearly one-third of its length longer than the new one; *i.e.*, the length of the new channel is only two-thirds that of the old one, so that there is a gain on the fall of one mile in three for improved drainage.

Surface levels of the river.

If the velocity of the river is uniform, its length and fall will give its inclination; or if its inclination is uniform, its fall and length will determine its velocity. But such conditions are never realised in a meandering river, the bends invariably proving the flow to be both accelerated and retarded; for a decrease of velocity at a bend involves an increase above, while as rivers descend toward their outfalls, their general velocities decrease. Hence the practice of taking the surface levels of the stream in order to determine how far it deviates from the hypotenusal plane of its inclination.

Although the course of the river is crooked, its longitudinal section may be represented as straight. Thus in the annexed diagram if AB is the length of the old channel, ED the length of the new one, and BC or FE the fall without correction for curvature, then the triangles ABC and EBC will be two right-angled triangles similar to *cix* in the diagram Example VI. The two normals in



the two cases are BC and FE in the above, and *ix* and *yc* in that of Ex. VI. The hypotenusal plane of the old channel is AB, and that of the new one EB.

The correction for curvature is generally so small as with difficulty to be represented on the section. At the same time, it must not be overlooked in plotting the dimensions, for reasons subsequently given.

If BC is found by trigonometry, then EC is the sine of the lesser arc of which Er (not shown) is radius, r being the centre of the great circle where the two normals BC and FE meet; Cn is the versed sine of the arc En, and represents the correction for curvature. The fall is thus Bn; but if the fall has been taken from the levelling staff EF, according to a former hypothesis, then the correction for curvature is sec. – rad. or FE – Fm = Em = Bn, the fall as before.

The correction for curvature on AB is greater than that on EB, although both lie between two stations on the survey, because the old channel traverses a curved surface. But the difference is less than it would be were the channel straight. The actual difference is, therefore, very small in the majority of examples. It requires however, to be attended to, for reasons which will be seen in the next paragraph, and in practice it is generally approximated on what is termed "the safe side."

If it is intended to give the new channel the same inclination as the old one (which is a general rule, the inclination being rather less than greater, so as not to wash away soil), then draw the line ED a distance below the parallel to AB through E equal to the difference between the corrections for curvature of AB and EB, which will give the fall Dn of the new channel, and BD the fall gained for drainage.

At any place between $+a$ and $+x$ the fall for drainage may be got by an intermediate normal at that place or a vertical line nearly parallel to BD between the two lines EB and ED.

The surface levels of the stream may at some places rise above AB, and at others fall below it. Thus, if no obstructions from piers of bridges, fords, and the like exist, then the surface water-line will be concave, falling below AB; but if side and bottom obstructions are numerous, the water-line will be convex, rising above AB. At acute bends of the river as at + 6 (page 136) a damming back process, with an elevated surface and retarded velocity, will be found. This involves a greater velocity at the narrow part above + 7, so that the former level may be above AB and the latter below it. These require surface level stations, one or two at each bend and narrow place.

Plotting the dimensions.

As the field operations of the vertical survey commence at station x, so do the office ones of plotting the dimensions. Thus draw the normal BC indefinite in the direction of C; the tangent BF indefinite in the direction of G; the diagonal BE according to dimensions found, making with BF the angle of depression also found, and the sine EC at an angle equal to FBE. If the work is thus far correct EC will cut BC in C, making a right angle at C, and BC equal to the dimension found by trigonometry. Next, EF, taken from the field-book, will cut BG in F, making the angle at E greater than a right angle, and the angle at F less than a right angle. Produce CE, in the direction EA indefinite, and with the measured length of the old channel from B cut CA in A. Draw AB and EB; also ED, nearly parallel to AB, as before directed. Mark off the surface level stations + 1, + 2, &c., on BA, commencing at B. The deviation of the water-line from BA may then be plotted from the readings of the levelling-staff at + 1, + 2, &c., the operation being similar to plotting a crooked fence on a horizontal survey from offsets taken from a line driven with the chain.

In the preceding example, from the shortness of the distance between + x and + a it is assumed that the latter station is seen from the former, and that the height EF is taken from the levelling staff at one operation : but when the survey extends over ten, twenty, or forty miles, this cannot be done.

In a comparatively level valley, forty miles, and even the length of a degree, or seventy miles, may be included between two normals as BC and FE, and two parallels BF and CE, the former, BF, a tangent to the greater radius Br and arc Bm, and the latter CE, a sine to the lesser radius Er and arc En.

In this case the distance EB, between the two extreme stations E and B, would be determined from the horizontal survey as before and so would BA. But besides BE and BA, there would now be a series of diagonals corresponding to the number of intervening levelling-staff stations. In most examples the position of the series of diagonals would be above the common diagonals BA and BE; in many places, however, they may be below, but in all their position and length can be easily determined, and plotted with the section according to constructive geometry, the series of diagonals being similar to the sides of a polygon on the horizontal plane, and the surface levels of the stream may be accurately drawn therefrom.

In more extensive surveys the longitudinal section should be divided into sectors, as in the diagram Example VI., which contains three, purposely drawn to show the principles of plotting.

The vertical offsets with the levelling-staff being taken above the diagonal AB, and not above the sine AC, the practice of laying down the surface levels from the latter, or from a datum line below it, is not correct, because AG is greater than BC; consequently the offsets above are greater than those below.

General rule for the inclination of the channel.

The angle of depression of the channel of a river should be that which will give to the flowing current a force sufficient to keep it clear of sediment in times of drought, when the surface level of the stream is low, but not such as will wash away its sides and bottom during periods of flood water, when the surface level is high.

If the fall and inclination are less than what is given in the rule, deposit will be formed, and the channel of the river gradually silted up. On the other hand, if it is greater, the force of the stream will wash away first the lighter materials of which the channel is formed, thus undermining the heavier, so that its proper shape will be destroyed, and its course made liable to change at every storm or heavy fall of rain.

In practice it is difficult to make provision for the two extremes, the minimum and maximum volumes of water, as sediment cannot be avoided during low water without injury being sustained in times of heavy floods. Practical rules are sometimes advanced relative to the proper fall per mile that will obviate both objections; but such can never be safely taught, unless upon the spot, and even then with much caution and allowance for differences in the quality of soil; for as such rules are deduced from the actual velocity of other rivers, because they are considered favourable examples, it follows that if there exists the slightest difference between the two soils of which the channels are composed, either silting or washing away will be the inevitable result, as shown in Example II.

When a new channel is proposed, as in the above example, the advisable plan for both the surveyor and drainage engineer is to make ample provision for flood waters, so as to obviate ruinous tear and wear. To effect this, all differences arising from corrections for curvature, refraction, &c., should be thrown upon the proper or safe side, so that the fall and inclination shall be rather less than calculated; and the practical reason for this is, that the small deposit of silt that takes place at low water will be removed during floods if the true form and direction of the channel are preserved.

In the generality of cases the old channel will be the best example as a practical rule for the new one, the soils opposite being for the most part equal, and the lowest velocity of the old river may be taken for the general inclination of the latter.

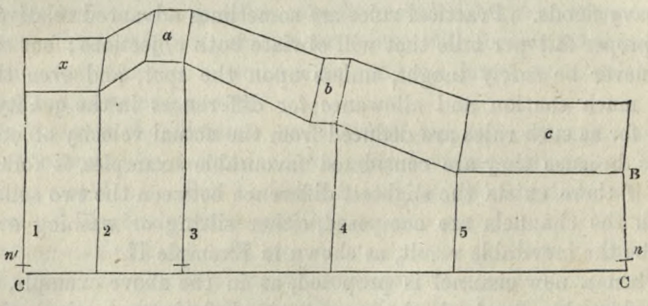
3. *Volume of water.*

The volume of water during the highest floods is that for which provision has to be made by the survey, and this has to be ascertained by measurement. In the same manner the transverse sectional capacity of a new channel may be determined from that of the old one. A diagram of a semi-transverse section will be given under the next head, Embankments.

4. *Area of land required for embankments.*

The areas of land required for the two embankments, including the two forelands, have generally to be computed separately from the area occupied by the channel between them and from the two areas outside : the several areas being shown upon the plan.

For the sake of illustration, let the annexed diagram represent a semi-transverse section of the river at station x, page 136. Then x will show the land outside at $+x$, a the top of the embankment, b the foreland between the embankment and edge of the channel, c the bottom of the channel, Bn the depth of the fall (see previous diagram), CC′ the horizontal plane of the sine EC, and 1, 2, 3, 4, and 5, the vertical offsets.



The inside capacity being uniform throughout the bottom c, the slopes of the channel and embankment, and the level of the foreland, will be similar at other sections; but the offsets will decrease towards A, and the surface level at x may be higher or lower.

By improved drainage rain-water is now much faster removed from land than formerly, and a corresponding provision should be made for such, by higher embankments and a greater breadth of foreland between the bottom of the inside slopes and edge of the channel of the river.

The foreland should be parallel to the top of the embankment and surface of the water, and not much above the ordinary level of the latter during the winter season.

The bends or curves of rivers should be of the greatest possible radius, and the slopes on the concave side opposed to the current at bends should be greater, or of a less angle, than where the river is straight; and the top of the embankment whose concave slope faces the current, should be higher than the opposite convex one.

A greater area of land is required for the embankments and channel at curves than where the river is straight; and the additional breadth, both for the embankments and channel, should, if practicable, be taken from the opposite or convex bank of the river.

5. *Subdivision and apportioning of land where the river is straightened.*

The method of apportioning land on the equitable principle of "take and give" is similar in this example to that described when land is intersected by a railway, Sect. II.

6. *Water power, irrigation works, bleaching grounds, &c.*

If levels are properly attended to, the straightening and deepening of rivers will give an increase of water-power for mills, proportionally to the increase of fall gained, but it will reduce facilities for irrigation, bleaching, and like purposes.

When the bottom of the channel is lowered, water-wheels require to be lowered to the same extent, and also the watercourses.

The two watercourses may also require to be of greater length, especially the one above the mill, in order to draw off the water from a higher level, so as to avoid an intercepting wire across the stream, which is always objectionable.

Such alterations involve a large amount of both field and office work, but, although the greatest degree of accuracy is required in taking the levels, the operation is not generally attended with much difficulty. If the water-power is large and of great value, longitudinal and transverse sections of the mill-courses may be included in the survey, but when it is small, the details may all be shown upon the plan, the fall being indicated by figures.

When water is taken from the river for irrigation, breweries, bleaching, &c., by gravitation, the lowering of the channel will render it necessary to draw off the water from a higher level, which will increase the length of the watercourse. Thus if the channel is lowered one foot, and if the fall per mile is one foot, then the increase of length for the watercourse will be one mile.

7. *Provision for the drainage of adjoining lands.*

All surface and spring water gathered above the level of the top of the embankments is generally discharged into the river between embankments, and what is collected below that level is discharged by sluices and similar contrivances.

As the water from elevated grounds may frequently be discharged into the river between embankments at a much higher level than what is requisite to drain the lower grounds, the side drains for the latter may have to run under the former, and be discharged into the main channel at a point farther down, in order to gain the proper fall.

The levels for such tributaries are generally included in the survey of the principal river; and when the works are large, sections of them may be required, but when small the vertical details may be shown upon the plan.

8. *New roads, bridges, fences, drainage, and levelling of the old channel.*

If, in straightening a river, the new channel shall cross a road, a new bridge will be required; and as old bridges are frequently not well situated, the old road may also have to be taken up, and a new one made. For similar reasons old fences will often have to be removed and new ones formed.

A new bridge, in such cases, is generally considered a favourite place for a transverse section; and if a new road is also required, the prolongation of the transverse level of the roadway on both sides of the new channel will form the surface-level of the longitudinal section of the road.

The foundation of the piers of bridges should be of such a depth as to be defended by a pool in a similar way to what is shown in the next example of mountain streams—the safety-line being considerably above the base-line of the masonry, or top of the piles, where such are used.

The superficial area of the old channel will have to be surveyed, and sometimes its capacity also measured, in order to ascertain the quantity of materials required to fill it up. This latter measurement is usually given in cubic yards. The expense of filling up the old course with the materials excavated from the new one, may likewise have to be estimated before the lands can be valued and apportioned.

When a tributary discharges itself into the river at the part straightened, it may in some cases be allowed to enter the new channel by the old one; or a new course may have to be surveyed and laid off for it, between its old mouth and the new river.

When the old river, or a portion of it, is to be filled up, the course of the whole, or the part, as the case may be, will have to be shown on the plan by dotted lines; but where the whole or the part is to be left open, such will require to be represented upon the plan accordingly.

When the old channel is to be filled up, and when a large covered sewer is placed in the bottom of the old channel for the purpose of drainage, its course may have to be represented upon the plan by a dotted line, its mouth where it enters the new channel being shown on the longitudinal section, when such is ordered.

The ranging of the poles for the horizontal survey, the measuring of the lines with the chain, the taking of the bearings with the theodolite, and the other minor details, are similar to what will be found under Prob. V. Part IV., and subsequently Part IX. Section VI.

EXAMPLE V.

GENERAL DIRECTIONS FOR THE SURVEY OF MOUNTAIN RIVERS.

That which chiefly distinguishes the rivers of mountainous districts from those of a comparatively level open country is their greater fall and velocity, and the peculiar phenomena to which such give rise, as the formation of pools, rapids, waterfalls, cascades, &c.

Pools and rapids are occasioned partly by the scooping-out process of the water, and partly by the formation of bars across the river. In all cases where the fall is great, the water, by its continuous scooping-out action, either cuts down to the rock, or sweeps before it, in heavy floods, stones and gravel to a less inclination, where such materials accumulate in a bar, forming a rapid below and a pool above. In this manner bar after bar rises in the channel, until the whole bed of the river, from its source to its confluence with the ocean or large river, in the campaign valley, is formed of an alternating series of pools and rapids, differing in length and depth according to the nature of the ground and force of the current.

In those cases where the water washes away the soil in the natural formation of a channel, and flows upon a rocky bottom, waterfalls and foaming cataracts are formed of various magnitudes.

In other examples rivers flow into and out of lakes, in channels consisting of pools and rapids, or of rocky bottoms.

The pools and rapids formed in gravelly soils are a natural protection to the bed of the river and the adjoining lands on both banks. The water, when it leaves the pool at the top of the rapid, begins to descend with a minimum velocity and force, which gradually increase to a maximum; but the accelerated velocity and force acquired in its descent is lost in the pool below. As the river thus pursues its course it leaves the pools above it at a minimum velocity, but enters those below at a maximum.

The depth and form of the pools at the bottom depend upon the force of the current and the nature of the subsoil. If water is gently decanted into a full vessel, the influx current will only penetrate a short distance into the stagnant fluid below; but if the decanter is elevated, and the volume and force of the water increased, it will sink to a greater depth. The river as it flows down the gradient forming the rapid into the pool below, is governed by the same hydrostatic law. When small it will penetrate to a short distance below the surface, but when swollen it will sink to a greater depth. If large stones are embedded in the subsoil, they may deflect the current, and thereby prevent the bottom being scooped out to so great a depth as otherwise it would have been. But where bottom soils are equal, the depth of the pool will be directly as the altitude and force of the swollen current during the greatest floods.

The elevation of a bar above the general inclination of the river will, like the depth of a pool, depend upon the force of the current and the nature of the accumulated debris of which it is formed. If the ground is full of large stones, consisting of fragments of rock, they will accumulate during the natural formation of the channel by the flowing stream, and thus form a more elevated, acute, and permanent ridge across the current than when the stones are of a less size and rounded by attrition. But when the bar is composed of finer gravel, it will be "more flat," will give to the rapid a greater length, and be much more "liable to shift" in times of heavy floods than the previous example.

The depths and water-level lengths of the pools; the depths and hypotenusal lengths of the rapids from smooth water to smooth water, and the heights and lengths of the bars, require to be ascertained in the survey, and shown upon the longitudinal section; the breadths and depths are represented upon the transverse section; and the breadth, horizontal length, and direction of the river are shown upon the plan.

Pools, rapids, waterfalls, and cataracts are frequently known to anglers and others by their names; and these may require to be shown both upon the plan and sections.

It is seldom found advisable to change the course of a large river in a hilly country; on the contrary, the difficulty experienced is to confine them within the channels they have naturally scooped out for themselves; for unless the pools are of sufficient depth, and the bars formed with proper materials, it is impossible to prevent harm in storms of more than ordinary magnitude and duration.

When a survey is undertaken for the express purpose of supplying the necessary data required for embanking a river, so as to prevent it from overflowing its banks, and for deepening the pools and improving the rapids, so as to obviate the shifting of the channel, the height and position of the embankments, and the materials of which the bar is formed, are the first matters of inquiry that engage the attention of the surveyor.

If the river has previously been allowed to overflow its banks during heavy floods, the confining of it within embankments will raise its surface level and increase its velocity and force.

The increase in the altitude of the river, at the top of the rapid, will be found in the majority of cases to endanger the stability of the bar, and hence the safety of the channel below. This arises from the increased velocity and force being produced by artificial means, whereas the defence is natural, the bar being formed by the river when flowing at a less depth.

Sometimes the stability of the bar and rapid may be increased by artificial means sufficient to counteract a greater force of current. This is done either by throwing in heavy stones in the smooth water immediately above the rapids whose bars are liable to be washed away, or by driving in piles and wattling them with small timber.

As the breadth and depth of the river at the top of the rapid during a heavy flood are determined by the height and position of the embankments, they should be placed as far from the river's edge on both sides as the circumstances of the case will permit, the depth being inversely as the breadth.

When a river has left its bed, the same data are applicable in the formation of new bars and embankments in order to direct it back into its natural channel. Wherever the surface of the water is raised on the top of a rapid, the stability of the bar must be increased, and also the depth of the pool below; and this has to be done for several pools and rapids down the river, so as to restore the deranged balance of things to a state of equilibrium, similar to what is found in natural examples of stability.

When arable, meadow, or pasture lands have been washed away or covered with gravel, they require to be measured, so as to enable the landowner to settle questions of value with his tenants. In such cases a section to show the subsoil and drift is not unfrequently ordered.

When the course of a river has to be changed, a pool of considerable depth and length is best for the purpose, more especially if the bend is quick. A rapid should in such cases be avoided, if possible, as the concave bank and embankment are difficult to protect at the top of the rapid when the river is much swollen.

A ferry is generally at the narrowest and smoothest part of a pool, and a ford at the first smooth water immediately above a rapid; both require to be shown upon the plan, and also the ferryman's house and boat.

The piers of bridges should be protected by pools of sufficient depth to prevent harm to their foundations during the greatest flood.

Where the foundations of the piers of old bridges are so high as to form rapids, such data should be carefully shown upon the sections, and special attention drawn in any report that may accompany the survey relative to details of tear and wear, &c., that cannot be otherwise represented.

Lakes are sounded and the depths shown upon the plan, as in the case of the ocean.

In surveying waterfalls and cataracts the chief points that require attention are the peculiar geological strata of the rocks that form the bed of the river, the fracture and angles thus formed, and the wearing away of the bottom by the continuous action of the water. Such involves a large amount of work, but it is simple and easily performed by those who have a taste for drawing.

EXAMPLE VI.

GENERAL DIRECTIONS FOR THE SURVEY OF CANALS, IRRIGATION-WORKS, AND WARP-LANDS.

Surveys under this example are either of works that are finished or else to furnish data whereby others may be executed at some future period.

Canals.

In Canal-Surveying the levels are necessarily "water-levels" or "dead-levels," as they are sometimes termed in contradistinction from horizontal levels on which water flows.

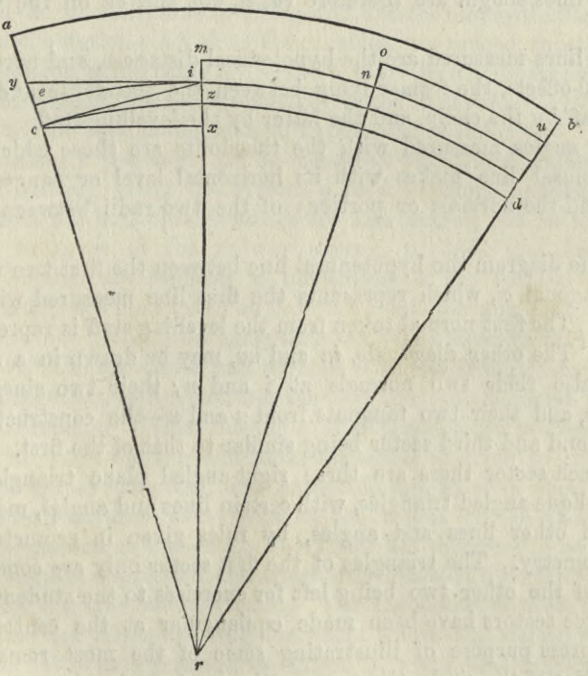
When the canal is formed and in operation, the details of the vertical survey for the sections are of the simplest kind, as the levels can then be taken from the water, the normals from the height of the locks, and the lengths and breadths by measurement with the chain.

The details of the horizontal survey for determining the direction of the line of the canal, the area of land which it occupies, including the position of basins, quays, &c., and the severance of property which it has effected, are similar to what has already been given, and to the corresponding data in Railway-Surveying, which will be found under that head, p. 385.

If the survey is made purposely to determine the levels and other data for a canal intended to be made, its details are more diversified and differ in many respects from those of a railway.

Under such conditions there are two levels given; viz., the lowest water level, and the highest one—the object of the canal being to raise the navigation from the former to the latter by means of a series of locks and dead levels, as from *c* to *b* in the annexed diagram.

The first step in the survey is to find the difference and distance between the two given levels. This may be done either by the common levels and chain, as in the case of the river (Example IV.), or by the theodolite and chain. We shall adopt the latter, as it will illustrate a survey on the vertical plane conducted on the principles of plane trigonometry. The difference between this practice and that with the level will also be so pointed out as to be easily understood, the theodolite being in point of fact a spirit level.



The object of taking the levels is to find the height of the upper level above the lower one, which determines the number of locks on the line, and that of measuring the hypotenusal distances with the chain to find data for ascertaining the true lengths of the dead levels or arcs that determine the length of the canal within the survey.

The survey may be represented as lying between two arcs of two concentric circles on the vertical plane.

If, in the foregoing diagram (drawn on an exaggerated scale purposely to illustrate details which otherwise could not have been shown and made intelligible to the private student), we suppose *abr* to represent a section of the vertical plane, then *ab* and *cd* will be the two arcs between which the survey lies; the former showing the upper water level line, and the latter the lower one.

If we further suppose four locks on the line, such data will give three intervening arcs; and if we again assume that the three arcs subtend equal angles, then *ce* in the diagram will show the first lock, *im* the second, *no* the third, and *ub* the fourth: *ei* will show the first intervening arc, *mn* the second, and *ou* the third.

The lines sought are therefore *bd*, *ei*, *mn*, and *ou* on the vertical plane.

The lines measured are the hypotenusal distances, and normals or vertical-offsets, the former lying between the theodolite and levelling-staff by the chain, and the latter by the levelling-staff.

The angles measured with the theodolite are those which each hypotenusal line makes with its horizontal level or tangent and sine, and the normals or portions of the two radii between which it lies.

In the diagram the hypotenusal line between the first two radii is the diagonal *ci*, which represents the first line measured with the chain. The first normal taken from the levelling-staff is represented by *cy*. The other diagonals, *in* and *nu*, may be drawn in a similar way; also their two normals at *i* and *n*; their two sines from *i* and *n*, and their two tangents from *n* and *u*—the construction of the second and third sector being similar to that of the first.

In each sector there are three right-angled plane triangles, and two oblique-angled triangles, with certain lines and angles, measured to find other lines and angles, by rules given in geometry and trigonometry. The triangles of the first sector only are completed, those of the other two being left for exercises to the student; and the three sectors have been made equiangular at the centre *r*, for the express purpose of illustrating some of the most remarkable properties of the circle, those properties most commonly met with in the practice of surveying. Two of the corresponding triangles of the three sectors are equiangular, but not equilateral, the sides increasing in length as the arcs and radii increase in length. The other three triangles are neither equiangular nor equilateral.

The three right-angled triangles in the first sector are (1) *cxi*, (2) *cxr*, and (3) *yir*: and the two oblique-angled triangles are (1) *cyi*, and (2) *icr*.

The two equianglar triangles are *yir* and *cxr*.

They are equiangular, *first*, because the two angles at *i* and *x* are right angles, and the angles at *y* and *c* each the complement of *r*, which is common to both; *second*, because *yi* is parallel to *cx*, the angle *iyr* is equal to the angle *xcr* (Theor. III. Part I.); and as the angle at *r* is common, and the remaining angles right angles, the triangles are therefore equiangular.

The angles of the remaining three triangles are dissimilar.

The two chief lines sought in this sector are the normal *ce*, representing the height of the first lock, and the arc *ei*, representing the length of the first level of the canal.

If we suppose the surveying-staff to be divided into two companies, one with the theodolite and levelling-staff, and the other with the chain, and that the station-poles along the canal-line are ranged, those shown on the vertical plane being *c*, *i*, *n*, and *b*, then the field operations will be performed and the measurements entered in the three field-books that would be open under such an hypothesis, as follows:—

At station *c* the angle *yci* is measured with the theodolite, the vertical bearing being taken from *y* to *i*. (Before removing the instrument the horizontal bearings are also taken; but as they are similar to those of the railway survey, p. 385, the directions in this section are confined to the survey of the vertical plane.) The bearings are next taken at station *i*; first the angle of depression *yic*, which gives at the same time *icx*. The measurement of the angles with the theodolite, and normals with the levelling-staff, now proceed together, *cy* representing the first one that is found by the latter instrument. When the angle of depression *yic* is accurately taken and entered in the field-book, the signal is given to those in charge of the levelling-staff, who enter the height *cy* in their field-book, which measurement includes the height of the theodolite above the ground at station *i*. This latter has therefore to be taken at each angle of depression, and afterwards deducted from the former, to obtain the true normal found. The theodolite is then turned round, and the vertical bearing *min* taken before the station-pole is replaced. It is next removed to station *n*, when the levelling-staff is taken to station *i*.

During the above operations, the chain may be employed in measuring the side lines of the horizontal survey; but when those working the levelling-staff replace station-pole *i*, and proceed to *n*, then the diagonal or base line *ci* may be driven.

This concludes the field operations for the first level *ei* or sector *eri*. Those of the other two sectors *mrn* and *oru* are performed in a similar manner.

From the two angles thus measured, viz., *yci* and *yic*, the other angles of the five triangles may be determined. Thus $180° - (yci + yic)$ gives *cyi* and *xcr*; $yic = icx$; $90° - icx = cix$; $90° - xcr = r$; and $icx + xcr = icr$.

From the two sides found, viz., *ci* and *cy*, and the above angles, the other sides of the triangles may now be determined by the following formulæ :—

To find *cx*	R : cos. *icx* : :		*ci* : *cx*
To find *ix*	R : sin. *icx* : :		*ci* : *ix*
To find *xr*	R : tan. *xcr* : :		*cx* : *xr*
To find *rc*	*rx* : R	: : cos.	*r* : *rc*
To find *yi*	R : tan. *r*	: : $rx + xi$	: *yi*
To find *yr*	R : sec. *r*	: : $rx + xi$	: *yr*

The two lines *yi* and *ir* may also be obtained by formulas for oblique-angled triangles, Part VIII. Section II., and the secants *yr* and *cr* by Theorem VII. Part I., as shown in Example IV. The line *ce* is now got by subtracting *cr* from *ir*; and *ey* representing the correction for curvature of the horizontal level *iy* from the arc *ei*, is obtained by subtracting *ir* from *yr*, or the radius of the arc from its secant.

The arc *ei*, representing the length of the first level of the canal, remains to be determined.

The difference between radius and secant is found with geometrical accuracy in several ways, but the true length of a curve or of an arc of a circle, as *ei*, is only approximated. Thus, it is less than its tangent, but greater than its chord, as will be seen from the diagram. Taking the diameter, for example, to be 1, the circumference of a circle has been calculated to be 3·14159; now 3·141 would lie within the circle, while 3·142 would lie without. Between these two lesser and greater differences, another one, viz., 3·141592265358979 lies; still it is upon the inner side of the curve: and in this manner the decimal has been extended to upwards of two hundred places, and might be carried to as many more. But for all practical purposes it may be closed at 3·1416 without, or 3·1415 within, the former being generally adopted. From such data the following rules have been deduced :—

To find the circumference, multiply the diameter by 3·1416.

To find an arc, multiply ·0174533 by radius, and the number of degrees which the arc contains.

Irrigation-works.

Irrigation-works may be considered with a view to their survey under the following three heads:—1. Bottom irrigation; 2. Surface irrigation; 3. The modern system of applying liquid manure to land.

1. Extensive examples of the first kind are to be found in our West India colonies, and many other parts of the world. The works consist principally of parallel canals and ditches, intersecting comparatively level land.

The nature of the survey is therefore similar to that under Example III., the canals and ditches being used for the twofold purpose of surface drainage in the rainy season and bottom irrigation during the intervening periods of drought.

2. Surface irrigation, the second example, is principally practised for the growth of rice and grass. The water is generally conveyed to the fields in both cases by gravitation, either from canals or rivers, naturally or artificially formed, but it is differently applied afterwards, paddy fields consisting of a series of dead levels, while water-meadows lie at various inclinations.

In surveys of this kind, field operations are simple, but multitudinous in character, the levels required for distributing the water being exceedingly numerous; and as they are represented upon the plan, they furnish a corresponding amount of office work.

3. The third practice of applying water to land is by forcing it through pipes, either by gravitation, or by steam, or other power. It may first be thrown, by means of pumping apparatus, into tanks or cisterns, situated on elevated ground, and from thence be distributed throughout the fields by gravitation; or it may be forced directly to the land. The pipes and hydrants are ramified throughout the fields in such a manner that each of the latter has a certain area of land assigned to it, so that the liquid may be showered equally over it by means of a hose of the proper length screwed on to the hydrant. When one area is finished, the liquid is turned off, the hose screwed on to another hydrant, and thus the work proceeds until the several hydrantal areas are gone over.

It does not belong to this work to describe the details of liquid manuring land on this plan. Enough has been said to show the student who may not be acquainted with the practice the nature of the levels that have to be taken, and the survey required in laying down the pipes and hydrants.

The vertical survey closely resembles that of water-works for supplying towns, castles, farm-homesteads, and fields, with water.

It embraces two levels, a lower one at the river or fountain-head, or liquid manure tank, and a higher one at the reservoir, or its equivalent, the height to which the liquid has to be forced. The work of taking the levels is therefore similar to that in canal surveying.

With regard to the horizontal survey, a hose screwed on to a hydrant or mouth of a stopcock would distribute water over a circular area of land. The areas, however, into which fields must be subdivided are necessarily square areas; consequently each square area has to be inscribed, as it were, within the circular area which the length of the hose and jet will cover. As gases are very liable to be disengaged from drainage water and liquid manure, and to collect in the bends of pipes, where they pass over elevated ground, hydrants should be placed at such, so as to draw off the gases which there collect.

Warping land.

When flowing water holding organic and inorganic matter in suspension is turned into an area of land inclosed by embankments, and then allowed to stagnate, such matter is deposited, and termed "warp," and the process "warping."

The annual overflowing of the Nile in Egypt furnishes a practical example on a large scale. Warping has also been extensively practised in this country, the warp being chiefly obtained from tidal rivers, some of which have been termed "muddy to excess," a depth of from six to twelve inches of warp being left by them upon the land in a single summer season.

The survey is similar in character to that for embanking land when the work is not attended with any difficulty. The height and breadth of the embankments at the base are generally shown by figures upon the plan, in the same manner as the area of land inclosed for warping. As the depth of the warp will be directly as that of the water, the depths of the surface of the land at the lowest and highest places below the top of the embankment may have to be taken and shown upon the plan, as different depths of the ocean are directed to be represented in Example III. Such bottom levels are easily found by setting station-poles at the places, and raising flags or marks upon them until such marks appear in a line or in the same plane with the top of the embankment levels.

SECTION IV.

THE METHOD OF REDUCING LOCAL OR CUSTOMARY MEASURES TO STATUTE MEASURE, AND VICE VERSÂ; ALSO, THE METHOD OF REDUCING SCOTCH AND IRISH MEASURES TO STATUTE MEASURE, AND VICE VERSÂ.

It has been already observed that formerly, by custom, the perch varied in different parts of England, and with it consequently the acre also varied in proportion.

In Devonshire and part of Somersetshire 15, in Cornwall 18, in Lancashire 21, and in Cheshire and Staffordshire 24 feet were accounted a perch.

In the common field-lands of Wiltshire, and in some other counties, there was a customary measure of a different nature, viz., of 120 instead of 160 statute perches to an acre; consequently 30 perches of statute measure made 1 rood of customary, or 3 statute roods made 1 customary acre, or 30 statute perches made 1 rood, and 4 such roods made 1 acre, customary measure.

In some places, an acre of this measure was called a day-work, or a day's work of land.

We may also observe that the customary measures of Scotland and Ireland differed very greatly from the English statute measure.

PROBLEM I.

To reduce customary measure to statute measure, or statute measure to customary measure.

GENERAL RULES.

RULE I. *To reduce customary measure to statute measure.*

Multiply the number of perches, customary measure, by the square feet in a square perch, customary measure; divide the product by the square feet in a square perch, statute measure, and the quotient will be the answer in square perches, which reduce to roods and acres by dividing by 40 and by 4 in the usual manner.

RULE II. As the square yards in an acre, statute measure, are to the square yards in an acre, customary measure; so are any number of acres and decimals, customary measure, to their equivalent in acres and decimals, statute measure. Then reduce the decimals to roods and perches by multiplying by 4 and by 40 in the usual manner.

NOTE 1. The roods and perches in the given quantity must be reduced to decimals of an acre before stating the question.

2. By reversing either of the preceding rules, we can reduce customary measure to statute measure.

3. It is scarcely necessary to remark that the length of any perch multiplied by itself will give the number of square feet in a square perch of the same measure; hence we have $16·5 \times 16·5 = 272·25$, the statute perch; $15 \times 15 = 225$, the Devonshire and Somersetshire perch; $18 \times 18 = 324$, the Cornwall perch; $21 \times 21 = 441$, the Lancashire perch; and $24 \times 24 = 576$, the Cheshire and Staffordshire perch.

4. It may also be observed that 4840 square yards make 1 statute acre; 4000 made 1 Devonshire or Somersetshire acre; 5760 made 1 Cornwall acre; 7840 made 1 Lancashire acre; and 10240 square yards made 1 acre of the customary measure of Cheshire or Staffordshire. Also, 3630 square yards made 1 acre of the customary measure of Wiltshire.

5. When it was intended to find the area of an estate in customary measure only, it was generally thought most convenient to take the dimensions by a chain properly adapted for that purpose. The Devonshire and Somerset chain was 60 feet; the Cornwall chain, 72 feet; the Lancashire chain, 84 feet; and the Cheshire and Staffordshire chain, 96 feet in length. Each of these chains was divided into 100 equal links, in the same manner as the statute-chain; consequently the customary measure was found by the same rules as the statute measure.

6. It may also be observed that the Devonshire and Somersetshire link was 7·2 inches; the Cornwall link 8·64 inches; the Lancashire link 10·08 inches; and the Cheshire and Staffordshire link was 11·52 inches in length.

Examples.

Ex. 1. In 32 acres 2 roods and 20 perches, Devonshire and Somersetshire customary measure, how many acres, &c., statute measure?

By Rule I. Here 32*a.* 2*r.* 20*p.* = 5220 customary perches; and $5220 \times 225 = 1174500$, the square feet in 5220 customary perches; then, $1174500·00 \div 272·25 = 4314$ statute perches = 26*a.* 3*r.* 34*p.*, statute measure.

By Rule II. Here 32*a.* 2*r.* 20*p.* = 32·625 acres; then, as 4840 *yds.* : 4000 *yds.* : : 32·625*a.* : 26·96281*a.* = 26*a.* 3*r.* 34*p.*, statute measure, the same as by Rule I.

Proof. As 4000 *yds.* : 4840 *yds.* : : 26·96281*a.* : 32·625*a.* = 32*a.* 2*r.* 20*p.*, customary measure, the same as given in the question; and obtained here by reversing Rule II.

Ex. 2. In 45 acres 3 roods and 15 perches, Wiltshire customary measure, how much statute measure?

By Rule II. Here 45*a.* 3*r.* 15*p.* = 45·875 customary acres; then, as 4840 *yds.* : 3630 *yds.* : : 45·875*a.* : 34·40625*a.* = 34*a.* 1*r.* 15*p.*, statute measure.

Proof. As 3630 *yds.* : 4840 *yds.* : : 34·40625*a.* : 45·875*a.* = 45*a.* 3*r.* 15*p.* customary measure, the same as given in the question.

NOTE.—In solving this and every similar question, the learner must recollect that 30 perches make 1 rood Wiltshire customary measure. (See the Introduction to this Section.)

Examples for practice.

1. Reduce 47*a.* 3*r.* 20*p.* Cornwall customary measure, to statute measure. *Ans.* 56*a.* 3*r.* 36*p.*

2. Reduce 22*a.* 1*r.* 27*p.* Lancashire customary measure, to statute measure. *Ans.* 36*a.* 1*r.* 10·3*p.*

3. Reduce 127*a.* 1*r.* 26*p.* Cheshire and Staffordshire customary measure, to statute measure. *Ans.* 269*a.* 2*r.* 1·07*p.*

4. How many statute acres are contained in 53*a.* 2*r.* 20*p.* of the customary measure of Wiltshire? *Ans.* 40*a.* 1*r.* 0*p.*

5. A gentleman's estate near Manchester, according to an ancient plan and survey, contains 345*a.* 2*r.* 30*p.* Lancashire customary measure, what is its content in statute measure?

Ans. 559*a.* 3*r.* 30*p.*

6. A gentleman's estate near Chester contains 436*a.* 3*r.* 36*p.* Cheshire customary measure, and is let at £8, 10*s.* 6*d.* per acre per annum; what would be an equivalent rent per acre if the estate were let by the statute acre?

Ans. £4, 0*s.* 7*d.* *per acre, statute measure.*

PROBLEM II.

To reduce Scotch and Irish land measures to English statute measure; and vice versâ.

The Scotch chain, by which land was formerly measured in Scotland, was divided into 100 links, in the same manner as the English statute chain; but the Scotch link was 8·88 inches in length; consequently the length of the chain was 888 inches, or 74 feet English measure.

The Scotch fall or perch was 18·5 feet in length; and hence the square fall or perch contained 342·25 square feet; and 40 falls or perches made 1 rood, and 4 roods made 1 acre. Also, 160 falls or perches made 1 acre; and consequently the *Scotch acre* contained $6084\frac{4}{9}$ square yards English measure.

The Irish chain was also divided into 100 links, each link being 10·08 inches in length; and consequently the length of the chain was 1008 inches, or 84 feet English measure.

The Irish perch was 21 feet in length, and hence the square perch contained 441 square feet, and 40 perches made 1 rood, and 4 roods made 1 acre. Also, 160 perches made 1 acre; and consequently the *Irish acre* contained 7840 square yards, English measure.

NOTE 1. The general rules given in the first problem may be applied to the reduction of both Scotch and Irish measures. Or, multiply Scotch acres by 1·25712, and Irish acres by 1·61983, and the respective products will be English statute acres. Also, if English statute acres be divided by these numbers, the respective quotients will be Scotch and Irish acres.

2. As both the Scotch and Irish chains were divided into 100 links, in the same manner as the English chain, it is manifest that the rules given in this work for finding the areas of different figures, and for laying-out, parting-off, and dividing land, were equally applicable in all cases of surveying, whether the dimensions were taken with the English, Scotch, or Irish chain.

3. The Scotch acre contained $1244\frac{4}{9}$ square yards, and the Irish acre 3000 square yards more than the English statute acre. It also appears that the Irish measure was the same as the Lancashire customary measure.

4. The Scotch mile was $1973\frac{1}{3}$, and the Irish mile 2240 English yards; and hence it appears that the Scotch mile was $213\frac{1}{3}$, and the Irish mile 480 yards more than the English mile.

Examples.

Ex. 1. Reduce 36*a.* 1*r.* 10*p.* Scotch measure to English statute measure.

By Rule I. Prob. I. Here, 36*a.* 1*r.* 10*p.* = 5810 Scotch perches; and $5810 \times 342{\cdot}25 = 1988472{\cdot}5$ square feet in 5810 perches; then $1988472{\cdot}5 \div 272{\cdot}25 = 7303{\cdot}8$ perches = 45*a.* 2*r.* 23·8*p.* statute measure.

By Note 1. Prob. II. Here 36*a.* 1*r.* 10*p.* = 36·3125*a.*, and $36{\cdot}3125 \times 1{\cdot}25712 = 45{\cdot}64917$*a.* = 45*a.* 2*r.* 23·8*p.* statute measure.

Ex. 2. Reduce 36*a.* 1*r.* 10*p.* Irish measure to English statute measure. *Ans.* 58*a.* 3*r.* 11*p.*

Ex. 3. The length of a rectangular field measured by the English statute chain is 1435 links, and its breadth 923 links; required the area of the field in English, Scotch, and Irish measures.

Ans. 13*a.* 0*r.* 39*p.* *English measure;* 10*a.* 2*r.* $5\frac{3}{4}$*p.* *Scotch measure; and* 8*a.* 0*r.* $28\frac{1}{4}$*p.* *Irish measure.*

REMARK.

The preceding problems and examples will be found useful, not only in reducing customary measures to English statute measure, but also in conveying information to our young surveyors relating to *ancient measures*, which will soon become only matters of history.

ESTIMATING LAND BY THE MILE.

The Method of Making a Rough Calculation of the Number of Acres contained in a Common, Moor, Lordship, County, or Kingdom.

Endeavour to ascertain in miles, as nearly as you can, either by your own observations or from the information of others, the mean length and breadth of the land to be estimated; then multiply the length by the breadth, and the product will be the area in square miles. Multiply this area by 640, the number of acres in a square mile, and the product thus obtained will be the area in acres, according to this method of calculating.

NOTE 1. The mean length and breadth of a county or a kingdom may be found from a map, in the following manner:—Measure several lengths by the scale of miles upon the map; add them together; and divide their sum by their number for a mean length. A mean breadth may be obtained by a similar process.

2. The foregoing method of finding the area of counties and kingdoms must, of course, be liable to considerable inaccuracy, not only as regards the method of taking the dimensions, but also as respects the correctness of the map and scale; for it is evident that if these be not truly delineated, the dimensions can never be obtained to any degree of accuracy.

3. When you have a correct map and scale of a county or a kingdom, its content may be found to a considerable degree of accuracy by the following method:—Divide the map into triangles and trapeziums in the most convenient manner, and straighten the crooked shores or coasts, either with a lantern horn, as directed in Part IV., or by the parallel ruler, as directed in Part V. Measure the bases, diagonals, and perpendiculars correctly, by the scale of miles belonging to the map; find the area of each figure separately; and the sum of these areas will be the whole area required.

Examples.

1. Suppose the mean length of a common or moor be estimated at $3\frac{3}{4}$ miles, and its mean breadth at $2\frac{1}{4}$ miles, what is the area in acres, according to this estimation?

$$\begin{array}{rl}
\text{miles.} & \\
3{\cdot}75 & \\
2{\cdot}25 & \\
\hline
1875 & \\
750 & \\
750 & \\
\hline
8{\cdot}4375 & \text{miles.} \\
640 & \\
\hline
3375000 & \\
506250 & \\
\hline
5400{\cdot}0000 & \text{acres.} \\
\hline
\textit{Ans. } 5400 & \textit{acres.}
\end{array}$$

2. If the mean length of a lordship be estimated at $4\frac{1}{4}$ miles, and its mean breadth at $2\frac{1}{2}$ miles; what is the content in miles and acres? *Ans.* 10·625 *miles and* 6800 *acres.*

3. The mean length of a county, found from a map, is 63 miles, and its mean breadth 42 miles; what is its area in miles and acres? *Ans.* 2646 *miles and* 1693440 *acres.*

4. According to Parliamentary Papers 1861, the content of Ireland is computed at 32·518 square miles; what is its area in acres? *Ans.* 20·811520 *acres.*

5. According to Parliamentary Papers, the content of Scotland is computed at 31·324 square miles; required its area in acres. *Ans.* 20·047360 *acres.*

6. According to Parliamentary Papers 1861, the extent of England and Wales is computed at 58,319 square miles; what is the area in acres? *Ans.* 37·324160 *acres.*

NOTE.—The county maps of the Ordnance Survey may be taken as examples for exercises.

PART VII.

THE METHOD OF MEASURING AND PLANNING MANSIONS, HOMESTEADS, VILLAGES, TOWNS, AND CITIES; DIRECTIONS FOR MEASURING AND PLANNING BUILDING GROUND, AND DIVIDING IT INTO CONVENIENT LOTS FOR SALE; AND MISCELLANEOUS QUESTIONS RELATING TO SURVEYING, LAYING-OUT, PARTING-OFF, AND DIVIDING LAND.

SECTION I.

THE METHOD OF MEASURING AND PLANNING VILLAGES, TOWNS, AND CITIES.

Villages, towns, or cities, mansions, and farm-homesteads, present themselves in almost every extensive survey, and are generally measured and planned with the adjoining or surrounding lands. The method of taking and laying down the dimensions of such places, and finishing the plans, will be given in the following sections.

Besides the plans of towns and cities are so essentially necessary for the purposes of commercial and general reference, that surveyors are not unfrequently employed in forming correct drawings of the same, in order to have them engraved and published in copperplates.

Without this art, we could not obtain the ichnography of towns and cities ; neither could we have any just idea of the shape, extent, and direction of the streets ; the size and number of the public buildings ; the local conveniences enjoyed by the inhabitants, &c., of those places which circumstances will not permit us to visit.

Directions for taking the Dimensions of Villages, Towns, and Cities.

The dimensions of villages, towns, and cities may generally be obtained by the chain only, as the streets are usually wide enough to admit of angles or tie-lines being taken at the meetings or intersections, in the same manner as directed in Problems IV. and V., Part IV. In these Problems the methods of measuring meres, woods, roads, rivers, and canals, are illustrated ; and as the learner becomes completely master of this branch of surveying, any difficulties that present themselves in measuring will be easily surmounted.

It sometimes happens that the tie-lines cannot be measured at a greater distance from the angular points than 30 or 40 links. In such cases the tie-lines must be taken to a quarter of a link, and both them and the angular distances must be multiplied by 2, 3, 4, or any larger numbers, as circumstances may require ; and the products used in laying down the chain-lines. (See Problem II. Part. IV.)

The notes taken in measuring towns and cities must be entered precisely in the same manner as in surveying estates ; and in measuring along the streets offsets must be taken to the houses on both sides of the chain-line, and particularly to every corner and projection ; even the small projections of bow-windows must not be omitted.

Sketches of the bases of the buildings, particularly the corners and projections, must be made in the margin of the note-book, in order to assist the surveyor in drawing a correct plan.

All public buildings, such as churches, prisons, castles, court-houses, market-places, halls, colleges, mansion-houses, &c., must be distinctly noticed ; and the range of the first line should be taken with the compass, in order that the draughtsman may be able to lay down every street in its true direction.

NOTE 1. In measuring along the streets, all the offsets to the buildings must be taken at right angles to the chain-lines. The bases of the buildings and all the projections must be sketched as you proceed ; and the breadths of the buildings, the lengths and breadths of the projections, &c., must be correctly measured and entered opposite to those parts of the sketch to which they respectively belong. The sign + (plus) is usually placed between the breadth of a building, at its perpendicular distance from the chain-line. The method of sketching the bases of buildings, and entering the notes, is exemplified in pages 4, 10, and 12, of the engraved field-book, to which the learner is referred. (See also Plate VII. No. 7, page 199.)

2. When a town and the surrounding or adjoining lands are both to be measured and planned together, the dimensions must be taken with Gunter's chain; and the lines measured along the streets must be properly connected with those measured in surveying the adjoining estates; but if the plan of a town only is required, it is more convenient to take the dimensions with a chain of 50 feet in length, divided into 50 links, and an offset-staff of 10 feet in length.

3. As station staves cannot always be fixed in the streets, in consequence of the pavement, they must either be set in pedestals made for that purpose, or two or more assistants must each hold a staff in those places that are pointed out by the surveyor.

4. Sometimes it is most convenient to measure external or main-lines on the outside of the town, as in surveying a mere or wood (Prob. IV. Part IV.); and in running such lines, stations must be left at the end of the streets, as you pass them, in order that lines may be run from one station to another in measuring the streets.

5. In some situations, and under certain circumstances, it is more eligible to measure the first line along one of the principal streets, and to intersect this line by another, measured along some other principal street, nearly at right angles with the former; then these two lines being tied together by a connecting line, measured in the most convenient manner, will divide the town into four parts, each of which may be measured separately by running lines in the most advantageous manner.

6. In putting down stations at the ends of the streets, &c., the number of the station may be made upon the wall of the opposite building (if there be one) with red or white chalk, in such a situation that an offset may be taken, at right angles to the building, from the station marked upon the wall to the station on the chain-line. This offset being entered in the book, and again measured from the station on the wall, at right angles to the building, will give you the station on the chain-line whenever you may want to find it.

7. When the foregoing method cannot be adopted, in consequence of not being able to take a right-angled offset from any building to the station which you wish to fix, then two lines may be measured from the station to the corners, or to any other parts of two adjoining buildings; and the intersection of these lines, when measured from the buildings, will give the station required.

8. After all the principal streets have been measured, then proceed to the smaller and intermediate streets; and lastly to the lanes, alleys, courts, yards, and every other part which it may be thought necessary to represent upon the plan.

9. When any of the streets are so narrow as not to admit of tie-lines being taken with the chain, the angles which the chain-lines make with each other, at the meetings or intersections of the streets, must be taken in degrees and minutes by a theodolite; and in planning, they must be laid down as directed in Problems XX. and XXI. Part I.

10. What has been advanced on this subject will, no doubt, be acceptable to learners; but as towns are built after such a variety of plans, and consequently vary so much in their forms, no directions can be given that will be applicable

to every particular case to be met with in practice. A great deal will always depend upon the skill and judgment of the surveyor, who should, after duly examining every part of the town, endeavour to run his lines in the most advantageous manner.

Examples.

Let it be required to measure the New Town, No. 7, Plate VII.

In order to follow the method described in Note 4, we shall begin at the south-west corner, as in Problem IV. Part IV.; although the survey would be conducted precisely in the same manner if we began at any other corner.

1*st Line.* Put down + 1, at the SW. corner, and proceed towards the SE. corner; taking offsets to the buildings, wherever it is necessary, and sketching their bases in the margin of the note-book. At the end of High Street, put down + 2; at Queen Street, + 3; at Low Street, + 4; at the SE. corner, + 5; and produce the line at pleasure to + 6.

2*nd Line.* From + 5 proceed towards the NE. corner, but when you arrive at the end of York Street, put down + 7, and thence run a tie-line to + 6. From + 7 proceed with the main-line, and at King Street put down + 8; at George Street, + 9; at the NE. corner, + 10; and continue the line to + 11.

3*rd Line.* From + 10 go towards the NW. corner; but when you come to the end of Low Street put down + 12, from which run a tie-line to + 11. Proceed from + 12, and at the end of Queen Street put down + 13; at High Street, + 14; and at the NW. corner, + 15.

4*th Line.* From + 15 proceed towards the SW. corner; and at the end of George Street put down + 16; at King Street, + 17; at York Street, + 18; and continuing the line to + 1, you will have circumscribed the town with four main-lines into which the lines measured along the streets must be run.

NOTE.—After the first three lines are laid down, it is evident that the fourth line will serve as a check, and will reach exactly from + 15 to + 1, if all the operations have been conducted with accuracy.

5*th Line.* From + 18, through York Street, to + 7.

6*th Line.* From + 8, along King Street, to + 17.

7*th Line.* From + 16, through George Street, to + 9.

8*th Line.* From + 12, along Low Street, to + 4.

9*th Line.* From + 3, through Queen Street, to + 13.

10*th Line.* From + 14, along High Street, to + 2; thus the survey of the town is completed.

NOTE 1. The chain-lines and stations do not appear upon the plan, as they could not have been conveniently entered without increasing its size; the learner will, however, find no difficulty in making a similar plan, two or three times as large; drawing the chain-lines, and putting down the stations in their proper places. Or he may take the dimensions of the given plan with a small scale, enter them in a note-book, and then draw a rough plan by a larger scale, and after that a finished one, which will be an exercise that will tend much to his improvement.

2. The survey of this town might have been carried on according to the directions given in Note 5, by measuring a line through King Street, and another through Queen Street; and then connecting these two lines together by tie-lines taken at the point of intersection.

3. Here it will be proper to observe that in taking an angle with the chain or theodolite, at the intersection or meeting of two lines, either the external or internal angle may be taken, as circumstances may make it most convenient; but it should always be remembered, that neither very acute, nor very obtuse angles should be measured, if it can be avoided, as both are liable to errors in laying down. Those angles which approach nearest to right angles should always be preferred as being most correct.

4. By way of proof it is an excellent plan to take both the angles. If they be taken by the chain, you will have a check-line by the scale; and if taken by the theodolite, their sum should be 180 degrees; and you will also have a proof in planning, in consequence of having measured an angle and its supplement. (See Definition 16, and Problems XX. and XXI.)

Directions for Surveying and Planning Mansions, Farm-buildings, Stack-yards, &c.

Landed estates, generally speaking, have mansions for their proprietors, and homesteads for tenants, and these occupy a prominent place on every plan.

In a survey, one of two ways may be ordered. *First*, a new site for a homestead or mansion may be proposed, involving the resubdivision of the land by new fences and roads; *second*, such improvements may be finished before the surveyor is employed.

Under the *first proposition*, the chief points that engage attention are, first, the form of position of buildings and fields; second, the site of the homestead and labourers' cottages; third, water for cattle, irrigation, and machinery; and fourth, roads: under the *second proposition*, surveying and planning only are involved.

1. A modern farm-homestead may be considered a manufactory consisting of a rectangular block of buildings, requiring for a site an area of land of a similar form, and in size according to the size of the form and number of cattle kept.

When the cattle-yards are roofed over, the position of the houses is of little moment, but when open they should have a southern exposure.

Rectanglar fields are to be preferred, and fences should run north and south, east and west. But this position, although the most favourable, is not always practicable, so that every individual case must be its own rule.

When the front of the houses does not run parallel to the fence opposite, there will be a triangular piece of land on each side to appropriate to some useful purpose. To throw such pieces into the adjoining fields would, in many cases, destroy their appearance and make them a perpetual eyesore, and what is worse, more expensive to labour. Small paddocks for calves, or for liquid manuring and soiling, if properly managed, are always the best paying land on the farm, and highly ornamental if well laid out; so that by such means every inch of land about the homestead can be turned to the best advantage.

2. The farm homestead and labourers' cottages should be centrally situated, but in practice the *third* and *fourth* points generally determine the sites of both upon a farm.

3. Water for the homestead, the cottages and fields being invaluable, is the first thing that should engage attention. The means of supply, be it pump or otherwise, should never be omitted, and when the water is conveyed from a distance, the fountain-head and direction of the pipes should also be indicated upon the plan. When a sufficient supply can be had for irrigation, or for water power, the same rules apply to water-courses, mill-dams, &c.

4. Roads and tramways for traction engines should be laid out on the same principles as railroads relative to levels. In many cases, therefore, their direction will not only determine the site of the homestead, but also the range of the fences.

When the farm has only to be surveyed and planned, it is usual to range one of the lines close past the homestead, so that the ground plan of the buildings can be laid down from it, as in the preceding example of a new town, No. 7, Plate VII. Part V.

The home farm of the landowner can be surveyed and planned as above, but improvements in connexion with the mansion, &c., involve much ornamental work, for which no general directions can be given. When the surrounding grounds are extensive and thickly wooded, some difficulty may be experienced in ranging lines and driving them with the chain, but the bearings can always be taken from some eminence by the theodolite, according to Section VI., Part IX., and the details easily worked out with the chain and plotting scales afterwards.

Directions for planning villages, towns, and cities.

All the main-lines must first be laid down, and the stations upon them marked off. The lines measured along the streets must then be drawn, and the stations upon them denoted. The bases of the buildings must next be laid down from the offsets, so as to form the streets, and shaded as directed in Part V., and exhibited in Plate VII. The rough plan must then be transferred to a clean sheet, by some of the methods described in Part V., in order to make a finished plan.

The bases of all public buildings, such as churches, castles, prisons, session-houses, market-places, infirmaries, hospitals, mansion-houses, monuments, &c., should be delineated upon the plan with the utmost correctness; and most surveyors draw the bases of the columns which support the roofs of market-crosses, the galleries of churches, &c., as exhibited in the plate to which we last referred.

The streets are usually left white; but some draughtsmen prefer colouring the causeways with a tint of blue to distinguish them from the carriage-roads, which are generally washed with a yellowish brown.

The grass-plots in gardens, public squares, &c., whether they be rectangles, rhombuses, circles, ovals or regular polygons, should be correctly delineated upon the plan; then shaded with Indian ink and washed with green, in the same manner as pasture grounds; and trees, water, pleasure-grounds, gardens, gravel-walks, &c., must be shaded and coloured as directed in Part V.

The name of the village, town, or city should be given in conspicuous characters, in some vacant part of the plan or map; and the names of all the streets, public squares, churches, colleges, halls, prisons, castles, court-houses, mansion-houses, market-places, lanes, alleys, courts, yards, &c., must be entered in their respective situations in the manner exhibited in Plate VII.

NOTE 1. If the dimensions be taken and laid down in feet, a scale of feet must be given; if in yards, a scale of yards must be given; if in chains and links, a scale of chains and links must be given; and if the town or city be very large, a scale of miles and furlongs may be given upon the plan, for the purpose of measuring distances; and as 220 yards make a furlong, the distance of one place from another may be easily obtained in miles and yards.

2. Any remarks or explanations that it may be thought necessary to give, may be entered in some vacant corner of the plan.

3. All plans, ornaments, &c., should first be drawn in pencil; and it will tend much to the improvement of the learner, if he form all his printing, German text, and large-hand letters by the pencil also, and then finish them with Indian ink.

4. In forming letters, ornaments, &c., with the pencil, the lines and strokes should be made as fine as possible, as the ink frequently runs upon the lead when the pencil has been used too freely; hence the necessity of applying Indian rubber after the outlines have been finely drawn with Indian ink, in order to remove the lead which is not covered by the ink, before we proceed to finish the letters, ornaments, &c.

5. If the pupil does not succeed well in his first attempt with the pencil, the letters, ornaments, &c., must be effaced with Indian rubber; and he must repeat the process until he can form all the letters, devices, &c., correctly.

6. Properly prepared lead pencils are essential for fine drawing, and will be found to answer well in making letters, ornaments, &c., when they are of a middling degree of hardness; because the marks made by them may be easily effaced.

7. After practice has made the learner a proficient in penmanship, he will be able to print text, and write more expeditiously without the use of the pencil.

8. Here it may not be improper to caution the learner against a very common fault of young draughtsmen; namely, that of making their lines and letters too strong, both with pencil and ink.

9. Engraven plans of towns, &c., as copy-books, may now be had in almost every county town in the kingdom, and if the learner can get possession of a plan of a place with which he is personally familiar, the example will be none the less interesting and instructive. And besides the ground plans of towns delineating the bases of houses, the areas of streets, parks, &c., those of large farmeries, with their outbuildings, gardens, and small paddocks adjoining, covering altogether an area of several acres of land, are not unfrequently called for on a larger scale than is exemplified on the general plan or atlas of the estate. This is more especially the case as to the ground plans and isometrical views of the mansion, gardens, pleasure-grounds, and buildings of the home farm of the proprietor. Engraven plans of many of these, and designs for an indefinite number more, are now accessible, and may be studied with advantage, and even transferred to drawing paper on a different scale. Nor is it advisable that the learner should exclusively confine his source of information to engraven plans, for many (especially farmers' and land-surveyors' sons) may very profitably get for an exercise to give in to their teachers a ground plan and view of their parents' homesteads or residences. Boys are generally fond of drawing, so that the more rudimentary period of education passes over smoothly, but when they advance to what may be termed *usefulness*, the measuring of the garden and the laying it down with mathematical accuracy upon a base line, is often surrounded with greater difficulty; but they should ever bear in mind that it is upon this latter that their professional eminence in surveying is to depend.

TO CLEAN PLANS OR MAPS.

It has already been intimated to the young draughtsman, that every precaution should be taken to keep plans and maps clean in executing them; but notwithstanding the greatest possible care, they will generally be somewhat soiled, both upon the face and back (perhaps in consequence of misfortunes), either by dust, ink, or colours; hence it is necessary to clean them before they are delivered.

To clean plans or maps that are soiled with dust, Indian ink, or colours.

Take a sharp penknife, with a roundish point, and scrape those parts gently which are besmeared with ink or colours, until you efface the blots; then use clean Indian rubber freely to those places that are soiled with dust; and lastly, rub the whole map well with white bread, taking care to pare the bread as it accumulates the dust.

NOTE 1. Indian rubber is made from the juice of a large tree, which grows in South America, Java, and India. The juice is obtained by making incisions through the bark of the tree. From the wounds thus formed the juice flows abundantly. It is usually brought to Europe in the form of pear-shaped bottles, which are made by spreading the juice over moulds of clay. These are dried in various ways according to the season. This done, the clay in the inside is then picked out by instruments.

2. When Indian rubber has become foul by frequent use, it may be cleaned by washing it in lukewarm water and soap.

To clean plans or maps that are blotted with common ink.

If the blots be light, they may be scraped out with a penknife, or effaced by rubbing them repeatedly with clean paper wet in water or saliva; but when they are deep, acid or salt of lemons must be used in the following manner: Dissolve a small portion of the acid in hot water, and with a clean hair-pencil, dipped in the solution, wash the blots until they are removed.

NOTE 1. Recent blots are easily obliterated; but when they are old, and very deep, it will be found necessary to let the paper dry, and repeat the wash several times. Salt of lemons is sold in small boxes by druggists.

2. When you have to write upon those places from which the blots have been emoved, the paper will bear the ink better if you rub a little pounce upon it with clean paper, and then smooth it with your folder, or with the haft of your penknife. Sometimes the paper will bear the ink pretty well, after a blot has been removed, by merely smoothing it as above directed.

3. Pounce or gum sandarach is a resinous gum which exudes from the juniper-tree, and is of a pale yellowish colour. It is imported in small pieces or tears, about the size of peas; and when reduced to powder, and passed through a fine sieve, it is used for rubbing upon writing paper, in places where any blot or writing has been scraped out.

4. Sometimes a spurious kind of pounce is made by reducing resin (the juice of the Scotch fir) to a powder; but this should never be used, as it is too gummy. It may be known from genuine pounce by being of a darker yellow, and having a stronger odour.

5. Large blots of Indian ink, common ink, or colours may be removed from maps or strong drawing paper by washing them repeatedly with a sponge and clear water, always taking care to squeeze and wash the sponge as it absorbs the ink or colours; repeating the process until the blots disappear.

6. Sponge is a kind of marine animal production, a species of the class polypi, found adhering to rocks, shells, &c., chiefly in the Mediterranean Sea. The best sponges are those which are white, light, and have the holes small, and near to each other.

7. Sponge is much used by landscape painters, in washing their pieces with water, after they have finished shading them with neutral tint. This process removes the superabounding shade, discharges the clouds, produces a remarkable softness, and prepares the pieces for taking the colours with ease and freedom.

SECTION II.

DIRECTIONS FOR MEASURING AND PLANNING BUILDING-GROUND, AND DIVIDING IT INTO CONVENIENT LOTS FOR SALE.

Directions for measuring building-ground.

Land lying in the vicinity of large towns is frequently sold by the square yard, for building-ground; and as it always bears a high price when the situation is eligible, it is of the greatest importance, both to the buyer and seller, to ascertain its contents with the utmost accuracy.

In order to accomplish this desirable object, the dimensions should be very correctly taken with a measuring-tape divided into yards, tenths, and hundredths; or with a tape divided into feet and tenths, or feet and inches.

When the dimensions are taken in feet and inches, the inches must be reduced to the decimal parts of a foot: and the area found from such dimensions must be divided by 9, to bring it into square yards.

Whatever be the shape of the ground to be measured, it must be divided into such squares, rectangles, trapezoids, trapeziums, or triangles as will give the true content of the whole: and if the side be crooked, offsets must be taken, as directed in Problem VI. Part III.

Narrow pieces of building-ground must be measured by Problem VII.; and if they be very irregular, their areas may be correctly found by the method of equidistant ordinates described in Problem IX., Part III.

NOTE 1. As a measuring tape is not so convenient in taking the dimensions of land as a chain, it is more eligible to use the latter when the land to be measured is extensive; the greatest care, however, must be used in order to obtain the dimensions correctly, which should be taken to a quarter of a link.

2. The chain must be completely stretched, and held at the bottom of the arrows in measuring; and if it be an inch or two overlong, an allowance must be made in the dimensions; thus, if a line 650 links be measured by a chain that is $2\frac{1}{2}$ inches above 66 feet, we shall have $6\frac{1}{2} \times 2\frac{1}{2} = 16\frac{1}{4}$ inches = 2 links *nearly;* hence the true length of the line will be 652 links.

3. The above method may also be adopted in measuring land, when it is found necessary to correct the dimensions taken by a chain that exceeds the proper length.

4. As 4840 square yards make 1 acre, 1210 square yards, 1 rood, and $30\frac{1}{4}$ square yards, 1 perch, we can easily reduce acres, roods, and perches to square yards, in the following manner: Multiply 4840 by the number of acres; 1210 by the number of roods; and 30·25 by the number of perches; then the sum of these three products will be the square yards required.

5. When the area is in square links, divide it by 20·6611, the number of square links in a square yard; and the quotient will be the area in square yards. (See the Table of Square Measures in Part III).

6. Building-ground is generally sold in small parcels. Sometimes, however, it is sold by whole fields together, which are afterwards divided by the buyer, and retailed out in small lots.

Examples.

1. The length of a rectangular piece of building-ground is 65·8 yards, and its breadth 32·6 yards; what is its area in square yards, and its value at 5*s.* 9*d.* per square yard?

```
    yds.
    65·8
    32·6
   -----
    3948
   1316
  1974
  -------
  2145·08 area
```

yd.		*s.*		*yds.*		*£*	*s.*	*d.*
As 1	:	5·75	: :	2145·08	:	616	14	$2\frac{1}{2}$ *the value.*

2. The length of a rectangle measures 85·36, and its breadth 43·28 yards; what is its area in square yards, and its value at 6*s.* 3*d.* per square yard?

Ans. The area is 1694·3808 square yards; and the value of the land 1154*l.* 9*s.* $10\frac{1}{2}d.$

3. The parallel sides of a piece of ground in the form of a trapezoid measure 84·63 and 72·78 yards, and the perpendicular distance between them 56·59 yards; what is its area in square yards?

Ans. 4453·91595 *square yards.*

4. The diagonal of a trapezium measures 236·5 feet, one of the perpendiculars 189·3 feet, and the other 127·9 feet; what is its area in square yards, and its value at 1*l.* 6*s.* 6*d.* per square yard?

Ans. The area is 4167·655 square yards; and the value of the ground 5522*l.* 2*s.* $10\frac{1}{4}d.$

5. The base of a triangle measures 369·9 feet, and the perpendicular 234·7 feet; what is its area in square yards, and its value at 2*s.* 6*d.* per square yard?

Ans. The area is 4823·085 square yards: and the value 602*l.* 17*s.* 8½*d.*

6. The three sides of a triangle measure 362 feet 3 inches, 316 feet 6 inches, and 284 feet 9 inches respectively; what is its area in square yards.

Ans. By Note 4, Part IV., you will find the area to be 4810 square yards.

7. Draw a plan of an irregular piece of land, and find its area in square yards, from the following dimensions, taken in feet.

	AB	
0	1286	145·6
247·6	1015	
	987	59·8
	790	136·5
317·6	720	
	560	164·2
223·5	465	
	346	124·8
345·2	268	
372·4	000	245·3
Begin at	A	and go West.

Answer.

Double areas

359147·3 offsets on the right
676164·8 ditto on the left

2)1035312·1 sum

9)517546·05 area in square feet

57517·33 ditto in square yards.

8. Required the plan of a piece of building-ground, and also its area in square yards, from the following equidistant ordinates, taken in feet.

	AB	
255·8	1000	
250·6	900	
246·3	800	
235·4	700	
221·2	600	
201·9	500	
176·8	400	
168·5	300	
157·7	200	
146·0	100	
139·1	000	
Begin at	A	and range East.

Answer.

394·9 the first and last ordinates
4009·6 four times the sum, &c.
1604·0 twice the sum, &c.
6008·5 sum total
100 the common distance
3)600850·0
9)200283·3 area in square feet
22253·7 ditto in square yards.

9. Required the plan of a piece of ground, and likewise its area in square yards, from the following dimensions, taken in feet.

	BC	
0·0	846	
33·2	700	
66·4	600	
74·8	500	
85·6	400	
62·3	300	
50·7	200	
25·9	100	
0·0	000	
	R. off B	
	AB	
	1253	
	1000	
	586	525·6 C
Begin at	A	and go West.

Answer.

Double areas
658576·8 triangle ABC
81307·2 offsets on BC
2)739884·0
9)369942·0 area in square feet
41104·6 ditto in square yards.

10. Required the plan of a portion of building-ground, and also its area in square yards, from the following dimensions, taken by Gunter's chain; likewise its value, supposing it to have been sold by auction at 14*s.* 9*d.* per square yard.

	BD	
	1235	
	1075	474½ A
C 482½	270	
	R. off B	
	AB	
22¼	1175	
25½	1100	
45¼	1000	
40½	900	
54¾	800	
68½	700	
70¼	600	
65¾	500	
60¼	400	
55½	300	
40¾	200	
32¼	100	
0	000	
Begin at	A	and go West.

NOTE.—In calculating the area, the quarter-links must be treated as decimals.

Answer.

Double areas
1181895·00 trapezium ABCD
112881·25 offsets on AB
2)1294776·25 sum
647388·125 area in square links.

By note 5, we have 647388·125 ÷ 20·6611 = 31333 67, the area in square yards; then, as 1 yd. : 14·75*s.* : : 31333·67 yds. : 462171·6325*s.* = 23108*l.* 11*s.* 7½*d.*, the value required.

Directions for planning building-ground, and dividing it into convenient lots for sale.

Building-ground may be laid down by a plotting-scale, whether the dimensions be taken in yards or in feet, by calling each chain upon the scale one yard, or one foot, as the case requires; and the intermediate divisions will evidently be tenths of a yard, or tenths of a foot.

If the plot of ground be small, the scale made choice of should be pretty large, so as to exhibit every part of the ground distinctly.

After the plan has been drawn, the ground must be judiciously divided and laid out, by making streets at a proper distance from each other; and then subdivided into convenient parcels or lots for sale, according to the situation of the place and the size of the houses for which the ground is best adapted.

Main or principal streets should be much wider than it is necessary to make back or intermediate streets; and the size of the house-steads adjoining main streets should exceed the size of those adjoining back streets.

When practicable, streets should be laid out in straight lines; and if possible on the rectangular plan shown Plate VII. Part V.

Streets are laid out of very different breadths, from 15 to 90 or 100 feet; but when ground is of great value, the breadth of the streets becomes an object of consideration, whether the ground occupied by them be given by the seller, or purchased by the buyer of the adjoining lots.

Building-ground may sometimes be very elegantly laid out in a square or a rectangle. When this is the case, the houses are built on the margin or outside of the square; and in the middle is left an open area, which is generally ornamented with grass-plots, gravel-walks, trees, &c.

If the ground will admit, it is very desirable for each house to have a garden laid out in the front; which must, of course, be sold with the house-stead. The open area may be divided into as many equal parts as there are house-steads in the square; one part may be sold with each house-stead; and the respective purchasers may occupy the whole as joint property.

Ground laid out in this manner generally fetches a good price, as most persons think it more pleasant and healthful living in squares than in streets.

After everything has been properly and judiciously arranged upon the plan, such dimensions must be taken by the scale as will enable the surveyor to stake out all the streets, squares, lots, house-steads, &c., in the field. This being done, the ground may then be considered as ready for inspection and sale.

NOTE 1. House-steads must be laid out of different sizes, according to the respectability of the intended buildings.

2. A plan of ground or buildings, intended for sale, is generally left for inspection at the office of the surveyor or solicitor employed on the occasion, from the time of advertising to the time of sale. Also, the special conditions of the sale, not specified in the advertisement, may commonly be known at those offices, or by applying to the proprietor, or to his agent, previously to the day of sale.

3. Building-ground is generally sold by auction; and if it be divided into small lots, it will tend much to promote the sale, as many persons may be desirous of purchasing a single house-stead, who would not find it convenient to purchase a lot containing two or three house-steads.

4. The price of building-ground varies from sixpence to upwards of two guineas per square yard, according to the eligibility of the situation.

5. The method of laying out building-ground so as to form straight streets at right angles to each other is exemplified in the Plan of a new Town, Plate VII.

SECTION III.

MISCELLANEOUS QUESTIONS RELATING TO SURVEYING, LAYING OUT, PARTING OFF, AND DIVIDING LAND.

1. The base of the largest Egyptian pyramid is a square whose side is 693 feet; how many acres of ground does it cover?
Ans. 11*a.* 0*r.* 4*p.*

2. Required the side of a square garden that cost 3*l.* 18*s.* $1\frac{1}{2}$*d.* trenching, at $1\frac{1}{2}$*d.* per square yard. *Ans.* 25 *yards.*

3. Required the area of a rectangle whose length is 1275 and breadth 675 links. *Ans.* 8*a.* 2*r.* 17*p.*

4. The area of a rectangular field is 14*a.* 2*r.* 11*p.*; what is its length, its breadth being 925 links? *Ans.* 1575 *links.*

5. A rectangular allotment upon a common cost 78*l.* 1*s.* $10\frac{1}{2}$*d.* digging and levelling, at 7*l.* 10*s.* per acre; what will be the expense of fencing it half round, at 5*s.* 6*d.* per rood; its length being 1225 links. *Ans.* 17*l.* 18*s.* 8*d.*

6. Measuring along the base of a field in the form of a rhomboid, I found the perpendicular to rise at 678, and its length 1264 links; the remainder of the base measured 2435 links; what is the area of the field? *Ans.* 39*a.* 1*r.* $15\frac{3}{4}$*p.*

7. A grass-plot, in a gentleman's pleasure-ground, cost 3*l.* 14*s.* 1*d.* making, at 4*d.* per square yard; what is the length of the base, the perpendicular being 40 feet, and the figure a rhombus?
Ans. 50 *feet.*

8. What is the area of a triangular field, the base of which measures 3568 links, the perpendicular 1589 links, and the distance between one end of the base and the place of the perpendicular 1495 links?
Ans 28*a.* 1*r.* $15\frac{1}{2}$*p.*

9. After measuring along the base of a triangle 895 links, I found the place of the perpendicular and the perpendicular itself = 994 links, the whole base measured 1958 links; what is the area of the triangle? *Ans.* 9*a.* 2*r.* 37*p.*

10. The area of a triangle is 6 acres, 2 roods, and 8 perches, and its perpendicular measures 826 links; what will be the expense of making a ditch the whole length of its base, at 2*s.* 6*d.* per rood?
Ans. 6*l.* 4*s.* $7\frac{1}{4}$*d.*

11. What is the area of a triangle whose 3 sides measure 15, 20, and 25 chains respectively? *Ans.* 15 *acres.*

12. Required the area of a grass-plot in the form of an equilateral triangle, whose side is 36 feet? *Ans.* 561·18446 *feet.*

13. What is the area of a triangular field whose 3 sides measure 2564, 2345, and 2139 links? *Ans.* 23*a.* 2*r.* 0½*p.*

14. The three sides of a trianglar fish-pond measure 293, 239, and 185 yards; what did the ground which it occupies cost, at 185*l.* per acre? *Ans.* 843*l.* 7*s.* 8*d.*

15. How many square yards of paving are there in a trapezium whose diagonal is found to measure 126 feet 3 inches, and perpendiculars 58 feet 6 inches and 65 feet 9 inches?

Ans. 871·47569 *yards.*

16. In taking the dimensions of a trapezium, I found the first perpendicular to rise at 568, and to measure 835 links; the second at 1865, and to measure 915 links; the whole diagonal measured 2543 links; what is the area of the trapezium?

Ans. 22*a.* 1*r.* 0*p.*

17. Lay down a trapezium, and find its area from the following dimensions; namely, the side AB measures 345, BC 156, CD 323, DA 192, and the diagonal AC 438 feet.

Ans. 52330·33406 *feet.*

18. What is the area of a trapezoid whose parallel sides measure 25 and 18 feet; and the perpendicular distance between them, 38 feet? *Ans.* 1197 *feet.*

19. The parallel sides of a piece of ground measure 856 and 684 links, and their perpendicular distance 985 links; what is its area?

Ans. 7*a.* 2*r.* 13½*p.*

20. If the parallel sides of a garden be 65 feet 6 inches and 49 feet 3 inches, and their perpendicular distance 56 feet 9 inches; what did it cost at 325*l.* 10*s.* per acre?

Ans. 24*l.* 6*s.* 7¼*d.*

21. It is required to lay down a pentangular field, and find its annual value at 2*l.* 5*s.* per acre, the first side measuring 926, the second 536, the third 835, the fourth 628, and the fifth 587 links; and the diagonal from the first angle to the third 1194, and that from the third to the fifth 1223 links?

Ans. 18*l.* 10*s.* 7½*d.*

22. The diameters of an elliptical piece of ground are 330 and 220 feet; and how many quicks will plant the fence forming the circumference, supposing them to be set 5 inches asunder?

Ans. 2073.

23. Given the lengths of 7 equidistant ordinates of an irregular

piece of ground, as follows; 15, 19, 20, 23, 25, 30, and 33 feet; and the length of the base 72 feet; require the plan and area.

Ans. 1704 *feet.*

24. What must be the length of a chord which will strike the circumference of a circular plantation that shall contain just an acre and a half of ground? *Ans.* 48·072 *yards.*

25. The annual rent of a triangular field is 43*l.* 15*s.*, its base measures 25, and perpendicular 14 chains; what is it let for per acre? *Ans.* 2*l.* 10*s.*

26. The transverse diameter of the ellipse in Grosvenor Square measures 840 links, and the conjugate 612, within the wall; the wall is 14 inches thick; what quantity of ground does it inclose, and how much does it occupy? *Ans. The wall incloses* 4*a.* 0*r.* 6*p.*, *and occupies* 1760·531 *square feet.*

27. Two sides of an obtuse-angled triangle are 5 and 10 chains; what must be the length of the third side that the triangle may contain just two acres of ground?

Ans. 8·06225 *or* 13·60147 *chains.*

28. What is the area of an isosceles triangle inscribed in a circle whose diameter is 24; the angle included by the equal sides of the triangle being 30 degrees? *Ans.* 134·3538.

29. The side AB of a triangular field is 40, BC 30, and CA 25 chains; required the sides of a triangle parted off by a division-fence made parallel to AB, and proceeding from a point in CA at the distance of 9 chains from the angle A.

Ans. 16, 19·2, *and* 25·6 *chains.*

30. A field in the form of a right angled triangle is to be divided between two persons, by a fence made from the right angle, meeting the hypotenuse perpendicularly, at the distance of 880 links from one end; required the area of each person's share, the length of the division-fence being 660 links?

Ans. 2*a.* 3*r.* $24\frac{1}{2}$*p.* *and* 1*a.* 2*r.* $21\frac{1}{4}$*p.*

31. It is required to part from a triangular field whose 3 sides measure 1200, 1000, and 800 links respectively, 1 acre, 2 roods, and 16 perches, by a line parallel to the longest side.

Ans. The sides of the remaining triangle are 927, $772\frac{1}{2}$, *and* 618 *links respectively.*

32. The base of a field, in the form of a trapezoid, is 30, and the two perpendiculars are 28 and 16 chains respectively; it is required to divide it equally between two persons, by a fence parallel to the perpendiculars. *Ans. The division fence is* 22·8035 *chains, and it divides the base into two parts, whose lengths are* 17·0087 *and* 12·9913 *chains respectively.*

33. A gentleman a garden had,
Five score feet long and four score broad;
A walk of equal width half round
He made, that took up half the ground;
Ye skilful in geometry,
Tell us how wide the walk must be.

Ans. 25·96876 *feet.*

NOTE 1. If the sum of the two diameters of an ellipse be multiplied by 1·5708, the product will be the circumference, exact enough for most practical purposes. (See Question 22.)

2. All the foregoing questions are taken from the author's Treatise on Practical Mensuration; consequently, their solutions may be found in the Key to that work.

PART VIII.

PLANE TRIGONOMETRY.

PLANE TRIGONOMETRY teaches how to determine certain unknown parts of a triangle from having the other parts given. The method of resolution and solution is based upon the relation that exists between the sides and angles of triangles. Thus, in the right-angled triangle ABE (fig. 1), if the sides AB and AE, together with the included angle EAB, are given, the side subtending the given angle and the angles adjacent to it may then be found.

SECTION I.

DEFINITIONS.

1. The circumference HBKE of a circle is divided into 360°, each degree into 60′, and each minute into 60″.

2. An arc is a part of the circumference lying between two radii, and is the measure of the angle which the latter form at the centre. Thus AB is the measure of the angle ACB (by construction = 60°).

3. When the two diameters HK and EB divide the circle into four equal parts, HCB, BCK, KCE, and ECH, they are termed quadrants of the circle.

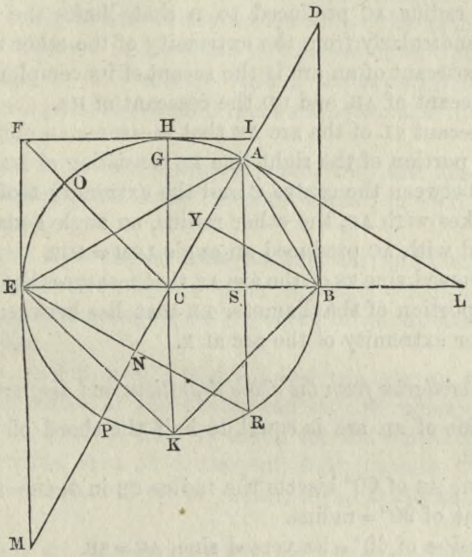
4. A right angle is a quadrant, and contains 90°.

5. The complement of an arc is what it wants of 90°. Thus AH and AB are the complements of each other.

6. The supplement of an arc is what it wants of 180°. Thus AB and AE are the supplements of each other.

7. The sine AS of the arc AB that measures an acute angle ACB is a right line drawn from the extremity A of the radius CA perpendicular to the radius CB that bounds the other extremity of the arc at B, and is parallel to BD.

Fig. 1.



8. The cosine of an arc that measures an acute angle is the sine of its complement. Thus AG is the cosine of AB, and AS the cosine of HA.

9. The versed sine of an arc that measures an acute angle is that segment of the radius which lies between the sine and the arc. Thus SB is the versed sine of AB, and GH the versed sine of HA.

10. The coversed sine of an arc is the versed sine of its complement. Thus GH is the coversed sine of AB, and SB the coversed sine of HA.

11. The sine EN of the arc AE that measures an obtuse angle as ACE, is a right line drawn from the extremity E of the radius CE perpendicular to the opposite radius AC produced to N, and is parallel to the tangent AL from the other extremity of the arc at A.

12. The tangent BD of the arc AB that measures the acute angle ACB is a perpendicular drawn from the extremity B of the radius CB that meets the other radius CA produced to D.

13. The cotangent of an arc that measures an acute angle is the

tangent of its complement. Thus HI is the cotangent of AB, and BD the cotangent of HA.

14. The tangent AL of the arc AE, which measures the obtuse angle ACE, is a perpendicular drawn from the extremity A of the radius CA, that meets the other radius EC produced to L, and that is parallel to the sine EN, drawn from the opposite extremity of the arc.

15. The secant CD of the arc AB that measures an acute angle, as ACB, is the radius AC produced to D that limits the tangent BD, drawn perpendicularly from the extremity of the other radius at B.

16. The cosecant of an arc is the secant of its complement. Thus CI is the cosecant of AB, and CD the cosecant of HA.

17. The secant CL of the arc AE that measures an obtuse angle, as ACE, is that portion of the right line EL consisting of EC produced to L, that lies between the centre C and the extremity L of the tangent AL, that makes with AC, the other radius, an angle equal to its supplement, and with AC produced an angle LCM = ACE.

18. The versed sine ES of the arc AE that measures the obtuse angle ACE is that portion of the diameter EB that lies between the sine AS and the other extremity of the arc at E.

Corollaries from the above definitions and diagram.

1. The sine of an arc is equal to half the chord of double that arc, $AS = \frac{1}{2}AR$.

2. The sine AS of 60° bisects the radius CB in S, $CS = SB$.

3. The sine of 90° = radius.

4. The cosine of 60° = its versed sine, $AG = SB$.

5. The chord of 60° and the tangent of 45° are each equal to radius, $AB = AC = HF$.

6. The sine, tangent, and secant of 45° are respectively equal to the cosine, cotangent, and cosecant of 45°.

7. The tangent of 60° is equal to the chord of 120°, $BD = AR$.

8. The secant of 60° = diameter, $CD = EB$.

9. The secant of 45° bisects the chord of 90°, and is equal to it, $CF = EK$.

10. The square of the diameter is equal to twice the square of the chord of 90° or secant of 45°, $EB^2 = EK^2 + BK^2$.

11. The square of the diameter is equal to the square of the chord of 120° added to the square of the radius, $EB^2 = AE^2 + AB^2$.

12. The square of the chord of 90° is equal to twice the square of the radius, $CF^2 = CH^2 + HF^2$.

13. The sum of the three angles of a triangle is equal to 180°.

14. If one of the acute angles of a right-angled triangle be given, the other acute angle is its complement, so that all the angles are

given. And if any two angles of a triangle = 90°, then the remaining angle = 90°, and the triangle is right-angled.

15. If one of the angles of an oblique-angled triangle be given, then that angle subtracted from 180° gives the sum of the other two angles; and if two angles are given, their sum subtracted from 180° gives the remaining angle.

16. If one of the equal angles of an isosceles triangle be found, then the other two angles are determined, because the sum of the two equal angles adjacent to the base subtracted from 180° gives the remaining angle at the vertex. And if the two angles at the base are equal, the sides that subtend them are equal, and the triangle is isosceles.

17. The two acute-angled triangles ABC and AER are each equilateral and equiangular, and therefore similar to each other, and each angle is 60°.

18. The three right-angled triangles ASB, ASC, and CGA are similar and equal to each other; ANE = ESA; also DBC = LAC = MEC.

19. A triangle standing upon the chord of an arc with two sides radii is isosceles.

20. In the resolution of a right-angled triangle, as CAL, if C is taken for the centre of the circle, and CA for radius, then AL is tangent and CL secant. If L is taken for the centre of the circle, and LA for radius, then AC is tangent, and LC secant. And if L is taken for the centre, and LC for radius, then CA will be sine and AL = cosine.

21. To every arc measuring either an acute or obtuse angle two sines, two tangents, and two secants can, and often require to be drawn.

22. In an arc subtending an acute angle the two sines and two tangents cross each other within the angle, so that the centre of the circle and the two points of intersection, the one where the two sines cross, and the other where the two tangents cross, are in a right line that bisects the arc, its chord, and the angle it measures.

23. Each of the two radii, which includes an acute angle at the centre, forms a portion of the two secants to an arc; and when the angle is 60° the portion of each that is inside the arc is equal to the remaining part outside the arc. Thus CB = BL, so that the tangent AL of 60° diverges from the arc AB, a distance equal to radius.

24. In an arc subtending an obtuse angle the two sines from its opposite extremities fall without the arc perpendicularly upon the two secants, now portions of other radii than those which include the angle. The two tangents are limited by the two secants, ranging from the centre at an angle equal to the supplement of the arc, and

in the opposite direction of the two radii that include the arc or angle. The secant therefore contains no part of the radii that include the angle, but the whole right line formed by each radius and its secant together is greater than the secant of its supplement by radius. Thus EL = CL + EC.

25. In arcs subtending obtuse angles the parallelism of sines with tangents, and *vice versâ*, lies on the opposite sides of the centre, the tangent being on one side, and its parallel sine on the other. Thus AS is parallel to EM, and EN to AL.

26. In arcs subtending acute angles the parallelism of the sines and tangents of an arc lies within the angle upon the same side of the centre. Thus AS is parallel to BD, and BV to AL.

27. The sines, tangents, and secants of an arc are equal in magnitude or length to the sines, tangents, and secants of the supplement of that arc; but their position is different, so that a sine, tangent, or secant of the one cannot properly be termed the sine, tangent, or secant of the other, while the versed sine of an obtuse angle is greater than the versed sine of its supplement.

28. It is the direction and position of the two sides of a triangle that determine the measure of the angle they include, and *vice versâ* the measure of the angle they include that fixes the position of any two sides.

29. When a right line is drawn from the obtuse angle of an oblique-angled triangle, perpendicular to the base or side subtending the obtuse angle, it resolves the oblique-angled triangle into two right-angled triangles; the perpendicular thus drawn becomes the sine to each of the two angles adjacent to the base; the sides that include the obtuse angle are radii; and the segments of the base are each equal to the cosine of its respective adjacent angle. Thus in the obtuse triangle EAL, AS is sine to E and L, ES = cos. E, and LS = cos. L, and ASE and ASL are each right-angled triangles.

NOTE 1. The sine, tangent, and secant of an arc, are in some books defined to be the sine, tangent, and secant of the supplement of that arc. This, however, is not correct: the two arcs measure two angles of a different species, the one being acute and the other obtuse, according to their geometrical definitions (see Def. 12, 13, 15, 18, 19, 20, 22, 23, and 24, Part I.); consequently each has a separate and independent existence distinct from the other. Thus BD, although defined to be the tangent of AE, is not its tangent; neither is EM the tangent of AB. Again BV is not the sine of AE, nor EN the sine of AB; much less is ES the versed sine of AE, viz., ES also the versed sine of AB; and lastly CD is not the secant of AE, nor CM the secant of AB. Hence the conclusion of dissimilarity as to position (cor. 27).

2. The diagram is drawn purposely to illustrate the dissimilarity of position that exists between the sines, tangents, secants, and versed sines of angles and their supplements, and also to show some of the properties of triangles formed by lines

in and about the circle that are of daily use in taking bearings and tie lines in surveying. It may also be added here that figs. 2 and 3 of the next section will still further show how absolutely necessary it is to attend to such distinctive characteristics as the position of lines, and the species of angles they subtend, in order to secure accuracy in the trigonometrical resolution of the triangles into which a survey may be divided, and to avoid ambiguity and error in plotting the same.

SECTION II.

ON SPECIES, CONSTRUCTION, RATIOS, RESOLUTION, AND SOLUTION, OF TRIANGLES.

1. Under Geometry, Part I., six kinds of plain triangles are defined, viz., (1) right-angled; (2) equilateral; (3) isosceles; (4) scalene; (5) acute-angled; and (6) obtuse-angled.

2. The angles of triangles are divided into three kinds, and defined (1) right angles; (2) acute angles; and (3) obtuse angles.

3. A plane triangle consists of six parts, viz., three sides and three angles; and if three of these, including one side, are given, the others, technically termed the sought parts, may be ascertained from the ratio or relation that subsists between them and the given parts.

4. In Plane Trigonometry triangles are divided into two classes, right-angled triangles and oblique-angled triangles. (The latter thus includes the five species of triangles that are not right-angled.)

5. In a trigonometrical survey the ground is divided into triangles. Thus any three station-poles not in a right line form a plane triangle.

6. Fields, farms, and estates may be divided and measured with the chain, either on the triangular method, as illustrated Plate III. Part V., or on the parallel method (Plate IV. Part V.), and yet both may be surveyed by the theodolite as if divided into triangles independently of those of the former survey. Thus the survey (Plate III.) is divided into two large triangles + 1 + 8 + 10, and + 1 + 8 + 15; or into four triangles + 1 + 5 + 10, + 5 + 10 + 8, + 8 + 5 + 15, and + 15 + 5 + 1. Again the estate (Plate IV.) may be divided into two triangles + 1 + 6 + 32, and + 1 + 32 + 40. Or the same station-poles shown on the plan may be used in taking the bearings, so as to give a much larger number of triangles in the trigonometrical survey. Thus each of the quadrilateral areas may be taken as forming two triangles.

7. In large surveys, where each of the interior triangles is bounded

by other three triangles, if the sides of one are determined, that will give one side in each of the adjacent triangles.

8. In trigonometrical surveying a side of one of the triangles is measured with the chain. This is termed the *main-line* or *base-line.* The other two sides may then be determined from the relation they bear to it (the measured line), and to the angles they subtend; and as a side of each of the adjacent triangles is thus found (by last article, 7), the sides of all the other triangles in the survey may therefore be ascertained from trigonometrical data.

9. The first triangle surveyed and plotted is the one that contains the base-line; and as this side is measured with the chain and the two adjacent angles with the theodolite, such data will give the other angle and species of triangle either by cor. 14, 15, or 16, Sect. I.

Right-angled triangles.

10. The field-book of the trigonometrical survey supplies only part of the dimensions requisite for plotting. Technically these are termed the "bearings," or "the given sides and angles," so that the remaining parts are found by calculation, or construction, or both, as illustrated in the following four examples.

Ex. 1. Let the base line be EB in the triangle AEB (fig. 1): let B = 60° and E 30°, then (by cor. 14) the triangle is right-angled and scalene. The side EB and angles B and E are given to find BA and EA: A is found by cor. 14.

Ex. 2. If the angles at the base are each 45° then EKB would represent the triangle; and as the angles E and B are equal, so are the sides BK and EK. The triangle is therefore both isosceles and right-angled, with a side and three angles given to find the remaining two sides.

Ex. 3. If one of the angles at the base is a right angle, then ASE would represent the triangle; ES the base line measured with the chain; E and S the angles taken with the theodolite; EA and SA the sides sought, and A the remaining angle, determined by cor. 14. Sect. I.

Ex. 4. In the interior triangles of a survey the two sides ES and SA, or EA and AS, or EA and ES, and the angle A = 90° may be found, when the remaining side EA and two acute angles E and A adjacent will be the parts sought: or ES and the angles E and A; or AS and the angles E and A, as the case may be.

11. The large triangles on the plans Part V. Plates III. and IV. may be considered copies of those in the field, so that plotting the first two bearings is simply a work of copying (Problems XIX. XX. and

XXI. Part I.), and when the two bearings or angles at the base are given the first triangle may be formed (cor. 28). Thus draw the base-line EB; next EA indefinite, making E = 30° (Problem XX. Part I.); BA in the same manner, making B = 60°: the two indefinite lines will, if the work is correctly performed, intersect each other at A, and form the triangle EAB right-angled at A. The lines EA and BA may then be measured by the scale on which EB has been laid down upon the plan, or they may be found by trigonometry. Thus, if EB is radius, as in the first two examples, then BA would be sine and EA cosine. The ratio would therefore in these two examples be that of radius to the sine and cosine of the angle E = 30°. In the *third example* ES is radius, SA tangent, and EA secant, consequently the ratio is that of radius to the tangent and secant of E = 30°. In the former case, either of the sides about the right angle may be sine or cosine; *i.e.*, if the one is sine the other is cosine, and in the latter, either side about the right angle may be radius or tangent; *i.e.*, if the one is radius the other is tangent.

12. The trigonometrical solution of the four examples would therefore be effected by the following formulas:—

Ex. 1.	To find BA	formula	1	Rad. : sin. E : : EB : BA
,,	To find EA	,,	2	Rad. : cos. E : : EB : EA
Ex. 2.	To find BK	,,	3	Rad. : sin. E : : EB : BK
,,	To find EK	,,	4	Rad. : sin. B : : BE : EK = BK
Ex. 3.	To find EA	,,	5	Rad. : sec. E : : ES : EA
,,	To find SA	,,	6	Rad. : tan. E : : ES : SA
Ex. 4.	To find EA	,,	7	Rad. : sec. E : : ES : EA
,,	To find E	,,	8	ES : SA : : rad. : tan. E
,,	To find ES	,,	9	Rad. : cos. E : : EA : ES
,,	To find E	,,	10	EA : AS : : rad. : sin. E
,,	To find SE	,,	11	sec. A : tan. A : : EA : SE
,,	To find A	,,	12	AS : SE : : rad. : tan. A

NOTE.—Under right-angled triangles it is common to consider the right angle as understood to be one of the parts given, although not so expressed. This however, is objectionable, for when articled pupils enter the field with such formulas stereotyped as it were upon their memories they are worse than useless to them, as they frequently lead to much confusion. In taking the bearings, right angles have to be found in the same way as acute and obtuse ones; and it may be further observed that unless the two acute angles = 90° as a *proof check*, right-angled triangles are treated as oblique ones; and as a general rule this is the more advisable one to follow in the trigonometrical survey. Thus the right-angled triangle EAB (Fig. 1) is resolved into two right-angled triangles by drawing the perpendicular AS.

Oblique-angled triangles.

13. When three parts of an oblique-angled triangle are given to find the remaining parts, it requires to be resolved into two or more right-angled triangles. This is done by drawing perpendiculars from the angles to the sides that subtend them (by cor. 29), or to the sides that subtend them produced. Thus AS (fig. 1) resolves the three triangles EAB (right-angled), EAL (isosceles), and CAB (equilateral) each into two right-angled triangles.

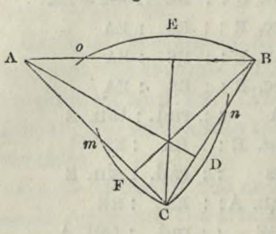
Ex. 1. If the angles at the base are each 60° then the remaining angle is 60° and the sides opposite the given angles each equal to the base (cor. 17). In this case resolution is unnecessary.

Ex. 2. If the angles at the base are equal, but greater or less than 60°, then the triangle is isosceles, and a perpendicular from the vertex to the base will resolve it into two equal right-angled triangles. In this case half the base may be taken for radius, when the side opposite the right angle would be secant of the given angle. Thus let EAL (fig. 1) be the triangle, EL the base ; E and L each 30° ; then ES would be radius, and EA found by formula 5 under article 12 ; AL and A by cor. 16.

Ex. 3. If the angles at the base are acute and unequal, the one being 45° and the other 60°, then the remaining angle opposite the base or given side is 75° and the triangle acute and scalene.

Let ABC (fig. 2) be the triangle, AB the base, A and B the angles taken with the theodolite, AC and BC the sides sought, C being found by cor. 15. Draw the perpendiculars AD, BF, and CE, which resolve AB into six right-angled triangles.

Fig. 2.

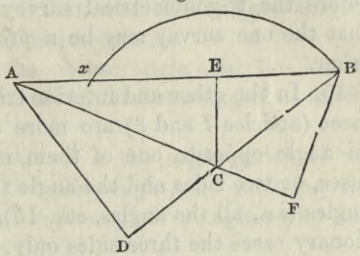


In the solution of this example each of the angles A, B, and C is common to two right-angled triangles ; their complements therefore are equal, or the remaining acute angle of each of the two triangles is equal each to each, so that the triangles are similar, each perpendicular being the sine of the two angles adjacent to the two segments of the base. The sides opposite the equal angles are thus proportional, and the ratio is that of a greater to a less magnitude, the given side being the greatest, or the given side being the radius of the greatest of the three circles now involved. *The sides therefore are to one another as the sines of the angles opposite.* Thus AB : AC : : sin. C : sin. B. To find AC, therefore, we have sin. C : rad. : : AD : AC ; sin. B : rad. : : AD : AB. Then inversely rad. : sin. B : : AB : AD ; and by *ex æquo* inversely sin. C : sin. B : : AB : AC. Again, to find CB we have C the centre with CB radius, and BF sine; and A the centre with AB as radius, and BF sin. Then sin. C : rad. : : BF : CB; sin. A : rad. : : BF : AB; by inversely rad. : sin. A : : AB : BF; and by *ex æquo* inversely sin. C : sin. A : : AB : CB.

Ex. 4. If the angles adjacent to the base-line are unequal and their sine less than 90°, then the triangle is obtuse-angled and scalene, the obtuse angle being opposite the base.

Let ABC (fig. 3) be the triangle, AB the base-line, A and B the angles adjacent whose sum is 60°; C = 120° (by cor. 15). Draw the perpendiculars, AD on BC produced, BF on AC produced, and CE on AB, the base. In this case because AC and BC are each less than AB, the sines AD and BF of the angles A and B when AB is radius, falls without the triangle; and because C is obtuse AD is the sine of C = 120°, when CA is radius; and BF the sine of C = 120° when CB is radius. But when A and B are the centres, and AC and BC the radii, then CE is the common sine; and the segments of the base the cosines of A and B. The sides AC and BC may thus be obtained by several methods of solution. Thus AB : AC : : sin. C : sin. B. Or rad. : sin. B : : BA : AD; and as the angle ACD is equal to BCF = 60°, then by making AD radius, rad. : sec. A : : AD : AC. BF and BC may be found in a similar manner. The area of the triangle ABC is obtained from CE as sine to A and B, AC and BC being respectively the two radii.

Fig. 3.



Ex. 5. If one of the angles at the base is obtuse, say 120°, and the other less than 30°, so that the acute angles found by cor. 15 are unequal, then the triangle is obtuse and scalene, as in Example 4, the greater side being opposite the greater angle, and the less one opposite the less angle.

Let ABC (fig. 3) be the triangle, AC the base-line, A and C the given angles, then B is also given (cor. 15). Draw the perpendicular CE, which will give the trigonometrical data for finding the segments AE and EB of the base AB, and also BC. Thus by making AC radius, CE is sine and AE cosine; CE may therefore be found by formula 1, art. 12, and AE by formula 2. Again, by making BE radius, EC would be tangent, and BC secant of the angle B, so that by formulas 6 and 11 conversely BE and BC would be found. Thus tan. B : rad. : : EC : BE; and tan. B : sec. B : : EC : BC. AE + EB = AB. The two sides AB and BC may also be found as by Examples 3 and 4.

Ex. 6. If one of the angles at the base is obtuse, and the other the greatest of the two acute angles, as B (fig. 3), then BC would be the base-line. The solution would be the same as in the last case (5), but as BC is the less side, the ratio is now that of a less magnitude to a greater.

14. The preceding examples have reference to the triangle on the base-line, when that side only and the adjacent angles, and hence by cor. 15 all the angles, are given. More frequently in practice the three sides of the first triangle are carefully measured with the chain before the trigonometrical survey begins. This is done purposely that the one survey may be a proof check on the accuracy of the other.

15. In the other and interior triangles the parts found by previous cases (articles 7 and 8) are more diversified. Thus two sides and an angle opposite one of them may be found from two previous cases, or two sides and the angle they include, or two sides and two angles (*i.e.*, all the angles, cor. 15), and in some central and exceptionary cases the three sides only.

16. In examples of this kind the construction, resolution, and solution, are more simple, provided the species and ratios are properly attended to in the trigonometrical survey.

Ex. 7. Given the two lesser sides and the angle opposite the least of them, to find the other side and two angles of an acute-angled scalene triangle.

Let ABC (fig. 2) be the triangle, AC and BC the given sides, A the angle $= 45°$. Then AB is the side sought, and C and B the angles. Draw CE and BF, which resolve the triangle into four right-angled triangles. In this case the triangle is scalene and acute-angled, and the ratio that of a less magnitude to a greater, because the lesser of the two given sides is opposite the given and smallest angle. The solution would therefore be to find B, BC : CA : : sin. A : sin. B, which also gives C (by cor. 15). Then sin. A : sin. C : : BC : BA.

Ex. 8. Given the two lesser sides and the angle opposite the greatest of them, to find the other side, and two angles of an acute-angled scalene triangle.

Let ABC (fig. 2) be the triangle, BC and AC the given sides, B the angle $= 60°$. Then AB is the side sought, and A and C the angles (any one of which will determine the other by cor. 15). Draw CE and BF, which resolve the triangle into four right-angled triangles. In this case the species and ratio are similar to those of the last example (7), as also the resolution. The construction and solution are also the same in principle, the only difference being that the greatest of the two given sides, instead of the less, is the last line in the construction; the following formula will give the angles sought; as CA : BC : : sin. B : sin. A.

Ex. 9. Given the two greater sides, and the angle opposite the greatest of them, to find the other side and two angles of an acute-angled scalene triangle.

Ex. 10. Given the greatest and least side, and the angle opposite the latter, to find the remaining side and angles of an acute-angled scalene triangle.

In both these examples, ABC (fig. 2) may represent the triangle, so that the species, construction, resolution, and solution are similar, but the ratio is from a greater to a less magnitude.

Ex. 11. Given the sides about the obtuse angle, and the less of the two acute angles, to find the other side and two angles of an obtuse-angled scalene triangle.

Let AC and BC be the two sides, and A the angle given of the triangle ABC (fig. 3). Then AB is the side sought, and B and C the angles. Draw CE, which gives two right-angled triangles, AEC and BEC.

In this example the sides AC and BC are by hypothesis already drawn upon the plan, and their extents or lengths known, as also the bearing of AB from C, which determines the direction and position of AB, B and C (cor. 28). But before AB is drawn, its length, and also the angles C and B, require to be found, as a check upon accuracy, the bearings having to be corrected for obliquity and refraction. Hence the object of the solution. To find CD, rad. : sin. A : : AC : CE; rad. : cos. A : : AC : AE. To find B, BC : CE : : rad. : sin. B; rad. : cos. B : : BC : BF; AE × EB = AB; $180° - A + B = C$ (cor. 15).

Ex. 12. Given the two greater sides, and the angle opposite the lesser of them, to find the other side, and two angles of an obtuse-angled scalene triangle.

Ex. 13. Given the greatest and least side, and the obtuse angle, to find the other side and angles of an obtuse-angled scalene triangle.

In both these cases ABC (fig. 3) may represent the triangle. In the former case, CE will resolve the triangle, as in Example 11, but in the latter (13), BF or AD is requisite, as in previous resolutions, Ex. 4, 5, and 6. The two solutions would therefore be accordingly, the ratios being from a greater to a less magnitude.

Ex. 14. Given the two greater sides, and the angle they include, to find the other side and two angles of an acute-angled scalene triangle.

Let AB and AC (fig. 2) be the two given sides, and A the angle of the triangle ABC; then BC and the adjacent angles B and C are the parts sought.

In this case, and in all examples of this kind, the given angle determines the position of the given sides that include it (by cor. 28), so that the construction and plotting is simple, and the parts sought easily measured by the scale and protractor.

The ratio for the trigonometrical solution is equally definite. Thus, as the given sides are opposite the unknown angles, and the unknown sides opposite the given angle, it follows that C must be greater than B, and BC less than AC, the least of the two given sides. The ratio, therefore, is that of a greater to a less magnitude.

Again, as $180° - A = B + C$, it follows that half the sum of the unknown angles is given. Now as half their difference added to half their sum gives the greater (C), and as half their difference taken from half their sum gives the less (B), it is usual to find half the difference. Thus :—

$$AB + AC : AB - AC :: \tan. \tfrac{1}{2}(C + B) : \tan. \tfrac{1}{2}(C - B).$$

The angles, however, may be otherwise found by resolving the triangle as in fig. 2. Then C is common to two right-angled triangles ADC and BFC, and B to other two. Either, therefore, can be determined in the process of finding BC. Thus, if AC is radius and A the centre, then CE is sin. A, and AE cos. A ; consequently, as AB is given, AB – cos. A = EB. To find B, make BE radius and EC tangent. The formula for solution would then be BE : EC : : rad. : tan. B; and to find BC, rad. : sec. B : : BE : BC; $180° - A + B = C$ (cor. 15).

Exercise 1. Given AC and CB, and the included angle C (fig. 2); required the formulas for finding AB, and the angles A and B.

2. Given CB and BA, and the included angle B (fig. 2); required the formulas for finding AC and the adjacent angles A and C.

Ex. 15. Given the two greater sides, and the angle they include to find the other side and two angles of an obtuse-angled scalene triangle.

Let AB and AC be the two given sides, and A the angle of the triangle ABC (fig. 3). Then BC is the side, and B and C the angles sought. Draw CE. The solution is similar to that of Example 14, B and BC being given in this case instead of A and AC in the previous one.

Exercise 1. Given AC and CB and the included angle C (fig. 3); required the formulas for finding the greater side AB and the acute angles A and B.

2. Given CB and BA and the included angle B (fig.3); required the formulas for finding AC, A and C.

Ex. 16. Given the three sides to find the angles of a scalene triangle.

Let ABC, either fig. 2 or 3, be the triangle. From C, the angle opposite the greater side, draw the perpendicular CE. In this example, according to hypothesis, the triangle is constructed by the plotting of the adjacent three triangles, and from the solutions of them the three sides AB, AC, and BC, were previously found. Such being the case, the perpendicular CE is drawn purposely for giving the content or area, and its length is easily found by the scale. Very generally this is the practice followed when none of the angles are taken in the field. There are, however, exceptions, for which provision requires to be made, *i.e.*, to find the angle A or B, of which CE is sine, AC or BC being radius. In the two right-angled triangles AEC and BEC, the right angles at E and the two sides opposite are given. There is thus only two parts given in each; a part is therefore wanting to effect the solution. This may be found by several propositions, from whence general rules are deduced. One will suffice for an example. Thus, to find the segments of the base AE and EB whose sum AB is given, AB : AC + BC : : AC − BC : AE − EB. Then half the difference of the segments, added to half their sum, gives the greater AE, and half their difference taken from half their sum, gives the less EB. Having thus found AE and EB, they are the cosines of A and B, AC and BC being the radii. To find A, AC : AE : : rad. : cos. A. To find B, BC : BE : : rad. : cos. B. To find CE, rad. : sin. A. : : AC : CE.

17. In the construction of all scalene triangles, whether acute-angled, obtuse-angled, or right-angled, the arcs of the lesser radii that measure the angles they include intersect in two points the lines that unite them with the longer radii. Thus, in the right-angled triangle EAB (fig. 1), with centre A and radius AB, the greater side is intersected in two points B and C. In fig. 2, AC, with centre A, intersects BC in C and *n;* BC, with centre B, intersects AC in C and *m;* CB, with centre C, cuts AB in B and *o.* And had A*o*C been the triangle, then *o*A with centre *o* would have cut AC in two points. In the obtuse-angled triangle ABC (fig. 3), CB, with centre C, intersects AB in B and *x.* And when the base of an isosceles triangle is less than one of the sides, and only one side, the base and vertical angle given the species of the triangle being unknown, then the base line may cut the unknown side in two points.

30. These examples show that in the construction of triangles, and hence in plotting the bearings of the trigonometrical survey, when two sides of a triangle and one angle only are given, then, either the species of the triangle and ratios of the sides must be known, or else the given angle requires to be the one that is included by the two given sides, so as to determine their position (cor. 28), and hence the position of the other line between their extremities, and of the

adjacent angles thus formed, otherwise the construction will be what has been termed *ambiguous*, *i.e.*, one of two triangles, as pointed out in Euclid's Data, Prop. XLVII., Case 3.

31. If, however, the species of the triangle, and ratios of the sides, and the position of the lesser given radius determined by the given angle (cor. 28) are known, such data will also determine the position of the intersecting radius or side that cuts the indefinite and first side drawn at the proper length. Thus, in fig. 1, because the triangle is right-angled at A, that fixes the direction and position of the lesser radius AB. But had the triangle been in species an obtuse-angled isosceles one, then the position of the lesser side or radius would have been AC, and the triangle ACE. In like manner ABC (fig. 2), being in species an acute-angled scalene triangle, that determines the position of the two lesser radii. But in the obtuse-angled scalene triangle ABC (fig. 3), C*x*A is also an obtuse-angled scalene triangle, so that species alone is not enough in this class of triangles, the ratios being also necessary. In other words, it is the position of CB, and the relation of the greater side to the other two sides, that determines the construction of the triangle.

32. In plane trigonometry triangles exist, and hence their species and also the ratios of the sides and angles exist. If, therefore, a side and two angles are given, or two sides and an angle, or any three parts including a side (art. 3), then the trigonometrical relation that subsists between them and the unknown parts determines the latter. According to the general definition given of trigonometry, it is this relation that supplies the mathematician and surveyor with data for finding the unknown parts. This is consequently the given hypothesis in every proposition in plane trigonometry. The sides and angles sought are therefore as definite and free from ambiguity as those given, otherwise they could not be determined.

33. The ambiguity referred to under (30) is, it will thus be seen (32), exclusively confined to constructive geometry. Applied to land-surveying it is wholly confined to the work of plotting the bearings or copying the triangles of the field on to the plan.

34. In the trigonometrical survey the species and ratios of the triangles should be carefully determined in the field; as this, practically speaking, is the principal part of the work, or the principal function of plane trigonometry. If this is done, no ambiguity will be experienced in plotting the bearings. (See Sect. VI. Part IX.)

SECTION III.

PLANE TRIANGLES.

The solution of plane triangles may be reduced to four cases, in which the given parts are the following:—

CASE I. A side and three angles.
CASE II. Two sides and an angle opposite one of them.
CASE III. Two sides and their included angle.
CASE IV. The three sides.

SOLUTION OF RIGHT-ANGLED PLANE TRIANGLES.

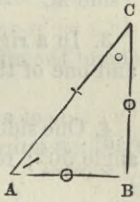
CASE I.

Given the angles and hypotenuse or a side, to determine the triangle.

RULE.—As sine of any angle : side opposite : : sine of any other angle : its opposite side.

1. Given * the hypotenuse AC = 720 links, and the angle A = 53° 8′; to find the sides AB and BC, and the angle C.

By construction.



Draw the line AB, of any length make the angle A = 53° 8′, draw the hypotenuse AC = 720 links; from C let fall the perpendicular CB; then ABC is the triangle required: and AB, being measured, will be found = 432 links, and BC = 576 links; also the angle C will be found = 36° 52′.

By calculation.

The angle C = 90° − 53° 8′ = 36° 52′.

To find BC.		To find AB.	
As sine ∠ B = 90°	10·00000	As sine ∠ B = 90°	10·00000
: hyp. AC = 720 links	2·85733	: hyp. AC = 720 links	2·85733
: : sine ∠ A = 53° 8′	2·90311	: : sine ∠ C = 36° 52′	9·77812
	12·76044		12·63545
	10·00000		10·00000
: side BC = 576 links	2·76044	: side AB = 432 links	2·63545

NOTE.—The logarithmic operation is performed by adding together the second and third terms of the above proportions, and subtracting the first from the respec-

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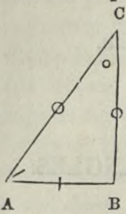
* The *given* parts of a triangle are generally marked with a dash (—), and the *required* parts with a circle (o), as in the figures annexed to the constructions.

tive sums, the remainders being the fourth terms: which operations must be understood to be performed in all the following examples, it being unnecessary to put down the work at length as above.

2. The side AB is = 4320 links, and the angle A = 53° 8′; required the other parts.



By construction.

Make AB = 4320 links, at B erect the indefinite perpendicular BC, make the angle A = 53° 8′, and draw AC, meeting BC in C; then AC will measure 7200 links, BC = 5760 links, and the angle C = 36° 52′.

By calculation.

The angle C = 90° − 53° 8′ = 36° 52′.

To find AC.			To find BC.		
As sin. ∠ C	= 36° 52′	9·77812	As sin. ∠ C	= 36° 52′	9·77812
: side AB	= 4320 links	3·63548	: side AB	= 4320 links	3·63548
: : sin. ∠ B	= 90°	10·00000	: : sin. ∠ A	= 53° 8′	9·90311
: side AC	= 7200 links	3·85736	: side BC	= 5760 links	3·76047

3. In a right-angled triangle are given the hypotenuse = 98 chains and one of the angles = 56° 48′, to find the sides and area.

Ans. Sides 82 *and* 53·66, *area* 22*a.* 0*r.* 1*p.*

4. One side of a right-angled triangle is 2350 links, and its opposite angle 50°; required the other side and hypotenuse.

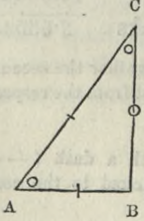
Ans. Side = 1977 *links hyp.* = 3067·5 *links.*

Case II.

Given the hypotenuse, a side, and the right angle, to determine the triangle.

Rule.—As hypotenuse : radius : : given side : to sine of opposite angle.

1. The hypotenuse AC is 24 chains, and the side AB = 14 chains 40 links; required the other side and angles.



By construction.

Make the side AB = 14·40 chains, erect the indefinite perpendicular BC; with centre A and radius AC = 24 chains, describe an arc cutting BC in C; join AC; then BC will be found = 19 chains 20 links, and the angles A and C respectively 53° 8′ and 36° 52′.

By calculation.

To find the angle C.		To find BC.	
As hyp. AC = 24	1·38021	As sin. ∠ B = 90°	10·00000
: sin. ∠ B = 90°	10·00000	: hyp. AC = 24	1·38021
: : side AB = 14·40	1·15836	: : sin. ∠ A = 53° 8′	9·90311
: sin. ∠ C = 36° 52′	9·77815	: side BC = 19·20	1·28332

Hence angle A = 90° − 36° 52′ = 53° 8′.

NOTE.—The side BC may be found by Euc. I. 47, thus, $\sqrt{AC^2 - AB^2} = \sqrt{24^2 - 19{\cdot}4^2} = \sqrt{368{\cdot}64}$ = 19 chains 20 links, the same as above.

2. The hypotenuse of a right-angled triangle is 960 links, and one of the sides 768 links; required the other side and angles.

Ans. Side 576 *links, angles* 53° 8′ *and* 36° 52′.

3. In a right-angled triangle are given the hypotenuse = 3350 links, and a side = 2631 links; required the other side and area.

Ans. Side 2074 *links, area* 27*a.* 1*r.* 5 *p.*

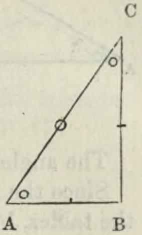
CASE III.

Given the right angle and the adjacent sides to find the other parts of the triangle.

RULE.—As a side : radius : : the other side : tangent of angle adjacent to the first side.

As the sine of this angle : rad. : : second side : hypotenuse.

1. The sides of a right-angled triangle are AB 2880 and BC 3840 links; required the hypotenuse AC and angles A and C.



By construction.

Make AB = 2880, perpendicular to which make BC = 3840 links; join AC; then AC will be found = 4800 links, angle A = 53° 8′, and angle C = 36° 52′.

By calculation.

To find angle A.		To find AC.	
As side AB = 2880	3·45939	As sin. A = 53° 8′	9·90311
: rad.	10·00000	: rad.	10·00000
: : side BC = 3840	3·58433	: : side BC = 3840	3·58433
: tan. A = 53° 8′1	0·12494	: hyp. AC = 4800	3·68122

Angle C = 90° − 53° 8′ = 36° 52′.

2. The sides adjacent the right angle of a right-angled triangle are 2683 and 4100 links; required the hypotenuse and angles.

Ans. Hyp. 4900 *links, angles* 56° 48′ *and* 33° 12′.

3. The sides of a right-angled triangle are 6250 and 1250 links required the hypotenuse, angles, and area.

Ans. Hyp. 6374 *links, angles* 78° 41′ *and* 11° 19′, *area* 39*a.* 0*r.* 10*p.*

OBLIQUE-ANGLED PLANE TRIANGLES.

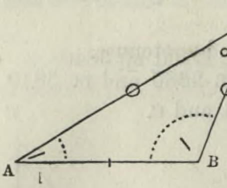
Case I. and II.

Given two sides and an angle opposite one of them, or given two angles and a side opposite one of them, to determine the triangle.

Rule.—As a side : sine of its opposite angle : : any other side : sine of its opposite angle.

And conversely, As sine of an angle : opposite side : : sine of any other angle : to its opposite side.

1. In an obtuse-angled scalene triangle are given the angle A = 32° 15′, the angle B = 114° 24′, and the side AB = 98 chains, to find the sides AC and BC, and the other angle.



By construction.

Make AB = 98 chains, draw BC, making the angle B = 114° 24′, also draw AC making the angle A = 32° 15′, and meeting BC in C ; then AC and BC, being measured, will be found respectively 162·34 and 95·12, and the angle C = 33° 21′.

By calculation.

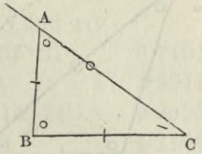
The angle C = 180° − (114° 24′ + 32° 15) = 33° 21′.

Since the angle B is greater than 90°, its sine cannot be found in the tables, but by taking it from 180°, its supplement 65° 36′ is obtained, the sine of which is also equal to the sine of the angle B.

To find AC.		To find BC.	
As sin. C = 33° 21′	9·74017	As sin. C = 33° 21	9·74017
: side AB = 98	1·99123	: side AB = 98	1·99123
: : sin. B = 114° 24′	9·95937	: : sin. A = 32° 15′	9·72723
: side AC = 162·34	2·21043	: side BC = 95·12	1·97829

2. In an acute-angled scalene triangle are given AB = 60 chains, BC = 95·12 chains, and the angle C = 33° 21′ ; required the greater side AC, and the angles A and B.

By construction.



Make BC = 95·12 chains, draw AC, making the angle C = 33° 21′; with the side AB = 60, as radius, and centre B, describe an arc cutting AC in A; then ABC is the triangle required.

AC = 108·87 chains, angle A = 60° 38′, angle B = 86° 1′.

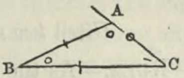
By calculation.

To find angle A.		To find AC.	
As AB = 60	1·77815	As sin. C = 33° 21′	9·74017
: sin. C = 33° 21′	9·74017	: AB = 60	1·77815
: : BC = 95·12	1·97827	: : sin. B = 86° 1′	9.99895
: sin. A = 60° 38′	9·94029	: AC = 108·87	2·03693

The sum of the angles C and A subtracted from 180°, leaves the angle B = 86° 1′.

3. In the obtuse-angled scalene triangle ABC are given the two greater sides BC = 95·12 chains, BA = 60 chains, and the angle C opposite the lesser of them = 33° 21′; required the side AC and the angles A and B.

By construction.



Make BC = 95·12 chains; draw CA indefinite, making the angle C = 33° 21′; then with BA = 60 chains as radius, and centre B, describe an arc cutting CA in A; draw BA; then ABC is the triangle required. AC = 50·03 chains; A = 119° 22′; and B = 27° 17′.

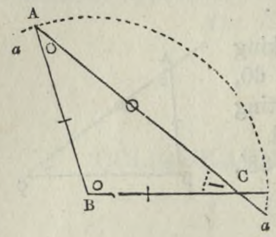
By calculation.

To find A.		To find AC.	
As BA = 60	1·77815	As sin. C = 33° 21′	9·74017
: sin. C = 33° 21′	9·74017	: BA = 60	1·77815
: : BC = 95·12	1·97827	: : sin. B = 27° 17′	9·66124
: sin. A = 119° 22′	9·94029	: AC = 50·03	1·69922

= 180° − (119° 22′ + 33° 21′)
= 27° 17′

4. In a plane triangle AB = 98, and BC = 95·12 chains, and angle C = 33° 21′; required the other parts.

By construction.



Make BC = 95·12 chains, draw AC indefinite, making the angle C = 33° 21′; with AB as radius and centre B describe the arc *aa*, cutting AC in A; then ABC is the triangle required. AC = 162·34 chains; A = 32° 15′; and B = 114° 24′.

By calculation.

To find angle A.		To find AC.	
As AB = 98	1·99123	As sin. C = 33° 21′	9·74017
: sin. C = 33° 21′	9·74017	: AB = 98	1·99123
: : BC = 95·12	1·97827	: : sin. B = 114° 24′	9·95937
: sin. A = 32° 15′	9·72721	: side AC = 162·34	2·21043

Angle B = 180° − (A + C) = 114° 24′.

5. In a plane triangle are given two angles 79° 23′ and 54° 22′, and a side 1250 links opposite the first angle, to find the other parts.

Ans. Sides 1033·6 *and* 918·7 *links.*

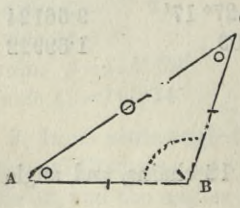
Case III.

Given two sides and their included angle to find the rest.

Rule.—As the sum of the given sides : their difference : : co-tangent of half the included angle : tangent of half the difference of the required angles. This angle, added to half the complement of the included angle, gives the greater required angle, and subtracted gives the lesser. The other side is then found by Case I.

1. Given the side AB = 9800 links, the side BC = 9512, and their included angle B = 114° 24′, to find the other side and angles.

By construction.



Make AB = 9800 links, and the angle B = 114° 24′; draw BC, which make = 9512 links: then ABC is the triangle required. The angles A and C measure 32° 15′ and 33° 21′, and AC 16234 links.

By calculation.

To find the angles.		To find AC.	
As AB + BC = 19312	4·28583	As sin. C = 33° 21′	9·74017
: AB − BC = 288	2·45939	: AB = 9800	3·99123
: : cot. $\frac{1}{2}$B = 57° 12′	9·80919	: : sin. B = 114° 24′	9·95937
: tan. $\frac{1}{2}$(C − A) 0° 33′	7·98275	: AC = 16234	4·21043
comp. $\frac{1}{2}$B = 32° 48′			
sum 33° 21′ = ∠C			
diff. 32° 15′ = ∠A			

2. Given the two sides 103 and 126 chains, and their contained angle 56° 30′, to determine the triangle.

Ans. The angles 72° 20′ *and* 51° 10′, *the side* 110·3 *chains.*

3. Two sides of a triangle are 34500 links and 17407, and their included angle 37° 20′; required the other angles and side.

Ans. The angles 27° 4′ *and* 115° 36′, *the side* 23200 *links.*

Case IV.

Given the three sides to find the angles.

Rule I. From half the sum of the three sides subtract the side opposite the angle sought, and the logarithms of the half sum and remainder, and increase the index of the sum by 20; from the sum thus increased, subtract the sum of the logarithms of the sides containing the angle sought; the remainder, divided by 2, is the log. cosine of half the angle sought.

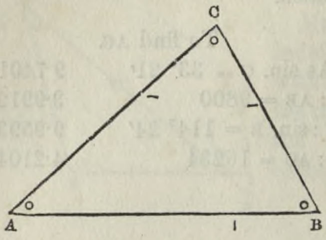
Rule II. From half the sum of the three sides subtract each of the sides containing the angle sought, add the logarithms of the two remainders, and increase the index of the sum by 20; also from half the sum of the three sides subtract the side opposite the angle sought, and add the logarithms of the half sum and remainder; then half the difference between these logarithmic sums is the log. tangent of half the angle sought.

The remaining angles may be found by Case I.

Note.—Rule II. is to be preferred when the angle sought is very small or near 180°.

1. Given the three sides AC = 98, AB = 162·34, and BC = 95·12 chains, to find the angles of the triangle.

By construction.



Draw the longest side AB = 162·34 chains; with AC = 98 as radius, and centre A, describe an arc; with BC = 95·12 as radius, and centre B, describe another arc cutting the former in C; draw AC and BC; then ABC is the triangle required. The angles being measured, will be found A = 32° 15′, B = 33° 21′, and C = 114° 24′.

By calculation. (RULE I.)

½(AB+AC+BC)	= 177·73	2·24976	sum + 20 =	24·16679
½(AB+AC+BC) − BC	= 82·61	1·91703		
AC	= 98	1·99123	sum =	4·20166
AB	= 162·34	2·21043		
				2)19·96513
cos. ½∠A	= 16° 7′ 30″			9·98257
	2			
∠A	= 32° 15′ 0″			

(RULE II.)

½(AB+AC+BC) − AB	= 15·39	1·18724	sum + 20 =	23·08886
½(AB+AC+BC) − AC	= 79·73	1·90162		
½(AB+AC+BC)	= 177·73	2·24976	sum =	4·16679
½(AB+AC+BC) − BC	= 82·61	1·91703		
				2)18·92207
tan. ½∠A	= 16° 7′ 30″			9·46104
	2			
∠A	= 32° 15′ 0″			

The remaining angles may be found by Case I.

2. In a plane triangle are given the three sides 7000, 10400, and 14202 links to find the angles.

Ans. 27° 59′, 44° 12′, and 107° 49′.

3. When the sides of the triangle are 2253, 2240, and 2400 links, what are the angles? *Ans.* 57° 27′, 57° 59′, and 64° 34′.

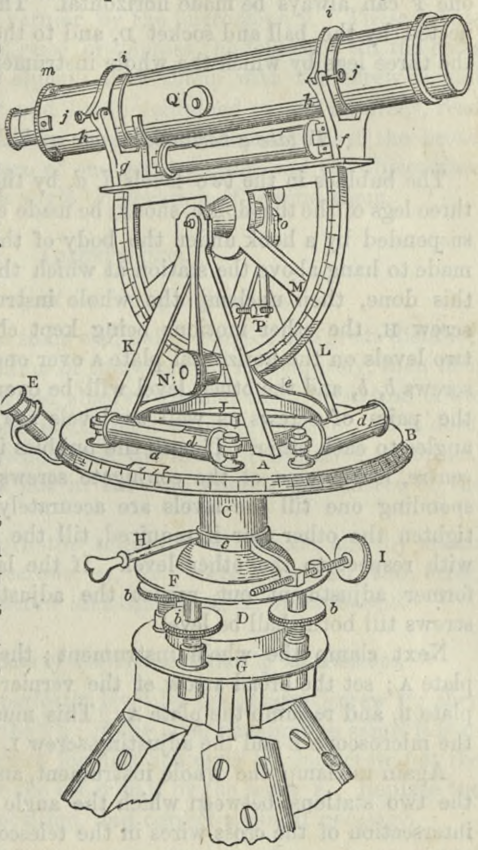
SECTION IV.

THE MEASUREMENT OF ANGLES.

The measurement of horizontal and vertical angles is usually taken with a theodolite.

THE THEODOLITE.

Two of the principal parts of a theodolite are the circular brass plates A and B, which turn one upon the other horizontally on the vertical axis C. Th circumference of the lower plate B is divided into 360°, and these again into half degrees. At the extremities of a diameter of the upper plate A are fixed two verniers for reading off the minutes in the divisions of the plate B. This plate has its clamp-screw H, and its adjusting-screw I, which clamp and adjust the whole instrument. To the plate A are fixed two spirit-levels, *d*, *d*, at right angles to each other; and a compass J concentric with it. It has also its clamp and adjusting screw (not shown in the figure). These two plates are used to take horizontal angles and lines.



The vertical semicircle M turns on an axis supported by the frame KL, resting on the plate A; its axis passes perpendicularly through the common axis of the instrument; its motion is therefore perpendicular to the horizontal plates. It is graduated like the plate B, and by means of a vernier, fixed above the compass, the divisons can be read off to minutes. It has its clamp-screw O, and its adjusting screw P, and is used for taking vertical angles and lines.

The telescope is placed above the vertical circle in two receptacles, called Y's from their shape, and secured above by two clips *i*, *i*. It has a long spirit-level fixed beneath and parallel to it. The object-glass is adjusted to the sight by the screw Q and the eyeglass by moving it backwards and forwards with the hand.

Between what are called the two parallel plates F and G are fixed two pairs of conjugate screws, *b*, *b*, &c., by means of which the upper one F can always be made horizontal. These two plates are connected by the ball and socket D, and to the lower one G are fixed the three legs by which the whole instrument is supported.

To take a horizontal angle with the theodolite.

The bubbles in the two levels *d*, *d*, by the proper opening of the three legs of the theodolite, should be made central, and the plummet, suspended by a hook under the body of the instrument, should be made to hang above the station at which the angle is to be taken : this done, then unclamp the whole instrument by means of the screw H, the other motions being kept clamped. Set one of the two levels on the horizontal plate A over one of the pairs of opposite screws *b*, *b*, and the other level will be over the other pair, because the pairs of screws as well as levels are conjugate, *i.e.*, at right angles to each other. If both the bubbles in the levels be not in the centre, loosen one of the conjugate screws, and tighten the corresponding one till the levels are accurately adjusted. Loosen and tighten the other pair if required, till the same result is obtained with respect to the other level. If the last operation throw the former adjustment out, repeat the adjustments on each pair of screws till both shall be level.

Next clamp the whole instrument; then unclamp the vernier plate A; set the broad arrow of the vernier to 360°, or zero, on the plate B, and reclamp the plate A. This must be carefully done by the microscope E and the adjusting-screw I.

Again unclamp the whole instrument, and turn it to the left of the two stations, between which the angle is to be taken, till the intersection of the cross wires in the telescope cut the flag or other object in the station as accurately as can be done with the hand; then clamp the screw H, and by slowly turning the screw I the greatest accuracy may be obtained.

Now unclamp the upper plate A, and turn it round till the cross wires in the telescope cut the object in the second station; then clamp and adjust with the two screws attached to the plate A, till having obtained perfect accuracy, read off the angle by means of the vernier with the glass E : in the same way, read off the angle with the other vernier, and the mean of the two will be the correct angle.

To take a vertical angle.

The horizontal plates being set level, as already explained, bring the bubble of the telescope level to the centre of its tube, observing at the same time whether the zero point of the vertical circle coincides with that of the vernier by the microscope N. These points being found to coincide, raise or depress the telescope till the cross wires cut the required object: then clamp with the screw O, and adjust with the screw P, and having obtained perfect accuracy, read off the angle, which will be an angle of depression, if the broad arrow be between the zero of the vertical circle and the object-glass of the telescope; and an angle of elevation, if beyond them.

ADJUSTMENTS.

1. *To adjust the line of collimation.*

Direct the telescope to some well-defined object at a great distance, adjust it till the intersection of the wires cut it accurately, then turn the telescope on its axis, and observe whether the intersection of the wires still continues to cut it during a whole revolution. If it does, it is in adjustment; if it does not, the line of collimation, or optical axis of the telescope, is not in the line joining the centres of the eye and object-glasses.

To correct this error, turn the telescope on its axis, and by means of the four conjugate screws, *m*, &c., correct for half the error, alternately loosing one screw and tightening its opposite one.

2. *To adjust the axis of the level with that of the telescope.*

Make the telescope perfectly level by the tangent screw P; then reverse the telescope in the Y's: if the level remains the same, it is in adjustment; if not, correct for half the error by the screw *f* at the end of the level, and for the other half by the screw P. Replace the telescope in its former position, and correct again, if necesary.

3. *To make the axes of the levels on the vernier plate parallel to it.*

Set one of the levels over one pair of the conjugate screws, *b*, *b*, &c., then the other level will be over the other pair; make both the bubbles come to their respective centres, turn the instrument half round, and if the bubbles deviate from their centres, correct half the error by the small screws on the levels, and the other half by the screws *b*, *b*, &c. Repeat this operation till the bubbles are central in every position throughout a whole revolution of the instrument.

THE VERNIER.

The vernier is a contrivance for measuring the fractional parts of the half-degrees on the lower plate of the theodolite. The space occupied by 29 or 31 of these half-degrees is used to form the scale of the vernier, and divided into 30 parts; whence it is obvious that the difference of one division on the vernier and one on the lower plate will be $\frac{1}{30}$th in excess or defect of the $\frac{1}{2}°$ on the lower plate, or a single minute; and the difference between 2, 3, 4, &c., divisions on the two plates will be 2, 3, 4, &c., minutes. Hence the following methods of finding the minutes corresponding to a fractional part of a degree on any given angle, &c.

1. *To find the minutes in the fractional part of a degree of a given angle by the vernier.*

If the broad arrow of the vernier is between a full degree and a half-degree, say between 40° and $40\frac{1}{2}°$, then the angle is 40° and as many minutes as are shown by the number of the first division-line of the vernier that coincides with a division on the lower plate, reading forward as the degrees are numbered. If the broad arrow be between $40\frac{1}{2}°$ and 41°, then the angle is 40° 30′ *plus* the number of minutes shown by the first coincidence of the divisions of the vernier and those on the lower plate. If the coincidence take place at the 13th division on the vernier, then the angle in the former case is 40° 30′, and in the latter $40° 30' + 13' = 40° 43'$.

2. *To set the theodolite to an angle of any number of degrees and minutes.*

If the required angle be 48° 28′, bring the vernier till the broad arrow shall be in such a position within the half-degree above 48°, that the division numbered 28 on the vernier shall coincide with the same numbered line on the lower plate; the theodolite is then set to the required angle.

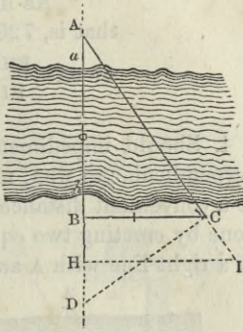
SECTION V.

TRIGONOMETRICAL SURVEY OF HEIGHTS AND DISTANCES.

PROBLEM I.

To measure that part of a base line which crosses a river.

Let DBA be the direction of the line, which has been measured to the edge *b* of the river; it is required to find the width *ba* of the river.



1. Having set up flags at any convenient points B and A (supposing sight not to be obstructed), range and measure the line BC perpendicular to BA, and let BC = 576 links, then take with the theodolite the angle ACB, which suppose to be 53° 8′; required the distance BA, and from thence the width of the river *ba*, the distance B*b* being 36 links, and *a*A 43 links.

sin. A = cos. C = 53° 8′ . .	9·77812
: side BC = 576	2·76042
: : sin. C = 53° 8′	9·90311
: BA = 768	2·88541

$ba = 768 - (36 \times 43) = 689$ links, the breadth of the river.

The distance BA may be found by construction. See solution of right-angled plane triangles, Case 1, Ex. 2.

NOTE.—If the angle ACB had been 45°, the angle A would also have been 45°, and therefore BA = BC. In this case no calculation or construction would be required. A cross-staff, having also sights placed at this angle, is sometimes used for thus expeditiously solving this problem, when the nature of the ground admits of the line BC being measured of a sufficient length for the purpose.

2. The distance BA may be found independently of trigonometry thus. Having measured the perpendicular BC = 576 links, as in the last example, range and measure a line CD perpendicular to CA, and meeting the base-line DBA in D; measure DB = 432 links: then (Euc. VI. 8)

As BD : BC : : BC : BA,

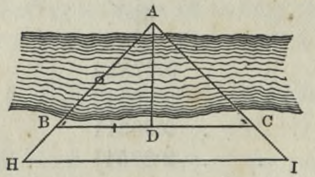
that is, 432 : 576 : : 576 : 768 = BA,

$$\text{or BA} = \frac{\text{BC}^2}{\text{B}} = \frac{(576)^2}{432} = 768 \text{ links.}$$

3. Should obstructions prevent the measurement of the perpendicular CD, let the line CD = 576 be measured as before; at any convenient distance BH = (suppose) 200 links on the base-line, range and measure HI perpendicular to DBA, till the point I shall be in a direct line with the flags at A and C, and let HI = 726 links; then

$$\text{As HI} - \text{BC} : \text{BH} :: \text{BC} : \text{BA},^*$$
$$\text{that is, } 726 - 576 : 200 :: 576 : 768 \text{ links} = \text{BA},$$
$$\text{or BA} = \frac{\text{BH} \times \text{BC}}{\text{HI} - \text{BC}} = \frac{200 \times 576}{150} = 768 \text{ links.}$$

4. Should impediments prevent the lines BC, HI, being measured perpendicular to HBA, measure BC to make any angle with HA, and at a convenient distance BH measure HI parallel to BC, which may be done by erecting two equal perpendiculars to BC, the point I being in a right line with A and C as before.



The rule for finding BA will be the same as in the last example; the calculation in both cases may be readily performed on the ground.

5. Lastly, if the nature of the ground be such that none of the four preceding methods can be adopted, neither to the right nor to the left of the base-line, measure BC in the most convenient direction, and take the angles ABC, ACB with the theodolite; then by taking the sum of these angles from 180°, the angle A will become known, whence, by Case I. of oblique-angled triangles :

$$\text{As sin.} \angle \text{A} : \text{BC} :: \text{sin.} \angle \text{C} : \text{BA}.$$

This problem may also be solved by construction.

Examples for practice.

1. BC = 308 links (fig. to first method) is measured perpendicular to BA, and CD is ranged perpendicular to CA, meeting DA in D, the distance BD being 400 links, and the flags at B and A being at the water's edge ; required the distance BA.

Ans. By the second method BA *is found* = 237 *links nearly.*

2. Given BC = 400 (fig. to first method), HI = 520, and BH = 180 links, to find BA.

Ans. By the third method BA *is found* = 600 *links.*

3. Given BC = 8 chains, and the angles at B and C respectively 30° and 45° ; required the distance BA, and the perpendicular width AD of the river.

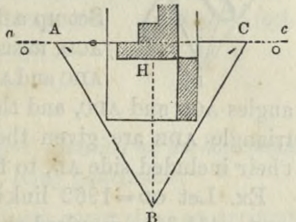
Ans. BA = 585 *links, and* AD = 292 *links.*

* This rule is derived from similar triangles ; for HI : BC : : HA : BA, and HI − BC : HA − BA : : BC : BA, *i.e.*, HI − BC : HB : : BC : BA.

PROBLEM II.

To continue the direction and measurement of a given line when buildings or other obstructions stand in the way, which can be avoided by going to right or left.

1. At any convenient point A in the given line *a*AC*c*, which is obstructed by the buildings H, take an angle *a*AB $= 120°$ (thus making the angle CAB $= 60°$); range and measure the line AB, till, by taking an angle B $= 60°$ the line BC will clear the obstructions at H; then measure BC = AB, and take the angle BC*c* $= 120°$. Then C*c* is in the range of the given line *a*AC*c*, and AC = AB or BC, the triangle ABC being thus made equilateral; hence the measurement of the line *a*AC*c* may proceed from C, after having added the distance thus found to the measure at A.



2. If the nature of the ground be such as not to admit line AB to make with *a*A an angle *a*AB $= 120°$, take any other angle, and suppose it to be $132°$, which taken from $180°$ leaves the angle CAB $= 48°$; measure AB, till by taking an angle at B $= 84°$ = twice the complement of the angle CAB, the line BC may clear the buildings; then measure BC = BA, taking the angle *c*CB also $= 132°$, and continue the line in the direction C*c*. The triangle ABC is thus made isosceles and a perpendicular let fall from B to H on AC will divide ABC into two equal right-angled triangles.

Hence as rad. or sin. $90°$: side AB : : sin. $\frac{1}{2} \angle$ B, or $42°$: AH = $\frac{1}{2}$ AC, whence AC becomes known.

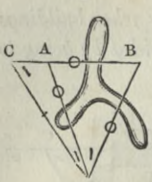
3. If it be convenient to take the angle *a*AB $= 135°$, the angle B $= 90°$, AB = BC = (suppose) 800 links, and lastly the angle *c*CB = $135°$, thus making each of the angles CAB, ACB $= 45°$, the triangle being right-angled at B, as well as isosceles. Hence, by Euc. I. 47,

$$AC = AB \times \sqrt{2} = 800 \times 1{\cdot}414 = 1131 \text{ links}.$$

NOTE.—By any of these methods the distance AC between any two given points A and C may be found, when sight is not obstructed, as when H is a lake or projection of a sea-coast, intervening between A and C, that cannot be measured across with the chain.

PROBLEM III.

To find the distances of three accessible and visible objects, when obstructions prevent the measurement of these distances in direct lines.



Let A, B, and D be the three objects, the distances of which, AB, AD, and BD, cannot be measured directly, on account of the obstruction shown in the figure. Set up a flag at C in the direction AB, take the angle ACD, measure the distance CD, and take the angles ADC and ADB. Then in the triangle ACD are given the angles ACD and ADC, and their included side CD, to find AD. In the triangle ADB are given the angles ADB and DAB = ACD + ADC, and their included side AD, to find AB and BD.

Ex. Let CD = 1962 links, and the angle C = 57°, CDA = 14°, and ADB = 41° 30′; required the distances AD, DB, and AB.

The angle DAC = $180° - (57° + 14) = 109°$,

The angle DAB = $57° + 14° = 71°$.

As sin. 109° : CD = 1962 : : sin. 57° : AD = 1740 links.

The angle B = $180 - (71° + 41°\ 30') = 67°\ 30'$.

As sin. 67° 30′ : AD = 1740 : : sin. 41° 30′ : AB = 1248 links,

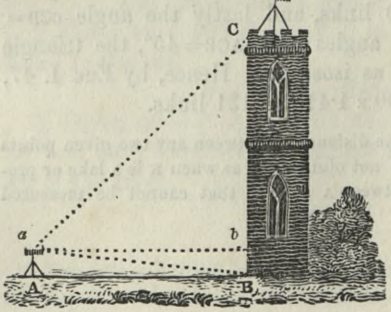
: : : sin. 71° : BD = 1781 links.

By construction.

Draw the indefinite line CB, make the angle C = 57; draw CD, which make = 1962 links, lay off the angles CDA = 14°, CDB = 14° + 41° 30′ = 55° 30′, and draw the lines DA and DB meeting CB in A and B, then the distances AD, AB, and BD will be found as above.

PROBLEM IV.

To find the height of a tower or any other object, standing on an horizontal plane, the base of the tower, &c., being accessible.



Let CB be a tower, standing on an horizontal plane. Measure any convenient distance, AB, in a direct line from the tower; fix the theodolite at A and take the vertical angle Cab, the line ab being in the level of centre of the vertical arc of the instrument. Then in the right-angled triangle aCb are given ab = AB, and the angle Cab, to find Cb; to which add Aa = Bb = height of the centre of the instrument, and the sum will be the height of the tower.

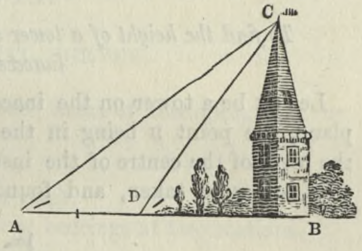
Ex. Let AB = 200 feet, the angle Cab = 46° 30′, and the height of the centre of the instrument 5 feet; required the height of the tower.

As rad. = sin. 90° : tan. Cab = 46° 30′ : : 200 = ab : Cb = 210·76
Aa = Bb = 5
The height of the tower BC = 215·76 feet.

PROBLEM V.

To find the height of a tower or any other object, standing on an horizontal plane, the base of the tower, &c., being inaccessible.

Let CB be a spire or steeple inaccessible on every side, on account of trees, a portion AD of the ground line AB being horizontal, and passing through the bottom of the spire. Take the angle CDB = 51° 30′, measure the distance AD = 75 feet, and take the angle A = 26° 30′. Required the height of the spire and the distance of the first station from the centre of its base.



Subtract the angle A from the angle CDB, the remainder 25° = angle ACD. Then in the oblique-angled triangle ADC, AD and all the angles are given to find DC = 79·18; and in the right-angled triangle DBC, the hypotenuse DC and angle BDC are given to find BC = 61·97, and DB equal 49·29 feet.

Or BC may be found by a single operation by the formula BC = $\frac{\text{AD} \times \text{sin. A} \times \text{sin. D}}{\text{rad.} \times \text{sin. (D} - \text{A)}}$, that is, add together the logarithms of AD, and of the sines of the angles A and D; from the sum subtract the log. sin. (D − A) with its index increased by 10, and the remainder is the log. of the height BC. Thus,

Log. AD	1·87506
sin. 51° 30′	9·89354
sin. 26° 30′	9·64953
	21·41813
sin. 25° + 10 . . .	19·62595
Log. BC = 61·97 . . .	1·79218

To this result the height of the instrument should be added.

Examples for practice on the two last problems.

1. At the distance of 45 feet from the bottom of a steeple, on an horizontal plane, the angle of elevation is 48° 12′, and the height of the instrument 5 feet. Required the height of the steeple.

Ans. 55 *feet* 4 *inches.*

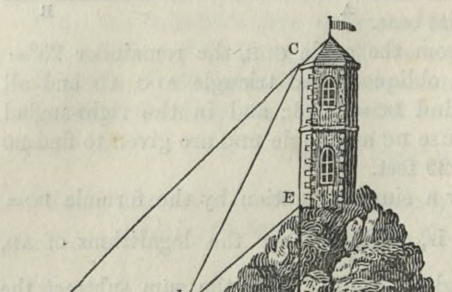
2. The angles taken at two stations A and D (see last fig.) on an horizontal plane passing through the base of an inaccessible tower CB, are 28° 34′ = CAB, 50° 9′ = CDB, the distance of stations being 30 yards = AD; required the height of the tower, and its distance from the station D, the height of the instrument being $5\frac{1}{4}$ feet.

Ans. Height of tower 31·69 *yards; distance from* D = 24·99.

PROBLEM VI.

To find the height of a tower standing on the side or top of an inaccessible hill.

Let CE be a tower on the inaccessible hill E, ADB the horizontal plane, the point B being in the prolongation of the line CE; *adb* the level of the centre of the instrument, with which the angles *a*. C*db*, E*db* were taken, and found to be respectively 44°, 67° 50′ and 51°. Required the heights of the tower and hill, the height of the instrument being 5 feet, and *ad* = 134 yards.



Subtract the angle *a* from C*db*, the remainder 23° 50′ = *a*C*d*: hence all the angles of the triangle *a*C*d* are given, and the side *ad*, to find C*d* = 230·36 yards.

In the triangle C*d*E all the angles are given, *viz.* C*d*E = 67° 50′ − 51° = 16° 50′, *d*CE = 90° − 67° 50′ = 22° 10′. Hence the angle CE*d* = 141°, and CE = 106 yards, the height of the tower.

In the right-angled triangle C*db*, the angle C*db* and DC are given to find C*b* = 213·34 yards: hence = *b*E *b*C − CE = 107·34 yards, to which add the height of the instrument = 5 feet = 1·66 yards, and there results EB = 109·01 yards, the height of the hill.

Examples for practice.

1. The angles of elevation of a castle standing on the top of a rock are CAD = 40°, CDB = 63° 20′, and CDE = 14° 30′, and the distance of the stations A and D, which are in a line with the castle, is 40 yards. Required the height of the castle.

Ans. 24·69 *yards.*

2. When the angles CAD, CDB, and CDE are respectively 30° 30′, 59° 5′, and 13° 52′, and the distance AD on the horizontal plane = 52 yards, what is the height of tower?

PART IX.

TRIGONOMETRICAL SURVEY.

SECTION I. Rules for finding the areas of triangles, &c., by logarithms.

SECTION II. To survey a wood.

SECTION III. To survey a road or river.

SECTION IV. The practice of taking bearings at two stations.

SECTION V. To survey a town or city.

SECTION VI. To range lines and take bearings in extensive surveys.

SECTION VII. To survey with the circumferentor.

SECTION I.

RULES FOR FINDING THE AREAS OF TRIANGLES, ETC., BY LOGARITHMS.

PROBLEM I.

To find the area of a triangle when two sides and their included angle are given.

To the logarithms of the two sides in links add the log. sine of the included angle, and from the sum subtract 15·30103, and the remainder is the log. of the area in acres and decimals of an acre.

Ex. The two sides of a triangle are 960 and 576 links, and their included angle 53° 8′; required the area.

Log. 960 . . 2·98227				
576 . . 2·76042				
Log. sin. 53° 8′ . . 9·90311				
15·64580				
15·30103				
Log. 2·212 . . . 0·34487				
4				
0·848				
40		A.	R.	P.
32·920	*Ans.*	2	0	33

PROBLEM II.

To find the area of a triangle when two angles and their included side are given.

From the sum of twice the log. of the given side in links and the log. sines of the given angles, subtract the log. sine of the sum of the given angles increased by 15·30103, and the remainder is the area in acres.

Ex. Two angles of a triangle are 59° 46′ and 60° 14′, and their included side 1000 links; required the area.

Log. 1000 . . 3·00000				
2				
6·00000				
Log. sin. 59° 46′ . . 9·93665				
60° 14′ . . 9·93855				
25·87520				
Log. sin. (59° 46′ + 60° 14′) = 120° . 9·93753, 15·30103 } . 25·23856				
Log. 4·332 · 0·63664				
4				
1·328				
40		A.	R.	P.
13·120	*Ans.*	4	1	13

PROBLEM III.

To find the area of a triangle when the three sides are given.

From the half sum of the three sides subtract each side separately, add together the logarithms of the half sum and the three remainders in links, and divide the sum by 2, diminishing the index by 5, and the result is the log. of the area in acres.

Ex. The three sides of a triangle are 4080, 5040, and 6100 links; required the area. *Ans.* 102*a*. 0*r*. 16*p*.

PROBLEM IV.

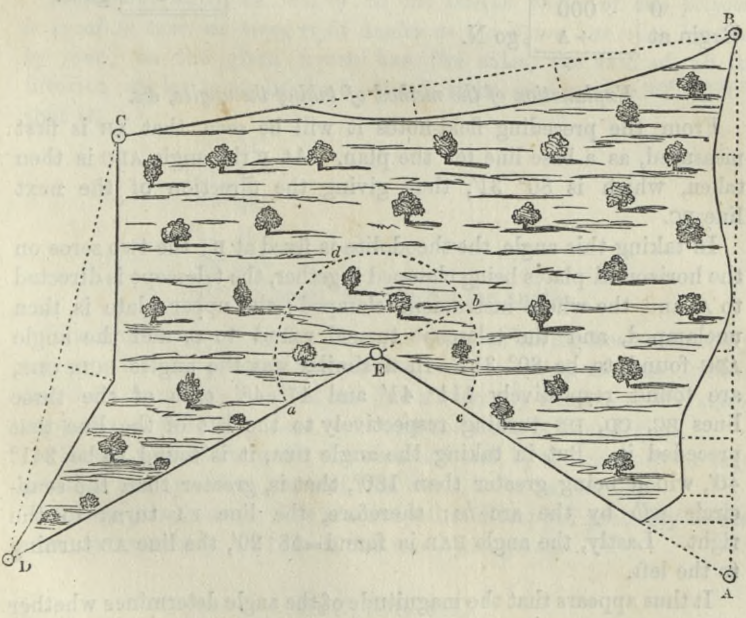
To find the area of a trapezium when the two diagonals and the angle made by their intersection are given.

The rule for the area is the same in this as in Prob. I., the diagonals being used as sides.

SECTION II.

TO SURVEY A WOOD.

The following figure represents a wood, the plan and area of which are required.



Fix station-flags so that the lines compassing the wood shall be as near it as possible, that the offsets may be conveniently taken, and that the stations, at the same time, are on ground proper for fixing the theodolite. Let the stations be A, B, C, D, and E, the field-notes being as below.

	to + C	
	1340	
0	1050	
+	1000	
50	700	
60	400	
0	000	
from + A	80° 31′	to + C
	+ B	go W.
	+ B	
0	1150	
+	1000	
50	900	
00	550	
100	300	
110	160	
188	110	
0	000	
Begin at	+ A	go N.

from + E	58° 20′	to + B
	to + A	
0	896	
0	675	
from + D	241° 40′	to + E
	+ E	go SE.
	to + E	
0	894	
0	000	
from + C	47° 48′	to + E
	+ D	go NE.
	to + D	
00	950	
120	500	
0	000	
from + B	111° 41′	to + D
	+ C	go S.

Explanation of the method of taking the angles, &c.

From the preceding field-notes it will be seen that AB is first measured, as a base line for the plan. At B the angle ABC is then taken, which is 80° 31′, thus giving the direction of the next line BC.

In taking this angle, the theodolite is fixed at B; the two zeros on the horizontal plates being clamped together, the telescope is directed to A, and the whole instrument clamped; the upper plate is then unclamped, and the telescope turned round to C, and the angle ABC found to be 80° 31′. In a similar way the angles BCD, CDE, are found respectively 111° 41′ and 47° 48′, each of the three lines BC, CD, DE turning respectively to the left of the line that preceded it. But in taking the angle DEA, it is found to be 241° 40′, which being greater than 180°, that is, greater than the semicircle *adb* by the arc *bc*; therefore, the line EA turns to the right. Lastly, the angle EAB is found = 58° 20′, the line AB turning to the left.

It thus appears that the magnitude of the angle determines whether the new line turns to the right or the left of the old one, that is, *the new line turns to the left of the old one if the angle is less than* 180°, *and to the right if greater than* 180°, *provided the zero of the instrument be directed towards the beginning of the old line;* therefore the bearings of the lines BC, CD, DE, AE, though put down by way of check, may be omitted in the field-notes.

The remainder of the field-notes are similar to those given in the preceding parts of this work, and therefore require no further explanation.

On plotting and proving the work.

Draw the line AB in the given direction, and of the given length, 1150 links, as a base line; place the centre of the protractor at B with its straight edge close against AB, and from the end towards A mark off $80\frac{1}{2}°$, then through B and the protractor-mark draw BC = 1340 links. Proceed similarly with the lines CD, CE, and the angles C, D. At E the angle is $241\frac{2}{3}°$: the next line EA must therefore turn to the right, and the angle to be marked off at E must be $360° - 241\frac{2}{3}° = 118\frac{1}{3}°$; or, if the protractor be a circular one, the whole angle may be marked off at once. At A, the commencement of the base line, the angle is $58\frac{1}{3}°$, which is a check angle to prove the accuracy of the previous work.

Moreover, since *the sum of all the interior angles of any polygon is equal to twice as many right angles as the figure has sides, lessened by four;* as the given figure has five sides, the sum of all its interior angles must be $2 \times 5 - 4 = 6$ right angles $= 6 \times 90° = 540°$, that is,

$$\begin{array}{r} 80° \ 31' \\ 111° \ 41' \\ 47° \ 48' \\ 241° \ 40' \\ 58° \ 20' \\ \hline 540° \ \ 0' \end{array}$$ proof, as respects the angles.

The angles may, therefore, be considered as having been accurately taken. But *the proof of the plotting being correct, is when the work is found to close,* that is, when the line EA exactly reaches to the starting-point A, or so very near to it that the error is immaterial.

Having plotted the work, the area is found, by the methods already given in the preceding parts of this work, to be 9*a.* 1*r.* 22*p.*

A large pond, mere, or lake is surveyed and mapped precisely in the same manner as in the case of the wood just given.

SECTION III.

TO SURVEY A ROAD OR RIVER.

1. The figure Problem V. Part IV., represents a road, the map of which is required from the following notes; and as the angles at

2, 6, 9, and 12 are taken by the theodolite, the tie-lines from 3 to 4, from 5 to 7, &c., will not be required.

	554	to fence
20	512	+ 6
50	450	
52	380	70
20	300	80
18	100	93
from + 1	108° 20′	to + 6
	+ 2	
	480	to fence
	400	+ 2
38	350	
15	300	
28	200	80
55	000	70
Begin at	+ 1	go N. ½ E.

	to + 14	
78	350	60
68	200	44
50	100	80
from + 1	234° 20′	to + 14 *ch*
from + 9	276° 8′	to + 14.
	+ 12	
	752	to fence
30	720	+ 12
70	650	
86	560	33
76	300	50
130	165	36
	135	cr. fence.
from + 6	58° 20′	to + 12.
	+ 9	
	+ 9	
	700	
	600	cr. fence.
38	480	84
40	300	60
52	180	65
50	100	
from + 2	297° 13′	to + 9.
	+ 6	

Explanatory remarks, &c.

Station-flags being set up at 1 and 2, and the line measured, a flag is then set up at 6, and the angle 2 taken, which being less than 180°, shows that the line 2 6 is on the left of 1 2. Having measured the line 2 6, the angle 2 6 9 is next taken, which, being 297° 13′, or greater than 180°, shows that the line 6 9 turns to the right. Similar observations will apply to the remaining lines. But at station 12, besides the angle to determine the position of the line 12 14, a check angle is taken between an imaginary line, drawn from 12 to 1 and the line 12 14, which is 234° 20′; therefore, after the work is plotted, the protractor will show this angle if the work be right.

If the station-flag at 1 cannot be seen from the station at 12, a flag may be fixed in any other given point in the line 1 2, or in its prolongation, for the purpose of taking the check angle; and if the road is very long, and has several bends, a proportionate number of check angles ought to be taken.

The method of plotting the preceding work will be sufficiently obvious from the example given in Section II. of the survey of a wood.

2. If the figure just referred to, *i.e.*, Problem V. Part IV., represent a *river*, it may be surveyed in the following way.

Measure a system of lines, *ac*, *cm*, &c., to *x*, similar to those used in the last example, and take the angles between them, but omitting *ab*, *nr*, &c., which being tie-lines are not required.

If the breadth of the river is very unequal, a similar system of lines must be used on the other side of it: but if its width is everywhere nearly equal, its breadth may be found by any of the methods given in Prob. I., Heights and Distances, or by Prob. X. Part III.

The map of the river being thus obtained, the area, if required, may be readily found by one or other of the methods already given.

SECTION IV.

THE PRACTICE OF TAKING BEARINGS AT TWO STATIONS.

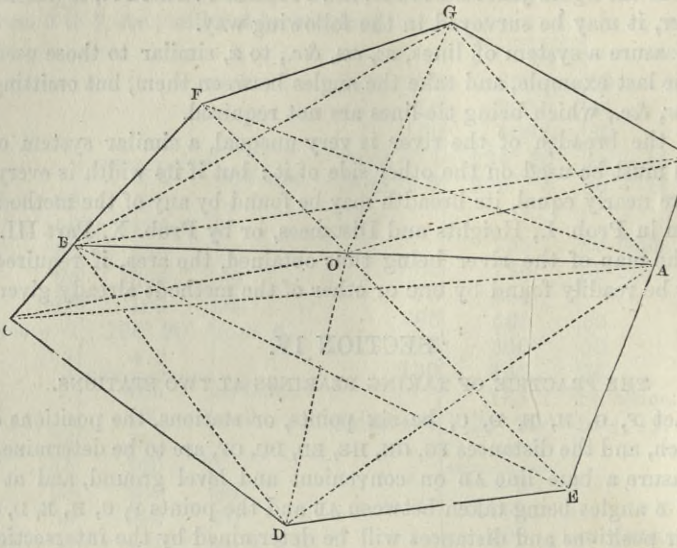
Let F, G, H, E, D, C, be six points, or stations, the positions of which, and the distances FG, GH, HE, ED, DC, CF, are to be determined. Measure a base line AB on convenient and level ground, and at A and B angles being taken between AB and the points F, G, H, E, D, C, their positions and distances will be determined by the intersection of the lines that include these angles, as shown in the figure.

The lines FG, GH, &c., being thus determined, may form new base lines for other portions of the survey.

NOTE.—It is not necessary that the stations A and B should be in the lines EH, CF, as in the figure, these positions being only assumed as being convenient for checking the work, which can be better done by taking angles at any given point O in AB.

The base AB, and the several angles taken at the stations A and B, are shown in the following field-notes, also the angles taken at O, a station in AB, as check angles.

	352° 28′	to + C		341° 11′	to + H
	322° 37′	to + D		292° 18′	to + G
	287° 14′	to + E		221° 14′	to + F
	107° 14′	to + H		167° 44′	to + C
	48° 18′	to + G		99° 59′	to + D
From + B	16° 56′	to + E	From + A	44° 17′	to + E
	+ A			+ 0	
	+ B			348° 28′	to + H
	4323			328° 38′	to + G
	+ 0			310° 56′	to + F
	2220			130° 56′	to + C
Begin at	+ A	go W.		48° 48′	to + D
			From + A	23° 40′	to + E
				+ B	



On laying down and proving the work.

Draw the base line AB in the given direction, and of the given length, 4323 links, making the station O thereon, for the check or proof angles. With the protractor, applied to the line AB, and its centre at A, lay off the angle BAF $= 16° \ 56'$, reckoning from B. Move its centre to B, and lay off the angle ABF$= 360° - 310° \ 56' = 49° \ 4'$, reckoning from A, and the intersection of the lines AF, BF, will give the point F. The position of the point G may be determined in the same way, by the intersection of AG, BG; as also the points H, E, D, C; recollecting that when the angles exceed 180° to take them from 360°, and to plot the remainder from right to left, looking from the centre of the instrument along the base line.

If the work be correct, the lines HE, CF will respectively pass through the stations A, B; this, however, does not check the points D and G: therefore join OD, and apply the protractor to AB with its centre at O, and if it show the angle AOD to be 99° 59′, the point D is in its right position. In the same way, if the angle AOG$= 360° - 292° \ 18' = 67° \ 42'$, the point G is in its correct position; and similarly with respect to the points E, C, F, H.

By calculation.

In the triangle ABF are given the side AB, and its adjacent angles BAF $= 16° \ 56'$, and ABF $= 360° - 310° \ 56' = 49° \ 4'$, from which

AF and BF may be found. Again, in the triangle ABG, are given AB, and the angles BAG = 48° 18′, and ABG = 360° − 328° 38′ = 31° 22′, from which AG may be found. Now in the triangle AFG, the two sides AF and AG, being determined by the two preceding operations, may be considered as given, and their included angle FAG is also given in the field-notes, being = 48° 18′ − 16° 56′ = 31° 22′, from which FG may be found. And similarly GH, HE, &c., may be found.

By either of the preceding methods are found CD = 2576, DE = 2102, EH = 2625, HG = 1977, GF = 1995, and FC = 2071 links.

TWO EXAMPLES FOR PRACTICE BY TWO STATIONS.

It is required to find the positions and distances of the points D, E, F and G, from the field-notes No. 1, also from the field-notes No. 2.

No. 1.

	331° 2′	to + G
	314° 0′	to + F
	70° 42′	to + E
From + A	27° 50′	to + D
	at + B	
	326° 0′	to + E
	269° 5′	to + D
	46° 25′	to + G
From + B	22° 0′	to + F
	at + A	
	to + B	
	8000	
	+ 0	
	4000	
Begin	at + A	

No. 2.

	336° 0′	to + G
	321° 30′	to + F
	68° 40′	to + E
From + A	31° 30′	to + D
	at + B	
	319° 26′	to + E
	381° 40′	to + D
	50° 14′	to + G
From + B	24° 10′	to + F
	at + A	
	to + B	
	15421	
	+ 0	
	6000	
Begin	at + A	

SECTION V.

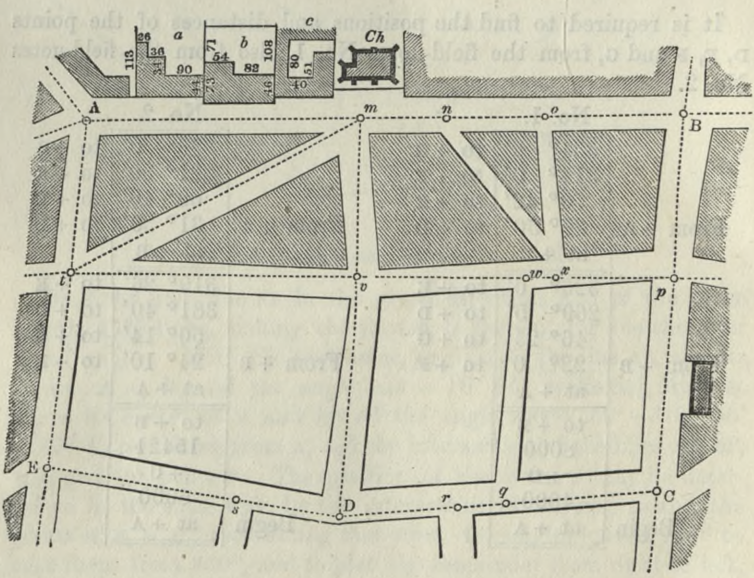
TO SURVEY A TOWN OR CITY.

Let the following figure represent a portion of a town or city, the map of which is required.

Begin the survey at the meeting of three or more principal streets, and select a station, as A, where the longest prospects can be obtained along all the streets diverging from it, in order to get the longest main or station lines. Having fixed the theodolite at that station, take the direction of one of the principal streets as a base line, as AB, which must be directed to some well-defined object, as to the right or left side of a door or window, or a projecting corner of a house, or to an angle, or upright post of railing, or any other mark that can be well remembered; a description of which should be also put down in the field-book.

Angles between this base line and the other lines, diverging from station A must then be taken, the objects to which these lines are directed being well defined, in the manner already explained with respect to the base line.

The lines thus set out must next be measured with the chain, taking offsets to all projecting corners of buildings, bends, or openings; and



to all public buildings, as churches, markets, &c., also to statues, obelisks, &c., leaving stations at the ends of the streets to the right and left of the line, and taking the angles of their several directions. The same process is to be repeated on the other lines till the whole is finished.

Thus, fix the theodolite at A, and take the bearings of all the streets meeting there; then measure AB, taking offsets to the openings, corners, &c., of buildings, leaving stations at *m*, *n*, *o*, opposite the directions of the streets on the right. If the survey is for a railway, for which some of the buildings are required to be taken down, the entries should be made at a sufficient distance apart to admit sketches of the buildings, gardens, yards, &c., belonging to the different proprietors, with the dimensions put upon them, as shown at *a*, *b*, and *c*; so that the several parts may be accurately mapped, and their areas determined, for the purpose of their valuation. The same should be done in parish maps, where the valuations of the different properties are required for the purpose of rating, &c.

The base line AB being measured, fix the theodolite at B, and take the angles of the several streets diverging from that point, then measure BC, taking the offsets, and making stations opposite the directions of the streets to the right and left, as at *p* and C. Make C the third station, at which take the angles and proceed to D, as before. From thence proceed in a similar manner to E, from which return to the first station A, and in measuring EA leave a station at *t*, from which the station *m* in AB may be joined by the line *tm*, and the station *p* in BC by the line *tp*, crossing the line joining the stations D and *m* in *v*, stations in all these lines having been left opposite the streets and lanes, to the right and left of them, as at *q*, *r*, *s*, *w*, *x*.

If the interior angles of the five-sided figure ABCDE have been correctly taken, their sum will be equal to $2 \times 5 - 4 = 6$ right angles, as shown in Sect. I. Also the line *tm* determines the position of AE, independent of the angles; in the same manner, the lines D*m*, *tp*, determine the positions of the line BC, CD, and DE. But lines can seldom be obtained in large towns thus effectually, to tie the main lines independently of the angles, this result being referable to the line *tm*, which is actually a tie-line; therefore, when the peculiar position of the street, in which the line *tm* is drawn, is wanting, the angles and the closing of the work must be altogether relied on, as they constitute a sufficient proof of themselves.

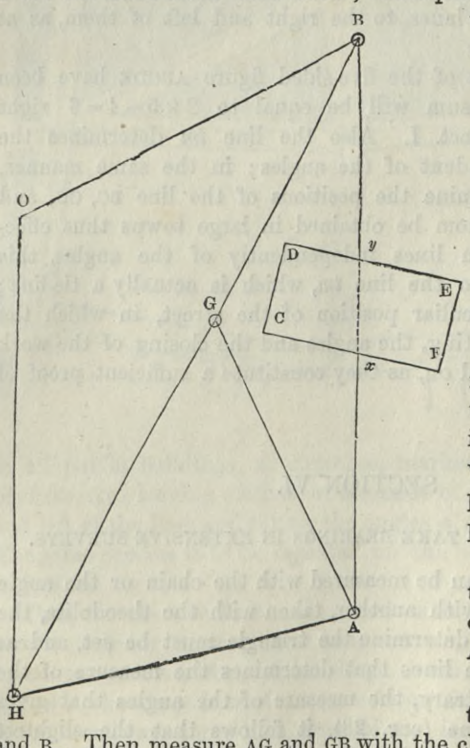
SECTION VI.

TO RANGE LINES AND TAKE BEARINGS IN EXTENSIVE SURVEYS.

Before a single line can be measured with the chain or the angle which one line makes with another, taken with the theodolite, the three station poles that determine the triangle must be set, and as it is the direction of the lines that determines the measure of the angles, and, on the contrary, the measure of the angles that gives the direction of the line (cor. 28), it follows that the slightest deviation in either will mar the trigonometrical relation that exists between them, so that the subsequent work of plotting cannot be performed with geometrical accuracy. In Constructive Geometry triangles have to be made, but in Trigonometry they are presumed to exist, and their sides and angles to have a definite relation to each other.

In every survey the first work is to set the poles at the principal stations, and thus determine the triangles of the trigonometrical survey, and to fix upon the main line, which should be measured with the chain before the taking of the bearings with the theodolite commences. This is done by the surveyor going over the ground, and either setting the poles or marking the best places for the stations.

Between the principal stations intervening poles require to be ranged afterwards, before the lines can be accurately measured with the chain. When the lines are long, this is generally done with the aid of a telescope only; but when sight is obstructed the theodolite may be necessary. Thus in the annexed diagram the stations A and B are not seen from each other; consequently, before the two intervening station poles x and y can be ranged so as to enable those in charge of the chain to measure the line, the angles A and B must first be determined, which is done in taking the bearings. An example will best illustrate this.



Ex. To range and measure a line between two stations when sight is obstructed by a town, village, farm-homestead, plantation, or other impediment.

Let A and B be the two stations, CDEF the obstruction to sight, and AB the line sought.

Take any third station G visible from A and B. Then measure AG and GB with the chain, and take the angle G with the theodolite. Having thus found two sides and the included angle, the other two angles and side may be got by Case III. Part VIII. Sect. III.

When the line AB has to be measured with the chain as is generally the case, find the angles A and B as before, and then either range $+x$ and $+y$ with the theodolite, or by Problem XXII. Part I. This done, the line may be measured with the chain by Problem XII. Part III.

In extensive surveys the bearings should be kept well in advance, in order not to give any hindrance to that division of the staff in charge of both chain and field-book on their arrival at stations such as A and B in the above example. And besides expediting the work, it is also in favour of accuracy; for when the principal lines, with the exception of the base line, are first determined by trigonometry, it serves as a check upon the measurement with the chain.

The principal stations should also be so organised that the bearings taken at them shall be a proof-check upon each other.

This is done by making each flagstaff visible from as many of the other principal stations on the survey as possible, so that the total area is thus twice taken, and the position of things several times determined.

In surveying thickly-wooded parks, game-preserves, deer-forests, and large plantations covering hundreds and even thousands of acres, it is often necessary to place station poles in any open grounds most convenient, and to take the bearings of such from other stations on elevated places at a distance. Such auxiliary stations are not unfrequently upon the estates of adjoining proprietors, whose leave must necessarily be obtained before field operations commence. Sometimes the smoke that issues from the chimneys of the shooting-box of the sportsman, or of the cottage of his gamekeeper, is the only thing that indicates their respective sites. In such cases a flagstaff may have to be fixed on the top of a tree. In other examples the chimney-tops may be visible from some elevated ground, and thus serve the purpose of flagstaffs.

On mountainous estates—as in some parts of Wales, north of England, and greater part of Scotland—a station pole on the top of a hill can seldom be seen from any part of the valley below, owing to the height and curvature of the former, and narrowness of the latter. In such cases the principal station poles will require to be placed at proper distances along the brows of the two opposite mountains, although only one of them may be included in the survey. The two sets of station poles on the opposite hills should be visible from as many of the stations on the open grounds of the valley as possible. Towards the top of the mountain being surveyed, other station poles may then be placed, visible from those on its brow farther down.

There are three principal reasons for placing station poles on opposite mountain-sides, as above directed, that require to be noticed.

First. The line between two flagstaffs, one on the brow of each hill, right and left, forms an office or plotting check, the bearings of the two being taken from a station in the valley below, and also the bearings of the stations in the valley from the two elevated ones above on each side.

Second. Lines along the brow of a hill can frequently be measured with the chain when those in the valley cannot, owing to lakes, rivers, or growing timber, being in the way.

Third. The bearings of stations on the opposite mountain, both above and below its brow, can be taken, and the various conspicuous objects thus laid down upon the plan in their true position—such as a lake, a waterfall, meanderings of a river, bridges, rocks, roads, residences, particular outstanding trees, &c.

In a campaign country, the station poles in the valleys may, with few exceptions, be seen from those on the tops of hills; but owing to the greater number of the latter, it is often advisable to continue the direction of lines rectilineally over them. This may be done by means of poles, but more easily by the theodolite placed on the top of the hill. In preliminary surveys taken with the telescope for the purpose of determining the best places for the stations of the trigonometrical survey, straight lines may thus be produced over hills, by simply laying down the offset-staff in the proper direction both ways, and then by directing the telescope first from the one end of the offset-staff, and then from the other, along its length. An approximation of the bearings may at the same time be taken by a protractor and the shadow of a staff, or by a pocket compass.

The work of taking the bearings commences first at the one end of the main line and then at the other. Thus, if we suppose BH in the diagram a main line and that at + B the bearings of O from H are first taken; then in going along the line to H, the angle AGH would be taken in passing, which would also give AGB. At H the angle BHO would be taken. The telescope would then be turned back again to B, and next round from B to A, in order to find the angle GHA, which, with AGH previously found, would give GAH.

The line BH, and consequently BG and GH, having been previously measured with the chain, the necessary trigonometrical data are now given to find the remaining parts of the three angles BOH, BGA, and AGH. Thus BH and the three angles are given to find BO and HO; GH and the three angles to find AG and AH; and AG and GB and the included angle AGB to find AB and the angles ABG and BAG.

In a similar manner all the sides and angles of the several triangles into which the survey may be divided are found. At every station there are always two lines ranging from it, more frequently three and four; and sometimes when the flagstaff occupies a central position, as at O in the diagram Section IV., a great many. And as the work proceeds, the readings from the compass and measure of the angles, horizontal and vertical, from the theodolite, are carefully recorded in the field-book in their respective columns; one for the horizontal angles, one for the vertical when taken, one for the compass, and the two outside spaces for remarks, &c., as usual.

In large surveys, the calculations and plottings should never be allowed to fall behind, as many things are liable to escape the memory that cannot be entered in the field-books. And besides this, it is necessary that the triangles be plotted to test their own accuracy, as well as that of the measurement with the chain.

With the exception of the main line, the chain should follow the theodolite. This is not an absolute rule, but one of expediency, which will be found for the most part advisable to observe.

When the details within the large triangles are left to be worked up in the office, those in charge of the field-book cannot be too careful to avoid omissions as to the proximate measure of angles not taken by the theodolite, and the range of lines not driven by the chain, that determine the species of the triangles and the ratios of their sides, otherwise they may fall into what is termed "the ambiguous case" already noticed (Section II. Part VIII.) In other words, when in the field they should take a note of the trigonometrical relationship of the sides and angles of the triangles to each other, more especially as to whether the sines are those of acute or obtuse angles, and whether the diameter includes that portion of it in which the centre of the circle lies (Art. 34, Part VIII. Section II.)

Omissions of the above kind are only experienced in plotting, which proves that they belong to Constructive Geometry, and not to Trigonometry or the mensuration of superficies, as formerly shown. To beginners, anxious to get quickly over the ground, such omissions are by no means unfrequent misfortunes. It must be borne in mind, however, that no ambiguity exists in the field, for there lines and angles bear a closer resemblance to the definition given them than do those of the office and class-room. If therefore, in driving the line AC (fig. 2, Section II. Part VIII.), it is omitted to enter in the field-book that the sine of C is within the triangle, and in driving

AC (fig. 3) that the sine of C is that of an obtuse angle, and consequently falls upon the diameter BCD, produced without the triangle, then ambiguity in the office is the natural concomitant of such an omission. And this, too, does not represent the full amount of negligence in the field, for in driving either of the two lines from C, the staff are proceeding to a definite point of the compass, one too far asunder from that of the other line to be mistaken for it.

The professional objection of surveyors to treating the ambiguous case as a proposition, either in trigonometry or the mensuration of superficies, is perhaps the most convincing proof that it belongs exclusively to Constructive Geometry. Thus, if admitted into the former, a triangular field would contain either the one or the other of two measures, either the area within the triangle ABC (Example II. Part VIII.), or that within ABC (Example 3); consequently two plans would be required, a greater and a less, for which payment would assuredly be received for neither. Hence the practical conclusion.

In field operations it is always preferable to determine either the three angles and one side or two sides of a triangle, or else two sides and the included angle. But this cannot always be complied with, and the following general rules will assist beginners in making provision for the exception, as by Article 34, Section II. Part VIII.

1. When the sum of the adjacent or opposite angles is equal to a right angle, the other angle is also a right angle.

2. When the sum of the two angles adjacent to any one of the given sides is less than a right angle, the other angle opposite that side is obtuse.

3. When the sum of the two angles adjacent to any one of the two given sides is greater than a right angle, the other angle opposite that side is acute.

4. When the angle included by the two given sides is obtuse, then the other two angles of the triangle are acute.

5. When the complement of the given angle is greater than the angle included by the two given sides, the other remaining angle is obtuse.

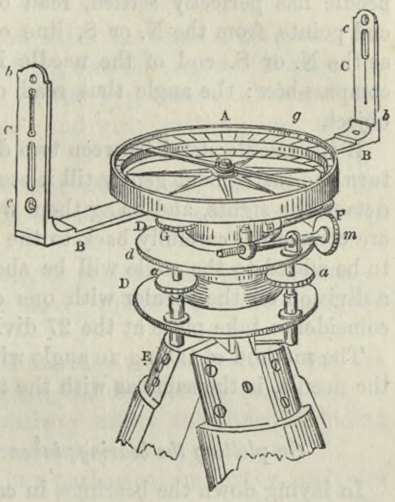
6. When the complement of the given angle is less than the angle included by the two given sides, the remaining angle is acute.

7. The nearer the angle opposite the given angle is to a right angle, the greater is the liability to error, in taking approximate estimates of angles.

SECTION VII.

TO SURVEY WITH THE CIRCUMFERENTOR.

The circumferentor is sometimes used in surveying. It consists of a flat bar of brass BB, about fifteen inches long, with sights CC at its opposite ends, and two narrow slits *b*, *c* for taking observations; in the middle of the bar is a circular brass box A, containing a magnetic needle, which, as usual, is covered with glass. The ends of the needle play over a brass circle *g*, which is divided into 360° in such a manner that the two quadrants are at right angles to the line drawn through the sights. The instrument is usually supported on a tripod E, and can be turned in any direction by means of its socket-joint. When the magnetic needle is properly balanced, and moves freely in its horizontal position, it will retain its position of magnetic north, while the sights are turned from one station to another; consequently the number of degrees which the angle contains can be read off. The great length of the magnetic needle increases the accuracy of the circumferentor, for, if it were short, and consequently the graduations on the brass circle proportionately small, the angle could not be read off with sufficient accuracy.



The instrument is chiefly used in surveying mines, coal-pits, woods, &c.

The circumferentor is sometimes provided with a spirit-level F, with adjusting screws *a*, *a*, &c., a tangent screw *m*, a vernier, &c., in which case it may be made to answer most of the purposes of a theodolite.

The needle should not be suffered to play longer than necessary, but be lifted off its centre, otherwise the delicate point on which it turns would soon be destroyed. The instrument usually has the east and west marked contrary to their true positions, in order that by the reading of the needle the actual direction of the line is shown.

To find the bearing of an object by the circumferentor.

Place the circumferentor over the station, turn N. sight to the object, and looking through the S. sight adjust with the tangent-screw till the hair in the N. sight exactly cuts the object—this part of the operation being the same as in the theodolite. When the needle has perfectly settled, read off the degrees to which its N. end points, from the N. or S. line of the compass-box, accordingly as the N. or S. end of the needle is in the N. or S. part of the compass-box; the angle thus read off is called the bearing of the object.

If the needle stand between two degrees, as between 40° and 41°, turn the instrument gently till it stand exactly at 40°, and clamp it; detach the sights, and bring them with the vernier, with which they are connected, carefully back to the object: the number of minutes to be added to the angle will be shown by the first coincidence of a division on the vernier with one on the horizontal plate; if the coincidence take place at the 27 division, the angle will be 40° 27′.

The method of taking an angle with this instrument without using the needle, is the same as with the theodolite.

On plotting the bearings taken by the circumferentor, &c.

In laying down the bearings, in cases where the angles are taken from the magnetic meridian of the circumferentor, that meridian is first drawn on the plan, and from a point in it the bearing of the first line from it is laid off with the protractor; the length of the line is next laid off, and through its extremity another meridian is drawn parallel to the former; the second bearing, and the length of the second line, are then laid off in the same manner; and so on till the work be completed, and the variation of the compass being known, the true meridian may then be drawn on the plan, that the work may have its proper position on the finished plan.

The above method is the same as that of laying down a wood, or road survey, by the theodolite, except in laying off the angles.

When the angles are taken with the circumferentor without reference to the magnetic needle, the method of laying down the work is the same as in theodolite surveying.

NOTE. In coal-mines, when a pit or shaft is required to be sunk from the surface to a given point in the works below, it is usual to take the several bearings and distances with this instrument, from the bottom of an existing shaft along the passages in the mine, to the required point; and then to repeat the same operation on the surface of the ground, without plotting the work on paper; by which means complete accuracy is frequently obtained.

RAILWAY SURVEYING.

A complete survey for an intended railway contains six problems, three of which are field operations, and the remainder office work. They are as follows :—

Prob. I. To range the station poles that form the chain and other lines.

Prob. II. To measure with the chain the chain lines; with the theodolite the angles which they form, also the angles of depression, to obtain data for the reduction of the hypotenusal planes of the earth's surface to a common plane; and with levelling instruments to take the levels of stations.

The field notes under this problem are in three field-books; the first for the chain, the second for the theodolite, and the third for the levelling instruments.

Prob. III. Given the field notes, to plot the plan and sections.

Prob. IV. To apportion the land required for the intended railway.

Prob. V. To find the field notes required for setting out the railway estate and the line, including the diversion of roads, &c.

Prob. VI. To set out the railway estate and line, including roads, &c., whose diversion is necessary.

In a preliminary survey, and in a parliamentary survey, and also in surveying a railway which passes through an estate being surveyed, only a portion of the above field and office operations are required.

In the following directions for a preliminary survey, the lines are ranged for laying down the railway from a series of connected base-lines. It is seldom, however, that lines thus ranged can be measured by the chain with sufficient accuracy, and when such is the case, even ground adjacent must be selected for base-lines, so as to obtain the true lengths of the former by trigonometry.

Another method is to range the right lines of the projected railway, and either to join them by the conveying tangents from the two tangential station poles, or else by the chord-line that lies between them (the two tangential stations), and then to take the bearings of these stations from the adjacent base-stations, so as to find their reduced distances or base-lengths required for plotting; and when an adjacent base-line is not perfectly level, its hypotenusal length must be reduced to base-measure, otherwise the field notes, both for plotting and setting out the railway, will be erroneous. (See ADVERTISEMENT, p. viii, under RAILWAY SURVEYING.)

Illustrated general directions for a preliminary survey of a portion of a projected railway.

Landowners (resident and absentee), the executors of minors, mortgagees, and capitalists, often order plans purposely to ascertain how far projected railways will interfere with their respective interests involved, and whether they would be justified in farther supporting such projects.

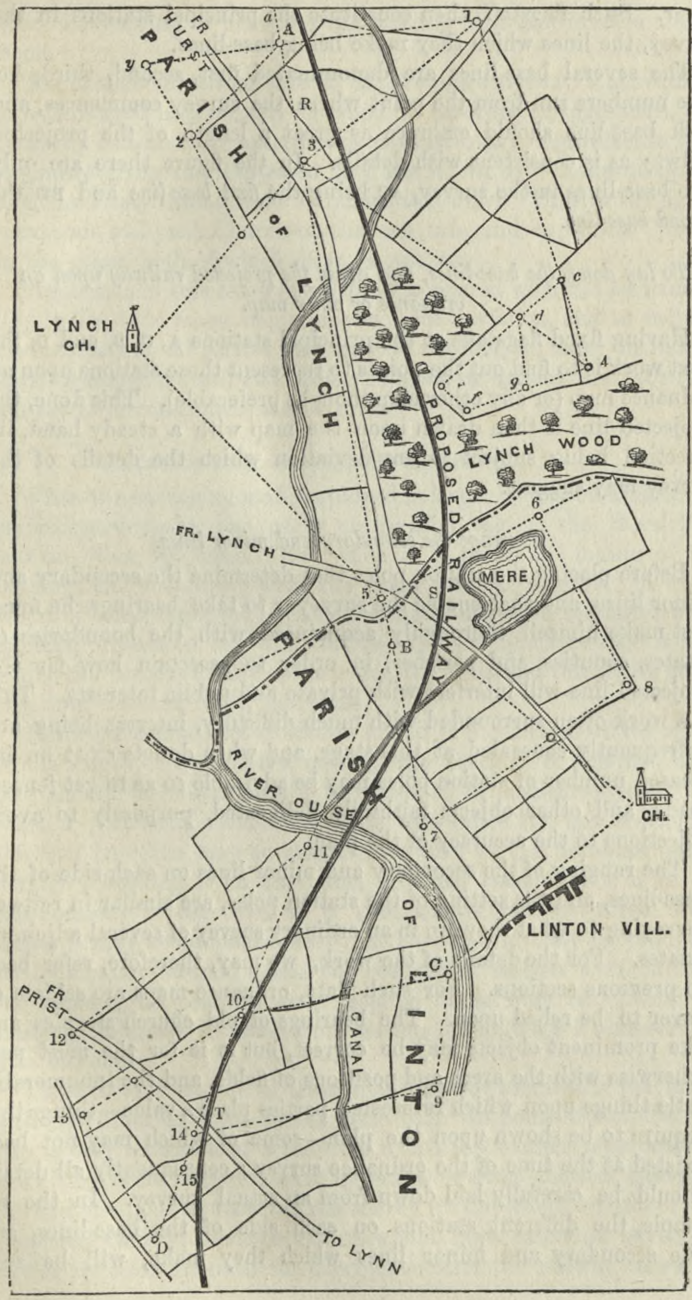
Plans of this kind are of the simplest character, consistent with the purpose they are intended to serve. The several resident agents in charge of estates proposed to be intersected watch such railway movements closely, and never fail to apply to their employers for instructions how to act. They are often by profession land-surveyors themselves, and, when such, they send to their absent employers a sketch of the projected line on tracing-paper, taken from the plans of their estates; or when otherwise, they employ their surveyors to do so for them, who may give a general sketch of the whole intended line upon an ordnance map.

Again, several parties may join and order a survey and plan of the whole district through which the projected railway is intended to pass, or only of that portion of it in which they are more immediately interested.

The annexed figure, already referred to, may be presumed to represent a plan of this latter kind. It is, in point of fact, a copy of a plan of a portion of a district surveyed for the purpose of the projected railway RST, the base-line AC being ranged so as to avoid Lynch Wood. This line terminates at + C, partly on account of its diverging too far from the line of the railway, and partly to avoid the River Ouse beyond + C. The next base-line commences at + B in AC, and runs close to the railway at T. Beyond + D (on the complete survey of the whole line) it continues the same direction till its deviation requires another base-line.

To range the base-lines.

The initiatory step in such a survey is to range the base-lines. This is done by going over the ground and carefully examining it from points where the projected railway may be presumed to be on a level with the surface. Such points are generally determined by the amount of excavation, on the one hand, that is required to fill up the hollows on the other. An experienced eye will approximate very closely to such data. Having come to a definite conclusion on these points, and the curvature to which they may give rise, the flagstaffs are then set so as to avoid obstructions, and be visible from each


FR HURST
PARISH OF LYNCH
LYNCH CH.
PROPOSED RAILWAY
LYNCH WOOD
MERE
FR. LYNCH
PARISH OF LINTON
RIVER OUSE
CH.
LINTON VILL.
CANAL
FR PRIST
TO LYNN
a
A
y
R
1
2
3
c
d
4
g
f
e
6
5
S
B
8
7
11
C
10
12
9
13
T
14
15
D

other. Such flagstaffs then constitute the principal stations in the survey, the lines which they make being base-lines.

The several base-lines are denominated first, second, third, &c. The numbers run from the point where the survey commences, and each base-line should embrace as great a length of the projected railway as is consistent with details. In the figure there are only two base-lines in the survey, AC being the *first base-line* and BD the *second base-line.*

To lay down the base-lines, and draw the projected railway upon an ordnance or other map.

Having fixed flagstaffs at the principal stations A, C, B, and D, the next work is to find out the points to represent those stations upon an ordnance map (or any other map thought preferable). This done, the projected line is then drawn upon the map with a steady hand, its direction being subject to any deviation which the details of the survey may suggest.

To range the secondary and minor lines.

Before placing the station poles that determine the secondary and minor lines, and that enable the surveyor to take bearings, he must first make himself thoroughly acquainted with the boundaries of estates, counties, and parishes, in order to ascertain how far the projected line will interfere with private and public interests. This is a work often surrounded with much difficulty, interest being not unfrequently concealed at this stage, and when doubts exist an increased number of station poles may be advisable so as to get fences, roads, and other objects faithfully delineated, purposely to avoid objections to the accuracy of the plan.

The ranging of the secondary and minor lines on each side of the base-lines, and the setting of the station poles, are similar in railway surveying to what they are in an ordinary survey of several adjoining estates. For the details of the work, we may, therefore, refer back to previous sections. For such data, ordnance maps are seldom or never to be relied upon. The bearings of old church steeples and like prominent objects may be correct, but it is for the most part otherwise with the areas and positions of fields, and the innumerable little things upon which interested parties place a value—things that require to be shown upon the plan—some of which may not have existed at the time of the ordnance survey ; consequently all details should be carefully laid down from an actual survey. In the example, the different stations on each side of the base-lines, and the secondary and minor lines which they make, will be seen upon the drawing or plan, and the plotting data they involve understood.

The distance on either side of the projected railway that requires to be surveyed is invariably specified in the order or agreement. It generally considerably exceeds the parliamentary requirements subsequently given and illustrated in Plate XIII., by "limits of deviation," and may extend from 30 chains to a mile. Thus in the figure it extends to Lynch Church on the one side, and to Linton Church on the other, both of which serve as stations.

The reason of this extra breadth, as compared with the parliamentary limits, is to make ample provision for deviation, and to include the interests of all parties that may be effected, in order to induce them to come forward, and either support or oppose the projected railway as circumstances may direct.

To take bearings and measure the first base-line AC.

When the surveying staff employed is large, it may be divided as in land-surveying; one party or division taking the theodolite, and the other the chain. But when it is small, the taking of the bearings and measuring with the chain proceed together, an extra labourer being employed to carry the theodolite when the latter work is being prosecuted between stations. This conjunct method is that which the present example illustrates, the chain and theodolite going together, the staff being undivided.

The work commences at station A, and in a complete survey of the whole length of a projected railway, the bearings of the first base-line should be taken from a meridian, as in land-surveying. But when a portion of such a survey is only called for, the bearing of the first base-line may be taken from that of the base-line immediately preceding it on that side, provided station A is in it. If the contrary is the case, as it generally is, two or three other methods may be adopted. *First*, the base-line AC may be produced to the auxiliary station *a*, and the bearings at station A taken from it. *Second*, the bearings at A may be taken from C. *Third*, the bearings at A may be taken from station 1, the line *y* 1 thus serving as a meridian.

The first and second of these methods are identically the same, although differing in the details indicated. In point of fact, the line CA*a* is only a portion of a base-line which the learner may presume has either been ranged from a meridian or from another base-line, for had the plan represented the survey of the whole length of the projected railway, then the line CA would have extended beyond A in the direction A*a* until its divergence from the railway called for another base-line; and we may further observe that, in a similar manner, the base BD would have proceeded till its deviation called for a base-line on that side, which may be taken as the *third base-line*, to range from station 15, in the same manner as the second base-line ranges from B on the first base-line.

From these observations, the learner will readily perceive how the total number of base-lines in a complete survey of the whole length of a projected railway may be ranged from a meridian, and their positions accurately delineated upon a plan. He will also be able to comprehend the proper relation that exists between the survey of a part and of the whole.

According to the above data, the range of the base-line AC with the meridian may be considered as having been previously taken. The bearings at A, therefore, commence either from the auxiliary station *a*, by the first method, or + 1 by the third method. The former is the more accurate of the two methods, as it gives the whole of the details required for rectilinear and angular proof; for if the reading at C is 180°, the line CA*a* is correct, and if the sum of the angles at A is 360, the whole of the bearings are true, as will be seen below. The readings of the bearings may be entered in the field-book thus:—

From + *a*	231° 40′	to + 2
in AC produced	180°	to + C
	90° 40′	to + 1
Begin	at + A	

The several angles at A are now found and read thus:—

*a*A1	=	90° 40′	
1AC	=	89° 20′	180
CA2	=	51° 40′	
2A*y*	=	39°	
*y*A*a*	=	89° 20′	180
	Proof	360	360

The bearings at A being taken, the work of measuring with the chain commences from that station to station 3. At + 3 the instrument is then placed, and the bearings from + A to + 1, from + 1 to + 4, at the north-eastern boundary of Lynch Wood, from + 4 to + C, and from + C to + Lynch Church, are each taken and entered in the field-book, as were the bearings at + A. The part of the base-line between + 3 and + 5 is next driven, the River Ouse being crossed by the bridge on the road from Hurst to Linton, that portion of the line across the river being measured according to Problem XII. Part III. At + 5 the bearings are taken from + 3 to + 6 to get the southern boundary of Lynch Wood, the northern boundary of Linton Mere, and the boundary between the two parishes shown by dotted lines, from + 6 to Linton Church, from Linton Church to + C, and from + C to Lynch Church; also from Lynch Church to + 3 as a check upon the last angle, so as to determine accurately the position of the church. When the ground is undulating, checks of this kind should never be omitted. The measurement now proceeds to + 7, where an angle is taken to + 8, the line passing over that from + 5 to Linton Church, a minor station pole being placed at the crossing of the two lines. The remaining portion of the first base-line is then driven, viz., from + 7 to + C, the distance across the River Ouse being measured by any of the methods given under Problem I. Part VIII. At C the bearings to Linton Church, and to stations 9 and 10, are taken, which conclude the work on the first base-line.

To take bearings and measure the second base-line BD.

The staff now go back to B, and from that station take the bearing from + C to + D, which determines the range of BD. The first portion of the line, viz., from + B to + 11, is then measured, the distance across the River Ouse being found, as before, where the first base-line crosses near Linton. The measuring of the remaining portions of the line, and the taking of the bearings at the several stations upon it, may advantageously be left for the learner to conclude as an exercise.

Details of the survey.

The measuring of the secondary and minor lines, the taking of the offsets, and the whole of the remaining portion of the work, with the exception of levelling, and reporting on the probable amount of damages along the projected line, is so exactly similar to land-surveying, that the more advisable course in an educational work of this kind is to leave it to be performed by the student. Such, when taken in detail, forms very suitable exercises, and by this time he must be considered qualified to go through the whole successfully.

With regard to levelling and reporting, all that is generally demanded of the surveyor, relative to the former, in a preliminary survey, is to take some points in the projected railway presumed to be on a level, or in the same horizontal plane with the surface of the ground, or nearly so, the difference between water-level lines and horizontal being nominal, and from these stations to take the angles of elevation and depression, so as to be able to give an approximate estimate of how much land it will occupy. On the severance of fields, the diversion of roads, watercourses, and other encroachments of this kind, the surveyor may report. But landowners and their agents, when once they get the plans, generally go over the ground along with their tenants and satisfy themselves in such points, and for the most part they are by no means strangers during the survey at the several station-flags.

PART X.

RAILWAY ENGINEERING.

SECTION I.

LEVELLING.

Levelling is the art of representing the inequalities of the upper boundary of any section of the earth's surface, and of determining the relative heights of any number of points in that boundary, above or below a line equidistant, at every point, from the earth's centre. This line is termed a level line,* and is that which water assumes when at rest.

The operation of levelling may be performed by various instruments, depending on the action of gravity: the most simple of which are those formed with a *plumb line*, such as the ordinary mason's level; but these are only suited for very limited operations.

The fluid or *water-level* consists principally of a glass tube, bent like the letter U, open at both ends, and partly filled with water; the surfaces of which, in the two arms of the tube, will stand at the same level by the action of gravity; and a line of sight taken over these two surfaces is therefore a level line. This instrument has been much improved, but, in all its modifications, it is found inconvenient in practice.

There are also the reflecting and cambrian levels, which are useful only for special operations of small extent, such as military engineering, &c.

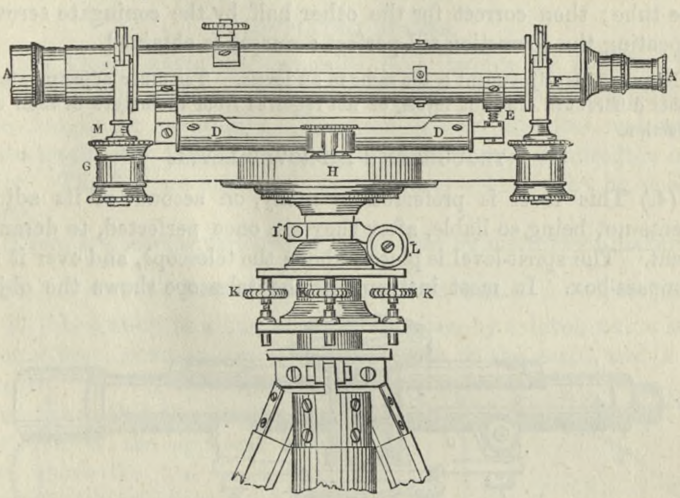
(1.) The instrument used for levelling by civil engineers is called a spirit-level, or simply, a level.

DESCRIPTION OF THE Y LEVEL.

(2.) This instrument consists of a portion of a theodolite brought to its utmost practical perfection, and constructed for levelling

* What is here termed a level line, is, in reality, a great circle of the earth, a small arc of which, on account of the great magnitude of its radius, may be considered as a straight and level line.

purposes only; its chief parts are the telescope AA with the spirit-level DD beneath it, and two parallel plates with four conjugate screws K, K, &c., between them; to the lower of these plates are fixed the legs that support the instrument; and on the upper one rests a pillar that supports a strong brass plate GG, in which is fixed the compass H. This plate supports the telescope by means of two pillars, F, F, called Y's from their forked shape, and



from which the instrument has its name. The telescope is provided with a screw C, to adjust the object-glass, and four other screws to adjust the cross wires within it. The spirit-level is adjusted by the screw E, having a joint at the other end. The instrument is clamped by the screw I, and brought accurately into the required line by the tangent screw L.

ADJUSTMENTS.

(3.) *There are three adjustments of the level;* firstly, *that of the line of collimation of the telescope;* secondly, *that of the axis of the level of the axis of the telescope;* thirdly, *that to make the telescope always at right angles to the vertical axis of the instrument.*

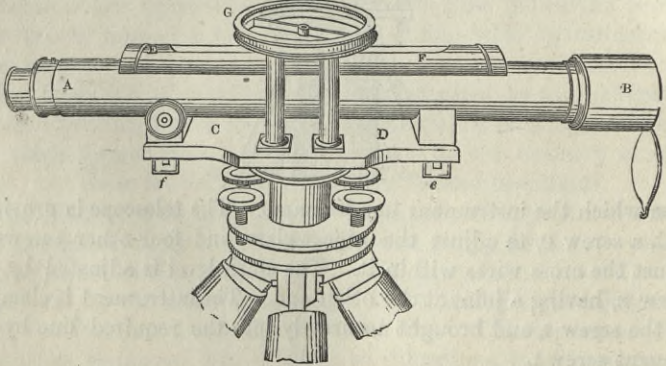
The two first adjustments are the same as those for the theodolite, which see: the last is as follows.

Having fixed the instrument on the ground as nearly level as can be judged by the eye, set the telescope over either pair of conjugate screws K, K, &c.; by means of which bring the bubble to the middle of the level tube; turn the telescope over the other pair of conjugate screws, and correct the level; turn it back to the other pair, and correct again, if necessary. Then reverse the direction of the telescope over either or both pairs of screws, alternately backwards and forwards; and if the bubble be found to stand in the middle in all directions, the instrument is then in adjustment; if not, the error arises from the axis of the level not being perpendicular to the vertical axis of the instrument. To correct this error, raise or depress the movable support of the telescope by the milled-headed screw beneath, till the bubble be brought half-way to the middle of the tube; then correct for the other half by the conjugate screws; repeating the correction till perfect accuracy is obtained.

NOTE.—This adjustment is the same in all levels. The other adjustments are either different in different levels, or not required from the nature of their construction.

TROUGHTON'S IMPROVED LEVEL.

(4.) This level is preferred by many, on account of its adjustments not being so liable, after they are once perfected, to derangement. The spirit-level is placed above the telescope, and over it the compass-box. In most instruments the telescope shows the object



inverted. The spirit-level is firmly fixed in its cell, and the adjustment of its axis with that of the telescope, or line of collimation, is given by two screws near the eye-glass.

GRAVATT'S LEVEL.

(5.) This level is another modification of the preceding one; it is usually called the Dumpy, from the comparative shortness of its telescope. It has an object-glass of larger aperture, and shorter focal length; its spirit-level, as in the preceding one, is fixed above the telescope; it has also a small mirror so fixed on a hinge that the position of the air-bubble can be seen by the observer while he reads off the staff.

Many varieties of levels are used by engineers; but the differences in construction are generally only slight. It is important to have one made by a good maker, or much trouble may be caused in its use. The telescope should be short and of large diameter, and *it should always reverse the image*, by which much light is gained. The reversal of the figures is only a little perplexing at first; it is very soon learnt, and no one who has been accustomed to a reversing level would willingly change to an upright one.

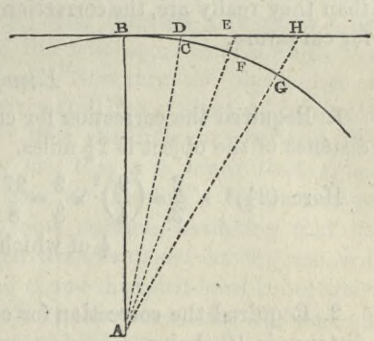
LEVELLING STAVES.

(6.) These staves are generally from twelve to fourteen feet in length, but made to slide telescopically into about one-third of this length for portability. They are divided into feet, and again into tenths of a foot; and, lastly, subdivided into hundredths of a foot. The feet are marked by large figures—the tenths by smaller ones.

There are several varieties of staves sold by instrument-makers.

CORRECTION FOR CURVATURE.

(7.) Let BDEH be a line of sight as given by a level, *i.e.*, a horizontal line; BCFG an arc of a great circle of the earth, and A its centre. It will be seen from the figure that the heights DC, EF, HG, of the apparent level BH, above the true level, continually increase from the point B. The height DC of the apparent level above the true, is nearly equal to the square of the distance BD, divided by twice the earth's radius AB, that is, $DC = \frac{BD^2}{2AB}$. Similarly $EF = \frac{BE^2}{2AB}$ &c.; therefore the corrections DC, EF, &c., are always proportional to the squares of the distances BD, BE, &c., seeing that 2AB is a constant quantity. By substituting the value of $\frac{1}{2AB}$ in miles, chains, &c., in the formula $DC = \frac{BD^2}{2AB}$, the rules for finding the corrections are derived.



Suppose the diameter of the earth to be 7958 miles, and that the distance $BC = 1$ mile, then the correction for curvature $DC = BC^2 \div 2AB = 1^2 \div 7958 = \frac{1}{7958}$ of a mile $= 7{\cdot}962$ inches = nearly 8 inches. If the distance $BE = 2$ miles, then the correction $EF = BE^2 \div 2AB = \frac{4}{7958}$ of a mile $= 31{\cdot}848$ inches.

The correction for curvature may be approximately found for any distance by the following practical rules :—

When the distance is in miles.

Rule I.—Take $\frac{2}{3}$ of the square of the distance in miles for the correction in feet.

When the distance is in chains.

Rule II.—Take $\frac{1}{8}$ of the square of the distance in chains, from which cut off two places for decimals, for the correction in inches.

When the distance is in yards.

Rule III.—Multiply the square of the distance in yards by ·00000257, for the correction in inches.

CORRECTION FOR REFRACTION.

(8.) The refraction varies according to the state of the atmosphere, but it may generally be taken at $\frac{1}{7}$ of the correction for curvature as an average; and since refraction makes objects appear higher than they really are, the correction for it must be deducted from that for curvature.

Examples.

1. Required the correction for curvature and refraction, when the distance of the object is $2\frac{1}{4}$ miles.

$$\text{Here } (2\tfrac{1}{4})^2 \times \frac{2}{3} = \left(\frac{9}{4}\right)^2 \times \frac{2}{3} = \frac{27}{8} = 3{\cdot}375 \text{ feet, cor. for curvature}$$
$$\tfrac{1}{7} \text{ of which} = \ {\cdot}482 \text{ —— cor. for refraction}$$
$$2{\cdot}893 \text{ —— cor. required.}$$

2. Required the correction for curvature and refraction, when the distance is 40 chains.

$$\text{Here } 40^2 \div 800 = 2{\cdot} \quad \text{inches, correction for curvature}$$
$$\tfrac{1}{7} \text{ of which} = \ {\cdot}285 \text{ —— cor. for refraction}$$
$$1{\cdot}715 \text{ —— cor. required.}$$

3. Required the correction, as in the last examples, when the distance of the object is 1000 yards.

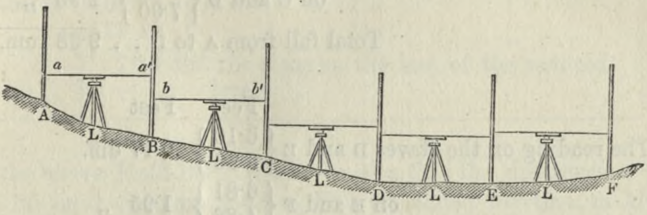
$$\text{Here } 1050^2 \times {\cdot}00000257 = 2{\cdot}57 \text{ inches, cor. for curvature}$$
$$\tfrac{1}{7} \text{ of which} = \ {\cdot}367 \text{ —— cor. for refraction}$$
$$2{\cdot}203 \text{ —— cor. required.}$$

NOTE.—These corrections, it will be seen, are very small at short distances, and are frequently obviated in practical levelling. See note to Art. 11.

THE PRACTICE OF LEVELLING.

(9.) *To find the differences of the levels of several points on the earth's surface, and to trace a sectional line of them.*

Let A, B, C, D, E, be the points on the earth's surface, a sectional line of which is required to be traced. Set up the levelling staves



perpendicularly at A and B; between which fix the level L on the ground; make the spirit-level horizontal, and turn the object-glass of the telescope to the back-staff A; then the staff A is cut at a by the horizontal line of sight aa' of the telescope, through which the observer can read off the height Aa. Now turn the object-glass of the telescope to the staff B, the horizontal line of sight aa' cutting it at a', and read off the height Ba': then the difference between the heights Ba', Aa, is the number of feet that B is lower than A, because the line aa' is horizontal. Now let the staff A be removed to C, the staff B remaining in the same position, excepting that its graduated side must now be turned towards C, and having removed the level L, place it between B and C, and the spirit-level being again made horizontal, the line of sight, in this case, being bb', read off the heights Bb, Cb' the difference of which is the number of feet that C is lower than B, as before.

In this manner, by alternately moving the levelling staves, placing the level between them, and reading off the heights below the line of sight or collimation, the operation is conducted to F.

Now, as there is a continued fall from A to D, the sum of the differences of the readings at B and A, at C and B, and at D and C, will give the actual fall *vertically* from A to D; in the same way the rise from D to F is estimated: if, therefore, the fall from A to D be greater than the rise from D to F, the difference of these two will be the *actual* fall from AF, or the distance in feet, estimated *vertically*, that the station A is higher than the station F.

EXAMPLE.

	Feet	Feet
Let the readings on the staves A and B be	$\left\{\begin{matrix}3\cdot50\\7\cdot57\end{matrix}\right\}$	4·07 diff.
on B and C	$\left\{\begin{matrix}5\cdot04\\7\cdot39\end{matrix}\right\}$	2·35 „
on C and D	$\left\{\begin{matrix}4\cdot04\\7\cdot00\end{matrix}\right\}$	2·96 „
Total fall from A to D . .		9·38 sum.

	Feet	Feet
The reading on the staves D and E	$\left\{\begin{matrix}6\cdot10\\5\cdot93\end{matrix}\right\}$	0·17 diff.
on E and F	$\left\{\begin{matrix}6\cdot81\\4\cdot86\end{matrix}\right\}$	1·95 „
Total rise from D to F . . .		2·12 sum.
Actual fall from A to F . . .		7·26 diff.

The difference of the sums of the back and front readings of the staves, or "back and fore sights," as they are termed, will also give the difference between A and F: thus—

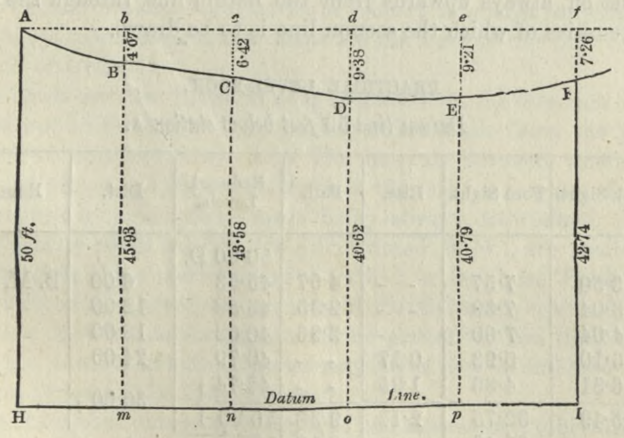
Back Sight	Fore Sight
3·50	7·57
5·04	7·39
4·04	7·00
6·10	7·93
6·81	7·86
25·49	32·75 sum of fore sights
	25·49 sum of back sights
	7·26 diff. the same as before.

But in order to draw the section to show the undulations of the ground between A and F, the *horizontal* distances of the several points B, C, D, E, and F, must be measured from the first station A; this is usually done with the chain during the operation of levelling. These distances with the back and fore sights may be arranged in a Field-Book of the following form, which, though not the practical form, may probably be better understood by the student.

FIELD-BOOK.

Back Sight.	Fore Sight.	Fall.	Rise.	Reduced Levels below Station A.	Distances.
					Chains.
3·50	7·57	4·07		4·07 at B	6·00
5·04	7·39	2·35		6·42 " C	12·00
4·04	7·00	2·96		9·38 " D	18·00
6·10	5·93		0·17	9·21 " E	24·00
6·81	4·86		1·95	7·26 " F	30·00
25·49	32·75				
	25·49				
	7·26 diff. the same as the last of the reduced levels				

In the above Field-Book it will be seen that the differences 4·07 and 2·35 in the column marked Fall are added together, making 6·42 for the fall at C, in the column of reduced levels: to this sum the next difference 2·96 is added for the fall at D. The other differences, in the column marked Rise, are successively subtracted from the last sum 9·38 for the falls at E and F: the latter of which, agreeing with the other difference, shows the accuracy of the castings. The last column shows the horizontal distances of the several points B, C, &c., from A.



In plotting from the above columns of reduced levels and distances, the former are always taken from a larger scale than the latter, otherwise the variations in the ground would, in a great many cases, be scarcely perceptible; the horizontal scale used in plotting the following section is 10 chains to an inch, and the vertical one for the reduced levels 25 feet to an inch, which latter scale has been thus chosen, that the section may be the same as that in the figure to Article 9, the vertical scale practically used being commonly 50 or 100 feet to an inch.

Having drawn the horizontal line A*f*, set off the distances A*b* = 6 chains, A*c* = 12 chains, &c., from the column of distances, on a scale of 10 chains to an inch; then set off the vertical lines *b*B, *c*C, &c., respectively, 4·07 feet, 6·42 feet, &c., from the column of reduced levels, on a scale of 25 feet to an inch, and through the points A, B, C, &c., draw the line ABCDEF, which is the section required.

THE DATUM LINE.

(10.) It is found inconvenient in practice to plot a section generally by the method already given, as the reduced levels, in extensive operations, would alternately rise and fall above and below A*f* (see last figure), and thus produce much confusion: therefore, a line HI, called a datum line, is assumed at 50, 100, or 200 feet, &c., below the first station A, so that it may be always *below* the sectional line ABCDEF. To or from this assumed distance of the datum line the rise or fall is respectively added or subtracted, and the next rise or fall added to or subtracted from the sum or difference, as in the following Level-Book; thus giving a series of vertical heights to be set off, always upwards from the datum line, through the upper extremities of which the section line is to be drawn.

PRACTICAL LEVEL-BOOK.

Datum line 50 *feet below station* A.

Back Sight.	Fore Sight.	Rise.	Fall.	Reduced Levels.	Dist.	Remarks.
				50·00 D.		
3·50	7·57	- -	4·07	45·93	6·00	B. M. at A.
5·04	7·39	- -	2·35	43·58	12·00	
4·04	7·00	- -	2·96	40·62	18·00	
6·10	5·93	0·17	- -	40·79	24·00	
6·81	4·86	1·95	- -	42·74	30·00	
25·49	32·75	2·12	9·38	50·00		
	25·49		2·12	7·26 diff. between last red. level and datum		
	7·26 differences = 7·26					

Having set off the distances on the datum line HI, draw the vertical lines HA, *m*B, *n*C, *o*D, *p*E, and IF, setting off their heights as in the column of *reduced levels*, and through the points A, B, &c., draw the required section ABCDEF.

(11.) It will be seen that the operation of levelling is extremely simple, as are also the castings of the Level-Book, and the plotting of the section. Great care is, however, necessary in reading off the staves, which ought, at the same time, to be held in a perfectly upright position ; *the castings in the Level-Book ought also to be carefully checked by taking the respective differences of the sums of the back and fore sights, and of the sums of the rises and falls, also of the datum number and the last reduced level ; which three differences must agree*, as shown in the preceding Level-Book, otherwise errors will be found in extensive operations to creep in imperceptibly.

NOTE.—Calculations to find the corrections for curvature and refraction are sometimes necessary in levelling, but as these would be extremely tedious, especially in extensive operations, they may be obviated by fixing the level always halfway between the staves, *as nearly as the eye can judge*, and thus the errors in both become equal and opposite, and consequently correct each other.

The general practice of levelling being now laid down in a brief manner, more details shall next be given.

TRIAL-LEVELS.

(12.) The general direction or route of a proposed line of railway between two places being determined upon, the line is marked out on such a map or plan of the district or country as is most convenient. (For this purpose the ordnance maps are the best, if the line be in England or Ireland.)

The levels are then taken, as near as possible to the direction thus marked out on the map ; if any deviations are made from the projected line, they are confined to the intervals between where it crosses turnpikes and other public roads, the relative heights of the points where it crosses these roads being always determined ; also marks, usually called *bench marks* (abbreviated B. M.), are made in or near them, these marks being described in the Level-Book for future reference. Cross-levels are also frequently taken, especially on the roads, to show the inclination of the ground to the right and left of the line, so that the engineer may know how to improve the line, and prepare it for further operations.

If the line, thus obtained, be thought not satisfactory, other directions, embracing either the whole, or only part of the distance between the two places, are tried in the same manner, and that direction which includes the fewest engineering difficulties is adopted.

CHECK-LEVELS AND SURVEY OF THE LINE.

(13.) Check-levels are sometimes taken to ascertain the accuracy of the trial-levels, if any doubt exist in this respect, and more especially if the line be a detached one; *i.e.*, not connected at its commencement or termination with existing lines, nor anywhere intersecting them. This, however, can rarely be the case in these times, when so many railways exist, the sections of which are published, and from and to which almost all new lines proceed. Hence the elevations of the commencement or termination of almost all new operations are so well known, not only with respect to these points, but also to other intermediate points of intersection with existing lines, as to obviate the use of check-levels, provided the trial-levels have been taken by a careful leveller.

(14.) The trial-levels being considered satisfactory, the survey of the surface is immediately commenced to the width of from 5 to 10 or 15 chains on each side of the projected line, accordingly as the property of individuals may appear to be affected, or the engineering difficulties or requirements of the line may demand. This survey must include every enclosure, building, road, &c., within the above-named limits; it is usually on a scale of from 5 to 10 chains to an inch. It is in every respect the same as in the preliminary survey, Section I., to which reference is given for the details of the work.

FINAL LEVELS.

(15.) The point of commencement of the proposed railway being determined upon by the previous operations, set one of the levelling staves on that point and call it the first back station, and select a point as near this station as possible, the height and position of which must be entered in the Level-Book as a mark of reference for future operations. This is called a *bench mark;* it is marked B. M. in the column of remarks in the Level-Book; it may be either between the first back station and the second, or between the second and third, accordingly as convenient objects for the purpose present themselves. The most proper places for bench marks are prominent stones on buildings, or other fixed stones, hinges of gates, milestones, notches near the roots of trees, &c., the positions of which must be described in the Level-Book. Fix the level firmly in the ground about half-way between the first and second stations, and make the axis of the spirit-level horizontal in all positions; read off the staff at the first back station; this reading is placed in the first column of the Level-Book, marked *Back Sights.* If neither the bench mark nor any other intermediate point requiring notice, occur between the two stations, turn the telescope to the second station and read off the height; this reading is placed in the third column of the Level-Book, marked *Fore Sights.* Remove the first staff to the bench mark, or intermediate station (if one occur), having at the time removed the instrument; read off the back sight and enter it in the Level-Book, as before; then read off the staff on the bench mark, and enter the reading in the column marked *Inter.;* lastly, read off the forestaff, as before: thus continuing and taking as many bench marks, and other intermediate sights, as may be thought necessary; the chaining of the line proceeding at the same time, the results of which are placed in the column of distances. It will readily be seen that four assistants will be required by the leveller, *i.e.*, two for the staves and two for the chaining. (For method of keeping Level-Book, see next page.)

From this Level-Book the method of proceeding, to any extent, may be sufficiently seen, with the help of the following remarks on the method of obtaining the reduced levels, at the same time.

NOTE.—That wheresoever the back or fore sight exceeds the usual height of the levelling-staff, that staff must be understood to have been placed on the top of a gate or rail-post, and its height afterwards measured, and added to the reading of the levelling-staff.

(16.) *Remarks on obtaining the reduced levels, &c., in the following Level-Book, &c.*

The method of casting the Level-Book referred to, is the same, with respect to the back and fore sights, as that given in Art. 10, page 353. These castings must be first done, and the intermediate ones, in the column marked Inter., afterwards.

The datum line is assumed 100 feet below the first station or commencement of the levels: this number is placed first in the column of reduced levels. The back sight is 2·20 feet, and its corresponding fore sight 13·40, which shows that the ground falls; the difference of these 11·20, is put in the column marked Fall, which, being subtracted from the datum 100 feet leaves *88·80, which is put under the datum, in the column marked Reduced Levels, being marked with an asterisk or cross, and shows the height of the forestaff above the datum line. The same remarks will apply to the back and fore sights 1·80 and 10·10, and to 7·30 and 1·80, observing that in the latter case the ground rises, and the difference must be put in the column marked Rise, and added to the next preceding reduced level that is marked thus *.

LEVEL-BOOK.

The projected railway from the Grand Junction Railway to Abbergelly. (See Section, Plate XIII.)

Back Sights.	Inter.	Fore Sights.	Rise.	Fall.	Reduced Levels.	Distances in Chains.	Remarks.
			Datum	...	Ft. 100.00	–	Below level of rails of the Grand Junction Railway, at its junction therewith.
On Rail 2.20 }		13.40		11.20	*88.80	0.45	On side of railway embankment.
1.80	10.16			8.36	80.44	0.80	B. M. on stone opposite Sheaf Inn, at foot of railway embankment.
		10.10		8.30	*80.50	3.00	
7.30	7.70			0.40	80.10	5.00	Edge of drain (3 ft. deep).
	4.80		2.50		83.00	7.00	
	4.60		2.70		83.20	10.00	Occupation road to Elm farm and Wingfield cottages.
		1.80	5.50		*86.00	11.10	At foot of perpendicular bank 12.00 2 ft. high.
12.00	10.00		2.00		88.00	11.13	At top of ditto.
	5.00		7.00		93.00	15.00	
		0.00	12.00		*98.00	17.00	
14.00	12.20		1.80		99.80	20.00	Centre of open occupation road to Hill farm.
		2.00	12.00		*110.00	23.86	On Bank top.
7.86	3.80		4.06		114.06	24.20	B. M. on flat square stone at corner of Hill farm barn, 1 ch. to left.
	2.20		5.66		115.66		
		0.00	7.86		*117.86	26.50	
7.20	5.00		2.20		120.06	28.00	
		2.00	5.20		*123.06 } 100.00 }	32.00	Footpath.
52.36 29.30 23.06		29.30			23.06		

The difference between the first back sight 1·80, and the first intermediate one 10·16, is 8·36, which shows that the ground falls; it is therefore put in the column of falls, under 11·20, and subtracted from the next preceding number in the column of reduced levels, marked thus*, and the remainder 80·44 is then put in that column. In like

LEVEL-BOOK—*continued.*

Back Sights.	Inter.	Fore Sights.	Rise.	Fall.	Reduced Levels.	Distances in Chains.	Remarks.
					Ft. *123.06		
9.83		8.89	0.94		*124.00	32.60	
13.50	8.50		5.00		129.00	33.00	In plantation.
		1.40	12.10		*136.10	35.00	Ditto.
10.60	4.70		5.90		142.00	37.00	Ditto.
	1.70		8.90		145.00	39.00	15 links over plantation hedge, on edge of bank of road to Westwick.
	6.70		3.90		140.00	39.40	Centre of road to Westwick.
		2.60	8.00		*144.10	40.00	On further bank of ditto. B. M. on square stone in line.
1.05	9.15			8.10	136.00	43.00	
		15.65		14.60	*129.50	45.20	
1.00	6.00			5.00	124.50	46.80	Crosses footpath to Westwick.
		13.50		12.50	*117.00	47.50	Edge of steep brow.
0.40	3.90			3.50	113.50	48.00	
		17.40		17.00	*100.00	49.00	
1.60	11.60			10.00	90.00	50.20	Bottom of brow.
		14.50		12.90	*87.10	51.15	Edge of brook.
5.00	14.00			9.00	78.10	51.65	Centre of do. (water 2 ft. deep).
		0.00	5.00		*92.10	52.00	Further bank of brook.
12.00		0.10	11.90		*104.00	53.50	
14.00		2.50	11.50		*115.50	55.45	
15.00	8.50		6.50		122.00	56.10	
		0.00	15.00		*130.50	56.95	
10.00		1.70	8.30		*138.80	58.75	Footpath to farm-house.
15.00	5.80		9.20		148.00	59.80	
		0.40	14.60		*153.40	60.65	
14.20		0.60	13.60		*167.00	61.60	
12.00	8.00		4.00		171.00	62.25	Centre of public road to Winston, road 50 links wide.
		1.00	11.00		*178.00	63.40	
135.18		80.24			123.06		
80.24					54.94		
54.94							

manner all intermediate differences are added to, or subtracted from, the next preceding reduced level number marked thus *, accordingly as they stand in the column marked Rise or Fall.

The proof of the accuracy of the work is by taking the differences of the sums of the back and fore sights, and of the first and last reduced levels, which differences must be equal; thus, in the latter portion of the Level-Book, the sums of the back and fore sights are 135·18 and 80·24, the difference of which is 54·94; and the first and last reduced levels are 123·06 and 178·00, the difference of which is also 54·94, which shows the work to be right.

(17.) As a means of checking the accuracy of trial-levels it has sometimes been recommended to employ two levellers, with separate instruments placed at a short distance from one another, to read from the same staves at the same time, and frequently to compare their reduced levels, correcting the errors immediately, if any occur; this would completely obviate the numerous mistakes that arise, in extensive operations, through wrong readings, wrong entries in the Level-Book, &c.; since the method of proving the Level-Book by differences only proves that the reduced levels are correct with respect to the back and fore sights entered in the book, and not the accuracy of the actual levels.

PLOTTING THE MAIN SECTION.

(18.) First draw the datum line, as in Plate XIII.; and from a scale of 5 chains to an inch lay off the $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1 mile distances, repeating them after every successive mile, and marking the miles 1, 2, 3, &c. Now lay off on the datum line, from the column of distances in the Level-Book, the several distances up to the first mile; do the same between the second and third line, between the third and fourth, &c., the distances always beginning afresh from the end of every mile in the Level-Book. Draw lines perpendicular to the datum line at the end of every mile, and parallel to these perpendiculars draw lines in pencil of the height given in the column of reduced levels, and at their corresponding distances, from a scale of 50 feet to an inch; and through the upper extremities of these lines draw the section, carefully noting the roads, paths, rivers, brooks, woods, &c., as given in the column of remarks in the Level-Books, which, it will be seen, are copied in the plate.

CROSS LEVELS.

(19.) Cross levels are taken principally on roads crossed by the line of railway, to show the position of the surface of the ground, partly with a view, if possible, of improving the main line, and partly to show the nature of the approaches of the cross roads to the bridges, where required, and the quantity in respect to depth and length of cutting, or of height and length of embankment required where the main line is crossed either over or under by bridges, or on the line of the rails.

The heights in the cross-section are usually taken at every chain's length, to the distance of 10 chains on each side of the line, as will be seen in the following notes; remembering always to take the levels *from the left to the right* of the onward direction of the main line, otherwise serious errors might arise by plotting the section in the wrong direction.

The cross-section is usually plotted on the same scale as the main section; some engineers adopt a larger one. The student can have no difficulty in plotting from the following notes. (See Plate XIII., Cross-Section No. 3.)

Cross levels No. 3 on road from Winston to Mold.

Back Sights	Inter.	Fore Sights	Distance.	Remarks.
1·13			0·00	
	1·86		1·00	
	2·89		2·00	
	6·70		4·00	
	8·83		5·00	
1·02		10·64	6·00	
	7·70		8·00	
2·83		10·76	9·00	
	6·60		10·00	
	7·31		10·31	Line crosses. Reduced level 104 feet
0·61		9·55	11·00	
0·48		8·32	13·00	
0·65		8·40	15·00	
1·63			17·00	
	6·70		19·00	
		8·06	20·00	

(20.) THE APPLICATION OF THE CORRECTION FOR CURVATURE AND REFRACTION IN LEVELLING.

In the preceding examples on levelling, the application of the correction for curvature and refraction has been avoided, by assuming that the levelling instrument was always placed in observing at or near the middle point between the staves; thus making the errors arising from these causes correct each other. (See note to Art. 11.) But in a case where a very long sight is taken in one direction, as, for instance, the fore sight, the back sight at the same time being a very short one, these corrections must be strictly attended to, as in the following

Example.—In taking levels for a projected railway, a swamp of 148 chains in width occurred in the line; the level was placed at the edge of the swamp, the back-staff was placed one chain behind the instrument, on the level of the swamp, the reading thereon being 5·56 feet; the level being reversed, was found to cut a notch in a post, standing nearly in the line, on the opposite edge of the swamp, and on going round the swamp to the post, and measuring the height of the notch thereon, it was found to be 7·36 feet above the level of the swamp; required the correction for curvature and refraction.

By Art. 8, Rule II. $(148)^2 \div 800 = 27 \cdot 38$ inches = cor. for cur.
deduct $\frac{1}{7} = 3 \cdot 91$ —— for ref.

Correction for curvature and ref. $= 23 \cdot 47 = 1 \cdot 95$ feet.

Therefore $7 \cdot 36 - 1 \cdot 95 = 5 \cdot 41$ feet, is the corrected fore sight; the back sight, being at so short a distance, needs no correction: hence the difference of the levels of the edges of the swamp is $5 \cdot 56 - 5 \cdot 41 = 0 \cdot 15$ feet.

NOTE.—From the above example it will be seen that had the correction not been made, the error in the levels would have been nearly 2 feet.

(21.) When several assistants are employed on the same section, a uniform system should be strictly adopted, and the superintendent of the work should fix upon the bench marks, occasionally checking the work of the others. When a section has been thus taken in several parts at the same time, after putting the parts carefully together, a common datum line must be assumed for them all, and the result checked by adding or subtracting the differences of the reduced levels at the points of junction, accordingly as they rise or fall.

Intermediate sights, in ordinary levelling, need only be taken to tenths of feet, as it would be a waste of time to attempt greater accuracy, excepting where the levels are taken to bench marks, in which the same accuracy should be observed as in taking back and fore sights. The height of the optical axis of the level, when on the line, or the level of the line of section, may always be put down as an intermediate sight, if required. It will here be proper to observe that it is not necessary that the levelling instrument should be placed directly on the sectional line, while making observations therewith, but in any convenient position either to the right or left of it.

In taking levels through towns, the operation frequently proceeds in a zigzag direction, such as the streets may present, the length of the required sectional line being determined from a map of the town, and the several heights of the points in the section obtained from the reduced levels, corresponding to the points in the streets where the sectional line crosses them. In a similar manner the elevation of the extreme points of the estates of proprietors and occupiers, who are hostile to engineering operations, are obtained by going round without the bounds of their premises, or by the nearest roads between the extreme points, the profile of the intermediate space being assumed.

NOTE.—All the methods of levelling, and of laying down sections therefrom, given in the preceding articles of this section, will apply to canals, roads, sewers, drains, &c., with the same facility as if they had been written for these purposes.

LEVELLING BY THE THEODOLITE.

(22.) When the levels are required to be taken over very high and rapidly rising summits, on the acclivities and declivities of which it is found very difficult to fix the levelling instrument, or over steep and almost perpendicular cliffs, where it cannot be fixed at all, the operation would be best performed by the theodolite, which must be set perfectly level, both with respect to the spirit-levels on their vernier plate, and that which is attached to the telescope; and the angles of elevation and depression of the required points, both before and behind the instrument, respectively, taken by means of the vertical arc; the distances on the slope of the observed points from the instrument being, at the same time, measured as correctly as circumstances may permit, from which *data* the perpendicular elevation and depression of the points, as well as their horizontal distances from the instrument, may be found by the rules of right-angled plane trigonometry. These operations may be repeated for any number of stations, recollecting to make the necessary corrections for curvature and refraction, and to take into account the heights of the instrument and of the objects placed in the observed points. Operations of this kind should be performed when the atmosphere is settled, otherwise the refraction will be found so extremely variable and deceptive as to produce considerable error.

LEVELLING BY THE ANEROID BAROMETER.

It is often very useful, in going over a line of country, to be able to form a general approximate idea of the levels or altitudes of the ground, without going to the trouble of levelling it in the usual way. For this purpose the aneroid barometer (which is now made small enough for the pocket) is admirably adapted, as it will indicate differences of altitude to a few feet with tolerable precision. Full directions for the use of the instrument are usually sold with it.

It must, however, be borne in mind that the pressure of the atmosphere, even in the same place, is constantly varying, and that, therefore, unless this source of error is provided against, the indications of the barometer may be exceedingly delusive. If accurate indications are desired, a duplicate instrument, remaining stationary in one place, should be constantly watched and registered during the survey, and its indications carefully compared with that carried about over the ground.

SECTION II.

PARLIAMENTARY PLAN AND SECTION, ETC.

(23.) When application is made to Parliament for authority to make a railway, it is necessary to prepare a survey of the intended line, showing both its horizontal and vertical position; the documents exhibiting these features being called the "Parliamentary Plan and Section." Copies of these and some other documents have to be *deposited,* in due form, at certain places, in compliance with certain rules established by Parliament for the purpose.

These rules are called the "Standing Orders of the Houses of Lords and Commons on Private Bills." They contain many directions not only for the preparation of the plans and sections, but also for the laying out of the lines, all which must be carefully adhered to by the engineer and surveyor, or he will risk the rejection of his Bill. The standing orders are altered and amended from time to time, and it is therefore highly necessary that any engineer who contemplates going to Parliament, should provide himself with a copy of the latest edition, which is sold by Parliamentary stationers at a moderate price.

According to the standing orders of 1863, the *Plan* must be drawn to a scale of not less than *four inches to a mile,* and where any buildings are included within the limits of deviation, enlarged plans must be given to a scale not less than $\frac{1}{4}$ of an inch to 100 feet.

The *Section* must be drawn to the same *horizontal* scale as the plan, and to a *vertical* scale of not less than 1 inch to every 100 feet. The line of the railway shown is to correspond with the upper surface of the rails.

A model plan and section usually accompany the published book of standing orders.

Method of preparing a plan and section of a railway, as required by the standing orders of the House of Commons, preparatory to obtaining an Act of Parliament for its construction.

Plate XIII. is a portion of a plan and section, with cross-sections &c., of a railway prepared for the above-named purpose. The plan or map of a portion of the country through which it passes occupies the lower portion of the plate, the proposed centre line of the railway being marked thereon by a strong black line, and by dots where it passes through a tunnel. The fields, &c., in the parish where the railway commences are numbered consecutively to the boundary of the next parish, where the numbers commence afresh, and so on through the successive parishes or townships; the numbers referring to corresponding numbers in a book of reference, in which are descriptions of the several properties, with the names of owners and occupiers. On each side of the centre line, and parallel to it, at the distance of 100 yards, or 454 links from it, are dotted lines called the *limits of deviation:* within the space included by these dotted lines, the engineer, on being empowered to construct the railway, is allowed to deviate from the line, as projected on the map, should he think it advisable for the sake of improving the line, or of avoiding expensive severance, &c.

The main section, with its accompanying cross-sections, occupies the upper portions of the plate (these sections are the same as those referred to in some of the preceding articles), to prepare which, for Parliamentary as well as practical purposes, it will be first necessary to explain the method of laying out gradients.

THE METHOD OF LAYING OUT GRADIENTS.

(24.) The gradient of any portion of a railway means the inclination of the surface of the rails with respect to a horizontal line. A level line of railway would, doubtless, be preferable to any other; but the unevenness of the earth's surface puts this out of the question in by far the greatest number of railways. It is well to make the gradients as flat as possible, consistent with economy in construction of the works; they were formerly limited to about 1 in 264, but owing to the improvements in the tractive power, they now frequently reach 1 in 100, beyond which they should not go except in very difficult cases. Parliamentary committees exercise a rigid investigation into the necessity of steep gradients in lines brought before them.

The usual practical method of laying out railway gradients is by applying one end of an extended silken thread to the commencement of the railway on the section, the other end being so applied that the extended thread may cut the curved boundary of the section or profile of the earth's surface, so as to leave an equal portion of space both above and below the thread, as nearly as can be judged by the eye, in order that the cuttings from the spaces or parts of the section above the thread, may produce materials sufficient to fill up the spaces or parts below the thread, or form the embankments of the railway. The position of the thread being thought satisfactory for the purposes required, its extremities are marked, and a line ruled in the place occupied by it for the first gradient. The second, third, &c., gradients are laid out in a similar manner to the end of the section. See Art. 26 and the two following notes.

NOTE 1.—The excavations and embankments of a railway are made about 2 feet lower than the level of the rails, thus giving a line parallel thereto, called the *balance or formation line;* the 2 feet filled up with gravel, to form the road and the beds for the sleepers of the rails.

2. If the position of the first gradient, though favourable in itself, cause the following gradient, or gradients, to be less favourable, with respect to the quantity of cuttings and embankments, it is advisable to alter the position of the first gradient to one less favourable, provided that the compound results of cuttings and embankments on the several successive gradients, as now altered, is more favourable than in the preceding case. In this manner, it is requisite to change the positions of the several gradients repeatedly till the *minimum*, or least possible quantity of cuttings and embankments, shall be required in the construction of the railway, keeping in view the required limit in the ascent and descent of the several gradients; the difficulty of making the excavations, throughout the whole length of the line, being supposed, at the same time, to be nearly equal. But where the geological character of the country through which the railway passes, differs considerably, presenting for excavation strata varying throughout the length of the line from loose sand to hard rock, and *vice versâ*, the facility or difficulty of making the excavations must be carefully considered in laying out the gradients; larger excavations being advisable where they can be easily made—and smaller where with difficulty. *The least possible expense will be incurred in the construction of a railway by taking into account all these circumstances.*

(25.) If anywhere in the section the excavations reach 60 feet in depth, and afterwards increase rapidly in depth, it is a more economical method of proceeding to make a subterraneous passage, called a *tunnel*, through the deep part, than to cut the whole open to the surface of the ground, which in many cases would be next to impossible. Tunnels are cut to the width and depth of 30 feet, for railways on the narrow gauge; if on the broad gauge, to the width and depth of 36 and 32 feet respectively, the width and depth being less in both cases, if the material to be cut be hard rock. The diminished quantity of cuttings, where tunnels occur, must be taken into account in laying out the gradients.

(26.) *To determine the rate of inclination of a gradient.*

AB, in Plate XIII., is the first gradient on the railway section, the cuttings or excavations above it being considered to be equal to the requirements of the embankments below it: at its commencement A, its height AO is 100 feet; and at its termination B, its height BC is 130 feet above the datum line; thus giving a rise of 130 – 100

= 30 feet; the horizontal length OC of the gradient is 65·60 chains = 4329·6 feet. There is, therefore, a vertical rise of 30 feet in a horizontal distance of 4329·6 : hence 30 : 4329·6 : : 1 : 144·32; or, in round numbers, a rise of 1 in 144, which is called the inclination of the gradient AB, and is thus noted on the section, INCLINATION 1 IN 144.

The rule for finding the rate of inclination of a gradient may be thus briefly enunciated:—

Multiply the horizontal length of the gradient in chains by 66, and divide the product by the difference of the heights of the gradient at its extremities, above the datum line, and the quotient is the horizontal distance to a rise of 1 foot, or the rate of inclination required.

(27.) The excavations at B being about 60 feet deep, and continuing to increase rapidly, a tunnel BD, to the length of 462 yards, is introduced on the next following gradient; the tunnel having passed beyond the summit, till its depth at D again becomes 60 feet; its height, as shown on the section, is 21 feet, being the height to which the cuttings are reduced by the ballasting below and the arch above. The gradient, of which the tunnel forms a part, is assumed to extend beyond the limits of the plate; its position, therefore, is not determined by the excavations and embankments shown thereon, but in conjunction with those beyond its limits.

(28.) *Method of determining the heights of the several roads passed over or under by the railway, and whether they should be* raised or lowered *to give sufficient height for viaducts or bridges;* or to be raised or lowered *to be passed on the level of the railway.*

When a road is passed over by the railway, the usual height allowed from the surface of the road to that of the rails is 18 or 20 feet; and when the road is passed under by the railway, the height allowed is 18 or 19 feet.

The heights or depths of the roads, rivers, &c., above or below the rails, or gradients, is sometimes found by measuring them carefully by the vertical scale; but they may be found more accurately by the following method:—

The distance of the middle of the first road from the commencement of the section is 10 chains, its height above the datum lines is 83·20 feet, the horizontal length OC of the first gradient is 65·60 chains, and the gradient rises 130 − 100 = 30 feet. There is, therefore, given the length and rise of the gradient to find its rise at any other given point; which may be done by similar triangles; thus—

As hor. length of gradient 65·60 ch. : 30 ft. : : 10 ch. the dist. of road : 4·57 ft., the rise of the gradient at *a* ; this added to 100 ft., the height of the gradient at A, gives 104·57 ft., the height of the point *a* above the datum line ; but the height of the road above the same line is 83·20 feet, therefore $104{\cdot}57 - 83{\cdot}20 = 21{\cdot}37$ ft. $= 21$ ft. 4 in. is the height of the rails at *a* above the road ; which, being above 20 ft., shows that the level of the road may remain unaltered, the height of the bridge required for the railway being 18 ft. ; thus leaving 3 ft. 4 in. for the thickness of the arch and ballasting, and its span being taken 24 ft., as being sufficient to allow carriages to pass one another on the road below the line.

The distance of the occupation road per Level-Book, is 24·06 ch., and the reduced level is 110 ft., therefore 65·60 ch. : 30 ft. : : 24·06 ch. : 10·91 ft., hence

$100 + 10{\cdot}91 = 110{\cdot}91$ ft. height of gradient at road
110·00 ft. do. of road

0·91 ft. 11 inches, the height the road must be raised to be passed over on level of rails. In raising or lowering roads for this purpose, a rise or fall of 1 foot in 20 is required.

In the same manner, the road from Westbrook to Hurst is found to be 22 feet above the gradient ; the hedge of the brook that forms the boundary of the parishes of Westbrook and Winston, 36 ft. 5 in. below the gradient ; and the road next following, 40 ft. 5 in. above it.

SECTION III.

RAILWAY CURVES.

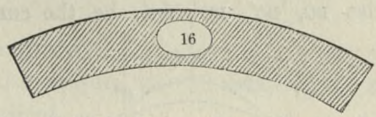
(1.) *On the use of curves in railways in general.*

The use of curves in railways is absolutely necessary on account of the natural unevenness of the earth's surface, it being desirable to attain the nearest possible practical level by avoiding hills, crags, mountains, &c., and by winding round them by means of curves. Curves are also equally necesssary in avoiding other natural and artificial obstructions, in many cases not materially affecting the level of the line, as lakes, swamps, the windings of sea-coasts, and of rivers ; also cities, towns, villages, parks, pleasure-grounds, &c. In this manner a great saving is effected in the expense of extensive excavations, embankments, tunnels, viaducts, &c., as well as the expense of the severance of valuable property, which would otherwise be required. Besides, it is frequently desirable to make a winding railway, in order that it may embrace in its route some important city, town, harbour, &c., or make a junction with another railway. Straight lines in railways are, however, much to be preferred to curves, and are therefore adopted as far as possible; they are first set out as bases for further operations; and every curve is subordinate to two straight portions of the line, which are tangents to it at its commencement and termination. The curves adopted in practice are always arcs of circles. Sometimes two, three, or more consecutive circular arcs, having a common tangent at their point or points of junction, are joined together, as in the case of the compound curve; and sometimes two circular arcs are connected, having their convexities turned different ways, with a common tangent at their point of junction, as in the case of the serpentine, or S curve.

NOTE.—Other curves beside the circular arc might sometimes with advantage be adopted as the curves of railways; but their construction is attended with difficulty.

(2.) *On mechanical railway curves, or curve-rulers.*

Mechanical railway curves, sometimes called curve-rulers, are a series of segments of circles made of hard wood, as box or mahogany, or strong pasteboard, with their radii in inches marked on them. These curves



commonly begin with a radius of $2\frac{1}{2}$ inches, and terminate with one of 160 inches, or upwards; the smallest radii usually increased by $\frac{1}{2}$ inch up to 10 inches; then by inches up to 20 inches; afterwards by 2 inches up to 80, including all numbers ending in 5; lastly, by 5 inches, till near the end of the series, when the radii increase by 10 inches.

The annexed figure represents the railway curve-ruler of 16 inches radius. If the scale of plan, to which this curve-ruler is applied be 5 chains to an inch, it will represent a curve or circular arc of $16 \times 5 = 80$ chains $= 1$ mile radius. If the scale of the plan be 12 chains to an inch, it will represent an arc of $16 \times 12 = 192$ chains $= 2$ miles and $3\frac{1}{5}$ furlongs radius; and similarly for other scales.

(3.) *The limit of the radii of railway curves.*

As a general rule the curves of a railway should be kept of as large a radius as possible consistently with the economical laying-out of the line. One mile radius or upwards is quite unobjectionable; but half a mile, or even less, will be sanctioned by Parliament in difficult cases, particularly near large stations, where the speed is usually slackened. In all sharp curves the outer rail requires to be raised above the level of the inner one, to counteract the effect of the centrifugal force. (See Formulæ in Section VII.)

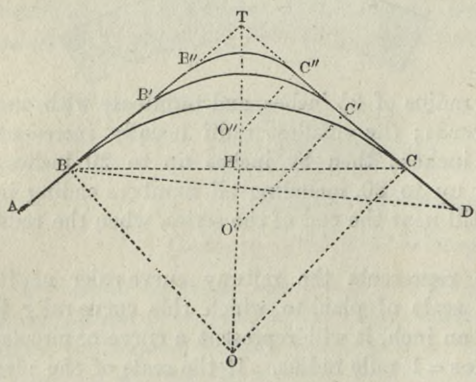
PROBLEM I.

CASE I.

To determine the radius of a railway curve mechanically, that is, by the curve-rulers; the positions of the straight or tangential portions of the railway being given.

This problem admits of an indefinite number of solutions, but the curve adopted is generally that which falls on ground presenting the fewest natural or artificial obstructions, provided its radius be not too great. (See Art. 3.)

Let AB, CD be two straight portions of a railway, the positions of which are given by an accurate plan; and which, when prolonged, meet at T. Apply several of the series of curve-rulers (see Art. 2) to touch the line AB, without cutting it, at the points B, B′, B″, &c., and also to touch CD, in like manner, at the points C, C′, C″, &c.; and let BC, BC′, BC″, &c., be the curves thus obtained. Then



whichsoever of these curves may appear, on a careful consideration, to be least obstructed by hills, lakes, rivers, buildings, &c., must be adopted as part of the line of railway, provided its radius be equal to, or exceed, the required limit. The radius is determined by multiplying the number on the curve-ruler by the number of chains per inch on the scale of the plan.

Example. If the curve BC be drawn by the curve-ruler, numbered 22, and be adopted as part of the line, and the scale of the plan be 5 chains to an inch, required the radius of the curve.

$22 \times 5 = 110$ chains = 1 mile 3 fur., the radius required.

NOTE.—By this method the radii of railway curves are commonly found in practice; but since a circular arc of large radius apparently coincides with its tangent for a considerable distance, it is difficult, in this manner, to determine the exact starting-points of the curve; besides, if the plan be not strictly accurate, as is too often the case, the positions of the tangents cannot be said to be given, and therefore this method should only be employed in roughly determining the radius and starting-points of the curve, so that it may fall, in its progress, on the most favourable ground.

CASE II.

To determine the radius of a railway curve geometrically, the starting-point B *being given.*

Prolong AB, DC till they meet at T; bisect the angle ATD by the line TO; and at B draw BO perpendicular to AT, meeting TO in O. Then O is the centre, and BO the required radius of the curve BC.

Or,

Make TC = TB; and draw the perpendiculars BO, CO, meeting in O. Then BO, or CO, is the radius of the curve.

In the same manner, the other radii C′O′, C″O″, &c., of the curves B′C′, B″C″, &c., may be found.

CASE III.

To find the radius of the curve by taking dimensions on the ground, the map being known to be inaccurate, and the starting-point B *being given.*

Range the tangents AB, DC till they meet at T; measure BT; then measure on TD till TC = BT; also measure BC. Then $BO = \frac{1}{2} BC . BT \div \sqrt{BT^2 - \frac{1}{4} BC^2}$* = radius required.

Ex. Let BT = 82·50 chains, and BC = 132·00 chains. Then BO = $\frac{1}{2}(132 \times 82{\cdot}5) \div \sqrt{(82{\cdot}5)^2 - \frac{1}{4}(132)^2} = 5445 \div 49{\cdot}5 = 110$ chains, the radius required.

Or by Trigonometry.

Range the tangents AB, DC till they meet at T, as before, then measure BT, and take the angle BTC, $\frac{1}{2}$ of which is the ∠BTO; then in the triangle BTO, right-angled at B, are given BT and the ∠BTO to find BO; whence by trigonometry,

cot. ∠BTO : rad. : : TC : BO, the radius required.

Ex. Let BT = 82·50 chains, and the angle BTC = 106° 16′.

Then cot. $\frac{1}{2}$ ∠BTC = cot. ∠BTO, 53° 8′ . .	9·87501
: rad. or sin. 90°	10·00000
: : BT = 82·50 chains, ...	1·91645
: BO = 110 chains, the required radius . .	2·04144

* *Demonstration.*—In the Δ BTH, $TH = \sqrt{BT^2 - BH^2} = \sqrt{BT^2 - \frac{1}{4} BC^2}$, and by the similar Δs BHT, BHO, $\overline{BT^2 - \frac{1}{4} BC^2}$ (=TH) : BT : : $\frac{1}{2}$ BC (=BH) : BO = $\frac{1}{2}$ BC : BT ÷ $\sqrt{BT - \frac{1}{4} BC^2}$. *Q. E. D.*

Case IV.

To find the radius of the curve, when the plan is known to be inaccurate, and the tangents AB, DC *cannot be prolonged to meet at* T *on account of obstructions, the starting-point* B *of the curve being given.*

From B measure a line BD, on the most convenient ground, to meet CD at D, taking the angles TBD, TDB; the sum of which taken from 180° gives the $\angle$ BTD, whence by trigonometry,

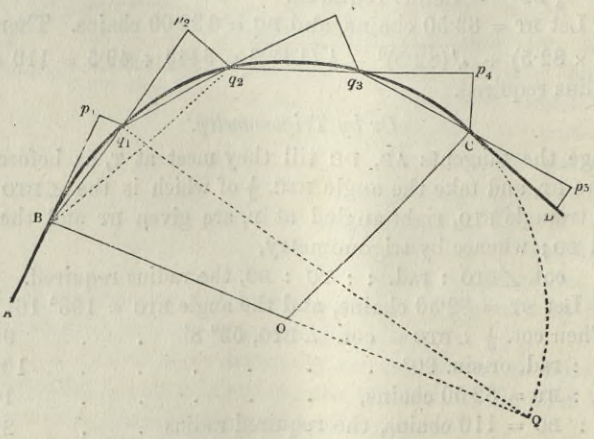
$$\sin. \angle \text{BTD} : \text{BD} :: \sin. \angle \text{TDB} : \text{BT}.$$

Now in the right-angled triangle BTO are given BT and the angle BTO $= \frac{1}{2} \angle$ BTD, to find the radius BO, which may be done as in Case III.

PROBLEM II.

To lay out on the ground a railway curve by the common method.

Let AB, CD be two straight or tangential portions of a railway, the points BC being required to be joined by a circular curve BC, to which AB, CD shall be tangents at the points B and C; and let BO be the radius of the curve, which is supposed to be determined by one or



other of the cases in Problem I. accordingly as the map is found to be accurate or inaccurate. Put the radius BO $= r$; measure on the tangent AB prolonged the distance Bp_1 = 1 chain, as is usual in practice; set off $p_1q_1 = \frac{1}{2r}$* at right angles to Bp_1; then q_1 is the first point in the curve. Through Bq_1 measure the right line Bq_1p_2 = twice Bp_1 = 2 chains, and set off $p_2q_2 = \frac{1}{r}$ = twice p_1q_1 at right angles to Bq_1p_2; then q_2 is the second point in the curve, repeating the last operation till the curve reach the point C. Lastly, q_3Cp_5 being measured = 2 chains the last offset p_5D will be found = the first offset p_1q_1, or half the preceding one p_4C, if the work be right. See the following notes and example.

NOTE 1. Only a small part of the operation for a curve of great length is shown in the figure; but as the whole of the work, except the first and last offsets, is alike, to show more would be unnecessary.

2. The values of $\frac{1}{2r}$ for all radii, from 15 chains to 540, are given in Table No. 4, at the end of the work.

EX. Let $BO = r = 80$ chains = 1 mile; then $\frac{1}{2r} = \frac{1}{160}$ of a chain, which being multiplied by 792, the number of inches in 1 chain, gives $\frac{1 \times 792}{160} = 4{\cdot}95$ inches, the first offset p_1q_1; whence $4{\cdot}95 \times 2 = 9{\cdot}9$ inches, the second offset p_2q_2.

Or by Table No. 4, opposite 80, in the column marked Radius, &c., stands 4·95 inches, in the column marked Offsets, &c., which is the value of p_1q_1, as above.

3. By this method the greater part of British, as well as foreign, railway curves have been laid out, having been invented by Mr. T. Baker, C.E., at a very early period, when the Stockton and Darlington Railway was laid out. It was eagerly adopted by railway surveyors, as it involves very little calculation, and does not require the use of an angular instrument. It is, however, defective in practice, on account of its requiring the coupling together of so very many short lines, as small errors will unavoidably creep in and multiply (and more especially so if the ground be rough), so that the curve has frequently to be retraced several times

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* *Demonstration.*—Complete the semicircle BCQ, prolong BO to Q, and join q_1Q, Bq_2. Then because Bp_1, Bp_2, Bq_2, are always, in practice, so very small, compared with BQ that they nearly coincide with the curve, and consequently Bp_1 is approximately $= Bq_1 = q_1p_2 = q_1q_2 = \frac{1}{2}Bq_2$; ∴ by the nature of the circle the Δs Bq_1Q, Bp_1q_1, Bp_2q_2, are similar, and

$$BQ : Bq_1 :: Bq_1 : p_1q_1 = \frac{Bq_1^2}{BQ} = \frac{Bp_1^2}{BQ} = \frac{\delta^2}{2r}.$$

$$\text{also} \quad : \quad :: Bq_2 : p_2q_2 = Bq_1 \times Bq_2 = \frac{2Bp_1^2}{BQ} = \frac{\delta^2}{r}.$$

$BO = \frac{1}{2}BQ$ being put $= r$, and $Bp_1 = Bq_1 = \&c. = \delta$; and when $\delta = 1$ chain, as in practice, $p_1q_1 = \frac{1}{2r}$, and $p_2q_2 = p_3q_3 = \&c. = \frac{1}{r}$. *Q. E. D.*

before it can be got right, especially if it be a long one. *This defect in the above method induced the author to prepare three other and more accurate methods, which are given in the following Problems.*

4. When the curve shall have been laid out with sufficient accuracy, the rods or quills that mark the ends of the offsets, q_1q_2, &c., must be taken out and their places supplied by strong wooden stumps, about 16 or 18 inches in length, and $1\frac{1}{2}$ inch square, each end of the curve being marked with three stumps, or one large one with a cross or some other conspicuous mark on it. The straight portions of the line must also be marked with stumps at the end of every chain.

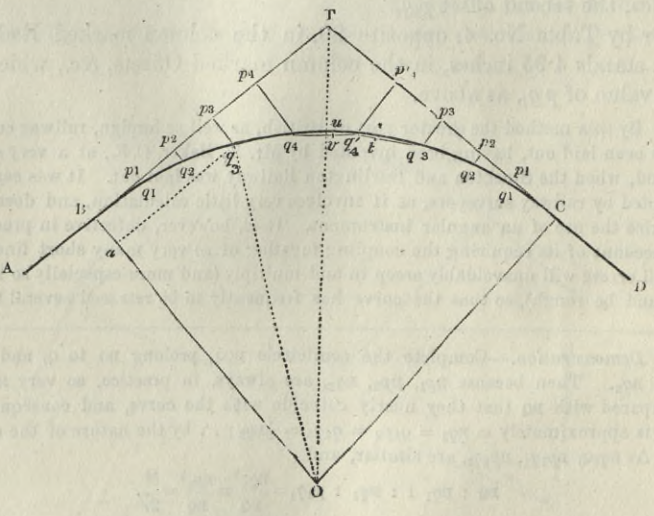
PROBLEM III.

To lay out a railway curve by ordinates or offsets from its tangents, no material obstructions being supposed to exist on the convex side of the curve, to prevent the use of the chain.

CASE I.

When the length of the curve is less than $\frac{1}{4}$ of its radius.

Let BC be the curve, AB, CD, tangents thereto, which are ranged till they meet at D, and BO the radius of the curve, which must be correctly determined by one or other of the cases in Prob. I.



according to circumstances. Measure on BT the usual distance Bp_1 = 1 chain, and lay off the offset $p_1q_1 = \frac{1}{2r}$ (r being the radius of the curve as before), then measure p_1p_2 = 1 chain, and lay off

$p_2q_2 = \frac{2^2}{2r}$. The successive offsets at the end of every chain being $\frac{1}{2r}, \frac{2^2}{2r}, \frac{3^2}{2r}, \frac{4^2}{2r}$, &c.,* or, what amounts to the same thing, the 2d, 3d, 4th, &c., offsets are respectively 2^2, 3^2, 4^2, &c., times the first offset, or 4, 9, 16, &c., times the first one.

Having laid out the offsets in this manner, till the last one, which suppose to be p_4q_4 is either within, or very little more than, a chain from T, make $p'_4\text{T} = p_4\text{T}$, and lay out the same offsets in an inverted order, on TC, as were laid out on BT, that is, beginning with the greatest first and ending with the least.

Ex. Let the radius of the curve be 80 chains, then $p_1q_1 = \frac{1 \times 792}{2 \times 80}$ $= 4{\cdot}95$ inches; whence $p_2q_2 = 4 \times 4{\cdot}95 = 19{\cdot}8$ inches, $p_3q_3 = 9 \times 4{\cdot}95$ $= 44{\cdot}55$ inches $= 3$ ft. $8{\cdot}55$ in., &c., or the value of the first offset may be taken from the table at the end of the book.

To make the distances of the stumps equal.

As it can rarely happen in practice that the distance $q_4q'_4$ will be $= 1$ chain, when this is the case it will be better to set out the curve from C as was done from B; then, if the distance $q_4q'_4$ be less than 1 chain, suppose m links less, lay the chain from q'_4 to q'_3, and at the distance $q'_4t = m$, lay off the offsets $ts = \frac{m\,(1-m)}{2r}$, then s will be the point for the stump, instead of q'_4, thus making $q'_4s = 1$ chain. This operation must be repeated between q_3 and q_2, q_2 and q_1, &c., the last point to be changed necessarily falling on the tangent CD. If the distance $q_4q'_4$ be greater than 1 chain by m links, lay the chain

* *Demonstration.*—Draw the radius q_3O, and $q_3a \perp$ to OB. Put $\text{OB} = \text{O}q^3 = r$, and $q_3a = p_3\text{B} = \delta$; then $\text{O}a = \sqrt{r^2 - \delta^2}$, and $\text{B}a = p_3q_3 = \text{OB} - \text{O}a = r - \sqrt{r^2\delta^2}$. Now, if δ be taken successively $= 1^2, 2^2, 3^2$, &c., chains, the values of p_1q_1, p_2q_2, p_3q_3, &c., will be respectively $r - \sqrt{r^2 - 1^2}$, $r - \sqrt{r^2 - 2^2}$, $r - \sqrt{r^2 - 3^2}$, &c. But as $1^2, 2^2, 3^2$, &c., are very small compared with r^2, within the limitation assigned to Case I. of this Problem, the values of $\sqrt{r^2 - 1^2}$, $\sqrt{r^2 - 2^2}$, &c., may be taken re- respectively $r - \frac{1^2}{2r}$ $r - \frac{2^2}{2r}$, &c., without material error, whence $p_1q_1 = r - \left(r - \frac{1^2}{2r}\right) = \frac{1^2}{2r}$, $p^2q^2 = r - \left(r - \frac{2^2}{2r}\right) = \frac{2^2}{2r}$, &c. But when the length of the curve exceeds $\frac{1}{4}$ of its radius, only 5 or 6 of the offsets ought to be taken in this manner, and the remainder, say, commencing with p_7q_7, ought to be taken $= r - \sqrt{r^2 - 7^2}$, &c.; although in a curve of 80 chains radius, the 10th offset by the contracted method exceeds the same offset by the correct method by only $\frac{1}{4}$ of a link out of $62\frac{1}{2}$ links. This difference, however, becomes gradually greater as the distance on the tangent approaches the 20th chain.

from q'_4 in the direction of q_4, and at the distance of $q'_4 \text{v} = m$ links, lay off the offset $vu = \frac{(1+m)m}{2r}$, then u will be the place for the stump; next lay the chain from q_3 in the direction of u, and repeat the previous operation: this must be done to the end of the curve, the last mark to be changed falling, in this case, within the curve. The stumps may now be put down as pointed out in Note 4, Prob. II.

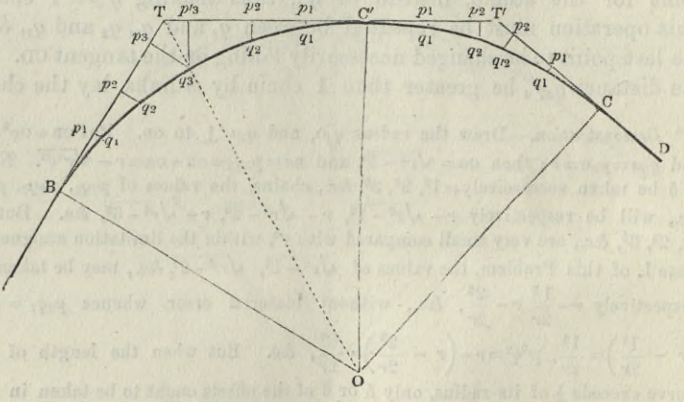
Ex. 1. When the distance $q_4q'_4$, is less than 1 chain by 40 links, the radius of the curve being 80 chains. Here $ts = \frac{(1-m)m}{2r} = \frac{(1-\cdot 40)\times\cdot 40}{160}$ ch. $= \frac{\cdot 60 \times \cdot 40 \times 792}{160} = 1\cdot 188$ in. $= 1\frac{1}{5}$ inch nearly.

Ex. 2. When the distance $q_4q'_4$ is more than 1 chain by 40 links, and rad. = 80 chains. Here $vu = \frac{(1+m)m}{2r} = \frac{(1+\cdot 40)\times\cdot 40}{160}$ ch. $\times$ $\frac{(1\cdot 40 \times \cdot 40)\times 792}{160} - 2\cdot 742$ in. $= 2\frac{3}{4}$ inches nearly.

Case II.

When the curve is any required length.

As it is inconvenient in practice to have the offsets greatly to exceed two chains in length; if, therefore, the curve be a long one, the offsets may be confined within proper limits by dividing the curve



into two or more parts, and introducing one or more additional tangents. In the annexed figure the curve BC is divided into two parts in C′, at which point the tangent TC′T′ is drawn, meeting the tangents BT, T′C in T and T′, the tangents BT, TC′ being each taken =

$Bp + \frac{1}{8}p_3q_3$,* wherein Bp_3 is taken the nearest whole number of chains to $\frac{1}{8}$BO, and p_3q_3 is the offset corresponding to Bp_3. By thus taking the length of BT, TC′, the last offset on BT and the first one on TC will meet at q_3, the middle point of BC′, and therefore the distances of the ends of the offsets that form the curve will all be sufficiently near to equality, *i.e.*, to one chain. Also by trigonometry, BT : BO : : rad. : tan. ∠ BTO = $\frac{1}{2}$ ∠ BTC′; whence the ∠ BTC′ becomes known, and consequently the direction of the new tangent TC′T′ is also known. Having, therefore, determined BT and ∠ BTC′, the offsets may be laid out to p_3q_3; and having made $Tp_3 = Tp'_3$, the same offsets may be laid out in an inverted order on TC′; the order of the offsets being a second time inverted on C′T′, and a third time on T′C. If the distance of the middle offsets $q_2q'_2$ be less or greater than 1 chain, that distance, and consequently the following ones, must be made equal to 1 chain, as in the preceding case.

EX. Let BO = 240 ch., then $Bp_3 = \frac{1}{8}$BO = 30 ch., $p_3q_3 = \frac{Bp^2_3}{2BO} = \frac{30^2}{480} = 1{\cdot}875$ ch. : whence BT. = TC′ = $Bp_3 + \frac{1}{8}p_3q_3 = 30 + \frac{1}{8}1{\cdot}875 = 30{\cdot}2344$ ch. = $30{\cdot}23\frac{1}{2}$ ch. nearly : and by trig.

As 30·2344 (BT) log. . .	1·48050
: 240 (BO).	2·38021
: : rad.	10·00000
: tan. $\frac{1}{2}$ ∠ BTC′ = 82° 49′ .	10·89971
2	
∠ BTC = 165° 38′,	

whence the position of the tangent TC′T′ becomes known; it will be unnecessary in this case to find ∠ TT′C.

By table 4, the first offset on BT, viz., $p_1q_1 = 1{\cdot}65$ in., from which the other offsets may be found by multiplying by 4, 9, 16, &c., which may be laid out as in the last case, p_3q_3 being the 30th offset from B, and meeting the corresponding offset p'_3q_3 at q_3, the middle of the curve BC′. The method of making the distances equal on the second part C′C of curve, if required, is the same as in Case I.

NOTE.—If the curve be a long one, it may be divided, in this manner, into 3, 4, or 5 parts, according to its length.

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* *Demonstration.*—By the similar triangles OBT, q_3p_3T, OB : $Bp_3 + p_3T$ (BT) : : $p_3q_3 : p_3T$; but since $Bp_3 = \frac{1}{n}$OB, or the nearest whole number to it, and since p_3T must always be very small compared with Bp_3, it may be rejected; ∴ OB : $\frac{1}{n}$OB (= very nearly to BT) : : $p_3q_3 : p_3T = \frac{1}{n}p_3q_3$, wherein n may be taken any whole number, 8 being the most convenient, as in the example.

Case III.

When the length of the curve exceeds the limit adopted in Case I., *but is considered too short to be divided into two parts, as in* Case II.

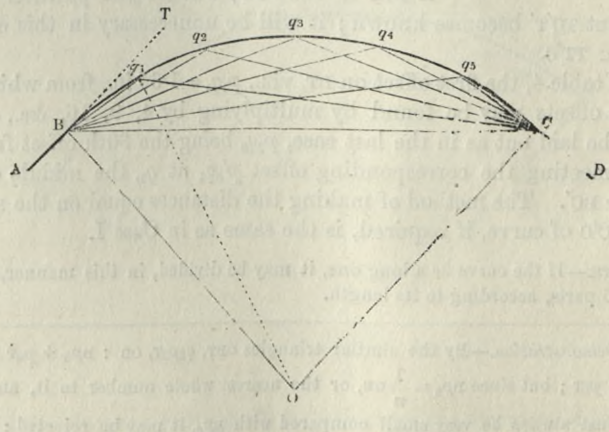
Let the curve be one of 80 chains radius, and between 32 and 38 chains in length, then to avoid the trouble of adding another tangent, the offsets beyond the 8th must be calculated from the following formulæ:—

$$p_7q_7 = r - \sqrt{r^2 - 9^2}$$
$$p_8q_8 = r - \sqrt{r^2 - 10}$$
$$\&c. = \&c.$$

If the radius of the curve be 160 chains, and its length about 50 or 60 chains, the offsets must be calculated, as above, beyond the 15th. The formula $\frac{1^2}{2r}$, $\frac{2^2}{2r}$, $\frac{3^2}{2r}$, &c., is a very near approximation to the true lengths of the offsets within the limits assigned, but beyond these limits the errors of the offsets begin gradually to augment, till they become too considerable to be overlooked. See Demonstration to Case I.

PROBLEM IV.

To lay out the curve, by two theodolites, where water, swamps, quarries, or other obstructions prevent the use of the chain.



The position of the tangents AB, DC to the curve at its extremities B and C, its radius OB or OC, and the ∠ BOC = supplement of the angle made by the tangents when prolonged, being determined; join

BC, and take several equidistant points q_1, q_2, q_3, &c., in the curve, from which points draw lines to B and C.

Take the $\angle\, q_1$CB = an arc whose sine is $\frac{\delta}{2r}$, wherein δ = Bq_1 = q_1q_2 = &c., and r = OB, or, because δ is, in practice, usually required to be = 1 chain, take $\angle\, q_1$CB = arc whose sine is $\frac{1}{2r}$. The several angles may then be arranged as below :—

$\angle\, q_1$CB = arc to sin. $\frac{1}{2r}$,	and	$\angle\, q_1$BC = $\frac{1}{2}\angle$ BOC − $\angle\, q_1$CB,
$\angle\, q_2$CB = 2 $\angle\, q_1$CB,		$\angle\, q_2$BC = $\frac{1}{2}\angle$ BOC − 2 $\angle\, q_1$CB,
$\angle\, q_3$CB = 3 $\angle\, q_1$CB,		$\angle\, q_3$BC = $\frac{1}{2}\angle$ BOC − 3 $\angle\, q_1$CB,
$\angle\, q_4$CB = 4 $\angle\, q_1$CB,		$\angle\, q_4$BC = $\frac{1}{2}\angle$ BOC − 4 $\angle\, q_1$CB,
&c. = &c.		&c. = &c.

Therefore if theodolites be fixed at B and C, and the angles q_1CB, q_1BC be taken at the same time, the intersection of Bq_1 and Cq_1 will give the point q_1. In the same manner by taking the angles q_2CB, q_2BC, the intersection of Bq_2 and Cq_2 will give the point q_2, &c.

Ex. Let the radius OB = 80 chains, and the angle between the tangents AB, DC, when prolonged to meet, be 160°; then its supplement = $\angle$ BOC = 80°, and $\frac{1}{2}\angle$ BOC = 40°. Also $\angle\, q_1$CB = arc to sine $\frac{1}{2r}\left(=\frac{1}{160}= \cdot 00625\right) = 0°\ 21'\ 29''$, whence $\angle\, q_1$BC = $\frac{1}{2}\angle$ BOC − $\angle\, q_1$CB = 40° − 0° 21′ 29″ = 39° 38′ 31″; $\angle\, q_2$CB = 2 $\angle\, q_1$CB = 2 × 0° 21′ 29″ = 0° 42′ 58″ and $\angle\, q_2$BC = $\frac{1}{2}\angle$ BOC − 2 $\angle\, q_1$CB = 40° − 0° 42′ 58″ = 39° 17′ 2″, &c. The angles would be best arranged for use as follows, the opposite ones to be taken at the same time.

$\angle q_1$CB = arc to sin. ·00625	= 0° 21′ 29″,	$\angle q_1$BC = $\frac{1}{2}\angle$BOC − $\angle q_1$CB = 39° 38′ 31″;
$\angle q_2$CB = 2 $\angle q_1$CB	= 0° 42′ 58″,	$\angle q_2$BC = $\frac{1}{2}\angle$BOC − 2 $\angle q_1$CB = 39° 17′ 2″;
$\angle q_3$CB = 3 $\angle q_1$CB	= 1° 4′ 27″,	$\angle q_3$BC = $\frac{1}{2}\angle$BOC − 3 $\angle q_1$CB = 38° 55′ 33″;
$\angle q_4$CB = 4 $\angle q_1$CB	= 1° 25′ 56″,	$\angle q_4$BC = $\frac{1}{2}\angle$BOC − 4 $\angle q_1$CB = 38° 34′ 4″;
&c. = &c.	= &c.	= &c. = &c.

This list of angles must be continued till $n\ \angle\, q_1$CB can no longer be taken from $\frac{1}{2}\angle$ BOC, and one angle in each column being taken at the same time, as $\angle\, q_1$CB at C and $\angle\, q_1$BC at B, will give the point q_1 : and so on for the other points q_2, q_3, &c. See the following notes.

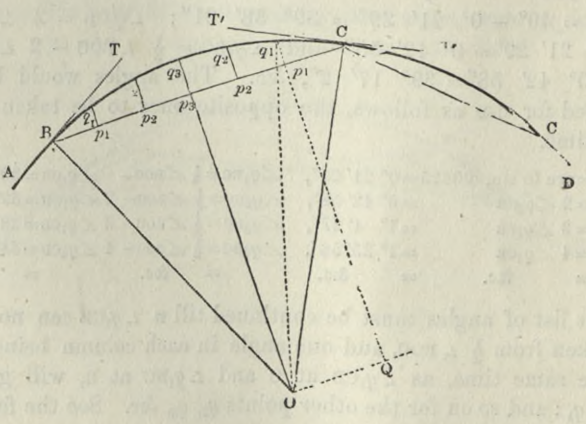
NOTE 1.—Where the obstructions are such as to prevent the stumps being put down at the consecutive points q_1, q_2, q_3, &c., it would be best to take every fourth angle, thus obtaining the points q_4, q_8, q_{12}, &c., which will be 4 chains apart on the curve, the intermediate points being left to be put in when the work of the line has progressed so far as to present a better opportunity. Or, if it should be thought preferable to avoid taking a multiplicity of angles, every fourth angle may be taken in any case, as this method is equally available whether obstructions exist or not.

Demonst. to Prob. IV.—The angles Bq_1C, Bq_2C, Bq_3C, &c., being in the same segment, are well known to be equal and constant, and also equal to half the supplement of $\angle BOC$, and consequently the sums of the angles of the triangles Bq_1C, Bq_2C, &c., adjacent to BC are equal to $\frac{1}{2}\angle BOC$, whence $\angle q_1BC = \frac{1}{2}\angle BOC - \angle q_1CB$, $\angle q_2BC = \frac{1}{2}\angle BOC - \angle q_2CB =$ (because equal angles stand on equal arcs) $\frac{1}{2}\angle BOC - 2\angle q_1CB$, &c. Also join Oq_1 and put $BO = r$, and $Bq_1 = \delta$, then it is well known that $\frac{1}{2}\angle BOq_1 = \angle q_1CB$, and because the sides BO, q_1O of the triangle BOq_1 are equal, $\sin. \frac{1}{2}\angle BOq_1 = \sin. \angle q_1CB = \frac{\text{rad.} \times \delta}{qr}$; or, by taking rad. $=1$, and $\delta = 1$ chain, as required in practice, $\sin. \angle q_1CB = \frac{1}{2r}$ or $\angle q_1CB =$ arc to $\text{sine}\frac{1}{2r}$. *Q. E. D.*

PROBLEM V.

To lay out a railway curve by means of ordinates or offsets from its chord or chords no material obstructions being supposed to exist on the concave side of the curve to prevent the use of the chain.

Let $BC'C$ be a portion, or the whole, of a curve of a railway; AT, $T'T'$ and CD tangents to the curve at B, C', and C; O the centre, OB the radius, BC' and $C'C$ chords of the curve; which chords must not exceed 40 chains, if the radius be 80 or 120 chains, but they may be 60



chains, if the radius exceed 120 (this limitation is necessary to prevent the offsets p_1q_1, p_2q_2, &c., being too long, as was observed with respect to the offsets from the tangents, in Prob. III.); and let the radius q_3o bisect the curve and chord in q_3 and p_3 respectively. Then from the right-angled triangle Bp_3O, in which BO and $Ap_3 =$

$\frac{1}{2}$BC′ are given, the $\angle$ BOp_3 = $\angle$ TBC may be found, which determines the position of the chord BC′; also Op_3 may be found from the same triangle being $= \sqrt{\text{OB}^2 - \text{BP}^2{}_3} = \sqrt{\text{OB}^2 - \frac{1}{4}\,\text{BC}'^2}$. Put OB $= r$, O$p_3 = s$, and the $\frac{1}{2}$ chord B$p_3 = n$ chains, then

$$q_1p_1 = \sqrt{r^2 - (n-1)^2} - s,^*$$
$$q_2p_2 = \sqrt{r^2 - (n-2)^2} - s,$$
$$\&\text{c.} = \quad \&\text{c.}$$
$$q_3p_3 = \sqrt{r^2 - (n-3)^2} - s = r - s.$$

q_3p_3 being supposed to be the offsets at the middle part of the curve BC′, not the third offset as shown in the figure, it being impossible to draw all the offsets without confusing the figure, or making it unnecessarily large. After reaching the middle point q_3 of the curve, the same offsets are repeated in an inverted order till the curve shall have been set out to C′. The same operation may be repeated as often as necessary, till the whole curve be completed, observing to make the $\angle$ BC′C = 2 complement of $\angle$ TBC′, which has been already found. See the following notes :—

NOTE 1. When BC′ is the whole curve, and its chord Bp_3C includes a fractional part of a chain, the distance of the offsets on each side of the middle of the curve will be less than one chain; therefore that distance, and consequently the following ones, must be made equal, as was shown with respect to the distances in Problem III.

2. If the last chord, which suppose to be C′C, be less than the preceding chord or chords, the $\angle$OC′C, must be found, and added to the $\angle$OBC′ or OC′B, which will give the $\angle$BC′C, showing the direction of the chord C′C.

3. This method of laying out the curve is seldom used, on account of the calculations it involves. It may, however, be used with advantage where a winding river or cliff is close to the convex side of the curve, or protrudes in some places a little through the curve; thus preventing the use of any other method, except that in Problem IV., which requires two theodolites.

Definition of the compound curve. (See figures to Prob. VI.)

The compound curve BCC′C″, joining the tangents AB, DC″, is composed of three circular arcs BC, CC′, C′C″, having common normals, OC, O′C′ at their points of junction C, C′; and therefore common tangents at the same points, the radii of the three portions of the curve being respectively OB = OC, O′C = O′C′, and O″C′ = O″C″. This kind of curve is adopted where the line is required to pass through

* *Demonstration.*—Draw OQ parallel to BC′, q_1Q $\perp$ to OQ, and join q_1O; and let n = number of chains in $\frac{1}{2}$BC′ or p_3C′, and C′p_1 = 1 chain. Then p_3p_1 = OQ $= n-1$; and (Euc. I. 47) $q_1\text{Q} = \sqrt{r^2-(n-1)^2}$; whence $q_1p_1 = \sqrt{r^2-(n^2-1)^2} - p_1\text{Q}$: but $p_1\text{Q} = p_3\text{O} = s$, $\therefore\ q_1p_1 = \sqrt{r^2-(n-1)^2} - s$. Similarly, q_2p_2 is found $= \sqrt{r^2-(n^2-2)^2} - s$, &c. *Q. E. D.*

given points, as C and C', to avoid obstructions, or where a principal station or *terminus* is at or near C''; in the latter case the radius $O''C''$ may, if required, be less than 80 chains.

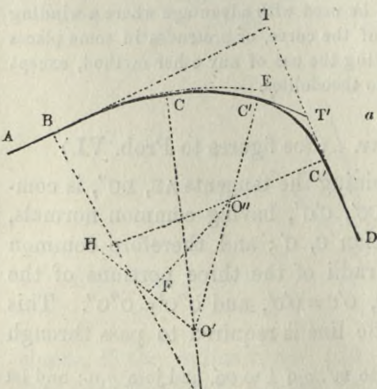
The compound curve may consist of two, three, or more portions of different arcs; thus the curve bcc' consists of two portions, bc, cc'.

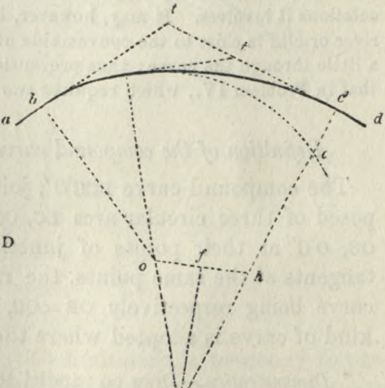
PROBLEM VI.

1. *To find the several radii of the compound curve mechanically.*

Let AB, DC'' be the tangential portions of the line, which are required to be joined by a curve passing through the points C, C', *the point* C'' *not being given.* Select a curve-ruler such that, being applied to touch AB at B, it may also pass through C; if this curve do not pass through C', but through some other point E, another curve-ruler of less radius, in this case, must be selected, and such that it may touch the arc BC at C without cutting it or its prolongation towards E, and also pass through the point C: if this curve cut the prolongation $C''T'$ of the tangent DC'', another curve-ruler of less radius than the last one must be selected, and such that it may touch curve CC' at C' and the tangent DC'' at C'': thus completing the curve $BCC'C''$. The radii OC, $O'C'$, $O''C''$ may be determined from the curve-rulers, as in Case I. Prob. I.

2. *To find the radius $c'o'$ of the compound curve bcc' geometrically, the starting point b and the radius bo being given.*





From the given point b in the tangent ab draw the given radius $bo \perp$ to ab; and draw the curve to some point c, where it is found convenient to change the radius: draw the radius oc, and from thereto draw ct', meeting the tangent dt in t'; make $t'c' = t'c$, and from c' draw $c'o' \perp$ to tc' meeting co, prolonged if necessary in o'; then o' is the centre of the arc cc' of the curve, conformable to the nature of tangents.

The method of constructing the curve, when it consists of three or more parts, is sufficiently obvious.

3. *One of the two radii of the compound curve and its starting and closing points being given, to find the other radius.*

Let ab, $c'd$, be the tangents, b and c' the starting and closing points of the curve. Draw the perpendiculars $bo = c'h$ = given radius to the tangents; join oh, and bisect it in f; draw $fo \perp$ to oh, meeting $c'h$ prolonged in o'; join $o'o$, and prolong it till $oc = c'h$: then the points o, o' are the centres of the arcs bc cc', which constitute the compound curve, $o'c = o'c'$ being the radius required.*

NOTE.—In the compound curve BCC′C″, where the radius C″O″, which is to be found, is less than the preceding radius CO, the $\perp$ C″H is made = CO; HO′ is joined and bisected in F; and FO″ drawn $\perp$ to HO′, meeting C″H in O″, which is the centre of the arc C′C″, &c.

Definition of the serpentine or S *curve.* (See figure to Prob. VII.)

The serpentine curve BGC is used in railways, when obstructions or some other cause render its adoption preferable; it consists of two circular arcs, having their convex sides turned in opposite directions, like the letter S, whence it is sometimes called the S curve; the two portions BG, GC of the curve having a common normal OGO′ at their point of junction G, and therefore a common tangent at the same point. This curve affords the most easy means of joining two parallel, or nearly parallel, portions of a line of railway.

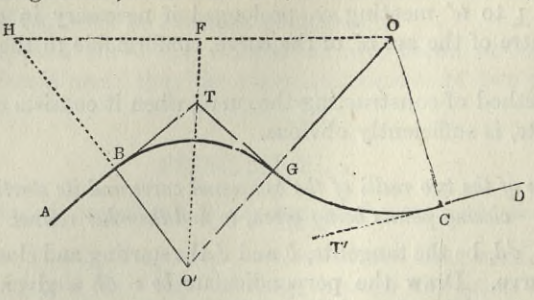
PROBLEM VII.

1. *When one radius and its tangential point are given, to find the other radius and tangential point of the serpentine curve geometrically.*

From the given tangential point C draw the given radius CO $\perp$ to the tangent CD, and draw the curve CG to some point G where it is found convenient that it should have its point of contrary flexure;

* *Demonstration.*—Since $of = fh$, and fo' is $\perp$ to $o\,h$, $oo' = o'h$; also bo was made $= c'h = oc$; $\therefore$ $o'c = o'c'$ and the normal $o'oc$ is common to the arcs of the curve. *Q. E. D.*

through OG draw the normal OGO′; from G draw GT ⊥ to OGO′, to meet the tangent AT; make TB = TG; and draw BO′ to AT, meeting



OGO′ in O′; then O is the centre, and O′B = OG is the radius of the curve BG, as is evident from the nature of tangents.

NOTE.—The radius O′B and tangential point B may be found mechanically, *i.e.*, by the curve-rulers, as in Problems I. and VI.

2. *When the tangential points and one of the radii of the serpentine curve are given, to find the other radius geometrically.*

From the given tangential points C and B draw CO, BH, respectively ⊥ to the tangents CD and BA, and equal to the given radius; join OH, and bisect it in F; draw FO′ ⊥ to OH, meeting HB prolonged in O′, and join O, O′; making O′G = O′B; then O′ is the centre, and O′B − O′G is the radius of the portion BG of the curve, as required.*

NOTE 1. The radius BO′ may be found mechanically as in the preceding case.

2. The radius O′B may be found from the following formula, wherein δ = BC, r = given radius OC, $\alpha = \angle$ TBC, and $\alpha' = \angle$ T′CB.

$$\text{O}'\text{B} = \frac{\delta\,(\delta - 2r\sin.\alpha')}{2\left(\delta\sin.\alpha + 2r\sin.\frac{\alpha-\alpha'}{2}\right)}.$$ See investigation and fig. to Problem VIII.

Ex. Let $\delta = 200$ chains, $r = 110$ chains, $\alpha = 45°$, and $\alpha' = 20°$; the sin. α = ·70711, sin. α' = ·34202, $\frac{\alpha-\alpha'}{2} = \frac{45°-20°}{2} = 12°\ 30'$, the sine of which is ·21644; and by the formula $\text{O}'\text{B} = \frac{200\ (200 - 220 \times \cdot 34202)}{2\ (200 \times \cdot 70711 \times 220 \times \cdot 21644^2} = \frac{200 \times 124\cdot 7556}{2 \times 151\cdot 7282} = 82\cdot 22$ chains.

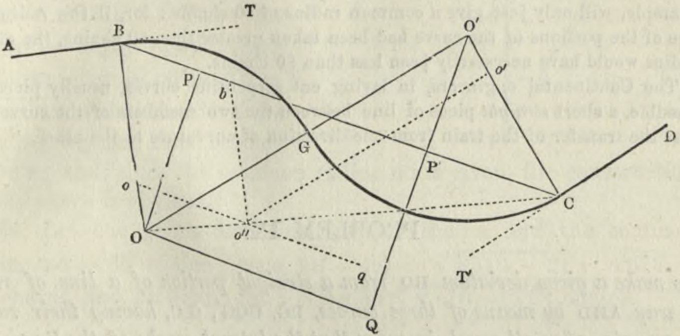
It may thus be readily ascertained whether the required radius O′B is greater, equal to, or less than 80 chains; if less, the given radius r ought either to be diminished, or the distance BC of the tangential points ought to be increased, according to circumstances, in order that O′B may be of the required length, assuming that the portion BG of the curve is not near a *terminus* or principal station.

* The *demonstration* in this case is similar to the one given to Problem VI.

PROBLEM VIII.

1. *When the two portions of the serpentine curve have the same radius, to determine that radius geometrically, the tangential points and their distance being given.*

Let AB, DC be the tangents, B and C the tangential points, and BC the given distance. Draw Bo = Co' respectively ⊥ to AB, DC, and of any convenient length; through o, parallel to BC, draw oq



indefinitely; with the compasses apply $o'o'' = 2$C$o' = 2$Bo; through C, o'' draw Co''O, meeting BO prolonged in O; and through O, parallel to $o''o'$, draw OO, meeting Co' prolonged in O; then O and O′ are the centres, and OB and O′C are the equal radii of the serpentine curve BGC, the common normal of the portions BG, GC of the curve being OGO′ = 2BO = 2CO′.*

2. *To find the common radius of the two portions of the serpentine curve by calculation, the same things being given as in the preceding case, and the angles* TBC, T′CB.

Put BC = δ, BO = CO′ = r, ∠ TBC = α, and ∠ T′CB = α'; then

$$r = \frac{\delta}{\sin.\ \alpha + \sin.\ \alpha' + 2\ \sin.\ \text{of arc to cos.}\ \tfrac{1}{2}\ (\cos.\ \alpha + \cos.\ \alpha')}\dagger$$

* *Demonstration.*—Draw $o''b$ parallel to OB; then by similar triangles,

Co' : $o'o''$ = 2Co' : : CO′ : 2CO′ = OO′,
Co'' : CO : : Co' = Bo = bo'' : CO′
∴ Co'' : CO : : Co' = Bo = bo'' : BO = CO′. *Q. E. D.*

† *Investigation.*—Draw PO, P′O′ ⊥ to BC; through O draw OQ parallel to BC, meeting O′P′ prolonged in Q: then OO′ = $2r$, BP = r sin. α, OP = P′Q = r cos. α, CP′ = r sin. α', O′P′ = r cos. α', O′Q = O′P′ + P′Q = r (cos. α + cos. α') : whence, from the right-angled triangle OQO′, sin. ∠ O′OQ $\frac{1}{2}$ (cos. α + cos. α'), which is therefore

Ex. Let $\delta=200$ chains, $\alpha=27°$, $\alpha'=50°$: then from a table of nat. sines sin. $\alpha=\cdot45399$, its cos. $=\cdot89101$; sin. $\alpha'=\cdot76604$, its cos. $=\cdot64279$: whence $\frac{1}{2}$ (cos. α + cos. α') $=\frac{1}{2}$ ($\cdot89101+\cdot64279$) $=\cdot76690=$ cos. and 2 sin. of arc to cos. $\cdot76690$ is $1\cdot28334$; therefore $r=\dfrac{200}{\cdot45399+\cdot76604+1\cdot28334}=\dfrac{200}{2\cdot50337}=79\cdot89$ chains, or nearly 80 chains, the radius required.

NOTE.—The method of forming the serpentine curve with a common radius is much to be preferred to any other, when the nature of the ground will admit of its being done; and more especially so, when the *data*, as in the preceding example, will only just give a common radius of 80 chains: for, if the radius of one of the portions of the curve had been taken greater than 80 chains, the other radius would have necessarily been less than 80 chains.

The Continental engineers, in laying out serpentine curves, usually place, if possible, a short *straight* piece of line between the two members of the curve, to ease the transfer of the train from one direction of curvature to the other.

PROBLEM IX.

To make a given deviation HQ *from a straight portion of a line of railway* AHD *by means of three curves*, BG, GQG′, G′C, *having their radii* OB, O′Q, O″C, *all equal, in order that the lateral works of the line may avoid the building, or other obstruction b, which is close to the centre of the straight portion of the line.*

1. *Construction.*—From the given point H draw HQ = given deviation ⊥ to AD; on QH prolonged, take QO′ = OP = given radius; with the compasses apply PB = QP = twice given radius; draw

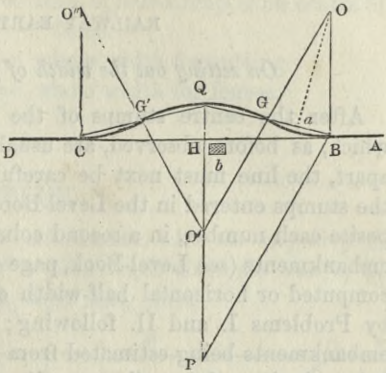
given, whence the comp. of ∠O′QO = ∠OO′Q is known, and may be thus expressed, sin. ∠OO′Q = sin. of arc to cos. $\frac{1}{2}$ (cos. α + cos. α'). Whence OQ = PP′ = OO′ × sin. ∠OO′Q = $2r$ × sin. of arc to cos. $\frac{1}{2}$ (cos. α + cos. α'), δ = BC = BP + PP′ + P′C = r sin. α + r sin. α' + $2r$ sin. of arc to cos. $\frac{1}{2}$ (cos. α + cos. α'), from which

$$r=\frac{\delta}{\text{sin. }\alpha+\text{sin. }\alpha'+2\text{ sin. of arc to cos. }\frac{1}{2}(\text{cos. }\alpha+\text{cos. }\alpha')}. \quad Q.\,E.\,I.$$

When the radii are unequal, and one of them, as r = O′C is given, and the other R = BO is required; then O′O = R + r, BP = R sin. α, PO = P′Q = R cos. α, O′Q = R cos. α + r cos. α', and OQ = PP′ = $\sqrt{O'O^2-O'Q^2}=\sqrt{(R+r)^2-(R\cos.\alpha+r\cos.\alpha')^2}$, $\delta=R\sin.\alpha+r\cos.\alpha'+\sqrt{(R+r)^2-(R\cos.\alpha+r\cos.\alpha')^2}$. By transposing and squaring, and remembering that sin.^{2} + cos.^{2} = 1, &c., there results $\delta^2-2\delta(R\sin.\alpha+r\sin.\alpha')=2Rr(1-\cos.\alpha\cos.\alpha-\sin.\alpha\sin.\alpha')=2Rr(1-\cos.\overline{\alpha-\alpha'})=4Rr\sin.^2\frac{\alpha-\alpha'}{2}$, whence

$$R=\frac{\delta(\delta-2r\sin.\alpha')}{2\left(\delta+2r\sin.^2\frac{\alpha-\alpha'}{2}\right)}.$$ This formula is used for finding the value of BO′, Note 2, Problem VII., where the symbols are defined. *Q. F. I.*

BO $\perp$ to AD; through O′ draw O′GO parallel to PB, meeting OB in O; make CH = HB; and join CQ, QB, the latter cutting OO′ in G, and the former cutting O′OO″ (which is similarly drawn to OO′) in G′: then B and C are the starting and closing points of the curve, of which the separate portions are BG, GQG′, G′C, and the chords BG, GQ, QG′, G′C are all equal.*



2. *Calculation.*—Take BH = HC = $\sqrt{\text{QH} \ (4\text{BO} - \text{QH})}$, and BG = GQ = QG′ = G′C = $\sqrt{\text{BO} \, . \, \text{QH}}$, which chords of the arcs BG, GQ, &c., thus become known; and, since the common radius BO is given, the construction of the curve is obvious.†

Ex. Let the given deviation QH = 2 chains, and the common radius BO = 85 chains; then BH = HC = $\sqrt{2(340 - 2)} = \sqrt{676} =$ 26 chains, and BG = GQ = &c. = $\sqrt{2 \times 85} = \sqrt{170} = 13{\cdot}04$ chains.

REMARKS ON LAYING OUT THE CURVES IN THE FOUR LAST PROBLEMS.

Having in the four last problems given various methods of determining the radii and common normals, indicating the positions of the tangent points of the parts of the compound, serpentine, and deviation curves, the method of laying out the curves themselves by Problems II., III., IV., or V., according to circumstances, will be readily seen, recollecting that when junction-points of curves of different radii occur, as CC′, first fig. to Prob. VI., to commence the operation afresh, by using the radii and tangents of the respective portions of the curve.

* *Demonstration.*—Because QP, BO are parallel, as are also O′O, PB; O′P = BO = (by const.) QO′ = given radius, and O′O = PB = 2QO′ = 2GO′; ∴ GO′ = GO = QO = BO, which are similarly proved to be = G′O′ = G′O″ = CO″. *Q. E. D.*

† *Demonstration.*—Draw Oα perpendicular to BQ, bisecting it in α; then by similar triangles BO : Bα = $\frac{1}{4}$BQ : : BQ : QH, whence BQ^2 = 4BO . QH, or BG^2 = $\frac{1}{2}$BQ^2 ≑ BO . QH, or BG = $\sqrt{\text{BO} \, . \, \text{QH}}$. Also BH^2 = HC^2 = BQ^2 − QH^2 = 4BO . QH − QH^2 = QH (4BO − QH), or BH = HC = $\sqrt{\text{QH} \ (4\text{BO} - \text{QH})}$. *Q. E. D.*

SECTION IV.

RAILWAY EARTH WORKS.

On setting out the width of ground for a railway.

After the centre stumps of the railway have been put down, which, as before observed, are usually at the distance of one chain apart, the line must next be carefully levelled, and the number of the stumps entered in the Level-Book, in a vertical column; and opposite each number, in a second column, the depth of the cuttings or embankments (see Level-Book, page 441); and in a third column, the computed or horizontal half-width of the surface cuttings as found by Problems I. and II. following; the depths of the cuttings and embankments being estimated from the balance line, which is 2 feet below the line of the rails or gradients, the 2 feet being filled up with gravel to form the way, and the beds for the sleepers of the rails.

The side stumps are next to be put down, which must be placed two on each side of every centre stump, in a direction perpendicular to the length of the line; or, if the line be curved, in a direction perpendicular to the tangent to the curve at the centre stump: the two interior stumps, *i.e.*, those next to the centre one, to mark the width of the cuttings, and the two exterior ones to mark the ditches of the side fences. The distances of every two of the interior stumps are to be entered in the Level-Book, opposite the number of the centre stump, in the main section, in order to ascertain the quantity of cuttings for the contractor; and the distances of every two of exterior stumps from the centre one are to be similarly entered, to ascertain the quantity of land which will be required from each proprietor for the works of the line; which last may be calculated accurately by the method of equidistant ordinates. (See Problem VI.)

PROBLEM I.

To set out the width when the surface of the ground is laterally on the same level as the intended railway.

From the centre stump, perpendicular to the direction of the line, set out half the bottom-width for the cutting, to which add the width of the side-fence, putting down a stump at each distance; then repeat the operation on the other side of the line.

Ex. If the bottom-width of the railway be 30 feet, and the breadth of one of the side fences be 12 feet, required the widths for cutting and for fences.

$30 \div 2 = 15$ ft. = dist. of side-stump from centre for cutting.

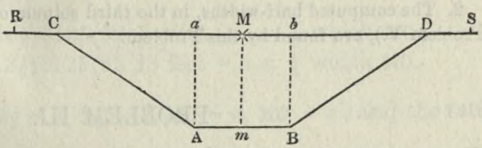
$15 + 12 = 27$ ft. = dist. of side-stump from centre for fence.

Therefore, $15 \times 2 = 30$ = whole width for cutting.

and $27 \times 2 = 54$ = whole width for fences.

PROBLEM II.

To set out the width of cuttings or embankments, when the surface of the ground is laterally level, and at a given height above the level of the intended railway, the ratio of the slopes being given.



Let ABCD be a cross-section of the cuttings, RS the horizontal surface of the ground, AB the bottom width, AC, BD the slopes, $\text{M}m = \text{A}a = \text{B}b$ the perpendicular depth, and M the middle stump. Multiply the given depth $\text{M}m$ by the ratio of the slopes, to which add half the bottom-width $\text{A}m$, or $a\text{M}$: set out this distance from M to C for the half-width of the cuttings, to which add the width of the side fence for the whole half-width, repeating the same operation on the other side of M.

Ex. If the bottom-width AB or ab be 30 feet, the depth of the cuttings 28 feet, and the ratio of the slopes $1\frac{1}{2}$: 1, and the width of one of the side fences 12 feet, required the width of the cuttings, and of the land for the works of the railway.

$(28 \times 1\frac{1}{2}) + \frac{30}{2} = 42 + 15 = 57$ feet $= \text{MC} = \text{MD} = \frac{1}{2}$ width of cuttings; and $57 + 12 = 69$ feet $= \text{MR} = \text{MS} = \frac{1}{2}$ width of land. The doubles of which are the whole widths.

If w = bottom-width = AB, a = depth of cuttings = $\text{M}m$, f = width of one of the side-fences = RC, and the ratio of the slopes $r : 1$. Then

$$1 : r :: a : a\text{C} = ar, \text{ hence } ar + \tfrac{1}{2} w = \text{MC} = \text{MD},$$

$$ar + \tfrac{1}{2} w + f = \text{MR} = \text{MS},$$

$$\text{and } 2\,(ar + \tfrac{1}{2} w + f) = w + 2ar + 2f = \text{RS}.$$

Construction.—Draw AB = given width = 30 feet, perpendicular to which, at its middle point m, draw $m\text{M}$ = given depth = 28 feet; through M parallel to AB draw RMS; through A and B draw $\text{A}a$, $\text{B}b$ parallel to $\text{M}m$; make $a\text{C} = b\text{D} = 1\frac{1}{2}$ times $\text{M}m$, join AC, BD, and make

RC, DS = width of one of the side-fences = 12 feet. Then ABDC is a cross section of the cuttings, CD the surface-width thereof, and RS the whole surface required for the railway.

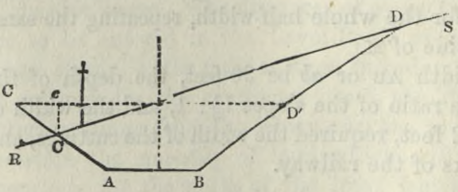
By inverting the last figure, so that ABDC may represent the cross-section of an embankment, it will be readily seen that the same method will apply, for setting out its half-widths, MC, MR, as that just given for the cuttings; as the dimensions are the same in both cases.

NOTE 1. The ratio of the slopes is the proportion of the horizontal line *ca* to the depth A*a* or M*m*. Thus, when the ratio is 1 : 1, *ca* = A*a*: when the ratio is 2 : 1, *ca*=2A*a*, &c. This ratio depends on the nature of the material through which the cuttings are made; if it is close-jointed hard rock, the ratio is ¼ or ½ to 1; if soft, or loose-jointed rock, or strong clay, the ratio is 1 or 1½ to 1; if springy ground, or loose sand, the ratio is 2 or 2½ to 1.

2. The computed half-widths, in the third column of the Level-Book (following Problem V.), are found by this Problem.

PROBLEM III.

To set out the width of the cuttings, when the surface of the ground is laterally sloping, the height of the centre stump above the level of the intended railway, the ratio of the slopes, and the lateral fall or rise of the ground in a given horizontal distance being given.



Let ABDC be a cross-section of the cuttings, RS the sloping surface of the ground, AB the bottom-width, M*m* the depth of the cuttings, M the middle stump, AC, BD the slopes, and C′D′ a horizontal line passing through M, MC′ = MD′ being the computed half-widths.

Having set the level so that by turning the telescope two or three chains' length of the line may be seen, if possible, on both sides of it, place a levelling-staff at M, and another at *q*, and observe the level readings on them, the difference of which will be *pq;* measure with the tape-line in feet the distance M*q* on the slope and the horizontal distance M*p;* take the computed half-width from the Level-Book, or find it by Problem II., and multiply it by the distance M*q* on the slope, and reserve the product. Add and subtract the product of the difference of the staff-readings, and the ratio of the slopes, to and from the horizontal distance M*p*, and reserve the sum and difference; divide the reserved product by the reserved sum for the corrected half-width MC, and by the reserved difference for the corrected half-width MD.

Ex. Let the depth of the cuttings at the centre-stump M be 20 feet, the bottom width AB = 30 feet, the distance Mq on the slope = 25 feet, the distance Mp on the level = 24 feet, the difference of the stave readings $pq = 7$ feet, and the ratio of the slopes $1\frac{1}{2} : 1$; required the corrected half-widths MC, MD.

$$20 \times 1\tfrac{1}{2} + \frac{30}{2} = 45 \text{ feet} = \text{computed half-width.}$$

$$\begin{array}{l} 25 \\ \hline 1125 \text{ reserved product.} \\ \hline \end{array}$$

$$24$$

$$7 \times 1\tfrac{1}{2} = 10\tfrac{1}{2}$$

reserved sum $34\frac{1}{2})1125(32{\cdot}6$ feet = cor. $\frac{1}{2}$ width MC

reserved diff. $13\frac{1}{2})1125(83{\cdot}33$ feet = cor. $\frac{1}{2}$ width MD.

Put MC$' = b$, $pq = h$, M$q = s$, M$p = l$, MC $= x$, MD $= x'$, and the ratio of the slopes $r : 1$.

$$\text{Then } x = \frac{bs}{l + rh} = \text{MC*}$$

$$\text{and } x' = \frac{bs}{l - rh} = \text{MD.}$$

If the numbers in the preceding example be substituted in these formulæ, the work will stand thus:

$$\text{MC} = \frac{bs}{l + rh} = \frac{45 \times 25}{24 \times (7 \times 1\frac{1}{2})} = \frac{1125}{34\frac{1}{2}} = \frac{2250}{69} .= 32{\cdot}6 \text{ feet.}$$

$$\text{MD} = \frac{bs}{b - rh} = \frac{45 \times 25}{24 - (7 \times 1\frac{1}{2})} = \frac{1125}{13\frac{1}{2}} = \frac{2250}{27} = 83{\cdot}33 \text{ feet;}$$

to each of which values of the corrected half-width, the width of

* *Demonstration.*—Draw cc $\perp$ to C'D'; then by similar triangles, $s : h :: x : \frac{hx}{s} =$ cc, hence C'$c = \frac{rhx}{s}$, and MC $=$ C'M $-$ C'$c = b - \frac{rhx}{s}$; again, by similar triangles $l : h :: b - \frac{rhx}{s}$: cc $= h$, $\frac{(bs - rhs)}{ls} = \frac{hx}{s}$, whence $x = \frac{bs}{l + rh} =$ MC; and in a similar manner is found $x' = \frac{bs}{l - rh} =$ MD. *Q. E. D.*

Cor. When the difference of the level readings is so large that the horizontal distance l cannot be conveniently measured, the value of $l = \sqrt{s^2 - h^2}$ must be substituted, thus giving

$$\text{MC} = \frac{bs}{\sqrt{s^2 - h^2} + rh}, \text{ and MD} = \frac{bs}{\sqrt{s^2 - h^2} - rh}.$$

one of the side-fences must be added for the whole breadth of land required.

NOTE 1. When the difference of level-readings is small, l may be taken $= s$ without material error.

2. A little practice will make this method of finding the corrected half-widths very easy. It is evidently much superior to the method of finding the same things by the blundering approximations often in use, especially where the difference of the levels of C and D is very considerable, and the surface of the ground has a regular slope, which often happens to be the case, or very nearly so; or, if the surface be a little uneven, a skilful engineer, by applying the tape-line in the general direction of the surface of the ground, will be readily able to correct the difference pq of the level-readings, so as to adapt it to his purpose. If, however, the surface of the ground be very uneven, which sometimes happens to be the case, it will be best to use the method of approximation, as in Problem V.

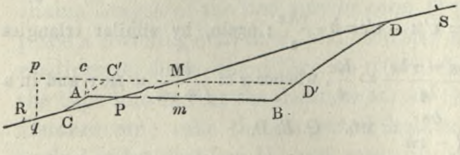
Construction.—Draw the cross-section ABD′C′ as in the preceding Problem; make Mp = given horizontal distance; draw pq perpendicular to C′D′ and equal to the difference of the level-readings; and through Mq draw RS, cutting AC′ in C and BD′ prolonged in D: then ABDC is the required cross-section of the cuttings, RC, DS being the breadths of the side fences.

By reversing the cross-section, it will be seen that the preceding calculation and construction will also apply to an embankment; but in this case the lesser distance MC must be measured up the slope, and the greater MD down it.

PROBLEM IV.

To set out the widths when the surface of the ground is laterally sloping, and when the cross section of the works consists partly of a cutting and partly of an embankment, the things given being the same as in the preceding Problem.

Let ACPBD be a cross-section of the works of a railway, consisting partly of the cutting BPD, and partly of the embankment APC; AB the bottom-width; M the centre stump, and Mm the depth of the cutting; RS the sloping surface of the ground, &c. When the embanked part, APC, is less than half the bottom-width Am, proceed to find, as in Problems II. and III. the computed half-width MD′, and from thence the corrected half-width MD. But to find the corrected half-width MC (which consists



partly of an embankment), multiply the distance MD (just found) by the difference of the bottom-width and the estimated half-width, and divide the product by the estimated half-width, and the quotient is the corrected half-width MC.*

Ex. Let the bottom-width AB = 30 ft., the depth Mm = 3 ft., the ratio of the slopes $1\frac{1}{2}:1$, and the difference of level-reading 7 ft., at the distances of 25 and 24 ft. from the centre-stump on the surface-slope, and on the level respectively; required the corrected half-widths MD and MC.

By Problem II. $3 \times 1\frac{1}{2} + \frac{30}{2} = 19\frac{1}{2}$ ft. = estimated half-width MD′,

By Problem III. $\frac{19\frac{1}{2} \times 25}{24-(7 \times 1\frac{1}{2})} = \frac{975}{27} = 36\cdot11$ ft. = corrected half-width MD.

By Problem IV. $\frac{36\cdot11 \times (30-19\frac{1}{2})}{19\frac{1}{2}} = 19\cdot44$ ft. = corrected half-width MC.

If the same symbols be used for the given parts in this example, as in Problem III., and w = bottom-width: then

$$\text{MD} = \frac{bs}{l-rh} = \frac{19\frac{1}{2} \times 25}{24-(7 \times 1\frac{1}{2})} = 36\cdot11 \text{ ft.} = \text{corrected half-width.}$$

$$\text{MC} = \frac{(w-b)s}{l-rh} = \frac{(30-19\frac{1}{2})25}{24-(7 \times 1\frac{1}{2})} = 19\cdot44 \text{ ft.} = \text{corrected half-width.}$$

NOTE.—This operation for finding MC, it will be seen, is different from the preceding one. See *Demonstration*.

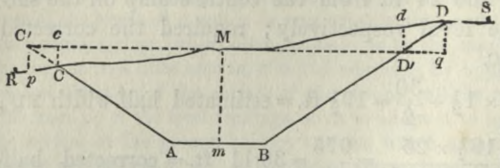
Construction. The operation for this Problem is the same as that for Problem III., except that AC is drawn parallel to BD.

By reversing the cross-section, it will be readily seen that the same calculation and construction will apply in the case of APC being a cutting, and PBD an embankment, observing that the corrected half-width for the cutting is the shorter distance, and *vice versâ*.

* *Demonstration.* Draw AC $\perp$ to C′D′ prolonged, and let pq, which is $\perp$ to pC′D′, be the difference of level-readings at M and q; then, if a = Mm, MC = Am = $\frac{1}{2}w$ = $\frac{1}{2}$ bottom width, the other symbols being the same as in the Demonstration to Problem III., ar = cC′ and C′M = MC − cC′ = $\frac{1}{2}w - ar$; whence, by the Demonstration above referred to, MC = $\frac{s(\frac{1}{2}w-ar)}{l-rh}$: but $b = \frac{1}{2}w + ar$, or $ar = b - \frac{1}{2}w$; this value being substituted in that of MC, gives $\frac{s(w-b)}{l-rh}$ = MC. Also $\frac{s(w-b)}{l-rh}$ = MC = $\frac{bs}{l-rh} \times \frac{w-b}{b}$, which is the rule given in words at length. *Q. E. D.*

PROBLEM V.

To find the width of the cuttings when the surface of the ground is laterally very uneven.



Let ABDMC be a cross-section of the cuttings, Mm the depth, C′M = MD′ = estimated half-width, CDM the uneven surface of the ground, &c. The solution of this Problem will be best effected by giving an example in numbers.

Let AB = 30 ft., Mm = 30 ft., and the ratio of the side-slopes as $1\frac{1}{2} : 1$; then MC′ = MD′ = $(30 \times 1\frac{1}{2}) + (\frac{1}{2} \times 30) = 60$ ft. Measure from M horizontally the distance Md = 60 ft.; the point d is directly above D′. Place levelling-staves at M and d, and observe the difference of the readings at M and d, which, in this base, is 7 ft.; whence $7 \times 1\frac{1}{2} = 10\frac{1}{2}$ ft. = approximate distance dD, and Md + dD = D = 60 + $10\frac{1}{2} = 70\frac{1}{2}$ ft. Now place a levelling-staff D, and observe the reading, which is found to be 7·8 ft. greater than that at M, or 0·8 ft. greater than that at d; whence $0{\cdot}8 \times 1\frac{1}{2} = 1{\cdot}2$ ft., and consequently $70\frac{1}{2} \times 1{\cdot}2 = 71{\cdot}7$ ft., which is a still nearer approximation to the true distance MD or Mq, it being measured horizontally. The operation for finding Mc, or MC measured horizontally, is the same as the preceding, excepting that the product of the staff-readings is subtracted from the estimated half-width, &c. In this manner the distance Mc = horizontal distance MC is found to be 52·6 feet.

NOTE 1. The widths RC, DS, of the side-fences, must be added to the above results for the whole width.

2. If the difference of the staff-readings at M and d be very large, it will require three or four approximations similar to those given in the preceding example to find the true corrected half-width.

Construction.—Take the levels of the several undulations of the surface CMD, making C′MD′ the *datum*-line, and draw the cross-section ABDMC by the methods already given.

By reversing the cross-section ABDMC, its application to an embankment is obvious, observing also to reverse the distances MC, MD, as previously noticed.

3. The truth of the method of approximation used in this example is too obvious to require a demonstration.

LEVEL-BOOK.

	No. of Stump.	Depth of Cuttings or Embankments.	Computed Half-width.	Corrected Half-widths for Edge of Cutting or Foot of Embankment.		Whole Widths including Fences, each Nine Feet in Width.
				North.	South.	
		Feet.	Feet.	Feet.	Feet.	Feet.
Cutting.	216	30·00	60·00	52·60	71·70	142·30
Cutting.	217	3·00	19·50	19·44	36·11	73·55
Cutting.	218	28·00	57·00	57·95	56·08	133·55
Cutting.	219	19·68	44·42	44·42	44·42	106·82
Cutting.	220	20·00	45·00	32·60	83·33	133 93
Embankt.	221	16·08	39·12	39·72	59·12	116·84
Embankt.	222	30·00	60·00	54·24	68·05	140·29
Embankt.	223	32·18	63·27	63·00	63·62	144·62

NOTE.—The depths of cuttings or embankments in the 2d column of the preceding Level-Book are found by calculation, or by carefully measuring them from the section by the vertical scale, but the latter method is not sufficiently correct. The computed half-widths in the 3d column are found by Problems I. and II. The corrected half-widths in columns 4th and 5th, by the five preceding Problems, according to the nature of the cuttings or embankments.

PROBLEM VI.

To find the quantity of land required for a projected railway.

CASE I.

In preparing the preliminary estimates for a projected railway, the quantity of land required for the purpose is usually found without paying any regard to the lateral inclination of the ground, by taking a considerable length of the section at once, especially if the surface thereof have a regular rise or fall, and by measuring the depth of the ends of such length with the vertical scale.

RULE.—Find the surface-widths, fences included, at each end of the given length, by Problems I. and II., add them together, multiply the sum by the length in chains, and divide the product by 1320 for the area in acres.

EX. Let the length of the sectional surface be 18 chains, and the depths at the ends 22 and 38 feet, the bottom-width of the railway 33 feet, and the ratio of the slopes $1\frac{1}{2}$ to 1; required the area of the surface, the width of the side-fences being 9 feet each.

By Prob. II. $w + 2ar + 2f = 33 + (3 \times 22) + (2 \times 9) = 117$

$$w + 2br + 2f = 33 + (3 \times 38) + (2 \times 9) = \underline{165}$$

$$\begin{array}{r} 282 \\ 18 \\ 1320 \left\{ \begin{array}{l|l} 12 & \overline{507 \cdot 6} \\ 11 & \overline{42 \cdot 3} \end{array} \right. \\ \hline 3 \cdot 84545 \end{array} \quad \begin{array}{l} \text{A. R. P.} \\ = 3\ 3\ 15 \end{array}$$

or by putting l = given length, and taking half the sum of the widths, there will result

$$\frac{(w + \overline{ra + b} + 2f)l}{660} = (33 + 1\tfrac{1}{2} \times \overline{22 + 38} + 2 \times 9)\frac{18}{660} = 3 \cdot 84545 \text{ acres.}$$

Case II.

When the exact quantity of land for the railway is required.

Rule.—Take the whole widths at the end of every chain, from the 6th column of the Level-Book, for the several widths; add continually together the first and last widths, and twice the sum of all the intermediate widths, and divide the whole sum by 1320 for the area in acres.

Ex. Required the area corresponding to the several widths in the Level-Book at the end of Problem V.

$$\begin{array}{rl} 73 \cdot 55 & \\ 133 \cdot 03 & \\ 106 \cdot 82 & \\ 133 \cdot 93 & \\ 116 \cdot 84 & \\ \underline{140 \cdot 29} & \\ 704 \cdot 46 & \\ \underline{2} & \\ 1408 \cdot 92 & = \text{twice sum of intermediate widths} \\ 142 \cdot 30 & = \text{first width} \\ 144 \cdot 62 & = \text{last width} \\ 1320 \left\{ \begin{array}{l|l} 12 & \overline{169 \cdot 584} \\ 11 & \overline{14 \cdot 132} \end{array} \right. & \\ \hline 1 \cdot 28473 & = 1\text{A. } 1\text{R. } 5\tfrac{1}{2}\text{P.} = \text{area required.} \end{array}$$

Note.—It is very common in practice to find the areas of the quantities of land required from the several proprietors, by actual measurement from the two chain maps, made for the use of the contractors, after the several widths have been laid down thereon: copies being taken, at the same time, from the maps, on tracing-paper, showing the position and quantity of land required from each proprietor.

ON RAILWAY CUTTINGS IN GENERAL, AND TABLES FOR FINDING THEIR CONTENTS.

In preparing the preliminary estimates for a railway, the contents of the cuttings are usually found by tables for the purpose, the surface of the ground in the several cross-sections being assumed to be on a level with the centre of the line. But when power has been granted for constructing the line, the cross-sections are carefully taken at the end of every prominent variation of the surface of the ground, or, if consistent with accuracy, at the end of every one, two, or three chains in length; and the several cross-sections are then plotted on a large scale, which may be done by the methods given in the preceding Problems. Their areas may then be found by actual measurement; or they may be reduced, where the surface of the ground is uneven, to horizontal sections. The contents are found, either by taking the mean of every two succeeding sections, *which method is very erroneous where the areas of the sections differ greatly*, or by finding the contents from the tables, by using the mean depths of the several sections, *which method is correct; but the mean depth used in this method cannot be accurately found in many cases without considerable calculation.* Some use a mean of the mean depths as the bases for a mean area, *which method is also very inaccurate, especially where the areas of the extreme sections differ greatly.* The magnitude of the errors in both cases will be pointed out in the investigations at the end of these Problems.

On Earthwork Tables.

Tables for this purpose have been published by Sir John M'Neill, Mr Bidder, Mr Bashforth, Messrs Sibley and Rutherford, and others; all of which are well adapted for finding the contents of cuttings, assuming the surface of the ground to be laterally level with respect to the direction of the cutting. But none of these tables are accompanied with directions for finding the contents from sectional areas, *i.e.*, from the areas of working drawings, excepting Mr Bashforth's tables; *but his mathematical investigation of the rule for using them in finding the contents from working drawings, where the surface of the ground is laterally sloping or uneven, is founded on a false assumption, and therefore his results are erroneous, and especially so where the sectional areas differ considerably.* (See the investigations at the end of these Problems.)

THE GENERAL EARTHWORK TABLE.

(At the end of the book.)

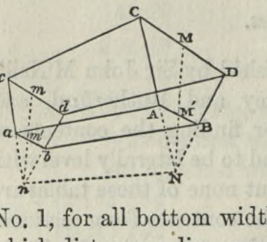
This Table, with the help of the Auxiliary Earthwork Tables, Nos. 1 and 2, on the same sheet, possesses the advantage of being general for all varieties of slopes and bottom widths in common use, as well as for decimal parts of feet in the depths. It may also, with a very trifling preliminary calculation, be made to extend to every variety of bottom width and ratio of slope that can occur, if even the slopes of the two sides differ in the same cutting; and with the help of a table of square roots it will apply, with all attainable mathematical accuracy, to cuttings where the surface of the ground is uneven. The investigation of the method of forming the Tables and using them, will be given at the end of these Problems. The contents in the General Table, and those in Table No. 2, are calculated to the nearest unit for one chain in length, and checked by differences; the side-slopes being assumed to be extended till they intersect. The auxiliary Table No. 1 gives the depths of the intersections of the side slopes below the balance-line, and the corresponding number of cubic yards to be deducted from the contents for each chain in length.

PROBLEM VII.

CASE I.

To find the contents of cuttings by the General Earthwork Table, and the Auxiliary Table No. 1, at the end of the book.

Let ABbdcC be a cutting, AB $= ab =$ bottom width on the formation level, MM′ and mm' the perpendicular depths at the middle of the two ends of the cutting; AC, BD, ac, bd, the side-slopes, which, being prolonged two and two will meet at the points N and n; also MM′ and mm', being prolonged, will meet at the same points. The distance M′N $= m'n$ in feet and decimals is given in the Auxiliary Earthwork Table



No. 1, for all bottom widths and ratios of slopes in common use, at which distance a line must be ruled on the section, parallel to the balance-line, or at the same distance + 2 feet from the line of the rails, in which latter case the balance-line need not be drawn. From the line thus ruled, the depths of the cutting must be measured to adapt them to the General Earthwork Table; or a mark might be made on the vertical scale with Indian ink (which is easily washed off) at the same distance, which mark might then be applied to the line of the rails in measuring off the depths. For measuring the depths of embankments, the line must be ruled at the same distance—2 feet above the line of the rails. When the several quantities of a cutting or embankment have been taken from the Table, and their sum multiplied by the ratio of the slopes, the cubic yards to be deducted for each chain in length, for the particular bottom width and ratio of slopes, must be taken from Table No. 1, and multiplied by the whole length of the cutting, and the product, being subtracted from the result obtained from the General Table, will give the content of the cutting in cubic yards.

The method of using the Tables will best appear from the following examples.

Ex. 1. Let the several depths of a railway cutting to the intersection of the slopes, at the end of every chain, be as in the annexed table, the bottom width 30 feet, and the ratio of the slopes $1\frac{1}{2}$ to 1; required the content of the cutting in cubic yards.

NOTE.—By the Table No. 1 the depth to be added to the depths of the cutting, for bottom width 30 ft. and ratio of slopes $1\frac{1}{2}$ to 1, is 10 ft., therefore the line from which the depths in the annexed table are measured is $10+2=12$ ft. below the line of the rails. The corresponding number of cubic yards to be subtracted is carried to two places of decimals, or, if the nearest whole number had been taken, the quantity would have been in excess or defect by several cubic yards, when the cutting is of a considerable length.

Dist. in Chains.	Depths	Qts. per Table.
0	10	
1·00	29	1003
2·00	32	2276
3·00	33	2582
4·00	39	3175
5·00	35	3350
6·00	10	1365

For slope 1 to 1 . .	13751
" $\frac{1}{2}$ to 1 . .	6875·5
" $1\frac{1}{2}$ to 1 . .	20626·5
Subtract $366·67 \times 6$	= 2200
Content in cubic yds.	=18426·5

Ex. 2. Let the several depths to the intersection of the slopes and their distances be as in the annexed table, the bottom width 30 feet, and the ratio of the slopes $1\frac{1}{2}$ to 1; required the content of the cutting.

NOTE.—When any of the distances is greater or less than 1 chain, the corresponding quantity must be multiplied by that distance; as in the cases of the distances between the depths 45 and 50, and between 30 and 10, the former distance being 3 chains and the latter 60 links.

Dist. in Chains.	Depths	Dist. greater than 1 Chain.	Qts. per Table.
0	10		
1·00	20		570
2·00	16		795
3·00	25		1044
4·00	32		1996
5·00	39		3091
6·00	45		4319
9·00	50	3 × 5520	16560
10·00	40		4971
11·00	30		3015
11·60	10	·60 × 1059	635·4

For slopes 1 to 1	36996·4
" $\frac{1}{2}$ to 1	18498·2
" $1\frac{1}{2}$ to 1	55494·6
$366·67 \times 11·60$ . . =	4253·4
Content in cubic yards . .	51241·2

Ex. 3. The depth of a cutting to the intersection of the slopes and their distances in feet are as in the annexed table; the bottom width is 36 feet, and the ratio of the slopes 2 to 1; required the content of the cutting in cubic yards.

NOTE.—When the distances are given in feet, the quantities from the General Table must be multiplied by their respective distances; also the quantity from Table No. 1 must be multiplied by the whole distance, and the final result divided by 66, as in the annexed example. See Demonstration at the end of these Problems.

Dist. in Feet.	Depths	Qts. × by Length.	Products.
0	39		
100	61	6210 × 100	621000
188	50	7554 × 88	664752
178	37	4660 × 90	419400
For slopes 1 to 1			1705152
			2
,, 2 to 1			3410304
		396 × 278 =	110088
		66)	3300216
Content in cubic yards . .			50003

CASE II.

To find the contents of cuttings by the Tables, when the depths are given in feet and decimals of feet.

RULE.—Let any two succeeding depths be denoted by a and b, and let the decimal parts of the depths be respectively denoted by a' and b'; find the quantity corresponding to a and b from the General Table, as in the former case; then

$\frac{2a+b}{10}$, or its nearest whole number, and the decimal a' will show the number of cubic yards to be added in Table No. 2, and

$\frac{2b+a}{10}$, or its nearest whole number, and the decimal b' will show the cubic yards to be added in the same Table.

Ex. 1. Let the depths to the intersection of the slopes be 61·6 and 39·4 feet, their distance 1 chain, the bottom width 36 feet, and the ratio of slopes 2 to 1; required the content of the cutting in cubic yards.

$$\text{Put } 61 = a, \text{ its decimal } \cdot 6 = a'$$
$$39 = b, \text{ its decimal } \cdot 4 = b'.$$

Then the depths a and b, per General Table, give .	6210
$\frac{2a+b}{10} = 16{\cdot}1$, or its nearest whole No. 16 and (a') ·6 per Table No. 2	78
$\frac{2b+a}{10} = 13{\cdot}9$, or its nearest whole No. 14 and (b) ·4 per Table No. 2	46
For slopes 1 to 1 . . .	6334
	2
For slopes 2 to 1 . . .	12668
By Table No. 1, for bottom width 36 feet and ratio of slopes 2 to 1, No. of cubic yds. to be deducted .	396
Content in cubic yards	12272

NOTE 1. Care must be taken to use the decimal a' with $\frac{2a+}{10}$, and the decimal b' with $\frac{2b+a}{10}$, in finding the quantities in Table No. 2, as a mistake might easily be made in this matter, which would lead to an erroneous result.

2. If any two succeeding depths be nearly equal, and the sum of their decimal parts be together equal to 1 foot, or nearly so, by adding 1 foot to the lesser depth, and rejecting the decimal in the larger depth, and using the depths thus altered as whole numbers, a result sufficiently correct will be obtained; as in the following example.

EX. 2. Let the depths of a cutting be 50·29 and 48·7 feet, their distance 1 chain, the bottom width 33 feet, and the ratio of the slopes $1\frac{1}{2}$ to 1; required the content.

By adding the decimal ·29, in the larger depth, to 48·7, the depths may be called 50 and 49, for which, by the General Table, the content is . . . . .

	5990 for 1 to 1
	2995 for $\frac{1}{2}$ to 1
	8985 for $1\frac{1}{2}$ to 1
From which deduct, from Table No. 1 .	443·67
	8541·33 cubic yards.

NOTE 1. The results of all the examples in the two preceding cases only differ by a very small fraction of a cubic yard from the true contents obtained by calculation from formula (1), page 456. The method of finding the contents to decimals or tenths of a foot in the depths has been particularly discussed, on account of its utility in finding the contents for actual contract-work from the working drawings, in which, as great accuracy is required, the contents should be found to two places of decimals, or to $\frac{1}{100}$ths of a foot in the depths, as in Case II. of the following Problem.

2. When one or both of the given depths exceed the limits of the table, find the content corresponding to half the two depths, and four times the result will be the content required.

PROBLEM VIII.

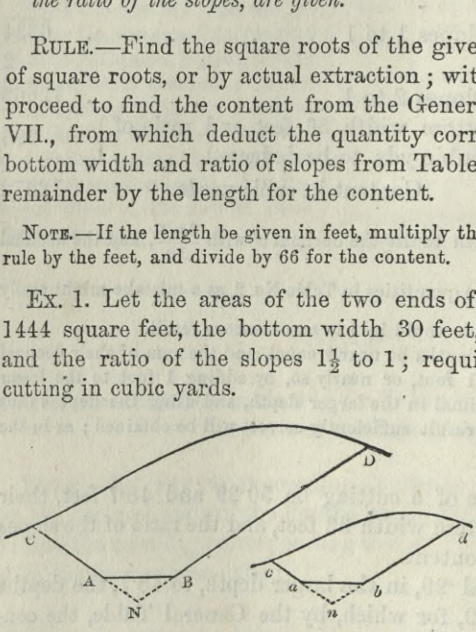
CASE I.

To find the content of a cutting between two cross-sections, the areas of which, in the intersection of the slopes, the length, the bottom width, and the ratio of the slopes, are given.

RULE.—Find the square roots of the given areas either by a table of square roots, or by actual extraction; with these roots, as depths, proceed to find the content from the General Table, as in Problem VII., from which deduct the quantity corresponding to the given bottom width and ratio of slopes from Table No. 1, and multiply the remainder by the length for the content.

NOTE.—If the length be given in feet, multiply the content found by the above rule by the feet, and divide by 66 for the content.

EX. 1. Let the areas of the two ends of a cutting be 5141 and 1444 square feet, the bottom width 30 feet, the length 1·60 chains and the ratio of the slopes $1\frac{1}{2}$ to 1; required the content of the cutting in cubic yards.



Let the annexed figures be the two cross-sections, the side slopes CA, DB; *ca*, *db*, being prolonged till they meet at N and *n*, the area CND = 5141, the area *cnd* = 1444; and since the bottom width AB = *ab*, and the ratio of the slopes are given, the solidity of the prism, the ends of which are ABN = *abn*, is given in Table No. 1, and is to be deducted from the content found by the General Table as in Problem VII.

The square roots of 5141 and 1444 are . .	71·7 and 38
By General Table, for 71 and 38 . . .	7483
By Table No. 2, for $\frac{(71 \times 2) + 38}{10} = 18$ and ·7 .	103
Content to intersection of slopes . . .	7586
By Table No. 1, for bottom width 30, and slopes $1\frac{1}{2}$ to 1	366·67.
Content for 1 chain in length . . .	7220·33
	1·60
Content for 1·60 chains in length . . .	11552·528
	cubic yards.

CASE II.

When great accuracy is required, especially in the measurement of contract work, the second decimals, or $\frac{1}{100}$ths, of feet, must be considered in the calculation by taking for them $\frac{1}{10}$th of their corresponding quantities in Table No. 2.

EX. Let the areas of the several cross-sections to the intersection of the slopes, and their distances, be as in the annexed table, the bottom width 36 feet, and the ratio of the slopes $1\frac{1}{2}$ to 1; required the content of the cutting in cubic yards.

Dist in Chains.	Areas in Sq. ft.	$\sqrt{\text{Areas.}}$	Product.	Contents.
0	2161	46·59		
1·00	3759	61·31	—	7147
2·00	5141	71·7	—	10832
4·20	4100	64·03	11271 × 2·20	24796
6·00	4221	64·97	10170 × 1·80	18306
8·00	3136	56·	8959 × 2·00	17918
9·00	2727	52·22	—	7160
				86159
From Table No. 1 for bottom width 36 ft. and slopes $1\frac{1}{2}$ to 1, 528 × 9·00				4752
Content in cubic yards				81407

PROBLEM IX.

To find the content of a railway cutting when the slopes of the two sides are different.

CASE I.

When the depths are given.

RULE.—Find the content by the tables, as in the two preceding Problems, multiply the result by half the sum of the side slopes, and from the product deduct half the sum of the cubic yards, corresponding to the given bottom width and the ratios of the slopes of the two sides, multiplied by the whole length of the cutting, and the remainder is the content in cubic yards.

EX. Let the depths and their distances be as in the annexed table, the bottom width 36 feet, and the ratios of the slopes of the two sides 1 to 1 and $1\frac{1}{2}$ to 1; required the content of the cutting.

NOTE.—Here the half sum of the first terms of the ratios of the slopes, viz., $\frac{1}{2}(1 + 1\frac{1}{2}) = 1\frac{1}{4}$ is the number by which the sum of the quantities, per tables, is to be multiplied, or $\frac{1}{4}$ added, as in the operation: and half the sum of the cubic yards, for the given bottom width and ratios of the two slopes, is multiplied by the whole length of the cutting, and deducted, agreeably to the rule.

Dist. in Chains.	Depths.	Qts. per Tables.
0	61	
1·00	52	7820
2·00	44·46	5473
For slopes 1 to 1		13293
,, $\frac{1}{4}$ to 1		3323·25
		16616·25
$\frac{1}{2}(528 + 792) \times 2$. . . .		1320
Content		15296·25

CASE II.

When the sectional areas are given.

RULE.—Find the content from the tables as in Problem VIII. and deduct for the quantity below the cutting, as in Case I. of this Problem.

NOTE.—This rule seems too obvious to require an example.

PROBLEM X.

To apply the General Table and Table No. 2 to such bottom widths and ratios of slopes as are not found in Table No. 1.

Put w = bottom width, and r to 1 the ratio of slopes.

Then $\frac{w}{2r}$ = feet to be added to depth of cutting, or distance from bottom of cutting to the intersection of slopes.

And $\frac{11w^2}{18r}$ = cubic yards to be deducted for each chain in length.

EX. If the bottom width be 28 feet, and the ratio of slopes $1\frac{1}{4}$ to 1; then $w \div 2r = 28 \div (2 \times 1\frac{1}{4}) = 11\frac{1}{5} = 11{\cdot}2$ feet = distance from bottom of cutting to intersection of slopes, and $11w^2 \div 18r = 11 \times 28^2 \div 18 \times 1\frac{1}{4} = 383{\cdot}29$ cubic yards to be deducted for each chain in length.

PROBLEM XI.

To find the content of the cutting of a tunnel.

1. *When the length is given in yards, and the width and height in feet.*

RULE.—Multiply continually together the length, width, and mean height, and divide the product by 9.

2. *When the length is given in chains, and the width and height in feet.*

RULE.—Multiply the continued product of the length, width, and mean height by 22, and divide by 9.

EX. The length of cutting of a tunnel is 1053 yards, its width 30 feet, and mean height 32 feet; required the content.

$$\frac{1053 \times 30 \times 32}{9} = 11232 \text{ cubic yards.}$$

The following example shows the magnitude of the errors of several methods, sometimes used, for calculating the contents of cuttings.

EX. The areas of the two cross-sections of a cutting, to the intersection of the slopes, are 10324 and 400 square feet, their distance 4 chains, the bottom width 36 feet, and the ratio of the slopes 1 to 1; required the content of the cutting by the General Table, &c., and by the erroneous methods sometimes used.

NOTE.—The great difference of the sectional areas in this example shows very prominently the errors of several methods of finding the contents of cuttings: at the same time it is proper to remark that similar differences in the sectional areas very frequently occur in practice, and that the erroneous methods give the contents very near the truth *only* when these areas are nearly equal.

$\left.\begin{array}{l}\sqrt{10324} = 101{\cdot}607 \\ \sqrt{400} = \ \ 20\end{array}\right\}$ Content by General Tab., &c. 10394

Deduction from Tab. No. 1 . . .	792
Content for 1 chain in length . . .	9602
	4
True content for 4 chains in length . .	38408 cub. yds.

By Mr Bashforth's Method.

(1.) By Table No. 1 the depth below the formation-level to the meeting of the slopes is 18 ft.; hence the area of the triangle below it is $\frac{1}{2}$ (36 × 18) = 324 square feet, which deducted from the given sectional areas, gives 10000 and 76 sq. ft. for the sectional areas, as used by Mr Bashforth.

Whence $\left.\begin{array}{l}\sqrt{10000} = 100 \\ \sqrt{76} = \ \ 8{\cdot}718\end{array}\right\}$ Content by Gen. Tab., &c. 8920·4

	4
Content for 4 chains in length . . .	35681·6 c. y.
Error in defect, compared with the true content given above, *being above* $7\frac{1}{2}$ *per cent., by Mr B.'s method.*	2726·4

(2.) *By taking a mean of the areas for a mean section.*

$\frac{1}{2}$ (10000 + 76) × $\frac{22}{4}$ × 4 = 49260·4 cubic yards; *which exceeds the true content by* 10852·4 *cubic yards, being above* 22 *per cent. in excess.*

(3.) *By taking a mean depth the error in defect is just half of the preceding error, or nearly* 11 *per cent.*

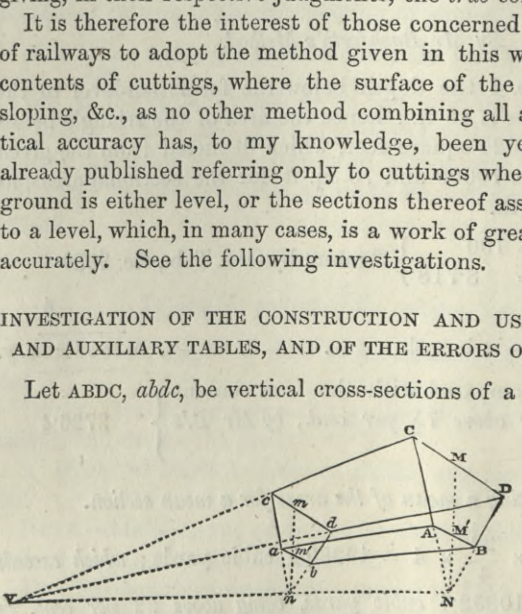
OBSERVATIONS IMPORTANT TO THOSE CONCERNED IN THE CONSTRUCTION OF RAILWAYS, ON THE ERRONEOUS METHODS OF CALCULATING THE CONTENTS OF CUTTINGS, WHERE THE SURFACE OF THE GROUND IS UNEVEN; ALSO ON THE METHOD GIVEN IN THIS WORK.

The magnitude of the errors of the methods (1), (2), and (3), in the last example, for calculating the contents of cuttings, where the surface of the ground is uneven, from sectional areas, is strikingly apparent; and since many tables are not accompanied by any directions for their practical application in this particular case, one or other of the last two of these defective methods is still frequently used by engineers and contractors; thus causing continual disputes concerning the contents in consequence of the different parties using irreconcilable methods, some preferring one and some the other, as giving, in their respective judgments, the *true* content.

It is therefore the interest of those concerned in the construction of railways to adopt the method given in this work for finding the contents of cuttings, where the surface of the ground is laterally sloping, &c., as no other method combining all attainable mathematical accuracy has, to my knowledge, been yet published: those already published referring only to cuttings where the surface of the ground is either level, or the sections thereof assumed to be reduced to a level, which, in many cases, is a work of great labour to perform accurately. See the following investigations.

INVESTIGATION OF THE CONSTRUCTION AND USE OF THE GENERAL AND AUXILIARY TABLES, AND OF THE ERRORS OF OTHER METHODS.

Let ABDC, *abdc*, be vertical cross-sections of a railway cutting, the surface widths CD, *cd*, being horizontal, AC*ca*, BD*db*, the side slopes, AB $= ab$ the bottom width. Prolong the planes of the side slopes downwards and to the left till they meet in N*n*V, and also meet the prolongation of the plane CD*dc* in *c*V, *d*V. Because AB $= ab$, the cross-sections are ⊥ to VN. Let fall the ⊥ NM, *nm* on CD, *cd* respectively, bisecting them in M and *m*, and also bisecting AB, *ab* in M′ and *m*′. Put $\text{MN} = a$, $mn = b$, $\text{N}n = \text{A}a = \text{B}b = l$, $\text{AB} = ab = w$, all in feet, and the ratio of the slopes, *i.e.*, CM : MN : : r : 1. Then



Δ CDN $= a^2r$, Δ $cdn = b^2r$. By similar figures $a : b :: \text{NV} : \text{NV} - l$, whence $\text{NV} = \dfrac{al}{a-b}$,

Pyramid VCND $= \frac{1}{3}$ NV $\times a^2r$.

. . . . . Vcdn $= \frac{1}{3}(\text{NV} - l)b^2r$; whence

Frustum CDN$ncd = \frac{1}{3}$ NV $\times a^2r - \frac{1}{3}$ (NV $- l$) $b^2r = \frac{1}{3}r$ $\overline{(a^2 - b^2\text{NV} + b^2l)} =$ (by substituting the value of NV)

$\frac{1}{3}rl(a^2 + ab + b^2)$ cubic feet

$= \dfrac{rl}{81}(a^2 + ab + b^2)$ cubic yards . . . . (1)

In the General Table $r = 1$, and $l = 1$ chain $= 66$ feet; and $\therefore$ the solidity

$$\text{S} = \frac{22}{27}(a^2 + ab + b^2) \text{ cubic yards} \quad . \quad . \quad . \quad (2)$$

wherein a and b have all integral values from 0 to 72.

Auxiliary Table No. 1.

$$r : 1 :: \tfrac{1}{2}\, w(= \text{AM}') : \text{NM}' = \frac{w}{2r} \quad . \quad . \quad . \quad . \quad (3)$$

whence Δ ABN $= \Delta$ $abn = \frac{1}{2}$ AB $\times$ NM$' = \dfrac{w^2}{4r}$ . . . . (4)

$$\therefore \text{Prism ABN}nab = \frac{w^2l}{4r} \text{ sq. ft.} = \frac{w^2l}{108r} = \text{cubic yds.} \quad . \quad (5)$$

From (3) and (5) the depths to be added, and the cubic yards to be deducted in Table No. 1, are calculated, by taking $l = 66$ feet, and w and r all the values most commonly used in practice.

$\therefore \dfrac{w^2l}{108r} = \dfrac{11w^2}{18r} =$ cubic yards to be deducted, Table No. 1. . (6)

$\therefore \dfrac{11w^2\text{L}}{18r} =$ cubic yards to be deducted for the length L.

Let $\Sigma =$ sum of all the solidities, S, S$'$, &c. (per General Table), the length of which is L, and $\varkappa = \dfrac{\text{H}w^2}{18r}$ from (6), then $\Sigma r =$ sum of solidities for slopes r to 1, from which deduct $\varkappa \times$ L, and there results

$\Sigma r - \varkappa\text{L} =$ cubic yards in the whole cutting . . . . (7)

Whence the method of finding the contents of cutting in Problem VII.

Cor 1. The content of a cutting having only two given depths is $(\text{S}r - \varkappa)\text{L}$ cubic yards. . . . . (8)

Cor. 2. If the ratio of the slopes of the two sides of a cutting be r to 1 and ρ to 1, and k, $\varkappa$ the corresponding cubic yards to be deducted. (Table No. 1.)

$$\tfrac{1}{2}\{\Sigma\,(r+\rho) - \text{L}(k+\varkappa)\} \text{ cubic yards.}$$

Let in (2) a and b, hitherto supposed to be integral numbers, have the increments or decimals α and β, so that a and b become respectively $a+\alpha$ and $b+\beta$, which being substituted for a and b, neglecting the squares and product of α and β, as being comparatively small, there results:

$$\frac{22}{27}(a^2+ab+b^2) + \frac{22}{27}\,(2a+b)\,\alpha + \frac{22}{27}(2b+a)\,\beta \quad . \quad . \quad . \quad (9)$$

By subtracting (2) from (9), there results the sum of

$$\frac{22}{27}\,(2a+b)\alpha \text{ and } \frac{22}{27}(2b+a)\beta, \text{ or}$$

$$\frac{22}{27}\left(\frac{2a\;+\;b}{10}\right)10\alpha \text{ and } \frac{22}{27}\left(\frac{2b\;+\;a}{10}\right)10\beta \quad . \quad . \quad . \quad (10)$$

which are the increments of the formula (2) arising from the addition of the decimal parts α and β to the depths a and b, from which Table No. 2 has been calculated, the quantities $2a+b$, and $2b+a$ being divided by 10, to prevent a too great extension of the Table; and the corresponding factors α and β multiplied by the same number, that the product might retain their original value, the decimal points being still affixed to the values of α and β in the horizontal line at the top of the Table.

In consulting Table No. 2 the nearest whole numbers to $\frac{2a+b}{10}$ and $\frac{2b+a}{10}$ are taken as the small errors, thus resulting, will usually balance one another in a long cutting, and can never in any case amount to much.

Let the plane VCD revolve on the side VC, so that the lines CD, cd may incline from their hitherto assumed horizontal position, thus making the side slopes unequal; and let the area CDN = A, and the area cdn = B, the other symbols being the same as before; then

$$\sqrt{\text{A}} : \sqrt{\text{B}} :: \text{VN} : \text{VN} - l, \text{ whence VN} = \frac{l\,\sqrt{\text{A}}}{\sqrt{\text{A}} - \sqrt{\text{B}}}.$$

Pyramid VCDN = $\frac{1}{3}$ VN × A, Pyramid Vcdn = $\frac{1}{3}$ (VN − l) × B, whence the frustum CDNncd = S = $\frac{1}{3}$ VN × A − $\frac{1}{3}$ (VN − l) × B = $\frac{1}{3}$ ($\overline{\text{A} - \text{B} \times \text{VN} + \text{B}l}$),

or S = $\frac{1}{3}$ l (A + B + $\sqrt{\text{A} + \text{B}}$), in cubic feet, and taking l = 66 feet

$$= \frac{22}{27} (\text{A} + \text{B} + \sqrt{\text{A}} \times \sqrt{\text{B}}) \text{ cubic yards} \quad . \quad . \quad . \quad . \quad (11)$$

Therefore the Table must be consulted for depths $\sqrt{\text{A}}$ and $\sqrt{\text{B}}$, the ratio of the slopes being included; for if a and b be the mean depths for the areas A and B respectively, the slopes being r to 1, then a^2r = A, b^2r = B and $\frac{22}{27}$ (A + B + $\sqrt{\text{A}}$ × $\sqrt{\text{B}}$) = $\frac{22r}{27}$ (a^2 + ab + b^2).

Therefore in reality the table is consulted for the depths $a\sqrt{r} = \sqrt{\text{A}}$, and $b\sqrt{r} = \sqrt{\text{B}}$.

If Σ' = sum of all the solidities S, S′, &c., for depths $\sqrt{\text{A}}$ and $\sqrt{\text{B}}$, &c., and length L; then by subtracting $\varkappa$L, as in (7), there results $\Sigma' - \varkappa$L cubic yards in the whole cutting . . . . . (12)

Whence the Rule in Problem VIII.

Cor. 1. The content of a cutting, having only two given sectional areas, is (S − $\varkappa$) L cubic yards . . . . (13).

Cor. 2. Formulæ (11), (12), and (13) will evidently hold, if, instead of the straight lines CD, cd, being the surface edges of the cross-sections CDN, cdn, of the cutting, these lines be the chords of similar curves forming the surface edges of the cross-sections, and the similarly situated points in the two curves be joined by right lines (which prolonged will all meet in the vertex V), thus forming the surface of the cutting, a necessary condition in taking cross-sections; or the small inequalities of the earth's surface between the cross-sections must be balanced, as nearly as can be judged by the eye, so that the surface may fulfil this condition, in order that all attainable mathematical accuracy may be arrived at in finding the contents.

THE ERRORS OF OTHER METHODS OF FINDING THE CONTENTS OF RAILWAY CUTTINGS.

Mr Bashforth, in his investigations for finding the contents of railway cuttings, where the surface of the ground is level, or assumed to be so, includes the prism ABNnba, and afterwards deducts it, which is mathematically accurate. But in finding the contents from sectional areas, where the surface of the ground is laterally sloping, or uneven, he altogether leaves out the above-named prism; for he says (Art. 15, p. 11 of his work), "The only way to proceed in such a case is—find the areas of the cross-sections in square feet, take out of Barlow's Tables the square root of each, and treat these square roots precisely as if they had been measured heights, excepting that there will be nothing to be deducted for the prism, and no multiplication for the slopes, these having been already accounted for in finding the areas."

Let A' and B' be the sectional areas CDBA, *cdba* respectively, as used by Mr B., and $l = Nn =$ length of the cutting (fig. page 452), then according to his method, the content of the cutting is

$$\frac{l}{81}(A' + \sqrt{A' \times B'} + B') \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

which is erroneous in every case, except where the areas represented by A' and B' are similar and equal. For when the equal areas ABN, *abn* are omitted, the areas represented by A' and B' are dissimilar, and $\therefore$ the proportion $\sqrt{A'} : \sqrt{B'} :: NV : nV$, on which Mr B.'s rule is founded, does not hold, it being true only in respect to the areas CDN, *cdn*, because the solids VCDN, V*cdn* are similar. Moreover, if the plane VCD revolve on the side CD so as to bring the lines *cd*, *ab*, to coincidence, or almost to coincidence, in the points *b* and *d*, the dissimilarity of the planes CDBA, *cdba* will become more strikingly evident, whereas with the addition of the Δs, ABN, *abn*, they still remain similar.

Let $\Delta ABN = abn = \alpha$, then $A' + \alpha$ and $B' + \alpha$ are the sectional areas corresponding to A and B in formula (11) of my investigation, whence by that formula the true content, including the prism ABN*nba*, is

$$\frac{l}{81}(A' + B' + 2\alpha + \sqrt{\overline{A' + \alpha} \times \overline{B' + \alpha}}) \text{ cubic yards.}$$

But the solidity of the prism is $\frac{al}{27} = \frac{3al}{81}$ cubic yards.

Whence the content of the prismoid or cutting is

$$\frac{l}{81}(A' + B' + 2\alpha + \sqrt{\overline{A' + \alpha} \times \overline{B' + \alpha}} - 3\alpha), \text{ or}$$

$$\frac{l}{81}(A' + B' + \sqrt{\overline{A' + \alpha} \times \overline{B' + \alpha}} - \alpha),$$

from which subtract the content according to Mr B. (14), and there results.

$$\frac{l}{81}\sqrt{\overline{A' + \alpha} \times \overline{B' + \alpha}} - \sqrt{A' \times B'} - \alpha) = \text{error in defect.}$$

Let $A' = \frac{1}{4}\alpha$, and $B' = 36\alpha$, which is a case that may occur in practice, then

$$\frac{l}{81}\left(\sqrt{\overline{A' + \alpha} \times \overline{B' + \alpha}} - \sqrt{A' \times B'} - \alpha\right) = \frac{\alpha l}{81}\left(\tfrac{1}{2}\sqrt{185} - 3 - 1\right)$$

$= \frac{\alpha l}{81} \times 2{\cdot}8007 =$ error in defect by Mr B.'s method.

and $\frac{l}{81}(A' + \sqrt{A' \times B'} + B') = \frac{\alpha l}{81}(\frac{1}{4} + 3 + 36) = \frac{\alpha l}{81} \times 39\frac{1}{4} =$

whole content by Mr B.'s method.

$\therefore$ whole content : error in defect :: $\frac{\alpha l}{81} \times 39\frac{1}{4} : \frac{\alpha l}{81} \times 2{\cdot}8007$

$39\frac{1}{4} : \quad 2{\cdot}8007$

$100 \quad : 7\frac{1}{9}$, or $7\frac{1}{9}$ *per cent.*

the error in defect . . . . . (15)

See Ex. at the end of Problem X., where the error of Mr Bashforth's method is shown by giving actual areas.

NOTE.—The errors of this method are not so prominent where the sectional areas approach near to equality, as in the case of the Burnley cross-sections, in the Ex. Art. 20, page 13, of Mr B.'s work: his method, however, is erroneous to a greater or lesser extent, in every case, except where the sectional areas are equal and similar; for they cannot be similar without being equal, while the bottom width remains the same.

Mr Bashforth says, in defence of his method of finding the contents of cuttings from sectional areas, in the "Mechanics' Magazine" for Sept. 11, 1847, p. 249, "In the case of contract estimates, numerous cross-sections ought to be taken; and it matters little whether we start from the intersection of the slopes or the formation-level." *The above-noticed error* (15) *shows that it "matters" so much as* $7\frac{1}{9}$ *per cent. in defect.* Besides, there is no need of taking "numerous cross-sections," as Mr B. recommends, especially where the surface of the ground is laterally sloping like a geometrical plane, or curved like a conical surface, one or other of which cases very frequently occurs, so that cross-sections, taken at a considerable distance from one another to the intersection of the slopes, may be considered as similar, or so very nearly so as not to induce any important mathematical error, which conditions may be easily determined by the eye. Moreover, the expense and trouble of taking "numerous cross-sections," plotting them, and finding their areas, are very considerable; and, therefore, ought to be avoided, together with all erroneous methods, such as Mr B.'s, of finding the contents of cuttings, as his "numerous cross-sections" only tend to diminish the errors, without wholly getting rid of them.

The error of the method of finding contents by mean areas.

Let A and B be the areas of two cross-sections of a cutting to the intersection of the slopes, and l its length: then the mean area is $\frac{1}{2}$ (A + B), and the content in cubic yards is $\frac{1}{27} l \times \frac{1}{2}(A+B) = \frac{1}{81} l \times \frac{3}{2}(A+B)$; from which subtract the true content, equation (11), p. 455, and there results

$\frac{1}{81} l \left\{\frac{3}{2}(A + B) - (A + B + \sqrt{AB})\right\} = \frac{1}{162} l (\sqrt{A} - \sqrt{B}) =$ *error in excess.*

Which error is very great when the areas A and B differ considerably. See (2), p. 451.

The error of the method of finding the content by mean depth.

Let a and b be the depths of two cross-sections to the intersection of the slopes, the ratio of which is r to 1, and l = length of the cutting; then $\frac{1}{2}(a + b)$ = mean depth, $\frac{1}{4}r(a + b)^2$ = mean area, and the content in cubic yards $= \frac{1}{27} l \times \frac{1}{4}r (a + b)^2 = \frac{1}{81} rl \times \frac{3}{4} (a+b)^2$;

which subtract from equation (1), p. 411, and there results

$\frac{1}{81} rl \left\{a^2 + ab + b^2 - \frac{3}{4}(a + b)^2\right\} = \frac{1}{324} rl (a - b)^2$ = error in defect:

Which error is very considerable, when the depths a and b differ greatly.

NOTE.—The errors of these methods, as here shown, are the same as if the areas and depths had only extended to the formation-level, since a common quantity, *i.e.*, the prism below the formation-level, is here included, and afterwards excluded, by taking the differences.

SECTION V.

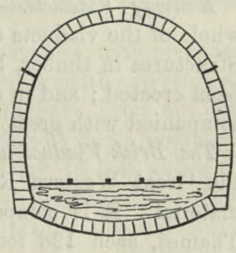
TUNNELLING.

(1.) Previous to setting out the earthwork of a tunnel, the levelling operation must be repeated with great care, and should also be checked by the method given in Art. (22), Section I., especially if the tunnel pass under a very high summit: for, if the section be incorrect, the gradient or gradients, on which the tunnel is formed, will not meet at the points shown thereon, and thus embarrass the mining operation.

(2.) If the tunnel is formed on one gradient, as BD, Plate XIII., the gradient must incline to one of the extremities of the tunnel, as at D, in order to discharge the water generated therein. Strong poles or masts must be firmly fixed on the surface, in the intended direction of the tunnel, of which one must be on the summit of the hill; at which place a temporary observatory is frequently erected, especially if the summit be a very high one, and the tunnel a very long one. Shafts must be sunk at the distance of four or five chains from one another, in the direction of the poles (and observatory, if there be one), in order to ventilate the tunnel, as well as to check the accuracy of the work as it proceeds. If the tunnel be a long one, it would be preferable, if convenient, to form it on two gradients, inclining to its opposite extremities to liberate the water, and thus to aid the mining operation, which is commonly commenced at both ends of the tunnel at the same time.

(3.) When it is necessary to have a curve in the direction of a part or of the whole of the tunnel, that direction must be carefully laid down on the surface, by the methods given in Section III., making allowance for acclivities and declivities, poles or masts being fixed therein, as pointed out in Art. (2), that the shafts may be sunk so as to meet the mining operations of the tunnel, as well as to check their accuracy in point of direction, and this will be the more especially necessary in the curved part of the tunnel.

(4.) The mining operation of the tunnel should commence when the depth of the cuttings at each end is about 60 feet. The width and depth of the excavation of a tunnel, on the narrow gauge, should be about 30 feet each, and must be dug 5 or 6 feet below the intended line of the rails, to give space for the inverted arch and the ballasting, excepting where the excavation is made through rock sufficiently hard to form the side-walls of the tunnel, in which case 22 or 24 feet in width, and about 26 feet in height, will be sufficient, the excavation in this case being terminated below by the balance-line, or formation-level. The depth and width of the excavation for a tunnel on the broad gauge must, in both cases, be proportionately larger.



The annexed figure is a cross-section of the masonry of a tunnel, which, of course, is such as is required where the tunnel is made through loose earth, only the arch above being required when made through hard rock.

NOTE.—The remarks at Articles (25) and (27), Section II., refer only to the projection of tunnels on Parliamentary maps.

A TABLE OF THE DIMENSIONS OF SEVERAL EXISTING TUNNELS.

Names of Railways and of Tunnels.		Length in Yards.	Width in Feet.	Height in Feet.	Shafts.
London and Birmingham	Primrose Hill	1120	22	22	5
	Weedon . .	418			
	Kilsby . . .	2398			10
Cromford and High Peak—Buxton		580	21	16	
Great Western—The Box Tunnel .		3227	35	29	13
Manchester and Leeds	Littleborough .	2869	24	$21\frac{1}{2}$	
North Midland	Claycross . .	1806	22	21	
	Melford . . .	836			
London and Brighton	Merstham	1780			
	Balcombe	1192			
Sheffield and Manchester	Five tunnels Total length	6245			
Chester and Holyhead	Three tunnels Total length	2130			
South-Western	Seven tunnels Total length	1998			

SECTION VI.

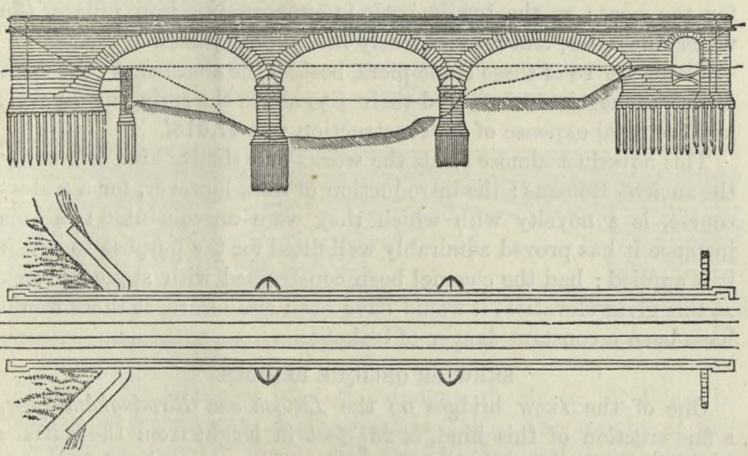
VIADUCTS, AQUEDUCTS, SKEW ARCHES, ETC.

What are here given on these subjects are necessarily only outlines, referring to some few existing structures, without entering into the details of specifications, working drawings, &c., which are foreign to the nature of this work, and properly belong to the department of the architect.

Railway Viaducts.—It would be impossible to enumerate the whole of the viaducts constructed since the introduction of railways. Structures in timber, brick, stone, and iron, of various designs, have been erected; and in some cases there is a novelty of principle accompanied with great boldness of execution.

The Brick Viaduct at Maidenhead, constructed by Mr. Brunel for the Great Western Railway, is one of the best examples in that material: it is composed of two elliptical arches spanning the Thames, each 128 feet, with a versed sine of $24\frac{1}{4}$ feet; the pier between the two arches is 30 feet in width. The arch in the middle is $5\frac{1}{4}$ feet in thickness, which gradually increases towards the abutments. Besides these two grand brick arches, on each bank of the river are four others; those on the abutments span 21 feet each, and the six others 28 feet each.

The Viaduct over the Ouse, near York, consists of three arches, each 66 feet span; the piers are 10 feet in thickness, and the width of



the arches $28\frac{1}{2}$ feet on the soffite, and their thickness at the keystone $3\frac{1}{2}$ feet, the voussoirs gradually increasing towards the springing.

This is a specimen of a stone viaduct combining great strength and elegance in its construction.

Aqueduct of Pont-y Cysyllte is 4 miles from Chirk, over the river Dee; its length is 1007 feet, and the river is 127 feet below the water-level of the canal carried over it.

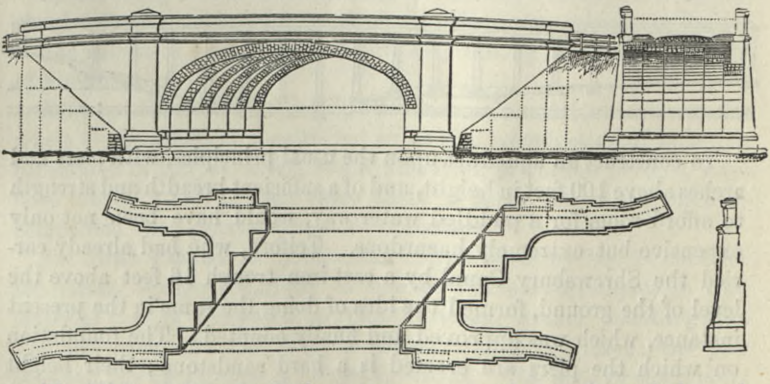
To construct an aqueduct upon the usual principles, with piers and arches above 100 feet in height, and of a sufficient breadth and strength to afford room for a puddled water-way, would have been not only expensive but extremely hazardous. Telford, who had already carried the Shrewsbury Canal by a cast-iron trough 16 feet above the level of the ground, formed the idea of doing the same in the present instance, which was approved and finally adopted. The foundation on which the piers are erected is a hard sandstone; their height above low water in the river is 121 feet: at the bottom they are 20 feet by 12, at the top 13 feet by $7\frac{1}{2}$. For a height of 70 feet above the foundations, they are built solid, and the remaining 50 feet hollow, the walls being only 2 feet in thickness, with one cross inner wall; by this means the centre of gravity of the piers is thrown lower, and the masonry economised. The width of the water-way is 12 feet; of which the towing-path covers 4½ feet, leaving 7½ feet for the boat: as the towing-path is supported by iron pillars, the water fluctuates, and recedes freely as the boat passes.

There are 18 of these stone piers, besides the abutments; the span of the arches is 45 feet, and their rise above the springing 7½ feet, and the total expense of its construction was 47,018*l*.

This aqueduct almost rivals the works of a similar kind left us by the ancient Romans: the introduction of iron, however, for a water-course, is a novelty with which they were unacquainted: in this instance it has proved admirably well fitted for the purpose to which it is applied: had the channel been constructed with stone or brick at this great elevation, it would have been less secure, as there would have been a constant danger of leakage.

SKEW OR OBLIQUE BRIDGES.

One of the skew bridges on the *London and Birmingham Line*, a fine erection of this kind, is 23½ feet in height from the surface of the road below to that of the rails, making an angle of 32° with the road: the direct span of the arch is 21 feet, and oblique span 40. The arch, which is 2½ feet thick, is the segment of a cylinder, the internal radius of which is 12½ feet, and the versed sine 5⅔ feet. The angle of which the coursing joints of the soffite cross the axis of the cylinder is 53° 25′, and the joints of the face of the arch all converge to a point 32½ feet below the axis of the cylinder, and 45 feet below the crown of the arch.

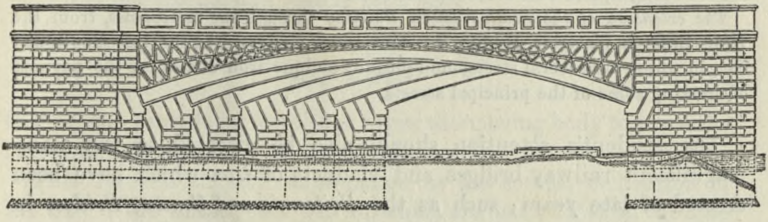


The Midland Counties Railway possesses another variety of skewed brick bridge, the span of which is 42½ feet, and versed sine 11 feet;

the arch consists of six ribs, 2 bricks in depth, and 4 feet in thickness, the total breadth of the bridge, measured at right angles to the face, being 24 feet.

Manchester and Birmingham Railway.—In Fairfield Street is a skew bridge, the oblique span of which is 128¾ feet, with a versed sine of 12 feet; the width from face to face is 31 feet.

Six ribs of iron abut on as many independent walls, and project before each other 13 feet; an ornamental stone parapet and cornice crown this viaduct, presenting a novel and agreeable appearance.



This bridge, executed after the designs of Mr G. W. Buck, is remarkable for its acute angle, which is 24½°. The weight of the ironwork employed on the six ribs was 540 tons, and the whole was admirably secured together; the remainder of the viaduct is formed with brick arches of 45 feet span.

DIMENSIONS OF VIADUCTS IN VARIOUS RAILWAYS.

In the Grand Junction Railway, at Vale Royal, is a viaduct of stone over the river Weaver, in which are five arches, each 63 feet in span and 60 in height; the length of the viaduct is 456 feet. Where this railway crosses the Mersey and Irwell Canal there is a viaduct of stone having 12 arches; the two in the centre are 75 feet in span, and the remainder from 40 feet to 12½.

In the Newcastle and Carlisle Railway, near Brampton, is a viaduct, which crosses the public road and the river Gelt, at the height of 80 feet above the bed of the river, over which it is carried in an oblique direction. The arches, which are three in number, are 33 feet in direct span, and are built at an angle of 45°.

In the Birmingham and Derby Junction Line, between Kingsbury and Tamworth, over the Anker, is a viaduct of 18 arches, each of 30 feet span, and one oblique arch of 60 feet span; its height above the river is 23 feet, and the cost was 18,000*l.* *In the same line,* between Tamworth and Burton-on-Trent, is a viaduct ¼ mile in length, built upon above 1000 piles, driven 15 feet below the bed of the river.

In the Newcastle and Shields Railway there is a viaduct over the Ouseburn of 9 arches, two of which at the ends are of stone, and the others are of timber, resting on stone piers; the three central arches have each a span of 116 feet, and two others 110; the total length of the viaduct is 750 feet, and its height above the water 180 feet. *In the same line,* the viaduct at Willingdon Dean has 7 timber arches, five of which span 120 feet each, and the two exterior each 115 feet; the whole length is 1050 feet, and the height 82 feet.

In the Taff Vale Railway, near Quaker's Yard, is a viaduct crossing the Taff, the length of which is 600 feet, and the height above the river 100 feet, having 6 arches. *In the same line*, at the conflux of the Rhondda and Taff, is a viaduct having an arch of 100 feet span, and 60 feet in height.

In the London and Brighton Railway, across the valley of the Ouse, is a viaduct 1437 feet in length, and the height varies from 40 to 96 feet; it is formed of 37 brick arches of 30 feet span.

The London and Greenwich Railway is a continuous viaduct of more than 1000 brick arches, each 18 feet span, 22 feet in height, and 25 feet in width. It is $3\frac{3}{4}$ miles in length, and cost 266,322*l.* per mile.

The extension of the South-Western Railway through the metropolis, from the Nine Elms to Waterloo Bridge, is a continuous viaduct, like the last-mentioned one, in which are several strong and elegant oblique iron arches for the purpose of crossing some of the principal streets.

The student's attention should also be directed to the many magnificent railway bridges and viaducts of *iron* which have been erected of late years; such as the *Britannia and Conway Bridges* in North Wales; the *High Level Bridge* at Newcastle-on-Tyne; and the great *Victoria Bridge* in Canada by Mr Robert Stephenson; the *Windsor*, *Chepstow*, and *Saltash* bridges by Mr Brunel; the new *Charing Cross Bridge* by Mr Hawkshaw; and the *Viaduct at Crumlin* in Monmouthshire, by Messrs. Kennard. Descriptions of these will be found in engineering works.

I shall conclude this section by recommending to those who wish for scientific and practical information of the first order on this subject, "The Theory, Practice, and Architecture of Bridges," by J. Hann and others; "Practical and Theoretical Essay on Oblique Bridges," by G. W. Buck, M. Inst. C.E.; "Treatise on the Equilibrium of Arches," by Joseph Gwilt, architect, F.S.A.; Cresy's "Encyclopædia;" and a "Practical Treatise on the Construction of Oblique Arches," by J. Hart.

SECTION VII.

SUPERELEVATION OF EXTERIOR RAIL IN CURVES.

The superelevation of the exterior rail in curves, the radii of which are within certain limits, is absolutely necessary to counteract the centrifugal force caused by the velocity of the train, since all moving bodies have a tendency to continue their motion in a direct line. From this cause the carriages of a railway train are driven towards the exterior rail, and would finally be thrown off the rails, were it not for the conical inclination of the tire and the flanges of the wheels.

Let W = weight of the moving body or train, V = its velocity per second, R = radius of the curve, and g = force of gravity at the earth's surface; then, by Dynamics, the centrifugal force

$$f = \frac{WV^2}{gR} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

When R = 1 mile = 5280 feet, V = velocity = 60 miles per hour = 88 feet per second, and $g = 32\frac{1}{6}$ feet, then

$$f = \frac{W \times 88^2}{32\frac{1}{6} \times 5280} = \frac{1}{22}W;$$

that is, the centrifugal force that urges the moving body to leave the curve, in this case, is $\frac{1}{22}$ of its weight.

This force is slightly counteracted by the conical inclination of the tire of the wheels, each pair of which are firmly fixed on the axle that turns with them. This inclination of the tire, together with the lateral play of the flanges of $\frac{1}{2}$ an inch on each side, and the centrifugal force impelling the carriages of the train, when moving in a curve, towards the exterior rail, enlarge the diameter of the exterior wheel, and diminish that of the interior, thus causing the train to roll on conical surfaces, which necessarily produces a centripetal force, the centre of which force is the vertex of the cone, of which the increased and diminished diameters of each pair of wheels are sections.

Let d be the outer diameter of the wheels, δ the increment and consequently the decrement that the diameters of the exterior and interior wheels respectively receive, through the joint action of the centrifugal force and the inclination of the tire: then under these circumstances the respective diameters of the exterior and interior wheels will be

$$d \times \delta \text{ and } d - \delta$$

also if R′ be the radius of a circle which the centre of the carriage would describe in consequence of the inclination of the tire of the wheels, and b the breadth of the road or gauge: then $R' + \frac{1}{2}b$ and $R' - \frac{1}{2}b$ are the radii which would be respectively described by the exterior and interior wheels; and by similar triangles

$$d + \delta : d - \delta : : R' + \tfrac{1}{2}b : R' - \tfrac{1}{2}b,$$

whence $\quad d : \delta : : 2R' : b$, and

$$R = \frac{bd}{2}$$

Or, if $\frac{1}{n}$ = inclination of the tire, and Δ the deviation of the wheels, the increment or decrement $\delta = \frac{2\Delta}{n}$; and, by substitution,

$$R = \frac{bdn}{4\Delta} \quad . \quad . \quad . \quad . \quad (2)$$

Also W and V representing the weight and velocity of the train, as in (1), and g the force of gravity, the centripetal force corresponding to the radius R′ will be

$$f' = \frac{WV^2}{gR'} \quad . \quad . \quad . \quad . \quad (3)$$

Or, by substituting the value of R′ from (2)

$$f' = \frac{WV^2}{g} \times \frac{4\Delta}{bdn} \quad . \quad . \quad . \quad . \quad (4)$$

Since the forces f and f' (1) and (3), act in contrary directions, they will hold each other in equilibrium when they become equal, and the train will cease to have a tendency to leave the curve; this takes place when

$$\frac{WV^2}{gR} = \frac{WV^2}{gR'}$$

$$\text{or } R = R'$$

Also from (1) and (4)

$$\frac{WV^2}{gR} = \frac{WV}{g} \times \frac{4\Delta}{bdn}$$

whence

$$\Delta = \frac{bdn}{4R} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

which is the deviation required to produce the equilibrum between the centripetal and centrifugal forces of the train. Therefore since $R = R'$, *i.e.*, the vertex of the imaginary cone of which each pair of wheels are sections will coincide with the centre of the curve, there will in consequence be no dragging of either of the wheels on the rail.

In practice, however, it is safer to neglect the effect of the coning of the tires, and to calculate the superelevation of the rail as if the tires were cylindrical.

The rule for this is as follows:—

Let R = radius of curve in chains.

b = breadth of gauge of the line in feet; or more properly, the distance from centre to centre of rails.

V = velocity of the train in miles per hour.

x = superelevation to be given to the outside rail in inches.

Then

$$x=\frac{b\mathrm{V}^2}{82{\cdot}5\ \mathrm{R}}$$

For the narrow gauge, where $b = 4$ ft. 11 in. $= 4{\cdot}91$,

$$x=\frac{\mathrm{V}^2}{17\mathrm{R}}\ \text{nearly.}$$

Example.—On a curve $\frac{1}{2}$ mile (= 40 chains) radius it is expected that trains will travel at 40 miles an hour. What ought to be the superelevation of the outside rail?

$$x=\frac{40^2}{17\times 40}=\frac{1600}{680}=2{\cdot}35\ \text{inches.}\quad \textit{Answer.}$$

TABLES FOR REDUCTION OF DISTANCES AND CORRECTION OF LEVELS FOR CURVATURE, ETC.

No. 1. Reduction for each Chain for the following Angles of elevation.

Angles.		Reduction in Links.
3°	0′	0·137
3	30	0·187
4	0	0·244
4	30	0·308
5	0	0·381
5	30	0·460
6	0	0·548
6	30	0·643
7	0	0·745
7	30	0·856
8	0	0·973
8	30	1·098
9	0	1·231
9	30	1·371
10	0	1·519
10	30	1·675
11	0	1·837
11	30	2·008
12	0	2·185
12	30	2·370
13	0	2·553
13	30	2·763
14	0	2·970
14	30	3·185
15	0	3·407
15	30	3·637
16	0	3·874
16	30	4·118
17	0	4·370
17	30	4·628
18	0	4·894
18	30	5·168
19	0	5·448
19	30	5·736
20	0	6 031
20	30	6·333
21	0	6·642
21	30	6·958
22	0	7·282
22	30	7·612
23	0	7 950
23	30	8·294
24	0	8·645
24	30	9·004
25	0	9·369
25	30	9·741
26	0	10·121
26	30	10·507
27	0	10·899
27	30	11·299
28	0	11·705
28	30	12·118
29	0	12·538
29	30	12·964
30	0	13·397

No. 2. Difference between apparent and True Level for distances in Chains. Correction in Decimals of Feet.

Distances in Chains.	For Curvature.	For Refraction.	For Curvature and Refraction.
3	·0009	·0001	·0008
3½	·0013	·0002	·0011
4	·0017	·0002	·0015
4½	·0021	·0003	·0018
5	·0026	·0004	·0022
5½	·0031	·0004	·0027
6	·0037	·0005	·0032
6½	·0044	·0006	·0038
7	·0051	·0007	·0044
7½	·0058	·0008	·0050
8	·0067	·0010	·0057
8½	·0075	·0011	·0064
9	·0084	·0012	·0072
9½	·0094	·0013	·0081
10	·0104	·0015	·0089
10½	·0115	·0016	·0099
11	·0126	·0018	·0108
11½	·0138	·0020	·0118
12	·0150	·0021	·0129
12½	·0163	·0023	·0140
13	·0176	·0025	·0151
13½	·0190	·0027	·0163
14	·0204	·0029	·0175
14½	·0219	·0031	·0188
15	·0234	·0033	·0201
15½	·0250	·0036	·0214
16	·0267	·0038	·0229
16½	·0284	·0041	·0243
17	·0301	·0043	·0258
17½	·0319	·0046	·0273
18	·0338	·0048	·0290
18½	·0357	·0051	·0306
19	·0376	·0054	·0322
19½	·0396	·0056	·0340
20	·0417	·0060	·0357
20½	·0438	·0063	·0375
21	·0459	·0066	·0393
21½	·0481	·0069	·0412
22	·0504	·0072	·0432
22½	·0527	·0075	·0452
23	·0551	·0079	·0472
23½	·0575	·0082	·0493
24	·0600	·0086	·0514
24½	·0625	·0089	·0536
25	·0651	·0093	·0558
25½	·0677	·0097	·0580
26	·0704	·0100	·0604
26½	·0731	·0104	·0627
27	·0759	·0108	·0651
27½	·0788	·0113	·0675
28	·0817	·0117	·0700
28½	·0846	·0121	·0725
29	·0876	·0125	·0751
29½	·0906	·0129	·0777
30	·0937	·0134	·0803

No. 3. Difference between apparent and True Level for distances in Miles. Correction in Feet and Decimals.

Distances in Miles.	For Curvature.	For Refraction.	For Curvature and Refraction.
¼	·042	·006	·036
½	·167	·024	·143
¾	·375	·054	·321
1	·667	·095	·572
1½	1·501	·215	1·286
2	2·668	·381	2·287
2½	4·169	·596	3·573
3	6·003	·856	5·147
3½	8·171	1·167	7·004
4	10·672	1·525	9·147
4½	13·547	1·930	11·517
5	16·675	2·382	14·293
5½	20·177	2·882	17·295
6	24·012	3·430	20·582
6½	28·181	4·026	24·155
7	32·683	4·669	28·014
7½	37·519	5·360	32·159
8	42·688	6·098	36·590
8½	48·191	6·884	41·307
9	54·027	7·718	46·309
9½	60·197	8·600	51·597
10	66·700	9 529	57·171
10½	71·520	10·217	61·303
11	80·687	11·527	69·160
11½	88·167	12·594	75·573
12	96·000	13·714	82·286
12½	104·167	14·881	89·286
13	112·667	16·095	96·572
13½	121·500	17·357	104·143
14	130·667	18·666	112·001
14½	140·167	20·024	120·143
15	150·000	21·428	128·572
15½	160·167	22·881	137·286
16	170·667	24·381	146·286
16½	181·500	25·928	155·572
17	192·667	27·524	165·143
17½	204·167	29·167	175·000
18	216·000	30·857	185·143
18½	228·167	32·581	195·586
19	240·667	34·381	206·286
19½	253·500	36·214	217·286
20	266·667	38·075	228·592
21	294·000	42·000	252·000
22	322·667	46·095	276·572
23	352·667	50·381	302·286
24	384·000	54·857	329·143
25	416·667	58·095	358·572
26	450·667	64·381	386·286
27	486·000	69·428	416·572
28	522·667	74·667	448·000
29	560·667	80·095	480·572
30	600·000	85·714	514·286

No. 4. Offsets or Ordinates at the end of the first Chain from tangent point of Railway Curves.								No. 5. Horizontal Distances to an Unit's Height to the following Angles of Elevation.	
Radius of Curve in Chains.	Offsets in Inches and Decimals.	Radius of Curve in Chains.	Offsets in Inches and Decimals.	Radius of Curve in Chains.	Offsets in Inches and Decimals.	Radius of Curve in Chains.	Offsets in Inches and Decimals.	Angle of Elevation.	Hor. Distance to one Perpendicular.
15	26·4000	65	6·0923	125	3·1680	225	1·7600	0° 5′	688
16	24·7500	66	6·0000	126	3·1428	230	1·7217	0 10	344
17	23·2941	67	5·9104	128	3·0937	235	1·6851	0 15	229
18	22·0000	68	5·8235	130	3·0461	240	1·6500	0 30	115
19	20·8421	69	5·7391	132	3·0000	245	1·6163	0 45	76
20	19·8000	70	5·6571	134	2·9552	250	1·5840	1 0	57
21	18·8571	71	5·5774	135	2·9333	255	1·5529	1 15	46
22	18·0000	72	5·5000	136	2·9117	260	1·5231	1 30	39
23	17·2174	73	5·4246	138	2·8645	265	1·4943	1 45	33
24	16·5000	74	5·3513	140	2·8285	270	1·4667	2 0	28
25	15·8400	75	5·2800	142	2·7887	275	1·4400	2 15	25
26	15·2307	76	5·2105	144	2·7500	280	1·4143	2 30	23
27	14·6666	77	5·1428	145	2·7310	285	1·3995	2 45	21
28	14·1428	78	5·0769	146	2·7123	290	1·3655	3 0	19
29	13·6551	79	5·0126	148	2·6756	295	1·3423	3 15	18
30	13·2000	80	4·9500	150	2·6400	300	1·3200	3 28	17
31	12·7742	81	4·8889	152	2·6052	305	1·2983	3 35	16
32	12·3750	82	4·8292	154	2·5714	310	1·2774	3 49	15
33	12·0000	83	4·7711	155	2·5548	315	1·2571	4 6	14
34	11·6470	84	4·7143	156	2·5384	320	1·2375	4 24	13
35	11·3142	85	4·6588	158	2·5063	325	1·2184	4 45	12
36	11·0000	86	4·6046	160	2·4750	330	1·2000	5 12	11
37	10·7026	87	4·5517	162	2·4444	335	1·1821	5 42	10
38	10·4210	88	4·5000	164	2·4146	340	1·1647	6 21	9
39	10·1538	89	4·4496	165	2·4000	345	1·1478	7 7	8
40	9·9000	90	4·4000	166	2·3855	350	1·1314	8 8	7
41	9·6588	91	4·3516	168	2·3571	355	1·1155	9 27	6
42	9·4285	92	4·3043	170	2·3294	360	1·1000	11 19	5
43	9·2093	93	4·2581	172	2·3023	365	1·0809	14 2	4
44	9·0000	94	4·2128	174	2·2758	370	1·0703	18 26	3
45	8·8000	95	4·1684	175	2·2628	375	1·0560	26 34	2
46	8·6087	96	4·1250	176	2·2500	380	1·0421	45 0	1
47	8·4255	97	4·0825	178	2·2248	385	1·0285		
48	8·2500	98	4·0408	180	2·2000	390	1·0154		
49	8·0816	99	4·0000	182	2·1758	395	1·0025		
50	7·9200	100	3·9600	184	2·1521	400	·9900		
51	7·7647	102	3·8824	185	2·1405	410	·9659		
52	7·6154	104	3·8077	186	2·1290	420	·9428		
53	7·4717	105	3·7714	188	2·1064	430	·9209		
54	7·3333	106	3·7358	190	2·0842	440	·9000		
55	7·2000	108	3·6667	192	2·0625	450	·8800		
56	7·0714	110	3·6000	194	2·0412	460	·8609		
57	6·9473	112	3·5352	195	2·0307	470	·8425		
58	6·8276	114	3·4736	196	2·0204	480	·8250		
59	6·7118	115	3·4435	198	2·0000	490	·8081		
60	6·6000	116	3·4138	200	1·9800	500	·7920		
61	6·4918	118	3·3559	205	1·9317	510	·7765		
62	6·3871	120	3·3000	210	1·8857	520	·7615		
63	6·2857	122	3·2459	215	1·8418	530	·7472		
64	6·1875	124	3·1935	220	1·8000	540	·7333		

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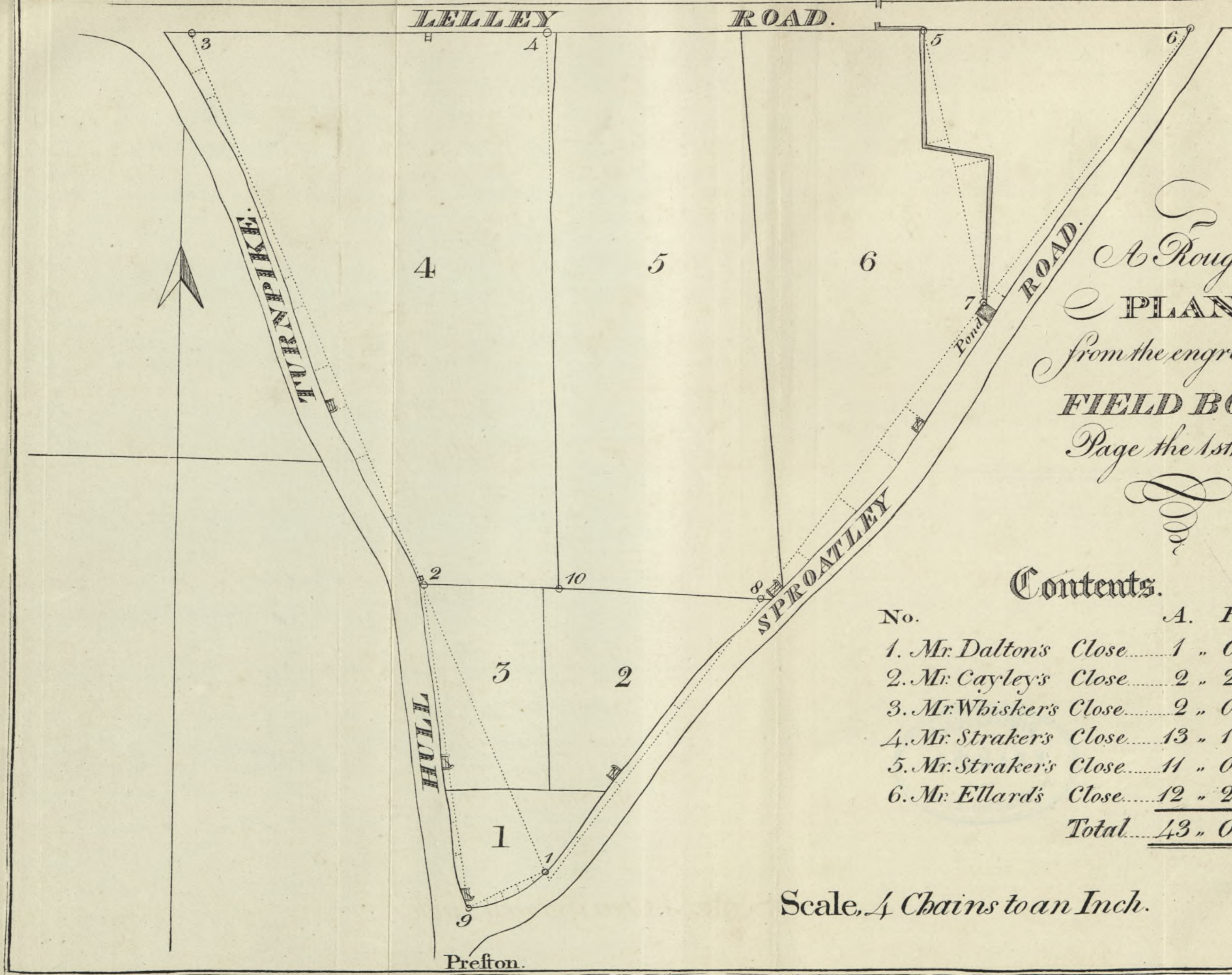


A Rough
PLAN
from the engraven
FIELD BOOK,
Page the 1st. &c.

Contents.

No.		A.	R.	P.
1. Mr. Dalton's	Close	1	0	20
2. Mr. Cayley's	Close	2	2	38
3. Mr. Whisker's	Close	2	0	10
4. Mr. Straker's	Close	13	1	28
5. Mr. Straker's	Close	11	0	9
6. Mr. Ellard's	Close	12	2	18
	Total	43	0	3

Scale, *4 Chains to an Inch.*

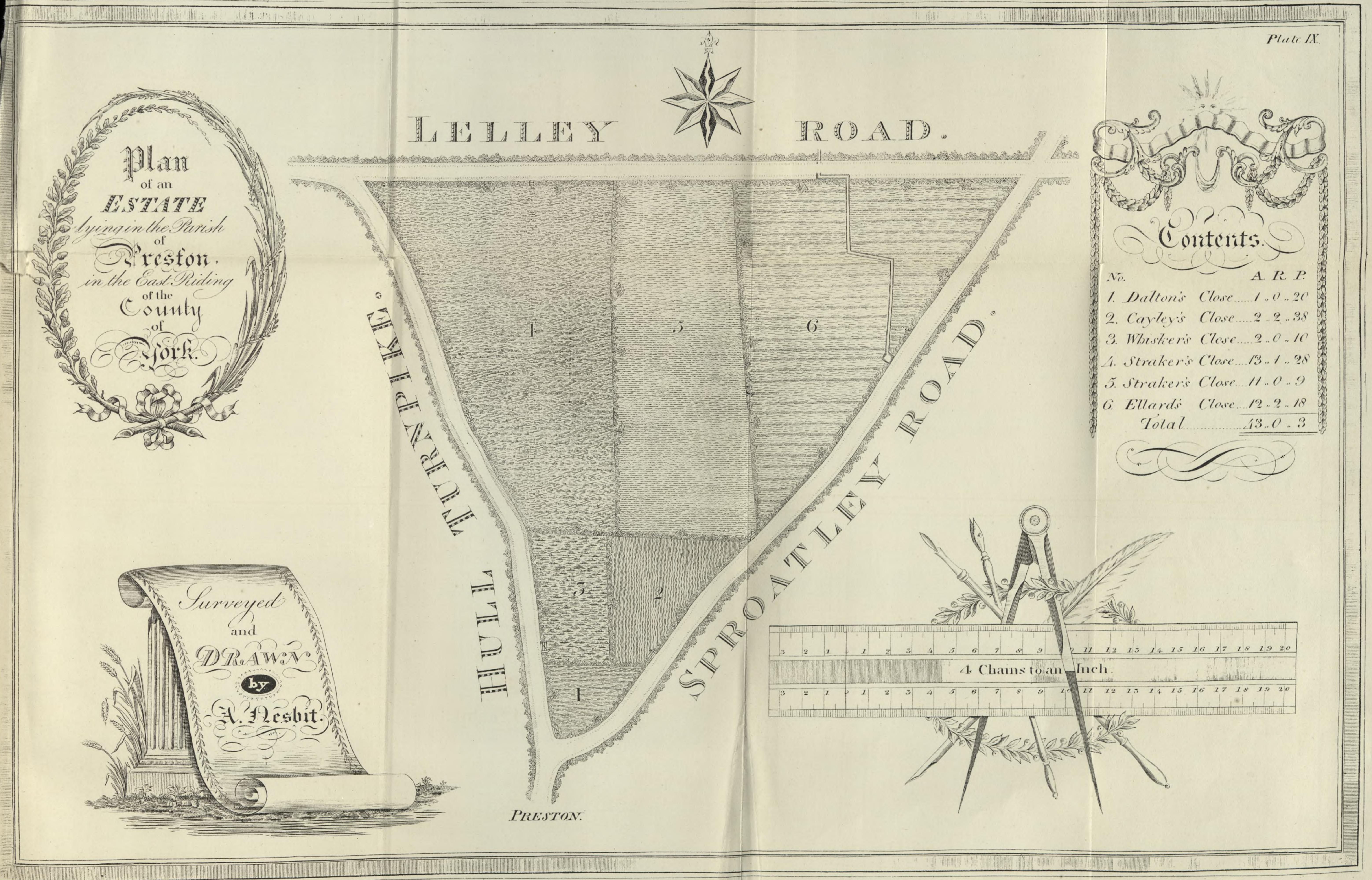
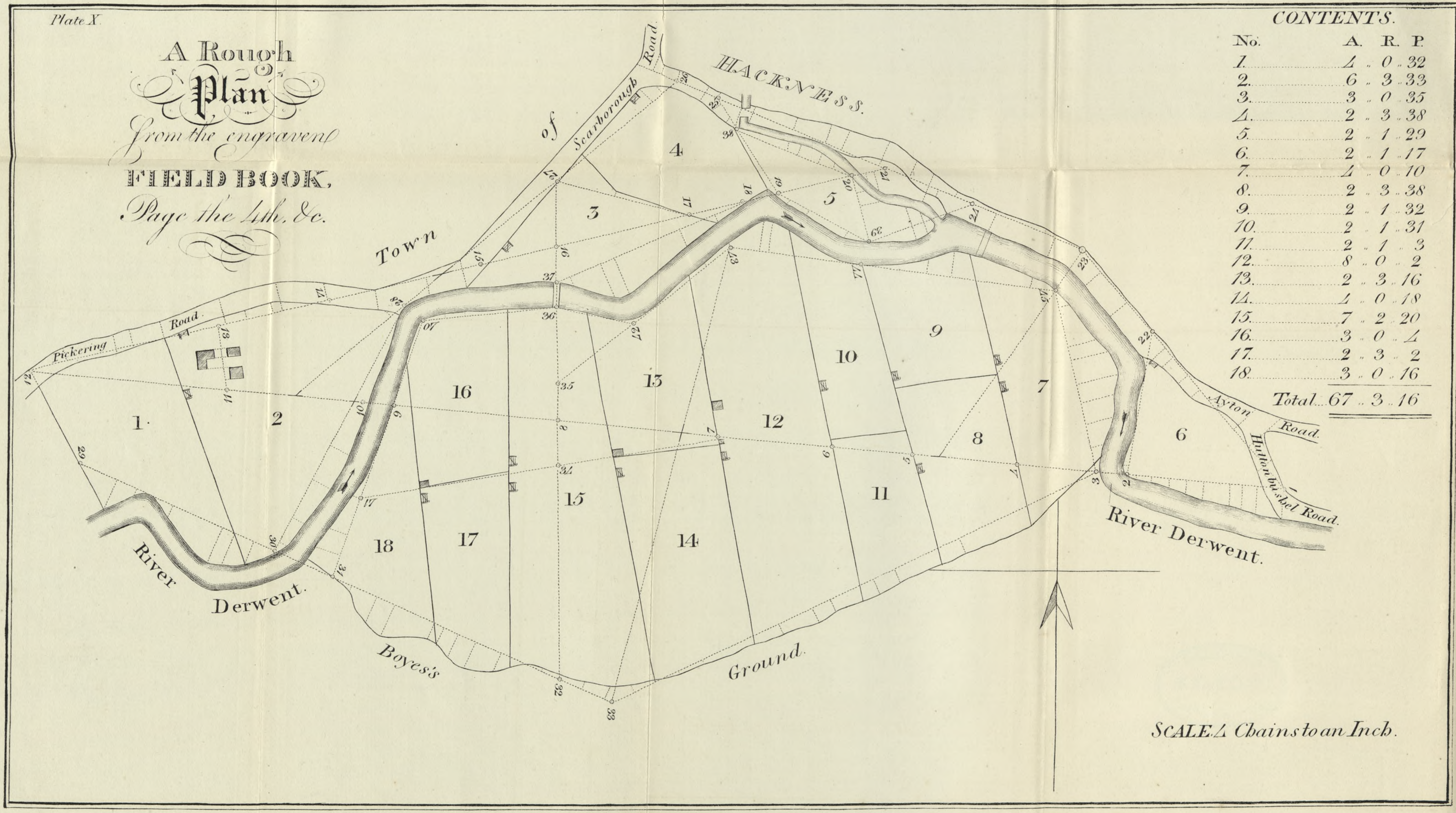


Plate IX.
Plan of an ESTATE lying in the Parish of Preston, in the East Riding of the County of York.
LELLEY ROAD.
HULL TURNPIKE.
SPROATLEY ROAD.
PRESTON.
1
2
3
4
5
6
Contents.
No. A. R. P.
1. Dalton's Close 1 . 0 . 20
2. Cayley's Close 2 . 2 . 38
3. Whisker's Close 2 . 0 . 10
4. Straker's Close 13 . 1 . 28
5. Straker's Close 11 . 0 . 9
6. Ellard's Close 12 . 2 . 18
Total 43 . 0 . 3
Surveyed and DRAWN by A. Nesbit.
4 Chains to an Inch.

Plate X.



CONTENTS.

No.	A.	R.	P.
1.	4	0	32
2.	6	3	33
3.	3	0	35
4.	2	3	38
5.	2	1	29
6.	2	1	17
7.	4	0	10
8.	2	3	38
9.	2	1	32
10.	2	1	31
11.	2	1	3
12.	8	0	2
13.	2	3	16
14.	4	0	18
15.	7	2	20
16.	3	0	4
17.	2	3	2
18.	3	0	16
Total	67	3	16

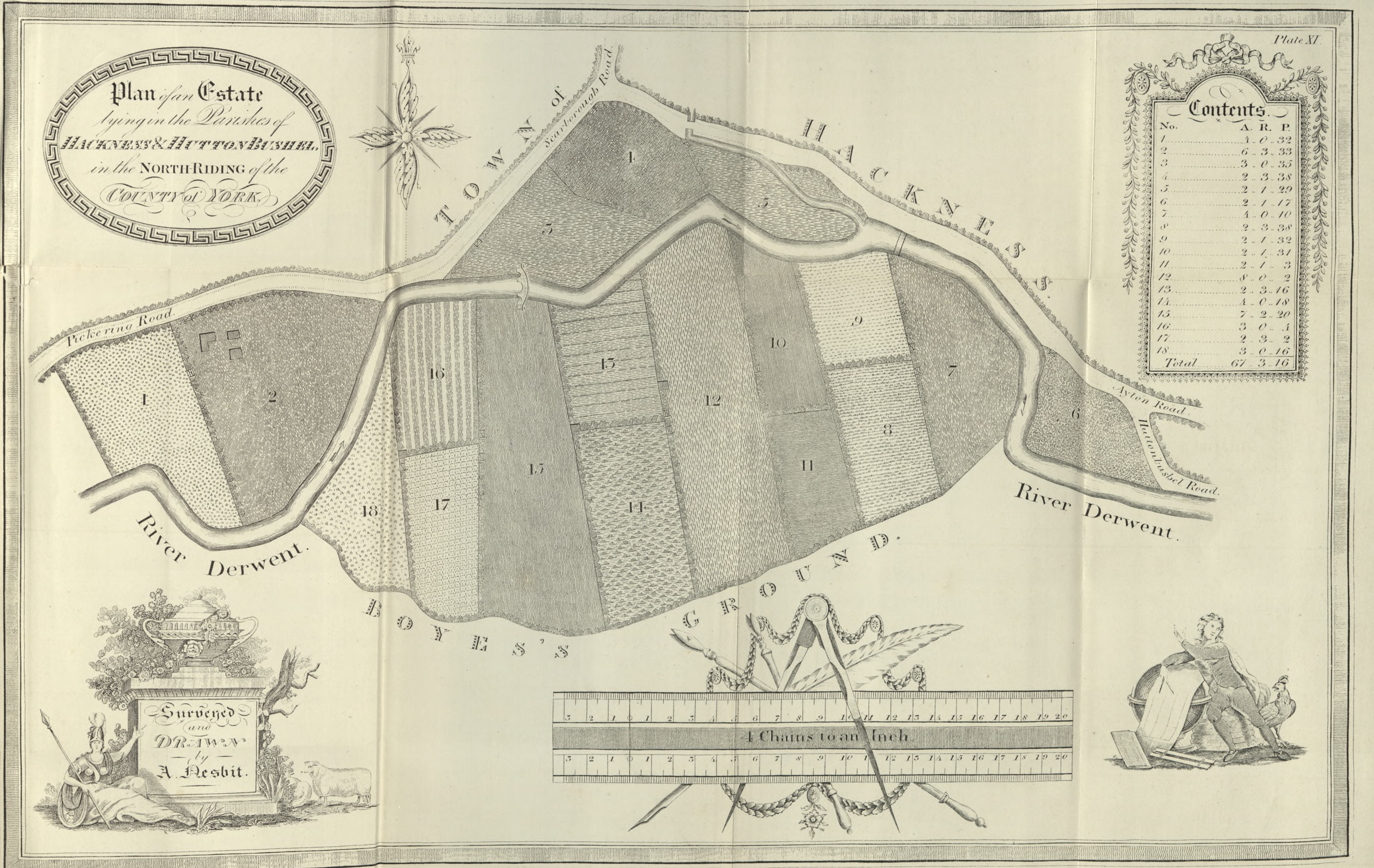
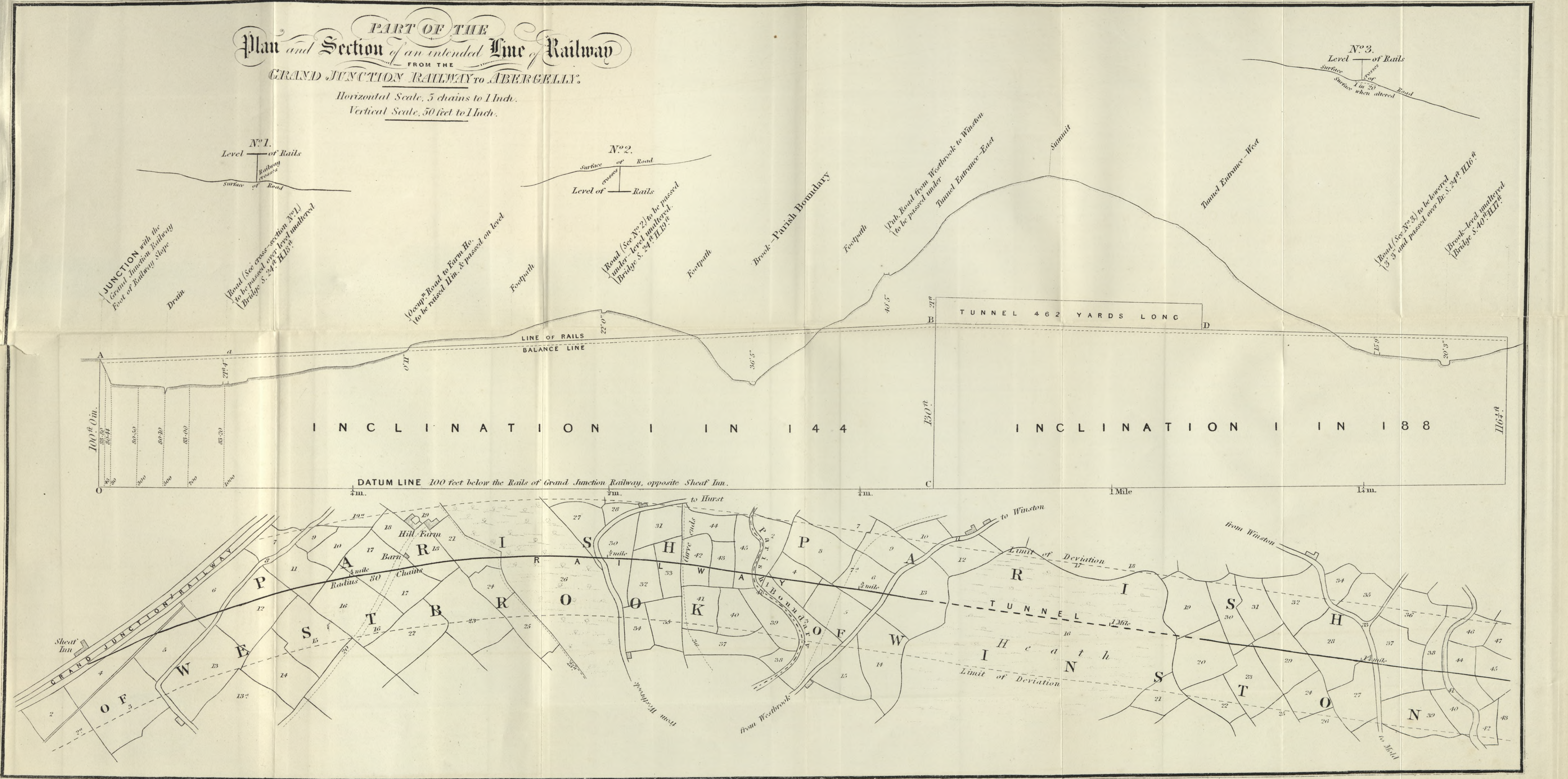


Plate XI.
Plan of an Estate lying in the Parishes of Hackness & Hutton Bushel, in the North-Riding of the County of York.
Contents.
No. A. R. P.
1 … 1 . 0 . 32
2 … 6 . 3 . 33
3 … 3 . 0 . 35
4 … 2 . 3 . 38
5 … 2 . 1 . 29
6 … 2 . 1 . 17
7 … 4 . 0 . 10
8 … 2 . 3 . 38
9 … 2 . 1 . 32
10 … 2 . 1 . 31
11 … 2 . 1 . 3
12 … 8 . 0 . 2
13 … 2 . 3 . 16
14 … 4 . 0 . 18
15 … 7 . 2 . 20
16 … 3 . 0 . 4
17 … 2 . 3 . 2
18 … 3 . 0 . 16
Total … 67 . 3 . 16
Town of Hackness.
Scarborough Road.
Pickering Road.
Ayton Road.
Huttonbushel Road.
River Derwent.
River Derwent.
Boyes's Ground.
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
Surveyed and Drawn by A. Nesbit.
4 Chains to an Inch
3 2 1 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20


PART OF THE
Plan and Section of an intended Line of Railway
FROM THE
GRAND JUNCTION RAILWAY to ABERGELLY.
Horizontal Scale, 5 chains to 1 Inch.
Vertical Scale, 50 feet to 1 Inch.
Nº 1.
Level of Rails
Railway crosses
Surface of Road
Nº 2.
Surface of Road
crosses
Level of Rails
Nº 3.
Level of Rails
Surface
Surface when altered
1 in 20
Road
JUNCTION with the Grand Junction Railway Foot of Railway Slope
Drain
Road (See cross-section Nº 1) to be passed over level unaltered Bridge S. 24 ft H. 19 ft
Occupn. Road to Farm Ho. to be raised 11 in. & passed on level
Footpath
Road (See Nº 2) to be passed under-level unaltered. Bridge S. 24 ft H. 19 ft
Footpath
Brook – Parish Boundary
Footpath
Pub. Road from Westbrook to Winston to be passed under
Tunnel Entrance–East
Summit
Tunnel Entrance–West
Road (See Nº 3) to be lowered 3' 3" and passed over Br. S. 24 ft H. 16 ft
Brook–level unaltered Bridge S. 40 ft H. 17 ft
TUNNEL 462 YARDS LONG
LINE OF RAILS
BALANCE LINE
INCLINATION 1 IN 144
INCLINATION 1 IN 188
100 ft 0 in.
130 ft
116 ft 4 in
DATUM LINE 100 feet below the Rails of Grand Junction Railway, opposite Sheaf Inn.
¼ m.
½ m.
¾ m.
1 Mile
1¼ m.
GRAND JUNCTION RAILWAY
Sheaf Inn
Hill Farm
Barn
Radius 80 Chains
PARISH OF WESTBROOK
RAILWAY
TUNNEL
Parish Boundary
PARISH OF WINSTON
Heath
Limit of Deviation
to Hurst
to Winston
from Winston
from Westbrook
to Mold

A GENERAL TABLE FOR THE CALCULATION OF EARTH-WORK FOR RAILWAYS, &c.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
1	1	2
2	3	6	10
3	7	11	15	22
4	13	17	23	30	39
5	20	25	32	40	50	61
6	29	35	42	51	62	74	88
7	40	46	55	64	76	89	103	120
8	52	59	68	79	91	105	121	138	156
9	66	74	84	95	108	123	139	157	177	198
10	81	90	101	113	127	143	160	178	199	221	244
11	99	108	120	133	147	164	182	201	222	245	270	296
12	117	128	140	154	169	187	205	226	248	271	297	323	352
13	138	149	162	177	193	211	231	252	275	299	325	353	382	413
14	160	172	186	201	218	237	257	279	303	328	355	384	414	446	479
15	183	196	211	227	245	265	286	309	333	359	387	416	447	480	514	550
16	209	222	238	255	274	294	316	340	365	392	420	451	482	516	551	587	626
17	235	250	266	284	304	325	348	372	398	426	455	486	519	553	589	627	666	706
18	264	279	297	315	336	358	381	407	433	462	492	524	557	592	629	667	707	749	792
19	294	310	328	348	369	392	416	442	470	499	530	563	597	633	671	710	750	793	837	882
20	326	343	362	382	404	428	453	480	508	539	570	604	639	675	714	754	795	838	883	930	978
21	359	377	397	418	441	465	491	519	548	579	612	646	682	719	759	799	842	886	931	979	1027	1078
22	394	413	433	455	479	504	531	560	590	622	655	690	727	765	805	847	890	935	981	1029	1079	1130	1183
23	431	451	472	495	519	545	573	602	633	666	700	736	773	812	853	895	939	985	1032	1081	1132	1184	1238	1293
24	469	490	512	535	561	587	616	646	678	711	746	783	821	861	903	946	991	1037	1085	1135	1186	1239	1294	1350	1408
25	509	530	553	578	604	631	661	692	724	759	794	832	871	912	954	998	1044	1091	1140	1190	1243	1296	1352	1409	1467	1528
26	551	573	596	622	649	677	707	739	772	807	844	882	922	964	1007	1052	1098	1146	1196	1247	1300	1355	1411	1469	1529	1590	1652
27	594	617	641	667	695	724	755	788	822	858	895	935	975	1018	1062	1107	1155	1203	1254	1306	1360	1415	1472	1531	1591	1653	1717	1782
28	639	662	688	715	743	773	805	838	873	910	948	988	1030	1073	1118	1164	1212	1262	1313	1366	1421	1477	1535	1595	1656	1718	1783	1849	1916
29	685	710	736	763	793	824	856	891	926	964	1003	1044	1086	1130	1176	1223	1272	1322	1375	1428	1484	1542	1599	1660	1722	1785	1850	1917	1986	2056
30	733	759	785	814	844	876	909	944	981	1019	1059	1101	1144	1189	1235	1283	1333	1384	1437	1492	1548	1606	1665	1727	1789	1854	1920	1987	2057	2127	2200
31	783	809	837	866	897	930	964	1000	1037	1076	1117	1159	1203	1249	1296	1345	1396	1448	1502	1557	1614	1673	1733	1795	1859	1924	1991	2059	2129	2201	2274	2349
32	834	861	890	920	952	985	1020	1057	1095	1135	1177	1220	1265	1311	1359	1409	1460	1513	1568	1624	1682	1741	1802	1865	1929	1996	2063	2132	2203	2276	2350	2426	2503
33	887	915	944	975	1008	1042	1078	1115	1155	1195	1238	1282	1327	1375	1423	1474	1526	1580	1635	1692	1751	1811	1873	1937	2002	2069	2137	2207	2279	2352	2427	2504	2582	2662
34	942	970	1001	1032	1066	1101	1137	1176	1216	1257	1300	1345	1392	1440	1489	1541	1594	1648	1705	1762	1822	1883	1946	2010	2076	2144	2213	2284	2356	2431	2506	2584	2663	2743	2826
35	998	1027	1058	1091	1125	1161	1199	1238	1278	1321	1365	1410	1458	1507	1557	1609	1663	1718	1775	1834	1894	1956	2020	2085	2152	2220	2290	2362	2435	2510	2587	2665	2745	2827	2910	2994
36	1056	1086	1118	1151	1186	1223	1261	1301	1343	1386	1431	1477	1525	1575	1626	1679	1734	1790	1848	1907	1969	2031	2096	2162	2229	2299	2369	2442	2516	2592	2669	2748	2829	2911	2995	3081	3168
37	1115	1146	1179	1213	1249	1287	1326	1366	1409	1453	1498	1546	1595	1645	1697	1751	1806	1863	1922	1982	2044	2108	2173	2240	2308	2378	2450	2523	2598	2675	2753	2833	2915	2998	3082	3169	3257	3346
38	1177	1208	1242	1277	1313	1352	1392	1433	1476	1521	1568	1616	1665	1717	1770	1824	1881	1938	1998	2059	2122	2168	2252	2320	2389	2460	2532	2607	2682	2760	2839	2919	3002	3086	3171	3258	3347	3438	3530
39	1239	1272	1306	1342	1379	1419	1459	1502	1546	1591	1639	1687	1738	1790	1844	1899	1956	2015	2075	2137	2201	2266	2333	2401	2471	2543	2616	2691	2768	2846	2926	3007	3091	3175	3262	3350	3439	3531	3623	3718
40	1304	1337	1372	1409	1447	1487	1529	1572	1617	1663	1711	1761	1812	1865	1920	1976	2034	2093	2154	2217	2281	2347	2415	2484	2555	2628	2702	2778	2855	2934	3015	3097	3181	3267	3354	3443	3533	3625	3719	3814	3911
41	1370	1404	1440	1477	1516	1557	1599	1643	1689	1736	1785	1836	1888	1942	1997	2054	2113	2173	2235	2299	2364	2431	2499	2569	2641	2714	2789	2866	2944	3024	3105	3188	3273	3359	3447	3537	3628	3721	3816	3912	4010	4109
42	1437	1472	1509	1547	1587	1629	1672	1717	1763	1811	1861	1912	1965	2020	2076	2134	2193	2255	2317	2382	2448	2515	2585	2655	2728	2802	2878	2955	3034	3115	3197	3281	3367	3454	3543	3633	3725	3819	3914	4011	4110	4210	4312
43	1507	1542	1580	1619	1660	1702	1746	1792	1839	1888	1938	1991	2044	2100	2157	2215	2276	2338	2401	2466	2533	2602	2672	2743	2817	2892	2968	3047	3126	3208	3291	3376	3462	3550	3640	3731	3824	3918	4015	4112	4212	4313	4415	4520
44	1577	1614	1652	1692	1734	1777	1822	1868	1916	1966	2017	2070	2125	2181	2239	2299	2360	2422	2487	2553	2620	2690	2761	2833	2907	2983	3060	3139	3220	3302	3386	3472	3559	3648	3738	3830	3924	4019	4116	4215	4315	4417	4521	4626	4732
45	1650	1687	1727	1767	1810	1854	1899	1947	1995	2046	2098	2152	2207	2264	2323	2383	2445	2509	2574	2641	2709	2779	2851	2924	2999	3076	3154	3234	3315	3399	3483	3570	3658	3747	3839	3931	4026	4122	4220	4319	4420	4523	4627	4733	4841	4950
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47	1800	1839	1880	1922	1966	2012	2059	2108	2158	2211	2264	2320	2377	2435	2496	2558	2621	2686	2753	2822	2892	2963	3037	3112	3188	3267	3346	3428	3511	3596	3682	3770	3860	3951	4044	4138	4235	4332	4432	4533	4635	4740	4846	4953	5062	5173	5286	5400
48	1877	1917	1959	2002	2047	2093	2141	2191	2242	2295	2350	2406	2464	2523	2585	2647	2712	2778	2845	2915	2985	3058	3132	3208	3285	3364	3445	3527	3611	3697	3784	3873	3963	4055	4149	4244	4341	4440	4540	4642	4745	4851	4957	5066	5176	5287	5401	5515	5632
49	1956	1997	2039	2083	2129	2176	2225	2276	2328	2382	2437	2494	2553	2613	2675	2739	2804	2871	2939	3009	3081	3154	3229	3306	3384	3464	3545	3628	3713	3799	3887	3977	4068	4161	4256	4352	4450	4549	4650	4753	4857	4963	5071	5180	5291	5403	5517	5633	5750	5869
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54	2376	2421	2467	2515	2565	2616	2669	2724	2780	2838	2897	2959	3021	3086	3152	3219	3289	3359	3432	3506	3582	3659	3738	3819	3901	3985	4071	4158	4247	4337	4429	4523	4618	4715	4814	4914	5016	5119	5225	5331	5440	5550	5661	5775	5889	6006	6124	6244	6365	6488	6613	6739	6867	6997	7128
55	2465	2510	2558	2607	2657	2709	2763	2818	2875	2934	2994	3056	3120	3185	3252	3320	3390	3462	3535	3610	3687	3765	3845	3927	4010	4094	4181	4269	4358	4450	4543	4637	4733	4831	4930	5031	5134	5238	5344	5452	5561	5672	5784	5898	6014	6131	6250	6371	6493	6617	6743	6870	6998	7129	7261	7394
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58	2741	2789	2839	2890	2943	2998	3054	3112	3171	3232	3295	3359	3425	3493	3562	3633	3706	3780	3856	3933	4012	4093	4175	4259	4345	4432	4521	4611	4703	4797	4892	4989	5088	5188	5290	5393	5498	5605	5713	5823	5935	6048	6163	6280	6398	6518	6639	6762	6887	7013	7141	7271	7402	7535	7669	7805	7943	8082	8223
59	2836	2885	2936	2988	3042	3097	3154	3213	3273	3335	3399	3464	3531	3599	3669	3741	3814	3889	3966	4044	4124	4205	4288	4373	4459	4547	4637	4728	4821	4916	5012	5110	5209	5310	5413	5517	5623	5731	5840	5951	6063	6177	6293	6410	6529	6650	6772	6896	7021	7148	7277	7407	7539	7673	7808	7945	8084	8224	8366	8509
60	2933	2983	3034	3087	3142	3198	3256	3315	3377	3439	3504	3570	3637	3707	3777	3850	3924	4000	4077	4156	4237	4319	4403	4489	4576	4665	4755	4847	4941	5036	5133	5232	5332	5434	5537	5643	5749	5858	5968	6079	6193	6307	6424	6542	6662	6783	6906	7031	7157	7285	7415	7546	7679	7813	7949	8087	8226	8367	8510	8654	8800
61	3032	3082	3135	3188	3244	3301	3359	3420	3482	3545	3610	3677	3746	3816	3887	3961	4036	4112	4191	4270	4352	4435	4520	4606	4694	4784	4875	4968	5062	5159	5256	5356	5457	5559	5664	5770	5877	5986	6097	6210	6324	6439	6557	6676	6796	6919	7042	7168	7295	7424	7554	7686	7820	7955	8092	8230	8371	8512	8656	8801	8947	9096
62	3132	3183	3236	3291	3347	3405	3465	3526	3588	3653	3719	3786	3856	3927	3999	4073	4149	4226	4305	4386	4468	4552	4638	4725	4814	4904	4996	5090	5185	5282	5381	5481	5583	5687	5792	5898	6007	6117	6228	6342	6457	6573	6691	6811	6932	7055	7180	7306	7434	7564	7695	7828	7962	8098	8236	8375	8516	8659	8803	8949	9097	9246	9396
63	3234	3286	3340	3395	3452	3511	3571	3633	3697	3762	3829	3897	3967	4039	4112	4187	4264	4342	4422	4503	4587	4671	4758	4846	4935	5027	5119	5214	5310	5408	5507	5608	5711	5815	5921	6029	6138	6249	6361	6475	6591	6708	6827	6948	7070	7194	7319	7447	7575	7706	7838	7971	8107	8243	8382	8522	8664	8807	8952	9099	9247	9397	9549	9702
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AUXILIARY EARTH-WORK TABLES.

No. 1.

A Table of Feet to be added to the Depths of Cuttings, and the Number of Cubic Yards to be deducted from the Content of each Chain in Length, for the most common Bottom Widths and Ratios of Slopes.

Ratio of Slopes.	Bottom Width 27 feet.		Bottom Width 30 feet.		Bottom Width 33 feet.		Bottom Width 36 feet.		Bottom Width 39 feet.	
	Feet to be added to Depths.	Cubic Yards to be deducted.	Feet to be added to Depths.	Cubic yards to be deducted.	Feet to be added to Depths.	Cubic Yards to be deducted.	Feet to be added to Depths.	Cubic Yards to be deducted.	Feet to be added to Depths.	Cubic Yards to be deducted.
3 to 1	4.50	148.50	5.00	183.33	5.50	221.83	6.00	264.00	6.50	309.83
2½ to 1	5.40	178.20	6.00	220.00	6.60	266.20	7.20	316.80	7.80	371.80
2 to 1	6.75	222.75	7.50	275.00	8.25	332.75	9.00	396.00	9.75	464.75
1½ to 1	9.00	297.00	10.00	366.67	11.00	443.67	12.00	528.00	13.00	619.67
1 to 1	13.50	445.50	15.00	550.00	16.50	665.50	18.00	792.00	19.50	929.50
¾ to 1	18.00	594.00	20.00	733.33	22.00	887.33	24.00	1056.00	26.00	1239.33
½ to 1	27.00	891.00	30.00	1100.00	33.00	1331.00	36.00	1584.00	39.00	1859.00
¼ to 1	54.00	1782.00	60.00	2200.00	66.00	2662.00	72.00	3168.00	78.00	3718.00

No. 2.

A Table of Cubic Yards to be added for the Decimal Parts of a Foot, to be used with the general Table for the Calculation of Earthwork, when required.

$\frac{1}{10}(2a+b)$ or $\frac{1}{10}(2b+a)$	·1	·2	·3	·4	·5	·6	·7	·8	·9
1	1	2	2	3	4	5	6	7	7
2	2	3	5	7	8	10	11	13	15
3	2	5	7	10	12	15	17	20	22
4	3	7	10	13	16	20	23	26	29
5	4	8	12	16	20	24	29	33	37
6	5	10	15	20	24	29	34	39	44
7	6	11	17	23	28	34	40	46	51
8	7	13	20	26	33	39	46	52	59
9	7	15	22	29	37	44	51	57	66
10	8	16	25	33	41	49	57	65	73
11	9	18	27	36	45	54	63	72	81
12	10	20	29	39	49	59	69	78	88
13	11	21	32	42	53	63	74	85	95
14	11	23	34	46	57	68	80	91	103
15	12	24	37	49	61	73	86	98	110
16	13	26	39	52	65	78	91	104	117
17	14	28	42	55	69	83	97	111	125
18	15	29	44	59	73	88	103	117	132
19	16	31	47	62	77	93	108	124	139
20	16	33	49	65	82	98	114	130	147
21	17	34	51	68	86	103	120	137	154

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