


## HYDROGRAPHICAL SURVEYING.

A DESCRIPTION OF THE MEANS AND METHODS EMPLOYED IN CONSTRUCTING

MARINE CHARTS.

By REAR-ADMIRAL
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## PREFACE TO FIRST EDITION.

I have endeavoured in the following pages to collect together information which has for the most part existed, but in a traditionary form, for many years.

Circumstances have led to a partial break in this tradition, and it is no disparagement to the more modern treatises on nautical surveying to say that the young surveyor has no book to which he can refer for information on all details, as they, confessedly, do not enter into them.

Belcher's, the former standard work, is out of print, and in many ways out of date, as though the main principles of chart-making must remain the same, the lapse of time and introduction of steam, \&c., have placed many additional means at the disposal of the marine surveyor of to-day.

Knowing that there are many older and more experienced hydrographical surveyors than myself, it has been with considerable diffidence that I have applied myself during a period of leisure to stop the gap, and I hope my brother surveyors will remember that I write not for them, but for those young officers who wish to become acquainted with the practical portion of our branch of the naval profession.

The Appendix, largely composed of reprints of tables either falling out of print, or scattered about in different works, will, it is hoped, be found to save labour and time, and be useful to the Surveying Service at large.

W. J. L. Wharton.

## H. M. S. Sylvia, March 29th, 1882.

## PREFACE TO SECOND EDITION.

In preparing a second edition of Hydrographical Surveying I have endeavoured to alter as little as possible, confining myself mainly to bringing the book up to date in matters connected with instruments and fittings which have changed since the first edition was published.

In deep-sea sounding there has been an entire revolution from the substitution of wire for hemp, and as my own experience with the former is very slight, I am greatly indebted to Captain A. M. Field and Captain W. U. Moore for information on the subject. I have also received from these officers many notes on other subjects treated of, and wish to express my acknowledgements for the valuable aid they have thus afforded me.

W. J. L. Wharton.

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\text { April, } 1898 .
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## HYDROGRAPHICAL SURVEYING.

## PRELIMINARY.

There is nothing mysteriously difficult in the art of Hydrographical or Marine Surveying. For the ordinary details, no deep theoretical or mathematical knowledge is needed ; on the contrary, it is an eminently practical branch of the Naval Profession.

An aspirant to its acquirement should have a quick eye, should possess the ordinary good common-sense that is necessary to secure success in all walks of life, but above all he must have a boundless capacity for taking pains in details at all times and seasons.

The advice, "Surtout, point de zèle," does not apply to surveying. Without zeal, and the utmost keenness for the progress of the work, the attention and interest will soon fail; and the necessity for constant application throughout long days, often extended into the night, will soon seem monotonous, and become a bore to one whose heart is not thoroughly in it.

Happily, it is a profession of volunteers, and the author's experience is, that in no branch of the public service can the juniors be more anxious to do their duty, not only to the letter, but to the utmost of the spirit, and to such as these no day seems long enough. To them, the interest is constantly kept up. Every day has its incidents. The accuracy of the
work of each assistant, when proved, is an infinite gratification to him, and he has also the continual satisfaction of feeling that of all he does a permanent record will remain, in the chart which is to guide hundreds of his fellow-seamen on their way.

For any naval officer, then, who is really anxious to learn, the practical part of surveying will soon be mastered. It will quickly become a labour of love, and the constant attention and trouble necessary will be merged in the interest taken in the work. Thorough honesty must always guide him, so that nothing may appear that is not known to be correct. Omissions there must always be, but let there be no sins of commission, that pains and care will prevent.

It is not of course suggested that all can become thorough good surveyors in all branches. One man will have a particular aptitude for astronomical observing; another will have a natural talent as a draughtsman, that no efforts on the part of another can compete with, and so on; but to become a good practical hand is within reach of all who seriously are desirous of being so, and will take the trouble to gain the necessary experience, without which all theory and book-teaching will be useless.

One crucial test of a surveyor's capability is his power of so planning and carrying out his work as to economise time. Even in a plan of an ordinary bay or harbour, which every naval officer should be able to make, the trained surveyor, by his experience of how to set about it, will accomplish it in a fraction of the time required by another. Nothing is more important than the knowledge of how to suit means to the end, and many hours are wasted by the anxious tyro in endeavouring to attain an accuracy in detail which cannot be utilised in the finished plan.

Inside of the broad principles of map making, marine surveying is made up of numerous dodges and details, for which there is nothing like practical exposition on the ground, and those who can get others to show them will need but little other help, but as in many cases this instructor will
not be at hand, it is hoped that the following pages may sometimes supply the information required.

It may seem to many that some points remarked on are too insignificant to be heeded, but those who are acquainted with the work will know how much time is lost by inattention to, or ignorance of, these little things, and a young surveyor will be a very few days at work before he finds this out.

We assume our reader to have the ordinary knowledge of the sextant that all naval officers are taught, and that he is not entirely ignorant of the first principles of making a plan from a base by means of angles.

We write mainly for those who join the Surveying Service, and shall speak throughout as though we had the resources of an ordinarily fitted surveying ship at command.

We have endeavoured to take things in the order that they will generally come in the prosecution of a survey.

In work of the nature of Hydrographical Surveying, it is impossible to give directions as to how to undertake every detail. Ordinary means fail now and again from exceptional, local, or other circumstances, and ready resource in overcoming difficulties is one of the most important requisites in a nautical surveyor. To invent or improvise a method of doing a particular piece of work is a most satisfactory achievement when successful, but it is scarcely necessary to say that this can only come to the most naturally talented with experience.

The following pages will then not be found to provide for every occasion, but will only describe the ordinary and accepted modes of setting about work.

## CHAPTER I.

## INSTRUMENTS AND FITTINGS.

Sextants and Stands-Horizon-Theodolite-Station Pointer-Scales-Straight-edges-Chains-Protractors-Pocket Aneroids-Heliostat-Ten-foot Pole-Drawing-Boards-Weights-Transfer Paper-Paper -Books - Chronometers - Marks - Boat's Fittings - Lead-linesBeacons.

Errors of instruments to be ascer-: tained.

In preparing for any surveying work, whether in a regularly fitted surveying ship or not, the first thing is to test all instruments and ascertain their errors. To do the former well, it is necessary to have an intimate knowledge of the points on which each instrument is liable to go wrong, which is only thoroughly to be learnt by experience ; but a few hints will assist the beginner.

A thorough acquaintance with the construction of instruments will save many an hour, lost by one whose instrument has gone wrong while in the middle of his work, and spent in fruitless efforts to make out where the fault lies.
No instru- No instrument, not even engine-divided protractors, can be ment. perfect. assumed to be without error, and are seldom found so, and though those errors may be small, in some cases they are of importance, and no work can be deemed satisfactory without the knowledge of how much correction should be applied, in such instances as it may be necessary to do so.
Contents We shall therefore commence by some observations on of Chapter. instruments, and on all materials and fittings required for conducting a regular marine survey, embodying in these such hints on using each instrument in general, as are likely to be
useful, and also some on choosing them that are not mentioned by Heather in his work on Instruments.*

This useful work, which should be in the hands of every $\underset{\text { Weather's }}{\text { Work on }}$ surveyor, goes so fully into the construction of instruments, Instruand in most cases into the methods of ascertaining, and, as ments. far as may be, correcting their errors, that we shall refer the reader to it on most points, adding only certain practical suggestions that are not therein mentioned.

## HADLEY'S SEXTANT.

It is not, perhaps, necessary to say much about the sextant, as so many works have already treated the subject; but there are several practical points not generally mentioned, which may be of value in selecting a sextant with a view to the work of a nautical surveyor.

Besides those noted by Heather, then,-

1. One of the eye-pieces of the inverting telescope should have a high magnifying power, about 15 diameters, as contacts of the sun's limbs in observations with the artificial horizon are far easier made the larger the suns.
2. Several dark eye-pieces should be provided, with neutral Dark eyetint glass in them of different intensities. These should be ${ }^{\text {pieces. }}$ fitted, not to screw on to the eye-piece, but ground conical, to slip on to a similar conically ground surface on the telescope eye-piece. These will be found very useful on cloudy days, as a little practice will soon enable the observer to substitute one shade for another in a fraction of a second, as clouds sweep on or off the sun, and many sights will thereby be saved. It is very important to have the suns in artificial horizon observations of the same brilliancy, and for this reason the hinged shades on the sextant should never be used for the purpose; as, in the first place, they introduce error, and also, if the shades have to be altered to suit the varying

[^1]brightness of the sun during the observation, the suns will be of different brilliancies, as these shades are never of the same tints.

By using the dark eye-pieces, the up-and-down piece,* when adjusted to equalise the suns, will bring the axis of the telescope nearly exactly in line with the edge of the silvered surface of the horizon-glass, which is the best position for observing, and from which it must never be moved until the equal altitudes or other observations are complete. No matter what depth of shade is then used by shifting the dark eye-pieces, the two images will be of the same tint.

The darker the shade used the better. Beginners are very apt to use too bright suns.

If in observing with the sun the observer can accustom himself to use one eye for taking the observation, and the other for reading and setting the vernier, he will find it very convenient, and it will tend to keep both his eyes in good order.

Position of up-anddown piece. the artificial horizon, if the up-and-down piece is so placed as to enable the observer to look over it into the horizonglass.

In many sextants the up-and-down piece is placed so close to the index glass that this is not possible, and regard should be had to this point.
4. An interrupted thread, to screw the telescope into the collar of the up-and-down piece, is a great convenience.
5. An extended vernier, i.e. a vernier whose divisions are twice the distance apart of those on the arc, will be found convenient for accurate observing.
6. A steel tangent screw will be found to last longer and work more evenly than a brass one.

The methods of ascertaining the index and other errors of Hadley's sextant, and correcting them, are so fully entered

[^2]into by Heather, that they are here omitted, with the exception of the following remarks on the centring error :-

This very important error of the sextant cannot be corrected Centring in the instrument, and it requires a considerable amount of Error. labour to settle its quantity, which in an indifferent instrument may be quite sufficient to vitiate the result of any observations on one side only of the zenith.

The centring error, pure and simple, arises from the noncoincidence of the centres of the index arm and of the graduated arc, so that the vernier does not move truly along the arc, and the angle read off will not be correct. This error varies with the angle, and is generally greater as the angle increases, but the same result of error appears from the index arm becoming bent; from any part of the frame receiving a blow which alters its shape; from the flexure of the instrument from varying temperature; and from defective graduation; but, as it is generally impossible to disentangle the errors arising from these different sources, they are all included in the one correction for centring.

Centring error is to be obtained by comparing the angle measured by the sextant with the true angle.

It is to be found roughly by measuring a series of angles carefully, by repetition, with a large theodolite, between welldefined objects on the horizontal plane at different angular distances, and then measuring the same with the sextant placed on a stand. The difference will be the centring error at each angle, index error being first applied.

The most accurate method, because it employs a large number of observations for the same, or nearly the same, angle, is by observation of pairs of circum-meridian stars in the artificial horizon, at various altitudes. Double the difference between the resulting latitude by each star, and the mean latitude, will be the centring error for an angle equal to the double altitude of that star, that is the angle actually measured by the sextant, index error being carefully determined and applied before working out.

The sign of the correction is easy to determine from a
consideration of whether the altitude is too little or too great. Thus in north latitude, if stars south of the zenith give a latitude too great, their altitudes have been too little, and the correction for centring will be plus.

It is hardly necessary to say that every precaution must be taken to eliminate other errors, such as choosing stars of a closely similar altitude, unless the latitude is already accurately known ; determining the roof error of the horizon, or eliminating it by reversion; carefully correcting the refraction for temperature, etc., and that it requires considerable accuracy of observation, and many sets to arrive at a good result. The agreement, or otherwise, of the mean latitude by each pair will form an excellent test of the general accuracy of observation, and the agreement of the resulting centring errors by different observations at the same altitudes will enable the observer to judge of the truth of his final errors. Thus every careful set of observations for latitude affords a means of testing this error.

Centring error may also be obtained by careful measurement of the angles between stars. The correct apparent distances must be found in the same manner as in clearing a lunar distance; the true distance being first calculated from their declinations and right ascensions, but if stars in the same vertical plane can be chosen, the apparent distance can be arrived at by simple application of the refractions.

There are other methods, involving more calculation, which need not be described.

The centring error is determined at Kew Observatory for certain angles by fixed collimators, and is given on every Kew certificate, but it must be remembered that in any case it can never be considered as determined for good. Including, as it does, errors from so many causes, it does not remain perfectly steady, but its amount should be ascertained from time to time for any sextant which is to be employed for accurate determination of positions, for circumstances often prevent the use of methods whereby it as well as other errors are eliminated. For instance, a latitude may have to
be obtained by altitude of the sun only, when, without knowledge of the centring error, it may easily be incorrect to as much as a minute, or even more.

As an example, the author's Troughton sextant had at $120^{\circ}$ a centring error of $-20^{\prime \prime}$. After a fall and repair by the maker, it was $+50^{\prime \prime}$.

To find the error caused by the refraction, through nonparallelism of the sides, of the coloured shades.

## Errors of

 hinged shades.Measure the diameter of the sun, with different combinations of the shades. Take out the pin which supports one set of the coloured shades, and replace the shades reversed, so that the face before next the index glass is now away from it. Remeasure the diameter of the sun with same combinations as before, and half the difference of the measurements of each set will be the error due to the shade reversed.

These errors can be neglected in sea observations, and if coloured eye-pieces are fitted as recommended above, the shades are not required when the artificial horizon is made use of.

## SOUNDING SEXTANT.

This useful form of sextant is made of various sizes. It chiefly differs from the observing sextant in being generally lighter and handier, in having the arc cut only to minutes, and having a tube of a bell shape so as to include a larger field in the telescope.

All angles in the frame of the instrument should be rounded off, especially that at the zero end of the arc. Considerable injuries may result to the face of the observer when using the sextant in a boat in a lively sea, if this is not done.

The graduation of the arc should be plain enough to read without a magnifying-glass.

The measurable angle should be as large as possible, i.e. about $140^{\circ}$.

The index glass should be large, so as easily to pick up objects.

Good tube invaluable.

The telescope should be of a high magnifying power and clear definition.

These sextants are now supplied by the Hydrographic Office with two telescopes-one for ordinary use, and another, of aluminium, with a larger object glass for occasions when faint objects are required to be seen. The collimation of these large telescopes is however a delicate matter, and when accuracy is required, should be tested.

When in good adjustment, a sounding sextant so fitted is invaluable for star observations with a faint sea horizon.

## RESILVERING MIRRORS.

On service, the mirrors of sextants, especially sounding sextants, frequently get dimmed by damp, and the surveyor must be able to resilver them himself.

A supply of tinfoil, of good quality, for this purpose, is one of the necessary stores. Mercury is always to be had. The operation has been frequently described, but it is perhaps better to repeat it.

Take a piece of tinfoil, a little larger than the glass to be silvered, and smooth it out on a perfectly flat surface, as a sheet of plate glass, or a thick smooth book-cover. This smoothing can be well done by a little pad of chamois leather, which can be kept for the purpose, or by the finger.

Drop a small bubble of mercury on to the foil, and by gentle rubbing with the pad, spread it over the former so that it shows a bright surface. Pour mercury on until the piece of foil is quite fluid, and brush any large spots of dross lightly off. Lay a piece of clean paper, long enough to handle easily, on the mercury, and the glass, previously well cleaned by means of spirits of wine, on the paper. Pressing on the glass with one hand, withdraw the paper with the other, slowly and steadily, and a pure surface
will appear under the glass, the dross all coming away with the paper.

Incline the book, or whatever surface we have been working on, so as to let superfluous mercury run off, placing strips of tinfoil at the lower edge to assist in sopping this up.

After from twelve to twenty-four hours, the amalgam will be dry, and firmly adhering to the glass. Cut the edges carefully round with a sharp knife, and varnish lightly over, either with the clear stuff used by the instrument makers, or with varnish that can be made on board, by dissolving sealing-wax in spirits of wine.

The glasses of some sextants seem fitted on purpose to invite the damp to penetrate between glass and silvered surface. These will want protection by sticking thin strips of paper along the edges exposed, and well varnishing. In some cases a stopping of thick amalgam, placed between the glass and the frame at the back, where there is one, will answer well, and prevent any damp getting at the back of the glass at all.

The mercury which remains will contain tinfoil in amal- Amalgagam, and should be preserved in a bottle by itself, mercury. draining off the thick of the amalgam by a sharply twisted paper funnel. It can then be used again for resilvering. Care must be taken not to allow any of this to get into the artificial horizon bottles, as the smallest quantity of it will spoil a whole bottle of pure mercury, and the amalgam can only be removed by evaporation.

Notwithstanding, mercury containing tin in amalgam can be used for artificial horizon work, by carefully sweeping the surface after it is poured out, with a piece of paper. Some observers have gone so far as to prefer amalgamated mercury for this purpose, but we do not agree, except when used in connection with the amalgamated trough described on page 13 .

In resilvering an horizon glass, only the portion required Horizon should be operated on, leaving one half clear. The edge of ${ }^{\text {Glass. }}$
the foil must be sharply and smoothly cut before applying the mercury, and not the smallest nick or cut permitted to remain in it.

## SEXTANT STAND.

Though a practised observer will get good observations in an artificial horizon, with a sextant without a stand, he will get them far better with one, and in all work where accuracy is aimed at, a stand should be used.

Unsteadiness of hand, to which all are so liable, from previous exertion, indisposition, and many other causes, is put out of the question by using a stand.

With star observations this is especially the case, as it is extremely difficult to hold the instrument in the hand firmly enough to prevent a little vibration of the images.

Sextant stands should be lacquered, not bright, and should have large heads to the foot screws, so as to be grasped easily while observing.

The bearing which carries the sextant should be accurately fitted into the socket in the handle, and should be very slightly conical. If too much so, it is liable to jam.

The counterbalances are usually too heavy for an ordinary sextant. They should be of such a weight as to balance the sextant without the screws at the ends of the pivot being set up too taut. Sometimes one weight is enough, or as much lead can be taken out of each as is necessary to reduce the weights to balance. The weights are now sometimes fitted to slide in and out, thus allowing of adjustment.

Stools for stand.

Small three-legged stools about 14 inches high, on which to place the sextant stand, should be made, and it will be found convenient to sink hollows in the top to correspond with the three foot screws to prevent slipping.

Other little hollows sunk in the top for the spare dark eyepieces to lie in, will also prevent these falling off, and by placing them in regular order, any one can be at once picked up without delay, when it is requisite to change them.

Another similar stool for the observer will make him comfortable, a great point for good observing.

## ARTIFICIAL HORIZON.

The glass in the roof should be of the best quality, and the faces of each pane accurately parallel.

A wooden trough to place inside the iron one is a convenience, as it raises the level of the mercury up to the height of the lower edge of the glasses in the horizon roof, a consideration where low altitudes have to be observed. The reduced area of mercury will not matter when observing the sun. When taking stars, the iron trough only should be used, as stars are more difficult to pick up, and its larger area will facilitate operations.

Three short wooden legs or buttons, fitted to the iron trough, will enable it to stand steadier on uneven ground than the four projections usually cast on the under side.

In connection with this, an artificial horizon stand is very Horizon useful. This consists of two iron plates; the lower one ${ }^{\text {stand. }}$ has three short legs on which it stands firmly; the upper one is pierced by three long large-headed screws, which serve as legs and fit into slight hollows on the lower plate, By adjusting these, the horizon laid on the upper plate can be levelled, when we have uneven ground. Four iron battens, screwed on to the upper plate so as just to permit the horizon roof to fit inside them, will prevent any wind getting to the mercury.

The horizon cover should be marked at one end, or side, Mark on and this mark should in most cases be in the same position ${ }^{\text {cover. }}$ with regard to the observer. Of this more is said under " Observations."

A new form of horizon is now being introduced, with the Amalgaobject of diminishing the waves set up in mercury by ${ }_{T}^{\mathrm{Trough}}$. vibrations.

It consists of a circular shallow trough, of metal gilt. This is amalgamated, after getting the surface absolutely
clean and free from grease, by wetting it with a few drops of dilute sulphuric acid, and then rubbing into it a drop of mercury until the whole surface is bright, when a very small quantity of mercury added will flow evenly and form a horizontal surface. The dross is wiped off with a broad camel's hair brush.

In this shallow trough waves are killed almost instantaneously. The trough should be thoroughly washed on each occasion before being used.

## THEODOLITE.

The less a theodolite is tampered with by unpractised hands the better, but they must be adjusted from time to time, and little things are constantly wanting attention.

The adjustments are well described by Heather, but as it is very important to know them, they are here given, in case the former work should not be at hand. The adjustments are-

1. Adjustments of the telescope, viz., for parallax and for collimation.
2. Adjustment of horizontal limb, viz., to set the levels on the horizontal limb to indicate the verticality of the azimuthal axis.
3. Adjustment of the vertical limb, viz., to set the level beneath the telescope to indicate the horizontality of the line of collimation.

Commence operations by setting up the theodolite as level as you can by eye, by moving the legs. See that the legs are firm, and everything tight. Set all levels as true as you can, by the parallel plate screws and vertical arc tangent screw.
Adjust- Parallax is occasioned by the image formed by the objectment for Parallax. glass not falling exactly on the cross-wires.

First adjust the moveable eye-piece until the cross-wires are sharply defined. Then obtain the proper focus for the object by moving the milled head on the telescope.

This will throw out the image of the cross-wires, and the eye-piece must be again adjusted, until cross-wires and object are both truly in focus.

This has to be done each time the theodolite is set up, and is therefore only a temporary adjustment. The others are more permanent.

Collimation is effected by directing the telescope on some Adjust-well-defined point, and bringing it to coincide with the intersection of the wires, with the level downwards. ment for Collimation.
Turn the telescope in the Y's, until the level is uppermost. If the object is still at the intersection of the wires, the collimation in altitude is correct.

If not, bring the wires half way towards the object by turning the screws holding the diaphragm. Then re-set the telescope by the tangent screw for the object, and bring the telescope round in the Y's to its former position, when any displacement still existing must be corrected in the same way, half by the diaphragm screws, and half by the tangent screw. After a few trials the error should be corrected.

Do the same with the telescope with level right, and level left, at right angles to its former positions in the Y's, for azimuth error. When this is done, the cross-wires, while the telescope is slowly revolved, should remain over the object.

* The collar being tightened by its clamping screw, unclamp Adjustthe vernier plate, and turn it round till the telescope is over ${ }_{\text {Horizontal }}^{\text {ment of }}$ two of the parallel plate screws. Bring the bubble of the level Limb. beneath the telescope to the centre of its run by turning the tangent screw of the vertical arc. Turn the vernier plate half round, bringing the telescope again over the same pair of the parallel plate screws ; and, if the bubble of the level be not still in the centre of its run, bring it back to the centre, half way, by turning the parallel plate screws over which it is placed, and half way by turning the tangent screw of the vertical arc. Repeat this operation till the bubble remains accurately in the centre of its run in both

[^3]positions of the telescope; and then turning the vernier plate round till the telescope is over the other pair of parallel plate screws, bring the bubble again to the centre of its run by turning these screws. The bubble will now retain its position, while the vernier plate is turned completely round, showing that the internal azimuthal axis, about which it turns, is truly vertical.

If the bubbles of the levels on the vernier plate are now brought to the centre of their tubes, by means of the screws fitted for the purpose, they will be adjusted to show the verticality of the internal azimuthal axis.

Now, having clamped the vernier plate, loosen the collar, by turning back the screw, and move the whole instrument slowly round upon the external azimuthal axis, and, if the bubble of the level beneath the telescope maintains its position during a complete revolution, the external azimuthal axis is truly parallel with the internal, and both are vertical at the same time; but, if the bubble does not maintain its position, it shows that the two parts of the axis have been inaccurately ground, and the fault can only be remedied by the instrument maker.

Adjustment for vertical limb.

To adjust for the vertical limb, the bubble of the level being in the centre of its run, reverse the telescope, end for end, in the Y's, and if the bubble does not remain in the same position, correct for one half the error by the capstan-headed adjusting screw at one end of the level, and for the other half, by the vertical tangent screw. Repeat the operation till the result is perfectly satisfactory. Next turn the telescope round a little, both to the right and to the left, and if the bubble does not still remain in the centre of its run, the level must be adjusted laterally by means of the screw at its other end. This adjustment will probably disturb the first, and the whole operation must then be carefully repeated. By means of a small screw, fastening the vernier of the vertical limb to the vernier plate over the compass box, the zero of this vernier may now be set to the zero of the limb, and the vertical limb will be adjusted for horizontality.

The vertical limb should move in a truly vertical plane. AdjustAny error can only be adjusted in the larger instruments, , mertical for but every theodolite must be tested for it, as, if much error plane. exists, the instrument requires alteration by the maker. It will introduce error into all angles to objects much elevated or depressed, and it is especially important for observations for true bearing to know that this adjustment is perfect.

To test it, direct the theodolite when horizontal to either the edge of a well-built wall, or still better, a steady plumbline. The cross-wires, when the instrument is elevated and depressed, should still intersect the line.

If they do not, in 6 -inch theodolites, the adjustment can generally be made by means of screws on one of the Y frames. In smaller theodolites we must accept the error, and take oare not to use them for true bearings.

These adjustments completed, the instrument will be ready for work.

There are, however, a variety of small points on which a Points theodolite may go wrong while away in the field, and a know- liable to ledge of the general causes of these temporary derangements ment. is very useful, and may prevent loss of a day's work, and much aggravation to all concerned.

The parts of a theodolite, especially in an old instrument, that most frequently get out of order, are the small screws which hold the milled heads of the tangent screws in their places. A young observer is often much bothered and puzzled by his instrument not coming back to zero, which may result from many things, but most frequently from one of the small screws above mentioned being loose. Screwing it up tight enough to prevent any play when the instrument is clamped, but not so tight as to make the tangent screw work hard, will often remove the difficulty.

Other causes of not coming back to zero are :-

1. Looseness of the sockets through which the tangent screws work, and which can be easily tightened by their screws.
2. Looseness of the fittings of the brass stand on the theodolite legs. There are many working parts here, and any
of them are liable to get loose. The leverage on the brass plate that fits on each leg head is enormous, if the leg should be allowed to swing out in taking off the rings; and as the screws that hold them on are small, looseness may easily take place here.
3. In an old instrument, the faces of the clamping plates may screw close together without clamping the instrument tightly. This is from the part that holds the instrument, and which gets all the friction, being much worn. The parts into which the clamping screw fits must be smoothly filed on their inner faces, so as to ensure the other parts coming into contact with the body of the instrument, before the faces of the clamping plates meet.
4. The upper plate will sometimes not revolve freely, but catches every now and then. This is from the piece of metal which clamps the two plates together either not fitting very well, or being dirty inside, or perhaps bent. Placing the finger underneath so as to press it up to the lower plate, whenever the plates are to be revolved, will ensure its working smoothly, and is a better thing for a young hand to do than to attempt to take it off. The same thing will happen to the reading-glass plate; but here it is often the little screw underneath which is loose, and simply screwing it up will relieve it. Lifting it with the finger will always assist it to run round easily.
Putting in An operation the nautical surveyor has frequently to pernew webs. form is replacing the wires, or rather cobwebs, of his theodolite telescope.

For this purpose catch a garden spider, as a house spider does not spin his rope taut enough. Having cut some holes, say two inches square, in a strip of cardboard about three inches wide, place the spider on it, and shake him off. As he throws out his web in falling, twist it up on the cardboard so as to cross the holes, and lay it on one side.

Having taken the diaphragm from the telescope, and scraped off the old balsam, lay it on the table and place the smallest drop of Canada balsam on its edges. With the aid
of a magnifying glass, place the cardboard across it, in such a manner that the web will lie in the notches cut in the diaphragm, when it will adhere to the balsam.

Heather gives a good description of measuring angles with the theodolite, to which we will add, that,

Regard must be had to the purpose for which the angles Measuring angles with the theodolite. are to be taken, in settling how many times, and in what manner, the angles shall be repeated. An error of two minutes will make no perceptible difference when plotted, unless the line be very long, say five feet. All objects, therefore, that are simply to be plotted, and do not come into the triangulation, can be taken round once with zero at $360^{\circ}$, and a second round taken after with another zero, say $100^{\circ}$ for convenience' sake, simply for the purpose of making sure that there are no gross errors, as no theodolite in adjustment should give an angle in error as much as two minutes.

Angles to main stations, however, will be very likely Repeating required to enter into the calculation, and the correctness of angles. the plotting will any way depend on them. These must therefore be repeated, the number of times varying according to the degree of accuracy required.

One method of repeating angles is thus given in Heather, somewhat altered.

Having taken the first measurement, loosen the clamp of First the lower plate, turn the theodolite bodily round until the telescope is directed upon the zero-object, and again clamping the instrument, perfect the bisection of the zero by the crosswires by means of the slow-motion screw on the neck of the instrument. The index of the vernier, together with the coincident division of the limb, will thus have been brought from the position in which it was when the telescope pointed at the object to be measured, round to the previous position of the $360^{\circ}$.

Now release the upper or vernier plate (looking again at the vernier first to see it has not been moved), turn it until the telescope is again directed towards the object, clamp and
perfect the bisection by the tangent screw moving the upper plate. The reading now on the vernier will be twice that formerly read off, or nearly so, and will be entered in the book under the former observation. This process can be repeated as often as required. The mean angle can be obtained by dividing the last reading (increased by as many three hundred and sixty degrees as the plate has revolved) by the number of observations, but it is better for our purposes to put down each individual reading. The difference between every two consecutive readings will give a value for the angle, and we can then see how they agree with one another. An example of this kind of repeating is given on page 78.

The above method is perhaps the most accurate ; but, when many angles are to be taken, requires much time, and we shall arrive at a conclusion quite near enough for any hydrographical triangulation by taking all the angles in succession with the vernier set to $360^{\circ}$, and then, changing the degree of the zero to some even submultiple of $360^{\circ}$, as $90^{\circ}, 180^{\circ}$, etc., take all objects again, repeating thus as often as necessary, which will be found much quicker. $360^{\circ}, 120^{\circ}$, and $240^{\circ}$ divide the arc equally and give three readings, which is often sufficient. The other method can be reserved for taking single angles, as, for example, a flash from a distant station.

If both verniers are read, any error arising from bad centring should be eliminated for any given position of the plates.

For practical hydrographical purposes if one vernier is read, with the zero in several positions, submultiples of $360^{\circ}$, it is as a rule sufficient.

Different to the sextant, the theodolite has no index error to apply to horizontal angles, but to the vertical are there is a correction to be found and applied, which will be mentioned in discussing the method of ascertaining heights.

A theodolite for hydrographical purposes should be fitted

Coloured shades to eye-pieces. with coloured shades to the eye-piece of the telescope for observing the sun for true bearing.

## STATION POINTER.

This useful instrument is of hourly service in nautical surveying.

Either in sounding, coast-lining, or topographical plotting, the position of the observer depends mainly on it.

The station pointer is used to plot a position on the chart, Theory of by means of angles taken at it, to other objects already fixed. $\begin{aligned} & \text { Station } \\ & \text { Pointer. }\end{aligned}$

Its construction depends upon the fact that the angles subtended by the chord of the segment of a circle measured from any point in the circumference, are equal. (Euclid III. 21.)


Thus, in the figure, the angles A D B, A E B , A F B are all equal, so that if we have observed the angle subtended by A B, we know at any rate that we are somewhere on the circumference of a circle, the size of which depends on the angle observed.

To draw this circle, we take advantage of the fact that the angle at the centre of any segment is double the angle at the circumference. (Euclid III. 20.) We lay off, therefore, from either end of the line whose subtended angle we have observed, the complement of the angle. The point where these lines meet is the centre of the circle, which we describe with the distance from this centre to either end of the line, as a radius.

Thus if our observed angle is $64^{\circ}$, we lay off A G, B G each making an angle of $26^{\circ}$ with A B, and describing the
circle with centre G and radius $\mathrm{A} G$ or $\mathrm{G} B$, we get the circle we want, for $A G B=180^{\circ}-(B A G+G B A)$
$=180^{\circ}-52^{\circ}$.
$=128^{\circ}$.
And as $\mathrm{A} \mathrm{GB}=2 \mathrm{AEB}$,
the angle A E B and all other angles on the circumference will be $64^{\circ}$.

If the angle observed is more than $90^{\circ}$, we describe the circle by laying off the number of degrees over $90^{\circ}$, on the opposite side of the line to that on which we know we are, and proceed as before.

If we can obtain, besides the angle subtended by $A B$, the

one subtended by BC , another line, one of whose ends is identical with A B, we can draw another circle on whose circumference we must also be, and the intersection of these two circles must be our exact position X , as it is the only one from which we could have obtained these two angles at the same time. See Fig. 2.

The station pointer obtains us this position X without the trouble of drawing the circles, as it is manifest that, if we have the angle A XB on one leg of the station pointer and BXC on the other, the only spot at which we can get the three legs to coincide with the points $\mathrm{A}, \mathrm{B}$, and C , will be X .

We place the station pointer, therefore, on the paper, bringing the chamfered edges of the three legs of the instrument to pass over the three points observed, and make a prick with a needle in the nick in the centre, which will then mark the spot.

A piece of tracing-paper on which the three angles are protracted will answer the same purpose, but, of course, this will entail more time, and in the open air will give trouble, as liable to be blown about by the wind. Nevertheless, this has often to be used, as when points are close together on a small scale, the central part of the station pointer will hide them, and prevent the use of the instrument.

A very useful instrument has been devised by Commander Cust for such occasions, and consists in a plate of transparent zylonite on which a graduated arc is engraved. The requisite angles are drawn on this with pencil, with the angles reversed, and the plate being turned over, so as to bring the pencil lines in contact with the paper to obviate parallax, is used as a station pointer.

This method of fixing is generally known as the "two Three circle" method, but it is really the "three circle" method, for the circle drawn through the two outer points and the observer's position is also involved. A comprehension of this is of value.

The chance of error in a "fix" varies greatly with the Position of position of the three points with regard to one another and "Points." the observer.

It is in general sufficient to realise that the more rectangular the intersection of the two circles, the less chance there is of any error in the resulting fix, but there are cases where the fix is admirable though these circles are almost tangential, because the third and larger circle produces a rectangular cut.

With points and the observer's position placed as in Fig. 2, the two circles give a good intersection, and the fix is good. Let us, however, take the same points with the observer's position close to the centre object B, as in Fig. 3. We there
see that the two circles are nearly tangential, but the third circle through the outer points and the observer, which the station pointer also gives us, cuts at a right angle, and as the position X cannot be off it, the fix is one of the best.


In such a case the whole angle between A and C should be observed, if not too large (as in our figure), as the accuracy of the fix depends entirely on this whole angle, and when so near to B a little movement may make considerable difference in BXA, and BXC are separately measured.


Let us now take three points and the observer's position as in Fig. 4, using the same letters.

The angles we have observed give us X as the point of intersection. It is evident that it is difficult to localise this point exactly, as all three circles so nearly coincide as make it impossible to say where the precise point is at which they
intersect, and, with the station pointer, we should find that we could move the centre of the instrument considerably, without materially affecting the coincidence of the legs with the three points.

When X is so placed as to fall on a circle passing through the three points ABC, there will be no intersection whatever, as the two circles will coincide; and we cannot tell where we are on the circumference of this practically single circle.

The nearer, therefore, we are to being on a circle, whose circumference will include all the three points and our own position, the worse will be what is technically called the "fix," and this must always be guarded against in selecting objects to observe.

When one object is farther from us than the central one, we shall, as a rule, have a good fix; but when the central object is the farthest, the two circles will begin to make a bad intersection.

There is nothing in the whole range of surveying that General requires so much attention and knowledge as the "fix," and ${ }^{\text {Rules. }}$ many are the errors which have crept into surveys from disregard of its conditions.

When moving along, as when sounding, and fixing from time to time, if both angles change slowly, the fix will be bad, for we must be moving nearly along the circumferences of both circles, and they must therefore nearly coincide.

In plotting the angles with the station pointer, the fix will be good if a very slight movement of the centre of the instrument throws one or more of the points away from the leg; but if this can be done without disturbing the coincidence of the legs and all three points appreciably, the fix is bad.

This is perhaps the most important thing for a beginner to remember and to practise, as it is a practical test involving no theory nor complications.

Theoretically, one of the best positions is inside the triangle formed by the objects, but in practice it is often
impossible to observe the large angles incidental to this position.

Practically the beginner will find the following rules safe :-

1. Never observe objects of which the central is the farthest.
2. Choose objects disposed as follows:-
(a) One outside object distant and the other two near, the angle between the two near objects being not less than $30^{\circ}$ or more than $140^{\circ}$. The amount of the angle

- between the middle and distant objects is immaterial.
(b) The three objects nearly in a straight line, the angle between any two being not less than $30^{\circ}$.
(c) As before remarked, that the observer is inside the triangle formed by the objects.
There are, nevertheless, cases where the middle object is very distant, when the fix will be good enough for many purposes, but such cases require a thorough grasp of the subject, and should not be adopted by the beginner unless forced to it.

Size of angles admissible.

The size of angles admissible in a good fix depends on the position of the three objects. If two objects are equidistant, the angle must not be small, for a slight error in the angle will make a great difference in the position; but if one object be much farther off than the other, a very small angle between these will suffice, so long as the third object is so placed as to make a fairly large angle.

An arrangement of the objects not yet considered, is when two of them are in line from the observer's position.
"Points" in Transit

This is technically called " transit," and no transit of known marks is allowed to take place without making use of it.

One angle to a third object is here enough to fix the position, which is one advantage, another being that if two angles are taken and placed on the station pointer, the coincidence of the position, as plotted by these two angles, with the transit line, gives an excellent check.

Here, Fig. 5, A and B are in line of transit $(\phi)$; C is a third object.

It will be evident that when the observed angle is on the
station pointer, and the latter is placed with one leg coinciding with the line A B, that we only have to move it up or down that line, until $C$ coincides with the other leg, which gives us X . Any other position, as $\mathrm{X}_{1}$, would not allow the leg to pass over H.

It will also be seen that the farther apart $A$ and $B$ are, the truer will be the direction of the transit line. If one object was at $B_{1}$, the position pricked through at $X$ might be a little right or left of the true transit line, without the deviation being visible on the leg of the station pointer.

Also, it will be seen that the angle to the third object should

be as near $90^{\circ}$ as possible, anything under $25^{\circ}$ being inadmissible, as the angle of intersection at X would permit of a false position without detection.

When using this method, the distance of the third object should also be considered. It must not be too far, or both theoretically and practically, by reason of the imperfection of instruments, the fix may be in error.

In choosing a station pointer, of which instrument Heather Choosing gives but a meagre account, the first important thing to look $\begin{gathered}\text { a Station } \\ \text { Pointer. }\end{gathered}$ at is that the smallest angle to be read on the leg which will not come to zero, is as small as it should be. A well-planned modern station pointer should allow this leg to return to $3^{\circ}$
or $4^{\circ}$, but old instruments frequently will not read less than $12^{\circ}$.

It is a great nuisance to find that the only angles you can take cannot be plotted by means of your station pointer, and the chance of this should therefore be minimised as much as possible.

Station pointers are generally made to allow the left angle to come to $0^{\circ}$.

When the angle on the right is too small to set, and the left angle is more than $90^{\circ}$, the difficulty can be got over by setting the small angle on the left leg and bringing the right leg round to the left until the required left angle is made between it and the left leg.

Another method when the above cannot be carried out, but care must be taken not to employ it when exact accuracy is required, is to set the angles on the station pointer reversed, i.e. the right angle on the left, and vice versâ. Place the legs containing the larger angle on its "points," with the centre near the supposed position, and make a prick. Move the centre a little, right and left of the first prick, keeping the points on, and make other pricks. A line drawn joining these pricks will form an arc of the circle for those "points."

Now do the same for the small angle and its "points," and the intersection of the two small arcs will give the position required.

It is, in fact, projecting the circles by means of the station pointer.

Station pointers are made with brass, and with silver arcs; the latter are of course more durable, but for many purposes the brass are to be preferred. When sounding, or doing any work in the open, the reflection from a silver are is often a bother, and hinders speedy setting of the vernier. The use of a reading-glass is almost a necessity with silver arcs for this reason, and also on account of the fineness of the cutting; whereas, with the brass arcs, a surveyor with good eyes can set his instrument quite correctly without one, a great point in a boat.

For chart-room use the silver arc is to be preferred.
The nick in the centre of the instrument should be small, i.e. just deep and wide enough to admit of a needle fairly catching in it. The needle-pricker should always be used for marking the position; not a pencil-point, which soon wears blunt, and will not mark truly in the centre. The prick also remains, and can be seen under the figure with a readingglass, when inked in.

The prick should be on the continuation of the edge of the bar in which is the nick.

For ordinary soundings and field work, a station pointer of about 5 inches diameter of are is most convenient. For ship sounding and chart-room work larger ones are supplied.

In testing a station pointer, the first thing is to see that the Testing a vernier of the leg which comes back to zero reads exactly $0^{\circ}$, Station using a magnifying-glass to read off accurately. If it does not, the screws which hold the vernier must be loosened slightly, and the vernier plate moved, until the arrow on the vernier corresponds exactly with $0^{\circ}$ of the arc, and the $30^{\prime}$ on the vernier with a division of the arc.

Take either a large sheet of backed paper, or a white Bristol Testing board, and mark out, by means of chords, lines radiating from a centre, and $10^{\circ}$ apart. These lines must be very carefully ruled, and in Indian ink, as this sheet must be kept as the test of all station pointers and protractors, which should be from time to time examined by its means. Screwing on the lengthening legs, and placing the station pointer on this sheet, with the nick in the centre of the arc corresponding exactly with the prick in the centre of your testing circle, and putting weights on the central part of the station pointer, each leg can be in turn moved to correspond with the ruled $10^{\circ}$ lines, and the reading of the vernier compared. The error at each $10^{\circ}$ should be written on a small piece of paper in the form of a table, and pasted on the inside of the box.

If the legs of the instrument are exactly centred, the readings will either be correct, or the same amount in error all round, for each leg; but as this is a degree of delicacy
rarely attained, it will usually be found that the error varies for different positions of the leg. The verniers should be set to minimise the errors between $0^{\circ}$ and $90^{\circ}$, which is the amount of angle most used in actual work.

The chamfered edge of the leg and lengthening piece should correspond exactly with the line in all its length; if it does not, it is also a result of bad centring or bad fitting on of the lengthening piece, but a good instrument should not have this error in any appreciable extent.

It need scarcely be added that if the instrument is found very badly centred, it should be returned to the maker, or not be chosen if buying; but when an instrument is sent to the other end of the world, you may have to make the best of it, and registering all the errors on the table, be careful to apply them when using the instrument.
Discretion The necessity for applying a small error depends upon
as to applying errors. circumstances, as in some cases, the position of the points used will admit of a difference of several minutes in the angle, without any appreciable alteration of the position of the observer ; in others, it is necessary to be exact. As the surveyor gains experience, he will learn when to apply the error, and when not. At the commencement, he must always apply the error.

Caution as to use of Station Pointer.

It may here be noted, that, if the points used to fix by are not correctly placed on the chart, the station pointer will not indicate anything wrong, unless a third, or "check" angle, be taken and plotted. This must always be remembered in using a station pointer on a published chart, or the adoption of this instrument may have a disastrous result.

In the first place, the chart may be from a rough survey, and there may be absolute errors in the points on it; and secondly, the distortion caused in printing with damp paper always changes the position of points, more or less, and with objects in certain positions, this alone may make an error in a station pointer fix.

In navigating, therefore, with a published chart, of the accuracy of which you are not certain, always use a bearing,
as well as the sextant angles plotted by station pointers, or use check angles to each fix. If the result is to show that the points are not correct relatively to one another, use the compass only, as it is less likely to get you into trouble with a defective chart, for the reason that the non-intersection of three bearings will at once indicate something wrong, and the navigator will choose the points of danger in his course ahead to steer by, rejecting the others whose positions with regard to him are of little moment.

## BRASS SCALES.

These must be examined by means of the beam compasses, to see that their divisions are correct, more especially the diagonal portion, as the makers are sometimes not careful enough. If a scale is found to vary, it should be rejected.

A brass scale should never be used for ruling, and never be taken out of its box. If it is, some day it will fall from the table, get bent, and its correctness is gone.

## STEEL STRAIGHT-EDGE.

This must be examined to see if its edge is exactly straight, by ruling a very fine line, and reversing the straight-edge, when, either ruling another line over the first, or examining the coincidence of the edge with the line already ruled by means of a reading-glass, will prove whether it is perfect.

Placing steel straight-edges edge to edge is another method when there are more than one; but great care must be taken with regard to the light if this is done, as it is difficult to detect a small error if the light falls across. They ought, of course, to touch throughout their whole length.

A steel straight-edge must be kept very clean, and carefully wiped before using, or the paper will soon become very dirty.

If kept bright, care must be taken that no emery is allowed to touch the chamfered edge, or it will get so sharp in time
as to cut the pencil, and even the fingers of the operator. When once clean, rubbing daily with a warm dry soft cloth will keep it so, with an occasional rub of emery in damp weather.

Straight-edges are now generally supplied nickelled.

## measuring CHAINs.

To test measuring chains, which are generally 100 feet long, a hundred feet should be accurately measured along a plank of the upper deck, and marked by nails driven in.
Always to Before measuring a base, all the links of the chains should be re- be examined, and bent ones straightened; the chains are then
tested when used. compared with this fixed length and the errors noted. The same reference should be made after measuring, and the mean of these errors applied to the distance measured.

It may be as well to note that, when the chain on comparison proves to be longer than 100 feet, the surplus is to be added to each length measured, and when it is shorter, subtracted.

Points of measurement.

The length is to be measured from the outer side of one handle, to the inner side of the other. This is to allow for the necessity of having a pin to put in the ground at each length.

Each link is a foot long, and every tenth link is marked by a brass label, with as many fingers on it as there are tens of feet from the nearest end.

## PROTRACTORS.

Protractors of all kinds must be tested for correctness of division by the same testing-sheet ruled for the station pointers.

This is especially necessary in the case of Bullock's protractors, which have extended arms, generally of very light construction, which a slight blow will bend out of the direct line. These sometimes admit of correction by means of
screws, which is easily accomplished by placing the protractor on the testing-sheet, with the opposite verniers exactly coinciding with the same line, and adjusting the extending points until they also prick precisely on the line.

If the divisions of an ordinary protractor are found to be incorrect, there is of course nothing for it but either to return it to the maker to be re-cut, or to mark the errors at each ten degrees, or wherever necessary, on small bits of paper pasted on the protractor.

A boxwood or vulcanite protractor is easily kept clean by rubbing it over with a piece of india-rubber, but a brass or electro-plated one is very apt to dirty the paper in plotting. It is a good plan to carefully paste a piece of tracing-paper on the under-side of these, when a rub with the india-rubber before use will ensure cleanliness. The thinness of the tracing-paper will not interfere with correctness in laying off the angles.

Vulcanite protractors are admirable for field work, as they are light, easily read, and when made thick do not chip like boxwood ones.

Large brass circular protractors of 10 inches or so radius are very useful for laying off the secondary points of a survey, saving the time involved by using chords.

## POCKET ANEROID BAROMETERS.

These are very useful when putting in the topography of a country, as they give sufficiently accurate results for minor heights, with but little loss of time; but for the more exact measurement of conspicuous hills, \&c., they are but of little use, and the theodolite and sextant must be had recourse to.

In choosing a pocket barometer for the above-named service, therefore, it is not necessary that it should read very low, as it will be but rarely that nautical surveyors have to deal with the intricacies of land over a few thousand feet high. For this purpose, 25 inches is quite low enough, and the five inches of barometric range thus obtained can be
so largely marked on the dial as greatly to facilitate the reading to two places of decimals.

## Index Needle.

An important point for delicate reading is the construction of the index needle. This should be very thin towards the point, and turned with its edge at right angles to the plane of the dial. In this position it should admit of very accurate reading, and moreover assists the observer to hold the instrument at right angles to his line of sight, and thereby to avoid parallactic errors. For this reason the point, though as thin and fine as possible one way, should be tolerably wide in the other, so as to show plainly by its apparent increase of width when it is being looked at from any direction but at right angles to the dial.

The point of the index needle should cover about half the graduation of the arc. If it is too short to reach to it, or so long as to project over it, it is not so easy to read accurately.

Reading

## an

Aneroid.

In reading, the best position for the aneroid to be held is upright, on a level with the eye, the index being vertical. Read with one eye only, or parallax will creep in. Tap gently each time before reading, and turn the instrument flat, and then vertical again for a second reading, to prevent mistakes, tapping as before. In whatever position, however, the instrument is read the first time, it must be always held for all other readings on that day, the reason being that the weights of the different parts in such a delicately made little instrument have considerable influence on its free movement, and that this influence must be so disposed as to act in the same manner at each reading.
Care of
Aneroid.
The surveyor will of course never let the pocket aneroid out of his own possession, and will place it about his person in such a manner as to minimise chances of shocks or blows in scrambling through rough country, getting wet in rainy weather, or tumbling out of the pocket. To prevent latter accident, always use a lanyard. The instrument should always be carried in its case.

If a small aneroid is not in regular use, the delicate internal
parts, especially the chain, are liable to stick from oxydization, when at length taken up a height. It is therefore convenient, if an air-pump be on board, to place the instrument under the receiver from time to time, so as to keep all working parts in order.

## HELIOSTAT.

This instrument, which is simply a mirror mounted in Most valugimbols, so as to turn and reflect the sun in every direction, is ${ }_{j}^{\text {abunct for }}$ of great use. In a survey where many assistants are at work together, it saves an immense amount of time. Smaller ing beacons or marks can be erected, and the position of a theodolite station that has to be made on the side of a hill, or with dense foliage behind it, is at once made apparent to another observer, who has to take angles to it by the flash, which can be seen a long distance by the naked eye.

Some heliostats supplied are mirrors in gimbols, mounted Fitting. either on stands or in portable cases, with a spike to drive into the ground.

Neither of these forms is satisfactory, as in many places from which it is desired to use them they cannot be conveniently and firmly placed. Tripod legs of some description on which to place the mirror are best, and a movable arm working round the centre, and carrying an adjustable ring through which to direct the flash, will be found very handy. If the surveyor has to trust to placing some separate object, such as a stick or another tripod, a few feet from the mirror, by which to direct his beam of light, he will soon find himself in some position where there is no standing-ground for such object, as when his theodolite is on the top of a sharp hill, or on a steep coast-line under cliffs at the edge of the sea.

A better instrument is the excellent and convenient Galton's Galton's Sun Signal, now also supplied. This is fitted with sun a telescope, by looking through which and adjusting the mirror, a dim image of the sun is seen covering the object
required to flash to. Nothing can be better adapted to the purposes of the nautical surveyor's work than this (when he is once accustomed to it, as at first it is a little awkward to manage), and when obtainable they should always be used. Care must be taken, however, that the instrument is in adjustment. This can be ascertained as follows: Place a board, with a sheet of white paper pinned on to it, about 50 yards off. Direct the sun signal flash on to it, and looking through the telescope, screen and unscreen rapidly with the hand the direct flash from the mirror. If the circular image formed by the direct flash on the sheet is not coincident with the image of the sun as seen through the telescope, take off the cap at the end of the tube and adjust with the screw that will be found underneath.

A very workable arrangement can be fitted on board any ship as follows:-

Instru-
ment
easily im provised on board.

A blacksmith will soon make a frame which will convert an ordinary looking-glass into a perfect instrument for surveying work, as it must be remembered that we do not propose to use it for talking, and therefore do not require the extreme accuracy in directing the beam necessary in the military heliograph.

The sketch annexed shows a looking-glass fitted in this manner by a ship's blacksmith. The standard can be made of any height as convenient; about two feet and a half is a good length. In soft ground the end of the legs can be pressed into the earth, and on rocky ground stones placed against the legs will hold the instrument steady. The arm, $m$, of light iron, is carried separately, and slips over the shaft of the standard, clamping where required with a screw.

Into a circular socket in head of standard shaft, the leg of the frame holding the mirror is shipped; this is also to be tightened by a retaining screw.

The mirror, which can be of any size from 2 to 6 inches or more in diameter, revolves on its retaining screws, as an ordinary toilet-table glass, and can be held in any position by tightening these screws.

The ring, of flat wood, is made as light as possible, so as to exert less strain in wind. Across it are nailed crossed strips of copper, with a white cardboard disc, about an inch in diameter, fastened to their centre.

The rod that carries this ring slips up and down in a hole at the end of the arm, and is clamped by a retaining screw.

FIC 6.


LOOKING-GLASS AS FITTED BY BLACKSMITH FOR HELICSTAT.
a. Sliding collar carrying arm $m$, revolving round S .
b. Wooden ring, painted black, with cross-wires and white cardboard centre, sliding vertically by means of rod through arm $m$.
c. Iron frame to hold mirror, fittling into socket in top of Standard S.
S. Iron standard with fixed tripod legs.
d. Blind spot in mirror.
e. Screw for c'amping mirror frame.
$f$. Screw for clamping arm.
g. Screw for clamping ring rod.

In the centre of the back of the mirror, a hole of about $\frac{3}{4}$-inch diameter is scraped in the tinfoil, being careful to leave a sharp edge. A similar hole is cut out of the wooden back of the glass frame. This we shall call the "blind spot."

To direct the flash to an object, bring the mirror vertical, and looking through the hole in the centre, revolve the arm until in the direction of the object nearly, clamp it, and
adjust the disc rod as nearly as may be, for elevation or depression. Then, slightly loosening the screw clamping the arm, finally adjust the latter, so that the object, as regarded through the hole in the mirror, is obscured by the white cardboard disc in centre of the ring. By turning the mirror so that the dark shade caused by the blind spot is thrown on to the disc, the flash will be truly directed, and must be kept so by slight alterations of the position of the mirror, which should therefore be clamped only sufficiently to hold it steady, and yet admit of gentle movement. The shadow of the blind spot should be slightly smaller than the disc, so as to ensure having it truly in the centre of the latter.
Best glass The mirror must be of the best glass, with its faces necessary. parallel, or the shadow of the blind spot will be very indistinct when the mirror is at a large angle, and also the beam of light will be dispersed before it has traversed many miles.

## Size of Mirror.

Power of penetration of flash.

It is well to have the mirror a fair size, say 6 inches square, as in practice it will be found generally necessary, in order to save time, after once adjusting the flash, to leave a bluejacket to keep it on, while the surveyor is taking his angles ; and although a man will soon pick up the knack, a larger mirror will allow for eccentricities on his part, and also, on a dull day, a faint flash will be detected from a large mirror, where a small one would not carry any distance.
On a bright day, a flash from a $3-\mathrm{in}$. by $2-\mathrm{in}$. mirror has been seen 55 miles and more.

In hazy weather, angles have been got when the place from which the flash was sent was entirely invisible ; and thus whole days have been saved by this simple contrivance.

Only those who have spent hours, or even days, in straining their eyes to see a distant mark, can appreciate the value of a heliostat.

## TEN-FOOT POLE.

For coast-lining, a pole of measured length is often required, to get distances by observing the angle subtended by it.

A convenient form is as follows -
Two oblong wooden frames, about 18 in . by 2 ft ., are made as light as possible, and covered with canvas. These will fit, by means of sockets at the back, on to the ends of a pole, and copper pins passed through socket and pole will keep them at a certain fixed distance apart. Ten feet is a convenient length for transport.

The face of the canvas on the frames is painted white, with a broad vertical black stripe in the centre, and the ten feet will be measured from centre to centre of the black stripes.

In measuring with a sextant the angle subtended by such a pole, the image of one stripe will be brought to cover the other stripe.

A table of distances, corresponding to the angle subtended
FIG 7.

by the length of the pole used, should be in each assistant's possession for reference on the spot.*

Fig. 7 represents a ten-foot pole.

## DRAWING-BOARDS.

In a surveying vessel it is convenient to have a considerable number of these, and of various sizes, so as to fit all scales. The largest may be about 29 in . by 25 in . The size of which most will be wanted will be about 27 in . to 25 in . by 20 in . to 22 in . There should be some smaller ones, 22 in . by 16 in.
Lightness, combined with sufficient strength not to warp, is the requisite, and seasoned wood is therefore necessary.

[^4]White pine, three-quarters of an inch thick, is as good as anything, though the smaller boards may be made of thinner mahogany.

Duck covers to fit the boards are necessary for field work, when the work is plotted at the time, to carry them in, and prevent rubbing and wetting from rain.

If it is intended to do most of the detail plotting in the field and boats, there should be about three or four boards to every assistant in the survey.

## WEIGHTS.

The weights supplied by the Stationery Office are of iron, flat, oblong, and covered with leather.

Drum-shaped leaden weights to supplement these, covered with duck or baize, will be found very handy. These can be of various sizes and weights, to suit all requirements. Cylinders $2 \frac{1}{2} \mathrm{in}$. in diameter and $1 \frac{1}{2} \mathrm{in}$. in height are an average size, and can be cast on board in a wooden mould.

A few others heavier are useful, and some flat weights, $2 \frac{1}{2} \mathrm{in}$. in diameter and $\frac{1}{3}$ of an inch thick, are good for keeping down small tracings.

## TRANSFER PAPER.

This must be made, not bought, as the stuff sold by stationers always has some oily material in it.

On to a damp sheet of tracing-paper scrape finely some blacklead, and rub it well in with the hand, a little at a time, allowing it to dry between each application. Rub off the loose particles before rubbing in more. The blacklead is only to be applied on one side of the tracing-paper. It must be done as evenly as possible, so as to ensure uniformity in the tint, but this is assisted by a good rub with a soft cloth when the sheet is finished. Two or three applications will be sufficient.

A lump of blacklead for this purpose is supplied to all
surveying vessels, but the lead from a soft pencil will answer as well.

It is a dirty process to the operator, but in a few hours enough can be made to satisfy the requirements of the survey for a long time.

## MOUNTING PAPER.

The consideration of what to plot the intending chart on must be undergone before commencing work. Two methods are in use.

To stretch, and firmly paste or glue the paper, on a drawing-board or table, where it must remain until the chart is complete; or to plot on a piece of drawing-paper mounted on holland or calico, and simply flattened out before use.

The advantage of the former is that the paper remains flat, Loose and free from wrinkles or movement, the whole time work is $\frac{\text { sheets }}{\text { best with }}$ being done on it; but for many reasons, the latter is most large staff. convenient for ship work. If many sheets are under weigh at one time, which frequently occurs in an extended survey and large staff, they take up less room, and interfere less with one another, when several persons are working at one table, than when sheets mounted on boards are used, and they are easier put out of the way. If the plotting sheet is very large, and formed of many pieces of paper, which it must often of necessity be, it is very difficult to stretch such a paper, and it would take up the whole of the table, where it would have to be placed, as a board of sufficient size would be very inconvenient in a ship,

The drawback is the constant stretching and taking up of the sheet, with the variation in temperature and dampness of the air, which is undoubtedly a source of annoyance in plotting long lines, as the radii measured the day before, or even a few hours before, will frequently be found so much altered in length as to necessitate remeasurement.

This variation of the sheet may also produce distortion ; but Distortion.
with good paper, well mounted, it will be nearly the same for all parts of the chart, and the distortion is so slight as to be of no practical inconvenience, and it is a question whether more distortion is not produced when, in using the other method, the stretched sheet is finally cut off the board when finished.

The fact is, that a material on which to draw charts free from the possibility of stretching and distortion has yet to be discovered, and we must put up with these inconveniences as long as we use the paper of the present day.

Taking one thing with the other, then, the author recommends the use of loose mounted sheets for general ship work.

## Care of paper.

Paper, whether mounted or not, in damp climates rapidly gets into a useless condition, and this even in hermetically sealed tins.

The stock of paper should be kept in tins in the driest place in the ship, which is probably in or near the engineroom.

The mounting of the backed sheets supplied from the Stationery Office is usually very well done, and it saves time to use these; but as it may be necessary for the surveyor to mount sheets himself, the method will be described.
Mounting
The holland or calico on which the paper is to be mounted, and which must be in one piece and larger than the board, must be lightly damped. It is then stretched over the board and tacked to the edges, care being taken to stretch it equally and squarely with the woof and warp. Rub plenty of strong paste into it with the hand, and see there are no lumps left. The sheet of paper must be well damped with a sponge on both sides, taking care to $d a b$ only, on the side on which the work is to be done, and not $r u b$ with the sponge. The sheet is then carefully lifted by the four corners, one edge laid on the holland while the rest is kept clear of it, and the paper gently rubbed on to the board with a soft handkerchief, the paper being gradually lowered, so as to allow air bubbles to escape. It will take two people to do this, and it must be done with great care. It must be left to
dry by itself, and no hot sun should be allowed to get to it, so that it may dry evenly.

If the plotting sheet is to be formed of more than one piece Joining of paper, the edges of the paper which will overlap must be Sheets. fined down. This is done in the first instance by scraping with a sharp knife, having drawn a line on the paper where the overlapping will come, and then finishing off with ink eraser. The piece that is to be uppermost must be scraped on the under-side only, and the undermost one on the upper side, so as to make, in fact, a scarph. This will lessen the appearance of a joint, and the inconvenience of ruling lines hereafter over it.

Drawing-paper is made of the following sizes :-


Atlas, Double Elephant, and Antiquarian are most used in chart-making, and are the sizes supplied by the Hydrographic Office.

For all rough work, as sounding sheets, ordinary field work, \&c., drawing-cartridge is used. It is quite good enough, and does not entail such expense as the use of an indefinite quantity of hot-pressed drawing-paper.

This cartridge-paper is mounted on the drawing boards by Field being wetted, and rubbed on to the well-pasted board in the manner described above for mounting on holland. It will dry quite flat. When this paper is done with, it must be floated off the board, which will cause it to distort and con-
tract considerably by the time it again dries; but as all the work on it will have been beforehand transferred to the tracing, this does not so much matter. The paper must be kept, however, as a record, for which it is just as valuable.

Where a field-board is wanted for delicate coast-line or other intricate work, a sheet of Atlas can be mounted, or white Bristol board used. The latter has many advantages, and can be tacked on to a board to keep it flat. If chart pins are used with thick Bristol board, they will not hold for long, and will give much bother by constantly falling out.

The Atlas paper, being good, may be mounted on to the board by merely pasting the edges, as described above. It can then be cut off without much distortion. If cartridge is treated this way, it is very apt to tear, being of a loose texture.

## BOOKS.

Blank books of various forms and dimensions for boat work, field work, \&c., are supplied from the Admiralty. These for the most part require no forms ruled in them; but there are a few purposes for which it is very convenient to have ruled forms bound up.

All such are now supplied by the Hydrographic Office, and it is not necessary to specify them. They are all enumerated in the Instructions to Surveyors. It is very necessary to record observations in these books in such form that they can be hereafter consulted as records.

## CHRONOMETERS.

About the care of the chronometers little need be said. Full instructions are issued by the Admiralty on the subject, to which reference can be made, and Capt. Shadwell's "Notes on Management of Chronometers " * contains all that

[^5]can be said on the subject. The box or "room" for the chronometers is now made after a fixed plan, the principle of which may be said to be that the solid block on which the chronometers rest, and which is, when practicable, bolted to the beams beneath, not the deck, can receive no blow or shock other than those communicated through the ship herself, which is done by surrounding it with a bulkhead, with a clear space between. Vibrations are lessened as much as possible by the interposition of sheets of india-rubber in building up the block, and by padding the partitions in which each chronometer rests with soft cushions.

The lid of each box is removed, and a general lid covers the whole.

A sheet of fearnought is laid over the chronometers, and has Uniflaps cut over each one, so that they can be uncovered in turn, $\begin{aligned} & \text { formity of } \\ & \text { Tempera- }\end{aligned}$ for purposes of comparison or winding. This is to assist to ture. keep the temperature uniform, and also deaden the ticking of other watches when comparing.

Winding is performed at the same hour daily, and com- Winding. paring also. There is no necessity that these hours should be identical; but it is generally the practice that they should be. If they are both done at an early hour, there is more chance of the same officer being on board to do it, which is of importance.

Always wind until the mechanism is felt to butt, to ensure the watch being fully wound up.

To prevent butting too sharply, the turns can be counted, which will warn the winder; but if winding is done delicately, this is scarcely necessary. When, however, any other but the officer in charge of the chronometers winds them, he should do this; to enable him to know the number of turns each watch requires, a piece of paper with the information can be pasted on each box.

The watch has to be reversed to wind, and it must be eased gently back when the operation is complete, not allowed to swing back. This daily reversing of the watch is said to be a good thing, as it distributes the oil in the bearings.

Accurate comparing only comes, like most other things of the kind, by practice. The comparing of watches is gone into at page 225 .

A comparison book and chronometer journal are kept ; the former being used to enter the comparisons at the time, with their checks, \&c., the latter as a fair book for a permanent record, and contains rates, and all data for noting the performance of each watch.

A maximum and a minimum thermometer are placed in the chronometer-room, and the reading of their indices is taken, and recorded in the comparison book, at the time of comparing. This is of importance when it is intended to take account of change of rate from changes of temperature, and, in any case, will enable us to estimate how far our endeavours to maintain a uniform temperature in the room are succeeding.

## MARKS.

It is of course necessary in making a survey of any description to have fixed objects, which are first plotted on to the sheet, and are technically known as "points." These vary, according to the description and scale of the survey, from mountain peaks, whose actual summits may be of considerable area, to thin staves.

Natural marks.

It is a great saving of time to the nautical surveyor to find plenty of natural marks, as peaks, conspicuous trees, houses, church spires, \&c., anything, in fact, which can be defined and recognised from the different directions it may be necessary to see them; but it is rare to find a sufficient number of these, properly placed, to be able to altogether do away with putting up his own marks for the details of the survey, and it is of these we now speak.
Whitewash.

Whitewash is the great friend of the surveyor. It has the advantage of being portable, showing generally very well, being cheap, and obtainable all over the world; it cannot disappear by being blown over, or by being stolen or knocked down by jealous inhabitants, who very naturally do
not understand what the meaning of different objects dotting their shores may be. Whitewash, therefore, is used wherever practicable, as on rocky cliffs and points, or tree-stems, angles of houses, \&c.; and is also used to whiten other objects put up by the surveyor, as cairns, canvas, \&c., where either there is no solid substance to whitewash, or it is necessary to see the mark from every direction, which it is evident cannot be the case when a cliff face is whitewashed, for example.

The nature of these marks must vary according to locality, and the distance it is necessary to see them.

Where there are stones, nothing is better than a cairn. Cairns. It is rather a question as to whether a cairn on a hill-top is better whitewashed or not. If the sun strikes on it, or there are higher dark hills behind, it shines like a star ; but on a dull day, against the sky, a white cairn will be so much the colour of it, that a dark object will show better.

A cairn on the beach should certainly be whitened.
Where there are no stones, tripods of rough poles or Tripods. stakes, about eight feet long, round which a bit of old canvas about six feet long, whitewashed, can be laced with spun-yarn, will be found good. The poles are easy to carry in the boat, and can be taken up hills without difficulty; they are easily taken down, and can be used over and over again. From their tripod form, they stand well in high wind, though it is as well to give them spun-yarn stays. The conical shape of the mark affords a capital object, and rough poles of the kind required can be got anywhere.

Where bamboos can be got they are very useful, from their lightness, to carry up hills, either to form tripods as just described, or as flag-poles.

Pieces of wide coarse white calico are useful for temporary marks, as where it is desired to see a station from another station, when there is not sun to use the heliostat, and it is not requisite to have a very large mark left for future use.

On a very flat low shore, where boat sounding has to be Flags. carried out a long distance, flagstaffs with large flags must be set up. Care should be taken in some parts of the world
that these flags are not national ones, or anything that can be mistaken for such, as difficulties have frequently occurred through such being hoisted. As the large old flags obtained from dockyards are always of this type, they should be cut up, and re-sewn with such an arrangement of colours as shall denote nothing.

The colours of red and white intermingled will be generally seen furthest.

Canvas.
On coasts lined with bush, like mangrove swamps for instance, square pieces of canvas, whitewashed and laced to the boughs, will be found to show very well.

In lacing these on, care must be taken to place them so that they will show as far round on both sides as possible, and always to have the lower part more to the front than the upper, as they will thereby catch more sun.

Whenever canvas is used, it is well to cut holes in it, sewing them round sufficiently to prevent tearing in the wind. This will make the canvas valueless for fishermen or natives of any kind, to whom, in all parts of the world, a good piece of stuff is a prize.

In preparing for a surveying cruise, therefore, provision of material for marks must not be forgotten.

## BOATS' FITTIINGS.

It is impossible to lay down any dogmatic rules for fitting boats for surveying work, as so much depends on individual tastes, and requirements of the locality; but a few points may be noted which have been found generally useful, and a list of articles which are always being wanted can be added, as some sort of guide.

> Steam Cutters.

> Steam cutters are of course the best boats for general sounding, as the engine never tires. The additional work that can be done with steam cutters at command is enormous, as they not only do their own work, but tow the pullingboats to and from their stations, and thereby save many hours that would otherwise be spent in beating up, or
pulling backwards and forwards to the ship morning and evening.

A little table may conveniently be fitted in the stern-sheets of a cutter, as there is plenty of room. The stern-sheet canopy, which will generally have to be in place when sounding, to prevent the spray injuring instruments, books, board, \&c., should not be too high, so that the officer standing in the stern-sheets may be able to take his angles over it.

Fittings for a small wire sounding machine are necessary.
Steam cutters for surveying work should be fully rigged, as accidents will happen, especially as the boiler gets. old, and it is awkward to find oneself broken down with the ship miles off, and probably out of sight, and nothing but a foresail, which is the present service-fitting for these boats. For this reason, unless working near the ship or in close harbours, the masts and sails should always be in the boat. Lumber irons should be fitted to carry these high up, so as not to interfere with the wash-streak of canvas, nor take up necessary room in the boat.

The usual fitting of a steam cutter, a canvas turtle-backed canopy forward, is inconvenient, as the leadsman cannot then stand in the bows, which is the best place for him. The canvas wash-streak must therefore be carried to the stem, and the stanchions on the bows must be higher than those amidships, to allow for plenty of pitching in a head sea.

It will be found absolutely necessary to use the bow and stern lockers for stowing gear, men's clothes, \&c.; but care must be taken that the lids are screwed down, whenever the boat is at work in open water.

Steam cutters must have little skiffs of some kind, to tow Light as tenders for landing, as the boat is too heavy and draws ${ }^{\text {Skiffs. }}$ too much water to be beached, and should always be kept off the ground, for fear of strains with the heavy boiler in her. When only sounding, the tender is of course not needed.

For pulling-boats, whalers will be found most generally Whalers.
useful; they employ fewer men, and have quite enough room in them.

The simpler the sail the better, as it may be often up and down, but a mizen is very useful.

Fixed wash-streaks forward and aft will keep out much water.

Crutches with a long shank, which will raise the crutch two inches, may be found useful in some types of whalers, as, with gear along the middle of the boat, and the low gunwale of an ordinary whaler, the loom of the oar cannot be depressed enough with a service crutch, when in broken water, for the blade to clear the tips of the waves.

Lockers, built in bow and stern, are useful for keeping gear and instruments in. These should be canvased over, for unless built of two thicknesses of wood, which is heavy, the tops will soon leak after a few months' hot sun. The top of the bow locker raises the leadsman, so that he can throw his lead well ahead, and he should have his foremost awning stanchion shipped, as a support in rough water.

The awning should be cut at the after-thwart, so as to enable the afterpart to be tipped, when it is necessary to stand up to take angles.
Catters. Cutters should also be fitted with bow and stern lockers, and a table can be arranged in stern-sheets if thought necessary. No other special fitting is required beyond those given below for all boats.

## FITTINGS AND GEAR FOR ALL BOATS USEFUL FOR GENERAI SUVEYING WORK.



A small galvanized-iron davit, with snatch block to place sounding sounding line in when in deep water, so that several men ${ }^{\text {Davit. }}$ can assist in hauling up the lead. In steam cutters it will be found handy for the leadsman always to use this.

A Massey's patent log, with the stray line between fan Patent and clock lengthened, so that the latter may be fastened out- ${ }^{\text {Log. }}$ side the gunwale for convenient reference, while the fan tows in the water behind.

A small galvanized-iron nun buoy, with a light chain and Buoy. weight to moor it by, is useful when sounding out a shoal patch.

The boat-hook should be marked in feet, with the marks Boat-hook. slightly cut and painted. This is useful for sounding in shallow water, and many other purposes.

A box containing some spare tins of preserved meat, in Spare Procase of accident detaining the boat beyond her time of return to the ship.

The following list of stores may be handy :-
Lead lines, 100 fathoms, 1.

| 25 | $"$ | 2. |
| ---: | :---: | :---: |
| 11 | lbs. | 2. |
| 7 | $"$ | 1. |

Anchor and cable. Latter should have a short ganger of chain in case of sharp rocky bottom, and be always made fast to crown, and stopped to the ring, in case of fouling.
Masts and sails.
Spare oar.
Awnings and stanchions.
Water barricoe.
Small portable ditto for carrying on shore, 2.
Axes, handy billy, 2.
Bag of lime.
Whitewash brush.
Box for arming for lead.
Tin pannikins.
Bag for biscuit.

Old canvas for mark.
Canvas cases for rifles.
(It is convenient always to have a rifle in the boat.)
Ensign, answering pendant, and signal book.
Tramping barricoes, 2.
Ammuni- Ammunition case, containing-
3 rockets with rope tails.
2 long lights.
1 handle and primers.
1 portfire.
1 porffir.
If a boat meets with an accident, this ammunition will come in handy to attract attention after dark.

Carpen- Carpenter's bag, containing-

Hammer.
Nails of sorts. Chisel.
Bradawl.
Gimlet.
Boatswain's bag, containingMarlinspike.
2 sail-needles, Twine.

Fearnought.
Lead.
Tallow.
Strips of copper.

Palm.
Bits of canvas.
Spun-yarn.

## LEAD-LINES.

The first thing in a newly commissioned ship is to get the lead-lines well stretched.

Until this is done it is only loss of time to mark the multitude of lines wanted for surveying.

As soon as the ship leaves port, tow seven or eight hundred fathoms of the new line astern, with a heavy lead on it, for some days. The line will then have got to its normal length, but lead-lines will always want re-marking from time to time.

Hand lead-lines, for ships or boats, should be marked in feet up to 5 fathoms, and at every fathom between 5 and 25 .

Ship lines for deeper sounding, and boat's 100 -fathom lines, should be marked the same as hand lines, to 25 fathoms.

After that, at every 5 fathoms, up to 100 fathoms.
Over 100 fathoms, at every 10 fathoms, to 200 fathoms.
Over 200, a mark at every 25 fathoms will be sufficient.
For deep-sea lines, a mark at every 25 fathoms, throughout.
There is no recognised system for these marks, but a list is appended of that used by the author.

## Marks in Lead-Lines for Surveying.

Faths. Feet.
2 line, 2 knots.
3 Blue.
4 line, 1 knot.
5 , 2 knots.
10 leather, one tail.
1 line, 1 knot.
2 " 2 knots.
3 Blue.
4 line, 1 knot.
5 , 2 knots.
20 leather, 2 tails.
1 line, 1 knot.
2 ", 2 knots.
3 Blue.
4 line, 1 knot.
5 ", 2 knots.
30 leather, 3 tails.
1 line, 1 knot.
2 ", 2 kncts.
3 Blue.
4 line, 1 knot.
5 " 2 knots.
40 Red, with line and 4 knots.
1 line, 1 knot.
2 " 2 knots.
3 Blue.
4 line, 1 knot.
5 , 2 knots.
50 White.
6 O Blue.
70 Red.
80 Canvas.

## Faths. Feet.

90 Blue.
100 leather, a hole.
110 " 1 tail.
120 " 2 tails.
13 O Blue.
140 Red.
150 White.
160 Blue.
17 O Red.
18 Canvas.
19 Blue.
20 line with 2 knots.
21 leather, 1 tail.
22 " 2 tails.
23 " 3 "
24 Red.
25 Red and White.
303 knots on line.
35 White.
404 kncts.
45 White.
50 Blue and White.
55 White.
606 knots.
65 White.
$70 \quad 7$ knots.
75 Red.
808 knots.
85 White.
$90 \quad 9$ knots.
95 White.
100 Blue and line, 1 knot.


## BEACONS.

Floating beacons are frequently of great service.
These are now generally made on board. A useful and convenient form is depicted in Fig. 8, which pretty well explains itself.

The heads of the two 27 -gallon casks should be filled up flush, and the planks above and below are screwed to the heads, the pole passing through the centre of each plank by a hole cut for the purpose. The planks can be hollowed out to fit the heads of the casks for further security.

Three casks can also be used if only small ones are available, by fitting the planks in triangle, with another plank across, through which the pole passes.

The strop for weighing should be of wire, which keeps well open from its own stiffness, and facilitates hooking on for hoisting in.

A slip by which the cable is attached to the mooring span assists in weighing the beacon.

Use a small kedge and light chain for anchoring, except in water, say, over 60 fathoms deep, when hemp should be employed, with some chain next to the anchor to take chafe. Beacons have been anchored in 3000 fathoms by means of sounding wire, and weight of 100 lbs .

In water of from 20 to 100 fathoms, about $1 \frac{1}{2}$ times the depth is necessary for the length of mooring rope. In deeper water, less.

This beacon will float nearly upright, and will carry in

## Fig. 8. Floating Beacon.

Flag twelve to sixteen feet square according to circumstances.

Wire sling strop for slipping \& weighing

## Moorings.

One 60 lb . boats anchor, four 56 lb . sinkers and one length of $1 / 2$ inch chain.
Length according to depth of water, attached to one length of $1 / 2$ inch chain at moorings.
moderate weather a flag 12 ft . square, of calico, which is lighter than bunting, and will be visible from the ship 10 miles, with a $30-\mathrm{ft}$. bamboo. Black, with other colours to distinguish one beacon from another, is recommended.

A piece of signal line should be fitted along the luff inclosed-in several folds of the calico, and the flag is stopped to the bamboo round this.

Another form of cask beacon is made by woulding three casks to the central spar with rope, which tautens when wet, but the beacon above described is more quickly fitted when the parts are ready beforehand.

Slipping a beacon is best accomplished from the mainyard, but the foreyard can be used. The anchor being over on the weather side, and beacon lowered into the water, slip by means of a large well-greased toggle.

Weighing is accomplished best from the foreyard. Having hooked to the weighing strop, run the beacon up, and having made fast a line from the hawse pipe to the wire strop at the upper end of mooring chain, knock off the slip, when the beacon can be landed inboard, and the mooring run up to the hawse pipe.

Fixed
Beacon,

A fixed beacon can be erected in shallow water of from 2 to 3 fathoms by constructing a tripod of spars of about 45 ft . long. The heads of two of them are lashed together, and the heels kept open at a fixed distance by a plank about 27 ft . long nailed on about 5 ft . above the heels of the span. These are taken cut by three boats, and the third tripod leg lashed in position on the boats, the heel in the opposite direction to the two others. The legs are weighted, and a gantline block lashed to the fork. The two first legs are let go first together, and the tripod hauled into position by guys. Weights can be added by slipping them down the legs, and the guys secured to anchors.

A vertical pole with bamboos can now be added, its weighted heel on the ground. It is placed by a jigger from the fork, to which it is afterwards lashed, and guys taken from the lower part to the tripod legs. A block and halyards from the bamboo permits a calico flag 14 ft . square to be hoisted.

## CHAPTER II.

## A marine survey in general.

There is a great variety in the methods that can be employed in making a marine survey, so much so that the task of describing any general scheme of operations is by no means easy.

In the first place, Marine Surveys may be divided into Different three heads.

1st. Preliminary or Sketch Surveys.
2nd. Surveys for the ordinary purposes of navigation.
3rd. Detailed Surveys.
The boundaries between these are by no means strongly marked, although each differs considerably from the other, and a finished sheet as sent home is not unfrequently a combination of all three, comprising pieces of work done after very different fashions, according to needs and circumstances.

A preliminary survey does not pretend to accuracy. The Sketch time expended on it, and the means used, cannot ensure it, Surveys. and it only represents what our second name for it indicates, a sketch. A sketch survey will be founded on a base of some kind, but this will generally be rough, and in some instances, as in many running surveys, will depend solely upon the speed of the ship as far as it can be ascertained by patent $\log$; so that the whole affair from beginning to end is only a rough approximation.

The necessity for sketch surveys may be said to be getting less and less every year. Most parts of the world have their coasts mapped at any rate as far as this; but there still
remain portions of our globe of which the coast-lines are not marked at all, or are extremely hazily delineated, and to these any sketch survey will be an improvement.

Ordinary Surveys.

The second head comprises the majority of charts now published, and many of those in course of construction in the present day, i.e. they are constructed on such a scale, and with such limitations of time, \&c., as to make it impossible either to show small details of land or sea, or to be perfectly certain that small inequalities of the bottom, or detached rocks, may not exist, unmarked. Everything, however, shown in such a survey should be correct, and it is only in its omissions that it should be imperfect.

Detailed Surveys.

A detailed survey is accurately constructed from the commencement, on a scale large enough to admit of close sounding, and time is given up to working out all minutiæ.

Detailed surveys are mostly confined to the more civilised shores of the world, where there is much trade, and to such ports, harbours, and channels as are largely used in navigation.

The necessity for these surveys increases to an enormous extent every year, with the prodigious strides trade, more especially trade by means of steam-vessels, is taking.

A steamer works against time; her paying capabilities largely depend on her getting quickly from port to port, and captains will take every practicable short cut that offers, and shave round capes and corners in a manner to be deprecated, but which will continue as long as celerity is an object. A channel which a sailing vessel will work through in perfect safety, from the obvious necessity of keeping a certain distance off shore, for fear of failing wind, missing stays, \&c., will be the scene of the wreck of many a steamer, from the inveterate love of shortening distances, and going too near to dangerous coasts only imperfectly surveyed. Better charts will not cure navigators of this propensity, but will save many disasters by revealing unknown dangers near the land.

Time, and the comparative scarcity of marine surveys, do not permit of keeping up to the rapid advance required in this style of survey; and unless the countries of the world
interested in ocean traffic largely increase their expenditure on these matters, it seems as if charts will get further and further behind requirements as years roll on.

Having settled of what description a chart is to be, there is still much diversity in the method of undertaking the details

Detail always variable. of it. The extent of the work, whether simply a plan of limited extent, or a large piece of open coast; the scale on which it is to be done; the nature of the coast and sea; the time and means at disposal ; the number of assistants ; will all be considered in determining exactly how to set about the work.

All this makes it very difficult to lay down rules for marine surveying. Experience alone can dictate what should be done in each particular instance. Though a plan may be produced, the time employed, and the result of the labour expended, will greatly vary according as to whether the work has been undertaken in the right way or not, apart from any personal qualities of the assistants, and nothing but the possession of the true surveying "knack," combined with experience, will point out this right way.

All surveys are, however, alike in this respect. They are, Trianguas it were, built up on a framework of triangles of some kind, the corners of which are the main "points" of the chart, and to obtain this framework is always the first thing to do, and how to set about it the first thing to consider.

The construction of this " triangulation," as it is termed, is of various kinds ; ranging, from the rough triangles obtained in a running survey, where the side is obtained by the distance it is supposed the ship has moved, and the angles are sextant angles, taken on board from a by no means stationary position; to the almost exactly formed triangles of a detailed survey, when carefully levelled theodolites observe the foundation of a regular trigonometrical network, which covers the whole portion to be mapped.

The term triangulation would seem to infer that this system of triangles would be always apparent; but in surveys irregularly plotted, and when working on a sheet previously
graduated, it will seem that there is no triangulation, and in the strict sense of the word there is none, but the framework of the chart is still built up on the system of triangles, and it is difficult to find any other name for the process.

For the present we will speak only of the second and third kinds of surveys, leaving Sketch Surveys to be described separately.

The system employed in Ordinary Surveys and Detailed Surveys is the same, and they really only differ in the scale of the chart, and the amount of time that is spent on them, especially with regard to closeness of soundings.

In a detailed survey, time must be subservient to the necessity for exactness, and for exploring every foot of the ground.

In an ordinary survey, judgment has to be exercised as to how far we must be satisfied with what we can get for triangulation, and how much time we can spend on details.

It is by no means necessary in an ordinary survey to observe the angles at each corner of the triangles. The happy fact that the sum of the three angles is $180^{\circ}$, enables us to manage whenever we have two of them, though it is of course more satisfactory to actually observe all three for the more important triangles.
Accuracy
necessary. The accuracy necessary in many details of a chart depends very much upon its scale. Over-accuracy is loss of time. Any time spent in obtaining what cannot be plotted on the chart is, as a rule, loss of time ; but it cannot be too strongly impressed upon the young nautical surveyor, that his work should be as correct as his scale will allow. Nothing should be put down of which he is not sure, and it is no loss of time to repeat angles to prevent mistakes. It is better to be overaccurate than to err in the opposite direction, and experience will soon show him when he must be very exact, and when a little latitude is permissible without interfering with the result.

The accuracy of the main triangles of a chart is most im-
portant, everything depends on them, and if they are incorrect, nothing will be satisfactory afterwards.

The general plan of a survey may be said to be this :General
1st. A base is obtained, either temporary, as in the case of plan of an extended survey; or absolute, as in a plan. This is the known side of the first triangle.

2nd. The main triangulation, that is the establishment by means of angles of a series of positions, at a considerable distance apart, from which, and to which, angles are afterwards taken, to fix other stations. These are the corners of our framework, and are known as the "main stations," the two ends of the base being the first two, on which everything is built.

3rd. The fixing by means of angles from these main stations of a sufficient number of secondary stations, and marks, to enable the detail of the chart to be filled in between them. In most cases angles will be required to be taken from the marks themselves as well.

4th. All these points, or those embracing a sufficient area to work on, being plotted on the chart, they are transferred to the field boards, either by pricking through the plotting sheet with a fine needle, or, what is a better way when carefully done, by making a tracing of them on tracing-cloth, and pricking through that on to the boards.

5th. Each assistant then has a certain portion told off to him to do. It must depend upon circumstances, but as a rule it is more satisfactory to have the coast-line put in first, and the soundings taken when this has been done. The topography, or detail of the land, can be done at any time.

6th. Each piece of work is inked by the assistant on his board, with all detail, and when complete, is carefully traced on the above tracing of the "points." All bits are thus collected together, and the total is retransferred to the plotting sheet by means of transfer paper, and inked in as the finished chart.

These details must not be taken as unalterable. Some prefer plotting everything on to the same original sheet, and
when a surveyor is by himself, or with one assistant, he would probably do this, but the method described is calculated for a number of assistants, and has been found to work well.

It is not absolutely necessary to get a base before starting a plan. Circumstances may make it imperative to wait a day or more for this, and in the meantime, a distance between two stations, to be finally measured, can be assumed and plotted, and the whole system of triangulation built upon this. But it must be remembered that no heights can be measured by means of angles, until a scale is obtained.

If an extended survey, and plenty of hands, some will carry on the triangulation and marking on ahead, while others are putting in the detail, and sounding the part already marked.

The deeper soundings will be taken from the ship, to a sufficient distance or depth off shore.

When to obtain the Astronomical Positions.

It will depend upon circumstances when the astronomical observations for latitude and longitude are taken. If only an isolated plan is being done, the observations to fix some definite point on it can be taken at any time.

When an extended survey is in progress, that has been commenced on a measured base, they can also be taken when convenient. In this case the final scale of the chart will always depend upon the observations taken at either extremity of the chart, and they must consequently be done very carefully. Circumstances of weather, time of year, \&c., will therefore influence the choice of the best time for these.

Sometimes an extended survey will be originally plotted on a base obtained by the astronomical positions, and in this case of course they will be the first thing to undertake. At any rate it will nearly always be convenient to obtain a true bearing at once, in order to have the meridian of the chart placed squarely on the paper from the commencement.

These separate steps in a survey will now be described in detail, following the order in which they will generally come, as far as can be done.

## CHAPTER III.

## BASES.

By Chain-By difference of Latitude-By Angle subtended by known length-By Measured Rope-By Sound.

Bases for marine charts or plans upon which to build Different the triangulation are obtained in several ways, according as kinds circumstances permit and accuracy requires.

1. By means of the $100-\mathrm{ft}$. chain or steel tape supplied for the purpose.
2. By difference of latitude, or difference of longitude.
3. By measuring with a micrometer or sextant the angle subtended by a known length, as two poles a measured distance apart, the ends of a long pole, or the masts of a ship.
4. By a measured rope, as a lead-line; or by the wire from a sounding machine.
5. By sound.

## CHAINED BASES.

The ground for a base, to be measured either by chain or rope, must be as level as can be found. Its length will be partly determined by the extent of the work to depend on it, varying from say 9000 feet to 1000 feet, or even less for a small harbour.

While it is certainly convenient to measure the base in one straight length, if convenient ground can be found, it is by no means necessary.

If several short lines making angles with one another are measured with the angles carefully observed, the terminal
points being visible from one another, the resulting distance calculated between the terminal points should be just as correct as if it had been measured direct.
Extension It is seldom that a chained base can be found, even for a of base. small plan, long enough to plot from directly, i.e., the measured length when protracted on the paper would be generally so short, that by placing that on the sheet first, and making it the starting line, errors would be sure to creep in, in increasing the size of the triangles, any little error being multiplied. It therefore is usually necessary to extend the base, as it is termed.


This consists simply in calculating a sufficient number of triangles, conveniently arranged, to obtain a side long enough to form a good start, so as to plot inwards as much as possible, when any little errors will be diminished, instead of increased.

As a commencement of this process, the base to be measured should, if possible, be placed so that there are two stations, one on each side of it, which can be used for the first triangles and consequent extension of the base.

Here, Fig. 9, A B is the measured base, C and D the two first stations. Angles are observed at A, B, C, D. The other
two sides in the triangles A B C, A B D being found, C D can be found in both the triangles A CD, B CD, which will check the result, and C D will be the extension of the base for further triangulation.

Of course this desired convenience will not always be found, but it is a thing to look out for.

It is by no means necessary to measure a long base, provided that convenient triangles can be found for extending the base by calculation. If the angles of these are of the necessary number of degrees, and they are carefully observed with theodolites, a short base, measured on flat smooth ground, will give a truer result than a longer one measured over inequalities. With a sextant survey it will be well to have as long a base as possible.

The ground having been walked over to ascertain it will Planting do, and that the base stations (the ends of the base) are so $\begin{gathered}\text { staves for } \\ \text { measuring }\end{gathered}$ placed that they see as much as possible on all sides: set by. up the theodolite at one end, and at the other a flagstaff or another theodolite, and let a man plant staves (boarding pikes make good ones) exactly in line between the two stations, giving him the position for the first two or three, by looking through the theodolite directed to the other station. After these are in place, he can plant the others in line by guiding himself by them.

Having the staves placed and in line, begin to measure from Method of one end. If two persons are to measure, begin from opposite ing. $_{\text {mea }}$ ends. A man is required for each end of the chain. The man at the foremost end of the chain carries ten pins, and the surveyor attends with his book to see the chain fairly placed in line between the staves, and to note down each length of chain measured. Do not let the men stretch the chain too tight, but it must lie straight on the ground between the two ends.

The chain being down for the first length, measuring from under the centre of the theodolite, put a pin in the ground, at the foremost end, inside the handle, and touching the flat side. Make a mark in the note book, and walk cn together,
the man at each end lifting the chain as much as he can, until the hindermost comes to the pin. He must then place the outside of his handle so as to touch the pin. Another pin is put in at the foremost end inside the handle, the second note made in the book, the first pin taken up by the hindermost man, and on you go.

The lengths are best noted by strokes, crossing every fifth over the four, as in ordinary tallying.

Check at every ten lengths by the number of pins. When the tenth stroke is made, the foremost man should have no pins left in his hand, and the other man should have nine, the tenth having been just put in.

The odd feet and inches in the last length are measured by counting the links, which are each a foot long.

In walking forward, take care that the hinder man does not overwalk the former, or the chain will have a bight dragging on the ground, links will catch in something and get bent, and the error of the chain will be very different when retested, to what it was before landing.
Repetition The number of times a base must be measured depends on necessary. circumstances. If for a harbour plan, only twice, if they agree to a foot or two, will be sufficient. For a survey of greater extent, three or four times will be more satisfactory, unless the two first measurements agree very well.
Inequalities.

Perfectly level ground can seldom be found, and the surveyor must make an allowance for inequalities by his judgment, which will be of course always subtracted from the measured length.

The chain must be tested for length, before and after measuring the base, to ascertain the error.

## BASE BY DIFFERENCE OF LATITUDE.

When two stations are available from twenty to thirty or forty miles apart, visible from one another, and bearing not more than two points from the meridian, having also a few
intermediate points visible from both, a very good base can be got by latitudes, and careful true bearings.

The base will then be diff. lat. $\times$ sec. Mercatorial bearing.

Similarly, if the stations bear nearly east and west from each other, the diff. long. may be obtained by chronometer or rockets. The true diff. long. by observation is converted into spherical diff. long. from which the departure is found. The length of base will be Dep. $\times$ Cosec. Mercatorial bearing.

By means of the intermediate points, triangles can be calculated down to a workable length of side for fixing marks.

Where no smooth ground for measuring a base can be found, and we want our scale to be near the truth from the first, this method is valuable.

The only drawback to it is the effect of local attraction on the pendulum, or, in other words, on the mercury in the artificial horizon. With high land behind a station and a deep sea in front it may result that there will be considerable error; and the difference in the distance between the terminal points of the survey, if it covers much ground, as deduced from such a base, and as determined from the observations at either end, may be much more than if starting from a measured base.

## BASE BY MAST-HEAD ANGLE.

This consists in measuring, with a micrometer or sextant, the angle between the mast-head of the ship and the hammock netting, or some other fixed line on the ship's side; not the water-line, as that varies.

The vertical circle of a theodolite being only marked to minutes, unless it be a much larger one than is generally available, is not sufficiently accurate for this.

It is well to use two sextants to check errors, and read them both on and off the arc.

The height of mast-head above the line must be accurately known to give a good result.

Working out a right-angled triangle gives the distance required.

A table should be formed of the distances corresponding to different angles of the mast-head of the ship, as this will be frequently used in sounding banks.

## BASE BY ANGLE OF SHORT MEASURED LENGTH.

Where the ship is not available, a base for a small plan can be obtained by measuring the angle between two welldefined marks placed in the ground at a carefully-measured distance apart, or that subtended by the ends of a long pole.

This must also be done with the sextant or micrometer.
If staves in the ground are used, care must be taken that they are at right angles to the required base. Similarly, if a pole is used, care must be taken to hold it at right angles to the observer, which can be ensured, either by having a pointer nailed on to the centre of the pole projecting at right angles, and which must be directed towards the observer by the man holding the pole in both hands horizontal, or by simply waving the pole, held in this position, backwards and forwards gently, when the observer will register the largest angle he observes as the correct one.

The angle observed should not be smaller than $1^{\circ}$, which with a distance of 20 feet, will give a base of over 1100 feet. It would be better, however, if practicable, to get a base by means of a longer distance, and larger angle than this, when a very trustworthy result will be obtained ; or to be content with a shorter base, and extend it by angles, as already described, to a longer working base.

Measurements must be made on and off the are, and it would be well to use more than one sextant.

Small lengths of this kind may also be measured by a micrometer, but a sextant will give just as good results, and is in a ship always handy.

No appreciable error will be introduced by taking distance $=$ length of pole $\times$ cot. angle .

## MEASURED BASE BY ROPE.

Measuring by a rope is of course not accurate. It is difficult to avoid stretching it more at one time than at another, and if it gets wet, it alters its length considerably. If measuring over ground where it is sure to get wet, it will be better to wet it well beforehand. Test it in that condition, and keep it well wet all the time of measuring.

## BASE BY SOUND.

This consists in counting the interval of time which elapses between the sight of the flash of a gun, and the arrival of the sound of the explosion, the gun being at one end of the required base, and the observer at the other.

Recourse is had to this method of obtaining a base when Final scale no flat ground can be found on which to measure. Its $\begin{gathered}\text { should not } \\ \text { depend on }\end{gathered}$ accuracy is not great by any means, but, if the final scale of base by the chart is to depend on astronomical positions, it is quite sound. near enough for working out details such as heights, small parts measured with ten-foot pole, \&c.

Its value is much increased by observing from both ends, Useful which should always be done if possible, and a surveying vessel should have two small brass Cohorn mortars supplied for this purpose, which can be sent away in a boat, and tumbled overboard without damage.

The ship is often used at one end of a base by sound, especially in work amongst small islands, and it is also necessary sometimes to have a boat at the other, but if at any rate one shore station can be obtained it will be better. If choice of direction can be had, measure with the wind across the base, as, though the error from increase and decrease of velocity is eliminated by measurement from both ends, the
sound may be difficult to hear against the breeze, if at all strong.

For either end choose positions for the hearers as much out of the wind as possible, as it is the whistling of it in the ears which disturbs the receiver more than anything.

A base of three miles is a very good length, but the surveyor will generally not have much choice in this matter. Needless to say, on a calm day the sound will be heard farthest and easiest, but the choice of days is seldom possible in practice. If we waited for the best opportunity for every detail of a survey, it would never get on, and the utmost that can be done is, when there is alternative work for which the day or opportunity is more suited, to take that in hand.

Signal to be made.

The guns from the two ends should be fired alternately, at regular intervals, and at some preconcerted signal, as dipping from the ship a flag visible from both stations, which should be hoisted a minute or half a minute before as warning, or rehoisting a dipped flag steadily, the gun being fired as the flag reaches the masthead. It is distracting to the receiver to be waiting an indefinite period for the flash.

Watch to be used.

A chronometer watch is the best, beating five ticks to the two seconds. An ordinary watch which beats nine ticks in the same period, goes at such a pace as to be rather confusing, especially when not in practice, though, if the observer is used to the process, he will measure as accurately with an ordinary watch, and possibly more so.
Preparation for counting.

When awaiting the flash, hold the watch to the ear and count to yourself-nought, nought, nought, \&c., continually, keeping time with the ticks ; you will then be ready to com-mence-one, two, three, \&c., as soon as you see the flash or smoke of the gun.

If going to use a telescope to watch for the warning signal, tie the watch over the ear with a handkerchief, which will leave both hands free.

Count only up to ten or twenty, and mark off each ten or twenty by putting down a finger of the unoccupied hand, or by some such means.

If time allows, three or four measurements should be made Repetieach way, or more if they do not agree with one another. $A^{\text {tions. }}$ signal must be arranged to ask for more than the number previously settled, if it be wanted.

In meaning the result, the arithmetical mean is not strictly Calculatcorrect, as the acceleration caused by travelling with the ing the wind is not so great as the retardation caused in the opposite direction, as in the latter case the disturbing cause has clearly acted for a longer period. The formula used is

$$
\mathrm{T}=\frac{2 t t^{1 *}}{t+t^{1}}
$$

when $T$ is the mean interval required, $t$ the interval observed one way, $t^{1}$ the interval the other way.
The mean interval thus found, multiplied into the velocity of sound for the temperature at the time, will give the required distance.

The velocity of sound varies considerably, and an accurate Velocity of law for all its causes of variation has not yet been discovered. The main cause is, however, temperature, and for this it can, to a certain extent, be corrected.

The most trustworthy experiments made, show that sound travels about 1090 feet in a second of time, at the temperature of $32^{\circ}$ Fahrenheit, and increases at the rate of $1 \cdot 15$ foot for each degree of temperature above the freezing-point, decreasing in the same proportion for temperatures lower than $32^{\circ}$.

This is the only correction that can be made, and a base measured in the manner described, with these data, will give an approximation sufficiently near for all practical purposes.

As an example, let us suppose A and B the two ends of the Example base to be measured.
of Base by sound.

[^6]At A have been observed
44 beats with watch beating 5 beats to 2 seconds 45
44
Mean $44 \cdot 33$ beats $=18 \cdot 532$ seconds.
81 beats with watch beating 9 beats to 2 seconds
82
83

Mean 82 beats $=18 \cdot 222$ seconds
Mean at $\mathrm{A}=18 \cdot 376$ seconds.
At B have been observed
85 beats with watch beating 9 beats to 2 seconds
87
88

Mean $86 \cdot 66$ beats $=19 \cdot 258$ seconds.
47 beats with watch beating 5 beats to 2 seconds
47
48

Mean $47 \cdot 33$ beats $=18 \cdot 932$ seconds.
Mean at $\mathrm{B}=19 \cdot 095$ seconds.
Then working $\mathrm{T}=\frac{2 t t^{1}}{t+t^{1}}$
we get $T=18 \cdot 728$ seconds.
Temperature is $80^{\circ}$, at which velocity of sound is $1145 \cdot 2$ feet per second.

This multiplied into the interval, gives 21448 feet for the length of our base.

The temperature must be taken in the open with the thermometer shaded from the direct rays of the sun, but not in too cool a spot, or it will not give the true temperature of the free air.

## CHAPTER IV.

## THE MAIN TRIANGULATION.

General-Making a Main Station-False Station-Sketch-Convergency
-Calculation.
The main triangulation has been already defined as "the Definiti)n establishment by means of angles of a series of positions, from which and to which angles are afterwards taken, to fix the secondary points of the survey."

All positions from which angles are taken, with the inten- Main tion of fixing other objects, are called "stations," the symbol ${ }^{\text {Stations. }}$ for which is $\Delta$, but the ones with which we are immediately concerned, that is, the first and important positions, are distinguished as "main stations," and these collectively form the " main triangulation."

The first object of main stations is to see other main stations, and with this in view their positions are chosen accordingly; but angles to everything useful, secondary stations, marks, \&c., are, of course, taken as well.

Secondary stations are those from which angles are taken Secondary solely to fix the smaller marks and details, \&c., of the survey. They will be nearer together than the main stations, and may often be perforce so placed as to be useless for any other object.

All objects fixed and plotted on the skeleton chart are Points. known as "points." A "point" may be a main station, a secondary station, or simply a mark; but when fixed and plotted on the sheet, with the intention of using them in the survey, they are one and all spoken of by the generic term of "points."

Main triangulations may be divided into two kinds:

Varieties of Triangulations.
"calculated," in which the triangles are all worked out, so that the length of any side, or the distance between any two main stations, can be found; and "plotted," in which the main stations are simply the first points laid down on the paper.
Calculated A calculated triangulation is used in any detailed survey, Triangula-
tions. to have different parts of the same survey on separate sheets, which can therefore afterwards be put together in the engraving, without any fear of their not fitting into one another.

Bases for plans, on a larger scale than the rest of the chart, can often be taken out of a calculated main triangulation without measuring separate small lengths.

Plotted Triang tions.

Plotted triangulations may further be subdivided into "regular" and "irregular."

A plotted regular triangulation will be when triangles have been obtained which could, if requisite, be calculated trigonometrically. As, however, a calculated triangulation is of great service as a record, and for future resurveys, it is expected to be furnished with every chart.

It is more satisfactory that triangulation should be regular if possible, but it very much depends upon the nature of the coast to be surveyed, in what manner it can be carried out.
Irregular In many extended surveys, where, for instance, the land is Plotted Triangulations. low and densely wooded, or perhaps bordered by reefs to a great distance from the shore, a regular triangulation is hardly possible, or would entail so much loss of time as would not justify its being undertaken.

The main points must be plotted in these cases by all sorts of means. The ship enters largely into the scheme, and frequently boats also. Stations may have to be fixed solely by angles observed at them. True bearings are freely used in the construction of the chart, and any regular system of triangles disappears.

A large proportion of existing charts have been, and man
more are now being, constructed, by means of irregular plotting.

A survey can often be commenced with a regular triangulation, when it will be found necessary, after having advanced a certain distance, to have recourse to irregular means to fix main stations.

Here it is, when ordinary rules and systems fail, that the skill of the chief of the survey is shown in overcoming these difficulties in the readiest and best method, and these are the circumstances on which we can give the fewest hints. Such as we do mention will be found in the next chapter on Plotting. In the present one we shall confine ourselves to regular triangulations.

The angles of the first few triangles in a triangulation, Great commenced on a measured base, will require to be extra- ${ }_{\text {in }}^{\text {accuracy }}$ carefully observed, and the theodolite must be carefully Triangles. placed exactly on the spot of the mark which will distinguish the station. For, as we shall be increasing our distances in each triangle, until sides long enough to carry on the triangulation without further enlargement are arrived at, any little error in an angle will give a larger error in the resulting side. These first triangles will nearly always require to be calculated, as already remarked under the head of Bases, in order to get a side long enough to plot from, whatever it may be the intention to do afterwards.

Although we are about to speak of triangulation from Triangula shore stations, as carried on by means of the theodolite, as ${ }_{\text {Sextant. }}^{\text {tion by }}$ this instrument is always available in a surveying ship, it must be understood that, with care, an excellent triangulation may be obtained with that invaluable instrument, the sextant.

The point on which care is principally needed, is that the Horizontal angles measured should be horizontal angles. A practised $\underset{\text { with }}{\text { angles }}$ surveyor will usually be able to note some small natural sextant. mark, directly above or below the object whose angle is required, and at his own level, to which to measure his angle, and in most cases of using the sextant this will give a
sufficiently near result, but if forced to use the sextant for triangulation, another means may be used.

From the end of a longish pole (boat-hook staff will do), planted at a slight angle from perpendicular, let a plumb line fall, and getting the object transit one point in the line, the angle can be taken to any other part of it. The plumb line must not be too close to the observer, or it will be difficult to keep the transit on, and parallax will creep in.

It is a question of circumstances as to whether the main triangulation is to be carried on by itself first of all, or in combination with the secondary stations and marks. This in noways affects the principle of the work, but only the detail of what is done when the angles at the main stations are observed.

The main triangles should be as large as possible. The fewer triangles there are, the fewer are the chances of errors of observation.

## MAKING A MAIN STATION.

## Choice of

 Zero.Observing angles at a station is technically called " making" it. Let us suppose a surveyor making a first station, probably one of the base stations.

He has been previously furnished with a list of the main stations visible from him, and has been told how many times his angles to them are to be repeated. He has also received instructions about the secondary stations and minor marks, if any have been selected and marked.

Having levelled the theodolite, the first thing is the choice of an object from which to measure all the angles, which is called the zero.

A zero should be, if possible, another main station. It must be at some distance, but not so far as to be easily obscured on a hazy day; well defined; so placed that the rays of the sun, when it moves from the position in which it happens to be when the station is commenced, will not obliterate it. It should be a fixed object, i.e. not likely to
be removed, or to tumble down, and not so high as to be covered with clouds, as a mountain peak.

A great deal of trouble is given when a zero has to be changed, or when on a subsequent visit to a station the same zero cannot be used. Attention to the above mentioned points is, therefore, of importance.

The bearing of the zero by the theodolite compass should always be entered in the book.

The zero fixed upon, and the theodolite directed upon it, observe observe the main angles, or those to the main stations, first, $\frac{\operatorname{Angin}}{\text { Angles }}$ repeating them the required number of times, by either of the first. two methods described under "Theodolite."

These completed, observe the secondary stations a sufficient number of times, as well as all marks and conspicuous objects.

It is important to remember that the position of the sun has a great effect on the visibiiity of objects, and therefore that those stations and objects on which the sun is shining should be secured first, because later, when they fall into shadow, they may be wholly invisible.

In most instances a sketch will be also necessary, on which the angles to conspicuous objects, tangents, \&c., will then be recorded, instead of in the book.

All angles should be read twice, in order to prevent mistakes; Repatition but to ensure accuracy when required, the angles must be reof Angles. peated on different parts of the circular arc, for the following reasons :-

A theodolite, however well turned out, is seldom exactly centred, hence arises error; as no matter how uniform the graduation of the circular are may be, a slight deviation of the axis from true centring will give a difference of reading for an angle on different parts of the arc.

The sum of the readings of the two verniers is supposed to correct errors of centring, but for remarks on this, see "Theodolite."

The reading itself of an angle can never be considered as perfectly correct. Slight parallax frequently exists, especially
when an instrument has been some time at work, and is getting worn. In small theodolites the marking of vernier and arc at any given angle will often not coincide exactly, and judgment may assign the wrong reading.

By multiplying readings, then, a mean will be obtained which will to a great extent eliminate these errors, and this must always be done in observing main angles.

Excepting for main angles, forms ruled in the angle book are unnecessary, and in this case the form is simple, consisting of columns to keep the figures separate, as under.

Ł July 4th, 1881, at Pagoda © , Theod. 77.

| $\Delta^{\text {s }}$ | L Observed. | Difference. |
| :---: | :---: | :---: |
| $\oplus$ Patero - ${ }^{-}$ | $\begin{array}{ccc} \circ \\ 360 & \prime \prime \\ \hline 1 \end{array}$ |  |
| Mango - (flash) $^{\text {a }}$ | $\begin{array}{llll}25 & 14 & 30\end{array}$ | 251430 |
|  | 502930 | $25 \quad 1500$ |
|  | 754400 | 1430 |
|  | 1005900 | 1500 |
|  | Mean | $25 \quad 1445$ |

ヶ July 4th, 1881, at Pagoda © $\oplus$ Patero © $\triangle$, Theod. 77.

| $\oplus$ | Prince $\triangle$ | Flag $\bullet$ | Snow ${ }^{\text {c }}$ |  |
| :---: | :---: | :---: | :---: | :---: |
| $\stackrel{\circ}{2}$ | $\begin{array}{lll} \circ & \prime \prime \prime \\ 24 & 18 & 00 \end{array}$ | $\begin{array}{lll} \hline 0 & \prime \prime \prime \\ 29 & 10 & 30 \end{array}$ | $\begin{array}{lll} \hline 0 & \prime \prime \prime \\ 48 & 26 & 00 \end{array}$ | Z. O. K. |
| 100 | 1241900 | 1291100 | 1482700 | Z. O. K. |
| 200 | 2000 | 1030 | 2700 | Z. O. K. |
| 3 CO | 1900 | 1100 | 2700 | Z. O. K. |
| Means | $24 \quad 19 \quad 00$ | 291045 | 482645 |  |

The first of these forms is adapted for the observation of main angles by repeating round and round singly; which is done when a solitary angle is required to be observed

accurately, but to obtain great accuracy the reading should be repeated right round the arc.

The second is for ordinary main angles. This method saves much time when there are a number of angles required, and is as correct as is generally necessary.

The weak point of the first method is that the zero cannot Comparibe referred to, but, as only one angle is taken each time, a ${ }^{\text {son }}$ Methods. theodolite must be very much out of order to introduce error.

If the angle to be observed is small, this method will not answer the purpose, as the theodolite will only be rotated through a small part of the circle, unless an inconveniently large number of repetitions be made.

The weak point of the second method is that any slight error in setting or reading the zero, affects every station observed; whereas in the other, the vernier being once set at the commencement, is afterwards read only.

By either method, the observer will see if his different observations of each angle are agreeing together, and can take more if requisite.

In all observations of angles with the theodolite (except the case referred to above), the zero must be looked at from time to time, and invariably at the conclusion of the set of angles, to make certain that the direction of the instrument has not changed by any unnoticed touch or shock. On every occasion of doing this it must be noted in the book, so as to know, in the event of the zero presently being discovered to be wrong, how far back the angles must be recommenced. A common form of notation of this is, Z.O.K., or Z.K., for zero correct.

If the zero is found continually getting displaced without Defects of any apparent cause, something is loose, and this must be Instrulooked to at once, or nothing will be satisfactory. The parts most liable to go wrong have been mentioned under the head of "Theodolite."

If using a heliostat, it must be place a front of the theo- Arrange-
dolite, in the direction of the station $t$ flash. When the stations are distant on
shich you mean to $\begin{gathered}\text { ment for } \\ \text { using }\end{gathered}$
rom the other, it is Heliostat. desirable to arrange who shall flash fisou; the receiver of
the flash, say at A from B, then takes his angles to it, and does not direct his flash to B until he has got the requisite number of repetitions. When he does flash to $B$, the latter will know A has done with him, and can direct his flash to some other station, while he observes $A$. When $B$ in turn has finished with A, he must give the latter another flash to acquaint him with the fact. A's turning off his flash will show B he understands.

Heliostat invaluable.

As already remarked, the amount of time saved when the sun is visible, by the use of a heliostat, is incalculable. It is useful for long distances, and short also, and on all sorts of occasions, and is, in fact, one of the surveyor's greatest friends.

## FALSE STATION.

It will often happen that a beacon having been erected, the theodolite cannot be placed exactly on the spot, at any rate without a great deal of trouble; or if a building or tree has

been selected as a station, that the observer finds on going there, that he has to make his station on one side of it in order to see what he wants, or has to make a supplementary station to see a few objects obscured by the building, \&c. This is called "False Station," and if the object is already plotted, or
it is desirable to plot it instead of the actual theodolite spot, the angles taken there must be corrected for the distance the theodolite was from such object.

The correction will vary according to the direction of the objects with regard to the true station, as the figures annexed will show.

In Fig. 10 let A be the true station at which angles are required; B the false station; D, E, F, H objects so placed as to illustrate all positions of false and true stations with regard to them. We have observed at B the angles to $\mathrm{D}, \mathrm{E}$, $\mathrm{F}, \mathrm{H}$, and A , and measured the distance BA .

Firstly. Required the angle EAD.
Produce B A towards C and Z .
Now E A C $=\mathrm{EBC}+\mathrm{BEA}$
and $\mathrm{DAC}=\mathrm{DBC}+\mathrm{BDA}$.
Subtracting we have

$$
\mathrm{EAC}-\mathrm{DAC}=\mathrm{EBC}-\mathrm{DBC}+\mathrm{BEA}-\mathrm{BDA}
$$

or

$$
E A D=E B D+(B E A-B D A)
$$

Secondly. Required the angle EAF.

Here
and
$Z \mathrm{AE}=\mathrm{ZBE}-\mathrm{BEA}$
$Z \mathrm{AF}=\mathrm{ZBF}-\mathrm{BFA}$.

Adding, we get $\mathrm{ZAE}+\mathrm{ZAF}=\mathrm{ZBE} \times \mathrm{ZBF}-(\mathrm{BEA}+$ BFA)
or

$$
E A F=E B F-(B E A+B F A)
$$

Thirdly. Required the angle DAH.
Here $\quad \mathrm{CAH}=\mathrm{CBH}+\mathrm{BHA}$
and $\quad \mathrm{CAD}=\mathrm{CBD}+\mathrm{BDA}$.
Adding, we get $\mathrm{D} \mathrm{AH}=\mathrm{D} \mathrm{BH}+(\mathrm{BH} \mathrm{A}+\mathrm{BDA})$.
These small angles, BD A, BE. A, \&c., the angles subtended by the distance between true and false station at each object observed, must be either calculated in each triangle, having two sides and the included angle (for the rough distances BD, BE, \&c., will answer the purpose), or else, which is simpler, have a table* made of the angles which are sub-

[^7]tended by different lengths at different distances, and take the required angles out, thus:

Calculating correction by Table.

Arrangements for working by the Table.

Let us suppose the theodolite angle in our book corresponding to A is $60^{\circ}, \mathrm{D} 160^{\circ}$, and $\mathrm{E} 220^{\circ}$. A B is 12 feet. E B is $1 \frac{1}{2}$ miles, and BD 2 miles, measured roughly on the sheet.

Required BE A by the table.
It will be evident that the angle BEA is that subtended by a chord drawn across to E A from B. This chord we get near enough by considering $B N$ as at right angles to both $E B$ and EA, and looking out in the traverse table with B A, or 12 feet, as a distance, and B AN or $20^{\circ}\left(180^{\circ}-160^{\circ}\right)$ as a course, and taking the departure for the length of $B N$, which in this case is $4 \cdot 1$ feet.

We then turn to our table, and see that four feet at $1 \frac{1}{2}$ miles subtends $1^{\prime} 31^{\prime \prime}$, which is the angle B E A. In a similar manner we can deduce any of the required angles, quite near enough for ordinary purposes.

Now this process becomes far simpler, and much time is saved if, in making a false station, a zero for the theodolite is chosen in a direction exactly opposite to the true station, as for example, in our figure at $Z$; for then each angle taken can easily be corrected separately for the error of the false station, and the true angle entered in the book. Difficulties as to whether the ultimate correction is + or - will be avoided, as in correcting the angles the error is subtracted from all theodolite angles up to $180^{\circ}$, and added to all angles between $180^{\circ}$ and $360^{\circ}$.

Thus, in the case as in the figure above, the angles will stand in the book-


In using the traverse table, take for the course-
Up to $90^{\circ}$ the observed theodolite angle itself
Between $90^{\circ}$ and $180^{\circ} \ldots \quad 180^{\circ}$-the obscrved angle

| $\#$ | $180^{\circ}$ | " | $270^{\circ}$ | $\ldots$ | observed angle- $180^{\circ}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $"$ | $270^{\circ}$ | $\#$ | $360^{\circ}$ | $\ldots$ | $360^{\circ}$-observed angle |

-and the departure is looked out in each instance.
The table of angles subtended by different lengths is useful Table for other purposes. As when an angle is taken to an object, ${ }_{\text {useful. }}^{\text {usenerall }}$ and it is afterwards decided to plot a station made near that object instead of the object itself, the angles to the station can be corrected by it, in precisely the same manner as described above, the distances and direction of the station from the object being known.

Distances or lengths, greater than those included in the Extension table, can be got by multiplication or division.

Thus, if the angle of 18 feet at 5 miles is required, it is double that of 9 feet.

Again, if the angle of 12 feet at 10 miles is wanted, it is half that at 5 miles.

## SKETCH.

A sketch taken from a station is made with the object of more easily identifying details to which it is necessary to take angles. By having a view of hills, islands, houses, trees, \&c., from two or three stations they can, if fairly placed in their proper positions, be easily recognised in the different sketches when plotting. No description in the angle book will do this so well unless, of course, there is something very remarkable in the object, but even then the position of it as shown in the sketch will assist materially to prevent mistakes, and a curt description is also written against it on the sketch.

Sketching to this extent is within anybody's reach. A fairly correct outline is all that is absolutely necessary, and a very little practice will enable the least likely draughtsman to make a sufficient sketch for practical purposes.

Checking Scale by angles.

Preservation of scale.

Useful hints for sketching.

It is well for a beginner to commence by taking some rough angles to check his scale, or, until he is used to it, he will probably have one part of his view two or three times as big as the other, which is confusing afterwards, although the proper angles will be written against the prominent objects when the sketch is finished.
Always put the most distant outline on the paper first, as it is far easier to keep the scale uniform if this is done.

Begin on the extreme left of your view, or if it is an all round view, choose a point, in the direction least required, to be the left, and always work to the right.

If the sketch is too long for one double page of the sketch book, when the right-hand end of it is reached, turn over, and turn one or two inches of the last page down, so as to show on the fresh page; this will give a commencement for the part to follow, and the sketch will be continuous.

Commence by settling whereabouts on the paper some two well-defined points of the distance are to be, and use these after as a scale from which to measure by eye the proper position of everything else.

If taking angles to assist correct drawing, as suggested above, a scale for the sketch must be decided on, say about one-third of an inch to a degree, but this will vary according to the complication of the sketch. If no divided scale is at hand, mark the edge of a strip of paper by eye, which will answer the purpose perfectly.

Take an angle from some definite point of the distance on the extreme left to some other, say about $20^{\circ}$ to its right. Make a dot for the first object, lay the scale or strip of paper on the sketch, and dot again at the proper number of degrees, and at the proper height, with regard to difference of altitude, for the second object. Other angles can be taken to other objects between these, and the view sketched in between these dots, commencing as already said with the outline most distant, and therefore highest in the sketch.

In sketching for this purpose, it is well to rather exaggerate the height of objects, as, where there are hills, range upon
range, or many objects, as houses, trees, \&c., at different altitudes, they will get so crowded up as to make the sketch difficult to decipher, unless this course is adopted.

The great thing in a sketch is to place objects fairly Important correctly with regard to each other horizontally considered; point to e.g., if there is a hill with a point nearly underneath it, take care that the latter is drawn on the correct side of the hill, right or left. Nothing is more calculated to confuse anybody plotting angles from a sketch, than to find that an object drawn apparently to one side of another object has an angle which shows it should have been on the other side. Doubt is at once thrown on the angle, when it is probably the drawing of the sketch which is incorrect.

When the sketch is finished resume the theodolite, using Descripthe same zero, and mark the angles on the sketch itself, objects in noting what the object is, when it may be doubtful, as for Sketch. instance-Chimney of red house, Right of two fir trees, Big white boulder, \&c. See example of sketch attached.

## PREPARATION FOR CALCULATING THE TRIANGULATION.

It is well that a true bearing be obtained between two a bearing distant stations, before plotting; but the method of doing this will be described under observations, and as far as absolute tation of necessity goes, a good compass bearing from a shore station is Chart. quite sufficient to begin on. The bearing is only wanted to plant the meridian fairly square on the paper, and the compass bearing will give us this, near enough to be able to lay off any bearings which may be taken in the course of mapping the detail. The compass will never be used in any of the important part of the chart, unless our survey partakes of the nature of a sketch or running survey.

If, however, regular triangulation is likely to fail, true bearings in the course of the work may be necessary to carry it on, and in this case we must begin with a careful true bearing.

Preparation of Triangles.

Spherical excess.

In preparing the triangles for working, they will of course never be found exactly correct, i.e., the three observed angles will be either more or less than $180^{\circ}$.
In dealing with this theoretically, the sum of the three theodolite angles taken at the corners of any triangle will be greater than $180^{\circ}$, in consequence of each angle observed being in a different plane. This is known as the spherical excess, and in extended triangulations for topographical purposes, as the survey of India, \&c., must be taken into account. For practical nautical work we need not regard it, as our instruments are not large enough to measure angles so exactly, nor is our work of sufficient extent.
Correcting In dealing with the amount the triangle is in error, for the $\xrightarrow{\text { the }}$ Triangles. three angles of the triangle must be corrected to make the precise $180^{\circ}$, before using them for calculation, circumstances must guide its distribution among the angles.

An angle observed with a large theodolite should have more value given to it than others. One station may have been more exposed to the wind than others, which would depreciate the value of the angles observed there.

Without any indications of this kind to guide, it is as well to divide the error equally among the angles; but it must be remembered, that an alteration in the small angle will make more difference in the resulting position than in either of the other two, so that if this angle at all approaches the limit which should be used for a receiving angle ( $30^{\circ}$ ) it is perhaps well to put the smallest amount of change into it, but it is of course impossible to guess where the error is. If the angles have been repeated often enough, the resulting error any way will be very small.
Error admissiblein

## Triangles.

No rule can be laid down with regard to the amount of deviation from the $180^{\circ}$ that can be admitted, it so much depends on the degree of accuracy required, but in an ordinary theodolite survey the error should not be more than three minutes, and ought to be under two, working with fiveinch theodolites, and repeating the angles three times if satisfactory, or more if they vary much.

In the first few triangles, the error should not be more than one minute.

Having corrected the triangles we come to the calculation.
The working out of the triangulation is the very simple affair of plane triangles which every naval officer understands. The rule of sines, and the rule to find the third side,* when two sides and the included angle are given, are all that are required.

Logarithms of all angles must be taken out to seconds, Logaso that the possession of tables giving these for every second ${ }_{\text {exact }}^{\text {rithms }}$ of are, will save much time and chance of mistake.

Into the final calculation of an extended calculated trian- Convergulation some other considerations enter.

The actual working of the triangles will be the same; but here we want the bearing of every side, as well as the distance, and the "convergency of the meridians" must be considered. This convergency will be explained before proceeding further.

## CONVERGENCY OF THE MERIDIANS.

The true bearing of any two points on the earth, taken one from the other, in both directions, will be found to differ by a quantity which is called the convergency, and varies with the latitude, distance apart, and position of the points in bearing, or in other words, with latitude and departure.

Thus, if $R$ and $L$ are two stations lying roughly N.E. Illustraand S.W. of one another, $R$ being nearest the pole, in this $\begin{gathered}\text { tion of } \\ \text { Conver- }\end{gathered}$ case the North Pole, the true bearing of L from R will be gency. found to be a greater number of degrees and minutes as measured from the meridian than the reverse bearing of R from $L$.

This results from the form of the earth. The true bearing Explanaof one position from another, is the angle which the arc of a ${ }^{\text {tion. }}$ great circle drawn between the two positions makes with the

[^8]meridian of the observing position. As meridians are not parallel, but converge at the poles, the great circle will cut each meridian it passes at a different angle, the amount of difference, for equal meridians, depending on the latitude.

To further the comprehension of this, let us consider the method of projection of the sphere used when graduating a map, made from the original data of angles and measurements.

Projec-
tions of the Sphere,

It will be evident to any one who considers the subject that as our globe is a sphere (speaking roughly), a portion of its surface cannot be shown on a flat piece of paper without distortion, more or less, according to the extent so shown.

There are a variety of methods used to delineate a portion of the earth's surface on a map, which are called "projections." Into this variety it is not proposed here to enter, as but one can be used when actually making a survey, which is the " Gnomonic Projection."
Gnomonic
This projection is the only one on which great circles Projection. of the earth are shown as straight lines. As it is on the chord of a great circle that we see one object from another, it is evident, that in graduating a map on which we have laid down, or are going to lay down, one position from another by drawing straight lines, we must use this projection.

A chart on the Gnomonic Projection is supposed to be drawn on a flat surface laid against the earth, touching it at the central point of the flat surface, and there only. From the centre of the earth lines are supposed to be drawn, passing through the different points to be shown on the map, until they pierce the flat surface.

The positions so indicated on the upper side of the flat surface, are those corresponding to the points required.

Here, in Fig. 11, P Q S is the globe, and A B C D a flat surface laid against it, touching at the point J, the centre of the flat surface, the under side of which is shown. P is the pole. M F are points taken on the same meridian as J . Imaginary lines drawn from the centre of the earth through these points will touch the flat surface in N and G, and the
line joining them, the central meridian of the chart, will be a straight one. K, another point on the globe east of the central meridian, will be projected at L , by the same method of drawing a line from the centre through $\mathrm{K} . \mathrm{X}$ is the point

in which the axis of the earth, produced, meets the central meridian of the chart also produced.

Let us again look at our flat surface, which we may now call the chart, from a different point of view, i.e. from a point in the extension of the line joining the centre of the earth and the central pcint of the chart.

In Fig. 12, $\mathrm{A} \mathrm{B} C \mathrm{D}$ is the chart as before, touching the spherical earth at the central point J. G and $N$ are the positions on the chart of the points on the earth's surface, F and M in the other figure. G J N is then the central meridian of the chart. X is, as before, the point where the extension of this meridian meets the extended axis of the earth. L is the position on our chart of K (see other

figure). $R$ is the position of a similar point, invisible in first figure, being on the other side of the earth. Meridians passing through $L$ and $R$ are projected on the chart by the same method as before, i.e. by drawing imaginary lines from the centre of the earth through different points in the required meridian; they will be found to lie as $\mathrm{T} L, \mathrm{R} \mathrm{O}$, and their extension will also pass through X , making an angle R X L, which is the Convergency of the meridians; and this
will be seen at once to be equal to the difference of the reverse bearings of $R$ and $L$, for,

$$
\begin{array}{ll} 
& \angle \mathrm{ORL}=\mathrm{OXL}+\mathrm{RLX} \\
\text { or } & \mathrm{OXL}=\mathrm{ORL}-\mathrm{RLX}
\end{array}
$$

i.e. Convergency $=$ Bearing of $L$ from $R-$ Bearing of $R$ from L.

A little consideration of this last figure will show, that the further towards the pole the central point J is, the greater will be the convergency of two meridians a fixed number of Converdegrees apart; that when the pole $P$ and $J$ coincide, the $\begin{gathered}\text { gency Equa- } \\ \text { gil }\end{gathered}$ meridians will radiate over the chart from that centre, and tor and the convergency will equal the distance between the equal to diff. long. meridians ; and that when $J$. is on the equator, the meridians ${ }^{\text {at Poles. }}$ will be parallel, and convergency will be nothing.

Parallels of latitude will appear on the chart as curves, Parallels. concave towards the poles, and cutting each meridian at right angles.

The equator being a great circle will be a straight line, and, generally, the further from the equator, i.e. the higher the latitude, the greater will be the degree of curvature in the parallels.

More consideration will show, that, the farther a part of Disthe flat surface is from the surface of the earth, the greater will be the distortion of the pósitions resulting from this method of delineating the globe; or in other words, that the distortion increases from the centre of a gnomonic chart, and will become very considerable towards the edges, if a large area of the earth is attempted to be shown on a flat surface. But in practice, a marine survey does not extend over a sufficient area to make this distortion in any way apparent. Our diagrams are of course much exaggerated in this respect.

It will be understood that the convergency is an actual fact, and does not result merely from the method employed in this projection. We have only considered it in connection with the projection, as it is thought that by so doing the nature of the convergency becomes more plainly apparent.

Mercatorial Bearing.

The mean of the two reverse bearings, or either one of them, plus or minus half the convergency, will give the Mercatorial Bearing, so called from being the bearing which each station will be from the other in a Mercator's chart, where, the meridians being all parallel, the line joining the stations will cut them at the same angle, this angle being also the one at which the line on our gnomonic chart will cut a meridian midway between the stations.

The actual observed bearing of a distant object, if protracted on a Mercator's Chart, will not pass through its position, in consequence of the existence of convergency. Mercator's charts are generally on such a small scale that, for navigating purposes, the error of taking the bearing swallows up the error introduced by not allowing for convergency.

The formula for Convergency is-
Conver- Tangent Convergency $=$ Tan departure $\times$ Tan Mid. lat. (1)
gency
Formulæ.

Or in anything but high latitude, or when the departure is great, it is correct enough to say-
Convergency (in mins.) $=$ dep. (in mins.) $\times$ Tan Mid. lat. (2) which can be converted into
Convergency $=\mathrm{d}$. long $\times$ Sin Mid. lat.
$=$ dist. $\times$ Sin Merc. Bearing $\times$ Tan Mid. lat. (4)
any of which can be used as convenient.
The proof of the formula is given in the Appendix A.

Convergency by Spherical Triangle.

The convergency can also be found when the latitudes of and difference of longitude between the two stations is known, by working out the spherical triangle, with the pole, and the two stations, as the three points. Here we have the two colatitudes as the sides, containing the difference of longitude as the polar angle, to find the other two angles, which will be the bearings of each station from the other. The difference of these will be, as before, the convergency.*

[^9]
## CALCULATING THE TRIANGULATION.

We now resume our remarks on working out a calculated main triangulation. All sides being calculated by the ordinary method of plane triangles, we now want the bearing, the mercatorial bearing, of each side, or, at any rate, a considerable number of them, in order that we can take any triangles or sides to work up details on, on a separate sheet, and that such sheet may be complete in itself as to bearing, distance, and position, with regard to other portions of the main triangulation.

We will take as an example the following :-

$$
\text { Lat. A, } 49^{\circ} 30^{\prime} 24^{\prime \prime} \mathrm{N} \text {. }
$$

True bearing observed B from A. N. $69^{\circ} 05^{\prime} 00^{\prime \prime} \mathrm{W}$.


|  |  | Observed. |  |  | ${ }^{\text {Corrected. }}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| B |  | 72 | 40 | 17 | 72 | 40 | 31 |
| C | . | 62 | 46 | 39 | 62 | 4 | 53 |
| E | .. | 44 | 32 | 22 | 44 | 3 | 36 |
|  |  |  | 59 |  | 180 |  |  |

FIG 13.


Example of Calculated Tri-angulation.

We have in the annexed figure (13) a portion of a triangulation, where all the angles have been observed at each station. The latitude of A is known, A B is the original long side obtained by extending the base, and the true bearing of $B$ and A have been taken from one another, from which we have deduced a mean bearing of $B$ from $A$ with which we intend to work. The length of each side has been calculated by ordinary trigonometry. We now want to calculate the bearings of the different sides, so as to be able to break up the triangulation into different sheets. We shall want also the latitude, and difference of longitude from A , of F , which is a station in a plan on a large scale we have made.

For the purposes of this plan we have obtained the side F C in the triangulation, which will serve as our base instead of measuring another.

We shall commence by calculating the convergency for ten miles of departure at the average latitude of the chart, as we shall want it directly.

In this case we find that-
Convergency for $10^{\prime}$ of departure $=11^{\prime} \cdot 92$.
Or for each mile of departure $=1^{\prime} \cdot 2$.
We then find approximate latitude of B by the formula-
Diff. lat. $=A \mathrm{~B} \times$ Cos rough mercatorial bearing.
We obtain the bearing, near enough for this purpose, by finding the rough convergency and applying half of it to the observed bearing of $B$ from $A$, thus-

Take departure from the traverse table, in this instance 9.5 . Multiply it by the convergency for a mile, just found to be $1 \cdot 2$, which gives us $11^{\prime} \cdot 4$ as the rough convergency. Adding half of this to the bearing of B from A , we get rough mercatorial bearing N. $69^{\circ} 11^{\prime}$ W., and working out the difference of latitude, we find it to be $3^{\prime} 38^{\prime \prime}$, which gives for the latitude of $\mathrm{B}, 49^{\circ} 34^{\prime} 02^{\prime \prime}$, and for middle latitude $49^{\circ} 32^{\prime} 13^{\prime \prime}$.

Then convergency $=$ dist. $\times$ Sin merc. bearing $\times \tan$ mid. lat.
Using the rough bearing just found, we get-
Convergency for $\mathrm{A} B=11^{\prime} 13^{\prime \prime} \cdot 8$.
This convergency, and half of it, added respectively to the bearing of $B$ from $A$, will give the reverse bearing of $A$ from B , and the mercatorial bearing, thus,

$$
\begin{array}{r}
\mathrm{B} \text { from } \mathrm{A}=\mathrm{N} .69^{\circ} 05^{\prime} 00^{\prime \prime} \mathrm{W} . \\
\mathrm{A} \quad, \quad \mathrm{~B}=\mathrm{S} .69 \\
69
\end{array} 14 \frac{14}{\mathrm{E} .} \begin{array}{r}
\text { Mercatorial bearing } \frac{\mathrm{N} .}{\mathrm{S}} 6910
\end{array} \begin{array}{llll}
67 & \frac{\mathrm{~W} .}{\mathrm{E} .}
\end{array}
$$

If this differs much from the rough mercatorial bearing, we must recalculate the latitude of B before proceeding further, but this should not be necessary.

Then to calculate bearing of BE, we have the bearing of A from $B$, just found, to start from. Adding the three angles, A B H, HBC, CBE, to it, we shall get the bearing of E from B. The convergency for $B E$ is calculated in the. same
manner as above, and we shall then have mercatorial bearing of B E. Thus :-
$\left.\begin{array}{cccccccccc}\text { A from B } & . & . & . & . & \text { S. } & 69^{\circ} & 16^{\prime} & 14^{\prime \prime} & \text { E. } \\ \text { A B H } & . & . & . & . & . & 38 & 51 & 47 \\ \text { H B C } & . & . & . & . & . & 59 & 33 & 27 \\ \text { C B E } & . & . & . & . & . & & 72 & 40 & 31\end{array}\right]$

Application of Convergency.

In like manner we must calculate the mercatorial bearing of all the sides we require, remembering that of the reverse bearings, the bearing of the station nearest the pole from the one farthest from the pole, is the smallest. In this case then, being in the northern hemisphere, where a bearing is measured from the north point, the convergency is added to obtain the reverse bearing.

Having obtained the bearing of each side, we can calculate the relative position of any two stations by working out the traverse between them.

Thus to get position of F we have,-


From which we calculate difference of latitude and departure in the ordinary manner.

We thus get the mercatorial bearing of A F, N. $12^{\circ} 32^{\prime} 45^{\prime \prime} \mathrm{W}$., and distance 25.5269 miles.

It will be understood that it is by no means necessary to work out all the triangulation as just described when commencing the plotting. All that is then required is as long a side as we can get on which to begin. The main triangulation can be calculated afterwards, and in many instances must be,
as the whole of the angles will not be obtained till later on. In some nautical surveys it will not be necessary to calculate any triangulation at all.

In the example of triangulation we have given we have supposed ourselves to be working from a measured base. If the survey is extensive, the ultimate scale of the chart will depend upon the astronomical positions. It is very unlikely that when these are obtained, the distance between the extreme points depending upon them will agree exactly with that deduced from the short side, and therefore all the sides will want correction in probably both bearing and distance.

The readiest way of doing this is to get a proportion between the two total distances, as found by the triangulation and by the astronomical positions respectively, in the shape of a logarithm, and multiply each side found by it, which will give the true value as dependent on observations. The bearing of every side will have to be corrected by the difference of the bearings of the extreme points.

Thus referring again to our example, (which, for the sake of brevity, we have confined to only a few sides,) let us suppose we find by observations that A F is N. $12^{\circ} 36^{\prime} \mathrm{W} .26 \cdot 248$ miles.

Dividing this distance by the former one, we get a proportion whose logarithm is 0.012097 . Adding this to the $\log$ of each side required to be corrected will give us the true value.

The difference of bearing is $3^{\prime} 15^{\prime \prime}$ more to the westward. The bearing of each side will then have to be corrected by this amount. Thus the bearing of $A B$ will stand N. $69^{\circ} 13^{\prime} 52^{\prime \prime}$ W.

This difference is somewhat exaggerated. It should seldom, when true bearings have been well observed, amount to so much, but in some climates it may be unavoidable.

In a case of this kind the result of both triangulation and astronomical observations would be transmitted home, as their concurrence or otherwise will form a good test of the value of the work generally.

In stating as we have that the ultimate scale of the chart
of an extended piece of coast will depend upon the astronomical positions at either end, it is not intended to lay down a too hard-and-fast rule. The conditions of each element, triangulation and positions by astronomical observations, must be considered. Both, under the ordinary circumstances of a marine survey, are liable to error. In a rigorous trigonometrical survey, the triangulation is more likely to be correct owing to the unknown error in the astronomical positions due to local attraction of the pendulum, or in other words of the mercury in the artificial horizon; but this very local attraction makes it necessary in a marine survey to regulate the distances by observations, as other surveys have to start from the same position, and subject to the same error, and moreover, in an ordinary marine survey, the triangulation is carried on under conditions which prevent the possibility of ensuring freedom from errors in it. Nevertheless, should the discrepancy in bearings be large, when we know our true bearings have been well observed and our triangulation to have carried it on within the limits of the discrepancy, it is desirable to adjust the astronomical positions so as to reduce the discrepancy in bearings.

Triangles containing small angles not always ill-conditioned.

It will be observed that we have a triangle CD F with a very small angle. This not being a receiving angle does not matter in the least. We are obtaining the position of F from C and D, which are already fixed, and the angle of intersection at F being nearly a right angle, the change of position in F, resulting from a small error in the angle at either $C$ or $D$, will be as small as is possible, and much less than if the angle at C being the same, that at D was $60^{\circ}$, which would result in the intersection at F being more acute, and any error would consequently change the position of F to a greater degree.

If we were obtaining $D$ from $C$ and $F$, such a small angle would not-be admissible for a moment, as it is evident that any small error at C or F would result in a great change of position in D.

It would be awkward and inconvenient to have many such
triangles in the main framework of the triangulation, as the small side is of no use in carrying on the chain, and we should be forced to multiply triangles in consequence; but we are, notwithstanding, sometimes obliged to include some such in our work, from the lie of the land and other causes, and as long as we use them as in the example they will not affect the result, as far as chance of accuracy goes, and should not be under these circumstances considered as "ill-conditioned."

In working out the diff. lat. and diff. long. of two posi- Correction tions from the triangulation geodetically, we have been $\frac{\text { for the }}{\text { spheroid. }}$ treating the earth as a sphere. This is not strictly the case, as the form of our globe is that of an oblate spheroid; but the error introduced by assuming it to be a sphere is small, and can often be disregarded in hydrographical work, as being swallowed up in the larger errors incident on imperfect triangulation.

When, however, a triangulation has been carefully done, and we wish to get the difference of longitude as near as we can, either for the scale of the chart, or for purposes of comparison with that deduced from the astronomical positions, or in latitudes far from the equator, the necessary correction for the spheroid should be applied.

This correction is $2 \operatorname{Cos}^{2}$ Mid. lat. $\times$ compression.
The compression of the earth is the proportion that the difference of the equatorial and polar diameters bears to the diameter, and can be taken as $\frac{1}{300}$.

The formula for correction for a given difference of longitude will then stand,-

$$
\text { Correction }=\text { diff. long. } \frac{\operatorname{Cos}^{2} \text { Mid. lat. }}{150}
$$

This is subtractive from the calculated difference of longitude by the triangulation.

In the latitude of $20^{\circ}$, this correction for a difference of longitude of $100^{\prime}$, amounts to $35^{\prime \prime}$, as will be seen by the following example:-

Example. In latitude $20^{\circ}$ the departure deduced from a triangulation was found to be $94^{\prime}$, required true difference of longitude.

$$
\begin{aligned}
& \text { Dep. .. .. .. 1•973128 } \\
& \text { Sec. lat. .. .. 0.027014 } \\
& \text { Spherical d. long } 2 \cdot 000142 \text {.. .. } 100^{\prime} \cdot 0327 \\
& \text { Cos }^{2} \text { lat. .. .. } 9 \cdot 945972 \\
& 11 \cdot 946114 \\
& 150 \text {.. } 2 \cdot 176091 \\
& \text { Reduction } \overline{1} \cdot 770023 \text {.. .. }-0.5889 \\
& \text { True diff. long. } \quad 99 \cdot 4438^{\prime} \\
& \text { or } \quad 1^{\circ} 39^{\prime} 26^{\prime \prime} \cdot 6
\end{aligned}
$$

The true difference of longitude can also be calculated from the tables of lengths of a minute of latitude and longitude in the Appendix M as follows :-

True diff. long. $=$ dep. No. of feet in minute of Lat.
Working out the above example this way, we have-

| Dep. .. .. | .. | 1.973128 |
| :---: | :---: | :---: |
| 6053 .. | .. | 3•781971 |
|  |  | 5.755099 |
| 5722 .. | .. | $3 \cdot 757548$ |
| True d. long | .. | 1.997551 |

which gives the same result as the other method.

## CHAPTER V.

## PLOTTING.

This chapter will comprise, besides a description of the Subjects method of placing the points on the paper, which is more in chapter. $_{\text {comprised }}$ generally understood by the term "plotting," an account of the different manners in which those points may be obtained, other than by a regular chain of triangles. This is, perhaps, more correctly, a part of triangulation, and for some reasons should be described under that article, but it is thought that it will tend to clearness of comprehension, if it is taken in connection with the mode of laying down the points as obtained, as it is not easy to separate the two steps in many instances.

In discussing the general question of Plotting, therefore, we will first take the placing of the points of an ordinary triangulated survey on paper, and then consider some other systems to be adopted when regular triangulation fails us.

Plotting the points is a most important operation, and one Great care requiring great care.

No matter on what scale, or on what system, a survey is ting. being made, equal pains must be bestowed on plotting the points. Indeed, it may almost be said that in proportion as the elements of a survey approach to the least accurate form, viz., a sketch survey, so does the necessity for careful plotting increase, as the numerous checks, which in a detailed triangulation will instantly make any error in plotting apparent, will be more or less absent in proportion to the departure from such regular triangulation; and not only will
the minor details of such a chart be inaccurate, which we expect, but the main and prominent points may be unnecessarily out of place unless care is bestowed on the plotting.

Plotting by chords.

Before describing in detail the different methods in plotting, it is necessary to understand the system of laying down angles by chords, and why this is done.

It will easily be seen that, where lines are to be drawn of considerable length, a protractor whose radius will be much shorter than the desired line, can hardly give the angle exact enough to ensure the extremity of the line being precisely placed; for the straight-edge, perhaps six feet in length, by which the required line is to be drawn, will only be directed by two pricks in the paper, which, with the largest protractor, will not be more than eighteen inches apart. However exactly the protractor has been placed, and the pricks made, the mere laying of the straight-edge so that the line drawn will pass precisely through the centre of the two pricks near together, is almost an impossibility, and an error, quite imperceptible at the pricks, will be very appreciable at the end of the straight-edge.

For this reason, we want our directing prick as far along the straight-edge as we can get it.

We accomplish this by using chords.
If two radii of a circle of given length of radius, containing between them a given angle $\theta$, be drawn to cut the circumference of the circle, the chord to the arc of the circumference thus cut off is 2 radius $\operatorname{Sin} \frac{\theta}{2}$.*

Thus, by reversing this and describing from the centre A, Fig. 14, an are of a circle of any radius, drawing the line A C, and measuring the chord C B (which will be done in practice by describing a short arc of a circle with the required chord as radius, from the centre $C$ ), the point $B$, where the chord cuts the circumference (or the two arcs intersect) joined to $A$, will give the required angle $\theta$.

[^10]A table of chords for a radius of 10 inches is given in Table of Appendix,* which saves much time and chance of errors, as Chords. the chord to the angle required can be taken from the table, and multiplied by the radius with which it is meant to lay off the angle, divided by ten ; but in case this is not at hand, we must calculate our own chords.

Tables of natural sines are not included in Inman's, the Calculattables generally in use at sea, and logarithms of sines are in ingChords. that work only given for every fifteen seconds, and we may want to take the angles out exactly. Moreover, by using the logsine, three logarithms will have to be taken out, and the process is somewhat longer. It is simpler, therefore, to use

## FIC 14.


the table of natural versines, which are given in Inman to seconds.
As $\sin . \frac{\theta}{2}=$ versine $\left(90+\frac{\theta}{2}\right)-1$, our required chord will be 2 radius (vers. $\left(90+\frac{\theta}{2}\right)-1$ ).

Versines are given for a radius of $1,000,000$, so we have to divide the versine taken out by that number. This reduces the rule in practice to this.

Look out the natural versine of $90^{\circ}+$ half the required angle, leaving out the left hand figure 1 , and putting a decimal point before the remaining six figures. Multiply this number by twice the radius, and the result will be the chord required.

Let us take now an example in practice.

[^11]At A, Fig. 15, the angle between B and C is $35^{\circ} 14^{\prime} 30^{\prime \prime}$. The line A L from A passing through B is already drawn. We want to lay off this angle, and requiring accuracy, we take a long radius, i.e., 45 in .

Forty-five inches must be carefully measured, by the brass diagonal scale, on to a pair of beam compasses, with the two steel points shipped. Flattening the paper down by placing the straight-edge close to the line A B, and putting weights on it, with the centre A describe a short arc of circle D E, scratching lightly the surface of the paper. Then moving the straight-edge into the direction of C (which can be ascertained roughly by a protractor), and again weighting it, make another small scratch F G. With the assistance of a readingglass, and by means of a needle mounted in a handle, and

FIG 15.

spoken of as the "Pricker," make a fine prick at the intersection of the lines A B, D E, i.e., at H.

Look out the versine of $107^{\circ} 37^{\prime} 15^{\prime \prime}(90+$ half the required angle), which is $1,302,717$. This becomes 302,717 , which multiplied by 90 , gives $27 \cdot 244$ inches as the chord.

Measure this distance on the beam compass, and flattening the paper as before, draw, with H as a centre, a short arc K M crossing F G. The point of intersection is to be pricked carefully as before, and the straight-edge can now be laid on A and it, and the line ruled will be at exactly the angle required. This seems a tedious operation, but it is the only way in which points can be got to go down satisfactorily, and in the end mach time will be saved.

It may be noted here, that it is preferable to make a mark


#### Abstract

with a steel point instead of a pencil, from the practical Steel difficulty of measuring accurately the required distance on boints to


 the beam compass when the pencil point is used, as, when the pencil point is cut sharp enough to make a fine line, it is almost impossible to prevent breakage in applying it to the brass scale divisions. It is also cleaner. In marking, the point must be held sloping, so as only to impress, and not actually to scratch the surface of the paper, which it will do if held perfectly upright.Of course, if the paper is stretched on a board instead of being loose on the table, the time and trouble of seeing the paper flat is saved, but this is seldom used in our work.

If the table of chords is available, look out the chord for $37^{\circ} 14^{\prime} 30^{\prime \prime}$ and multiply it by $4 \cdot 5$, as the table is made out for a radius of 10 in . This will give the same quantity of $27 \cdot 244$ inches as found above.

C may be of course anywhere on the line $A B$, and supposing ourselves to be plotting from an original base A B, will probably be much nearer to A than to F G , but by taking such a long radius we get a straight line in the true direction of the angle laid off, and when we want to measure another angle on to another object, perhaps three times the distance of C from A , we have a long line we are certain of, to do it from.

Here let it be impressed upon the surveyor that all lines Always drawn for plotting the main points, and indeed all points draw long (except very minor ones, on which the position of nothing else will depend), must be drawn as long as possible, and with more or less long chords, if we desire correctness. If we have a line drawn between two stations which lie, say six inches apart on the paper, and it only projects a few inches beyond each, and we hereafter require to lay off an angle from one, having the other as zero, to a station which will be, say two feet or more distant, we cannot do it correctly, as this longer line will have to be directed by a prick which cannot be farther off than the length of the zero line; but by drawing long lines with long chords, we are ready for
anything, and it will not matter whether the station we take for zero be near or far, as we use, not it, but the long line ruled through it.

Length of radius.

In no case should a line to a station be laid off with a protractor or chord whose radius is less than the distance of the station, excepting in a rough plan which we want to do rapidly, or in most parts of a running survey, where pretensions to accuracy being thrown to the winds, we get points near enough for our purpose down with a protractor.
Lengthen-
It is difficult to extend correctly a short line once drawn, by simply ruling on with the straight-edge. If a longer line is wanted, it is better to lay off the angle to it again from some other long line, with a sufficient radius.

## Ruling a

 straight line.Angles
over $60^{\circ}$.
To rule a true straight line which will pass exactly over the centre of the pricks is by no means an easy thing. The ruling pencil, which should be of the hardest lead manufactured, should be cut to an edge, not a point, and the straight-edge being placed in position, and weighted to keep it in contact with the paper throughout its length, the flat side of the pencil is placed against it, and tried at both points, to see whether the line will pass truly over them. Care must then be taken to hold the pencil in the same position while drawing the whole line.

In laying off by chords an angle over $60^{\circ}$, or a little under $60^{\circ}$, it will be found best to mark off $60^{\circ}$ first, and measure the remainder of the angle from the $60^{\circ}$ prick. This is done by drawing short ares with the radius used, from the station from which it is desired to lay off the angle, and from the radius prick ( $H$ in last figure), the intersection of these must be pricked off as $60^{\circ}$, and another short are being drawn with the originating station as centre, the chord of the difference of the angle from $60^{\circ}$ is measured from the $60^{\circ}$ prick to the last short arc, as in Fig. 16 (p. 107).

This is done not from any incorrectness of the principle if the angle were laid off at once, but because it is inconvenient to be measuring long distances as chords, as there is a greater chance of some little inequality of the paper causing error,
and also, the longer the chord measured, the more acute will be the angle between the two intersecting arcs, and consequently the greater the difficulty of pricking in accurately at the intersection.

Understanding then how to lay off angles by chords, and Comhaving obtained by calculation as long a side as we can for a $\frac{\text { mence- }}{\text { ment }}$ of plotting base line, so as to plot as much as possible inwards, Plotting. or with decreasing distances, and not outwards to stations farther distant than the original two, and having settled whereabouts on the sheet this base line shall be placed, draw a meridian line, parallel with the side of the paper, and passing

at one end of where the base is to be. Make a prick on this line for one end of the base, using, as always for pricking, a reading-glass, to ensure getting the prick exactly on the line. Let us call this A.

From A, lay off, with as long a chord as can be commanded, the true bearing of the base, and having ruled this line of bearing as long as possible, make another prick on it, at the required distance from $A$, for the other end of the base. From the two base stations lay off angles to two other main positions, and choose the one of these where the intersection of the lines makes the nearest angle to $90^{\circ}$ as the third station
to prick in, doing so with great care on the intersection of the two lines. Then from this third station lay off an angle to the fourth, and if this, when ruled, passes exactly over the intersection of the two lines from the base stations, it can be pricked in. All four stations are correct, and the groundwork of the chart is laid ; but if there is any little triangle visible with the reading-glass, all must be plotted over again, for unless these first four stations are exactly right, nothing will ever go right afterwards.

These four stations settled, proceed in like manner with other main stations ; but now we shall of course have three intersecting lines for each station, and care must be taken that these lines do truly intersect, and no station must be pricked in, that has not got three such converging lines through it.

The main stations down, smaller chords may be used for secondary theodolite stations, and the protractors will come in in plotting the marks and other minor points, the necessary angles for which we may suppose some of the party are getting, whilst the first main points are being carefully plotted.

As the chart fills, there will be many lines from which the angle to a new point can be measured, and it is well to remember that as a standing rule the smallest angles both give less trouble, and prevent least chance of error.

Marking
"Points." circle of carmine round them. Larger circles can conveniently be used to distinguish the main stations.
Stretching of the ) Paper.

Calculat-
ing third Angle.

The ordinary way of marking the points is to ring a small It will be found in the course of plotting that the paper will vary so much, expanding at one time and contracting at another, that the arcs of radius once measured and scratched on the paper, cannot be considered as so done once for all. If some hours have elapsed since marking any radius, it must be remeasured, to ascertain if it has altered.
In getting angles for plotting stations of all kinds, it must be remembered that two angles of a triangle will always give the third, and that as far as mere plotting goes, it is not
necessary to waste time in observing the third angle. If the two observed angles have been got fairly accurately, the double error which will be thrown into the third angle deduced from them should not be enough to show in plotting, and if it does, it will soon make itself apparent by not intersecting. An angle from a fourth station will show which of the other three angles is wrong.

Thus if we have observed at a station C, which we want to plot, the angle between $A$ and $B$, and also the angle at $A$ between B and C , the angle at B which is wanted to draw a line to C can be calculated without the trouble of visiting B . It is indeed a blessed circumstance for the marine surveyor that the three angles of any triangle equal $180^{\circ}$.

Some surveyors have preferred to plot main stations by distances. In this case the triangulation must necessarily be Plotting calculated beforehand. We do not consider that much is gained by this method. Three distances must be measured to obtain an intersection, as three angles must be laid off for the same result. A distance is sometimes useful as a check.

## IRREGULAR METHODS OF PLOTTING.

We have up to the present been considering the plotting of stations for a regularly triangulated survey. Let us now look at some other methods.

In plotting the points of a chart which is being constructed a Position on the principle of do-with-what-you-can-get, which is very $\begin{aligned} & \text { by its own } \\ & \text { Angles. }\end{aligned}$ often what has to be done in marine surveys, it is frequently found necessary to plot a position by its own angles, as, for instance, where the ship, anchored or moored off a low coast, has to be a main station, and only angles from aloft can be obtained to objects inland, such as hills, conspicuous trees, \&c., already fixed.

A station pointer, generally, has some small errors of Use of centring, \&c., that prevent it being used where exactness is Tracingrequired, and, moreover, only two angles can be laid off at a time by this instrument. In this case then it is better to
plot all the angles obtainable on to tracing-paper, using chords for the purpose, and being very careful to make a very minute hole at the centre from which they radiate. If the objects are fairly well placed, a very exact position will be obtained, by laying this tracing on the sheet, and pricking through for the position. This will be much assisted if but one line can be got from a fixed station, as the angles can then be plotted on this line, supposing that in this case back angles cannot be calculated.

Only two Angles available.

Again, it may sometimes be found necessary to carry on the main stations with a point plotted by only two angles; but if this happens, efforts must be made to check this, by getting an angle back from stations plotted on by means of this doubtful position, to some old well-fixed station, as a distant mountain ; or if this is not to be had, a regular beginning must be made again by plotting two stations with two angles, pricking one, and then laying the angle from that to the fourth, as practised at the commencement of the chart, which will give a certain amount of check.
Mountains A well-defined mountain, though miles inland and never invaluable. visited by the surveyors, will often prove the very keystone of a chart that cannot be regularly and theoretically triangulated. When once well fixed, it will remain to get angles to, long after all the other first points of the survey have sunk below the horizon as the work progresses.
Use ofTrue The bearings of this will often be useful, and these can be Bearings. laid off from the mountain by applying the convergency.

Let us take an example, which will perhaps explain what is required easier by means of a diagram.

We hope that we have made it plain, by what has gone before, that if a distant object bears, say, N. $47^{\circ} 20^{\prime}$ W., we do not bear from such object S. $47^{\circ} 20^{\prime}$ E., but so much less or more by the convergency; and that in all cases of fixing ourselves by means of true bearings observed from our own position, the amount of convergency, due to the bearing and distance of the object, must be calculated and applied to our bearing, before we can use it as a bearing from the object.

Here, Fig. 17, let B A be the original meridian drawn at the commencement of plotting through any station A . M is the distant mountain. At X our main points are falling short from some reason or another, and we are obliged to have recourse to a true bearing of M , which we accordingly obtain. Required to draw this true bearing from the fixed point M. If we have the sheet graduated, it will not much simplify matters, as it is a great chance if a meridian passes close enough to M to use it without further correction ; but let us suppose that we have no other meridian on the chart but A B. We must lay off the true bearing from M., with A as

FIG 17.

the zero, so we require the angle A M X. If M has been observed from A, whence we had a true bearing by which the meridian A B is directed, we have the bearing or angle B A M. If not, we must measure it from the sheet by reversing the chord method; drawing a line from $A$ to $M$, and measuring the chord to the line A B at a given radius with beam compasses, and calculating the angle which corresponds to it, or B A M.

Now consider the figure again, $\mathrm{M} \mathrm{C}, \mathrm{X} \mathrm{D}$, being imaginary meridians to assist conception.

The bearing of $A$ from $M=$ bearing of $M$ from $A+$ the convergency, as M is nearer the pole than A , or

C $M A=B A M+$ convergency for difference of departure of A and M .

In like manner :
$\mathrm{CMX}=\mathrm{MXD}$ (the observed bearing from X$)+$ convergency for difference of departure of M X.

Adding, we have C M A + C M X = B A M +MXD + convergency for A X.

Or $\mathrm{A} M \mathrm{X}=$ bearing M from $\mathrm{A}+$ bearing M from $\mathrm{X}+$ convergency for A X.

To get convergency in this case, we must assume a position for X , which we can roughly plot for the purpose, and measure the distance A X and bearing B A X.

We can then from this calculate the convergency required, knowing roughly the latitude of $A$, for

Convergency $=$ distance $\times$ Sin merc. bearing $\times$ Tan Mid. lat.

Drawing true Meridian.

If M is likely to be used much in this way, it will be worth while to lay a meridian off through M by plotting the bearing A M C or BAM + the convergency for A M; from which meridian subsequent bearings can then be laid off, duly corrected for convergency, for the distance between M and the station from which the bearing is observed.
Neglect of Of course it will depend on the latitude how much error Convergency. will be introduced by neglecting the convergency; but when it is considered that in latitude $45^{\circ}$ the convergency is equal to the departure, it will be seen that a large error will result by not applying it; for in this latitude, supposing $A$ and $X$ are 30 miles apart, an error of half a degree would be made by drawing a meridian parallel to A B, and laying off the bearing observed at X from M .

As a rule, therefore, it can never be safely neglected except very near the Equator.

If it is intended to lay off the true bearing of an object from a station plotted on the chart, the convergency must likewise be borne in mind, and the meridian to be ruled through X (in this case considered as fixed) from which to measure the bearing, must be, in transferring it from $A B$,
corrected for the convergency due to the distance $A X$, by, after ruling a line through X parallel to A B, laying off at X , from the parallel just ruled, towards the pole, and on the side of $A$, an angle equal to the convergency required, which will give the direction of the true meridian.

The system of true bearings may be used in many ways Further whilst carrying on an irregular triangulation. It is impos- $\frac{\text { use of } \operatorname{True} \text {. }}{\text { Bearings. }}$ sible to give instances of all the difficulties which may be surmounted by this means, but an example, taken from actual practice, will show the style of use to which true bearings may be put.

Let us suppose ourselves employed in the survey of a piece of coast which offers no facilities for obtaining a base by

measurement; but it is the season for observations, and we have points so placed that we can work directly from the astronomical base, instead of obtaining a base by sound or other doubtful methods, which we should otherwise have to do.

In Fig. 18, A and B are two positions invisible from one another near the confines of our chart; C is a distant inaccessible mountain visible from both A and $\mathrm{B} ; \mathrm{D}$ is an elevation visible from $B$, but not from $A$, and from which C can also be seen.

To utilise this arrangement we take observations for latitude at $A$ and $B$, and run the meridian distance ; we also get the true bearing of $C$ from $A, B$, and $D$.

Calculate the bearing and distance A B astronomically, and place this line on the paper. The lines B C, A C can be now drawn by the difference of the bearings observed and calculated from $A$ and $B$, which will give us $C$ with two cuts. B D is drawn from B, and the back bearing of $D$ from C (calculated from the observed bearing of C from D, with convergency applied) drawn, by which we shall get $D$, also with two cuts only.

If we can find a point $E$ which can be seen both from $B$ and D , and from which C can be seen, we can lay it down with three cuts, as the angle from C can be calculated in either triangle C E B or C E D, and the intersection of these three will prove the exactness of our work. B E will then be our base for working, as we are supposing B A to be about 60 miles, which, as we have drawn it, will make B E about 15 miles, which is a workable base.

In the case which we have put it is very unlikely that, after all these different bearings, the intersection of the three lines at the point $\mathbf{E}$ will be a perfect one. If it is not good, the best way to obtain the base B E may be to calculate it in as many triangles as we can command, and, taking the mean of these results, to commence the actual plotting from this mean base. This would depend however upon circumstances. It is impossible to lay down any hard-and-fast rule with respect to this kind of work, and the case is simply given as an instance of the uses to which true bearings may be put.

Graduation of Sheet before Plotting.

In some extensive surveys on a small scale it may be necessary to graduate the sheet first, when positions can be placed on it by their latitudes and longitudes, and the intervening parts plotted or triangulated by means of bases measured at each of these astronomical positions. This will be done when coasts are low and marks scarce. We can scarcely hope that when these different bits meet, they will agree exactly; but with a small scale, say half an inch to the mile, the discrepancy ought not to be sufficient to introduce much error, if we square in five or six miles of
the points worked up from either end, when they meet and disagree.

This undoubtedly partakes of the nature of " cooking;" but when we undertake to map a coast on such a small scale we cannot pretend to much accuracy in detail, and shall only do this when it has been considered advisable to lay down a large extent of coast in the time available, with the intention of presenting its more salient features as correctly as we can.

Work amongst islands (as portions of the Pacific) would be done in this manner.

## FIXING MARKS.

It is not possible to lay down any dogmatic plan for fixing the marks which have to be erected. In many cases it is well to put them all up first, and then get angles to them afterwards ; but if non-surveyors are deputed to make the marks, they will seldom be placed in the right spots. A whitewash, for instance, will be so placed that it cannot be seen in certain directions. A tripod or pole will not be in the most convenient position for the officer who afterwards puts in the coast-line, and numerous small errors of this description will be made by one who is not capable of taking in all the little requirements.

It is therefore more satisfactory to send a surveyor to do systemthis, and while he is there he may just as well take angles, so that the writer has found it saves time in the end, in general, to have a surveyor at some main or secondary station, whence he can see most of the marks, and let the officer who erects the mark take angles at it to the above station, which we may call the "shooting up" station, and to a sufficient number of other stations which can be seen from the "shooting up" station also, to fix himself. The angles from these other stations can then be calculated. In this way two or three officers can be at work putting up marks, and fixing them at
the same time. The officer who erects a mark gives it a name, and notes the time by his watch when he is there. The officer at the shooting up station also takes the time, and notes the position and kind of mark put up, to which he takes his angles, writing the name against it in his book when he returns to the ship and meets the other officers.

Officer marking responsible for sufficiency of Angles.

The officer marking must think for himself whether he has enough angles to fix the point; and in case any mark cannot be seen from the shooting up station, he must get an angle from some other of his marks, which will be then-used to calculate the other angles in the same manner.

Use of Heliostat.

A heliostat is invaluable here. In hazy weather, and when the shooting up station is distant especially, a flash will be seen when neither mark, nor boat, nor anything to direct where to look for the mark, will be visible. The officer shooting up should also return the flash, to show he sees the station, as well as give a well-defined object to get the angles to.

Of course circumstances may not render this system advisable, but it is here suggested as having worked very well in many places, a long extent of coast being " marked," and all marks fixed in a short time.

Triangulating, and fixing Marks by Ship.

Frequently the minor marks must be fixed by angles from the ship, or a boat at anchor, as on a straight coast where nothing behind can be seen from the marks. When this is necessary it will often be also necessary to carry on the main triangulation as well by means of ships and boats, so that a description of one serves for the other.

The ship, anchored short, or moored if necessary, should be shot up from one or more shore stations. If the angles taken from the ship are indispensable to fix her own position, try calculating the back angles from other objects first, and lay them off as cuts to the position, as if they agree it will be the most satisfactory manner ; but often back angles, calculated from sextant angles, will not be correct enough to give a good intersection, especially if the points are distant. In this case,
let all the angles taken at the ship or boat be plotted on tracing-paper as before described, and the position pricked through on the guiding line from the shore station. A signal should be made when the angle to the ship is to be observed, and the angles from the ship taken at the same time.

The ship angles should be observed from the fore part of the ship, and frequently the foretop will be found the best place. Whatever spot is used it must, of course, be arranged beforehand, so that the observer's exact position on board may be taken from the shore station.

From the ship the main angles, that is the angles to the Taking positions already plotted, which are to be taken for the Angles from Ship. purpose of fixing the ship, must be observed first, using some well-defined station as zero, and measuring all the main angles from this with the sextant. Some other station must be chosen as the zero with which to measure the angles to the marks, and the angle to this second zero observed from the main station zero.

This second zero is wanted to be in such a position with regard to the marks, that any slight movement in the ship will make the least possible difference in the angles to be observed between it and the marks. It must be, therefore, at about the average distance of the marks. It will not do to choose some object miles away behind the marks, as the least swing of the ship will at once alter the whole of the angles. Generally speaking, the central mark to be fixed will answer the purpose best, but in many cases it will be found necessary to change this zero for some marks, measuring from some other object at an equal distance from the ship.

When the minor angles have been taken, repeat the main Repeating angles to see if the ship has moved, giving another signal to Angles. the shore station for another angle from it. All mark angles should then be observed again to check errors.

It need scarcely be said that the more rapidly these angles are taken, the less the chance of any error arising from variation of ship's position, by change of direction of current,
wind, \&c. An experienced hand should therefore be chosen for this work.

Telescope of Sextant.

A sextant with a telescope of high magnifying power is most useful. On this head see page 10 .

## CALCULATING A POSITION FROM TWO ANGLES TO THREE KNOWN OBJECTS.

It may be sometimes required, in the course of a survey not regularly triangulated, to calculate the distance of the observer from an object, from the two angles he has observed between three known "points," one of them being the object whose distance is required. Or he may require the angle, at the object observed, to him, from the same data.

This is, perhaps, best accomplished by using the one-circle method, so called in contradistinction to the method of protraction by three circles already explained under "Station Pointer."

The three figures 19-21 give the three possible positions of the objects, viz. : When the observer is inside the triangle formed by the objects; when he is outside, and the centre object is nearer than one of the others; and when, under similar circumstances, it is the farthest.

If the angles between the three objects are known, which is most probable, the calculation of the second formula will be unnecessary.

Let ABC be the objects observed. X the position of observer to be determined. $\mathrm{AB}=c, \mathrm{BC}=a, \mathrm{AC}=b$, are the sides known, $\mathrm{A} \times \mathrm{B}=m$ and $\mathrm{BXC}=n$, the angles observed. Required XA and the angle BAX.

At A, in A C, draw, on the side remote from $\mathrm{X}, \mathrm{AD}$, making $\mathrm{CAD}=n$. At C , in AC , draw in like manner CD , making $\mathrm{ACD}=m$.

When X is inside the triangle (Fig. 19) CAD, and ACD must be drawn to equal $180^{\circ}-n$ and $180^{\circ}-m$ respectively.

Describe a circle to pass through the points A, D, C.
Join D B, and produce it until it cuts the circumference of the circle in X .

Then X is the position required.


For ACD, AXD, being angles in the same segment, are equal, and $A C D$ is drawn $=m$.

$$
\begin{array}{r}
\therefore \mathrm{AXD}=m \\
\text { or } \mathrm{AXB}=m \\
\text { Similarly } \mathrm{BXC}=n .
\end{array}
$$

Then $\mathrm{A} \mathrm{D}=6 \operatorname{Sin} m . \operatorname{Cosec}(m+n)$. . . . . . . (1)
$\operatorname{Cos} \frac{\mathrm{BA} \mathrm{C}}{2}=\sqrt{\frac{\mathrm{S(S-a)}}{b c}}$
$\mathrm{BAD}=\mathrm{BAC} \pm \mathrm{CAD}$
$\operatorname{Tan} \frac{1}{2}(\mathrm{ABD}-\mathrm{ADB})=\frac{c-\mathrm{A}}{c+\mathrm{AD}} \operatorname{Tan} \frac{1}{2}(\mathrm{ABD}+\mathrm{ADB})$
$\mathrm{XA}=c . \quad \operatorname{Sin} \mathrm{ABD} . \quad \operatorname{Cosec} m$.
$\operatorname{Sin} \mathrm{ABX}=\frac{\mathrm{XA} \cdot \operatorname{Sin} m}{\mathrm{AB}}$
$\mathrm{BAX}=180-(m+\mathrm{ABX})$.
X can now be plotted by the angles from $\mathrm{A}, \mathrm{B}, \mathrm{C}$, if required.

## DRAWING RECTANGULAR LINES.

The methods of drawing a line perpendicular to another line are well known, but are here repeated.


Erecting a Perpendicular to a line from any point not near its extremity.

Measure from the point $A$ with the beam compass any equal distances right and left of $A$, as $A B, A C$.


From B and C draw, with a radius about half as much"again as $A B$, short arcs intersecting one another. A line drawn
through this intersection $D$, from $A$, will be at right angles to AB .

Take any point B, Fig. 23, in a direction about $45^{\circ}$ from Erecting a A, and from it as centre, with the radius B A describe a $\begin{gathered}\text { Perpendi- } \\ \text { cular from }\end{gathered}$ short arc intersecting AD in C, and likewise a short arc E F the end of in the opposite direction. Join CB and produce it to intersect EF in G. A line joining A and $G$ will be at right angles to AB .

In all careful work, these operations should be checked by repetition, with different radii.

## CHAPTER VI.

RUNNING SURVEY.
A running survey, the least accurate form of "sketch" survey, is one where the best part of the work is done from the ship running along the coast, fixing points, sketching in the coast-line and prominent parts of the land, and sounding, at the same time.

It is capable of many modifications, more especially with regard to the fixing of the main points.

Roughest form of Survey.

The rudest form of running survey is where, beginning upon nothing, everything is eventually put on paper by observations, angles, and soundings taken from the ship without anchoring.

At the other extreme comes a running survey made upon some main points already fixed by triangulation of some kind, and which has for its object only the sketching of coastline and detail of an inaccessible coast, which is assisted by occasional anchoring, and where sounding would be carried on in the boats as well as the ship, after enough natural objects have been fixed by the angles from ship stations.
Graduating beforehand.

In making an extensive running survey of the simplest kind, i.e. where we commence on nothing, and only run past the coast once, it is well to have the paper graduated (see p. 270), as astronomical observations from time to time will fix the scale of the chart, and it is easier to plot these positions when the sheet is graduated.

The course and the distance run by the ship between each


#### Abstract

position where series of angles are taken, as given by patent Method of logs, will form a series of bases, which will have to be, how- ordinary ever, modified afterwards to agree with the positions astro- Survey. nomically fixed, which must be taken as the fundamental points of the chart.


A running survey must be roughly plotted, and everything sketched in, as we go on, putting down position after position by course and distance, and cutting in the objects we choose for marks, giving them names by which to recognise them, and to record in the sounding book. Assistants should be told off for separate duties. One to look after the sounding; another to sketch in the coast-line and hills between each object chosen, on another sheet or sheets of paper ; the chief and some assistants getting the angles; one writing down; another plotting the stations and drawing the lines to the points, so as to see what angles are wanted at the next station to objects already chosen, and how far on the next station should be. Bearings should be taken of all prominent points in transit.

At each position, as laid down by course and distance, commence plotting by laying down the bearing of the object we have selected for zero for the round of angles. From this, the other angles can then be laid down.

It follows that a bearing must be obtained, as a necessity, from each position. This should be taken to the zero selected.

Distant hills are a great help in a running survey, as, when Hills of replotting from the astronomical positions, if these hills can value be fixed by bearings (true or compass) from them, the angles taken to the hills, at a position now and then, may possibly be used as fixes, which may be plotted by station pointer, and so get intermediate positions independent of the patent $\log$ positions, which are so liable to error by the action of currents.

A running survey will nearly always have to be replotted, as the astronomical positions and those by patent $\log$ will never agree.

FIG. 24.


Having plotted the positions where astronomical observa- Replottions have been taken, if the intermediate stations are to be put in by hearing and distance, they must be squared in so in. as to agree in total distance and bearing with the astronomical positions. Thus, in Fig. 24, let A be the position from which we start; B, C, \&c., to H, are positions of the ship as plotted by course and distance on the rough chart; $a, h$, are the same positions as $\mathrm{A}, \mathrm{H}$, but as given by the astronomical observations.

To bring the intermediate positions to agree with $a, h$, as plotted on the graduated sheet, we join A H and $a h$. Drop perpendiculars from B, C, \&c., to the normal line A H. With the proportional compasses set to correspond to the different lengths $a h$, A H, measure the corresponding distances along $a h$ for the points where the perpendiculars will cut, and lay off perpendiculars along which the corresponding distances can be measured, and so we obtain $b, c, d, \& c$.

If any mountains have been observed both from A and H , their positions should next be put down by these two bearings. The angles taken from the first positions are now laid off, and as objects are fixed, they can be used as checks to the next positions. If we can rely upon the bearings taken to the mountains we shall use them to fix the intermediate positions in preference to course and distance, so that $b, c$, \&c., may be again shifted, especially if the ship has not been accurately steered on her courses, or we have reason to think currents have varied at different parts of our run.

Nothing will agree exactly in a running survey of this No exactkind, but a very fair approximation to the relative positions $\begin{gathered}\text { nexpected. }\end{gathered}$ of conspicuous objects may be got.

The amount of detail possible will not be very great, but Amount of will vary with the quickness and accuracy of eye and hand Detail. of the officer sketching it in. There is nothing that requires the knack which distinguishes a good surveyor so much as this sketching in fairly accurately of a coast-line in a running survey, and good judgment as to depth of bays, and other points that must be mainly put in by eye, is most valuable.

It is well to have one officer aloft, who will be able to get a better view of river mouths, \&c., and make little sketches of bits not seen from deck. He can also take angles to objects that have sunk, or not yet risen above the horizon of the deck.

Compass-bearings are of great use, as direction of valleys, \&c., may be noted without making a position.

The whole course of a running survey will have to be one of compromise between discordant results, and only long practice will enable the surveyor to decide what to throw out, and what to accept.

Modified Running Survey.

It may often occur in a survey, that a portion of the coast is inaccessible for landing by reason of heavy surf; or the shore is so cliffy or densely thick with jungle, that stations cannot be made without loss of more time than they are worth. A running survey of this piece may be as much as is requisite, but the probability is that we shall be able to fix on some main points from the triangulation of the other and more important part of the survey, and these will greatly help us to make the best chart of the portion we can under the circumstances.

In such a case, the best course to pursue is to pass along the coast at some distance, stopping at convenient positions, where the ship can get station-pointer fixes by the main points, anchoring, if possible, for this purpose, and cutting in from these positions other secondary points nearer together, and nearer the coast than the first. Then pass along again closer to the land, and fix points on the shore itself, using the secondary points to fix the ship with. Boats may then be sent to sound, if required, or to sketch in more details of little bays, \&c., if they can get near enough. Compromise will be required here too, probably, in plotting the points, as, unless the ship is absolutely motionless, it is unlikely the angles will intersect exactly, but it is astonishing what good results can be obtained with a number of officers taking angles at the same time, with the ship's way stopped, each being told off to take two or three angles as quickly as
possible; the most important angles, and those that change most rapidly, being taken first.

Advantageous use may be made of beacons in a running survey.

As an example we will give some details of a method employed with success on the south-east coast of Africa * on an open coast, beacons dropped in from 20 to 60 fathoms.

At a distance of from 2 to 4 miles from the shore drop a beacon abreast of some conspicuous object, called the First Breastmark (1 B in Fig. 25), which should be if possible some 3 or 4 miles back from the coast. Note the time, and shape a course parallel to the coast. Put over logs, and steam about 10 miles down it, sounding and fixing with objects selected, until the beacon gets indistinct from aloft, and you are abreast the second Breastmark. Stop, haul in logs, note time, and drop Beacon II. During the run down the coast, three primary objects and other secondary ones have been selected and named, and on arriving at II. a Provisional Breastmark, 10 miles or so ahead must be selected, and also the middle Primary Point of the next fleet.

At II. simultaneous angles are taken between I., the First Breastmark, A, B, C, Primary Marks, Second Breastmark, Provisional Breastmark, and Middle Primary of fleet 3. The Secondary Objects are next taken, using any of the abovementioned points which are most conveniently situated as regards distance, so that any small change of position of the ship shall make the smallest possible alteration in the angle. Each officer is told off to a primary object and some secondary ones, and is responsible that his secondary objects are taken to a suitable zero.

On taking angles, the rough bearing of Beacon II. (which will be as close to as is safe), and also its distance by elevation of its staff, is noted. Take also compass bearing of Breastmark I. as a check.

Now steam straight out for, say, a mile or so. Turn ship's

[^12]
head ready for the run back, and stop. Take simultaneous angles as before, at Corner II., Beacon II. being observed instead of Beacon I. which there is no occasion to look for.

Note time and bearing of II., put over logs, and shape a course parallel to 1st Run.

Run back about one-third of the distance. Stop, and make Intermediate Fix I.

When I. is on the same bearing as was II. when logs were put over, note time, read $\log s$, and stop for the Corner Fix I.

Then run into I., take angles, \&c., as at II., and pick up Beacon I.

The primaries and breastmark can be plotted roughly from what we have whilst beacon is being picked up, and will furnish enough to sound upon, and ensure filling up properly and not crossing the old line, for the distance between Beacons I. and II. is obtained from the runs up and down.

Whilst sounding up to II., the coast and topography are shot in and a rough sketch of the coast and hills are put into the deck-book, which at the end of the fleet is sent down to the officer in the chartroom for plotting.

On passing close to Beacon II., put over logs and shape a straight course roughly parallel again to the shore, until abreast of Breastmark III., when stop, put over Beacon IIL., and take angles as described at II.

Turn outwards again and get Corner Fix III., and run back parallel to old course to Intermediate Fix II., which, and all subsequent intermediate fixes, should be at twothirds of the distance back. At this position it is important to be able to get a good fix on the points of Fleet 2, in other words, that the angle between the Breastmark 2 and Beacon II. shall be over $30^{\circ}$. It matters not whether Beacon II. is outside or inside of Breastmark 1.

On dropping a beacon, the essential angles are the primary and breastmark of the fleet, and the other beacon, the provisional breastmark ahead and the middle primary ahead.

At the corner fixes, obtain the same angles as in dropping
the beacon, only using the beacon just dropped instead of the other beacon. Take elevations for heights.

At the intermediate fixes, get the primary and breastmark of the fleet, the beacon towards which you are running, the breastmark of that beacon, and the next primary behind.

On picking up a beacon, get the primary and breastmark of the fleet ahead, the other beacon, the breastmark abreast of you, and if visible, the breastmarks behind, as well as primaries.

It will not always of course be possible to distinguish the breastmarks of fleets so far ahead and astern, but whenever possible they should be taken, as points so far distant give excellent zeros for plotting, and the bearing is preserved. Any conspicuous object, whether used as breastmark or not, will answer this purpose.

A continuous series of true bearings is necessary, and it is more convenient if they are taken at the beacons.

Simultaneous angles must be observed at the same time to correct the bearing to points as far as possible up and down the coast.

With a Thomson's compass in a favourable position, i.e., a position where it is not liable to change, a compass bearing will be admissible now and then.

The first true bearing should be taken from the last beacon from which the first breastmark is visible. No other bearing is necessary till the points fixed from this position are passed, when another is wanted to carry on and preserve the bearing.

When the coast trends nearly north and south, latitudes by twilight stars north and south are advisable every 30 or 40 miles to check the scale; and similarly when coast is nearer east and west, longitude observations east and west.

Obtain shore observations when possible.
To plot, begin when the first true bearing is attained. Draw the meridian through the beacon from which it is observed, and lay off the true bearing of the first breastmark. Let us suppose we begin at Beacon III., and we are going to plot just the Fleet III.-II., and then II.-I.

Lay off the angles to II. and breastmark 2 B, drawing long lines. Calculate the patent log base III.-II. from the two runs, and prick off Beacon II.

Lay off all angles from III., first making sure that the whole angle between first breastmark and the breastmark ahead is the same both at dropping and picking up the beacon.

Then lay off the angles from II., and prick in the two breastmarks and primary points.

Lay off on tracing-paper the angles from intermediate Fix II., and all points should intersect, and be pricked in. Angles to points in Fleet II.-I. can be laid off from this position.

Plot intermediate Fix I. by the points already fixed, and lay off angles.

Lay off angles from II., using III. as zero.
Prick in Beacon I. by distance, and test it by a station pointer fix, using II. and Breastmark 2, the centre being or the line drawn from II. to I. If this agrees, there should be: an intersection at Breastmark I. of five lines, viz., from III., II., the intermediate fixes, and from I.

Plot the corner fixes and lay off all angles to secondary points.

All points being down, the soundings can be fixed and coast-line and topographical features sketched in, getting additional angles when necessary.

It is desirable to use two deck books, entering all angles for each alternate fleet in one or the other. The plotting can then go on in the chart-room, while the angles for the next fleet are being obtained, and recorded.

It is necessary to remember that on obtaining the true bearing of an object a long way ahead, such object is eventually plotted on that line.

If care be taken over the details described, the objects should plot very closely, when the tide is not too strong, and precautions are taken not to run the bases when the streams are changing.

At the extremities of such a survey shore observations
should be obtained if possible. From these positions and from the shore true bearings obtained, corrections to scale and bearing can be made as already explained on p. 97.

In calculating the length of the different patent $\log$ bases the following formula should be used:-

$$
\mathrm{X}=\frac{t(b-a)}{t+t^{\prime}}
$$

Where X is the current in time $t$ with the stream, $\alpha$ is distance shown by the $\log$ with the stream, $b$ is distance shown by the log against the stream, $t^{\prime}$ is time occupied in run against the stream.
From X the true distance can be deduced.

## CHAPTER VII.

## COAST-LINING.

When and how the putting in of the coast-line is done, must depend much upon circumstances.

If making a chart with pretensions to accuracy in the $\operatorname{In}$ a details, it is better to do it before the soundings are taken, detailed as, for the inshore soundings, the little points and bays, not distinguished by marks, will be very valuable. In this case, too, every yard of the coast that can be walked over should be. If the surveyor pull along the coast in his boat, from one spot to another, he will be liable to miss little details, such as stream entrances, which may be blocked by the sand beach in summer ; lagoons behind the shore, etc. The boat should therefore only be used to pass rocky points and cliffs that cannot be walked along, or to make stations in, at anchor off the coast, if it is necessary to do so, to shoot up the details.

The method of putting the coast-line on to the sheet also Plotting varies. The angles can be taken, and the details between the coastsubsidiary fixes on the beach sketched into the angle book, using always a larger scale than that of the chart, and then these fixes and angles plotted on to the chart after return on board; or the surveyor can take a field board, with the points on it, with him, and plot the coast as he goes along it on to his board.

Of these two the latter method is by far the best, and Plotting should always be employed as a rule. There is no chance of ground. having necessary angles omitted if the fixes are plotted at
the time, and any little error is easier detected on the spot than when plotting afterwards on board. Of course rainy weather or other circumstances will sometimes prevent the work being plotted at the time, but unless some good reason exists, it should be done.

Instruments required.

If conveniently situated marks are plentiful, the coastliner will only want his theodolite or sextant, or both, to take his angles, and a station pointer and tracing-paper for plotting, with protractor, etc. But if the coast has no objects off it to seaward, and landward marks are also short, or invisible from the shore, he will require, very probably, a pole of measured length, whereby to ascertain, by observing the angle subtended by its extremities, the distance of points, etc., from one another.

A convenient form of this pole is described under "Tenfoot Pole," page 39.

Each assistant should have a copy of the Ten-foot Pole Table,* on a piece of cardboard, always in his angle book, ready for reference in the field.

General method of Coastlining.

Let us suppose an officer landed with his board of points to do coast-line.

He will start at some point already plotted on the chart, and will take angles from it to all the objects he can distinguish between him and the next fixed point, and beyond, if necessary.

He will then walk on to another spot, where he will make a supplementary station, fixing himself by angles to known points, either by theodolite or sextant, according to circumstances.

He will then plot this, his No. 2 ©, on his board, by station pointer or tracing-paper, taking care to check his position by his line from the 1 st $\triangle$, or by a third or " check" angle from his present position. His No. 2 plotted, he will sketch in on the board, the coast-line between that and the first, having noted any peculiarities as he walked along.

[^13]The scale of the chart will largely influence the distance between the subsidiary stations to be made by the coastliner, as will also the character of the shore line, and the intended nature of the chart as to exactitude of detail.

If the work is to be plotted on return on board, the system is precisely the same, only the detail of coast between the stations must be sketched in the angle-book, instead of directly on to the board.

When the coast-liner sees that at the next station he will not be able to fix himself by angles, he must use his ten-foot pole, sending a man on with it with instructions where to stand, or going on himself to some point solicited, to which he will first take the angle; leaving the pole behind with a man at his present station, with directions, when signalled, to hold the pole horizontal, and at right angles to the observer.

To ensure the latter, either a rough pointer of some kind can be attached to the centre of the pole, so as to project at right angles, in which case the holder will be directed to point this to the observer, or, he will be told to sway it gently backwards and forwards, and the observer will read the largest angle he can measure.

The angle observed, and the corresponding distance looked out of the table, the latter is measured on the scale of the chart, and applied by a pair of compasses, as a distance from the last station along the line laid off from that last station in the direction of the required station.

If necessary, the whole coast can be carried on in this way; but if the marks are a long way apart, great care must be taken in observing the angles on to the positions to be measured, as there is no check on the work, and each error will be accumulative. In this case the man must be sent on, and must mark the exact place he stood when the angle was observed to him, and the coast-liner must make his next station precisely on that spot.

The azimuth compass may sometimes be employed in this work with advantage. Any little error, when a properly fixed station is reached, can be squared in.

It will be understood that this ten-foot pole method is only used for the smaller detail, where sufficient angles to fix cannot be obtained. It is especially useful in delineating the shores of islands, or of small bays which have no fixed point in them.

For instance, in Fig. 26, let us suppose the two points, marked Ash and Lime, are fixed, but in between them is the small bay shown.

At Ash we obtain the angle between Lime and $A$, the next point visible, and also the distance by our ten-foot pole. If we can make out that $B$ is a point, and can see any prominent spot on it, we shall get an angle to that also.

We then go to A, sketching in between on the way. At A
FIG. 26.

we become aware of the little bay, and we send the pole over to C, pointing out to the man with it where to stand, and telling him to put a stick or stone there, when he is signalled to go on to B .

At A we get all we can, angles from Lime as zero, to Ash, $\mathrm{B}, \mathrm{C}$, tangent of bay on towards D , and anything prominent, and the distance to C by the pole.

Leaving a little mark at our station at $A$, we go to Lime, and take angles from Ash to $A, B$, and distance to $B$ by the pole now there.

We then go back to $B$, and send the pole over to $D$, and again get all angles we can, and distance to D .

We now sit down and plot our data. We have two angles to A from Ash and Lime, and a distance to A from

Ash. These ought to agree, and we prick in A. We have the line to B from Lime, and perhaps from Ash as well, but we will suppose not, and will plot $B$ by the distance from Lime. Then placing our protractor on B, lay off the angle observed there to Ash, which ought to go through, and make a check for $B$.

We plot C and D by their distances on their respective lines from $A$ and $B$. We then walk round the bay, sketching it in, and can get an angle at $C$, from $A$ to $D$, as another check, and any other angles to assist in sketching in details.

The coast-liner will generally be responsible for all the coastdetails of topography close to the coast such as follow, the liner toposcale of the chart being taken into consideration as to with what degree of accuracy detail can be laid down.

Heights of cliffs must either be measured with a lead-line, or by getting an elevation to some definite point, which must afterwards be fixed, from one of the stations, or may merely be estimated and entered in the angle book.

The height of a cliff can be readily calculated on the spot from an angle, by the formula :-

Height in feet $=\frac{\text { Angle in seconds } \times \text { distance in miles } .}{34}$
Cliffs have generally to be exaggerated on the chart, to show distinctly. The height in feet should be written against them.

The directions of lower parts of streams, or rivers, must either be walked up, and fixed, a certain distance back, or can merely have their entrances fixed, and an angle taken up for their general direction.

Lower spurs of abrupt hills must be sketched in, assisted by angles to them from different points.

Houses standing back from the shore must be put in. These can usually be fixed by angles to them without visiting them, unless it is necessary to get their dimensions, names, etc., or perhaps to ascertain if a good well or spring of water may be near, that would do for watering on an emergency.

Swamps near the coast should be sketched in as far as necessary, and a look out kept for evidences of any extension of their area in winter. Information on these points can be picked up from passing inhabitants.

Angles should be got also to any conspicuous objects farther inland, as they will be very useful when the topography is sketched, and the surveyor should always look ahead, and seize any opportunity of the kind for helping on other parts of the work than those he may be immediately engaged in.

Roads near the coast should be walked back to, and fixed here and there, sketching in between.

Rocks above water, or breaking, should be fixed. Though these come into the province of the sounding, it is often useful to have them down first; and in the case of a break only, it may be very much so indeed, as it may be an isolated head, which a boat sounding near high water may miss.

Low-
water Shore-line.

Though it is the high-water line that the coast-liner is more immediately concerned with, he should mark at low water the position of the dry line, especially where this runs off a long way at points, etc.

In a detailed survey on a large scale, it may be necessary to send some one round the water-line at low tide to get it accurately, but this is more usually obtained by the soundings, for by reducing these to the low-water level of springs, a series of points will be obtained, where each line of soundings crosses the low-water line, which can then be drawn in as a line passing through these points.

## Elevations of hills.

General Information for Directions.

Angles of elevation for heights of the hills should be taken when getting the angles for fixing the points of the chart, from main and secondary stations, or any well-fixed points; but if the coast-liner gets some more elevations from marks on the water-line, they will never come amiss, as long as the position is well fixed.

The officer coast-lining will make note of anything worth recording in the sailing directions, as little nooks for landing, convenient places for watering, etc., letting his captain know
on return on board, in order that they may be, if necessary, again looked at, or entered in the latter's notes.

It may be convenient to keep a book for the purpose, in which any useful information can be entered.

As an instance of the application of the ten-foot pole Further method, we may mention the following, which is adapted for tion of use on shores with fringing coral reefs, or broad sand or mud Ten-foot flats, which dry sufficiently at low water to enable people to Pole Method. walk on them, and when either the steepness of the hills or the denseness of the vegetation prevent marks being fixed on the coast.

FIG. 27.


Let annexed diagram, Fig. 27, represent an island of this kind.

A long measured lead-line, say of 500 feet, is provided. This is taken by an officer we will call B , who has a prismatic compass. Another officer, A, is provided with theodolite, or sextant, or micrometer, and prismatic compass, according to circumstances, sextant and compass being quite sufficient.

Starting at $a$, B remains there while A walks to $b$. B stretches his line out at right angles to $a, b$, and plants a flag at the extremity. A observes angle subtended by flag and

- $a$, with his micrometer or sextant, and both A and B observe the bearing of $a b$.

A waves to B , who goes on to $c$, when the operation is repeated.

A then moves on to $d$, B pivoting his line round $c$, so as to be rectangular to $c d$; and so on, until $f$ is reached. We will here suppose that, from $a$ to $f$, we have been able to triangulate, the reef being broader. We have therefore the correct bearing and distance of $a f$.

To plot this, the mean compass bearings and distances $a, b, c$, etc., will be put on a separate sheet of paper on a larger scale than the chart, and the positions $a f$ being joined on both, the other stations will be squared in on to the chart.

Marks will be left at each station, if required for sounding, or delineating the outer edge of the reef. Subsidiary marks can be made at other points, as $x, y, z$, and fixed by angles from $b, d$, etc., with distances measured by the angle of the line.

The shore line can either be sketched by A, as he walks from station to station; or can be put in afterwards, if greater correctness is required, using the ordinary 10 -foot pole to fill in between $a, b, c$, etc.

If a theodolite is used, which it is well to do in a case where we have not been able to get any measured base at all, and must consequently work back to $a$, it must be set up first at $a$, and the angle to $b$ taken from some fixed object, whose true bearing we should obtain, as we in this case must not be dependent on the compass. B will be at $b$ with his line, and when A has finished, will walk on to $c$, so that $A$, when he arrives at $b$, can take the angle from $a$ as zero, to $c$. With a theodolite, then, A must visit every station, unless B has one also.

At every new position, the last $\triangle$ will be used as zero.
The readiest way for B to direct his line so as to be at right angles is to use the so-called "cord-triangle," which is simply a triangle formed of a piece of line whose sides are in the proportion of $3,4,5$, the angles being marked by knots.

When stretched on the ground, with the corner between 3 and 4 at the $\bullet$, and the 4 side coincident with the direction of the other $\triangle$, the direction of the 3 side is at the right angle required. Any similar contrivance will serve the purpose.

Note.-This method was largely used by Lieutenant W. U. Moore in the survey of the Fiji Islands, and
is a good example of the dudges that have to be improvised to meet circumstances.

## CHAPTER VIII.

## SOUNDING.

Boat Sounding-Ship Sounding-Searching for Vigias,
Import- IT is difficult to say that any one step in the construction of
ance of Sounding. a chart is more important than another, as each is necessary for the completion of the whole, and an error anywhere may cause a disaster ; but if any particular item is to be picked out, perhaps the sounding should rank in the highest place.

The operation of sounding is the least pleasant part of a marine surveyor's work, especially when the weather is against him, and the sounding uninteresting, that is, where the depths are regular, and there is no excitement in the way of discovering, and working out, shoals and reefs; but the notion that it is therefore always to be relegated to the juniors of a survey, is not only hard upon them, but may introduce errors into the very part of the chart which, as we have already said, is the most directly important.

As soon as the points are down, i.e., plotted, the sounding can be commenced; but, as before remarked, on an intricate piece of coast it is better if the coast-line is put in first.

## Ordinary

Method of
Sounding.
The ordinary main plan of sounding is thus. The boat proceeds in straight lines in a direction, of a length, and at distances previously decided on, with a man in the bow constantly sounding. Every so many soundings, as the case may be, the officer takes angles with a sextant to fix the position of the boat, always doing this at the beginning and ending of every line,

It is evident that this main plan may be largely varied in its details.

In the first place rises the question as to whether it is better to plot fixes, and enter soundings on the sheet, regularly, in the boat, or leave them until return on board, merely putting down an occasional fix to see where you are. The writer says, certainly, as a rule, plot them at once. It can be done in ordinary circumstances just as correctly, and gives more information to the officer sounding as to little bits which may want additional casts, and it also gives the men at the oars a little rest from time to time. In very rough water it of course cannot be well done, and must be left till return on board to the comparatively motionless ship; but when you can, plot at once. In harbour work on large scales, again, it will be better to plot afterwards, as great accuracy will be required.

The extent to which the soundings themselves can be entered at the time on the chart, depends of course upon the state of our knowledge of the tide. If the tidal range is small, or the motions of the tide are sufficiently known to form a table of reduction beforehand, the reduced sounding can be written on the board at once. If not, the soundings as taken can be written down, and reduced on inking on return on board, or, only the sounding taken at each fix can be written against the prick of the fix, and intermediate soundings left to be entered on board. The latter will generally be found most convenient.

The pace at which the boat may go, and the necessity, or Circumnot, for stopping at the casts, will depend on the depth of stances water and the capacity of the leadsman.

Whether it is necessary to stop to get the angles depends upon the convenience and visibility of the marks, and the quickness of the angle-taker. A beginner will of course do everything deliberately, until he feels capable of combining speed with correctness.

Whether each fix shall be plotted at once, or whether to wait until two or three have been got, and then lay on oars,
or anchor for a few minutes, must also vary with circumstances.

If laying on oars, keep the lead on the bottom with a slack line, and let the coxswain keep the boat in position.

What the distance should be between each fix will depend largely upon the scale of the chart, and the nature of the bottom. On an evenly sloping bottom many soundings can be got without another fix; but where depths vary or increase rapidly, the fixes must be closer together.

The soundings which will be joined together on the finished chart by fathom lines, e.g., the three, five, ten fathoms, etc., should always be fixed, and in doing this it must be remembered that it is the outer sounding of any of the same depth that will be on the fathom line, and also the tide reduction must be taken into consideration. This latter will of course be in many cases only approximately known, so that exactly the right sounding may not be fixed.

Direction of lines.

The sounding lines should be in ordinary cases at right angles to the coast, and parallel to one another, as not only will a better line be got for tracing the fathom lines, but the boat will easier be kept in her right direction by observing two objects which have been seen to be in transit, in the right direction, at the commencement of the line.

Marks in transit for directing lines.

In nice work on large scales it is generally necessary to place two marks in line for this purpose ; but, for ordinary surveying, changing them from one line to the other will take far too much time for the purpose, and marks to answer all practical purposes may usually be found placed by Nature already.

Before starting in towards the coast on a line, it is frequently desirable to take off by the protractor the angle to the point on the shore where the next line out will commence, and placing this on the sextant, try to find some object on the shore which can be utilised as a mark to warn when to turn out. When close in shore it is often impossible to fix, and the lines may therefore be irregular, without some such assistance.

In sounding out a small harbour, circumstances must guide the direction of the lines.

The depth to which the boat soundings are to be carried Depth to will depend upon circumstances. When soundings of over bhich 20 fathoms are taken from a boat, it gives a great deal of Soundings labour, unless a small sounding machine is carried.

When the boat gets to the end of her line, and turns to pull along to the end of the next one to return, soundings should still be carried on, as before.

The method of using the station pointer has been explained under the head of " Station Pointer."

It only remains to note that it must be recollected, in construcgetting the fix, that the right or left angle (according to $\frac{\text { tion of }}{\text { Station }}$ whether a right-handed or left-handed station pointer is in Pointer to the boat) must be observed of a sufficient number of degrees be rem. to be measured on the instrument, if possible. If this cannot be got, recourse must be had to tracing-paper for plotting the position.

The sounding book need not be ruled. There are several ways of writing down the objects used for fixing and the

Entering soundings. in Book.. angles between them, but the best, if space permits, as it does in the sounding book supplied by the Admiralty, is to put them down as you look at them, the right-hand object to the right, the middle one in the middle of the page, and the left one on the left-hand side. The hour, in Roman numerals, and minutes should be entered from time to time, to know the reduction for tide; the sounding at the fix goes on the extreme right, and subsequent soundings up to the next fix, in a row underneath, thus-
$\left.\begin{array}{lccccccc}\text { X } 14 & \text { Pagoda } 28^{\circ} 31^{\prime} & \text { Mat } 62^{\circ} 14^{\prime} & \text { Can } & 7^{\frac{1}{2}} \\ & 7 \frac{1}{2} & 8 \times & \times & 8 \frac{1}{2} & 9 \times & \times & 10\end{array}\right)$

The cross $(\times)$ signifies the same sounding as before; and

> All casts to be entered.

Space for reducing.

Check angles.

Nature of bottom.
it may here be mentioned that all soundings must be put down, even though there may not be room for half of them eventually; as, the man heaving regularly, if all his casts are not registered, the change of fathom will not come in its true place when interpolating between the fixes.

Space must be left under each line for the soundings, as reduced to low water, to be written in in red ink.

A check angle should be taken, from time to time, to make certain that things are right, as is noted above at the last cast, in the example Can to Pea. This is especially necessary at the commencement of work with new points, as mistakes will occur in plotting points occasionally. A check will show at once if points are true, and if the angles have been taken correctly.
The nature of the bottom must be taken every few casts, and recorded, the officer having a look at it from time to time himself, to make certain that the leadsman is calling the stuff he brings up by its right name. For instance, many men will insist on calling "stones," rock, which is of course quite a different thing.
Same
"Points" to be used.

The same objects should be taken for the fix as long as possible. It tends to check errors in reading off, as the angles at each fix will bear a definite proportion to the last set. For instance, if we are pulling off shore with both Mat and Pagoda astern of us, the angle will be less each time, and a reading of say $33^{\circ}$ instead of $23^{\circ}$ would be at once detected as erroneous, before the disjointing of the line when the fix was plotted showed there was "something wrong somewhere."

The variation in the angles will also enable us to see if the "fix" is remaining good. This plan also saves time in setting the station pointer verniers.

> Necessity for assistance.

> When assistants are not thoroughly used to the work of sounding, it will be necessary to have two in each boat, to ensure no mistake ; but when not only officers, but men get used to it, one officer will in most cases be able to carry on the work by himself, with the assistance of a man to write down for him. Now that seamen are all taught to write, there
is seldom any difficulty in finding one of the boat's crew, the coxswain if possible, to write down fairly. The same man will steer generally, and so permit the officer to keep his eyes for other matters.

In deep water the boat must of course be stopped, and the leadsman will only heave when told. The interval can be timed by watch, or, in very open deep soundings, by the Massey's log towing astern, fitted as described on page 51.

The distance between the lines of sounding will depend upon the scale and the character of the survey, also upon whether the place is inhabited or not, for where there are natives, information can be picked up as to shoals, \&c., from the fishermen. The value of this, however, largely depends upon the intelligence of the informant, and often cannot be trusted.

If the coast or harbour be unknown, and the land of certain geological formations, it takes a great deal of sounding to be certain no stray rocks exist undiscovered; and, as was pointed out in our preliminary remarks, the majority of marine surveys are not on a sufficient scale, nor will time at disposal allow us, to sound so close as to be absolutely certain nothing is missed. The surveyor must make up for this by keeping his eye ever on the look-out for discoloured water, and by examining every suspicious spot.

It must always be remembered that in the ordinary scales used for surveying, figures may look close together, and yet be, in nature, quite far enough apart for a rock or bank to exist, without giving any indication in the lines of soundings passing on either side of it. On a scale of 3 inches to the mile, each figure will occupy a space of 50 yards nearly.

It will depend upon the orders received from the chief of Suspicious the survey whether suspicious ground is searched at once, or ground. merely pointed out on return on board for further examination. As a general rule, whenever the soundings, in pulling off shore say, decrease, it is suspicious, and the spot must be examined by intermediate lines, which in many cases should
be at right angles to those previously run, and looking out sharp with the eye as well.

In calm weather, when there is a tide, a sharp eye may detect a pinnacle rock by the ripple it may form.

It is in looking out for, and utilising such small indications that the genius of the true surveyor displays itself, and many are the rocks that have been missed for want of such sharp. intelligence.

Small buoy.

Doubling the Shoaler lines.

A small nun buoy, with light chain and a weight to anchor it by, is useful in the sounding boat, to drop over on a shoal spot, so as to guide a boat working round and round while trying for still shoaler water.
In many cases it is convenient to run double the number FIG 28.

of lines in shoal water, (say out to 7 fathoms,) that are required in greater depths. In this case, one set of lines will be run first, and when the boat gets to the end of her allotted space, she will return in the opposite direction, and run intermediate lines.

See Fig. 28, where we suppose the boat to start at A, work along the long lines to B , and then return to C along the intermediate lines, crossing the old work at every line, and thereby getting a check on it.
Sounding sections.

In sounding a harbour channel on a large scale, it is often convenient to stretch a lead-line across from side to side, and sound at regular distances apart by this line, shifting it for each section required.
sweeping. Sweeping for a reported pinnacle rock is resorted to when
sounding fails to discover it. Two or more boats, pulling abreast, tow a lead-line between them, well weighted under the stern of each boat. If one weight in the centre is used, the rock may very likely be missed. The size of the boats will govern the length of line between them. An iron bar is still better. It is by no means an easy thing to do efficiently, so that all the ground shall be traversed without unnecessarily going over it again and again. If steam-cutters are used, care must be taken not to go too fast for the weights attached, or the bight of line will be towed nearer the surface than is intended.

Shoal banks, out of sight of land, or too far off to use Sounding marks, can be sounded by starring round the ship, at of sight of anchor on it, or off its edge. For these, compass bearings land. of the ship taken from the boat, with distance measured by the masthead-angle, will probably suffice in accuracy, the boats sounding in lines radiating from the ship in all directions.

A large canvas ball or cylinder, on a light framework of iron and painted black, will be found very useful at the masthead when taking the angle for this purpose, as it will clearly define the masthead, and also indicates, "Ship in position."

Boats or beacons can be moored in convenient positions, and fixed by angles to one another, and to and from the ship, also at anchor, and the base obtained by masthead-angle, if it is necessary to sound a bank a little more accurately. These will then be used as marks, and the soundings fixed by angles in the ordinary way.

When surveying a large bank, where accuracy is desired, the beacons should be placed on a regular plan, and nothing is better from every point of view than anchoring them in two lines, so as to form equilateral triangles and a series of parallelograms, the beacons being about 5 miles apart. This distance permits the corners of the parallelograms being seen from one another. Bases are obtained by patent $\log$, and astronomical observations fix the extreme points.

If sounding out the triangles by boats, a mark boat, flying
a flag from a bamboo lashed to the mast, can be moored halfway between the lines to aid fixing.

## Reef <br> Sections.

Sectional lines off coral reefs are sometimes now required to show the exact slopes for scientific purposes, or for cables.

It is not an easy operation, and cannot be hurried.
The soundings must be close to show the exact slope, when it is, as in many cases, steep.

The section must be run on a transit line, and there are many ways of fixing the distance.

A boat anchored on the edge of the reef for the outer transit mark, with a long bamboo or other light spar stepped, will, by means of vertical angles, afford a means of ascertaining the distance up to perhaps half a mile, but beyond it will be necessary to have another boat or mark on the reef at a fixed distance from the transit line, to which horizontal angles can be taken, making, in fact, an exaggerated ten-foot pole. Other methods will suggest themselves to the surveyor.

The diagram should be drawn on a true scale, i.e., the vertical and horizontal scales equal, an inch to 30 fathoms, and the slope to the left, so as to facilitate comparisons with other diagrams.

## Measuring

 "Leadlines."In all sounding, the lead-lines should be measured on return on board, and a note made in the book, "Lead line correct," or so much out. When the line has not been used for some time, it should be measured before leaving in the morning also ; but if it has been examined the evening before, this will not be necessary.

While on this subject, it may be noted that new lead-line should never be used for boats' soundings. At the beginning of the commission it may be necessary to do so, but afterwards make lead-lines out of old well-stretched stuff that has been used for deep lines for ships sounding, and measure and mark them when wet.

## Necessity for fractions.

The soundings must be put into the book to the exact depth obtained, but it will depend upon the scale, the general accuracy of the chart, and the thickness of the soundings, how far halves and quarters will be placed on the sheet. As
a rule, fractions should be retained up to 6 fathoms, and over that depth only the even fathom, taking of course the fathom under the depth. Thus a sounding which, when reduced to low water, is $9 \frac{3}{4}$, will appear as 9 fathoms.

The necessity for accuracy in reducing soundings to low Reducing water will also very much depend on the scale of the chart ings. and the depths. It is evident that with soundings of over 6 fathoms at low water, if we are using a small scale, where the size of the figure placed on the chart will, in ruality, cover ground on which we have taken five or six soundings, any nicety of reduction is an absurdity, and labour thrown away; but in shallow water the reduction will be just as necessary in a small scale as a large, as a sounding of 5 fathoms will be a danger or not, according to what amount of reduction we apply.

It is usual in surveying vessels to depart from the time- Calling honoured habit of calling soundings, and to call simply "six $\begin{gathered}\text { Sound } \\ \text { ings. }\end{gathered}$ and three-quarters," "five and a half," and so on. This is simpler, and saves time. The men should also be trained to call out sharply, and on no account allowed to drawl.

There are, however, two exceptions to this. "Seven" and "Eleven" have a great similarity when called from the chains, and to prevent mistakes, "Deep eleven" should be called. Similarly "Nine" and "Five" sound much alike, and "Deep nine" should be given. "Five" and "Seven" are given simply.

On all occasions, whether in ship or boats, when the leads- "Shoal man suddenly gets a shoaler cast than expected from his previous soundings, he should call out "Shoal water," without waiting to complete his usually fruitless endeavours to gather in the slack line, and find out the depth. The author has been on shore from the neglect of this, the leadsman being foolish enough to wait until he had repeated his cast, so as to give the correct depth, and gave no warning to the officer on the bridge until too late.

Belcher proposes a plan for ascertaining the depth on a bar Belcher's which it is desired to cross, without risking a capsize, which "Sound- ing a Bar.:
may be quoted, though we have no knowledge of its having been practically tried. He suggests anchoring the boat as close to the bar as is safe, with the tide at flood, and veering away a barricoe with a grapnel hanging at a given length of rope. The barricoe is permitted to drift freely over the bar, when the anchor catching, will give a shock to the barricoe that will be seen by the watcher in the boat, and will indicate that a less depth than the length of the cable allowed to the anchor is on that part of the bar.

The line attached to the barricoe, with presumably a tripping connection with the grapnel, will bring the apparatus back to the boat, when she can test another part of the bar in the same manner.

## SHIP SOUNDING.

The soundings over a certain depth, about 20 fathoms, can generally be most advantageously done from the ship.

Usual plan.

## Arrange-

 ment for expeditious sounding.Where a steam winch is fitted, soundings can be got with great rapidity ; and by dropping the lead from forward and heaving it up to a davit fitted on the taffrail, up-and-down casts can be got in 40 fathoms at a speed of about four-and-a-half knots without stopping, with a 100 lb . lead.

If a long spar be fitted as a derrick aft, soundings can be obtained in water up to 20 fathoms, by merely swinging the lead and letting it go without heaving forward.

For deeper water a variety of methods have been devised for getting the lead forward and dropping it rapidly.

The following is now generally used.
The lower boom is got out and topped to an angle of about $40^{\circ}$.

An endless rounding line of lead-line is carried through a block at the end of the boom, by leading blocks to the steam winch, and to the derrick or davit aft. A slip is attached to this with a broad projecting palm of sheet iron to the catch. See Fig. 29.

To this the lead is attached, and hove forward by the winch. When up to the loom, the rounding line is let go, and on
striking the water the palm releases the catch, and the lead falls free to the bottom.

The rounding line is at once rounded aft again, ready for the slip to be again attached when the lead comes up.

There are varieties in the detail of the fittings, according to different ideas.

By dropping the lead well away from the ship, the chances

of the lead-line fouling the screw, if the helm is over, are much lessened.

A variety of instruments have been invented for giving Instruthe accurate depth when the line cannot be got up and down; ments for some depending on a fan which works a series of cogged depth. wheels, as Massey's; others, on pressure at different depths. These are all useful up to a certain point, and when their
errors have been obtained, may sometimes be attached to the lead with advantage.

Recorders, however, of great value to navigation, are of no use in surveying operations, and the majority of these navigational inventions are liable to small errors, which we must not have in depths which are to be placed on charts.

Burt's bag and nipper are useful when the ship drifts away from the vertical position over the lead, and one should always be handy when sounding, but great care is necessary that it bites the line.

Perfect machine yet to be made.

It is evident that a perfect machine is more trustworthy than the record of an up-and-down cast with the ship in motion, as given by a fallible man; and when such perfect machine is invented, it will be gladly adopted by surveyors ; but, up to the present time, the machines are more liable to error than a trained man, under most circumstances.
Long lines In localities where currents are prevalent and vary, when of soundings off shore.
we are running long lines of soundings in the ship off shore, out of sight of land, it is very important to get, on the return line towards the shore, a fix as soon as possible. The soundings we are obtaining may be hereafter used, especially where fogs are frequent (as, e.g., British Channel, Bay of Fundy), to give vessels a notion of their position, and we must therefore use every dodge to get our true position at the earliest opportunity, so as to depend upon dead reckoning as little as we can.

Two theodolite stations, from which a large flag at the masthead can be observed as soon as it appears above the horizon, is a plan sometimes employed. These need not see one another. As long as their relative positions on the chart are known, and the true bearing of the zero employed has been established, the angles to the ship can be plotted. The smoke from the ship when she herself is below the horizon will often enable a valuable angle to be observed. A single theodolite line is often of great value, as it can be utilised in conjunction with angles, bearings, or observations from the ship herself.

It is scarcely necessary to say that an observer will be on Surveyor the topgallant yard of the ship, as he may, from atmospheric alo't. or local causes, be able to see something on the land before the theodolite observers catch sight of the ship.

True bearings come in useful again. The angular distance between the sun and the mountain, or other object, seen from aloft, will be taken by the observer aloft, while the sun's altitude is taken from deck for the azimuth.

Another method is to have one or two ships anchored as Tenders. far from the land as they can fix, which observe, and are observed from, the sounding ship, as she runs in and out on her lines. A ship can easily, in light winds, anchor in 100 fathoms, and even in deeper water.

When land stations are employed, heliostats are useful, as Use of informing the running ship that she is seen from the station. heliostat. A flash will tell the officer aloft that a sounding can be taken, with the certainty of an angle being got to the ship, for which, perhaps, she has been waiting.

If not able to return and pick up the land before nightfall, Position blue lights and rockets are useful, both from stations and moving ship.

A true bearing of a light, or mountain, if visible, as it often Use of is a great distance on moonlight nights, can be obtained in the northern hemisphere very conveniently by the angular distance from the Pole star, as described on page 281. This angular distance can again be taken from aloft; but in the case of Polaris, we require no altitude.

If Polaris is not available, a time azimuth of a star near the prime vertical will give a good result. Should the resulting longitude differ much from the assumed, it may be necessary to re-calculate. Altitude azimuths cannot be much trusted in at night.

When objects are visible from deck at night, and we can Compass rely on the compass, very good bearings can be taken with bearings. the standard, if the lighting arrangements are properly fitted.

A ruled "Deck Book," as now supplied, is convenient for Deck ship's sounding. In this everything taken from the ship book.
should be recorded, as, rounds of angles when the ship is used as a station in main triangulation; elevations with sextant, and the corresponding fix; sketches of little bits of coast, \&c.

## SEARCHING FOR VIGIAS.

Difficulty of disproof.

Area of search.

In searching for a "vigia" it is difficult to say when its existence is to be considered as disproved. Although experience shows that nine out of ten of these bugbears and blots on the oceanic charts have been mistakenly placed there, from reports of floating whales, wrecks, and patches of confervce taken for discoloured water over a bank, \&c., still the apparently astounding manuer in which coral banks rise from very deep water must always make us careful of assuming from a hasty search, that no shoal water exists near a given locality.

The area over which to search must always be large, as the reckoning of the reporting ship, especially as regards longitude, may often be considerably in error. In the vicinity of such reefs also, currents are generally accelerated, and altogether we must allow a large margin, in undertaking to search for a danger reported in a particular spot.
In clear bright weather, coral banks will show some miles with the sun in the right direction: but under other circumstances it is quite possible for a ship to pass within a mile of a bank with as little water as three fathoms on it, without its being detected

Assuming that coral reefs are built on submerged mountainpeaks, a little consideration will show that there is nothing extraordinary in a shoal near the surface standing in 2000 fathoms water, on a base of not more than three miles diameter.

The annexed sketch, Fig. 30, will show that we are not assuming an improbable steepness of side to the submerged island.

This allows us to pass little more than $1 \frac{1}{2}$ miles from such a shoal, and still get a cast of 2000 fathoms, so that even a
positive sounding of great depth will only cover a comparatively small area, and soundings of a hundred fathoms, no bottom, do not assure us of anything to a certainty, except that the reef does not exist within a few hundred yards of that cast.*

FIG. 30.


Nevertheless banks can frequently be diagnosed from a sounding, though deep, a little less than others around, and the only way to make certain that a bank does not exist is to follow up the direction of the slope with positive soundings. No bottoms are of little value.

Experience has shown that, in coral waters, the edge of a bank is a favourite spot for the growth of a small danger. This part of a bank should therefore be closely searched.

The difficulty of fixing the position at sea to within three Doubt as miles or so, adds another element of uncertainty to our search, ${ }^{\text {to ousition. }}$ so that it is only by crossing and recrossing the area to be examined that we can at length say, positively, nothing is there.

This is especially the case where the reported danger is Credence out of the usual track of ships, as there is nothing improbable ${ }^{\text {in reports. }}$ then in its having escaped notice up to that time. Where the locality is frequently passed over, there is more prima facie reason for doubting the report, and in many instances, a crossexamination of the person making the report, will show how very slight is the ground for it. An actual cast of the lead seems a fact impossible to make a mistake about, but instances have occurred where this also has proved to be so, even with so-called " bottom" brought up. In cases, however, where a sounding has been obtained, we must conclude the report to

[^14]be true, and a rigorous search must be made before the vigia can be obliterated.

Preliminary search.

Lying-to near Vigias at night.

Decision rests with Hydrographer.

As a general rule, for the first commencement it is best to run lines east and west in or near the latitude reported, as this is more likely to be near the truth than the longitude.

When going to make an exhaustive search, the first day is perhaps best spent in doing this without getting more than may be one positive sounding, as we can cover more ground, and, if the danger exists, we have a good chance of finding it by sight, or by the soundings taken when the ship is running, as of course the deep-sea lead will be kept, going constantly.
It is rather unpleasant to be drifting about at night with reported reefs in the vicinity, and by no means a bad precaution is to ease a kedge anchor down to 100 fathoms or so, which may bring the ship up, or at any rate show, by drawing ahead, that bottom is reached, before she strikes on the reef.

At night the vicinity of a reef in open ocean may be indicated by fish, which invariably frequent these isolated spots, and here phosphorescence will help greatly in making their presence very apparent.

In daytime, birds, which generally congregate wherever fish are plentiful, may be an indication.

In every case, of course, the surveyor transmits home a plan of his track and soundings, as it is at headquarters only that a decision on the matter can be arrived at.

Under the head of "Sea Observations" will be found hints as to early ascertaining of the ship's position, a most important matter on each morning.

The submarine sontry, now supplied to all surveying ships, should be constantly towing, set to about 30 fathoms. With kites fitted to float with the ship stopped, the chance of fouling the screw is much diminished, and the apparatus is of the greatest use in furnishing indications of small banks that may otherwise be missed, though it sometimes gives false alarms.

As the wire generally carries away close to the kite, it saves loss if a preventer of slack wire, secured to the tail of the kite, be spliced into the main wire about 4 fathoms up it.

## CHAPTER IX.

TIDES.
All soundings in published charts are given for low water Tidal at ordinary spring-tides; we therefore want all the infor- observamation we can get about the tides, and the very first thing menced at to be done on arriving on the surveying ground is to com- ${ }^{\text {once. }}$ mence observations on them.

There are very few parts of the world in which we have absolutely no knowledge of the tidal movement, so we have generally something to commence upon. That is, we usually know within an hour or two the time of high water at full and new moon, called H. W. F. and C. on the charts, as, except in estuaries or peculiarly shaped coasts, this will not differ greatly from places near at hand, and the same may be said for the range of the tide.

It will altogether depend upon our length of stay in any Different locality, as to what we can hope to find out about the tides. observaTo get full information requires observation during months in succession, as in many parts the tides vary considerably at require $\begin{aligned} & \text { rents. }\end{aligned}$ different times of year. The number of high and low tides in a day, in certain places, departs from the normal phase of from 6 to 7 hours for each rise or fall; in others, the tide will take longer to rise or fall, than vice versá, \&c., \&c. A long series of this kind is therefore very valuable, as the tidal theories are at present far from fulfilling all the requirements of observation all over the world, and good data are much wanted; but it is not often that the surveyor can obtain such a series.

It will be seen, then, that tidal observations for the practical reduction of soundings for purposes of navigation are one thing, and those for obtaining additional data for scientific investigation are another.

We shall mainly concern ourselves with the former, where much rougher observations are usually admissible ; but here, again, it must depend upon the scale and nature of our chart what degree of nicety is requisite.

A few words on the Theory of the Tides will be given at the end of the chapter.

Tide
Tables.

Local circumstances.

The reader is referred to Dr. Whewell's Treatise on the Tides, published in the preliminary part of the Admiralty Tide Tables, for much information respecting their movement.

A regular series of observations, even for our practical work, should be taken if possible; but in many cases the necessity for leaving tide-watchers encamped is inconvenient, and may be unhealthy, and we may have to be satisfied by obtaining what will be sufficient to enable us to construct the chart, which is our immediate business.

In other cases we may only be staying a few days at a place, as when making a plan of a small isolated harbour.

## Observa-

 tion indispensable.What we absolutely require in making a chart is to know the height of the water, whilst sounding is going on, above the level of low-water springs, which is called the "datum for reduction."

We shall also wish to ascertain, if possible, the " establishment," which is the time of high water at full and new moon, called in the charts, "High water at full and change;" the rise of spring-tides above our datum; and the range of the tides at neaps, and the time occupied by the rise and the fall of each tide, as these will give valuable information to the navigator.

We may here give definitions of some of the terms used in speaking of the tides.

[^15]low-water levels of any one tide, without any reference to the datum.

The "semimenstrual inéquality of heights" is the difference between the heights of spring and neap tides above mean water-level.

The "diurnal inequality of heights" is, in irregular tides, the difference between the height of high water of each successive tide.

The "age of the tide" is the interval between the time of new or full moon, and the time of the next spring-tide, and varies from $1 \frac{1}{2}$ to 3 days.

The "lunitidal interval" is the time that elapses each day, between the transit of the moon over the meridian, and high water.

The "establishment" may be also defined as the lunitidal interval when the time of moon's mer. pass. is $0^{\mathrm{h}} .0^{\mathrm{m}}$. or $12^{\mathrm{h}} .00^{\mathrm{m}}$. This is called the "vulgar establishment."

The "mean establishment" is the mean of all the lunitidal intervals in a semilunation, and may differ considerably from the vulgar establishment. The latter is the high-water full and change given in the charts.

The "semimenstrual inequality of time" is the difference between the greatest and smallest lunitidal interval.

The "diurnal inequality of time" is, in irregular tides, the difference between the lunitidal intervals of each successive tide.

The time and height of the tide is ever changing, caused Cause of by the relative positions of the sun and moon, and these in Tides. more or less regular variations are further affected by winds, and by the height of the barometer. The difference in level due to the latter may be taken roughly as a foot for every inch of the barometer above or below the mean barometer, high barometer causing lower tides.

The time of the moon's transit over the meridian gives us a Observarough measurement of the relative position of sun and moon $\begin{gathered}\text { tions re- } \\ \text { ferred to }\end{gathered}$ in right ascension, and it is therefore to this meridian passage of the moon that we refer all calculations of the tides.

If the tides are regular, we shall find that on days on which the moon passes the meridian at the same time, the times and heights of high and low water will be the same.

This knowledge is very valuable in many surveys where from local causes we cannot always have a tide pole going, as from previous observation we can, when the tides have been found to be regular, construct a table founded on moon's meridian passage, from which we can take out a reduction for soundings, when working on a small scale.

When we arrive on our surveying ground, then, one of the first things to do will be to set up a tide pole, whatever is going to be the character of our observations.

Position for Tide Pole.

For this we want a sheltered spot, if we can find one, and also firm ground on which to place it, as nothing is more annoying than to find the pole down, especially when out of sight of the ship, when the tide-watchers, unassisted, generally succeed in putting it up again in a different position.

If a pier is available, there is nothing so simple and satisfactory as a plank secured to it, marked in feet and inches, the former being painted red, white, and blue alternately, with bold black figures.
Tide Poles. If we have no pier, an ordinary spar, shod with an iron spike and painted as above, driven as far into the ground as possible and well stayed to heavy weights, anchors, rocks, or whatever we can get, will stand well, and generally answers our practical purposes. This may sometimes be so placed as to be read from the ship with a glass.

If however there is no shelter and much wash of the sea, and accurate observations are required, we must use a tube of some kind.

A square one of deals can be knocked up on board; but it must not be too small, as we shall want a slit down one side through which an indicator fixed to a rod carried by a float inside may work, and the water washing in by this slit will destroy the value of the tube, unless the area of it be large enough to make the water thus admitted too in-
significant in quantity to disturb practically the surface of the water inside. Where there is not much range of tide, the slit can be dispensed with, and the rise and fall marked by an indicator protruding from the top of the tube (which in this case could be a boiler tube), and marking on a scale lashed so as to project above the tube. The water would be admitted by holes bored near the bottom of the tube, if it is to be placed on muddy ground.

A good portable automatic tide-gauge, suitable for all Automatic requirements, has not yet been made. A pneumatic gauge gideis now (1897) under trial, and promises well, but it cannot yet be said whether it will answer.

Whenever it can be done, a mark should be made on some Fixed fixed object near the tide pole, corresponding to some mark ${ }_{\text {reference. }}^{m a r k}$ for on the pole, which can then be replaced in the same position if it accidentally gets displaced.

Levels should also be carried to some permanent mark in the vicinity, and the difference of level between this mark and the datum given in the chart, with the object of enabling future surveys to be reduced to the same datum level. This is most important. When, as is the case of many civilised countries, there is a fixed plane of reference for land surveys, and level marks are available, the tidal datum should always be connected with such fixed plane.

The level of the water on the tide-gauge should be noted every hour of, if we are going to make a regular series, both

Time of Observations. night and day, if simply to get a datum for soundings, only of the day, except at springs, when it is as well to get the high and low water at night also, as night tides in some places and at some seasons are lower or higher than the day ones.

It is not amiss in any case, when nothing is known of the tides, to observe for twenty-four hours, at half-hour intervals, as a commencement, as this will tell us whether the tides are regular or not, and we can take observations accordingly.

To get the time and height of high and low water accu- High and rately, observe every ten minutes, for half an hour or so, waw M 2

Observa- before and after high and low water, and calculate from these
tions.
Graphic
Method. records the exact time and height required.

This is best done by projecting graphically thus :-Divide a line into equal parts to represent hours and minutes, and from this, at the corresponding time, set off at right angles distances, on any chosen scale, to represent the height of tide registered at that time. These spots, joined by a curve, will enable the time and height of high or low water to be arrived at much nearer than by simple observation.

Thus, suppose we have noted-

| X. 00 A.m. | .. | .. | .. | .. | 12 | in 3 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | .. | .. | .. | .. | 12 | 8 |
| 20 | .. | .. | .. | .. | 13 | 0 |
| 30 | .. | .. | .. | .. | 13 | 1 |
| 40 | .. | .. | .. | .. | 13 | 1 |
| 50 | .. | .. | .. | .. | 12 | 10 |
| XI. 00 | .. | .. | .. | .. | 12 | 5 |

FIG 31.


We project these as in accompanying Fig. 31, and by drawing a horizontal line from the X. 10 position to the opposite side of the curve, bisecting it, and letting fall a perpendicular to the line of time, we find X. 32 as the time of high water. The compasses, measuring the highest point of the curve, gives a little over 13 feet 1 inch as the height marked on the pole.

Approximating a Datum.

If we are at the place during the spring-tides, we can get a fair low-water datum by observation, and all soundings will be reduced to that, by the height marked on the pole above this datum, at the time the soundings were taken each day
being subtracted from them. But it may happen that we arrive at the place a few days after a spring-tide, and leave again before the next one. The only thing to do is to note the high-water mark on the shore, and ascertain by measurement how far it is above the high tide of the day as marked also on the shore, subtract the same quantity from the lowwater mark on the pole of that day, and call that the lowwater spring datum, subtracting perhaps a foot or two extra, to be on the safe side.

Thus, suppose at high water our pole marks 13 feet 1 inch, and the high-water mark on the beach is 2 feet 6 inches above the level of the sea at that time; at low water the pole marks 5 feet 8 inches. This will give us 3 feet 2 inches as the probable low-water spring mark. If we reduce our soundings 2 feet below this to the 1 -foot mark, we shall be pretty certain not to give too much water on shoal spots.

An approximation of this kind would be of course noted on the chart when sent home, and also the manner in which the rise of spring-tides, which would be given as 14 feet, has been obtained.

In still rougher work, an approximation of the rise of Rougher the tide may be got by having a marked boat-hook held mation of upright at the water-line at time of low water; the observer Datum. then places his eye at the high-water mark on the beach, and reads the mark on the boat-hook, where the horizon line cuts the latter, which will be the fall of the tide that day below high-water mark. If it is the high-water mark of the day that is so used, the result is the range of the tide for the day; and if the distance that the springs' mark is above the day high-tide mark can be measured, we can arrive at the full rise and fall, as in the last article.

This may be very useful in making a hurried plan of a bay, and thus the height of the water can be got by the officer putting in the coast-line from time to time during the day, without delaying him much, and to the great advantage of the correctness of the soundings being taken at the time.

The " vulgar establishment" is an exceedingly loose term, as

Estimation of Establishment.
given on the charts. As it is strictly only on days when the moon's mer. pass. is $12^{\mathrm{h}}$. or $0^{\mathrm{h}}$., that it can be directly observed, the surveyor is obliged to approximate to it in most cases. This perhaps matters the less from the fact that the establishment, even when correctly obtained, is seldom invariable.

The best way to approximate is to project the line of lunitidal intervals, and measure the length of the abscissæ from XII ${ }^{\mathrm{h}}$. and $0^{\mathrm{h}}$. for the vulgar establishment, meaning them if we get more than one.

FIG 32.


If the tides are regular, especially as regards the semimenstrual inequality, the establishment may be roughly determined by a method given in the article in the Tide Tables, from an observation of the tide at any period of the moon's transit, but which we shall not further discuss, as, in a case where it would be required, we should not know whether the tides are regular or not, and any assumption would probably end in very erroneous results.

Interpolating height of Tide.

In a case where only the high and low waters on any day are obtained, the height of the tide, at times between, is best to be got at by drawing a curve in the imagined course of the
tide, after the manner of Fig. 32 ; the height on the pole at any time can then be taken from this, and a table of reduction formed for the day.

Thus, in Fig. 32, where we have only got the times and heights at high and low water on that day, viz. H. W. at VI. 50, mark on pole 15 feet 7 inches; L. W. at I. 10, mark on pole 8 feet.

On a piece of paper, either ruled in squares for the purpose, or on ready-printed squared paper, which is very useful to have by one for these like occasions, we draw a curve after the fashion shown. Then, supposing our datum for reduction to have been settled as 4 feet on the pole, our table of reduction for the day will stand as follows :-


| I | 4 | ft . or |  |
| :---: | :---: | :---: | :---: |
| II | $4 \frac{1}{2}$ | ", or $\frac{3}{4}$ |  |
| III | $5 \frac{1}{2}$ | , or 1 |  |
| IV | 7 | \% or $1 \frac{1}{4}$ | " |
| V | $8 \frac{1}{2}$ | ", or $1 \frac{1}{2}$ | " |
| VI | 102 | , or $1 \frac{3}{4}$ | , |
| VII | $11_{2}^{1}$ | " or 2 |  |

This will of course be, as all attempts at arriving at anything with insufficient data, only an approximation, but will probably be near enough for the purposes we want. If we intend great accuracy, we shall make arrangements to have the tide carefully observed throughout the day, whenever sounding is going on, and take every precaution not to be reduced to these straits.

When tides are found to be regular, a table of reduction Table of may be formed from the observations during one or more Reduotion complete lunations, by tabulating the tides according to the certain time of moon's upper transit. Such a table may be very stances. useful when the scale of the chart is small; and sounding can be carried on under these circumstances, viz. regular tides, and scale of chart small, when no direct observations can be got.

In many places external circumstances control our wishes. For instance, it was found on the East Coast of Africa that if
men were landed to make a regular series of day and night observations on the tides, fever generally ensued, and consequently the record was restricted to day tides. Again, the tide pole may have to be so placed on a shelving shore, or among reefs, that a boat would have to be used to go out at high water to read it, and this may not be convenient. Often, for considerable tracts among reefs, observations may be impossible, and a table deduced, as above suggested, from former observations may be then used.

Graphic Projection of Tidal Movement.

In forming such a table, it is best to project the tidal curves as shown in Fig. 32, but by the hourly observations. It will then be seen whether the tides are regular, and days of similar time of upper transit of the moon can be compared, to see whether a table will give us the reduction near enough for practical purposes.

Surveying ships are now supplied with abstract forms for the projection of high and low waters in such a manner that the regularity or otherwise of a tide can at once be seen.

They provide for the record of the lunitidal interval, the moon's mer. passage : the declination of the sun and moon, apogee and perigee, and the mean time of the high water following the superior transit, and of the highest tide in the 24 hours.

A horizontal line in the upper part of the diagram is divided into hours with vertical lines drawn through them. These hours are those of the moon's superior transit. Directly under the position on the line of the local mean time of the moon's superior transit is plotted, as a dot, the high water next following that transit, at its proper position for height of the tide, as measured by the side scale in feet. When the next day's similar high water has been similarly plotted, the intermediate high water is plotted half-way between them at its proper height and the two low waters similarly interpolated.

The high waters following the superior transit are joined by a red line, and the intermediate high waters by a blue line,

Similarly with the low waters, taking care to note which are the low waters next following the superior transit. Each dot is joined to its next successive high or low water by a black line.

The lunitidal interval for each high water is plotted in the place provided, and the dots joined. By adding the moon's superior transit to the lunitidal interval as plotted, the mean time of each high water can, if necessary, be ascertained.

When the curves of the sun's and moon's declination, and of the moon's parallax are plotted, the general movement of the tide, and its relation to the positions of the sun and moon in declination can at once be seen.

The mean between each high and low-water height will give roughly the mean water-level.

To obtain the true mean water-level in a few days, other True Mean observations must be taken, and we subjoin an extract from ${ }_{\text {Level. }}^{\text {Water- }}$ the "Instructions to H.M.S. Challenger," which contains full directions, only adding that these observations must be made when the tide is moving normally, that is, when there are no strong winds to raise or depress the water-level.
"A good determination of the mean level-sea by the simple operation of taking means may be made, in less than two days, with even a moderate number of observations properly distributed so as to subdivide both solar and lunar days into not less than three equal parts. Suppose, for example, we choose 8 -hour intervals, both solar and lunar. Take a lunar day at 24 hours 48 minutes solar time, which is near enough, and is convenient for division; and choosing any convenient hour for commencement, let the height of the water be observed at the following times, reckoned from the com-mencement:-

| h. m. | h. m. | h. m. |
| ---: | ---: | ---: | ---: |
| 00 | 80 | 160 |
| 816 | 1616 | 2416 |
| 1632 | 2432 | 3232 |

[^16]8 hours solar or lunar, while one group is separated from the next by 8 hours lunar or solar. In the mean of the 9 results the lunar and solar semidiurnal and diurnal inequalities are all four eliminated.
"Nine is the smallest number of observations which can form a complete series. If the solar day be divided into $m$ and the lunar into $n$ equal parts, where $m$ and $n$ must both be greater than 2 , there will be $m n$ observations in the series; and if either $m$ or $n$ be a multiple of 3 , or of a larger number, the whole series may be divided into two or more series having no observation in common, and each complete in itself. The accuracy of the method can thus be tested, by comparing the means obtained from the separate sub-series of which the whole is made up.
"Should the ship's stay not permit of the employment of the above method, a very fair determination may be made in less than a day, by taking the mean of $n$ observations taken at intervals of the $n$th part of a lunar day, $n$ being greater than 2. Thus if $n=3$, these observations require a total interval of time amounting to only 16 hours 32 minutes. The theoretical error of this method is very small, and the result thus obtained is decidedly to be preferred to the mere mean of the heights at high and low water.
" The mean level thus determined is subject to meteorological influences, and it would be desirable, should there be an opportunity, to redetermine it at the same place at a different time of year. Should a regular series of observations for a fortnight be instituted, it would be superfluous to make an independent determination of the mean sea-level by either of the above methods at the same time."

In some cases the mean level of the water may be made use of as a temporary datum for reducing the soundings.

If, for instance, we commence soundings in a place where we do not yet know the spring's range, but intend to get it accurately after some months' observations, we may find it convenient to reduce all soundings to the mean level, as found by meaning each day's high and low water. Then,
when we have ascertained the level of low springs below this mean level, one uniform quantity will have to be subtracted from every sounding, which will save a good deal of complication and waiting, as the soundings may all be plotted without fear of mistakes in reducing them afterwards.

This mode will be mostly used for shallow channels, where a difference of a foot or two is an important matter, but it is liable to the error caused by variation in the height of the mean tide-level.

The direction and rate of the tidal streams and other Tidal currents must be observed.

This is best done under ordinary circumstances from the ship at anchor by means of a current log, which is simply a very large log-ship, and is worked in the ordinary manner,

Streams and Surface Currents. Current Log. but with a longer interval of time. The line, which is small, is marked at every 10 feet, and is permitted to run out for an even number of minutes, varying according to the velocity of the current.

Then the rate per hour of the current=number of feet run out, divided by one hundred times the number of minutes.

Thus, if the $\log$-ship is permitted to run for three minutes, and 220 feet of line pass out,-

Rate per hour $=\frac{220}{300}=0.73$ knots.
This current $\log$ should be hove at stated times, whenever the ship is on her surveying ground, and at anchor, and an entry made in the current $\log$ whether there is anything recorded or not, as negative results are in some ways as valuable as positive ones. Where the tidal range is great, and streams change their direction, these observations will be made at comparatively short intervals, in order to ascertain the movement of the water at different times of the tide. Where streams are strong, and of importance in navigation, assistants will be sent to heave the current $\log$ from a boat at anchor in different positions.

The current $\log$ can be kept by quartermasters, with supervision. A watch or clock with a seconds-hand is a requisite.

In the Current Book will be entered the position, time, direction of drift of the log-ship, number of minutes it was allowed to run out, and number of feet of line run out, wind and force. Blank columns for rate per hour, and time of tide, will be filled up afterwards by the officer discussing the currents.

Time of change of direction of Tidal Streams.

The direction of the tidal stream will frequently change after high or low water, and when this occurs, we must endeavour to find out whether the change of stream occurs at a regular time of the tide, as this is an important point in the navigation of channels.

In channels connecting two open areas of sea, the general law is that the stream will run for some hours, often for three hours, after the tide, as indicated by the rise or fall on the shore, has turned. This makes it very confusing to speak of a stream as the flood or ebb stream, and the term east-going or south-going, or whatever the main direction of the stream may be, should always be used in preference; for in such a case the direction of a stream may be the same for the last three hours of the flood, and the first three hours of the ebb.

Theory of the Tides.

The Theory of the Tides is one of the most complicated subjects that can be considered.

Recent investigations by Sir W. Thomson and Professor G. H. Darwin have shown that the tidal movement may be considered to be the resultant of as many as thirty-three separate tidal waves. Some are dependent upon the moon, others on the sun. Some occur once in the day, or are diurnal ; others twice, or semi-diurnal; some have a period of a lunar month dependent on the moon's position in her orbit as regards the sun; others have six monthly periods dependent on the sun's declination. The moon's declination, a variable quantity with a long period of years, controls others. The position of the moon's node, and her varying distance from the earth, are responsible for considerable waves, the Perigee tide being always larger than one in Apogee.

The system of harmonic analysis has been adopted for
the clearing of these different waves, and is, when tides are very variable, the only method by which the calculation and forecasting of tides is possible.

It is not proposed to enter into this, which forms no part of the necessary work of a marine surveyor in the field, and is a subject in itself; but it may be mentioned that while in a few regions the time-honoured practice of calculating the time of the tide from the moon's meridian passage, and its height from the mean of tides observed in connection with that meridian passage may serve for practical purposes ; in most parts of the world the movement is so complicated that for a satisfactory forecast the employment of harmonic analysis is necessary.

From what has been said it is evident that observation of General the tide for a short period will, when tides are complicated, Remarks afford no means of predicting them. The movement may be wholly different when the sun is north of the equator and when it is south, and many other factors make it necessary to observe for at least a year to gather an idea of the behaviour of the tides at all seasons.

The variety in complicated tides is infinite. In some cases the water will rise to nearly the same level every tide, while the low water shows great differences; in others it is vice versa. In some cases the diurnal inequality, the difference in height of each successive tide, will be equally distributed between both high and low water levels; in others it affects only one, or may vary with the season.

One feature of a tide when there is much inequality is generally regular, i.e., the succession of the inequality in height. At some places the higher high water is followed by a fall to the lower low water; the tide then rises to the lower high water, then falls to the higher low water, and finally completes its round by a rise to the higher high water again. At others this succession is reversed, but for most places the succession, whichever way it takes place, is the same throughout the year. Fig. 33 will show the two movements.

When there is great but regular inequality, the higher tide
will always be in the day time when the sun is on one side of the equator, and in the night time when it is on the other.

When the diurnal tides are great, the inequality in height will sometimes be such as to cause a mere stand in the tide during either the rise or fall of one tide, giving the effect of only one high and low water in the twenty-four hours.

As a general rule, it may be stated that in temperate latitudes the highest tides take place at the equinoxes, whereas in the tropics these occur at the solstices.

Curiously enough, and it has affected many of our notions about tides, the tides about the British Isles are the most simple that are anywhere found, that is to say, so far as the individual movement of the tide at any one place is con-


FIG. 33.
cerned. But they are largely affected by a further complication, known as "Interference." By this is meant the appearance of another tidal wave, or perhaps more than one, which affects the height and time of the resultant tide at any place.

This is caused by either a tidal wave coming round from an opposite direction, or by one reflected from another coast, and it will be at once seen that if the crests of such tidal waves coincide at any point with the crest of the primary wave, the resultant tide will be higher, and if the crest of one reaches a point at the same time as the hollow of another the range of the tide may be, if the waves are of equal height, nothing.

To this is to be attributed the variety of the height of the tide at different parts of a coast that appears fairly open.

Thus, in the English Channel, the height of the tide varies at different places on the same day from 5 feet to 25 feet; and the same phenomenon occurs in many parts of the world.

The tidal streams are not, however, directly affected in the same way, and there are many places where the rise of the tide is insignificant, but the tidal streams are very strong.

This is because the horizontal flow of the water is not determinable by the rise at the spot, but by the rise at other places, possibly at some distance, and by the fact that water once set in motion is not easily arrested.

The variations caused by Interference are wholly distinct from the differences in the height of tide and velocity of streams caused by the conformation of the land.

The vertical movement of the water in the deep open ocean is not great, probably not more than two feet, and the horizontal motion is practically nil, being a merely insignificant oscillation. It is only when the water shoals that the friction of the bottom, and the constriction caused by the water, which is in motion throughout its depth, being forced into a shorter column, cause the wave to become unnaturally heightened; and horizontal movements, which we know as tidal streams, are set up, by reason of water flowing from the higher to the lower level.

Thus, on banks in mid-ocean regular tidal streams are found, and could the height of the water be measured, it would be found to vary more than in the deep water around.

The opposition of a coast, and the shape of deep bays, gulfs and channels, accentuate these effects, and the height of the tide and velocity of the tidal streams varies in different places exceedingly. The oceanic tidal wave, thousands of miles in length, may have the distance between its crests shortened to hundreds, and the width of the portion that approaches the shore continuously narrowed as it passes up funnel-shaped passages.

Enough has been said to show that the tidal movements are exceedingly complicated, and though long-continued observation at one spot may enable predictions for that spot to be made, the variations are sometimes so great that at a short distance the phenomena are entirely different.

All prediction may be upset by what is known as the meteorological tide, that is the variation in the height caused by winds, and by the difference in pressure of the air on the surface of the water.

The inconsistencies in the tide thus caused affect the height of the water, more than the time of high or low water, and as they affect the mean level of the water, an unusually high tide induced by them does not mean an unusually low tide, but the reverse.

Wind will naturally have more effect when there is a funnel-shaped estuary than when it blows on to a straight, open coast, the heaped-up water at the wide mouth being forced higher and higher as it advances, for under such circumstances water will run up hill.

Mean
Level.

Bore,

From the foregoing it is evident that the mean level of the water will considerably vary.

When steady winds blow at certain times of the year, the variation in mean level will be seasonal ; in other places it may be constantly varying with the direction of the wind.

This variation in the mean level is important as regards navigation in some places. For instance, when a shallow flat exists which must be crossed to gain access to a harbour, and which at ordinary high water affords just sufficient depth, a change in the mean level may cause the high-water level to be one, two, or even three feet less than usual.

It is, therefore, most necessary for surveyors to acquaint themselves with the effect of wind at such places, and to record it.

In cases where the rise of tide is great, in a funnel-shaped estuary much encumbered by sandbanks, and where there is a continuous outflow caused by a large river, the phenomena of the "bore" appears.

This consists in the face of the rising tidal wave becoming so steep that it rushes up the estuary in the shape of a sudden wave, sometimes almost a wall of water, breaking as it advances, and the tide thus rises many feet in a few seconds, followed by a still rapid, but more gradual, further rise, so that the whole flood may only last one or two hours, the ebb prevailing for nine or ten hours.

The main factor in the production of a bore is the retardation of the lower part of the inflowing water by the friction of the bottom in shallow water, and by the action of the down-flowing river, so that a high tide rises quicker than it can flow forward, until its height and momentum enables it to overcome these obstacles by a final grand rush.

Bores are rare, but whenever encountered the surveyor should investigate the conditions, as but little is known of the details of most of them.

## CHAPTER X.

## TOPOGRAPHY.

The sketching in of the topography, or detail of the land, is a point on which there is more variation, as to the manner in which it is done, than in any other of the steps of a survey.

It is the least necessary part of a chart, which is destined mainly to guide over the water and not on the land; but as we are guided over the water by the land, a perfect chart should have the features of the country correctly delineated, so as to assist the mariner in recognising the land by the mutual positions of peaks and other conspicuous objects. Furthermore, with our universal presence and interest all over the globe, it is impossible to say that an expedition may not want to start from some point on our chart, when information for a short distance inland will, in such a case, be most useful.

Width of Topography from Coast.

As a general rule, the land should be put in as far back from the shore as it is visible from the sea; but this is only a very general guide, and must depend upon the distance of the back ranges, and the size of our sheet of paper. When the most distant mountains are very far back, we cannot spare time to do more than fix their summits by angles, get their heights and the extent of the range, and the country between must be a perfect blank.
Rough Topography.

Often, in savage lands, the country will be too dense with jungle to be able to do much to the topography by walking over it, which is of course the only way to get it correctly
mapped, and we must then be content to sketch what we can see from the sea, and from the coast. By making stations in the ship, drawing a sketch at each, and getting angles to all prominent parts, such as spurs of hills, valleys, ravines, smaller peaks, \&c., which will be entered on the sketch, a very fair approximation of the position and shape of the more conspicuous elevations in the land, visible from seaward, will be made. The officer coast-lining will have got the entrances of all little streams fixed, and from the ship off shore we can recognise which ravines, or at any rate which of the larger ones, join on to these entrances. Topography put in in this way will present a somewhat detached appearance, and we can only fill up the hiatus by writing on the chart the general appearance of the land intervening between the hills, as far as we can see it from aloft, as, " rolling grassy plain," "densely wooded and undulating," \&c. Sometimes, on a coast of this description, we can get back from time to time to an elevation we see from the ship to be partially clear, and a sketch from a position of that kind will materially improve our knowledge of the topography.

By referring to the sketch at page 84 it will be seen how, with similar views from different points, ravines and valleys may be cut in, and roughly drawn on the chart.

When, however, we can spare the time to perfect our Regular chart, and the nature of the country permits it, we should Traph Topowalk all over it, and sketch the topography on the ground. To do this, we must have as many conspicuous objects as possible fixed beforehand, and pricked on to a board, as for sounding or coast-line. Topography can be plotted afterwards, the same as can be done with coast-line or any other work, but it will be much more satisfactorily done if plotted at the time.

We then walk over our country, fixing ourselves with angles on commanding spots, plotting the stations by the station pointer or tracing-paper, and drawing lines from them to all things we want to plot, spurs of hills, houses, valleys, \&c., \&c., and sketching the details immediately
around us. To fix details for this purpose we shall often have to content ourselves with two angles only, and as long as we do not use such points to carry on our stations with, this will be sufficient. A good deal of judgment is necessary in selecting spots to make stations, which cannot come without experience.

In placing the details on the paper on the rough board, sketch in the line of a valley first by the stream at the bottom, and then the adjacent hills or spurs.

A ten-foot or longer pole may be used with advantage in sketching topography.

Difference of level can be at once obtained from a theodolite angle of elevation or depression by the formula :Diff. of level in feet $=\frac{\text { angle in secs. } \times \text { dist. in miles. }}{34}$

Contouring.

Hills are best shown by contours. We do not of course pretend that our contours are a fixed distance apart, but we must endeavour to draw them approximately so, calling each contour line, 25,50 , or 100 feet apart, as the scale may require, and estimating the height of each spur, with the assistance of a pocket barometer, if we have one, which will give us roughly the height of each station above the sea, if we read it when we land, and whenever we have occasion to do so. Each contour must be continued on from one hill to the other, or until it meets itself again round the hill; and as their number and closeness together will roughly indicate the height of the hill, we must be careful not to get more on one side of a hill than another, or the value of this method will be lost, and the contours will simply show the shape of each spur, without reference to its relation in height, steepness, \&c., to the next one, which is what we want to show as well. These contours will perhaps not appear in the finished chart, in which the mountains may be delineated in a different manner, but they will form an excellent guide for the amount of shade to be put on to the different hills and slopes, and it is the readiest and quickest method of showing this at the time.


Red and blue Pencils.

Red and blue pencils are useful for topography. With the blue we show streams, and the red is used for marking roads. With only a black-lead pencil, the markings of these details are apt to get confused with the contour-lines to express the hills.

Much topography can be done with the pocket sextant and compass only, the latter being only used, however, when three objects to fix by cannot be got. The magnetic meridian, or several magnetic meridians, must be ruled on to the rough board, to permit the use of bearings. When the only objects available are much above us or below us, correct angles cannot be got with the sextant; and though we allow ourselves a certain amount of latitude in our angles for the purpose of topography, it will often be necessary to take a small theodolite for the purpose. A pocket sextant can be taken as well, and the theodolite, which requires more time to set up and arrange, only be used when the sextant angles will be too erroneous.

If we have a theodolite, we must take advantage of good opportunities to get a series of elevations and depressions for heights.

| Diffi- | In taking angles with a sextant to objects on different |
| :--- | :--- |
| culties |  |
| with |  |
| sextant |  |
| levels, try to find some natural mark which is exactly above |  |
| Angles. |  |
| or below, as the case may be, the object the farthest from |  |
| your level, and nearly on a level with the other object, and |  |
| take this instead of the object itself. But it must be noted |  |
| that unless this second object is nearly at the level of the |  |
| observer, the angle will still be incorrect. |  |
| Fig. 34 shows a rough field board, before any shading is |  |
| placed on the sides of the valleys, which will be done with |  |
| the brush before the work is considered completed. |  |

## CHAPTER XI.

## HEIGHTS.

By Theodolite-By Sextant-Obtaining Distance from Elevation of a known Height-Levelling.

For obtaining heights, we must mainly depend on angles of Means elevation with sextant from afloat, and of elevation and used. depression with theodolite from shore stations. The pocket aneroid, though useful, as described under "Topography," to get subsidiary heights, and assist in delineation of hills, is not to be depended upon.

At all main stations, and, in fact, any station well fixed and conveniently placed, angles of elevation and depression to the objects whose heights we want, should be taken Heights. throughout the course of the work. These are entered into the "Height Book," and worked out when we can get the distances, and occasion offers, the results being tabulated and meaned.

Elevations and depressions can be taken from any station whose height we shall eventually know; but it is evident that any slight error in the true height of the observing station will be carried on into all heights deduced from it, and therefore it is well to get as many observations as we can from stations at the water-level, or so placed that the height above the water-level can be measured with a line.

In observing elevations and depressions with a theodolite, Use of the instrument must be in fair adjustment, and carefully ${ }_{\text {lite. }}^{\text {Theo }}$ levelled, and it is further necessary to take into account the errors of level and collimation.

There are two ways of doing this. One is to take a series of observations with the telescope in its ordinary position, and then another with the telescope reversed, end for end, in the Y's, when the mean of these two observations for each object will be the correct amount of elevation or depression. This is the best way, and eliminates all error. It may, however, be sometimes convenient to proceed as follows :-

Ascertain the collimation error by directing the telescope on to an object in elevation, reading the vernier, then turning the telescope round until the level is uppermost, and again adjusting for the object and reading the vernier again. Half the difference between the readings is the collimation error, which, when the reading taken with the level uppermost is greatest, will be added to observations of elevations made with the telescope in its normal position, and subtracted from depressions. This collimation error is permanent for all positions of the horizontal arc.

For level error, at each observation of each separate object, the telescope must be brought horizontal by the level attached to it, and the vernier of the vertical are read. Whatever it reads will be the level error.

The sign of the correction to be applied for this error is, for elevation, + , when the $0^{\circ}$ of the are is above the zero of the vernier when the tube is level, and -, when below. For depressions the signs will be reversed. Care must be taken that no mistakes are made as to these signs. For a tyro it is slightly confusing.

Both level and collimation error must be applied to each observation.

Sextant Elevations.

When the ship can be well fixed, sextant angles of elevation from her with a sea-horizon will be very good, as good, in fact, as elevations with a small theodolite, as they are free from all possible errors of levelling, \&c., and a sextant measures angles to 10 seconds, whereas a small theodolite is only cut to minutes. Even when the ship is within the limits of the sea-horizon, the results will be good, providing the distance of the shore line is well known, and is not
under half-a-mile. By observing from the lowest step of the accommodation-ladder, we can use a shore horizon at even less distances.

Sextant elevations, then, are very useful, but we do not generally get so many opportunities of obtaining series of heights by it, and when at any distance from the land, only the skyline of hills will be clearly seen, so that it is principally to the theodolite that we must look to give us a sufficiency of elevations.

Before dealing with the method of calculation of heights, Refracwe must refer to the effects of refraction.

The apparent position of one object from another, as seen through our atmosphere, appears higher, whether we look up or down. The amount varies with the difference of densities of the various strata of the air, which are constantly changing.

All we can do is to take the mean refraction, and it has been found by experiment, that by taking $\frac{1}{12}$ of the distance, regarded as minutes and seconds of arc, and applying this to the observed angle of elevation, it will give us a fair mean result for the true angle of elevation, when this is small, as in all practical cases it is. It follows from this unknown amount of error in the coefficient of refraction that, when possible, objects should not be observed for elevation or depression at more than a few miles distance. We cannot always command the maintenance of this limit, any more than we can many other theoretical points in practical hydrographical work, but when circumstances are favourable, they must be regarded.

Looking upwards, or from a denser into a rarer medium, the effect of refraction is to increase the apparent elevation. This correction is therefore to be subtracted from elevations. As the effect, when looking downwards, is also to raise the object, or, in other words, to decrease the angle of depression, the correction for refraction must be added to angles of depression.

The angle of elevation measured by a theodolite, or the

Result of Spherical Form of the Earth.
sextant angle when corrected for height of eye above the sea, is the angle between the tangent to the earth's surface at the observer's position, and the line drawn from him to the object. If the surface of the earth was a plane, all that would be necessary to obtain the height would be to work out in a right-angle triangle, Perp. $=$ base $\times$ Tan angle of elevation, after the latter had been first corrected for the effects of refraction; but as the earth is a sphere, the tangent to it, produced, will cut the line representing the height we want, not at the point where it leaves the earth, but somewhere above that, depending upon the distance ; the perpen-

dicular, therefore, as worked out, will only give us a portion of the height required, the other portion being that below the tangent.

Explanation of "Dip."

Thus, in Fig. 35, A is the position of the observer, A H the tangent to the earth's surface at his position, B a mountain peak whose height, B D, we want to obtain. The angle of elevation measured by a theodolite is BAH , and it is evident that the height we shall obtain by working out the triangle will be BH , leaving H D to be found independently. It will be seen that we are going to treat the angle BH A as if it was a right angle, when it is evidently
more than $90^{\circ}$ by the angle D C A at the centre of the earth; but our figure is much exaggerated to show things clearly, and in practice the distances we use to get elevations are so insignificant, comparatively to the diameter of the earth, and consequently the angle D C A so small, that we can neglect this quantity without introducing any error in the result. With a distance of 60 miles, when the angle is a degree, the discrepancy introduced into a height of 6000 feet is only 2 feet.

We require, then, to get H D to add on to B H in order to get the full height B D. This quantity, H D, is called "dip," an awkward nomenclature, as it is the same used at sea to express the angular quantity we apply to elevations taken with a sextant from a height, to reduce them to the tangent to the earth, whereas here it is used to express a linear quantity.

The problem can be solved in two ways. Either by finding H, D independently, or by adding the angle H A D to H A B to get the angle BA D , when a right-angled triangle gives us B D.

The latter method is the shorter and is now employed, and the former is therefore not described, but a table giving H D , the Dip, or the height of the part of the object observed obscured by the horizon, is given in Appendix Table O, as it may be sometimes useful to know how much of a mountain is below the horizon.

In the method now used, the angle H A D is found as follows :-

Angle H A B is the elevation, corrected for refraction, and the angle H A D (between the chord and tangent) is equal to half the angle at the centre, i.e., half the distance in arc.

Suppose A B to be 60 miles,

$$
\text { Then H A D }=+30^{\prime}
$$

Correction for refraction ( $\frac{1}{12}$ of the distance $=-5^{\prime}$

- to elevation, + to depression).

Whole correction $=+25^{\prime}$

Chart of

|  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \dot{y} \\ & 0 \\ & 0 \end{aligned}$ |  |  |  |  |
| $\begin{aligned} & \text { H } \\ & \text { s } \\ & 0 \\ & 0 \end{aligned}$ |  |  |  |  |
|  |  |  <br> 20 <br> $\stackrel{8}{0}$ <br>  |  |  |

Consequently, as $60^{\prime}: 25^{\prime}$
or $1^{\prime}: 25^{\prime \prime}$ is a constant proportion for all angles.
Therefore the total correction, for Dip and Refraction, in seconds of arc, to the observed angle of elevation or depression is :-

## Distance in sea miles $\times 100$

## 4.

This correction is to be added to angles of elevation, and subtracted from angles of depression.

## Height

 Form bound into Book.Entering and calculating Elevations.

A ruled form is supplied by the Hydrographic Office, which much facilitates the calculation of heights. This form, bound into a book, constitutes the Height Book. A specimen is given on last page, which nearly speaks for itself.
The angle observed to the object is entered under the head of either elevations or depressions, as the case may be; as observed, in the case of theodolite; minus the correction for height of eye, if with the sextant; and the distance in miles and decimals is entered under its head.

To get this distance, if we happen to have it worked out in the triangulation, we shall of course use the calculated distance; but if not, which will be the case generally, we must measure it on our sheet, and enter the corresponding distance according to the scale.

In the column headed Corr ${ }^{\mathrm{n}}$, enter the correction for Dip and Refraction, obtained as above by multiplying the distance by 100 and dividing by 4 .

We then work out the difference of height with these data on the opposite side of the page, the constant $\log$ being the $\log$ of feet in a mile, by which the distance must be multiplied to bring result out in feet. The $\log$ given is that for 6075 feet, the number of feet in a mile in Lat. $44^{\circ}$. Theoretically we should have a different $\log$ for different latitudes, but, as the utmost extent of error by neglect of this is 22 feet in a height of 6000 feet, we need not regard it. This difference of height is entered in its proper column.

Tables, computed by Commander Purey Cust, for taking
out the difference of height for any angle and distance, are now supplied. If these are at hand this computation is dispensed with.

The column for height of theodolite is a little confusing, as sometimes it will be merely the height of the theodolitetelescope above the ground, and sometimes the height of it above the sea-level, which we shall enter, according as we want the height of observer's position, or of object observed as will be presently explained.

We have now all the data necessary to obtain heights.
When we have accumulated enough observations, we set about getting out results.

There are four problems for obtaining heights, and the data Height we have for each observation will be combined according to what we want to arrive at.

These four problems are as follows :-
1st. To find height of object observed, when height of observer is known, and the angle is one of elevation.
2nd. Ditto, when angle is one of depression.
3rd. To find height of observer, when height of object observed is known, and the angle is one of elevation.
4th. Ditto, when angle is one of depression.
To understand the mode of combining the data, let us consider the Figs. 36 and 37.

In Fig. 36, which is the case where the angle observed is of elevation, and comprises Problems 1 and 3, we may have either X or Y known, and wish to obtain the other.

Suppose X to be known (Problem 1), to find Y, we have

$$
\mathrm{Y}=\mathrm{X}+(t+h) \ldots \ldots
$$

Height Formulæ.

If Y is known (Problem 3), to find X ,

$$
\mathrm{X}=\mathrm{Y}-(t+h) \ldots 3
$$

In Fig. 37, the case where the observed angle is one of depression, and comprises Problems 2 and 4,

Suppose X known (Problem 2), to find Y ,

$$
\mathrm{Y}=(\mathrm{X}+t)-h \ldots . . . .
$$

Suppose Y known (Problem 4), to find X,

$$
\mathrm{X}=\mathrm{Y}+h-t \ldots . . . .
$$

These four formulæ, which it is also convenient to have written for reference in the Height Book, will enable us to solve any of the problems.

When we are getting the height of the Object observed, we shall enter in the column of "Height of theodolite," X $+t$, or the height of theodolite above the sea; but when the observation is used to obtain the height of Observer, only $t$, the height of the theodolite above the ground, will be inserted.
Elevations from Sealevel first Meaned.

Column
Height of Theod.

$\mathrm{X}=$ Height of observer's position. $\quad h=$ Difference of height.
$\mathrm{Y}=$ Height of observed position. $\quad t=$ Heighto of theodoilite above ground.
from stations at the sea-level, or from stations whose height above the sea-level has been measured, which will give us the heights of objects observed; and also with depressions from stations to the sea-level, or to stations whose height above sea-level has been measured; which will give us the height of the observing-stations.

## Absolute Heights.

Heights so obtained are termed " absolute," as being calculated directly from the sea-level.

All such heights must be obtained first, then, meaning the heights of one station which has the most observations, or of which the results agree best, we can work out all other
observations from that station to other objects. We then mean another, and so on, using our observations either to obtain height of observer or of observed object, as is most convenient, as we proceed.

These heights will be "dependent," as resting on the Dependent ascertained height of other stations.

No height can be considered as exact, that is not the result of both elevations and depressions, as no matter how nicely a set, of depressions say, comes out, they will all include the refraction error, for the refraction correction is only approximate. This is with reference to detailed surveys only.

Sextant angles of elevation must be corrected for the sextant height of eye before being entered in the Height Book as ${ }_{\text {tions. }}^{\text {Eleva- }}$ angle observed. They are then treated in precisely the same manner as the theodolite elevations.

The pocket aneroids should be tested up a known height, Aneroids. to get the value of each tenth, which will be from 92 to 100 feet for a tenth, each instrument varying slightly. As before mentioned, they are useless in getting accurate heights, but will give very good approximations up to about 4000 feet, if in good order and constantly worked; but their delicate chain-work is so liable to rust slightly at sea, that the links will frequently stick if the instrument is not carried up heights continually to work it. Placing under an air-pump will serve the same purpose. See Barometer, page 35.

It is useless to enter into intricate calculations of data only for obtained by so small a scaled instrument as a pocket aneroid; approxithe impossibility of reading it exactly precludes any but Heights. approximate results, and a simple multiplication of the decimals of inches by the value of a tenth, as obtained above, is quite sufficient for the purposes for which we use the instrument, the differences being taken from the barometer as observed on board.

To obtain distance from an angle of elevation of a known Obtaining height is like using a lever with the ends reversed, and is Distance seldom had recourse to in surveying, as not being correct vation of enough.
known
Height

As it may be, however, sometimes useful, we give a formula.
$\left.\begin{array}{l}\text { Distance in } \\ \text { nautical miles }\end{array}\right\}=\frac{34 h}{E}$
When $\quad \hbar$ is height of mountain in feet
$E$ is the angle of elevation in seconds, reduced to water-level, and corrected by the addition of the dip, as explained in the rules for obtaining heights.
The same formula in rougher terms is
$\left.\begin{array}{l}\text { Distance in } \\ \text { nautical miles }\end{array}\right\}=\frac{100 \mathrm{~h}}{3 E}$
If the estimated distance should differ much from that given by the calculation, it should be recalculated with the correct allowance for dip and refraction.

An example is appended.


The rougher formula will give the distance as $29 \cdot 4$ miles.

## LEVELLING.

Simple Levelling with a staff is not very much required in marine Levelling. surveying. Ascertaining the height above the sea of the fixed mark used for reference for the tidal datum is the purpose for which it is most used, but it is also required to find the height of the base of a lighthouse, \&c.

This is called simple levelling, and gives us the height
between the two required points only, without any regard to distance.

A levelling staff and level is usually supplied to surveying ships; but a theodolite and marked boat-hook or pole will answer the purpose, if we have not got the regular apparatus.

Holding the staff at high-water mark, we place the instrument (level or theodolite) so far up the slope that we shall when it is carefully levelled by the level attached to the telescope, read off near the top of the staff. The reading of this, called the back station, being taken, the staff is taken above us, and planted so that we can read just above zero of the staff, which is now at the fore station. The theodolite is now moved up the hill until we shall again, when levelled, read near the top of the staff; this will be another observation of back station, and so on until the levelled telescope reads the staff on the spot whose height we want.

There is no necessity to keep in one line directly for the spot whose height we wish to measure; we shall do so if we can, as it is the shortest way, but in practice we are generally forced to zigzag.

The difference of the sums of the readings of back and fore stations will be the height required. Where former are the greatest, we are going up hill, and it is called rise ; vice vers $\hat{a}$ is called a fall.

For Height of - Light-house.

| Back - |  |  |  | Fore 4 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Reading of Staff. |  |  |  |  | ing | Staff |  |
| Water-level | . |  | $12 \cdot 64$ | (1) |  |  | $0 \cdot 63$ |
| (1) .. | .. |  | $13 \cdot 42$ | (2) |  |  | $1 \cdot 22$ |
| (2) .. |  |  | $13 \cdot 81$ | (3) |  |  | $1 \cdot 52$ |
| (3) |  |  | $12 \cdot 50$ | (4) |  |  | $0 \cdot 32$ |
| (4) |  |  | 13.06 | (5) |  |  | 0.87 |
|  |  |  | $12 \cdot 18$ | Base |  | .. | $3 \cdot 45$ |
|  |  |  | $\begin{array}{r} 77 \cdot 61 \\ 8.01 \end{array}$ |  |  |  | 8.01 |
| $69 \cdot 60$ |  |  |  |  |  |  |  |
| Height of base of Light-house above High-water level |  |  |  |  |  | .. $69 \cdot 6 \mathrm{ft}$. |  |

## Corrections, when necessary.

For our purposes, when the distance of the staff from the theodolite is not great, or when the distance of fore and back station from the theodolite are nearly the same, it will be sufficient to observe readings with the telescope in one position only; but when the rise of the hill is slight and distances increase, especially when the difference of distance between fore and back station is great, and we require accuracy, the telescope should be reversed in the Y's, and being again brought level by the bubble, readings should be taken a second time. The mean will be the true reading.

If the axis of the telescope and the attached level are perfectly parallel, and therefore in adjustment, it will be shown by the readings agreeing when reversed at the first station, and we shall know that we need not take this trouble; but it is necessary to ascertain this, as theodolites continually undergoing carriage by boat are liable to many accidents.

This method enables us, if necessary, to calculate the height of any of the stations where the pole is erected, but gives us no information as to the height of the spots where the theodolite stands. This can be obtained, if wished, by measuring the height of the axis of the telescope above the ground, when,

Height of theodolite position $=$ height of back station + present reading of said back station - height of eye (back station being below us).

Distances measured will enable us to make a section of the ground traversed, but, as already remarked, this is not often required from the marine surveyor, and will not be enlarged upon here.

## CHAPTER XII.

## OBSERVATIONS FOR LATITUDE.

By Circum-meridian Altitudes of Stars-By Circum-meridian Altitudes of Sun.

Astronomical observations are largely used in all descrip- General tions of marine surveying. In all but small plans the even- Remarks. tual scale of the chart is decided by the latitude and longitude as obtained by observations of sun or stars, and we have seen that true bearings often enter largely into the construction of charts. In running surveys, or in searching for, or sounding over shoals, in mid-ocean, everything depends on the positions astronomically found, and every method of correctly finding the latitude and longitude is in requisition. In considering this subject, we will take first shore observations with artificial horizons, where we require results as accurate as we can obtain with the sextant, to which instrument remarks will be confined, excepting so far as the theodolite is used for true bearings; and afterwards, sea observations.

In all observations of the heavenly bodies, instrumental Eliminaerrors, atmospheric effects, and personal differences, largely $\frac{\text { tion of }}{\text { Errors. }}$ influence the results. No matter how correctly we may take the actual observations, unless we can eliminate these variable quantities, the positions obtained will be in error.

On every occasion, therefore, where accuracy is aimed at, the mode in which this elimination can be best carried out must be considered. The general principle used in doing this is to get two sets of observations for one result, in such a manner that the errors of all kinds will act in opposite
directions in each set, and therefore disappear when the mean is taken. The precise way in which this is done will be described under each different observation.

## LATITUDE BY CIRCUM-MERIDIAN ALTITUDES OF STARS.

Latitude by Observations "Absolute."

But more difficult.

Determinations of latitude are more simple in one respect than those for longitude, as they are "absolute," that is to say, they depend solely upon themselves; whereas longitude has to be obtained by the difference of two sets of observations at two different places, and is further complicated by the eccentricities of the chronometers upon which, when there is no telegraph, we have to rely.

But, on the other hand, the observations required for correct latitude are more difficuit to take, as, to arrive at anything like an exact result, we must use stars, and each step of the observation of these in an artificial horizon is rendered less easy by the fact of their being made at night. It is much easier to become a good day observer than a good night one.
Elimiñtion of Errors in observations for Latitude.

The errors to be eliminated as far as possible in observing for latitude are, firstly, errors of observation; secondly, instrumental errors, as centering error, index error of sextant, error caused by refraction in the rays passing through the: glasses of the roof of the horizon, \&c.; thirdly, atmospheric refraction, which varies much, and for which no known rule of correction thoroughly suffices ; fourthly, personal errors, caused by each individual's mode of observing the contacts.

Errors of observation are eliminated by taking as many observations of altitude as we can, and we must therefore observe off the meridian, or what are known as circummeridian altitudes ; which consist in observing from a short time before the meridian passage to a short time after it, and adding a certain correction to each altitude to make it equal to the meridian altitude, and thus get a mean meridian
altitude, which, if we can calculate the correction exactly, will be of much more value than actual observation on the meridian only.

There remain the other errors, some of which may be directly allowed for, but only approximately; others cannot be corrected at all, and the latitude resulting from observations of a single body, as e.g. the sun, will be therefore always more or less in error.

The only way satisfactorily to clear these errors is to observe stars, in pairs, of as equal altitude as can be found, one Pairs of Stars. north, and one south, of the zenith. These errors will then act in opposite directions, as everything tending to increase or diminish the altitude on one side of the zenith, will act similarly on the other ; but, in working out the latitude, the resulting error will increase the latitude in one case, and decrease it in the other, so that the mean of the latitudes obtained by each star of such a pair will approximate very closely to the correct one.

To eliminate the artificial horizon roof error when observing pairs of stars, the roof must always be in the same position with respect to the observer, and therefore must be reversed when changing from face north to face south, and vice versâ. If observing a single object, as the sun, the roof must be reversed when halfway through the observation.

The use of a sextant stand, when once the observer has got sextant thoroughly accustomed to it, is an immense assistance to good Stand. observations, as the images of the stars, instead of quavering and shaking with every slight motion of the hand of the observer, remain perfectly still, and can be made to pass over one another with great accuracy.

Certain preparations are necessary for good star observa- Preparations, for all scurry that can be avoided, should be.

In the first place stars must be selected and arranged for observations according to their pairs.

If stars given in the 'Nautical Almanac' only, are used, the chances are very much against a sufficient number of pairs being obtainable, as only a small proportion of observable
stars are there included, though the number has been lately much increased.

## Star Cata- A surveying vessel will have the Greenwich and Cape logues.

 Observatory Catalogues of Stars, and out of these enough pairs can nearly always be picked to enable us to get a satisfactory latitude in one night, including stars down to the fourth magnitude, which can be easily observed on an average night by a practised observer with good instruments.A special list of stars is supplied to all surveying ships, which includes all stars in the Greenwich and Cape Catalogues, and also a diagram with an ingenious method of pairing stars.*

## Choosing Stars.

The approximate altitude of each star must be calculated and inserted in a list in the angle book, together with the time of its meridian passage, its magnitude, the time that will be shown by the pocket chronometer that is to be used for taking time, whether it is N. or S. of the zenith; and each pair must be numbered. The nearer together in point of time the two stars of a pair can be placed, the greater will be the chances of the elimination of the refraction errors, as in a few hours temperature often varies much, dews form, and many differences may arise in the atmospherical conditions.

Stars over $60^{\circ}$ of altitude are not usually good to observe; as, though a sextant will measure over $120^{\circ}$, the image of the star will not be sharp when reflected from the index glass at such a large angle, unless the glass be unusually good.

Altitudes of stars selected for pairs should not differ more than four degrees if possible. Generally pairs within this limit can be found.

If one star of a pair be lost, it is useless taking the other, unless a substitute for the one lost can be had. It is well, therefore, to be provided with spare stars for pairs, as this may often happen from clouds intervening, \&c.

Care must be taken, in choosing pairs, to leave sufficient

[^17]time between each meridian passage for the due observation of each star before and after culmination.

This will vary with the latitude and declination, as a star should not be observed so far from the meridian as to bring the mean of the altitudes observed less than a minute or so under the meridian altitude. Fifteen minutes, elapsing between each passage, will give plenty of time under most circumstances.

Time must also be allowed for changing the position from north to south, and vice versâ; but all this will vary with the quickness and experience of the observer. Beginners must be satisfied with a few stars, and must allow more time.

In preparing the ground we must look out for a spot Preparing whence we can see clear in the line of the meridian north ground. and south, and one far enough from the beach to be beyond the distance where surf will shake our quicksilver. The latter point is sometimes, as for instance where jungle comes down to the very beach, difficult to find, but it is well worth looking for, and going inland a bit to get it; as otherwise good observations may be rendered impossible from the vibration set up. The more solid the ground the better, as it is astonishing what slight causes will suffice to set the surface of the mercury in motion. The use of the amalgamated trough, mentioned on page 13, will, however, enable observations to be obtained when impossible with the older form of horizon.

Wind is a frequent source of quaking mercury, and care should be taken to have the horizon trough firmly placed, and the roof so fitted that the wind cannot get under its lower edge.*

A screen of canvas to windward is sometimes a good thing, but on some ground this causes such vibration of the earth as to be worse than the free blast of the wind.

If more than one officer is to observe, a screen of some kind

[^18]should be put up north and south between the observers to keep the lights out of one another's eyes.

In the tropics, and some other localities, mosquitoes must. not be forgotten. In places where these plagues abound, it is preferable to court the wind instead of shutting it out, in order to free ourselves if possible of them. Sandflies are perhaps worse, as nothing will get rid of them, and many an otherwise favourable opportunity of getting stars has been spoilt by these wretched little insects.

The spot for the artificial horizon being settled on, and the direction of the meridian taken with a compass, it is a good plan to dig holes, if the nature of the soil permits, in which to place the lantern used for reading off when not required, so as to avoid unnecessary glare.* The best place for these will be on the left side of, and a little behind the observer's seat, and two will be wanted for each observer, one for the north stars' position, the other when facing south. If the ground will not admit of digging holes, buckets will answer the same. purpose well enough, but not so well.

If special lanterns can be got, these precautions will not be necessary ; but we are assuming observations with ordinary ships' lanterns.

All these kinds of preparations should be made before sunset, if possible ; confusion will be sure to occur if things are delayed till after dark.

For observations with a sextant stand, a small stool is wanted, as described under "Sextant Stand," and another for the observer's seat. It much facilitates good observations for the observer to be comfortable, especially when he is about to observe for several hours consecutively.
Star Mar. A good star map is very useful to assist in recognising the objects chosen.

Error of chronometer require.

It will have been necessary to obtain the error of the chronometer on the day of our star observations (by single

[^19]altitude is sufficient), unless we have recently obtained error at the same place, and have confidence that the chronometers are going sufficiently well to give us the true time of place to, say, two seconds.

Having thus made all preparations, and compared the pocket watches with the standard chronometer, and another as a check, we land to observe. We may here remark that we must again compare on returning on board.

Placing the artificial horizon north and south, we put on Observing the roof, with the mark on it in the settled direction, according as our first star is north or south of the zenith.

We then place the sextant on its stand, having first screwed in the inverting tube with the weakest-powered eyepiece. This should be adjusted to focus, as near as possible, before screwing into the collar.

Place the sextant and stand on the stool, so that one of the three legs which support the stand is at right angles to the meridian, and on the right of the observer.

Set the vernier to a few minutes less than the estimated double altitude of the star.

Move the stool with the instrument on it, so that, looking over the tube, we can see the reflection of our star in the artificial horizon.

Point the telescope at this, and set taut the screw that fixes the handle of the sextant on to the bearing of the stand, then working the sextant right and left in the stand pivots, the other image of the star will soon dash across the field.

Unless the star is moving rapidly in altitude there will be no further need to move the sextant on its bearing, and care should be taken that the screw is tight enough to prevent its moving while turning the sextant up to read, or the telescope will not point to the star, when directed to the horizon again, which it should do at once, and so save time in redirecting. Beginners often neglect this, failing to see the necessity for it; and, losing time in looking again for the star after each reading, miss one of the great advantages of a stand, viz. that
once fixed, the stars will always be in the field without any bother.

With faint stars, when there are several of nearly the same brightness close together, it is sometimes rather confusing, and a beginner will find it very difficult to be sure of his star; but comparison with the star atlas, and consideration of how the star wanted lies with regard to the others, will, after a little experience, clear up the difficulty.

If the star be a faint one, it is difficult to bring it down to meet its image in the horizon by taking the sextant off its stand; and if it is a bright one, there is no need to do so, as it cannot be mistaken if the estimated altitude is anywhere near the truth. There is then, in regularly pre-arranged observations, no necessity for doing this, and we shall trust entirely to the vernier being set to the calculated altitude for finding the star.*

Having brought the two images into proximity by hand, place the right hand on the screw at the end of the stand-leg that has been arranged at right angles, and the left hand on the tangent screw of the sextant, when, by working these two screws, the images of the stars can be made to pass over one another exactly, and the word "stop" given.

At this signal the attendant bluejacket will hold the lantern up for reading off. The light should be thrown on to the are from a direction as nearly at right angles to the plane of the sextant as possible, to avoid parallax.

## Resetting

 for each observation.In taking the next observation, turn the tangent screw on or back alternately, before commencing to bring the images together again, so as to be entirely free from bias as to whether the star is rising or falling.
Time from Meridian.

The amount of time to observe before and after culmination varies with the position of the star and the latitude of the place as before mentioned; but, as a rule, commencing six

[^20]minutes before the calculated time of meridian passage, and continuing for a like time afterwards, will be ample, as we do not wish the correction eventually to be applied to the mean of the observed altitudes to bring it up to the meridian altitude to be more than one minute, if we can manage it.

Observing as close to the meridian as is recommended above, the decimal parts of seconds need not be recorded in taking the time.

The observed altitude of a heavenly body can be corrected
Decimals unnecessary. to the meridian when the hour angle and the latitude are eduction to Meriknown; but if the hour angle is large, the calculation of the quantity to be added to the altitude is a complicated process, and it is only when the hour angle is small, and very little in error, that the formula assumes a simple and practical form. The error introduced by working with an assumed latitude also increases rapidly with the hour angle, so that we are confined in using this method to about twenty minutes of the time of meridian passage in ordinary latitudes; but in observations such as we are discussing now, which have for their object as correct a determination of the latitude as we can obtain, we must not observe more than about ten minutes from the meridian, and perhaps less.

The best method to use in reducing observations to the meridian is that known as Raper's, in which the principle is to add on to the observed altitude the amount necessary to make it equal to the meridian altitude, and then to calculate the latitude as in a meridian observation.

This amount is known as the "Reduction to the Meridian," and is so called because it is subtractive from the observed zenith distance.

The formula used by Raper is *-


Raper gives a table of $\frac{\text { Vers H A }}{\operatorname{Sin} 1^{\prime \prime}}$ for every minute and

[^21]second of hour angle up to thirty minutes from the meridian, which is very convenient, and is given in Appendix M. It saves a considerable amount of figures in the calculation, and thereby diminishes the chances of clerical errors ; but the formula, as given above, can be worked out, if Raper's table is not at hand.

## Sidereal

 Hour Angle.The hour angle as marked by a watch beating mean time, or nearly mean time, will not be strictly correct either for a star or the sun; as, for the former, it should be taken by a watch beating sidereal time, which gets over its 24 hours while a mean solar watch has only advanced $23^{\mathrm{h}} 56^{\mathrm{m}} 04^{\mathrm{s}}$ nearly ; and apparent solar time varies from day to day as the speed of the earth in her orbit varies; but, within the limits we observe, the difference in ordinary latitudes is scarcely perceptible, and if we observe the stars of a pair about the same distance from the meridian, any little discrepancy will disappear in the mean.

If it be desired to correct for the difference when observing stars, a constant $\log$ of 0.002000 added to the other parts of the equation will give a close approximation to the exact reduction.

Calculating the Reduction.

In working out the circum-meridian observations of a star, it is not necessary to calculate the reduction for each individual observation. As $\frac{\text { Vers H A }}{\text { Sin } 1^{\prime \prime}}$ is the only variable quantity in the equation, and as it varies with the hour angle, by taking this out for each observation and meaning these quantities, we obtain a mean value of $\frac{\text { Vers H A }}{\operatorname{Sin} 1^{\prime \prime}}$ which we insert in the equation, and add the mean reduction so found, to the mean of the altitudes. If working with Raper's tables, we take out the whole quantity $\frac{\text { Vers H A }}{\operatorname{Sin} 1^{\prime \prime}}$; if without them, we look out the Vers H A only, and introduce the $\log \operatorname{Sin} 1^{\prime \prime}$ into the calculation with the other logarithms.

Knowing then the error of the watch used on mean time of place, and the approximate latitude and longitude, the rule for
working out circum-meridian observations of stars will stand thus:

1. Calculate Greenwich date.

Rule for
2. Correct right ascension of mean sun (sidereal time of calcula"Nautical Almanac') for this Greenwich date, and subtract it duction. from the right ascension of the star, which will give the mean time of star's meridian passage.
3. Apply to this the error of the watch, which will give the time shown by the watch at the star's meridian passage.
4. Mean the observed altitudes, correct this mean for index error, and divide it by two. To this apply refraction (corrected for thermometer and barometer), which will give true altitude.

5 . Write down the times of each observation in a column, and taking the difference between each, and the time shown by the watch at meridian passage, we get the hour angle at each observation, which place in another parallel column.
6. Take out for each of these hour angles the quantity from Raper's Reduction Table, or, if we have not got that, the natural versine.
7. Add these quantities together, and divide the sum by the number of observations, to get a mean.
8. Add together $\log$ cosine declination, $\log$ cosine estimated latitude, $\log$ secant true altitude, and the logarithm of the result of No. 7. If we are using versines, a constant $\log$ $9 \cdot 316400$ is also to be added. (This is $\log$ cosec $1^{\prime \prime}+4$ $+0 \cdot 002000$.)
9. Look out the sum of these logs as a natural number, which will be the number of seconds of reduction required.
10. Add this to the mean observed true altitude, which will give the calculated meridian altitude, from which the latitude is obtained in the usual manner.

The following is given as an example.

On July 11th, 1879. At Buyuk Chekmejeh © the following observations were taken. Index error of sextant-35" Latitude (approx.) $40^{\circ} 57^{\prime} 45^{\prime \prime} \mathrm{N}$. Longitude $28^{\circ} 30^{\prime} \mathrm{E}$. Mean Time of Transit of star calculated, Xh. 12 m .


The calculation will appear as follows :-

| Watch Times. | Hour Angle. | $\frac{\text { Vers H. A. }}{\operatorname{Sin} .1^{\prime \prime}}$ |
| :---: | :---: | :---: |
| $\begin{array}{ccc}\text { h. } & \mathrm{m} . & \mathrm{s} . \\ 10 & 09 & 36\end{array}$ | $\begin{array}{ll}\text { m. } & \text { s. } \\ 4 & 27\end{array}$ | $38 \cdot 9$ |
| 1011 | $3 \quad 52$ | $29 \cdot 4$ |
| $10 \quad 42$ | $3 \quad 21$ | $22 \cdot 0$ |
| $11 \quad 14$ | $2 \quad 49$ | $15 \cdot 6$ |
| 1143 | 220 | $10 \cdot 7$ |
| $12 \quad 09$ | 154 | $7 \cdot 1$ |
| 1250 | $1 \quad 13$ | $2 \cdot 9$ |
| 1314 | 049 | $1 \cdot 3$ |
| 13 38 | 025 | $0 \cdot 3$ |
| 1406 | $0 \quad 03$ | $0 \cdot 0$ |
| 1433 | 030 | $0 \cdot 5$ |
| 1455 | $0 \quad 42$ | $1 \cdot 0$ |




When the pole star is observed, it must be worked out by Pole Star. the rule given in the Nautical Almanac, care being taken to take out all the quantities from the tables exactly by interpolation.

Moon not convenient.

The moon is but of little use for observations of any kind. Its rapid motion necessitates very careful corrections, which take more time than they are worth, and besides we have nothing to put against it to eliminate errors.
Results of The separate stars being worked out, we mean the result of
Pairs. each pair, and the mean of these again will give us the mean latitude.

Although from circumstances many more observations may be got of one star of a pair than of the other, no value can be assigned to one over the other, and the direct mean must be taken; but in meaning up the results of pairs, less value would be given to a pair in which the observations of one star are few in number, than to a pair where a proper number of observations of each star has been obtained. The necessity for assigning this value is increased where the observations are not only few but indifferent; but it would be a question whether it would not be better to omit such a pair from the final result altogether, and certainly it would be best to do so, if there are several other good pairs.

An example of the method of tabulating the different observations and pairs is given on next page.

The sets of stars given as an example were taken under favourable circumstances of sky and weather, and are not meant as a standard which we must expect always to get. Here no value is given, as each pair were nearly equally good, and the observations were nearly the same as to number.

It is, however, evident on an inspection of this set of observations, that the sextant had a centering error. It will be remarked that the latitude obtained from south stars diminishes as the altitude increases, and vice versa with the north stars. This is wholly attributable to the centering error, and from these observations a very fair idea of that error for each altitude may be obtained, as explained on p. 7. It is therefore well, though the result will not be affected, to apply the centering errors if known.

In the example given only the direct mean was taken, though the second pair was open to suspicion, the seconds

of both north and south stars being somewhat out of the order observable in the remainder.

## Valuing Results.

If, however, the observations had been less uniform, and there were not sufficient pairs manifestly better than others, by which we could elect to stand, value would be assigned by some such system as the following.

Assume a number as perfection, say 10, and give to each pair its value in that scale. Then the sum of the products of the value and the seconds of latitude by each pair, divided by the sum of the values, will give the mean seconds of latitude. The values should not be given by the results, or we shall arrive at pretty much the same conclusion as we should by assuming the latitude directly, but by the circumstances of observation of each star, and the number of observations, remembering that reasons to doubt one star will as equally affect the pair, as if both stars were bad.

Values must also be given when meaning the results by different observers; but here again, if one observer is admittedly the best and his observations are also good, a more probable result will be obtained by ignoring the latitudes of the others altogether. They will not be totally lost, as experience in good observing is only to be obtained by plenty of practice, and a young observer is more likely to take pains when there is a chance, if his observations turn out well, of having them included in the operations of the survey, than when merely observing for what one may term barren practice.

This question of giving values is a very difficult one, and many experienced observers prefer to take a direct mean both for the result by each individual, and when finally meaning the different observers' results, omitting those altogether which seem least worthy of trust, and giving an equal value to everything else. It is very much a case of judgment under the circumstances ; on some occasions one system will be best, and at others the other, and we must so leave it, simply remarking that whenever observations vary enough to make the consideration of this question necessary, the final result must always be regarded wi:h suspicion.



An example is given on the previous page of latitude arrived at by giving values.
Stars In observing stars under the pole, we must not forget that
under Pole. Planets. the reduction will be subtractive from the observed altitude.

The larger planets are not good for observation, as they are so much bigger and brighter than the point which a star shows. On occasions, however, they must be used. The R.A. and Dec. must be calculated exactly.

In using stars from the Greenwich or Cape Catalogues, it is necessary to calculate the apparent place of the star for the day. The method of doing this is given in the Nautical Almanac, in the explanation under the head of "Stars," where examples of a north and of a south star are shown worked out. Care must be taken to give the proper signs + or - to each logarithm.

Next to the observation of stars in pairs, the circum-

Latitude by Circummeridian Altitudes of Sun. meridian observation of the sun in the artificial horizon is the most correct and simple method we have of obtaining latitude ; but it is evident that we cannot use it when the altitude exceeds $65^{\circ}$, as a sextant will not measure the double angle. We must, in the case of the sun, be doubly careful in correcting the refraction, if we wish to get as near the truth as possible. There is nothing to be gained by observing both limbs of the sun, as the motion in altitude will
be so small that it will not matter whether the images are opening or closing.

The roof of the horizon should be reversed at about noon and the sights worked out as two sets, roof one way and roof the other.

However careful we may be, we shall not expect our latitude by the sun only to be exact, and in many cases where we are going to be satisfied with this observation, it will not matter if the latitude be a quarter of a mile or so in error, and the reversal of the roof may often be dispensed with.

If however we know our centering error, and can depend upon the sextant, we can by its application get a vastly improved result, and none of these precautions should be omitted.

An observation of the sun cannot be meaned with an observation of a star the other side of the zenith, as all refraction errors, as well as errors introduced into the instrument by the heat of the sun, will be entirely different.

Circum-meridian observations of the sun are worked out in sun and Stars can not be paired. precisely the same manner as those of a star, the only difference being in the ordinary corrections to declination, \&c. We want the error of the watch on apparent time to calculate what it shows at apparent noon, the time the sun will be on the meridian.

An example is given on next page.
On November 15th, 1876, circum-meridian altitudes of the © were observed with artifical horizon at Maghabiyeh Id. Approx. lat. $18^{\circ} 15^{\prime}$ N., long. $40^{\circ} 44^{\prime}$ E. Bar. $30 \cdot 00$ ins. Ther. $80^{\circ}$. Mean of observed altitudes Sun's Upper Limb, $106^{\circ} 47^{\prime} 09^{\prime \prime} \cdot 1$.

There is a source of error in obtaining an accurate latitude Local which must not be forgotten, especially when the scale of a $a$ tion. chart depends on a difference of latitude, and that is the local attraction due to the irregular disposition of masses of land in the vicinity of an observation spot. A mountain mass on one side, or deep sea, will cause the local direction

Latitude by Altitude of Sun.

of gravity to slightly diverge from the vertical. The surface of the mercury will not in such a case be truly horizontal, and the altitude will be in error. This may often largely account for differences between triangulation and observation, and when the former is good, it may be in certain cases desirable to rest on it rather than the scale by observations. Formulæ have been drawn up for correction, but they rest largely on assumptions of mass which cannot be verified.

This attraction will also affect determination of time and therefore longitude.

## CHAPTER XIII.

## OBSERVATIONS FOR ERROR OF CHRONOMETER.

General remarks on obtaining Longitude-Error by Equal AltitudesError by two Sta:s at Equal Altitudes.

Shadwell's The whole question of obtaining longitude by means of
"Notes on Chronometers."

## Absolute Longitudes.

chronometers is so ably and exhaustively treated by Captain Shadwell in his "Notes on the Mauagement of Chronometers," both as regards the treatment of the watches, the method of observation, and the various systems of obtaining meridian distances, that we refer the reader to that work for full information on the subject. Here we do not pretend to give more than the broader principles of the general question, but a work of this kind, intended for the perusal of young surveyors, would be incomplete without some reference to it.
The methods of obtaining longitude, called "absolute methods," which give the longitude of the place as measured from the first meridian, directly and independently, such as observations of occultations of the stars by the moon, moon culminating stars, eclipses of Jupiter satellites, \&c., are now rarely employed in nautical surveying, and may be said to be decidedly inferior in value to the results of good chronometric runs.

Similarly, altazimuths, portable transits, and other like astronomical instruments, are now seldom or never supplied to a surveying vessel. The sextant in a practised hand will give results equal to those obtainable by fixed instruments of small size, and has the great advantage of being more portable, and always ready.

[^22] longitude of places in their vicinity depend.

When therefore a secondary meridian is changed in its value as regards its distance from the first meridian, all places whose longitude have been measured from it are changed also.

This is the work of the Hydrographic Office, which receives and collates all information. The nautical surveyor simply finds the difference of longitude, and transmits that informa= tion only.

A list of secondary meridians is given in the Instructions for Hydrographic Surveyors.

For our purposes we may look upon difference of longitude Cases as divided into two main cases. The first, where the scale of a chart we are making depends on the astronomical observations for latitude and longitude at either extremity of our piece of coast. The second, when we wish to determine the relative positions of places more or less far apart, which are mainly required for the purposes of navigation.

In the first, we can nearly always use the system, hereafter described, of "travelling rates," which much adds to the accuracy of the result.

In the second, time, distance, and general circumstances often prevent our obtaining these, and compel us to use what we can get.

In obtaining the meridian distance between two places, Principle either by means of a telegraph, or by carrying chronometers of diffebetween them, the principle is the same, and is this, viz. Longitude. that the difference of mean time of place at any moment at the two places is their difference of longitude in time.

If, therefore, we can find out that at the time that at a position A it is 9 o'clock, it is 8 o'clock at B , we know that the difference of longitude is equal to one hour of time, and that $A$ is east of $B$. The telegraph enables us to do this in its simplest form, as, ascertaining the exact time at each end by astronomical observations, we can find out by an exchange of signals what is the difference of time.

Chronometric Difference of Longitude.

In chronometric difference of longitude we have literally to carry the time from one place to the other. We ascertain the time at one place on a certain day, or, what is the same thing, we find out the Error of our chronometer on local time.

Supposing for the moment that our chronometer is keeping exact mean time, by carrying it to the other place and finding out its Error on local time there, we can deduce the difference of longitude by the difference of the two Errors. Thus, if our chronometer is four hours slow on mean time at $A$, and we find when we get to $B$ that it is three hours slow, we know the difference of longitude is one hour, and that $A$ is east of B. Unfortunately, chronometers do not keep mean time, and the problem is complicated by having to ascertain what time they do keep, or, in other words, what they gain or lose in each day, which is called the rate. If we can find this, we shall be able to get the difference of longitude just as accurately as if the chronometer was keeping mean time, as we can correct for this rate; but here, again, chronometers are not, and probably never will be, perfect instruments, and are liable to change of rate, and it by no means stands to reason that because a chronometer gains five seconds a day one week, it will do so the next, especially when the ship has been at anchor during one period, and under weigh for the other, and the temperature has not been invariable.

Chronometric runs are therefore liable to the errors arising from change of rate. To overcome this as far as possible, a number of chronometers are carried instead of one, and, if possible, what is called the travelling rate is obtained.
Travelling If the rate of chronometers is obtained at a station A, and Rate. we then go to another station, $B$, and obtain rate again there,
and apply the mean of these rates as the assumed rate of the chronometers while being carried from A to B, we have no guarantee whatever that this assumption is correct, as the time employed in carrying the chronometers does not enter into the calculation at all, and they may have been going quite differently when the ship was at sea, with the vibration of the engines, motion of the ship, \&c., to influence them, to what they were when the ship was at anchor, besides the important factor of change of temperature. If, however, we can return at once to A, and obtain the Error again, we can positively say that the chronometers have gained or lost so much between the first and second observations at $A$. Assuming this loss or gain to have been uniformly carried on throughout the interval, we shall have a travelling rate which will give a far nearer result than by using rates obtained at either end of our required base. By this means we only obtain one meridian distance for our double run forwards and backwards, but it will be of more value than two separate meridian distances obtained by fixed rates.

Even if we have to stop at B a few days, by observing on arrival, and immediately before departure, we can eliminate the gain or loss of the chronometers during the stay there, by Rate. subtracting it from the total gain or loss during the time of our absence from $A$, and dividing the remainder by the number of days actually travelling. We shall thus still get a fair travelling rate, if the chronometers are at all trustworthy as timekeepers.

This, then, is the system of travelling rates, which can be generally, and always should be, if possible, used in determining difference of longitude for the scale of a chart.

Whatever be the system of rates employed, good observations must of course be regarded as the foundation of all of them. We cannot control the irregularities of our chronometers, but we can, to a certain extent, make sure of getting fairly correct time by using the proper means.

To ascertain the Error of the chronometer as exactly as we Eliminacan with sextant and artificial horizon, we must endeavour to

Equal Al-
titudes. vations of stars for latitude, which in this case is attained by observing at equal altitudes east and west of the meridian. It will be evident that, whatever be the instrumental and other errors, (excepting those of observation,) supposing them to remain unaltered, the middle time between the observations will be the same, as whatever tends to make the observed altitude more or less in the forenoon, will act in the same manner in the afternoon, and as we do not want to know at all what that altitude is, but merely to ensure that it is equal, A.m. and. P.m., the amount of the errors is immaterial.

The method of equal altitudes, therefore, must be used whenever we wish to get Error exactly. The Error of watch obtained by single altitudes, called "absolute observations," will depend for its accuracy upon the corrections for each source of error, which, as we have before stated, can only be considered as approximate.

Superior and Inferior Transit.

Principle of Equal Altitudes.

Equal altitudes of the sun can be taken either in the forenoon and afternoon of the same day, so as to find the Error at noon, called Error at superior transit; or in the afternoon of one day and the forenoon of the next, by which means we obtain the Error at midnight, or at inferior transit. Theoretically, these are equally correct, but in practice it is better to get Error at noon, if we can, as the elapsed time being less, gives less latitude to the chronometers or hack watches for eccentricity. The alternative is, however, very valuable, and saves many a day, as when, for instance, we arrive at the place we wish to observe at, an hour or two too late for forenoon sights. We can then begin our set in the afternoon, and get away, if we wish to do so, the next morning after forenoon sights, and thus save several hours, a considerable consideration in running meridian distances.

The principle of finding the Error of a timekeeper by observation of equal altitudes is, that the earth revolving at a uniform rate, equal altitudes of a body, on either side of the meridion, will be found at equal intervals from the time of transit of that body over the meridian, and that therefore the
mean of the times of such equal altitudes will give the time at transit.

In the case of a body whose declination is practically invariable, as a star, this is strictly true, and the calculation of the Error of the watch is confined to taking the difference of the time shown by the watch, and the true calculated time of transit.

In the case of the sun, however, the declination is constantly Equation changing; the altitudes are thereby affected, and an altitude equal to that observed before transit will be reached after transit, sooner or later, according to the direction of change in the declination.

It is therefore necessary to make a calculation of the correc-
tion, resulting from this change of declination, to be applied to the middle time, to reduce it to apparent noon, which correction is termed the "equation of equal altitudes."

The observation of stars at equal altitudes will therefore be, theoretically, the best to use, as being the simplest, and they of Equal Altitudes. will indeed give as good results as those of the sun; but practically, the latter has generally been observed in marine surveying for the purpose of obtaining time. In many cases the inconvenience of landing, and carrying watches backwards and forwards for comparison, \&c., by night, besides the increased difficulties of observing, and reading instruments by lamp-light, lead to the choice of day observations; but in places where clouds persistently veil the sun in forenoon or afternoon, the nights are often clear, and equal altitudes of stars become most valuable.

There are two other methods of obtaining error by stars, that are both good. The first, observing stars of corresponding

Other Methods by Stars. altitude on either side of the meridian, working them out as absolute altitudes and meaning the results. The other is a method to which attention has been drawn by Captain A. M. Field, R.N., of observing two different stars of nearly the same declination at equal altitudes. This is published as a pamphlet by the Hydrographic Office, but is briefly described on p. 238. The great advantage of either of these
is that the complete observation is made in a very short time, thereby obviating the disadvantages of star observations, mentioned above.

Limitations of Observetions.

Sets of Observations.

In taking the observations of equal altitudes in the artificial horizon, we are limited, as always, to altitudes between $20^{\circ}$ and $60^{\circ}$, as the horizon will not permit us to observe a lower altitude than $20^{\circ}$, and the sextant will not measure much more than $120^{\circ}$. These ristrictions will, however, only be inconveniences, as regards the sun, in extreme latitudes, as we must choose, as our time of observation, so as to minimise the effects of errors of observation, the period at which its motion in altitude is the greatest, i.e. when it is near the prime vertical, at which time, in all but high latitudes, the altitude will come between our limits. When the place of observation is near the equator, and the latitude and declination are nearly the same, we could observe up to a very short time of noon, the sun's motion in altitude being nearly uniform throughout the day; but we are in this case limited by the range of the sextant.

It is difficult to lay down any rule as to what is the smallest rate of motion in altitude we should observe at, as the greatest motion in altitude during the day varies so much with the latitude and declination. We can only say that we should, when we have any choice, not observe beyond an hour when the time of changing $10^{\prime}$ of double altitude exceeds 30 seconds.

Opinions have much differed on the number of consecutive observations that it is best to take to comprise in a set. The only theoretical limit is that the equation of equal altitudes should be practically the same throughout the set, as the variation in the time required by the sun to traverse the number of minutes of altitude between observations at the beginning and end of the set will not matter, as we do not care whether the mean of the times agrees exactly with the mean of the altitudes.

It seems well, therefore, to observe tolerably long sets, as errors of observation are thereby eliminated. The same result
in the end will be attained by a large number of shorter sets; but the value of each set is much enhanced if composed of a considerable number of observations, and it saves time and trouble in the calculations.

Too long sets are to be deprecated as wearying to the eye and hand, and the observations will therefore suffer from that cause, especially in hot countries, where the necessity for observing in the full glare of the sun makes it a trying operation.

We prefer to take eleven observations in a set. This allows the observer to commence his second set, of lower limbs, (A.M. observations) at exactly one degree more altitude than his first one, of upper limbs. It does not much matter, as each one has his own plans for these details, and soon falls into a regular method, which is the great thing to prevent mistakes.

The only point in fact of importance is, always to observe Observain the same way. Not only does it save time and errors, but $\begin{aligned} & \text { tions must } \\ & \text { be similar. }\end{aligned}$ it is necessary in combining observations, whether for rate or meridian distance, that they be as similar in all respects as we can get them. The whole system is a system of differences, and it is manifest that the result is the better the more like the observations are. It follows from this that the observers employed in any string of meridian distances should be the same, the instruments and watches the same, the temperature and time of observing the same, as far as possible. Also, supposing temperature to be the same, that a rate will be probably more correct if obtained by combining single altitudes of different days, both either A.M. or P.M., than by taking equal altitudes one day and single altitudes the other.

If clouds prevent observation at precisely the same altitude, after transit, the mean of Error obtained by absolute sights, A.M. and P.M., at nearly the same altitudes, will be almost as good as equal altitude sights.

When the observers are good, the greatest error is frequently Comparing introduced in comparing the hack watches to be used for watches.
taking time with the chronometers, and great pains should therefore be taken with this operation.

The watches used for taking time must be compared before, and after, both forenoon and afternoon sights, with the standard and another chronometer; and at noon, all the chronometers should be compared with the standard, and the hack watches with the same two chronometers.

In the case of stars, when return cannot be made to compare, the proper comparison for middle time must be deduced by interpolation from the comparisons before leaving and on returning.

Defects in Pocket Chronometers.

Before saying more about comparing, we must remark that the seconds-hands of pocket chronometers are rarely placed symmetrically in the centre of the dial on which the seconds are marked.

These watches beat five times in two seconds, commencing with the even minute. The beat of the watch should therefore coincide exactly with every even second; the first beat from the minute being 0.4 sec ., the second 0.8 , the third 1.2 , the fourth $1 \cdot 6$, the fifth at $2 \cdot 0$, and so on throughout the whole 60 seconds.

But it will be found, in nearly every watch, that the hand does not fall over the even second on some parts of the dial, although it may on others, and each watch must be examined by counting the beats from the even minute, to ascertain how the hand falls in different parts of the dial, or the time-taker will be at a loss to know what is the exact decimal which his watch is beating. For instance, supposing that at the 40 seconds' mark on the dial, the hand falls a little short of the mark one beat, and a little in advance at the next; unless he knows which of those beats is meant for the 40 seconds, he may be giving the time four-tenths of a second wrong. This, of course, refers both to comparing and taking time for the observations.
Method of In comparing, which is perhaps best done by two persons, Comparing. the "Stop" is given on an exact second of the box-chronometer (which beats half seconds), and the time by the pocket-
watch noted. A check is then taken, by comparing the reverse way, calling the "stop" at an exact second by the pocket-watch, and noting the seconds and parts of seconds of the box-chronometer. As parts of seconds have to be estimated on hoth watches, these two comparisons will frequently differ two-tenths of a second. This is as near as we shall probably be able to arrive at the truth; but if the difference exceeds this, more checks should be taken, until we are satisfied which was wrong.

No operation requires more care or more practice than comparing, and while the simple method given above will give good results when observers are experienced; varying the method by stopping on odd seconds, will in the case of the less experienced, prevent any bias on the part of the observer taking the corresponding time.

In the same manner checks should be taken when comparing one box-chronometer with another.

If two pocket-watches are available, it will not be amiss to use both, even when there is only one observer, as it helps to eliminate errors of comparison. In this case half the sets will be taken with one watch, and the other half with the other.

Preparation of the ground is not necessary, as in the case of observing stars, excepting so far as selecting the spot, to ensure being able to see in both the A.M. and P.M. directions.

Having compared the watches, we land to observe.
The watches to be used on the ground should always be Care in carried in their boxes, and great care must be taken not to jerk them, and above all to avoid any circular motion.

The method of observation for time differs from that Method of already described of stars for latitude, inasmuch as we ob- observaserve at stated altitudes, generally at every $10^{\prime}$, setting the sextant for the purpose, and noting when the contact takes place. In observing with the stand, therefore, we only need to work the screw of the stand leg, to get the suns vertically under one another.

## Observing both limbs.

It is well to observe both upper and lower limbs, as though it will make no difference to the result, it is good to have constant practice at both opening and closing suns, and not have all one way in the forenoon and the other in the afternoon. If we begin by a set of upper limbs, and immediately after take a set of lower, as an invariable practice, there will be no confusion, and we shall soon naturally fall into the system.

It may be here noted that with the inverting tube, the movable sun (the sun reflected from index and horizonglass) is above the other, when we are observing upper limb, and below when lower limb. Also that upper limbs in the forenoon are closing suns, and in the afternoon opening suns. It is necessarily vice versâ for lower limb.

Dark eyepieces to be used.

Always use the dark eye-pieces, of which there should be several of different degrees of shade, as, if the brilliancy of the sun varies by passing clouds, no inherent error is introduced by changing these, which is the case with the hinged shades on the sextant. Moreover, the suns having been once equalised as to brilliancy with one eye-piece, by moving the up-and-down piece screw, they will remain equal, no matter what shade of eye-piece we use; but with the hinged shades, the position of up-and-down piece which equalises the suns as seen through one set of them, will be different to that required for others, besides the possibility of error thus introduced.

The suns should be as dark as possible. If too light shades are used, the irradiation spoils the sharpness of the limb.

## Should be as large as possible. <br> Settingthe vernier.

to be coincidence with the minute of are required, glance right and left to see that the marks on vernier and arc are displaced in a symmetrical manner on either side. The eye will easier catch any inaccuracy in the setting by this means.

In setting, turn the tangent screw for the final adjustment the same way both forenoon and afternoon. Thus, if with altitudes increasing the tangent screw is turned to the right to attain coincidence, with altitudes diminishing the vernier must be set back below the required altitude, so as again to turn the screw to the right for final adjustment. This tends to eliminate error from slackness of screw.

Some observers, after giving a preliminary "Ready" at Warning the commencement of each set, give no warning after, and ${ }^{\text {calls. }}$ simply "Stop" at each observation. With very careful time-takers this is sufficient, but experience of human nature leads us to say that it is better to call "Ready" about three seconds or so before each "Stop," and thereby avoid all chance of the time-taker having his eye and ear off the watch.

We should take more observations in our first half of equal allowanoe altitudes than will be absolutely needed, so as to allow of for clouds some losses in the observations after transit, from obscurations Transit. of the sun.

At the conclusion of observations it is always well to take Index the index error. It tells us whether our sextant is keeping a steady error, and also, by calculation of semi-diameter therefrom, whether the instrument is in adjustment for side error, and also, if we lose the other half of equal altitudes, and decide to work single altitudes instead, we shall have the index error observed at the time.

After sights before transit, we must calculate the time the Caloulatobservations after transit will commence. By far the simplest $\begin{gathered}\text { ing time } \\ \text { for obser- }\end{gathered}$ plan, when engaged in observations, is to have an ordinary vations watch set to apparent time, which time the ship herself will Transit. in many cases be keeping, when by noting the time by this watch at the last observation, the time of commencement of
the first observation after transit will be found by taking the time noted from twelve hours.

If we do not do this, and the ship be keeping mean time, we must find the mean time of the last observation by applying the approximate Error of the watch. Subtract this from twelve hours, and apply twice the equation of time, subtracting if apparent noon is before mean noon, and adding if vice versâ. This will give mean time of the first observation after transit, which can be re-transferred to the watch by the application of the Error.

We want the time by the ship's clock, to ensure leaving her at the right time, and the time by our watch, to avoid the chance of being too late on one hand, and scurry after reaching the observation spot, on the other.

Algebraically we can express this,

$$
\mathrm{T}=12-(t+e)+2 q
$$

where T is mean time of first observation required.
$t$ is time by watch of last observation.
$e$ is Error of watch slow of mean time.
$q$ is equation of time.

Form in rough
Sight
Book.

Noting
Intervals between sights.

The book in which the observations are registered should be ruled as in annexed specimen.
The first column is for the intervals between each observation of the first set of sights. The second, the time taken at those sights. The third, the double altitude. The fourth, the sum of the seconds of the two times. The fifth, the time at second set of sights; and the sixth for the intervals between the latter.

On next page is an example of a set of sights as written in the angle book.
When the time-taker is practised, it is well for him to note down the interval between the sights as he notes down the time, as it enables the observer at once to know, when the set is over, whether he has been getting good observations or not, as the intervals should theoretically be precisely the same.

Ћ April 3rd, 1880, at Nagara Light-house $\triangle$.
Time by Breguet (2086).

| Interval in second - | Time by watch. | ఠ | Sum of secs. | Time by watch. | Interval in seconds. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & 28 \cdot 0 \\ & 27 \cdot 6 \\ & 27 \cdot 2 \\ & 28 \cdot 0 \\ & 27 \cdot 2 \\ & 27 \cdot 4 \\ & 28 \cdot 2 \\ & 27 \cdot 2 \\ & 28 \cdot 0 \\ & 28 \cdot 0 \end{aligned}$ | h. m.   <br> 8 03  | - 5720 | $\stackrel{\text { s. }}{14} \cdot 8$ |  | $27 \cdot 6$ |
|  | 38.8 | 30 | $15 \cdot 2$ | $36 \cdot 4$ | $28 \cdot 2$ |
|  | $0406 \cdot 4$ | 40 | $14 \cdot 6$ | $3908 \cdot 2$ | $27 \cdot 2$ |
|  | $33 \cdot 6$ | 50 | $14 \cdot 6$ | 41.0 | $27 \cdot 8$ |
|  | $0501 \cdot 6$ | 5800 | $14 \cdot 8$ | $3813 \cdot 2$ | $28 \cdot 0$ |
|  | $28 \cdot 8$ | 10 | $14 \cdot 0$ | $45 \cdot 2$ | $27 \cdot 6$ |
|  | $56 \cdot 2$ | 20 | $13 \cdot 8$ | $3717 \cdot 6$ | $28 \cdot 0$ |
|  | $0624 \cdot 4$ | 30 | $14 \cdot 0$ | - $49 \cdot 6$ | $27 \cdot 4$ |
|  | $51 \cdot 6$ | 40 | $13 \cdot 8$ | $3622 \cdot 2$ | $27 \cdot 2$ |
|  | $0719 \cdot 6$ | 50 | $14 \cdot 6$ | $55 \cdot 0$ | $27 \cdot 8$ |
|  | $47 \cdot 6$ | $59 \cdot 00$ | $14 \cdot 8$ | $3527 \cdot 2$ |  |
|  |  |  | $5 \cdot 0$ |  |  |
|  |  |  | $14 \cdot 45$ |  |  |
|  | Times at mi | e sigh | $\begin{array}{r} \text { h. } \mathrm{m} . \\ 805 \\ 1507 \end{array}$ | $\begin{array}{r} \text { s. } \\ 28 \cdot 8 \\ 45 \cdot 2 \end{array}$ |  |
|  |  |  | )23 43 | $14 \cdot 0$ |  |
|  | Mean mid. time by watch $115137 \cdot 22$ |  |  |  |  |

N.B.-The seconds of the result are obtained by halving the mean of Column 4.

We do not usually attempt to estimate time with a pocketwatch to less than two-tenths of a second, so we shall not find the intervals agreeing exactly, even supposing no errors of observation to exist. Taking everything together, if the difference in these intervals does not exceed one second and a half, we may consider that we have obtained very good observations. Another reason for noting these intervals is that we shall see if the sun's motion is becoming too slow.

In working out sights for equal altitudes, it is merely the Method of mean of the middle times of each set that we wish to get, so Meaning. that we need not mean up each column of times, but merely the sum of the seconds of each corresponding times, which we have in the fo urth column of our sight book. Then taking
the times of the middle observation, meaning those, and substituting, for the seconds of this mean, the mean of the seconds just found from the fourth column, we shall have the true mean middle time of our observations. Thus, in our example the mean of the fourth column is $14^{8.5}$. We substitute this for the $14^{s .0}$ obtained by adding the two times at the middle observation, and dividing by two, we get the mean middle time by the set as $11 \mathrm{~h} .51 \mathrm{~m} .37 \cdot 22 \mathrm{~s}$.

In cases where the equation of equal altitudes is varying rapidly, we shall not find the middle times of two successive sets agreeing exactly, as they should differ by the amount of the variation of the equation of equal altitudes in the time.

Theoretically, this is an objection to long sets of observation, but practically, the errors of observation will exceed any little discrepancy introduced by assuming the equation to be uniformly variable during a set of 11 to 15 .

Sights missed.

If a contact is missed in either half set, it is no use to interpolate a time. The sight must be missed out of the double set altogether. It will not affect anything but the number of observations in the set, except when it happens to be the first or last of a set, when the set will become one with an even number of observations, and to get elapsed time we must, instead of the central observation, take the mean of the two times corresponding to the two central observations.

Four sets of eleven observations each, half upper limb and half lower, ought to give accurate time ; but it must be understood that equal altitudes do not eliminate personal errors, only instrumental and atmospherical ones. It is evident that, if an observer is, for instance, habitually too slow in recording his contacts, the resulting middle time will be so much after the true time of transit. Taking the case of an observer recording the time of contacts of opening limbs correctly, and of closing ones habitually too late; the middle time will still be in error. It is only when the observer records opening limbs in error one way, and closing in error in the contrary way, that these personal errors can be eliminated. This is of course not likely to happen with many people, and we must
consider the time resulting from any observer's sights as being always in error by the amount of his personal equation.

In running a meridian distance, however, this personal Diserror, supposing it to be tolerably constant, will disappear, as his time being equally in error, and in the same direction, appears in Meridian (either too fast or too slow) at both places, the difference of time, on which alone longitude depends, will not be affected. From this it results that in running a meridian distance, the same observers must always be employed.

The formula for finding the equation of equal altitudes is as follows:-

Equation $=\mathrm{A}+\mathrm{B}$.

Formula
for EquaEqual Altitudes. B ( do. $)=\frac{1}{15} \times \frac{c}{2} \operatorname{Tan} \operatorname{dec} \times \operatorname{Cot} \frac{\text { elapsed time. }}{2}$ where $\frac{c}{2}$ is half the change of declination in the elapsed time, or, as we use it in the computation, the change of declination in half the elapsed time.

The rules for noting the algebraic signs of $A$ and $B$ will be given hereafter.

In making the observations it is most convenient to ascer- Error tain the Error of the hack watch, and thence, by using the ${ }_{\text {of hack }}^{\text {obtained }}$ comparisons, to arrive at the Error of the standard. In the watch. case where the watch has a large rate, as shown by the comparisons before and after sights, the elapsed time must be corrected for the amount gained or lost by the watch in the interval on mean time, which can be roughly calculated from the known rate of the standard.

Having meaned the sights, and obtained the mean middle time for each set, and knowing the estimated latitude and longitude, the rule for working a set of equal altitudes at superior transit will stand thus:-

1. Ascertain elapsed time by subtracting the central time Practical of observation before transit from the central time after transit, ${ }^{\text {B }}$ increased, if necessary, by 12 hours. Halve this, and if there
are decimals, they can be rejected, the nearest even second being used.
2. To 0 hours apply longitude to find Greenwich date of apparent noon at place; this is first Greenwich date.
3. Find second Greenwich date, by subtracting the half elapsed time from the first Greenwich date.
4. Correct declination at apparent noon in Nautical Almanac for the second Greenwich date.
5. Correct equation of time at apparent noon for first Greenwich date.
6. Multiply the variation in declination for one hour, by the half elapsed time, to get $\frac{c}{2}$.
N.B.-The variation we want is that at Greenwich time of local noon, we must therefore correct the variation given in the Nautical Almanac for the longitude.
7. For A, add together the logarithms of $\frac{c}{2}$, the tangent of the latitude, and cosecant of half elapsed time; and for B the logarithms of $\frac{c}{2}$, the tangent of the declination, and the co-tangent of half elapsed time. Either subtract the log. of 15 from each of these sums, to reduce the results to time at onee, or take out the natural numbers of the sums as they stand, and when A and B have been added or subtracted, divide the result by 15 , to reduce it to time.
N.B.-Tables are given in various works on nautical astronomy to facilitate the calculation of A and B ; but as these are only made out for every so many minutes of elapsed time, interpolation is necessary when working with any pretence to accuracy, and very little is gained by their use in their present form.
8. To the mean middle time of the set, apply the equation of equal altitudes with its proper sign (rule given below), which will give the time shown by the watch at apparent noon. N.B.-When working several sets, calculate them
simultaneously as far as this, and mean the results, thus getting the mean time shown by the watch at apparent noon.
9. Find the mean time of apparent noon, by applying the equation of time with its proper sign to 0 or 24 hours, and take the difference between this and mean time shown by the watch, for the Error of the latter, subtracting one from the other, according as it is intended to show the watch as fast or slow on mean time.

An universal system must be adopted of showing all all chronometers and hack watches as fast or slow of the standard and on mean time, not some one way and some another, which leads to confusion. It does not much matter which is taken. The writer has always shown them as slow on mean time. watohes to be either slow or fast of Mean Time. Thus all chronometers are shown slow of the standard, and the standard and all others slow on mean time of place, or of Greenwich, as the case may be.

The rule for giving A and B their proper algebraic signs is as follows :-
\(\left.$$
\begin{array}{l}\text { At } \\
\begin{array}{l}\text { Superior } \\
\text { Transit. }\end{array}\left\{\begin{array}{c}\text { A is }\left\{\begin{array}{c}+ \text { if declination is decreasing and of same } \\
\text { name. } \\
+ \text { if declination is increasing and of diffe- } \\
\text { rent name. }\end{array}\right. \\
- \text { if otherwise. }\end{array}\right. \\
\text { B is }\left\{\begin{array}{c}\text { Signs of } \\
\text { Equation. }\end{array}
$$\right. <br>
\begin{array}{c}+ if declination is <br>
increasing <br>
if declination is <br>

decreasing\end{array}\end{array}\right\}\)| When elapsed time is |
| :---: |
| 12 hours. |

At
nferior
ransit. $\left\{\begin{array}{l}\mathrm{A} \text { is reversed from what it would be at superior } \\ \mathrm{B} \text { is thansit. } \\ \text { 就e as at superior transit. }\end{array}\right.$
In working with inferior transit, whereby we find the Error Change of at midnight, there is no difference in the rule, except that in tion in calculating the change during the half-elapsed time, we use Inferior the variation of declination found by interpolation for the Greenwich time of local midnight.

The next step is to calculate, from the comparisons taken Calenwith the standard before and after sights, a mean comparison

Comparison for Hack Watch.
to apply to the Error of watch found above, to arrive at the Error of the standard.

To do this, we take any sight, and by interpolation calculate the comparison at the A.M., and also the P.M. time corresponding. The mean of these two will give the comparison at noon. This should, if the watch has been going well, correspond very closely with the comparison actually taken at noon, and it will be satisfactory if it does so. If it does not, we cannot help it; but we shall know that the Error of the standard will be slightly incorrect from a jump in the watch, and shall be prepared to give the result a smaller value in consequence, in event of discrepancies with others.
Noon com. The mean comparison, as found above, must always be parison not to be used. used, not the comparison taken at noon, which is done solely to ascertain how the watch has been going.

An example of the calculation follows.

## At Merale ld ${ }^{\text {© }}$ Aug. 31st, 1878.

Sights obtained for Error by Equal Altitudes. Lat. $5^{\circ} 14 \mathrm{~S}$. Long. $39^{\circ} 40^{\prime} \mathrm{E}$.


h. m. s.


To calculate the comparison between standard (A) and watch at noon, we have the following comparisons observed:-


|  | Noon. |  |  | Check. |  | Before P.M. Sights. |  |  | Checks. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | ${ }_{7}^{\mathrm{h}} .$ | $\frac{\mathrm{m}}{50}$ | $50^{8 .}$ | secs. $06 \cdot 2$ |  |  | $\begin{aligned} & \mathrm{m} . \\ & 47 \end{aligned}$ | $25: 0$ | $\begin{gathered} \text { secs. } \\ 20.0 \end{gathered}$ | secs. $50 \cdot 0$ |
| Breguet | 7 | 03 | $55 \cdot 8$ | $12 \cdot 0$ | .. | 8 | 50 | $30 \cdot 6$ | $44^{\circ} 0$ | 55.4 |
|  | 0 |  | $\begin{aligned} & 54 \cdot 2 \\ & \text { ean } 54 \end{aligned}$ | $54 \cdot 2$ | . | 0 |  | $\begin{array}{r} 54 \cdot 4 \\ \operatorname{ean} 54 \end{array}$ | $54 \cdot 8$ | $54 \cdot 6$ |



Taking any sight, say the middle times at the set we have shown worked out, we find by interpolation that

$$
\begin{aligned}
& \text { At } 3.56 \text { by watch, comparisan is } \begin{array}{c}
\mathrm{m} . \\
46 \\
52.74
\end{array} \\
& \text { At } 10 \cdot 16 \text { " ", " } 46 \quad 55 \cdot 07
\end{aligned}
$$

and as these are equal times from noon, the noon-comparison will be the mean of these, or $46^{\mathrm{m} \cdot} 53^{\mathrm{s} \cdot 90}$. This differs $0^{\mathrm{s} \cdot} 3$ from the observed noon-comparison, which, supposing the comparisons to have been carefully observed, means that there has been a slight irregularity in the motion of the watch, which must be remembered in comparing any meridian distance founded on these sights with others ; but in this case it is so small as scarcely to be taken into consideration.

We now apply this mean comparison to Error of watch.-


We next take the comparisons observed at noon between A and all the other chronometers; and applying them to A's Error, we get the Error of each.

Error by Equal Altitudes of two Stars on opposite sides of the Meridian, by Captain A. M. Field, R.N.

The principle of this method depends upon the sidereal time of passing the meridian of a place, by an imaginary star having the mean right ascension of the two stars selected, being compared with the time shown by a sidereal chronometer at that instant; the difference is its error on sidereal time. A mean solar chronometer can be used equally well. The sidereal time required is the mean of the right ascensions of the two selected stars. The chronometer time at that instant (mean or sidereal) is the mean of the times at which the eastern and western stars had equal altitudes, with the "equation of equal altitudes" applied with its proper sign.

When preparing a list of stars for observing, it will be necessary to first find the R. A. of the meridian for the times between which it is required to carry on the observations. As stars will generally be observed within $4^{\mathrm{h}}$ of the meridian, the limits of $R$. A. of the stars falling within the required period may be obtained by subtracting $4^{\mathrm{h}}$ from the first R. A. of the meridian, and adding $4^{\mathrm{h}}$ to the last R. A. of the. meridian as found above.

To arrange the stars in pairs, it is necessary to select two bright stars of nearly the same declination, not differing much from the latitude, but differing in R.A. by from $4^{\text {h }}$ to $8^{\mathrm{h}}$. The time at which they will be simultaneously of equal altitude will be about the time when the mean of their R. A.'s is on the meridian.

The time at which it will be necessary to begin observing will be governed by this, and the observations of one star should be completed shortly before that time, in order to allow an interval to prepare for observing the other.

It may be found that when two stars are of equal altitudes, that they are too near the meridian for obtaining the best results, or that the altitudes are too great for the sextant; in which case it would be necessary to commence observations of the eastern star as much as an hour earlier, and the: western star will then have to be observed the same time later than when they would be simultaneously of equal altitude.

As a general rule, if the difference in right ascension is less than $6^{\mathrm{h}}$, the eastern star should be observed first; if it exceeds $6^{\mathrm{h}}$, then the western star; this is in order that the stars may be observed as favourably as possible with respect to the "prime vertical," but it will vary according to the latitude and declination. It will be noticed that if the observations are commenced with the eastern star, then they are, as a whole, taken further from the meridian than in the other case.

If the difference in R. A. exceeds $8^{\mathrm{h}}$, then there will probably be an interval between finishing the observations of
one star and beginning those of the other, and part of the advantages of the method are lost; the same remark applies if the difference in R. A. is less than $4^{\mathrm{h}}$.

Having decided on which star to begin with, observe it continuously in the ordinary way, until the sidereal time is nearly equal to the mean R. A. of the two stars (the error of chronometer on sidereal time should be roughly known), when prepare to observe the other star, commencing at the same altitude as the last observation of the first star, and complete the series, which may be divided into sets in the usual way.

Owing to the more rapid change in "equation of equal altitudes" when there is a large difference in the declinations of the stars, it will be remarked that the "middle times" vary more rapidly than in the case of the sun; and the rapidity of this change increases as the observations get further away from the sidereal time at which the "imaginary star" passes the meridian.

If a mean solar chronometer be used the chronometric interval (corrected for rate) must be turned into a sidereal interval, and the resulting "error of chronometer" will be the error on sidereal time at that particular instant, from which the error on mean time can be readily deduced.

## Equation of Equal Altitudes.

The rigorous expression, according to Chauvenet, is :-
$\operatorname{Sin} a=\cot \frac{1}{2}$ E.T. $\tan d . \tan \delta . \cos . a-\operatorname{cosec} \cdot \frac{1}{2} \mathrm{E} . \mathrm{T} . \tan . l . \tan \delta$.
where $a=$ equation of equal altitudes $=\frac{h-h^{1}}{2}$

$$
\begin{aligned}
\frac{1}{2} \text { E.T. } & =\frac{1}{2} \text { elapsed time }=\frac{h+h^{1}}{2} \\
d & =\text { declination at upper meridian passage. } \\
d-\delta & =\text { ", observation E. of meridian. } \\
d+\delta & =\text { " " " " W. "" } \\
\delta & =\frac{1}{2} \text { difference of the declination at the two times } \\
& \text { of observation. }
\end{aligned}
$$

$h$ and $h^{1}=$ Hour angles from noon at East and West observations respectively.

In the case of equal altitudes of two stars, one East and the other West of meridian, the above formula is strictly accurate, whatever may be the difference in the declinations; the half elapsed time being found as follows :-

$$
\begin{aligned}
\frac{1}{2} \text { E.T. } & =\frac{h+h^{1}}{2} \\
& =\frac{(\mathrm{R}-\mathrm{S})+\left(\mathrm{S}^{1}-\mathrm{R}^{1}\right)}{2} \\
& =\frac{\left(\mathrm{R}-\mathrm{R}^{1}\right)+\left(\mathrm{S}^{1}-\mathrm{S}\right)}{2}
\end{aligned}
$$

Where, $R$ and $R^{1}=$ Right ascensions of star E . and star W. S and $S^{1}=$ Sidereal time of observation of star $E$. and star W. respectively.
$S^{1}-\mathrm{S}$, the difference of the sidereal times, may be taken as equal to the difference of times (as shown by a sidereal chronometer) of the observation of the two stars ; the chronometer being of course corrected for rate.

$$
\text { If } \mathrm{R}^{1}>\mathrm{R} \text {, add } 24^{\mathrm{h}} \text { to } \mathrm{R} \text {. }
$$

If the western star be observed first, in which case $S>S^{1}$, then $S$ and $S^{1}$ are treated algebraically.
$d=$ the mean of the declinations of the two stars.
$\delta=\frac{1}{2}$ difference " " "
In working the rigorous expression, an approximate value for $a$ is first obtained, disregarding the term Cos. $a$, and then rework, using the value of $a$ thus found.

## Notes on the Computation.

The constant logarithm 4.138339 is an abridgment of Chauvenet's formula, by adopting it to circular measure, and is correct so long as the equation of equal altitude is small; certainly up to 4 minutes, no error is introduced by its use.

The cosine of $a$ with an equation of equal altitude under 4 minutes changes so slowly that it is practically the same for each set unless separated by a very long interval.

The R. A. M. S. must be found for the Greenwich date corresponding to the chronometer middle time. In cases of a small equation of equal altitude and changing slowly, this is the same for each set ; but where it is large and therefore changing rapidly, as in the second example, the R. A. M. S. must be corrected for every set, but this merely involves applying the acceleration due to the difference of the chronometer middle time of each successive set to the R. A. M. S. for the first set.

A more or less accurate knowledge of the G. M. T. is therefore necessary, but this is inherent to the use of stars for time under all circumstances, and a second approximation must be made if the error on local time has been assumed more than three or four seconds in error.

It is worthy of remark that $\tan d \times \tan \delta \times 4.138339$ and $\tan l \times \tan \delta \times 4.138339$ form absolutely constant logarithms for the same stars, at the same place, for any night in the year, and it is only necessary to add the logarithm of $\frac{1}{2} \mathrm{E}$. T. to each to obtain the equation of equal altitude corresponding to that $\frac{1}{2} \mathrm{E}$. T.

The equation of equal altitude is always so large as to exclude all possibility of doubt as to which way to apply it, but the investigation gives the rule. Two examples are given; the first illustrates the case of two stars differing by $1^{\circ} 40^{\prime}$ in declination, and the second where they differ by $8^{\circ} 25^{\prime}$.

In the first instance the equation of equal altitudes hardly changes between the first observation and the last, and, therefore, the sums of the middle times do not vary. This is due to the difference in declination being small, and to the fact that the middle time of observation is only $10^{\mathrm{m}} 16^{8}$ from the "time of crossing," or the time at which the stars had equal altitudes, the one rising and the other setting, and is represented by the " $\frac{1}{2}$ interval."

In the second instance the equation of equal altitude changes very rapidly; the difference in declination is large, and the " $\frac{1}{2}$ interval" is also somewhat large, viz., $44^{\mathrm{m}} 20^{\mathrm{s}}$; but the " $\frac{1}{2}$ interval" may, nevertheless, be extended to upwards of an hour if necessary, to wait for clouds, \&c., without affecting the accuracy of the result.

The acceleration in the change of equation of equal altitudes may be considered as practically uniform for the short intervals of four or five minutes necessary to obtain sets of observations, and therefore, although it will be noticed that the sums of the middle times change rapidly, yet it will nevertheless be perfectly accurate to take the means and calculate the equation of equal altitude corresponding to the " $\frac{1}{2}$ interval" for the middle observation, no matter how rapidly the equations may change.
Example 1.
longitude $114^{\circ} 9^{\prime} 49^{\prime \prime}$ E., the following observations of Markab (West of meridian) and Aldebaran (East of meridian) were obtained :


Example 2.
On the evening of 13 th November, 1892, at Fullerton Battery, Singapore, latitude $1^{\circ} 1 \tau^{\prime} 11^{\prime \prime}$ N., longitude $103^{\circ} 51^{\prime} 15^{\prime \prime}$ E., the following observations of Markab (West of meridian) and $\gamma$



## CHAPTER XIV.

## MERIDIAN DISTANCES.

Telegraphic-Chronometric.
Under this head we shall consider all the methods, available for our purposes, of obtaining difference of longitude.

## TELEGRAPHIC MERIDIAN DISTANCE.

Where a telegraph can be used, it is of course the best, and at the same time the simplest, means of obtaining difference of longitude.

This method consists in sending a current through the wire at a known local time from one place, the local time of arrival at the other place being noted. The difference of these is the difference of longitude.
Retardation.

In theory, the passage of the current through the wire is instantaneous; but in practice it takes an appreciable time, when the distance is considerable, and the electrical condition of the wire is not first-rate ; and to eliminate this, and also to decrease errors of sending and receiving, we must send several sets of signals in both directions equally, the mean of which will give the true time.

A little consideration will show that if a signal is sent from $A$ to $B$, a place to the westward, and it takes two seconds to traverse the wire, the time at $B$ will have had those two seconds in which to catch up the A time, which is so much
ahead; or, in other words, the difference of the two times as shown will be two seconds too little. Whereas, if the signal is sent from B to $A$, the watch at $A$, already ahead, will advance another two seconds before the signal arrives, and the difference of the two times will be two seconds too much. The mean of the two will therefore be correct, and half the difference of the two will give the "retardation of the wire," a matter, however, purely of curiosity as far as our results are concerned. In land lines the retardation is about $\frac{1}{10}$ of a second per thousand miles, with submarine cables it is larger and varies.

To eliminate all personal errors, the only satisfactory method is for the observers to change ends. When this is not possible, personal errors when obtainable should be applied. These can only be determined by a considerable series of observations for time, and the error of each observer or the person chosen as standard being recorded and meaned. The steadiness, and therefore the value, of the personal error can thus be judged.

In this case a number of chronometers is not necessary, only one All that is wanted is one good time-keeper. If, however, two watches are at hand, it is not amiss to ascertain the Errors of each separately, and use them both in transmitting the signals.

A box-chronometer is the best for sending and receiving signals by, and if practicable, it may be a good plan to land one, and let it stand in the telegraph office for a few days beforehand to settle down, comparing the watch actually used at sights with it, before and after observations.

Sights must be obtained on the day of sending the signals, Time to and the latter should be transmitted at or about noon or exchange midnight. Where the places are far apart in longitude, it can only be near noon at one place, and Error must be obtained at the other, either on the day before or after, as well, so as to be able to correct the Error to the time of interchange of signals. Using land lines, day-time is most favourable for exchange of signals.

Observa-
tion Spot.

Galvanometer.

Prearranged method of sending signals.

If the observation spot can be at, or close to, the telegraph office, it is convenient, as the watch will not have to be carried about; but in many instances the local arrangements will not admit of this.

Telegraphic instruments differ very much; but it does not much matter which are used, as long as they are similar at both ends. The deflection of an ordinary galvanometer-needle of Wheatstone's instrument, or of the Morse recorder, or of the more delicate mirror of long submarine cables, will all serve our purpose. Preference is given to one or the other by different observers. The writer prefers an instrument giving a sound, to the silent movement of the suspended mirror.

Each signal will consist of one deflection, and the key should be kept pressed down for about a second.
In sending the signals, it must be clearly arranged beforehand what is going to be done.

A good plan is as follows :-
In commencing, give a warning, say of three rapid signals, at ten seconds before an even minute by the sender's watch. The first signal will then go at the even minute, and at every ten seconds another, missing the fifty seconds, to marle the even minute, for three minutes, ending with another even minute.

After an interval of three or four minutes, a similar set will be sent in the reverse direction.

If at the receiving ends the signals agree, this will be quite sufficient, unless we intend to use another watch.

An example of a telegraphic meridian distance is appended,
It will be seen that the resistance of the wire and instrumental retardation was less on one day than on the other, amounting at one time to nearly a tenth of a second, and at the other to only twenty-five thousandths.

Meridian Distance by Telegrapi Signals exchanged between Constantinople and Dardanelles.

April 3rd and 18th, 1880.
Observation spot at Constantinople was at Leander's Tower, observers having to go $1 \frac{1}{2}$ miles to Telegraph Office by caïque.
Observation spot at Dardanelles at Nagara Light-house, observers having to go three miles to Telegraph Station by steam pinnace.



April 18th, 1880.


## CHRONOMETRIC MERIDIAN DISTANCES.

When we have no telegraph we must have recourse to chronometers for conveying the time.

Having obtained sights at the two places whose meridian distance we require, we come to the consideration of the rate to be used.

If we have been able to run backwards and forwards, as recommended on page 220, we shall use a travelling rate.

The algebraic formula for finding the meridian distance by Formula travelling rate, when we return at once to the original station, $\underset{T}{\text { Travelling }}$ is as follows:-

$$
\mathrm{M}=\beta-\alpha-n \frac{a^{\prime}-\alpha}{m+n},
$$

where $\mathrm{M}=$ meridian distance,
$a=$ error at place A, before starting,
$a^{\prime}=\quad, \quad$, on returning,
$\beta=\quad$, B,
$n=$ No. of days between first observations at $A$ and those at B,
$m=$ No. of days between observations at B and those at A, on returning.
Then $\frac{a-a}{m+n}=$ travelling rate.
This can be put another way, by example, as follows :-
Let us suppose we have obtained Error at Maziwi at noon, August 27th; we have been to Mesale, and there obtained

Example
of Travelling Rate. Error at noon on the 31st ; and then, returning to Maziwi, have obtained another Error there at midnight of September 1st-2nd.

To find rate in this case, we simply divide the difference of the Errors ascertained on the 27 th and 1st by $5 \frac{1}{2}$ (the interval between them). This rate, multiplied by 4 (the interval between sights at Maziwi on 27th and Mesale on 31st), will give the quantity to be applied to the error of the chronometer in question at Maziwi on the 27th, to give the error
on the 31st on mean time of Maziwi. The difference of this and the error of the same chronometer on mean time of Mesale, as ascertained on that day, will be the meridian distance by that chronometer.

## Form for

 Meridian Distance.In working out a meridian distance with several chronometers, it is convenient to use a form, as shown in the example of the above-cited instance (page 255).
Rejection of results.

Here so many of the chronometers agree closely, that the result by D seems doubtful, and looking at the comparisons taken every day with the standard, we see it has been going very irregularly; we therefore reject it. This should not be done without some independent evidence of this kind; and in a meridian distance, where the interval of time is great, or where all the chronometers have been going but fairly, as shown by the daily comparisons, it is very unsafe to reject chronometers solely because they vary from a small majority of the others.

> Another case of ${ }_{1}$ Travelling Rates.

> Supposing that we had had to stay at Mesale for a few days before returning to Maziwi, we can still find a fair travelling rate.

The formula for this is as follows:-

$$
\mathrm{M}=\beta-a-n \frac{\left(\alpha^{1}-a\right)-\left(\beta^{1}-\beta\right)}{m+n}
$$

where, the other letters representing the same values, $\beta^{1}$ is the error at place $B$ before leaving.
Here the travelling rate is,

$$
\frac{\left(a^{\prime}-a\right)-\left(\beta^{1}-\beta\right)}{m+n}
$$

This can be exemplified thus. We obtain sights at Maziwi on 27th and at Mesale on 31st as before. Then sights again at Mesale on the 4th noon, and again at Maziwi on the 6th noon.

From the difference of the errors on the 27 th and 6 th we should deduct the difference of the errors on the 31st and 4th, and divide the remainder by 6 , the sum of the intervals from the 27 th to the 31 st, and of the 4 th to the 6 th, or, in other

| Meridian Distance between Maziwi $\triangle$ and Mesale $\triangle$, using Travelling Rates. Chrons. slow on M. T. Place. |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | B | C | D | E | F | G | H |
|  | $\begin{array}{rl} \text { h. m. } & \mathrm{s} . \\ 4 \mathrm{OF} & 01 \cdot 33 \\ & 11 \cdot 05 \end{array}$ |  | $\begin{array}{rrr} \text { h. m. } & \text { s. } \\ 4 & 55 & 26 \cdot 58 \\ 56 & 12 \cdot 55 \end{array}$ | $\begin{array}{r} \text { h. m. s. } \\ 348 \\ \hline 58 \cdot 53 \\ 49 \\ \hline 29 \cdot 20 \end{array}$ | $\begin{array}{rrr} \text { h. m. } & \text { s. } \\ 3 & 08 & 42 \cdot 63 \\ & 44 \cdot 10 \end{array}$ | $\begin{array}{r} \text { h. m. } \\ \begin{array}{l} \text { s. } \\ 2 \end{array} 59 \\ \\ \\ \\ 10 \cdot 9 \cdot \\ 16 \cdot 55 \end{array}$ | $\begin{array}{r} \text { h. m. } \\ 5 \mathrm{~m} . \\ 52 \\ 45 \cdot 73 \\ 53 \\ 53 \end{array}$ | $\begin{array}{rrr} \text { h. m. } \\ 3 & 10 & 56 \cdot 18 \\ 11 & 09 \cdot 75 \end{array}$ |
|  | $\frac{09 \cdot 72}{-1 \cdot 767}$ | $115 \cdot 43$ $+13 \cdot 714$ | $\frac{45 \cdot 97}{-8 \cdot 358}$ | $\frac{30 \cdot 67}{-5 \cdot 576}$ | $\frac{01 \cdot 47}{-0.267}$ | $\frac{05 \cdot 57}{-1 \cdot 012}$ | $\frac{19 \cdot 32}{-3 \cdot 512}$ | $\frac{13 \cdot 57}{-2.467}$ |
|  | $\begin{array}{rr}  & 07 \cdot 07 \\ 4 & 05 \\ 4 & 08 \cdot 40 \\ 4 & 07 \\ \hline \end{array}$ | $\begin{array}{lll}  & & 54 \cdot 86 \\ 0 & 38 & 04 \cdot 27 \\ 0 & 40 & 13 \cdot 02 \end{array}$ | $\begin{array}{r}  \\ \\ 43 \cdot 43 \\ 4560001 \\ 458 \\ 4 \\ 58 \cdot 47 \end{array}$ | $\begin{array}{lrr}  & & 22 \cdot 30 \\ 3 & 49 & 20 \cdot 83 \\ 3 & 51 & 30 \cdot 07 \end{array}$ | $\begin{array}{rr}  & \\ & 01 \cdot 07 \\ 3 & 08 \\ 3 & 43 \cdot 70 \\ 3 & 10 \end{array} 51 \cdot 97$ | $\begin{array}{lr}  & \\ & 04 \cdot 05 \\ 2 & 59 \\ 3 & 15 \cdot 01 \\ 3 & 23 \cdot 42 \end{array}$ |   $14 \cdot 05$ <br> 5 52 $59 \cdot 78$ <br> 5 55 $08 \cdot 37$ | $\begin{array}{lll}  & & 09 \cdot 87 \\ 3 & 11 & 06 \cdot 05 \\ 3 & 13 & 14.52 \end{array}$ |
|  | $208 \cdot 37$ | $208 \cdot 75$ | 208.46 | $\begin{aligned} & 209 \cdot 24 \\ & \text { Reject. } \end{aligned}$ | $208 \cdot 27$ | $208 \cdot 39$ | $208 \cdot 59$ | 208.47 |
| $\begin{array}{rlll} \text { Mean Meridian Distance } & . . & . . & 2^{\mathrm{m}} 08^{\circ} \cdot 47 \text { E. of Maziwi. } \\ \text { Range } & \text {.. } & \text {.. } & 0^{s} .48 . \end{array}$ |  |  |  |  |  |  |  |  |

words, the number of days actually travelling, which will give us the rate. We then proceed as before.

The travelling rate obtained in this instance will not be as good as in the former case, as the chronometers will have had two disturbances instead of one, and the rate they may settle into on starting the second time, after four days' quiet at anchor, may not be the same as before; but it will still be better than obtainable by any other method, and if circumstances of weather, sea, and temperature are nearly alike on both journeys, and the intervals are not long, we shall probably get a very good result.

Travelling rates, obtained thus, should always, as already remarked, be used when the scale of the chart depends on the observations.

The method is very simple, and, used for this purpose, none of the considerations of temperature, \&c., hereafter mentioned need be thought of, as the time is short.

## Other Rates.

## Why Watches change their rates.

We now come to the consideration of the rates to be used on other occasions, especially when voyages are long, and circumstances change much during them.

This is a very wide subject, and besides the fact that it has already been fully discussed by Captain Shadwell, in his masterly treatise before referred to, neither space nor the intention of these pages permits our going very far into it, and we shall content ourselves with giving general descriptions of cases, together with formulæ for them, with just sufficient reasons to allow of their being understood.

The whole question rests on,-What makes chronometers vary?
The labours of many observers show us that the answer is :-

1. Imperfection in the workmanship of the watch.
2. Changes of temperature.
3. The quality of the oil in the pivots, and its age (i.e. the time elapsed since the watch was last cleaned).
4. Accidental shocks or vibrations imparted to the watch.

A supplementary question may be asked-Which of these
is the most important? To which the general answer is that, according to circumstances, any one may be.

First, Imperfection of workmanship.
For this manifestly there is nothing to be done. A badly Imperfeomade chronometer will go so erratically that we shall soon tion. lose confidence in it, and reject it from all results, returning it as soon as we can. There are, however, but few chronometers that pass through the hands of the Royal Observatory which will come under this head, and doubtless many a chronometer has been classed in this category from ignorance of the circumstances of its compensation, and its resulting variation under change of temperature. If on a voyage, during which temperature is uniform, a chronometer placed with others, under the same conditions of protection from injury, \&c., goes erratically, while the others maintain their rates pretty steadily, we may fairly conclude it to be inferior.

The uniformity of rate of a chronometer while on shore, or when the ship is at rest, cannot be taken as a conclusive test.

Secondly. Change of temperature.
A chronometer is supposed to be compensated in such a Change of manner that at two temperatures, a varying number of degrees Temperaapart, the rates will be equal. At all other temperatures the rates will vary, reaching a maximum at about the mean temperature between the other two.

Let us, for brevity, call this temperature of maximum rate T.

If we then examine the rates of a chronometer we should Temporafind a steady change of rate in one direction (nearly always ${ }_{\text {Maximum }}^{\text {ture of }}$ in the direction of acceleration of gaining) from low tempera- Rate. tures to high, until we reach T, when the change of rate should vary in the opposite direction.

For every chronometer we shall have a different quantity for T , and different coefficients of change. Many chronometers are supposed to be compensated for $\mathrm{T}=60^{\circ}$, the mean temperature generally experienced over the globe; but it would seem that makers cannot command the point T; any-
way many have T over $90^{\circ}$, so that for such a watch, in practice, the direction of change is invariable, which will result in a great accumulation of difference of rate when passing through hot and cold climates, and where the coefficient is large, in great absolute change of rate.

Different conclusions.

No strict law.

Practical experiments.

Liverpool Observatory.

Different observers on the performances of chronometers have come to different conclusions on the subject of the law of change for a degree on all parts of the scale, which can only be accounted for by supposing that they have experimented on different classes of time-keopers.

Some have stated that they vary regularly, so as to liave the same rate at an equal number of degrees above or below $T$, and have established the proportion of variation at the square of the difference of $T$, and the temperature required.

Other experiments have shown that the manner in which watches vary is not quite so regular as this, and that the coefficient of change is generally less at temperatures higher than T, than at those below.
The fact is, that there is no invariable law on the subject, a watch being too complicated a machine to admit of any practical conclusion, uubased on actual experiment with each individual watch.

Experiment, however, does give results that can be practically used, and tables of rates can be formed from observation of the watch at different fixed temperatures, which, with some watches, will undoubtedly give better results than by using invariable rates.

Tables of this kind are now furnished to ships sailing from Liverpool, whose chronometers are rated at the Bidston Observatory, the director of whish, Mr. Hartnup, has studied the question for many years.

The rate of the watch to be used for determining the position at sea is then taken day by day from the table, according to the temperature experienced, and added to the accumulated rate since departure, obtained in a similar manner.

It seems pretty well established that the coefficient of
change for a degree remains the same, or nearly the same, for each individual watch, although the absolute rates of the watch (which depend upon many things) may vary.

The chronometers issued to H.M. ships have no such Chronoinformation sent with them, for this reason.

The timekeepers are carefully chosen from many sent to Ships. the Royal Observatory by different makers for trial, and only those whose compensation is such that they show very little change of rate at a great variation of temperature, or, in other words, whose compensation is as perfect as may be, are taken, the limit allowed being one and a quarter second of change of daily rate for forty-five degrees of temperature.

This reduces the variation of rate, arising from change of temperature probable in a voyage, to very small quantities, which would be lost in the variation arising from other causes, and it is not considered necessary under these circumstances to give data for allowing it.

Thirdly. The oil in the pivots. With good oil the in- Quality equality arising from age shows itself in the shape of $a^{\text {oil. }}$ gradual and tolerably uniform acceleration of rate, generally in the direction of gaining, with a new chronometer, and when the instrument is older and all parts somewhat worn, in the contrary direction. It should be excessively small, and our opinion is that in the practical question of meridian distances, the labour of ascertaining it is not repaid by the result. It is difficult to separate the error due to this from that originating in defective mechanism, and though formulæ have been elaborated for its detection, we do not propose to give them here.

Fourthly. Vibration and shocks. However well chrono- Vibrations meters may be stowed, the jars from seas striking the ship, ${ }_{\text {shocks. }}^{\text {and }}$ and other like accidents, must be communicated more or less to the chronometers. The vibration of the screw is in some vessels sufficient to pass through all the soft cushions in which they may lie, and must have its effect, mora especi lly from the fact that the watches themselves are hanging in the metal gimbols, in which there must be play sufficient to
allow them to swing easily, and therefore enough to set up small shocks on any violent movements of the ship.*


#### Abstract

* In connection with the observations of the Transit of Venus of 1874, Lord Lindsay conveyed nearly sixty chronometers to Mauritius. These were kindly permitted by him to be used in assisting to determine the meridian distance between Mauritius and Rodriguez, when they were shipped on board H.M.S. "Shearwater," under the author's command. As the results by these watches, both of the distance between Mauritius and Rodriguez, and Mauritius and Aden (between which latter places they were conveyed in the mail steamer), were remarkably


 good, and as the results by the "Shearwater's" chronometers which were admitted into the distance Mauritius to Rodriguez were not so satisfactory, a description of the manner in which Lord Lindsay's watches were stowed may not be out of place. We may add that the "Shearwater" had to beat up for eight days against a strong tradewind on one occasion, and was a very lively ship.The watches were taken out of their gimbols and placed in square boxes, which held nine of them each. I he partitions of these boxes were thickly stuffed with very soft material (cotton wool) covered with satin, so that each watch lay in a bed of down which was made exactly to fit it.

Each box was fitted with a metal framework after the fashion of gimbols, the outer pivots of which fitted into carefully turned sockets, in two upright columns of wood,
which were firmly screwed to the deck. Each pair of uprights carried three boxes of watches.

The effect of this was that any slight shocks to the boxes caused by seas striking the ship, or by longitudinal slipping of the pivots, were entirely deadened before reaching the watches themselves.

This mode of stowing necessitated taking the watch up bodily in the hand to wind, which at first sight seems dangerous, and undoubtedly does present more opportunity for accident than the ordinary method; but, as far as the author is aware, none took place during the five or six months the watches were thus treated, and the admirable agreement of the results seems to show that this system was unusually successful.

Whether it could be adopted on board men-of-war, especially small ones, which are usually employed in surveying duties, is another matter, as it certainly demands more space, both for the swinging of the box and to allow of free access for handling the watches.

It was very convenient for comparing, as cne watch could be held to the ear while the eye took the time by the standard.

It seems probable, also, that the temperature would be more constant, from the fact of the watch being imbedded in thick soft material. The lid of each box was. also stuffid softly, and, when in place, pressed on the glass of each watch, excluding all air.

In our opinion the variation of rate arising from these causes is, with the generality of Admiralty watches, the larger proportion of the total change.

The only notice that can be taken of variation of rate due to this, is to consider it as detracting from the general value of the meridian distance; and the nature of the passage, whether rough or smooth, should therefore be noted in the returns.

Magnetism is another disturbing cause, to which irregularities of chronometers have been referred. As no trustworthy conclusion as to this has been arrived at, we do no more than mention it.

It follows as a matter of course, from the preceding observations, that not only will the rate of a chronometer as ascertained before leaving a port be different to that found on arrival at another port, but that the sea rate for the interval will probably be different from either of them.

We have now to consider the means at our disposal for approximating to the true rate under different circumstances.

The most satisfactory circumstance under which we can Interpoladetermine meridian distance (after those already described) is when, having left a port A called at B (the position we places is when, having left a port A, called at B (the position we places want), where we have only stayed long enough to get Error, whose longitude and eventually arrived at K without further stoppage, the longitudes of $A$ and $K$ are sufficiently well known to take them as secondary meridians.

In this case, by applying the known difference of longitude between A and K to the observations at A, we find the Error on mean time at K at the epoch of starting from A . The difference between this and the Error ascertained on arrival at K , divided by the duration of the voyage, will give us a fair sea rate, which we shall assume to be uniform and invariable during the voyage. A simple application, then, of accumulated rate up to the time of observations at B, will give us the meridian distance from A to B , dependent upon A and K being in certain longitudes.

We can use the same means if we call at more places than
one on the way between $A$ and $K$, but each stoppage will probably detract from the value of the sea rate.

We are here using the sea rate only, and therefore shall take the date of the last observations at departure, and first on arrival, as the epochs for calculation. If we have obtained rate on departure and arrival, we shall gain valuable information about our chronometers, as we shall be able to see how far they have obeyed any theory as to gradual or uniform change of rate, according to the ordinary assumption that the sea rate is the mean of the two harbour rates.

The value of a meridian distance by this method will, as always, be influenced by the conditions of temperature, fair passage, \&c., which must therefore be taken into consideration and recorded.

It will be remarked that by this method, a large amount of time is saved, and opportunities otherwise wasted are utilised to their full extent. Instead of the necessity of waiting, certainly at $A$ and $K$, and perhaps at $B$ as well, for from five to eight days, a simple call of a few hours at each is sufficient to obtain an excellent result. Moreover, instead of involving the eccentricities of chronometers during the time in harbour at each end, we only include in the calculation the actual time while travelling at sea, and thereby save the irregularities of a good many extra days.
Shadwell's Captain Shadwell, in treating of this case, does not use an

Treatment. invariable sea rate pure and simple, but supposes that the rate of departure has gradually and uniformly changed into the sea rate, which he considers as the rate on the middle day of the passage only. He therefore applies for his determination of $B$ from $A$ an intermediate rate between the sea rate and rate of departure ; but our experience does not lead us to think that this is an advantage, although by doing the same to the sea rate and rate of arrival, he gets a second meridian distance from B to K , and takes the mean of the two as his result. Our opinion is that, temperature being left out of the question, a better result is likely by using a uniform sea rate.

The algebraic formula for meridian distance by above Formula method of uniform sea rate, is, for Interpolation.

$$
\mathrm{M}_{1}=\lambda_{1}-\lambda+\tau \frac{\lambda_{2}-\lambda-\mathrm{M}}{t}
$$

Where M is meridian distance between the terminal points A and K of the voyage.
$M_{1}$ is meridian distance between port of departure, and a port B touched at on the voyage.
$\lambda$ is error at $A$ on leaving.
$\lambda_{1} \quad, \quad B$ on toucbing.
$\lambda_{2} \quad$, K on arriving.
$t$ is interval between observations at $A$ and $K$.
$\tau \quad$ " $\quad, \quad$ A and B.
In any case of a ship's calling at a place as an intermediate port on her voyage between two other places, it may be well to send home, beside the meridian distance obtained in the ordinary manner, the information which would enable the office, if or when it possesses the true difference of longitude between the terminal ports, to calculate the difference of longitude of the intermediate place by the last formula.

This necessary information will be-

$$
\lambda, \lambda_{1}, \lambda_{2}, t, \text { and } \tau
$$

In transmitting this information we could, for the facilita- Meaning tion of computation afterwards, give only the mean of the the Errors. Errors of all the chronometers, instead of the individual error of each, or in other words, assume an imaginary watch, the result of which will give the same meridian distance, as the mean of the meridian distances by each chronometer; but the adoption of this method will of course preclude any estimation of the value of the distance by the concurrence of individual results, and should be therefore only adopted when we have reason to believe from inter-comparisons during the voyage that the watches have been going well together.*

[^23]There is another adaptation of the methods of sea rates as obtained by Error at two places whose longitude is known, which is often useful.

Adaptation of foregoing.

If we obtain Error before leaving A, and after some days call at $B$, whose difference of longitude from $A$ is known, and there obtain Error again, we get a very good sea rate for the subsequent part of our voyage, which we can utilise to determine the position of C, any third place at which we may hereafter soon call, with a probable better result than by means of harbour rates. If absolute altitudes are used, they must of course be both either A.M. or P.M.

This method is especially useful for navigational purposes. Suppose a ship to leave Portsmouth and to call at Gibraltar for a few hours only. Error can be obtained, and by means of the known difference of longitude a sea rate deduced, which will give a better landfall for Malta, than the harbour rates at Portsmouth.

## Mean Harbour <br> Rates.

When our voyage is simply from one port to another, and we wish to find the meridian distance between them, we must depend mainly upon the harbour rates ascertained before departure and on arrival.

The ordinary and rougher method is to assume that the rate has changed uniformly from the rate of departure to that of arrival, and that therefore the mean of the two rates will represent the mean rate during the passage. We believe that (owing to the many causes of variation impossible to formulate) in most cases, and especially where temperature has been, in the chronometer room, fairly uniform, this method will give as good a result as any other; but where temperature has changed much, the result of long meridian distances with such rates will have but very little value, and that a correction for temperature will much improve the result, if we can apply it.

French naval officers have done much in working out this question, and Captain Shadwell gives their separate theories and formulæ. To our mind the method of M. Mouchez is the most practical; and not undertaking to enter into the
question of acceleration, nor depending on observations on the watches while in the Observatory, it is more adapted to actual work.

Mouchez proceeds on the assumption, which is near enough to truth for the method, that the rate varies uniformly with the temperature ; but in working on this hypothesis, we must not forget that for each chronometer there is a point of temperature at which the rate is at a maximum, and that the sign of the variation will change as we pass it.

He ascertains by observations for rate at different temperatures, undertaken by the officers when the chronometers are embarked, the coefficient for temperature by simply dividing the difference of rate by the difference of mean temperatures during the intervals of rating.

This coefficient of change will remain constant for some period, though the actual rates themselves will alter from other causes ; nevertheless, the more these observations are multiplied the better, and the latest determinations will be used in practice.

In determining the sea rate for a meridian distance, he applies to the rate of departure the change of rate due to the difference between the mean temperature during rating, and the mean temperature during the passage, which gives one value for the sea rate. Doing the same for the rates of arrival, he gets another value for sea rate. The mean of these two he takes as the final mean sea rate to be used. One weak point here is that the mean temperature, T, of the compensation will not be indicated, unless many observations at different temperatures are made. It will therefore add considerably to the value of this method if we can find T.

It will be more satisfactory if we can get this from the Liensson's Observatory; but a formula for ascertaining it is given by $\begin{gathered}\text { Formula } \\ \text { foscer- }\end{gathered}$ Capt. Shadwell, from M. Lieussou, which we here append, taining T. but we apprehend that in practice not many opportunities. will present themselves for making use of it. It depends on the results of four observations for rates, at equal intervals of time, and at different temperatures, a difficult condition
to satisfy except with artificial aid for the temperature. M. Lieussou remarks, " that four rates and four temperatures, observed at intervals of ten days, determine the constants for each chronometer with a precision sufficiently remarkable." With the other constants we do not propose to deal, but solely to give his formula for ascertaining T, which is
$\mathrm{T}=\frac{1}{2} \cdot \frac{\left(m_{1}-2 m_{2}+m_{3}\right)\left(t_{2}^{2}-2 t_{3}^{2}+t_{4}^{2}\right)-\left(m_{2}-2 m_{3}+m_{4}\right)\left(t_{1}^{2}-2 t_{2}^{2}+t_{3}^{2}\right)}{\left(m_{1}-2 m_{2}+m_{3}\right)\left(t_{2}-2 t_{3}+t_{4}\right)-\left(m_{2}-2 m_{3}+m_{4}\right)\left(t_{1}-2 t_{2}+t_{3}\right)}$
Here $\mathrm{T}=$ mean temperature of compensation required. $m_{1} m_{2} m_{3} m_{4}$ are the four observed rates corresponding to $t_{1} t_{2} t_{3} t_{4}$ the four temperatures.
The intervals between the sets of observations for rates should be between 10 and 30 days.

Hartnup's Formulæ.

Mr. Hartnup's formulæ are somewhat different, and do not give exactly the same results with the same data.

He observes the rate at three different temperatures not less than $15^{\circ}$ apart, but there must be an equal number of degrees between them.

The same remark already made as to M. Lieussou's method will apply here, viz. that in service afloat it will be difficult to fulfil the conditions of observation. His formulæ are as follows :-

$$
\begin{aligned}
& \mathrm{C}=\frac{2\left(d-d_{1}\right)}{p^{2}} \\
& \mathrm{~T}=t_{1}+\frac{d+d_{1}}{2 \mathrm{C} p} \\
& \mathrm{R}=r_{1}-\left(\mathrm{T}-t_{1}\right) \frac{d+d_{1}}{2 p}
\end{aligned}
$$

Where C is the coefficient of change of rate,
T is temperature of maximum rate,
R is rate at that temperature,
$t_{1}$ is the middle temperature,
$r_{1}$ is observed rate at temperature $t_{1}$,
$d$ is difference of rate between that at lowest temperature and $t_{1}$,
> $d_{1}$ is difference of rate between $t_{1}$, and that at highest temperature,
> $p$ is difference between highest and lowest temperatures observed at.

Then to find the rate in any required temperature.
If $\mathrm{N}=$ any number of degrees from T .
Rate at $T \pm N=R+C N^{2}$.
In using the rates of departure and arrival in calculating a Epochs of meridian distance, the Errors at the last observation at de- Caleulation. parture and first at arrival should not be taken for the epochs of calculation, but the mean of the two should be used for the purpose, for it is at the mean date between the two observations for each rate at which the latter is actually fixed. Thus, if we observe at a place A on the 2nd and 8th, and again on arrival at B on the 20th and 27th, we should take the mean of the two Errors on 2nd and 8th and call it the Error at A on the 5th, and similarly at B on the $23^{\text {rd }} 5$, and use the interval between these two epochs for the multiplication of the mean rate.

The formula given by Tiarks, and generally adopted, for calculating the meridian distance between two places by rates at departure and arrival, without any consideration of temperature is

$$
\mathrm{M}=\lambda^{1}-\left\{\lambda+t\left(a+\frac{b}{2}\right)\right\}
$$

Where M is meridian distance required,
$\lambda$ the Error at mean epoch of departure,
$\lambda^{1} \quad "$ arrival,
$t$ the interval between the two epochs,
$a$ the rate at departure,
$b$ the difference between rate at departure and arrival.
In calculating $t$, the difference of time, due to difference of Caloulatlongitude between the two places, must not be forgotten; but, Interval. being reduced to the decimal of a day, must be added or subtracted to the interval between the epochs, according as we have moved westward or eastward.

Thus, if our mean epoch at A is at noon on the 20th, and
at another place, B, 30 degrees to the westward, at noon on the 30th, the interval of time for accumulated rate will not be ten days, but ten plus the difference of longitude of the two places, or $10^{\text {d. }} 08$; for the sun, having completed the ten days by returning to the meridian of A , will take yet another $\cdot 08$ of a day to be on the meridian of B .

Similarly, in calculating sea rate from observations at different places where longitude is known, we must allow for this difference of time.

Thus, having taken sights at A at noon on the 2nd, and at B, 20 degrees eastward, on the 11th at noon, the interval with which to divide the difference of Error at A (corrected for difference of longitude) and Error at B, to ascertain the daily rate, will be $8^{d .94}$. as the sun will be on the meridian at B $\cdot 06$ of a day earlier than at A.

Tiarks' Formula with temperature correction.

The same formula, when intending to correct for temperature, will stand thus:

$$
\mathrm{M}=\lambda^{1}-\left\{\lambda+t\left(a+\frac{b}{2}\right)+t\left(\frac{\theta+\theta^{1}}{2}-\theta_{2}\right) y\right\}
$$

Where, the other letters signifying as before, $\theta$ is mean temperature during rating at departure, $\begin{array}{lll}\theta^{1} & " & " \\ \theta_{2} & " & \text { during the passage. }\end{array}$ $y$ is the coefficient for temperature found from previous observations.
Algebraio In all cases of correction for temperature the algebraic sign signs. of $y$ must be remembered, that is, it must be applied according to the observed effect in altering the rate.

The same remark applies to the algebraic signs of all quantities in the formulæ.

Thus in the formula:

$$
M=\lambda^{1}-\left\{\lambda+t\left(a+\frac{b}{2}\right)\right\}
$$

the signs which are here given, as throughout, for chronometers slow of mean time and losing rates, will only be true
under those circumstances with increasing losing rates and when moving eastward. A consideration of the facts, and obvious effects of the corrections, is perhaps the best course to take to determine these signs.

A meridian distance, founded only upon rates obtained at one end, without any further correction, cannot be considered as of any value whatever, unless the voyage be very short.

When using the combination of harbour rates at each end of a voyage, $A$ to $K$, to determine the position of some intermediate place, B , we must, to be consistent, remember that we are assuming that the rate has gradually and uniformly changed from that of departure to that of arrival, and that the rate to be used for a portion of the voyage will not therefore be the same as that for the whole of it.

Tiarks, interpreted by Capt. Shadwell, gives us the following formula.

$$
\mathrm{M}_{1}=\lambda_{1}-\left\{\lambda+\left(\tau a+\frac{\tau^{2}}{2} t^{b}\right)\right\}
$$

Where $M_{1}$ is meridian distance $A$ to $B$.
$\lambda_{1}$, Error at B.
$\lambda$,, Error at mean epoch at A.
a ", rate of departure.
$b$,, difference of rates of departure and arrival.
$t$," interval between mean epochs of rating at A and K .
$\tau$,, interval between mean epoch at $A$ and observations at B.
It is to this case that our observations on page 263 refer, to the effect that the data for calculating the position of B, as interpolated between A and K , may be also transmitted home.

A very good way of measuring meridian distance for the Use of scale of a chart, when the actual distance between the Rockets. stations is not too far, is by rockets. Parties landed at either end of the base whose difference of longitude is to be measured, ascertain the Error of their pocket chronometers. The ship, midway between the two, fires rockets vertically,
and the bursting of these, an instantaneous phenomenon, is noted by the watches at either end.

An ordinary service signal rocket can be depended on to mount 1200 feet, and should reach 1600 . The bursting, if it occurs, as it should, at the highest point, will therefore be visible nearly 40 miles on either side, which will permit a base of 75 miles to be measured under very favourable circumstances of dark night, and clear atmosphere, when the stations are east and west of one another.

Rockets will not often, however, be seen this full distance; the balls of fire, released on bursting, are scarcely bright enough; and supposing the observers to be at the sea-level, the burst of the rockets will only just be above the horizon, in which position atmospheric disturbances are greatest, and may disperse the rays of light before they can reach the observer. Ascending a hill, therefore, will greatly assist clear vision, and the use of a pair of field glasses will do wonders. Twenty-five miles, on either side, should be measured in this way without any great difficulty.
Transmitting Results.

It is important, in transmitting to the Hydrographic Office the results of a Meridian Distance, that sufficient information is given to enable it to be valued and compared with others between the same places. The form appended on page 271, is that now employed.

No. 24.
Return of Meridian Distance, H.M.S. 1874.

Captain.
From Seychelles.
To Zanzibar.
Observation spot, Syychelles-Hondouls Jetty, Mahé .. Lat. 43715 S . " "Zanzibar—Old British Consulate Garden " 60945 S . Rates used-Mean rates of departure and arrival.
Error at Seychelles on Jan. 13th by Eq. Alts. ©

| $"$ | $"$ | $"$ | $"$ 18th ", "Zanzibar | $"$ Feb. 1st | $"$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $"$ |  |  |  |  |  |
| $"$ | $"$ | $"$ |  |  |  |
| $"$ | $"$ | $"$ 9th | $"$ | $"$ | $"$ |

Duration of passage, Jan. 18th, 6 P.m. to Jan. 30th, 4 p.m.
Epochs for calculating accumulated rate, Jan. $15 \frac{1}{2}$, Feb. 5 th $=20.545$ days.


## CHAPTER XV.

## TRUE BEARING.

By Theodolite-By Sextant-Variation.
In nearly all descriptions of surveys true bearings will be used.

By theodolite and sextant.

The most correct method, from a shore station, is to use the theodolite, which will alone give a very good result for azimuth; but it is better to get the altitudes with a sextant and artificial horizon, when two observers are available.

The theodolite in this case is only used for taking the horizontal angle between the sun and the zero.

There are three principal methods in use for obtaining the azimuth. By observations at equal altitude A.M. and P.M., by observations A.M. and P.M. at nearly the same altitudes, or by single observations.
Single altitude generally sufficient.

The former is theoretically the more correct, as many errors are eliminated; the second is nearly as good; but our experience is that with single observations taken with the sun near the prime vertical, with instruments in good order, the result is quite as near the truth as is generally requisite in marine surveys. When an extensive piece of coast is being surveyed, we shall, as before stated, depend mainly upon the astronomical positions for the scale and bearing of the chart, but nevertheless accuracy in obtaining the original bearing for working is necessary.

A very important point is careful levelling, which should be done with the telescope pointing in the direction of the sun, and the accuracy of the movement of the telescope in a
vertical plane should be tested, as no method will eliminate error due to want of such accuracy.

In the first method, the sun will be observed at an even stated altitude, and the sextant will be set beforehand, the observer using it giving the "stop" to the theodolite observer.

In the others, the theodolite observer generally calls the "stop," and the sextant observer takes whatever altitude it happens to be.

To arrive at a satisfactory result in either case, it is neces- Changing sary to take several sets, with a different degree of the are degre. pointed at the zero in each, so as to eliminate the errors of the horizontal arc of the instrument.

As it is the bearing of the sun's centre which we obtain Correeting by working out the azimuth, the aim of the theodolite to sun's observations is to get the horizontal angle between that centre and our zero; but it is manifest that we cannot trust our eye to place the cross-wires of the telescope exactly on the centre of the sun, nor can we place the wires truly vertical and horizontal.

If we could do the latter, we could arrive at the angles to the centre by merely observing the sun in one quadrant, and applying the semi-diameter $\times$ sec. alt. ; but we must not trust this, if we want fair accuracy.

In equal altitude observations, the method is to fix on an Methodiof altitude for both sextant and theodolite, and set the vertical bearing by arc of latter at it. In the forenoon, bring the sun so that it altitudes. is in the lower half of the field, and approaching the vertical wire. The theodolite observer then keeps the limb of the sun in contact with the vertical wire, and below the horizontal one. If the theodolite is truly levelled, he will not need to touch his vertical tangent screw, but if necessary he must do so, to keep the upper limb of the sun as nearly touching the horizontal wire as he can. When the upper limbs of the sun in the artificial horizon are in contact, the observer calls "stop," and the motion of both tangent screws of the theodolite ceases. The horizontal arc is then read.

Then, without moving the theodolite in altitude, the other limb of the sun is brought on the other side of the vertical wire, and the reading made when the artificial horizon observer gives "stop," on the lower limbs of the sun coming in contact.

The sun will thus have passed between opposite quadrants of the cross-wires, as in the diagram Fig. 38.

Similar observations are made at the same altitude in the afternoon, the lower limb coming first. Each set will thus consist of two observations A,M, and two P.M. In this method the time must be taken exactly, which is a drawback, as it either requires three persons, or that one should take time as well as his observation. There is, however, no necessity to know the local time very exactly, all we want is the true elapsed time.


FIG 38.


Calculating Bearing by Equal Altitudes.

To work out the equal altitude observation, the means of the times, and of the horizontal angles of A.M. and P.M. respectively, are taken.

If the sun had no motion in declination, the mean of A.M. and P.M. horizontal angle would be the angle on the horizontal arc corresponding to the true meridian, or, in other words, the bearing of the zero; but as this is not so, we must work a correction similar to the Equation of equal altitudes when obtaining time, to be applied to this mean of the angles.

The formula for this is-

$$
\text { Correction }=\frac{c}{2} \operatorname{Cosec} \frac{\text { time elapsed }}{2} \text { Sec lat., }
$$

where $\frac{c}{2}$ is half the change of declination in elapsed time.
This correction is additive to the angle when the sun is
moving from the nearest pole, and subtractive when moving towards it.

Let us take the following example-


Lat. $30^{\circ} \mathrm{N}$. Declination corrected to Greenwich time of A.M. observation $18^{\circ} 14^{\prime} \mathrm{N}$. Sun moving north.!


A number of similar sets, taken with different degrees as zero, will give a very correct result, and though all instrumental errors will not be eliminated, the majority of them will disappear.

In "single" observations, each set will consist of four Method by contacts, in each of which the sun will be tangential to the Single vertical wire in a different quadrant of the field.

The mean of these will then be the angle to the sun's centre, corresponding to the mean of the four altitudes.

When the altitude is being taken by a sextant, it will only be necessary for the theodolite observer to be very exact with the contact of the side-limb of the sun; but his upper or lower limb, as the case may be, should be as nearly touching the horizontal wire as possible, to insure the elimination of the wire error.

It is quite immaterial in which quadrant the observer commences; but whatever plan he adopts, he should always
fig 39.

observe in the same manner, as it prevents contusion and mistakes. The sun will appear as in the diagrams in Fig. 39.

When taking the observation with the theodolite alone, it will of course be necessary to see that both the horizontal and vertical wires are truly tangential to the sun's limbs.

Six sets should give a very good bearing; but if the theodolite is a very small one, or is known to be badly graduated, more may be necessary.

Half the altitudes in the artificial horizon may be taken with upper limb and half with the lower; but this is not
important, as if the observation be made when the sun is near the prime vertical, a small error in the altitude will but slightly affect the azimuth.

The azimuth of the sun having been obtained by the ordinary rule of nautical astronomy, the true bearing of the object is found by applying the mean of the theodolite angles of that set.

In finding a true bearing with sextant only, it will be Method by more accurate if two observers are employed-one to take Sextant. the altitude, the other to measure the angular distance at the same instant.

If only one observer is available, he must take altitude and angular distance alternately, taking care to end with the same

observation as that with which he begins, so that the mean of each kind will correspond as nearly as may be in time. Thus, if he begins with altitude, he must also end with altitude.

This method should not, however, ever be used when a theodolite is available, and is only adopted for true bearings from the ship, in an irregular survey.

In this instance we have to calculate the horizontal angle, Calonlawhich with the theodolite we obtained directly.

The object should be so chosen that the line joining it with Angle. the sun should not make a larger angle with the horizon than $20^{\circ}$, and the less the better, as any inaccuracies of observation will not then be much increased when the horizontalengle is deduced. If we take an object $90^{\circ}$ or more from the sun, these
conditions will be fulfilled, the sun being of course comparatively low, and near the prime vertical.
Two Cases. There are two separate cases :-
First, when the object whose bearing is desired is on the horizon; and secondly, when it has a sensible altitude, as a mountain top.

In the first we have to solve a quadrantal triangle as shown in Fig. 40.

Object on horizon.

In this Fig. $Z$ is zenith, $S$ is sun, and $O$ the object on the horizon.

We have $Z O=90^{\circ}$. Z S the apparent zenith distance, and OS the observed angular distance, to find 0 ZS , the horizontal angle required, or

Cos horiz. angle $=$ Cos ang. dist. $\times$ Sec. app. alt.

## Example.

(Object on horizon, two observers with sextants and artificial horizon.)
On June 1st, 1881, at Ship IV. $7^{\mathrm{h}} 24^{\mathrm{m}}$ A.m. mean time of place, observed altitude of $\varrho 30^{\circ} 13^{\prime} 50^{\prime \prime}$, mean angular distance of $\propto$ to Pine $\triangle$ on horizon $84^{\circ} 26^{\prime} 20^{\prime \prime}$, object right of $\odot$. Lat. $40^{\circ} 26^{\prime} 15^{\prime \prime} \mathrm{N}$. Long. $28^{\circ} 00^{\prime}$ E. Index Errors - $35^{\prime \prime}$ and $0^{\prime \prime}$. H.E. 20 ft .



Azimuth of sun .. .. .. N. $85^{\circ} 49^{\prime} 25^{\prime \prime}$ E.
Cos, true Ang. dist... $\quad 8 \cdot 965353$
Sec. app. alt. .. -064294
Cos. Hor. Ang. .. $\overline{9 \cdot 029647} \quad 8351 \quad 14$ Hor. ang.


In the second case, we have a spherical triangle with three object sides known, as in Figure 41.

Here, Fig. 41, we have Z O, the zenith distance of the object,


Z S, and 0 S , as before, the apparent zenith distance of sun, and angular distance; to find OZS , the horizontal angle
required, which can be done by any of the applications of the formula

$$
\operatorname{Cos} 0 Z S=\frac{\operatorname{Cos} O S-\operatorname{Cos} Z S \cdot \operatorname{Cos} Z O}{\operatorname{Sin} Z S \cdot \operatorname{Sin} Z O}
$$

## Example.

(One observer with sextant, sea horizon, alternate observations, object elevated.)
At S. Ann's ©, October 5th, 1881, Lat. $5^{\circ} 10^{\prime}$ S. Long. $57^{\circ} 14^{\prime}$ E., the following observations were taken for true bearing of Snow Peak. Height of eye 10 feet, object right of $\odot$. M.T. place 8.00 A.M. I.E. $-50^{\prime \prime}$.




The Pole star may be used in the northern hemisphere to Use of obtain true bearings at night.

Circumstances under which this is useful are related at page 155 , which see.

The Greenwich time must be known, and the angle between the Pole star and object whose bearing is required, must be large.

Measure the angle and take the time.
Ascertaining the sidereal time of observation as in ordinary Pole star calculation, add six hours to it for a second sidereal time.

Out of Table I. in Nautical Almanac, take the correction with first sidereal time, which, applied with the reverse sign to the latitude, will give the altitude at the time.

Take out a second correction with second sidereal time, which will be the rectangular deviation of Polaris from the meridian.

To calculate the horizontal angle answering to this, the formula is

Sin horizontal angle $=$ Sin correction $\times$ Sec. alt.
which will give the true bearing of Polaris, east of meridian when first sidereal time is between 13 h .20 m . and 1 h .20 m ., west when otherwise.

## Example.

August 10th, 1881, Lat. $43^{\circ} 30^{\prime}$ N., Long. $66^{\circ} 30^{\prime}$ W., at 13.34 G.M.T. Observed angle from Polaris to Seal I ${ }^{\text {d }}$. Light $80^{\circ} 10^{\prime}$, right of Polaris.

| G. M. T. | $\begin{array}{rl}\text { h. m. } \\ 13 & 34\end{array}$ | Cor for 1st Sid. T. | $\circ$ $-\quad 017$ |
| :---: | :---: | :---: | :---: |
| Long. .. | 426 | Latitude | 4330 |
| M. T. ship | 908 | Altitude Polaris .. | 4313 |
| Sid T. noon | 916 |  |  |
| Acceler. | 2 |  |  |
| 1st S. T. obs. | 1826 | Sin. Corr. for 2 nd S. | $8 \cdot 35018$ |
|  | + 6 | Sec. Alt. | -13741 |
| 2nd " | 026 | Sine True B | $8 \cdot 48759$ |
| Corr. for 2nd S. T. .. $1^{\circ} 17^{\prime}$ |  | Polaris .. N. $1^{\circ} 45^{\prime}$ E. |  |



Seal 1 ${ }^{\text {d }}$. L. .. .. .. .. N. 78 12 E.

## VARIATION.

Accurate variations are very useful in all parts of the world, as from them the lines of equal variation shown on
charts are drawn; but to enable them to be so used, they must be trustworthy.

Variations obtained by swinging the ship carefully with a sea obsersmooth sea, and in water of, say, over 50 fathoms are most ${ }^{\text {vations. }}$ useful, as fear of local attraction is thereby removed.

The bearing of the sun, or of an object sufficiently distant to maintain the same direction whilst steaming round, and of which the true bearing is obtained, should be observed on evenly distributed points. There is no necessity to observe more than on every other point, and good results will be obtained from the cardinal and quadrantal points.

In the mean of the total errors the deviation will be eliminated, and the result is the variation.

A full report of the observations should be transmitted home.

Shore variations are also of value, when taken on ground free from suspicion of local attraction, for the determination of the true variation, and, in other cases, for the information they afford on the amount of the local attraction as obtained by comparison with the variation found from sea observations in the vicinity, a point of much interest, and often of practical importance.

The requirements for a good shore variation, that the Hydrographic Office can put confidence in, are as follows:

1. The true bearing of different points (about six) as equally distributed as possible round the circle whose centre is the observation spot, must be well and accurately observed with a theodolite.
2. Bearings of all these points must be taken by the compass from the observation spot.
3. Different sets of observations must be made with different pivots and with different cards.
4. The spot on which the observation is made should be free from every suspicion of any iron in the vicinity, and the nature of the rock, or whatever the formation may be near the observation spot, should be mentioned in the return transmitted home.

Points 1 and 2 are necessary precautions against the errors of the card caused either by bad graduation, or from accidental bending of the card. In ascertaining the true bearings, it will only be necessary to observe one object, when theodolite angles to the others will give their difference of bearing.

As regards No. 3, all compass cards have an error caused by inaccurate affixing of the magnetic needles, and it is necessary to multiply observations, and make certain the card is working properly.

Shore observations should be obtained at stations where the variation is already well known, when opportunity offers, as these will enable the Office to calculate the change of variation.

An example of observation for variation is appended.

Variation deduced at Office.

Although the variation is here deduced to show the method, this would not be done in forwarding these observations to the Admiralty, as there are certain card-errors to be applied first.
Variation.

| Date. | Observation Spot, Nature of ground, \&c. | Nature of Observation. | Objects. | True Bearing. | Magnetic Bearings. |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | Standard Compass B 154. |  |  | Dover. H. 0. |  |
|  |  |  |  |  | Card A. | J. | Spare A. | A. | B. |
| $\begin{gathered} 11 \text { Dec. } \\ 1878 . \end{gathered}$ | West Base. <br> Pasha Liman I ${ }^{\text {d }}$. <br> Sea of Marmara. $\qquad$ Alluvial soil. <br> Lat. $40^{\circ} 28^{\prime} \mathrm{N}$. <br> Long. $27^{\circ} 34^{\prime} \mathrm{E}$. |  |  |  | $\circ$ <br> N. <br> 8 05 E.  <br> 62 10 <br> 120 15 <br> 186 20 <br> 249 15 <br> 309 45 <br> 348 50 | $\circ$ <br> N. <br> 8 05 E.  <br> 62 20 <br> 120 15 <br> 186 10 <br> 249 05 <br> 309 42 <br> 348 40 | N. 815 E. $6218$ <br> 12012 <br> 18618 <br> 24915 <br> 30952 <br> 34842 | $\begin{array}{rl} \text { N. } 8 & 25 \text { E. } \\ 62 & 40 \\ 120 & 42 \\ 186 & 10 \\ 249 & 10 \\ 310 & 08 \\ 349 & 05 \end{array}$ | $\begin{array}{cc} \begin{array}{c} \circ \\ \mathrm{N} .8 \\ 8 \end{array} 0^{\prime} \\ 62 & 35 \\ 120 & 33 \\ 186 & 15 \\ 249 & 33 \\ 310 & 05 \\ 348 & 50 \end{array}$ |
|  |  |  | Mean. | $17724$ | $\circ$   <br> 183 31 26 <br> 177 24  | $\begin{array}{\|ccc\|} \hline \circ & \prime & \prime \prime \\ 183 & 28 & 26 \\ 177 & 24 & \\ \hline \end{array}$ | $\begin{array}{\|ccc\|} \hline \circ & \prime & \prime \prime \\ 188 & 33 & 09 \\ 177 & 24 \end{array}$ | $\circ$  <br> 183 45 <br> 13  <br> 177 24 | $\begin{array}{ccc} \hline \circ & \prime & \prime \prime \\ 183 & 42 & 43 \\ 177 & 24 & \end{array}$ |
|  |  |  | Rough (without | Variation.. card errors) | W. 60726 | 60426 | 60909 | 62143 | 61843 |

## CHAPTER XVI.

SEA OBSERVATIONS.
Double Altitude-Sumner's Method-New Navigation-Short Equal Altitude-Circum-meridian Altitudes of Sun.

As regards For surveying purposes, observations at sea are mainly reSurveying Operations. quired for fixing the ship's position when sounding banks, or looking for vigias.

We cannot hope to attain to any very great accuracy, and are much dependent on weather and the state of the sea and clearness of the horizon. As longitude must depend entirely on the chronometers, we must in cases where we require all the accuracy we can get, as in fixing the position of banks far away in mid-ocean, wait until we can again obtain Error and rates to give the final positions; but with ordinarily good chronometers our daily positions obtained whilst sounding will be correct, comparatively one with the other, so that, we can at once plot and delineate the shape of the banks, which is what we want at the time.

Refraction.

Position early in the day.

In all observations at sea it must be remembered that the horizon may be affected by abnormal refraction, and no dependence can be placed on a latitude or longitude deduced from altitude observed on one side alone. In certain cases the error may amount to $2^{\prime}$ or even $3^{\prime}$.

With a high sun at noon, when accuracy is aimed at, it is well to observe the opposite side of the horizon; an awkward observation at first, which practice will render easy.

One great object when sounding or looking for banks is to obtain a position as early as possible in the day, after lying-
to probably all night, as in the vicinity of banks currents are nearly always set up, and in variable directions, so that we cannot at all depend upon dead reckoning, or upon finding ourselves where we laid-to the night before, and in many instances, unless we know in which direction to go, it is useless to move at all.

In such cases observations should be taken throughout the Star Latinight; for, though they will give but approximate results, latitudes by pairs of stars north and south of zenith by the same observer should give under favourable circumstances a position within five miles of the correct latitude, which will tell us if we are drifting much in the line of the meridian, and also affords us an approximate latitude to work longitude by, in the morning.

The worst of it is, that circumstances apparently favourable are often not really so, as the great source of error in night observations is the impossibility of being certain of the horizon. A false horizon will frequently look so well defined as to mislead the best observer, and will of course throw out the resulting latitude greatly. Thus we can put no great faith in latitude by stars, and none whatever in a single observation, or even in a single pair, as the horizon in one direction may be true and in another false. It is only in a series of pairs of stars through several hours that we can have any confidence, as if the result of these agree fairly, or steadily show movement in one direction (the effect of a current) we may then feel pretty sure of our position as far as latitude goes.

Night observations at sea for longitude are not of much use ; star but, under unusually good circumstances of horizon, the mean observaof two star chronometers, one east, the other west of meridian, longitude. may be better than nothing.

If, however, when the day has sufficiently broken to stars at enable the horizon to be clearly seen, we can get observa- daybreak. tions of bright stars or planets on different bearings, we can obtain an excellent position by Sumner's method from which to start our day's work.

Or, if we can only get one daybreak star, as soon as we
can get an observation of the sun, we can combine it with the daybreak observation.

Venus by day.

Different Methods of obtaining Position.

The planet Venus can, when near quadrature, be observed all day. The altitude can be calculated, when she will easily be found in the field of the telescope. This observation is but too little used, as it is a most valuable one.

As observations must be carried on throughout the day, in order to get as many positions as we can, we now come to the different methods of obtaining latitude and longitude other than by the ordinary means of longitude by chronometer and latitude at noon.

There are three methods of finding latitude and longitude at the same time, viz. by Ivory's rule for double altitude; by Sumner's method ; and by a short equal altitude ; and for latitude only, we have circum-meridian altitudes. These are all of service under different circumstances, which will be hereafter described.

## DOUBLE ALTITUDE.

Ivory's

## Double

 Altitude.Ivory's rule for working a double altitude, with Riddle's extension, by which the longitude is also obtained, is too well known to require any special remarks.
Conditions The condition requisite to make the position obtained by for good Double Altitude. double altitude trustworthy, is mainly that the sun should change in azimuth a fair amount, otherwise one of the triangles will be so ill-conditioned that a small error in either altitude or time will have a great effect on the result.

In the generality of cases we have this condition by allowing about two hours to elapse between the observations, and we can therefore get a fair position by about half-past nine or ten o'clock.

In low latitudes, however, when the declination and latitude are nearly the same, the sun will rise so nearly on a circle of altitude as to change the azimuth very slowly, and we must wait till nearly noon before we can put any confidence in the observation. Unless then we lose our meridian or
circum-meridian observation, a double altitude is under these circumstances of little use to us. It is, however, more to be trusted than a Sumner, when change of azimuth is small; but it may be broadly stated that unless we are very ignorant (from lack of previous observations or other causes) of our position in latitude, we cannot under these circumstances do much before noon with observations of the sun alone; but we can get a good position by combining a daybreak observation of a star with one of the sun, as soon as it has sufficiently risen, by Sumner's method.

## SUMNER'S METHOD.

Sumner's method of obtaining the latitude and longitude at any one moment, which is but too little used in ordinary navigation, depends upon the fact that a heavenly body at any moment will be seen at an equal altitude from any part of a small circle of the earth, circumscribed round the spot where the body is vertical, with a radius equal to the zenith distance.

Knowing, therefore, the altitude of the sun, and the point on the earth whose zenith it is in, an observer will always be able to say that he is somewhere on the circumference of that circle. This alone would not give us much information, but it is seldom that we do not know our latitude to twenty or thirty miles. We shall know then that we are on that part of the circumference which includes these latitudes, and as when the sun is not very high the circle will be of large diameter, the portion of it within our limits may be, without much error, taken as a straight line.

In practice, then, having obtained an altitude of the sun, or any other body, we assume a latitude from our dead reckoning, and work out the longitude. By the aid of the azimuth tables we obtain the bearing of the body observed, and having plotted the position obtained, we draw a line at right angles to the bearing, which is the "position line." We now know that we are somewhere on that line. We must know
the Greenwich time, and therefore our positions will be as much dependent on the chronometers as any ordinary longitude.

The position line can be also obtained by working with two assumed latitudes and joining the positions resulting, but the above is shorter.

Waiting until the earth has revolved a sufficient amount to alter the bearing of the sun, we repeat the operation, and obtain another line, the direction of which will differ from the former by the difference in the azimuths of the sun at the two observations.

If we have been motionless in the interval, the intersection of these two lines will give us our exact position (always dependent on the chronometers) ; but if we have moved, we must so far trust our dead reckoning as to transfer the first line in the direction, and for the distance, we have run, when the intersection of the second position of the first line with the second line will be our position at the second observation.

## Plotting

 Sheet.H.M. ships are provided with a large sheet on which longitude is marked, leaving the navigator to complete the Mercator's projection by measuring the meridional parts for the latitude he is in.

On this, or a similar sheet graduated on board, the Sumner lines will be plotted, as it will both spare the charts, and from the increased scale provided, will give a better position than if the plotting was done on the usual small scale general chart of an ocean.

It is obvious that the value of a position will largely depend on the angle between the two lines, or, in other words, on the change in azimuth between the observations.

In low latitudes, therefore, when declination and latitude are nearly alike, we cannot use this method with the sun alone early in the day, as the sun will rise nearly vertically from the horizon, and we want a change in azimuth of at least $20^{\circ}$ to give us a trustworthy position. The same circumstances will much detract from the value of a double altitude,
as has been remarked, so that in such a case neither the one nor the other are much use as an absolute determination of position.

But Sumner's method has other resources. We can com- other bine lines obtained from two or more stars, or a line obtained from the sun with one from the moon or other heavenly body, as, for instance, a star obtained at daybreak when the horizon is sufficiently defined for us to trust it, or Venus in daylight, as before remarked. All we need is that the bearings of the two bodies differ sufficiently to give a good intersection.

By this means we can often get a good position early in the day, which we cannot get in any latitude with the sun alone, without a considerable interval of time elapsing.

This combination, therefore, of stars and the sun affords us the best and earliest opportunity of determining our position, and we should always endeavour to obtain it.

Should we be able to get a good meridian or circummeridian altitude, of a star, we shall of course use the resulting latitude.

A sun-Sumner requires the same circumstances and obser- Advanvations as a double altitude, but it has several advantages over the latter.

In the first place, the first half of the observation can be worked out at once; by which means we not only obtain the line on which we know we must be, and so have an approximation to our position at once, but also, having worked half of the calculation, it will not require many minutes after the second observation is taken, to complete it and obtain the true position.

Secondly, errors of calculation are less likely to be made-in a Sumner, as it involves merely the ordinary "chronometer" problem.

Thirdly, the fact of obtaining a line of position is of great value in many cases, as we can always tell roughly in what direction to go to shorten our distance to any given point, unless it should fall on or near the line, and when searching
for a vigia this knowledge, early in the day after a night's lying-to, will be invaluable.

Fourthly, we can repeat the observations a third time, and so check our first position with but little labour of calculation long before noon, especially in the case where we have combined a star with the sun, and are, perhaps, doubtful of the star observation, either from faintness of the star or indistinctness of the horizon,
Bearing of The true bearing of a distant mountain whose position is land and known, will also give a position by combination with a Sumner Line.


FIG. 42.
line, if its direction is such as to make a good cut with the latter.

Example of Sumner.

In Fig. 42, let us suppose A to be the position found by assuming a latitude and working out the altitude of a star obtained at daybreak. Drawing a line at right angles to the bearing, we get our first Sumner line E F, and we know we are somewhere on it. Having run W. b. S. $6 \cdot 2$ miles, we get an altitude of the sun; and assuming in this case a latitude a little south, we get another position B, and draw another position line G H. To project the run, we draw a line in the
required direction, and for the distance run, from any part of the line A, and draw another line parallel to the line E F, through the end of the run line. The position S, where this last intersects line $G H$, is the position of ship at second observation.

Running on in the same direction for 12 miles, we get another altitude of the sun, and another resulting Sumner line CD. Transferring the two first lines by the run as before, we now have three lines intersecting, or nearly so, at $P$, and by their coincidence or not we can measure the accuracy of our former positions-to a certain extent, that is, for it must be remembered that as the intersection of our lines is governed by the run allowed, a current, or constant error in calculating the run, might give an apparently good position which may really be considerably in error, even when the third intersection is obtained, with certain arrangements of the lines and the run.

Sumner's method is, in fact, the means by which all individual observations can be combined, and is from every point of view invaluable.

## NEW NAVIGATION.

There is a method of obtaining position known as the "New Navigation," which is highly thought of by some surveyors. It is claimed that position obtained by this method from observations of a heavenly body between the azimuths of $20^{\circ}$ and $70^{\prime}$ is more accurate than by the ordinary "chronometer" method.

Having had no personal experience of this method we are unable to recommend it.

It is, therefore, considered sufficient to mention that it consists in assuming a latitude and longitude, and in calculating a correction to this assumed position. A line of position can then be drawn through the true position thus obtained, and the results combined by Sumner's method with
other observations. The calculation is given in the book quoted in the footnote.*

## SHORT EQUAL ALTITUDE.

In low latitudes, where the motion of the sun in altitude is rapid nearly to the time of transit, a very good longitude may be obtained at noon, by a short equal altitude, taking observations about 20 minutes before and after noon. The change of declination in this short interval will not affect the time, so that the middle time between the observations as shown by the watch, can be taken for the time by the watch at apparent noon. All we have to do, therefore, is to take the difference between mean time of apparent noon and the Greenwich time, as shown by our chronometer, which gives us longitude directly.

## CIRCUM-MERIDIAN ALTITUDES OF SUN.

These are of great value, as, when the observations are within the limits of time from noon, the resulting latitude is as correct as from a meridian observation, which may be lost from clouds. They should be worked in precisely the same manner as the shore observations of the same description, and should be obtained as near noon as possible. If more than four or five minutes have to be added to the observed altitude, they will not be of much value.

If Raper's most valuable book is at hand, a short and correet rule, in connection with two of his tables, will be found at page 232 of the thirteenth edition, which will give the reduction as nearly as requisite for sea work.

[^24]
## CHAPTER XVII.

## THE COMPLETED CHART. <br> Fair Chart-Reducing Plans-Delineation-Symbols-ColouringGraduation.

The work is sent home to be published in several ways, Transaccording to circumstances.

When the detail, as it proceeds, is inked on the original sheet itself, it may be necessary to transmit a portion home before the survey is all complete, and a tracing is often used for this purpose, as the original sheet, with the "points" still accumulating, must be retained on board; but, if possible, it is better to send work home on drawing-paper, which is not liable to so many accidents from tearing, \&c., can be more fully worked up as regards detail, and can be better kept as a record, though the originals will in the end be transmitted to the Admiralty in any case.

When the detail is placed directly on the original sheet, it Original is very difficult to keep it clean enough for everything to be Chart. clear and distinct, as straight-edges, protractors, \&c., will be constantly placed on the chart over the completed part, and lines must be often drawn over it. It can be kept clean enough for transmission home as the finished chart, and by doing so, all errors arising from imperfect transferring will be avoided; but the surface of the paper must get so rubbed by constant cleanings, that, if a large sheet, it is seldom satisfactory. Several hands may have been employed in it, and the chart will then bear a piecemeal look. If this original sheet is not sent home, a copy has to be made on another sheet of paper, which will be the fair chart.

Fair
Chart.
The usual mode of making this is to place the new sheet under the old one, and prick the "points" through the latter, on to the former. A careful tracing having been made of the working sheet, it is placed on to the fair sheet, so that the points all correspond, and by means of transfer paper is traced on to the fair sheet, and inked in.

Great care is requisite, in transferring in this manner, that the tracing does not move from its proper position, and heavy weights must be used to prevent it from so doing. Errors have often crept in from careless transferring and want of proper examination and comparison afterwards.

In working with the method recommended by the writer, viz. each assistant's work plotted and inked on to his own separate board, and all then placed on one tracing, the final sheet can either be the original on which all the points have been plotted, if that has been kept clean enough ; or a sheet may be pricked through, as mentioned above, for the purpose; but in either case only one complete chart will be made, the general tracing sufficing to show whether the work of different assistants has met, and what is wanted to complete.

This, or these (as in a large sheet there will be several tracings for different parts), will be the tracing used for making the final chart in this case.

These tracings should not be too large, as they are apt to get distorted. For fine work it is desirable to make small tracings on paper for the special work of transferring.

This chart will also be the work of one hand, who will, after transferring outline, soundings, \&c., from the general tracings, have the original little bits before him while inking in; these little bits having been taken off their boards, and so reduced, by having superfluous paper cut off, as to be handy to lay on the sheet.

Original Field Sheets.

By washing off the field boards, the paper will have become distorted and contracted, but not to a sufficient degree to interfere with the small detail of sinuosity of the coast, which is what we mainly want them for. Everything will have been traced on the general tracings before the paper has been re-
moved, and care must be taken that this is so, as it cannot be done afterwards.

In whatever manner the final chart is sent in to the office, "Points" all "points" must be distinctly marked on it, especially main ${ }_{\text {shown. }}^{\text {to be }}$ points. These latter are often distinguished by the triangle which means theodolite station, and in surveys where the sextant has also been employed in triangulation, should cer tainly be so. The " points" are necessary to join one chart to another, and also, in case of future revision of the chart, they afford means to the reviser of measuring the accuracy of his predecessor's groundwork.

Plans sent home by officers in general service ships often lose much of their value from neglect of this. The existence of the "points," and their proper position, will at once give a confidence in the detail of the plan, that it is impossible to accord to the work of an officer, however zealous, of whom nothing is known as to his hydrographical capability, and who fails to give any indication in his chart of how it has been constructed.

## REDUCING PLANS.

In a survey of an extensive nature, bays, harbours, \&c., will often be done on a larger scale than the rest of the sheet. These must be either left blank on the coast sheet, or else reduced from the large scale plans.

It may sometimes happen that a portion of an anchorage is surveyed on the small scale before it is decided to make a large plan of it, on discovering it to be worth while to do so. This must not appear, however, on the completed chart, it must be all reduced from the larger scale.

Instruments for reducing, e.g. eidographs, are not supplied, Reduction and the reduction is accomplished by "squaring." by Squar-

This consists of ruling similar lines on both sheets, forming squares and diagonals all over the part to be reduced.

The two stations farthest apart on the plan, which must
also be plotted on the small scale chart, are joined by a line on both sheets, the "directing line." Then, taking the smaller first, divide this line into as many equal parts as is thought necessary. These parts will be from a quarter to an eighth of an inch long. Set off lines at right angles to the directing line from each point measured, and then lines parallel to the directing line, at the same distances apart as the others. The portion of the sheet required is now covered with squares. Rule also the diagonals. These will check the correctness of the squares, as they should, of course, pass exactly through each corner.

Now do the same for the large scale, making an equal number of squares.

It will be seen that nothing is measured, everything being done by subdivision of the directing line.

Great care is necessary to rule all these lines truly rectangular and equidistant.

Number the lines on each plan, to prevent mistakes, giving the same number to similar lines. Letters may be put to one set of lines, and numbers to those at right angles.

Then, taking proportional compasses, set to the difference of the scale as ascertained by measuring the distances apart of similar lines, the distance of each little detail of the plan from the nearest lines, can be put down by the same distance from the similar lines on the small scale.

Reducing is an operation demanding even more patience and trouble than usual, and it is better to leave the space blank than to reduce it carelessly.

## DELINEATION, SYMBOLS, AND COLOURING.

The annexed specimen chart, taken from the 'Admiralty Manual,' shows the method of delineation employed in fair chart work.

The following symbols are in use in surveying, in field books, and rough charts.


The days of the week are thus symbolised by the astrono- Signs for mical signs of the planets.

Days.


The following signs are useful in the field books.
Other
Symbols.


Some charts are worked up by indian-ink alone in all parts; in others, colour is used to assist the delineation of the different parts, indian-ink being always used over the colour, in exactly the same manner as if there was no groundwork.

A wash of some colour on the land helps to throw it up Colouring. very much; but care is very necessary in giving this edging that it be not too deep, and that too much water is not used, or the paper will distort, and the tracing will not fit. Also in drying, that it does so gradually and generally, not allowing a streak of sunlight, for example, to fall across one part of the sheet.

If using colour, the following tints should be used for the different parts.

$\left.\begin{array}{l}\text { Margroves, Cultivated Ground, } \\ \text { Grass, Meadows, Trees.. }\end{array}\right\}$ Prussian Green.
Swamps and Marshy Land .. Prussian Blue.
The three and five fathom lines should be coloured with cobalt; the former with a light tint all over the space included between it and low water, and the latter with a narrow edge inside the fathom line.
N.B.-To make Indian-ink perfectly black, mix a little indigo with it.

When the country is mountainous, no general wash, but only a local green in the valleys, and on flat ground, has a good effect.
Hills. As hills in most large scale charts are now engraved in contours, it is best to use this system in the fair chart.

Shading of indian-ink, put on with a brush, is done quickly, and shows up very well.

Simple contour lines will enable the chart to be engraved almost as well as the other modes, but does not look so well.

In charts issued by the British Admiralty, the shading is put on hills as though it were a raised map, with the light coming from the north-west.
Names. In inserting the names, care should be taken that no letters are upside down. Thus, it is often necessary to write a name in nearly a meridional direction, and it will depend upon whether the trend of the name is east or west of the meridian, whether it is written from south to north or north to south.

Thus in the two instances given here, Fig. 43, if the names had been written in the opposite direction, part of them would have been upside down. All names should be readable by turning the head, without the necessity of moving the chart.*

All names of capes, \&c., should be as much on the land as possible. The soundings being the most important part of a chart, they should be kept as clear and distinct as practicable.

Different characters should be used for the names of different classes of objects. Thus, one style for bays, another for points, another for shoals, and so on.

The scale of the chart is got from the longest calculated side for distance on it. This will, in cases of plans, generally be the ${ }^{\text {scale. }}$ same as that we originally plotted from, in which case we

FIG. 43.

already know our scale. But if we were obliged to plot from a short side, and have since obtained data which will enable us to calculate a longer distance, we must measure the distance between the two points on our chart, and dividing this number of inches and decimals by the distance as calculated, we shall get the true scale.

It is well to indicate the two stations from which the scale is derived, by drawing a red line between them, and writing, either against it, or elsewhere on the chart, the calculated distance and bearing. If a long distance, this last should be the Mercatorial bearing.

In the case of extended surveys, or when there is no regular triangulation, the scale will depend upon the distance obtained between two stations by astronomical observations. This

[^25]distance being calculated, the scale will be obtained as before.

## Soundings thick.

## Natural

 Scale.The soundings in the chart sent home should be as thick as possible, without sacrificing legibility. There is always a great temptation to thin them out, so as to look better ; but that is the work of the Office, and will probably have to be done again there in any case, as the scale on which the chart is published is usually smaller than that on which it is constructed, and if so, will not permit all soundings in the original to appear.

The natural scale, or the proportion which our chart lineally bears to the actual size of the portion of the globe it represents, is obtained by dividing the number of inches corresponding to one mile on our chart, obtained as above, by the number of inches in the nautical mile at the latitude. It is given in the form of a fraction, whose numerator is one.

Thus, supposing our scale is found to be 1.8 inches to a mile, in latitude $3^{\circ}$, we divide 72552 (the number of inches in a mile) by $1 \cdot 8$.

This gives $\frac{1}{40306}$ as the natural scale.
This natural scale should be noted on all sheets that are not graduated.

When the chart includes a considerable extent of coast-line that is intended to form part of a navigational sheet, it will have eventually to be redrawn on Mercator's projection, as it is on that projection all charts are published.
To do this, the sheet must be graduated, i.e. have the meridians and parallels placed upon it, as it is by means of them that a chart on one projection is redrawn on another.

## GRADUATION OF THE SHEET.

Gnomonic
Projection.

We have before said that a chart constructed by drawing right lines from one object to another, when graduated, has to be considered as being on the Gnomonic projection, and the
general features of this projection have been explained.* It now remains to consider how to graduate such a chart.

A sheet may be graduated either before or after the chart is drawn on it. The methods are substantially the same, and will differ only in some preparatory work necessary in the latter case.

We will first consider the case of graduation after the chart is complete, and to do this we must suppose our observations to be obtained, and that we know the latitudes and longitudes of two stations on our chart as far apart as possible in opposite corners of the chart.


We require, first of all, the reciprocal bearings of each from the other, and the distance between them.

In Fig. 44 let A and B be two stations whose latitudes and longitudes we have obtained; P is the pole. Add to the diff. long. the spheroidal correction, and use this, corrected diff. long. in the calculations. Calculate by spherical trigonometry the bearing of each station from the other.

We have PB, PA the co-latitudes, and BPA the diff. longitude. P B A and BAP are the angles required. The latter subtracted from $180^{\circ}$ will give us BAQ, and the.

[^26]difference between PBA and BAQ is the convergency. Find also the distance A B, to get the scale.

Now in Fig. 45 let AB be these same stations plotted on our chart. Required to graduate it.

Join A B, and from A and B lay off (by chords) the reciprocal bearings of one another, ascertained as above, as A N, B M, which will be meridians passing through those points.

From A and B measure, on the meridians, A H, BE, the distance, according to scale, to the nearest even minute of latitude (as $1^{\prime}, 5^{\prime}, 10^{\prime}$, \&c., as convenient).

At H and E lay off short perpendiculars to the meridians,

and on these measure the distances $\mathrm{HC}, \mathrm{E} \mathrm{D}$, the lengths of departure, according to scale, to the nearest even minutes of longitude that may be convenient.

In high latitudes and large scales, if the even meridian required is many miles distant, error will be introduced by this latter operation. It will only be correct for short distances, as the curve of the parallel, on which we ought to measure this departure, will not coincide with the perpendicular to the meridian for more than a mile or two in such a case.

We have now C and D, two stations on even meridians and
even parallels, which we shall take as our points for graduation. This is exactly the case when we wish to graduate the sheet first, so that henceforward the methods are identical. In the case of after-graduation, when these even points have

FIG. 46.

been obtained, we can rub out on our chart all lines already ruled, to prevent confusion, and we will take a new figure for the similar purpose of facilitating comprehension.

In Fig. 46 let C and D be the positions for graduation.

Calculate spherically, as before, the bearings of C and D from one another, and lay off the meridians C N, D M.

From C and D lay off the perpendiculars C H, D F, and from these perpendiculars lay off, on the side of the pole, half the convergency calculated for the difference of longitude in the latitude of C and D respectively, as C 11, D 5, cutting the opposite meridians respectively in J and G. Then J will be on the same parallel as $C$, and $G$ as $D$, and $J D, C G$, should be equal.*

To get the central meridian of the chart, bisect J C and D G in A and B , and join them.

Then J G joined should intersect CD in the central meridian. This is a capital check for our correctness so far.

To get other meridians, divide J C and D G as many times as there are meridians required, and join them, as O S, P T, Q V, \&c.

To get the parallels, which it will be remembered are curves, divide the half convergency chord, already measured, into as many parts as we have meridians.

In our figure we want five meridians from D to G , therefore we divide the chord into five parts, as $1,2,3,4$. Draw a small portion of D 4, cutting $\mathrm{R} W$ in $Z$. $Z$ will then be the position on R W of the parallel of D G . By similarly drawing D 3 to cut Q V in E, D 2 to cut P T in H, and D 1 to cut OS in F, we obtain a series of points on the meridians, which, connected together, will form the curve of the parallel D G required. In high latitudes we want more meridians, to draw the curve exactly, than in low, and we must therefore be guided by circumstances as to the number of them.

Similarly, we obtain the curve of the parallel J C.
To draw more parallels, divide each meridian between the parallels obtained into as many parts as required, and join them.

This process demands considerable care and accuracy in

[^27]drawing every line, and should be checked wherever practicable.

The margin of the chart is marked by subdividing the distance between each parallel or degree to the unit required.

There are other ways of drawing this graduation, all founded on the same principle. As this is, in the writer's opinion, the best of them, it is here given.

Finally, every original chart must have a memoir written Memoir of on it, giving a brief description of how the chart has been $\begin{gathered}\text { Construc- } \\ \text { tion. }\end{gathered}$ constructed, the base used, observations for latitude and longitude, \&c., \&c., enabling the authorities at home to put the proper value on the work.

It is scarcely necessary to describe the construction of a TransferMercator's Chart, as every naval officer learns it as part of Mercator's his education.

To redraw a survey on Mercator's projection, similar meridians and parallels must be drawn on both charts, and enough of them to make the parallelograms formed by them small enough to reduce the discrepancy between the shape of any parallelograms on either chart to as little as possible. The soundings, coast-line, \&c., in each parallelogram of the gnomonic chart are then transferred to the same parallelogram of the Mercator, by the latitude and longitude of each detail.

## CHAPTER XVIII.

DEEP SEA SOUNDINGS.*<br>Wire Sounding-Dredging.

In the first edition of this work the method of deep sea sounding with a hempen line was alone described. Hemp has now been entirely superseded by wire, and therefore the machines employed in wire sounding and the methods of using wire will alone be treated of.
Advantage Besides the advantage of weight, greater compactness of of Wire. the apparatus, the celerity with which the weight descends, and the greater speed at which it can be reeled in, wire, from its small size, and the smoothness of its surface, enables in many cases soundings of greater accuracy to be obtained. In sounding in a surface current with hemp, the line was carried along with the current, and it was impossible to keep the ship over the lead. The result was that when the lead reached the bottom the ship was a long way astern, and an empirical correction had to be made to arrive at the vertical depth. With the fine wire now used the friction is so slight that the ship can be kept over the lead without the wire getting under the bottom, and the length of wire out is the depth.

The machine used in surveying vessels for wire deep sea sounding is that devised by Mr. Lucas, of the Telegraphic Construction and Maintenance Company, and has undergone several modifications.

[^28]The large machine now supplied holds over 5000 fathoms of 20 -gauge wire, and is very compact. It is fitted with two brakes: one a screw brake for holding the reel when required, the other an automatic brake for stopping the reel when the weights strike the bottom. A guider for the purpose of winding the wire uniformly on to the reel is also attached, and is worked by a small handle.

After leaving the reel the wire passes over a registering
FIG. 47.


References.
A Reel or drum.
B Brake.
C Brake lever.
D Springs.

E Regulating screw. F Hand wheel.
G Swivelling frame.
H Measuring wheel.
$J$ Indicator.

K Stop.
L Wire guiding rollers.
M Handle for working roller.
N Bolt.
0 Screw brake.
wheel, the dial of which indicates the amount of wire run out, no matter how little or how much wire is on the reel.

A machine of smaller size, but very similar in type, is supplied for use in boats for soundings of, say, more than 15 fathoms, and is also useful from the ship for serial temperatures and other purposes.

The larger machine is represented in Fig. 47, but further detailed description will not be given, as the type may be further altered.

Wire.
The wire used is galvanised steel wire of 20 -gauge. It is supplied on drums in 5000 fathoms lengths, which are sometimes in one piece, but often have a splice in them. The drums are in hermetically-sealed tins. Though galvanised, the wire requires looking after. The galvanising process is not perfect, and it may be thin in some places, and even actually bare spots may occur. The wire should therefore be passed through an oily rag as often as possible, and oily cloths kept on the machine to protect the outer layers from damp air. After a long sounding cruise it is probably safer to condemn the wire on the machine.

The wire when new has a breaking strain of 240 lbs .
Smaller wire of 21 -gauge has also been supplied, for the purpose of allowing a sufficient length to be on the reel for very deep soundings, but with the larger machines now supplied, will probably be discontinued. Its breaking strain is 190 lbs.
Splices.
Splices are made about five feet in length, one wire being laid round the other in a long spiral of about one turn per inch. The ends are soldered, and a seizing of fine wire laid over the end and for two or three inches up the splice. No end must project. Solder is then applied along the whole length of the splice. A third seizing can be placed in the centre.

Splices are the weakest points of the wire. They should be frequently examined, and their positions noted, so that, both in running out and heaving in, thev may be eased round the wheels.

Sounding

## Rods and

 Sinkers.For depths of 1000 fathoms and under, the lead can be recovered, and no detaching rod is necessary. A lead of 40 or 50 lbs. weight is suitable.

For greater depths two kinds of rods for slipping sinkers are supplied, the "Baillie" and the "Driver." Both are fitted with tubes to bring up a specimen of the bottom, and the same sinkers fit them both.

The sinkers are conical and cylindrical iron weights of 25 and 20 lbs., with cylindrical holes cast in them, through
which the rod is passed. The sinkers are attached to the slipping arrangement of the rod by wire or cod line, the length of which should be so adjusted that as much of the rod as possible should project under the weight, in order to permit the rod to penetrate well into the mud. A rope grummet or iron ring fits round the bottom of the lower weight, to which is attached the suspending wire.

For water under 2000 fathoms, twc conical weights are sufficient. In deeper water, a third cylindrical weight should be put between them.

It is important to have a piece of hemp-line, some ten Splicing fathoms long, interposed between the end of the wire and wire into the lead or rod. This is for the purpose of preventing the wire from kinking when the lead strikes. A piece of sheet lead about 1 lb . in weight wrapped round the hemp just below the junction, keeps the wire taut, while the hemp slacks. To splice the hemp to the wire, lay the wire in the lay of the hemp for about six feet, putting on a good racking seizing of well-waxed twine at about every foot. Test this splice well.

Before splicing, the wire must be led from the reel of the machine, between the jaws of the guiding lever, through the hollow spindle of the swivelling frame, and over the registering wheel.

The wire must be carefully transferred from the drum on Winding which it is supplied, to the reel of the machine, by mounting the Wire. the former on a temporary spindle, and fitting a brake, by which the wire can be kept taut. Winding must be even, the wire passing through a piece of greased canvas in a man's hand.

Small brass screw stoppers are provided for holding the wire wire, if necessary, during a sounding. These should be fitted Stoppers. with a hempen tail to make fast to a cleat or other fixture.

For the greater depths it is usual to sound from forward, Method of but some officers have successfully accomplished it from aft Morking in fine weather. A projecting platform is fitted on the fore-
castle, to which the machine is bolted so as to plumb the water, being pointed in a direction slightly on the bow. An endless hemp swifter of 2 -inch rope connects the deck engine and sounding machine. This is led through blocks to the forecastle, and so to the machine. One or two turns are taken round the drum of the deck engine, and the bight passes through a leading block with a jigger attached, which is placed abaft the deck engine. By means of the jigger the swifter can be kept to the requisite amount of tautness. The details of this arrangement will of course vary in different ships, and with individual tastes. Specially made sister blocks for guiding the swifter are now supplied.

As the wire runs out, the regulating screw of the brake must be gradually screwed up, so as to increase the power of the brake in proportion to the amount of wire out. The regulating screw is marked for each 500 fathoms. In fairly smooth water the brake will at once act when the weights strike the bottom, and the reel stops.

When sounding in depths of less than 3000 fathoms it is best to use only one spring, but beyond that depth two springs are required. The marks on the regulating screw are only intended as a guide; the real test is that the brake is just on the balance so as to act when the strain lessens, which may be known by the swivelling frame being just lifted off the stop. As the wire weighs $7 \frac{1}{2}$ lbs. for each 500 fathoms, the 500 -fathom mark on the screw should be at the position in which the screw has to be to sustain a weight of $7 \frac{1}{2} \mathrm{lbs}$. ; the 1000 -fathom mark 15 lbs ., and so on. This can be tested and the marks verified.

A spring balance is supplied for attachment to the brake lever when heaving in, by which the amount of strain can be seen, and the deck engine worked accordingly.
Signals. It is necessary to establish some system of signals by which the officer on the forecastle, who is carrying out the sounding, can control the helm, main engines, and deck engine, both by day and night.

The signals given in Fig. 48 have been used with success for helm for day work :-

By night.
Green light for starboard helm. Red light for port helm. No light for amidships. If lights are waved, hard over.

Fig. 48.
Helm. Red Flag.


Fig. 49 gives signals for Main Engines :-
Fig. 49.
Main Engines.
Yellow Flag.


By night, a white light in starboard fore rigging for ahead, and a white light in port fore rigging for astern ; the height indicates the speed.

## For Deel Engine.

Blue flag held vertically downwards-Ordinary speed for heaving in.
Blue flag held horizontally-Slow.
Blue flag held over man's head-Stop.
Bynight, pass the word or arrange a bell near the deck engine.

Preparing
for a
Sounding.

See head rails cleared away as necessary.
Have ready wire stoppers, weights, sounding rod, grummet or ring, and sling, oil can and spanners, dish and spoon for collecting bottom specimen.

See that sounding machine is properly placed; and that the swifter runs fair, and is put round both deck engine and sounding machine wheel in the right directions; jigger in place, but not taut. Place indicator at zero.

Hook the springs downward to brake lever, and see regulating screw set to zero, and screw brake screwed down. Wire guiding rollers must be turned back, by taking out the bolt, and slewing the rollers clear, so as to allow the wire to run clear. All parts of machine should be well oiled, and the winch handles unshipped.

See that the wire is evenly wound, and taut.

## Letting

go.

Ease down the weights, using a stick with hook at the end to prevent a jerk, as the strain comes on the machine.

Attend the screw brake, and ease down gently and carefully to the first 100 fathoms or so, according to the weather, after which the screw brake is no longer necessary, and may be lifted clear of the rim of the reel.

As the wire runs out, screw up the regulating screw of the brake.

When the bottom is reached the springs come into action, the reel will stop, and the depth can be read on the indicator. The length of stray hempen line, less the drift to the water's edge, must be added.

When bottom is struck, ship one of the winch handles; Heaving then press the brake lever outwards to free the reel, and reel up some 10 or 20 fathoms as quickly as possible to get the rod off the bottom. If the sinkers have not detached, the effort required to reel up by hand will indicate that they are still on, and the operation of letting go and heaving up by hand must be repeated until it is certain that the weights are off, using the screw brake each time for letting go.

When the weights are gone, screw down the screw brake, and holding the brake lever by hand, run out the regulating screw, disconnect the springs and hook the spring balance, stopping the legs of the springs loosely to the balance to keep them out of the way.

See the guiding lever in gear.
Bring to and set up the swifter, which may be ready in position round the grooved wheel, the latter being secured to the shaft by means of the T head screw when required to connect.

Unship the winch handle, and heave in by the deck engine, taking care not to heave in so fast as to bring a greater strain, as shown by the spring balance, than 100 or 130 lbs . In smooth water and depth less than 2000 fathoms the strain may be as little as 70 or 80 lbs . with the wire coming in at a good pace.

When the indicator shows 30 fathoms, stop the deck engine, and heave the remainder in by hand.

When the hemp appears at the surface, heave very slowly, and when up to the machine secure the reel by screwing down the screw brake, and lift the rod in by hand.

It is very important to guide the wire on to the reel carefully and evenly by the guiding apparatus. Wire badly wound is sure to develop slack turns on running out, and will probably kink and snap.

When the rod is up disconnect the guiding gear, and get everything ready for another cast.

During the whole of the operation the ship must be care-
fully looked after, in order to keep the wire up and down, or as nearly up and down as possible. Ship.

Though perfectly simple to effect after a little practice, inexperienced officers frequently part the wire or get erroneous soundings, and the following notes may be of service until experience is gained.

Sounding from forward, when there is little or no current, all sails are furled except the spanker, which should be set with the sheet to windward.

Keep the wind slightly on that bow on which the sounding machine is fixed ; and never let the ship get more than two points off the wind unless a weather tide necessitates it.

Always endeavour to keep the position by small changes of speed and helm, avoiding high speed. This demands the closest watch, and the moment the wire is seen to be getting out of the vertical apply the brake and steam her up with the helm in the necessary position. Bear in mind that the helm is of little use, even with slight headway, unless the screw is working.

If you have failed to catch her in time, act decisively with engines and helm before matters get too bad, using high speed if necessary. A few quick revolutions of the screw with helm hard over is the best plan, checking the headway thus unavoidably given by a back turn when she has sufficient swing on, remembering that going astern always causes the ship's head to fall off from the wind, but to a less extent if the wind is on that side to which the ship's head naturally turns in a calm on reversing the engines.

The worst position is when the wire gets under the bottom, as it may catch the copper, and everything must be done to prevent this occurring. It will generally happen from allowing the wind to get on the wrong bow. In such a case it will probably be necessary to steam her rapidly round till the wind is on the right bow, and then let her drop down until the wire is again clear.

Should the wire foul the copper, or, as it may do, the gangway wire used for serial temperatures, it is sometimes possible
to clear it by getting out the lower boom and using long hook ropes of ordinary sounding line with weighted smooth hooks.

A surface current across the wind complicates matters considerably. In order to get an up and down sounding the ship must be moved against the current, and the wind must then be more or less on the beam ; with a weatherly current the wind may be nearly aft. Under such conditions it requires a practised eye and hand to manage the ship.

It must be understood that unless the wire is nearly up and down, it may be very difficult to say when the bottom is reached, and the depth, as given by the wire out, will not be accurate. If wire is run out after the lead has reached the bottom, kinks will result, and the wire will part.

It not infrequently happens that the wire parts for no apparent cause, but this may be due to a kink in the wire from a previous sounding.

Unless conditions of current, as above mentioned, necessitate it, it is generally fatal to let the ship fall off broadside to the wind. Should she be allowed to get round with the wind aft, there is probably no remedy but to heave in again, and commence afresh.

Care is necessary in heaving in the last 50 fathoms, so as to stop the deck engine in time.

The time interval with wire, when not pitching heavily, Time up to depths of between 2000 and 3000 fathoms, is about one minute per 100 fathoms. Reeling in may be accomplished at nearly the same rate.

A sounding of 1000 fathoms may be obtained in 25 minutes from the time the weight is lowered to the time the order is given to put the ship on her course. 2000 fathoms will require 45 minutes, and 3000 fathoms 75 minutes.

Though the time of running out each hundred fathoms is no longer required, as with hemp, for ascertaining when the sinkers strike the bottom, it is well to take the intervals, as they assist in the regulation of the brake.

If a second wire machine is available (a boat's machine will Serial do), serial temperatures can be conveniently taken from the tures.
gangway whilst the sounding is being obtained forward, thus gaining time.

A 30 lb . sinker is attached to the end of the wire, and the thermometers are secured to the wire by the metal clips at the back of the cases, at the required distances. See that the indices are down before attaching the thermometers.

There is a certain amount of extra risk in thus working from the gangway while the other wire is over, as the two wires may foul deep down, when the fact of the thermometers acting as toggles may make them difficult to clear. The time saved, however, justifies it in fine weather, and when experience in sounding is gained. To avoid heavy loss, however, not more than four thermometers should be on the wire.

Sounding

## on

Voyages.

Deep sea soundings on every voyage are now a recognised part of a surveying ship's routine. It is only in this way that depths so useful for submarine cable, as well as for scientific purposes, can be accumulated without the expenditure of time involved in special sounding cruises, though those are occasionally necessary. As a rough rule a sounding after daybreak, and before sunset, should be obtained daily, when observations can be got.
Dredging. Connected with deep sea sounding, though not such a common part of a surveyor's duty, is dredging, on which a few words may be useful.

The dredge consists of a strong iron frame, the sides forming lips, which are connected at each end by an iron bar, and are chamfered off to fairly fine edges. These edges slightly incline outwards, as seen in the sketch. On the iron bars arms are fitted, and to the eye at the extremity of one of them the dredging hawser is bent, the eye of the other arm being seized to it to form a span of such a strength that the seizing will carry away if the dredge catches.

Attached by wire seizings to holes in the lower part of each lip is a stout canvas bag, perforated with holes in its lower portion to permit the water to flow through.

A stout iron bar, to which three long swabs are secured,
is suspended by ropes from the iron end bars immediately below the canvas bag.

Dredges are of various dimensions, but a convenient size is as follows, as illustrated by the sketch Fig. 50 :-

Fig. 50.


The Dredge.
$A$ and $B$, arms, $2 \frac{1}{2}$ feet long.
C, hawser.
D E and F G, lips of the dredge, $2 \frac{1}{2}$ feet long.
H , holes to which bag is laced.
I I, perforations in the canvas bag. K K, swab bar.
L, bar of dredge mouth, 6 inches.

The hawser is weighted with about 60 lbs ., at 5 fathoms from the dredge.

On a sandy bottom, a net bag is substituted for the canvas bag, which gets full of sand.

On a rough bottom an iron triangle carrying swabs only is used, the arms being stopped lightly together so as to carry away, if caught ; or even one large swab at the end of a rope, weighted two feet above it to keep it down. These are especially useful on coral banks, where a regular dredge may very likely be lost.

To dredge, turn the ship away from the wind or current, and drop the dredge from aft with slight headway on, taking care that the bag and swabs do not capsize and foul the mouth of the dredge. Ease out the hawser to about three times the depth of water, and let the ship drift for about 20 or 30 minutes. The dredge can be hoisted in by a burton from the mizzen gaff.

## CHAPTER XIX.

## MISCELLANEOUS. <br> Distortion of Printed Charts-Observations on Under-Currents-Exploring a River-Swinging Ship.

In printing charts from an engraved plate, the paper has to Distortion be damped. This results in distortion on the sheet drying, $\begin{aligned} & \text { in Print } \\ & \text { Charts. }\end{aligned}$ and angles laid off on a published sheet will never be found to agree exactly, especially if the sheet is large. This must always be borne in mind, in trying angles on a published chart.

For this reason, when a published plan is to be examined, a "dry proof" is supplied to the surveyor from the Admiralty. This is an impression " pulled," as it is termed, on to a dry sheet. It is much fainter than a damp-pulled copy, and would not do for ordinary use; but being an exact facsimile of the copper plate, all angles, bearings, \&c., should agree precisely, if the original survey is correct.

This fact of the distortion of published charts is not generally known, and many reports of so-called inaccuracies have been made in ignorance of it. The amount of it varies with the goodness of the paper, and the trouble bestowed by the printer in damping his paper uniformly. It is a fact much to be deplored, and the man who invents a means of obviating it, will bestow a great boon on cartography.

## OBSERVATIONS ON UNDER-CURRENTS.

Though not in the ordinary run of surveying operations, a slight description of the method of discovering the direction and approximate rate of under-currents may be useful.

To ascertain these satisfactorily, special gear is necessary.

## General Principle.

The general principle is to expose a large surface to the action of the under-current, and to support this in the water by a floating buoy which will present as small a surface as possible to the action of the surface stream.

The experiments must be carried on from boats, and therefore the gear must be as light as possible, for easy handling.

A series of observations on the under-currents in the Bosporus and Dardanelles resulted in the author's adopting the following:-*
Apparatus A light, flat wooden board, 6 feet square, with a wing used. 2 feet in length, at right angles to the rest of the frame, was used as the submerged drag. (See Sketch, Fig. 51.)

To the extremities of the wing the sling, $a a$, was made fast, and to this sling the supporting line to the buoy was bent, at such a point as kept the surface of the drag vertical when the strain came on.

It weighed 70 lbs . in air, and took 120 lbs . of lead to sink it satisfactorily. These leads were made fast with a little drift, and another line, $c$, was bent, both to them and to the lifting sling, $b b$, so that the weight of the leads could be taken off the drag, when pulled up to the surface, before finally hoisting it into the boat.

An iron buoy, 1 foot in diameter and 5 feet long, supported this structure well when the surface current was not very strong, and only presented an area of less than one square foot to pull through the water.

When the surface current was swift, other buoys had to be added, attached in line to the upper end of the first, for under

[^29]FIG. 51.

these circumstances the single buoy was dragged under water, and its motion could not be followed. Several disappeared in this way, some to reappear when the apparatus got into slacker water, some for good and all.
Method of To ascertain the movement of the floating buoy, and there-ascertaining Rate fore the direction in which the drag was carried by the underand Direc- current, a " fix" was taken to shore objects, and plotted on a
tion. large scale sheet of points, when the drag was let go free from

FIG. 52.

the boat. Subsequent fixes and times taken enabled the course and distance of the buoy in the intervals to be recorded on this sheet.

## Surface Current.

A small buoy, weighted so as to float awash, put into the water at the same spot and time, and followed by another boat, afforded means of ascertaining the surface current.
Defects.
This arrangement worked very satisfactorily altogether, but there are several defects in it.

The depth of the submerged drag will not be the length of the line allowed, but some unknown quantity less, as will be seen by accompanying sketch, Fig. 52.

This must be estimated.
The force expended in dragging the buoy through the Rate less surface water, and overcoming the friction of the suspending ${ }_{\text {Rate }}^{\text {than true }}$ line, is also an unknown quantity, but will always have the effect of retarding the motion of the submerged drag. The rate therefore recorded, by the movement of the surface buoy, will always be less than the true rate of the under-current.

We do not imagine that the apparatus described above may not be much improved upon, but we give it as a startingpoint for any officer employed in future investigations of a similar character.

Several instrument makers now turn out " Current Meters" Current Meter. of various forms. Doubtless these could be, with a little ingenuity, adapted to sea work, at least to show the true rate of an under-current.

A deep sea current meter devised by Lieutenant Pillsbury, U.S.N., has, with several modifications, been (1897) under trial, but is not yet brought into general use.

The observations are made more complete by ascertaining Temperathe temperature and density of the water, at the depths $\begin{gathered}\text { ture and } \\ \text { Density. }\end{gathered}$ experimented on.

## EXPLORING A RIVER.

Narrow rivers, navigable for boats, will generally be suffi- Running ciently laid down on a marine chart by a sketch survey, made ${ }^{\text {Survey. }}$ from the boat (a steam pinnace, if possible), while passing up and down. Patent log and compass will be the instruments mainly used for putting down the direction and length of each reach; though if we have objects that we can use for a sextant fix, we shall of course use them in preference, at any rate from time to time. We must endeavour in every case to get a good fix at our furthest point, and the course of the river, as mapped by patent $\log$ and compass, will then be squared
in on that, and the fixed points at the entrance and any other fixes we may have got. Any elevated points near, which we can ascend and fix, and from them get angles to bends and reaches of the river, will much assist us, especially when, which is so often the case, the river is thickly lined with trees and jungle.

The patent $\log$ will be fitted, as already described, with the dial on the gunwale and the fan towing astern. Theodolite legs standing in the stern-sheets make an excellent stand for a prismatic compass, and enable us to get a better bearing than by holding it in the hand.

It is in rounding the bends that the greatest error in mapping a river is introduced, as the distance run over while gradually altering course must be estimated by eye, which requires considerable experience at judging distances.

Current must be taken into consideration, and may be obtained, if time allows, by anchoring the boat for half an hour, and reading the patent $\log$, or in shorter time, by heaving the current log.

In a river where the tide extends some distance up, and where the land is low and jungly, as in so many mangrove rivers, our difficulties are much increased, as the velocity of the current will be constantly varying, and we cannot hope to obtain any sextant fixes to check our position. In cases of this kind, if it is desired to have any degree of accuracy in the sketch, the only way is to run over the work again, making an independent map, and squaring in afterwards a mean of the two.

Plotting in Boat.

It is best always to plot as we go. Mistakes are thus rendered less likely, and the vexed question of the bends can best be solved by placing their shape on the paper at once. We can also look at our work on the way down again, and correct little inaccuracies more readily.
Survey on If it is desired to make a large scale plan of a river of larger scale. greater width, the best method is to employ several boats at once, four if possible, which will triangulate their way up, two on either side. Starting from two fixed points at the
mouth of the river, two boats will remain there while the other two go up to convenient positions, whence they can see the boats remaining at the first points. Angles will then be taken from all, to everything conspicuous, and to one another, and the lower boats will, leaving marks at their old stations, move up to two new positions above the other boats, when the angles will be repeated, and so on, the lower boats moving on each time.

The shore line can either be sketched by the boats as they go up, or done afterwards more correctly when the marks are all up and fixed. Soundings, in the same way, can either be taken from the boats as they move from station to station, in which case they would cross over each time so as to get a diagonal line across the channel, or can be more regularly taken afterwards, as the circumstances of the case may require.

Everything must be plotted afterwards, and communication between the boats as they pass one another, when names can be given and objects pointed out for mutual observation, will greatly facilitate the comprehension of one another's angles, when putting down the points.

## SWINGING SHIP.

Though the compass is but little employed in surveying, it is occasionally unavoidably brought into use.

As deviation varies, with lapse of time and change of latitude, it must be constantly ascertained by swinging ship.

The methods in use in swinging ship are well known, but perhaps a repetition may not be thrown away.

They are two in number.
One, by observing the compass-bearing of a distant object whose true magnetic bearing is known.

The other, by reciprocal bearings of the compass on board, and another on shore.

The first is the best and simplest when the magnetic By distant bearing of the distant object can be well determined.

The object should be, at the least, six miles distant, and the more the better.

Its bearing can be obtained from observations on shore from a spot in line with the ship, or by true bearing with known variation applied.

Objects for this purpose are sometimes indicated on the charts or in the sailing directions, and the bearings given.

The deviation, for each position of ship's head, is then the difference between this fixed magnetic bearing and the observed bearing by compass.

The ship can either be hauled round with hawsers, at anchor, or, if the object be far enough off, can be steamed round a circle small enough to make no difference in the bearing.

If steamed round, it is well to repeat the operation, turning in the opposite direction, as the compass may partake of the swing of the ship, which will introduce error, The mean of the two will then be the bearing to use.

By re-
ciprocal ciprocal Bearings.

The second method is perhaps the one generally employed, and is very convenient with a theodolite at hand.

An officer is landed with azimuth compass and theodolite.
He obtains the bearing with the compass of some welldefined object, and setting up his theodolite, takes it for his zero.

In arranging the theodolite on zero, it saves calculation to point the degree and minute of the magnetic bearing to the zero instead of $360^{\circ}$. Thus, if the zero bears by compass S. $44^{\circ} 20^{\prime}$ E. (supposed to be unaffected), set the vernier to $134^{\circ} 20^{\prime}$. The angles read to the ship will then be the angle east of the magnetic north.

A flag on a long staff is held behind the theodolite, when all is ready.

The ship, under steam, and with a flag placed exactly over the standard compass, steams slowly round, hoisting a large flag close up to the mast-head just before the ship's head comes to each point, which is dipped at the moment of observation, when the bearing of the shore station is taken.

The flag on shore is dipped, to show that the angle of the
FIG 53.

flag over the compass has been obtained by the theodolite, and is again shown as a response, when the flag is mast-headed for the next observation.

The time of each observation is taken by previously compared watches.

In this case, too, the ship should be swung in the opposite direction, if it is deemed necessary.

The difference of the reciprocal bearings is the deviation at each observation.

If more than one observation at any or all points has been obtained, the results are meaned for the accepted deviation.

It is usual to observe at every point of the compass, for the ship's head, but in some vessels it may be necessary to subdivide this.

The readiest way of examining the results of our observations is by use of the Graphic method. (Fig. 53.)

Drawing a long line, measure off equal parts along it, for the points of ship's head, and at each point on this normal lay off, at right angles, a line equal to the degrees and minutes of the deviation, on any scale we choose-easterly deviation to the right, westerly to the left of the normal.

Lines drawn through the extremities of these abscissæ will denote the curve of deviation observed.

By the irregularities of this curve, we can judge of the correctness of the observations very fairly; and for our final table of deviation, we can draw a mean curve, if there are many irregularities, and measure to that for the amount of deviation for each point.

The valuable results for variation obtainable from swinging have already been mentioned at p. 283.

## APPENDIX.

A.-To prove that Tan Convergency = Tan Dep. Tan Mid Lat.


Here C is the centre of the earth, P is the pole, E P, Q P, two meridians a known distance apart. B L, E L, are two tangents to the meridians, at the middle latitude known, in the same plane as the meridian, and meeting one another and the axis of the earth C P, produced, in L.

Then B L D is the Convergency required, and D L C is the middle latitude, and BCD the departure. D C is a radius of the earth $=r$.

Now as B D is small, it can be taken as a straight line without sensible error.

We can also assume B L D and BCD to be right-angled triangles.
Then B D $=\mathrm{D} \mathrm{L} \times$ Tan BLD.
Similarly B D $=r \times$ Tan B C D.
Equating, we have $\mathrm{D} \mathrm{L} \times \operatorname{Tan} \mathrm{BL} \mathrm{D}=r \times \operatorname{Tan} \mathrm{BCD}$.

```
But D L =r }\times\operatorname{Cot D L C;
    \thereforer\times\operatorname{Cot D L C }\times Tan B L D =r }\times\mathrm{ Tan B C D,
    or Tan B L D = Tan B C D }\times\operatorname{Tan}DLLC
    or Tan Convergency = Tan dep }\times\mathrm{ Tan Mid Lat.,
        and when Convergency is very small, we can say
            Convergency = Dep }\times\mathrm{ Tan Mid Lat.
```


## B.-In Graduating a Chart on the Gnomonic Projection.

To show that the angle of half convergency laid off from the rectangle intersects the opposite meridian on the parallel, and also that the further subdivisions of the convergency intersect their respective meridians on the same parallel.


From K and H , the graduating positions, draw the true bearings, lines K P, P G, which are meridians and will meet at $P$, the pole of the projection, making the angle K P C , or the Convergency.

Make $\mathrm{HC}=$ difference of latitude of H and K . Then PC will equal PK .
Join K C, bisect it in D, and join P D, the central meridian. Lay off K G perpendicular to K P .

Then $\angle \mathrm{CK} G$ is the half convergency;
For in $\triangle$ P D K . . . D K P $=90-$ D P K, and as F K P is drawn $=90$;

$$
\therefore \text {. D K P }=90-\text { D K F }
$$

$\therefore \mathrm{D} \mathrm{PK}=\mathrm{D}$ K F.
But D P K $=\frac{1}{2} \mathrm{~K}$ P C the convergency;
$\therefore$ D K F or C K G $=\frac{1}{2}$ convergency.
Q. E. D.

Bisect C K G in K N, making G K N or X K Z $=\frac{1}{4}$ convergency, Then E where K N intersects P F is on the parallel K C, or P E = P K.

Bisect K P E in P X.
Now $\quad \mathrm{MKZ}=\mathrm{K} \mathrm{Z} \mathrm{P}+\mathrm{KPZ}$,
but M K Z $=90^{\circ}+\frac{1}{4}$ Convergency (by construction) and K P Z $=\frac{1}{4}$ Convergency;
$\therefore 90^{\circ}+\frac{1}{4}$ Conv $=\mathrm{KZ} \mathrm{P}+\frac{1}{4}$ Conv;
$\therefore \mathrm{K} \mathrm{Z} \mathrm{P}=90^{\circ}=\mathrm{P} Z \mathrm{E}$,
and as $K P Z=Z P E$ and $P Z$ is common, the $\triangle s K Z P \& P Z E$ are equal and similar;

$$
\therefore \mathrm{PE}=\mathrm{PK}
$$

Q. E. D.

$$
\text { C.-To prove Chord }=2 \mathrm{rad}\left\{\operatorname{Vers}\left(90+\frac{\theta}{2}\right)-1\right\} \text {. }
$$

FIG 56.


Let $\mathrm{CA} \mathrm{B}=\theta$, the angle whose chord is required.
At any radius $\mathrm{AC}=r$, describe arc $\mathrm{C} B$.
Join $C B$, then $C B$ is Chord required.
Bisect B C in D and join A D.
Then $\mathrm{DAB}=\frac{\theta}{2}$.
Now $\mathrm{DB}=\mathrm{AB}, \operatorname{Sin} \mathrm{DAB}$

$$
=r \cdot \operatorname{Sin} \frac{\theta}{2}
$$

but $\mathrm{BC}=2 \mathrm{DB}$;

$$
\therefore \mathrm{BC}=2 r, \operatorname{Sin} \frac{\theta}{2} \cdot \cdots . . . . .(a)
$$

But Versine $\frac{\theta}{2}=1-\operatorname{Cos} \frac{\theta}{2}$;

$$
\begin{aligned}
\therefore \text { Versine }\left(90+\frac{\theta}{2}\right) & =1-\operatorname{Cos}\left(90+\frac{\theta}{2}\right) \\
& =1-\left(-\operatorname{Sin} \frac{\theta}{2}\right) \\
& =1+\operatorname{Sin} \frac{\theta}{2} \\
\therefore \operatorname{Sin} \frac{\theta}{2} & =\operatorname{Vers}\left(90+\frac{\theta}{2}\right)
\end{aligned}
$$

Substituting this in (a) we get

$$
\mathrm{BC}=2 r\left\{\operatorname{Vers}\left(90+\frac{\theta}{2}\right)-1\right\}
$$

D.-To prove Reduction to the Meridian $=\frac{\operatorname{Cos} l \cdot \operatorname{Cos} d}{\operatorname{Sin} z} \cdot \frac{\text { Vers } h}{\operatorname{Sin} 1^{\prime \prime}}$


Let X be a heavenly body near the Meridian, P the pole, Z the Zenith.
Let Hour Angle ZPX $=h$, Latitude $=90-\mathrm{PZ}=l$, Zenith distance $\mathrm{X} Z=z$, Declination $=90-\mathrm{P} \mathrm{X}=d$.

Then $\operatorname{Cos} Z \mathrm{P} \mathrm{X}=\frac{\operatorname{Cos} \mathrm{XZ}-\operatorname{Cos} \mathrm{PX} \cdot \operatorname{Cos} \mathrm{PZ}}{\operatorname{Sin} \mathrm{PX} \cdot \operatorname{Sin} \mathrm{PZ}}$,

$$
\text { or } \operatorname{Cos} h=\frac{\operatorname{Cos} z-\operatorname{Sin} l \cdot \operatorname{Sin} d}{\operatorname{Cos} l \cdot \operatorname{Cos} d} ;
$$

$\therefore \operatorname{Cos} \mathrm{z}-\operatorname{Sin} l . \operatorname{Sin} d=\operatorname{Cos} l . \operatorname{Cos} d . \operatorname{Cos} h$

$$
\begin{aligned}
& =\operatorname{Cos} l \cdot \operatorname{Cos} d \cdot(1-\operatorname{Vers} h) \\
& =\operatorname{Cos} l \cdot \operatorname{Cos} d-\operatorname{Cos} l \cdot \operatorname{Cos} d \cdot \text { Vers } h ;
\end{aligned}
$$

$\therefore \operatorname{Cos} z+\operatorname{Cos} l . \operatorname{Cos} d$. Vers $h=\operatorname{Cos} l . \operatorname{Cos} d+\operatorname{Sin} l . \operatorname{Sin} d$

$$
=\operatorname{Cos}(l \sim d)
$$

$$
=1-\operatorname{Vers}(l \sim d) \text {; }
$$

$\therefore \quad$ Vers $(l \sim d)=1-\operatorname{Cos} z-\operatorname{Cos} l . \operatorname{Cos} d$. Vers $\%$

$$
=\text { Vers } z-\operatorname{Cos} l . \operatorname{Cos} d . \text { Vers } h .
$$

Working with Declination $=90+\mathrm{PX}$, we shall get

$$
=\operatorname{Vers} z-\operatorname{Cos} l . \operatorname{Cos} d . \text { Vers } h .
$$

But $l \sim d$ or $l+d$ is the Meridian Zenith Distance $=Z$.
Then Vers Z $=$ Vers $z-\operatorname{Cos} l$. $\operatorname{Cos} d$. Vers $h$.
$-\operatorname{Cos} l$. $\operatorname{Cos} d$. Vers $h=$ Vers Z - Vers $z$

$$
\begin{aligned}
& =1-\operatorname{Cos} Z-1+\operatorname{Cos} z \\
& =\operatorname{Cos} z-\operatorname{Cos} Z \\
& =-2 \operatorname{Sin} \frac{z+Z}{2} \cdot \operatorname{Sin} \frac{z-Z}{2} ;
\end{aligned}
$$

but $z$ and $Z$ are nearly alike, so $\frac{z+Z}{2}$ may be taken $=z$ and $z-Z$ is very small $\therefore 2 \operatorname{Sin} \frac{z-Z}{2}$ may be taken $=(z-Z) \operatorname{Sin} 1^{\prime \prime}$ :
$\therefore \operatorname{Cos} l \cdot \operatorname{Cos} d \cdot$ Vers $h=\operatorname{Sin} z \cdot(z-Z) \operatorname{Sin} 1^{\prime \prime}$,

$$
\text { or } z-Z=\frac{\operatorname{Cos} l \cdot \operatorname{Cos} d}{\operatorname{Sin} z} \cdot \frac{\text { Vers } h}{\operatorname{Sin} 1^{\prime \prime}}
$$

but $z-\mathrm{Z}$ is the Reduction to the Meridian;
$\therefore$ Reduction to Mer. $=\frac{\operatorname{Cos} l \cdot \operatorname{Cos} d}{\operatorname{Sin} z} \cdot \frac{\text { Vers } h}{\operatorname{Sin} 1^{\prime \prime}}$.

## E.-To show that the Distance of Horizon in English Miles

$$
=\sqrt{\frac{3}{2} \text { height in feet. }}
$$

FIG 58.


Let $r$ be radius of earth.
$h$ height of observer in feet.
d distance of horizon.

$$
\begin{aligned}
d^{2}+r^{2} & =(h+r)^{2} \\
& =h^{2}+r^{2}+2 h r
\end{aligned}
$$

$$
h^{2} \text { being small may be omitted. }
$$

$$
d^{2}=2 h r
$$

$$
\text { but } h \text { in Eng. miles is } \frac{\mathrm{h}}{5280}
$$

$$
\text { and } 2 r \quad, \quad \text { is } 7910
$$

$$
\begin{aligned}
\begin{aligned}
d^{2} & =\frac{7910}{52 ४ 0} h \\
& =\frac{3}{2} h \text { very nearly } \\
\therefore d & =\sqrt{\frac{3}{2} h}
\end{aligned}
\end{aligned}
$$

This is the distance disregarding refraction, which has the effect of increasing the distance of the visible horizon. If having found $d$ as above, we subtract $\frac{1}{18}$ of itself from it, the remainder will be the true distance in sea miles very nearly, with the effects of refraction taken into consideration.

## F.-Base by Sound.

To prove that $\mathrm{T}=\frac{2 t t^{1}}{t+t^{1}}$
Let $d$ be distance in feet between stations, $v$ the velocity of sound, $\quad$ in feet per $x \quad "$ of the wind, $\}$ second, $t$ the observed interval in seconds with the wind, $t_{1} \quad, \quad$ against the wind, T the mean interval required.

Then $d=v$ T.
By observation $d=v t+x t$
and $\quad d=v t_{1}-x t_{1}$.
Dividing by $t$ and $t_{1}$ we get

$$
\begin{aligned}
& \frac{d}{t}=v+x \\
& \frac{d}{t_{1}}=v-x
\end{aligned}
$$

Adding, we have

$$
\begin{aligned}
\frac{d}{t}+\frac{d}{t^{1}} & =2 v \\
d\left(\frac{1}{t}+\frac{1}{t^{1}}\right) & =2 v \\
d \frac{t^{1}+t}{t t^{1}} & =2 v \\
d & =v \frac{2 t t^{1}}{t+t^{1}}
\end{aligned}
$$

but we have also

$$
d=v \mathrm{~T},
$$

and by equating

$$
\mathrm{T}=\frac{2 t t^{1}}{t+t_{1}}
$$

Form G.
Object.
Soundings after.
.


Form H.
Chronometer Comparison Book.
Max.
Ther.
Min.

| Chrons. | Time. | Check. | Slow on A. | 2nd Diff. |  |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A |  |  |  |  |  |
| B |  |  |  |  |  |
| A |  |  |  |  |  |
| C |  |  |  |  |  |
| A |  |  |  |  |  |
| A |  |  |  |  |  |
| A |  |  |  |  |  |
| A |  |  |  |  |  |
| A |  |  |  |  |  |
| A |  |  |  |  |  |
| A |  |  |  |  |  |
| A |  |  |  |  |  |
| A |  |  |  |  |  |
| A |  |  |  |  |  |

Table J. - Table of Chords of Arcs from $0^{\circ}$ to $60^{\circ}$, to facilitate the Projection of Angles. Radius =10. By T. H. Tizard, Navigating Lieut. R.N.

| Min. | $0^{\circ}$ | Parts for | $1^{\circ}$ | Parts for | $2^{\circ}$ | Parts for | $3^{\circ}$ | Parts <br> for | $4^{\circ}$ | Parts <br> for |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0.00000 | $\bigcirc$ | 0.17453 | $\bigcirc$ | 0.34905 | $\bigcirc$ | 0.52354 | 0 | - 0.69799 | $\bigcirc$ |
| 1 | -00291 | 5 | - 17744 | 5 | - 35195 | 5 | -52644 | 5 | - 70080 | 5 |
| 2 | -00582 | 10 | -18035 | 10 | - 35486 | 10 | - 52935 | 10 | - 70380 | 10 |
| 3 | -00873 | 15 | -18326 | 15 | - 35777 | 15 | - 53226 | 15 | - 70671 | 15 |
| 4 | -01164 | 20 | -18617 | 20 | - 36068 | 20 | -53517 | 20 | -70962 | 20 |
| 5 | -01455 | 25 | $\cdot 18907$ | 25 | - 36359 | 25 | -53808 | 25 | . 71252 | 25 |
| 6 | - or 746 | 30 | -19198 | 30 | - 36650 | 30 | -54098 | 30 | -71543 | 30 |
| 7 | -02037 | 35 | - 19489 | 35 | -36941 | 35 | - 54389 | 35 | -71834 | 35 |
| 8 | - 02328 | 39 | -19780 | 39 | . 37231 | 39 | -54680 | 39 | -72129 | 39 |
| 9 | -02618 | 44 | - 20070 | 44 | -37522 | 44 | - 54970 | 44 | -72455 | 44 |
| 10 | -02908 | 49 | -2036I | 49 | -37813 | 49 | 55261 | 49 | - 72706 | 49 |
| II | 0.03199 | 53 | 0. 20652 | 53 | $0 \cdot 38104$ | 53 | 0.55552 | 53 | - 72996 | 53 |
| 12 | -03490 | 58 | - 20943 | 58 | - 38395 | 58 | - 55843 | 58 | - 73287 | 58 |
| 13 | -03781 | 63 | - 21234 | 63 | - 38685 | 63 | -56134 | 63 | - 73578 | 63 |
| 14 | . 04072 | 68 | - 21525 | 68 | - 38976 | 68 | - 56425 | 68 | - 73869 | 68 |
| 15 | -04363 | 73 | - 21816 | 73 | - 39267 | 73 | - 56715 | 73 | -74159 | 73 |
| 16 | -04654 | 78 | -22107 | 78 | - 39558 | 78 | - 57006 | 78 | -74450 | 78 |
| 17 | -04945 | 82 | - 22398 | 82 | - 39849 | 82 | $\cdot 57297$ | 82 | - 74741 | 82 |
| 18 | .05236 | 87 | - 22689 | 87 | . 40140 | 87 | - 57588 | 87 | -75032 | 87 |
| 19 | -05527 | 92 | - 22979 | 92 | - 40430 | 92 | - 57878 | 92 | - 75322 | 92 |
| 20 | -05818 | 97 | - 23270 | 97 | -4072 | 97 | -58169 | 97 | -75613 | 97 |
| 21 | 0.06109 | 102 | 0.2356 I | 102 | 0.41012 | 102 | 0.58460 | 102 | $0 \cdot 75903$ | 102 |
| 22 | - 06400 | 107 | -23852 | 107 | -41303 | 107 | $\cdot 58751$ | 107 | -76194 | 107 |
| 23 | -06691 | II2 | -24143 | 112 | -41594 | 112 | - 59042 | 112 | - 76485 | 112 |
| 24 | -06982 | 117 | - 24434 | 117 | -41884 | 117 | -59333 | 117 | - 76775 | 117 |
| 25 | . 07272 | 122 | - 24725 | 122 | '42175 | 122 | - 59623 | 122 | - 77066 | 122 |
| 26 | -07563 | 127 | - 25016 | 127 | -42466 | 127 | - 59914 | 127 | - 77356 | 127 |
| 27 | -07854 | 132 | - 25306 | 132 | . 42757 | 132 | -60205 | 132 | - 77647 | 132 |
| 28 | -08145 | 137 | - 25597 | 137 | . 43048 | 137 | -60495 | 137 | - 77938 | 137 |
| 29 | -08436 | 141 | - 25888 | 14 I | - 43339 | 141 | -60786 | 141 | -78229 | 141 |
| 30 | -08727 | 145 | - 26179 | 145 | -43629 | 145 | -61077 | 145 | -78519 | 145 |
| 31 | 0.09017 | 150 | $0 \cdot 26470$ | 150 | 0.43920 | 150 | 0.61368 | 150 | 0.78810 | 150 |
| 32 | -09308 | 155 | - 26761 | 155 | - 44211 | 155 | -61658 | 155 | -79107 | 155 |
| 33 | -09599 | 160 | - 27052 | 160 | - 44502 | 160 | -61949 | 160 | -79392 | 160 |
| 34 | -09808 | 165 | - 27342 | 165 | - 44793 | 165 | -62240 | 165 | -79682 | 165 |
| 35 | - 10181 | 170 | -27633 | 170 | -45084 | 170 | -62531 | 170 | - 79973 | 170 |
| 36 | - 10472 | 175 | - 27924 | 175 | -45374 | 175 | -6282I | 175 | - 80264 | 175 |
| 37 | -10763 | 180 | - 28215 | 180 | - 45665 | 180 | . 63112 | 180 | - 80554 | 180 |
| 38 | -11054 | 185 | - 28506 | 185 | - 45956 | 185 | -63403 | 185 | . 80845 | 185 |
| 39 | - 11344 | 190 | - 28797 | 190 | - 46247 | 190 | - 63694 | 190 | -81135 | 190 |
| 40 | - 11635 | I94 | - 29088 | 194 | - 46538 | 194 | . 63984 | 194 | -81426 | 194 |
| 41 | $0 \cdot 11926$ | r99 | - 29378 | 199 | 0.46828 | 199 | 0.64275 | 199 | -81717 | 199 |
| 42 | - 12217 | 204 | - 29669 | 204 | -47119 | 204 | - 64566 | 204 | -82007 | 204 |
| 43 | - 12508 | 209 | - 29960 | 209 | - 41410 | 209 | -64857 | 209 | - 82297 | 209 |
| 44 | - 12799 | 214 | . 30251 | 214 | -47701 | 214 | . 65147 | 214 | - 82588 | 214 |
| 45 | - 13090 | 219 | - 30542 | 219 | - 47992 | 219 | -65438 | 219 | -82879 | 219 |
| 46 | -13381 | 223 | - 30833 | 223 | - 48283 | 223 | -65728 | 223 | -83170 | 223 |
| 47 | -13672 | 228 | - 31123 | 228 | - 48574 | 228 | -66019 | 228 | -8346I | 228 |
| 48 | -13963 | 232 | -31414 | 232 | -48864 | 232 | -66310 | 232 | -83751 | 232 |
| 49 | -14153 | 237 | - 31705 | 237 | -49155 | 237 | -66601 | 237 | -84042 | 237 |
| 50 | - 14544 | 242 | -31996 | 242 | - 49446 | 242 | -66892 | 242 | -84332 | 242 |
| 51 | 0.14835 | 247 | 0.32287 | 247 | 0.49737 | 247 | 0.67182 | 247 | 0.84623 | 247 |
| 52 | -15126 | 252 | - 32278 | 252 | -50027 | 252 | -67473 | 252 | -84913 | 252 |
| 53 | -15417 | 257 | - 32869 | 257 | - 50318 | 257 | -67764 | 257 | -85204 | 257 |
| 54 | - 15708 | 262 | - 33160 | 262 | - 50609 | 262 | - 68055 | 262 | -85495 | 262 |
| 55 | - 15999 | 266 | - 33450 | 266 | - 50900 | 266 | -68345 | 266 | . 85785 | 266 |
| 56 | -16290 | 271 | - 33741 | 271 | -51191 | 271 | -68636 | 271 | -86076 | 271 |
| 57 | - 16580 | 276 | - 34032 | 276 | -51481 | 276 | -68927 | 276 | -86367 | 276 |
| 58 | -16871 | 281 | - 34323 | 281 | -51772 | 281 | -69217 | 281 | -86657 | 281 |
| 59 | -17162 | 286 | - 34614 | 286 | - 52063 | 286 | -69508 | 286 | -86948 | 226 |
| 60 | - 17453 | 291 | 34905 | 291 | -52354 | 291 | -69799 | 291 | -87239 | 291 |


|  |  | $5^{\circ}$ | $\underset{\substack{\text { Parts } \\ \text { for } \\ \text { for }}}{ }$ | $6^{\circ}$ | $\begin{gathered} \text { Parts } \\ \text { for } \\ \prime \prime \end{gathered}$ | $7 \times$ | $\begin{aligned} & \text { Parts } \\ & \text { for } \end{aligned}$ | $8{ }^{\circ}$ | $\begin{gathered} \text { Parts } \\ \text { for } \\ \text { for } \end{gathered}$ | $9^{\circ}$ | $\begin{aligned} & \text { Parts } \\ & \text { for } \\ & \text { /" } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 0.87239 | - | 1.04672 | $\bigcirc$ | I 22097 | - | I-39513 | $\bigcirc$ | 1-56918 |  |
|  | 1 | - 87529 | 5 | -04962 | 5 | - 22387 | 5 | -39803 | 5 | -57208 |  |
|  | 2 | -87820 | 10 | . 05252 | 10 | -22677 | 10 | -40093 | 10 | -57498 | 0 |
|  | 3 | -88110 | 15 | -05543 | 15 | - 22968 | 15 | - 40383 | 15 | $\cdot 57788$ | 15 |
|  | 4 | -88401 | 20 | . 05833 | 20 | - 23258 | 20 | -40673 | 20 | - 58078 | 20 |
|  | 5 | -88691 | 25 | -06124 | 25 | - 23548 | 25 | -40964 | 25 | -58368 | 25 |
|  | 6 | -88982 | 30 | -06414 | 30 | -23839 | 30 | -41254 | 30 | -58658 | 30 |
|  | 7 | -89273 | 35 | -. 06705 | 35 | -24129 | 35 | - 41544 | 35 | - 58948 | 35 |
|  | 8 | -89563 | 39 | -06995 | 39 | - 24419 | 39 | - 41834 | 39 | -59238 | 39 |
|  | 9 | -89854 | 44 | -07286 | 44 | -24710 | 44 | -42124 | 44 | - 59528 | 44 |
|  |  | - 90144 | 49 | -07576 | 48 | -25000 | 48 | . 42415 | 48 | -59818 | 48 |
| II |  | - 90435 | 53 | 1. 07867 | 53 | - 25292 | 53 | 1.42705 | 53 | -60108 | 53 |
| 12 |  | -90726 | 58 | -08157 | 58 | -25581 | 58 | -42995 | 58 | -60398 | 58 |
| 13 |  | -91016 | 63 | -08448 | 63 | - 25871 | 63 | -43285 | 63 | -60688 | 63 |
| 14 |  | -91307 | 68 | -08738 | 68 | -26161 | 68 | -43575 | 68 | -60978 | 68 |
| 15 |  | -91597 | 73 | -09029 | 73 | - 26452 | 73 | . 43866 | 73 | .61267 | 73 |
| If |  | -91888 | 78 | -09319 | 78 | - 26742 | 78 | -44156 | 78 | -61557 | 78 |
| 17 |  | -92178 | 82 | -09610 | 82 | -27032 | 82 | -44446 | 82 | -61847 | 82 |
| 18 |  | -92469 | 87 | -09900 | 87 | -27323 | 87 | -44736 | 87 | - 62137 | 87 |
| 19 |  | -92759 | 92 | - 10190 | 92 | -27613 | 92 | -45026 | 92 | -62427 | 92 |
| 20 |  | -93050 | 97 | -10481 | 97 | -27903 | 97 | 45316 | 97 | -62717 | 97 |
| 21 |  | -933341 | 102 | -10771 | 02 | 1-28 194 | 102 | 1.45607 | 102 | 1.63007 | 102 |
| 22 |  | -93631 | 107 | 11062 | 107 | - 28484 | 107 | -45897 | 107 | -63297 | 107 |
| 23 |  | -93922 | 112 | -11352 | 112 | -28774 | 112 | -46187 | 112 | -63587 | I12 |
| 24 |  | -94212 | 117 | -11643 | 117 | - 29064 | 117 | -46477 | 117 | - 63876 | 117 |
| 25 |  | -94503 | 122 | -11933 | 22 | - 29355 | 122 | $\cdot 46767$ | 122 | - 64166 | 122 |
| 26 |  | -94794 | 127 | -1223 | 127 | - 29645 | 127 | -47057 | 127 | - 64556 | 127 |
| 7 |  | -95084 | 132 | - 12514 | 132 | - 29935 | 132 | -47347 | 132 | - 64746 | 132 |
| 28 |  | . 95375 | 137 | -12804 | 137 | -30225 | 137 | -47637 | 137 | . 65036 | 137 |
| 9 |  | . 05665 | 141 | -13095 | 141 | - 30516 | 141 | -47927 | 141 | . 65326 | 141 |
| 30 |  | -95956 | 145 | -13385 | 145 | 308 | 45 | 48217 | 145 | . 63616 | 45 |
| 31 |  | -. 96246 | 150 | 1.13676 | 149 | 1•31096 | 149 | 1.48;07 | 149 | I.65906 | 149 |
| 2 |  | -96537 | 155 | -13966 | 154 | - 31387 | 154 | $\cdot 48797$ | 154 | -66196 | 154 |
| 33 |  | -96827 | 160 | -14257 | 159 | -31677 | 159 | - 49088 | 159 | - 66486 | 159 |
| 34 |  | -97118 | 165 | -14547 | 164 | - 31967 | 164 | - 49378 | 164 | . 66776 | 164 |
| 5 |  | - 97409 | 170 | -14837 | 169 | - 32257 | 169 | - 49668 | 169 | . 67065 | 169 |
| 36 |  | -97699 | 175 | -15128 | 174 | - 32547 | 174 | -49958 | 174 | . 67355 | ${ }^{1} 74$ |
| 37 |  | -97990 | 180 | -15418 | 179 | - 32838 | 179 | . 50248 | 179 | - 67645 | r 79 |
| 8 |  | -98280 | 185 | -15709 | 184 | -33128 | 184 | -50538 | 184 | - 67935 | 184 |
| 39 |  | -98571 | 190 | -15999 | 189 | -33418 | 189 | 50828 | 189 | -68225 | 189 |
| 40 |  | -98861 | 194 | 16289 | 193 | -33709 | 193 | 51178 | 193 | - 68515 | 193 |
| 41 |  | -99152 | 199 | I-16580 | 198 | - 33999 | 198 | r. 51408 | 198 | - 688805 | 198 |
| 42 |  | -99442 | 204 | -16870 | 203 | -34289 | 203 | 51698 | 203 | -69095 | 203 |
| 43 |  | -99733 | 209 | -17160 | 208 | - 34579 | 208 | - 51988 | 208 | -69384 | 208 |
| 44 |  | -00023 | 2 I 4 | -1745 | 213 | - 34869 | 213 | 52278 | 213 | -69674 | 213 |
| 45 |  | -00314 | 219 | -17441 | 218 | - 35160 | 218 | - 52568 | 218 | -69964 | 218 |
| 46 |  | -00605 | 223 | -18031 | 222 | - 35450 | 222 | - 52858 | 222 | -70254 | 222 |
| 47 |  | -00895 | 228 | -18322 | 227 | - 35740 | 227 | - 53148 | 227 | -70544 | 227 |
| 48 |  | . 01185 | 232 | -18612 | 231 | - 36030 | 231 | - 53438 | 231 | -70833 | 231 |
| 49 |  | . 01476 | 237 | -18903 | 236 | -36321 | 236 | - 53728 | 236 | -71123 | 236 |
| 50 |  | -01766 | 242 | -19193 | 24.1 | -36611 | 24 I | 18 | 241 | -71413 | 241 |
| 51 |  | . 02057 | 247 | r-19483 | 246 | . 36901 | 246 | - 54308 | 246 | - 71703 | 246 |
| 52 |  | -02348 | 252 | - 97774 | 251 | -37191 | 251 | - 54598 | 251 | -71993 | 251 |
| 53 |  | -02638 | 257 | - 20064 | 256 | -37481 | 256 | - 54888 | 256 | - 72283 | 256 |
| 54 |  | -02929 | 262 | - 20354 | 261 | - 37771 | 261 | -51178 | 261 | - 72572 | 261 |
| 55 |  | -03219 | 266 | - 20645 | 265 | - 38062 | 265 | - 55468 | 265 | - 72862 | 265 |
| 56 |  | -03510 | 271 | - 20935 | 270 | - 38352 | 270 | - 55758 | 270 | -73152 | 270 |
| 57 |  | -03800 | 276 | - 21226 | 275 | - 38642 | 275 | - 56048 | 275 | -73442 | 275 |
| 38 |  | - 04090 | 281 | - 21516 | 280 | -38932 | 280 | - 56338 | 280 | - 73732 | 280 |
| 59 |  | -04381 | 286 | 21806 | 285 | - 39222 | 285 | - 56628 | 285 | $\cdot 7402 \mathrm{I}$ | 285 |
| 60 |  | -04672 | 291 | 22097 | 290 | -39513 | 290 | -56918 | 290 | -74311 | 290 |


| Min. | $10^{\circ}$ | $\begin{gathered} \text { Parts } \\ \text { for } \\ \prime \end{gathered}$ | $11^{\circ}$ | $\begin{gathered} \text { Parts } \\ \text { for } \\ \prime \prime \\ \hline \end{gathered}$ | $12^{\circ}$ | $\underset{\substack{\text { Parts } \\ \text { for } \\ \hline}}{\text { Par }}$ | $13^{\circ}$ | $\begin{aligned} & \text { Parts } \\ & \text { for } \end{aligned}$ | $14^{\circ}$ | $\begin{aligned} & \text { Parts } \\ & \text { for } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - | 1-74311 | $\bigcirc$ | - 91691 | $\bigcirc$ | 2.09057 | $\bigcirc$ | $2 \cdot 26407$ | $\bigcirc$ | 2.43738 |  |
| 1 | - 74601 | 5 | -91980 | 5 | 09346 | 5 | - 26696 | 5 | - 44026 |  |
| 2 | -74891 | 10 | -92270 | 10 | -09635 | 10 | - 26985 | 10 | - 44315 | 10 |
| 3 | -7518¢ | 15 | -92560 | 15 | -09924 | 15 | - 27274 | 15 | -44604 | 15 |
| 4 | - 75470 | 20 | -92849 | 20 | -10214 | 20 | - 27563 | 20 | -44892 | 20 |
| 5 | - 75760 | 25 | -93139 | 25 | -10503 | 25 | - 27852 | 25 | - 4518 I | 25 |
| 6 | - 76050 | 30 | -93428 | 30 | -10792 | 30 | -28141 | 30 | - 45470 | 30 |
| 7 | -76340 | 33 | -93718 | 35 | . 11082 | 35 | - 28430 | 35 | - 45758 | 35 |
| 8 | - 76630 | 39 | -94008 | 39 | -11371 | 39 | - 28719 | $3 n$ | - 46047 | 39 |
| 9 | - 76919 | 44 | -94297 | 44 | -11660 | 44 | - 29008 | 44 | . 46336 |  |
| - | -77209 | 48 | -94587 | 48 | 11950 | 48 | - 29297 | 48 | -46625 | 48 |
| 11 | - $\cdot 77499$ | 53 | - 94876 | 53 | 2-12239 | 53 | 2. 29586 | 53 | $2 \cdot 46913$ | 53 |
| 12 | - 77789 | 58 | $\cdot 95166$ | 58 | -12528 | 58 | - 29875 | 58 | -47202 | 58 |
| 13 | - 78078 | 62 | -95455 | 62 | -12817 | 62 | -30164 | 62 | -47491 | 62 |
| 14 | - 78368 | 67 | -95745 | 67 | -13106 | 67 | - 30453 | 67 | -47779 | 67 |
| 15 | -78658 | 72 | -96034 | 72 | - 13396 | 72 | -30742 | 72 | -48068 | 72 |
| 16 | -78947 | 77 | -96324 | 77 | -13685 | 77 | -31031 | 77 | -48357 | 77 |
| 17 | -79237 | 81 | -96613 | 81 | - 13974 | 81 | -31320 | 81 | -48645 | 81 |
| 18 | -79527 | 86 | -96903 | 86 | - 14263 | 86 | -31609 | 86 | -48934 | 86 |
| 19 | -79816 | 91 | -97112 | 91 | -14552 | 91 | -31898 | 91 | -49223 | 91 |
| 20 | -80106 | 96 | -97482 | 96 | -14842 | 96 | -32187 | 96 | -49512 | 96 |
| 21 | 1.80396 | ror | 97771 | 1 | 2.15131 | ror | $2 \cdot 32475$ | Ior | - 49800 | 101 |
| 22 | -80686 | 106 | -98060 | ro6 | -15420 | 106 | - 32764 | 106 | - 50089 | 106 |
| 23 | -80975 | 115 | -98350 | 111 | -15709 | III | - 33053 | III | -50377 | III |
| 24 | -81265 | 116 | '98669 | 16 | -15998 | 116 | -33342 | 116 | 50666 | 116 |
| 25 | -81555 | 121 | -98929 | 121 | -16288 | 121 | - 33631 | 121 | - 50955 | 121 |
| 26 | -81844 | 126 | -99218 | 126 | -16577 | 126 | -33919 | 26 | 51243 | 126 |
| 27 | -82134 | 131 | -99507 | 131 | -16866 | 131 | -34208 | 131 | -51532 | 131 |
| 28 | - 82242 | 136 | -99797 | 136 | - 17155 | 136 | -34497 | 136 | - 5182 I | 136 |
| 29 | -82713 | 140 | -00086 | 140 | -17444 | 140 | -34786 | 140 | 52110 | 140 |
| 30 | -83003 | 144 | -00376 | 4 | - 17734 | 144 | - 35075 | 144 | -52399 | 144 |
| 3 I | $1 \cdot 83292$ | 148 | 2.00665 | 48 | $2 \cdot 18023$ | 148 | $2 \cdot 35363$ | 148 | - 52687 | 148 |
| 32 | -83582 | 153 | -00954 | 153 | -18312 | 153 | - 35652 | 153 | - 52976 | 153 |
| 33 | -83872 | 158 | -01244 | 158 | -18601 | 158 | -3594r | 158 | - 53264 | 158 |
| 34 | -84161 | 163 | -01533 | 163 | -18890 | 163 | - 36230 | 163 | - 53553 | 163 |
| 35 | -8445 1 | 168 | -01823 | 168 | -19179 | 168 | - 36519 | 168 | - 53848 | 168 |
| 36 | - 84740 | 173 | -02112 | 173 | -19468 | ${ }^{1} 73$ | - 36807 | 173 | -54130 | 173 |
| 37 | -85030 | 178 | -02401 | 178 | -19757 | 178 | -37096 | 178 | -54418 | 178 |
| 38 | -85320 | 183 | -02691 | 183 | -20046 | 183 | - 37385 | 183 | - 54707 | 183 |
| 39 | $\cdot 85609$ | 188 | -02980 | 188 | - 20335 | 188 | - 37674 | 188 | - 54995 | 188 |
| 40 | -85899 | 192 | -03270 | 92 | - 20625 | 192 | - 37963 | 192 | -55284 | 192 |
| 41 | 1-86188 | 197 | 2.03559 | 197 | 2.20914 | 197 | $2 \cdot 38251$ | 197 | . 55572 | 197 |
| 42 | -86478 | 202 | -03848 | 202 | - 21203 | 202 | - 38540 | 202 | '5586r | 202 |
| 43 | -86768 | 207 | . 04128 | 207 | - 21492 | 207 | - 38829 | 207 | -56149 | 207 |
| 44 | - 87056 | 212 | -04427 | 212 | -21781 | 212 | - 39118 | 212 | - $5643^{8}$ | 212 |
| 45 | -87347 | 217 | -04717 | 217 | -22070 | ${ }^{1} 7$ | -39407 | 217 | 56726 | 217 |
| 46 | - 87637 | 22 I | -05006 | 22 I | - 22359 | 221 | -39695 | 22 I | -57015 | 221 |
| 47 | -87926 | 226 | $\cdot 05295$ | 226 | - 22648 | 226 | -39984 | 226 | -57303 | 22 |
| 48 | -88216 | 230 | . 05585 | 230 | -22937 | 230 | -40273 | 230 | - 57592 | 230 |
| 49 | -88506 | 235 | -05874 | 235 | -23226 | 235 | -40562 | 235 | - 57880 | 235 |
| 50 | -88796 | 240 |  | 240 | - 23516 | 240 | -4085 1 | 240 | -58169 | 240 |
| 51 | $1 \cdot 89085$ | 245 | 2.06453 | 245 | $2 \cdot 23805$ | 245 | 2.41139 | 245 | $2 \cdot 58457$ | 25 |
| 52 | -89375 | 250 | -06742 | 250 | -24094 | 250 | -41428 | 250 | - 58745 | 250 |
| 53 | -89664 | 255 | -07031 | 255 | - 24383 | 255 | -41717 | 255 | - 59034 | 255 |
| 54 | -89954 | 260 | -07321 | 260 | $\cdot 24672$ | 260 | - 42005 | 260 | -59322 | 260 |
| 55 | -90243 | 264 | -07610 | 264 | - 24961 | 264 | -42294 | 264 | 59611 | 264 |
| 56 | -90533 | 269 | -07899 | 269 | - 25250 | 269 | -42583 | 269 | 59899 | 269 |
| 57 | -90822 | 274 | -08189 | 274 | - 25539 | 274 | $\cdot 42871$ | 274 | -60187 | 274 |
| 58 | -91112 | 279 | -08478 | 279 | $\cdot 25828$ | 279 | - 43160 | 279 | -60476 | 279 |
| 5 | -9140r | 284 | -08767 | 284 | -26117 | 284 | -43449 | 284 | -60764 | 284 288 |
| 60 | -9r69r | 289 | -09057 | 289 | - 26407 | 289 | -43738 | 289 | -61053 | 288 |


| Min. | $15^{\circ}$ |  | $16^{\circ}$ | $\underset{\substack{\text { Parts } \\ \text { for }}}{\text { Pr }}$ | $17^{\circ}$ | $\underset{\substack{\text { Parts } \\ \text { for } \\ \prime}}{\text { Pat }}$ | $18^{\circ}$ | $\underset{\substack{\text { Parts } \\ \text { for }}}{\text { Pat }}$ | $19^{\circ}$ | $\underset{\substack{\text { Parts } \\ \text { for } \\ \prime \prime}}{\text { Par }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - | $2 \cdot 6105$ | - | 2.783 | - | $2 \cdot 956$ | - | 3.12868 | $\bigcirc$ | $3 \cdot 30095$ |  |
| 1 | -61341 | 5 | -78634 | 5 | -95906 | 5 | -13155 | 5 | -30382 | 5 |
| 2 | -61629 | 10 | - 78922 | 10 | -96194 | 10 | -13442 | 10 | 30669 | \% |
| 3 | -61917 | 15 | -79210 | 15 | -96482 | 15 | -13729 | 15 | - 30955 | 15 |
| 4 | - 62206 | 20 | - 79498 | 20 | - 96769 | 20 | -14017 | 20 | -31242 | 0 |
| 5 | - 62494 | 25 | - 79786 | 25 | -97057 | 25 | - 14304 | 25 | - 31529 | 25 |
| 6 | -62782 | 30 | -80074 | 30 | -97345 | 30 | -14591 | 30 | -31816 | 30 |
| 7 | -63071 | 35 | - 80362 | 35 | -97632 | 35 | -14879 | 35 | -32103 | 35 |
| 8 | - 63359 | 39 | -80651 | 39 | -97920 | 39 | -15166 | 39 | - 32390 | 39 |
| 9 | -6364 | 44 | -80939 | 44 | -98208 | 44 | -15453 | 44 | - 32677 | 44 |
| 10 | -6393 | 48 | -81227 | 48 | -9849 | 48 | -15741 | 48 | -329 | 48 |
| 11 | $2 \cdot 64224$ | 53 | $2 \cdot 81515$ | 53 | 2.98783 | 53 | 3.16028 |  | 3.33251 | 53 |
| 12 | - 64512 | 58 | -81803 | 58 | -99071 | 58 | -16315 | 58 | - 33537 | 58 |
| 13 | - 64800 | 62 | -82091 | 62 | -99358 | 62 | -16602 | 62 | - 33824 | 62 |
| 14 | - 65089 | 67 | -82379 | 67 | -99646 | 67 | -16889 | 67 | -34111 | 67 |
| 15 | - 65377 | 72 | -82667 | 72 | 99934 | 72 | -17177 | 72 | - 34398 | 72 |
| 16 | - 6566 | 77 | -82955 | 77 | 3.00221 | 77 | - 17464 | 77 | - 34685 | 77 |
| 17 | - 65954 | 81 | -83243 | 81 | -00509 | 81 | -1775 | 81 | - 34971 | I |
| 18 | - 66242 | 86 | -83531 | 86 | -00796 | 86 | -18038 | 86 | - 35258 | 86 |
| 19 | -66530 | 91 | -83819 | 91 | -01084 | 91 | - 18325 | 91 | 3545 | 91 |
| 20 | -6681 | 96 | -84107 | 96 | -01372 | $9^{6}$ | -18613 | 96 | -35832 | 95 |
| 21 | $2 \cdot 67107$ | O | $2 \cdot 843$ | IOI | 3-01659 | or | 3. 18900 | IOI | 3. 36118 | 100 |
| 22 | -67395 | 106 | -84682 | 106 | . 01947 | 106 | $\cdot 19187$ | 106 | - 36405 | 05 |
| 23 | -67683 | 111 | - 84970 | 11 | . 02234 | 111 | -19474 | III | -36692 | 10 |
| 24 | -67971 | 116 | -85258 | 116 | . 02522 | 116 | -19762 | 116 | - 36979 | 115 |
| 25 | - 68260 | 121 | -85546 | 121 | - 02809 | 121 | - 20049 | 121 | $\cdot 37265$ | 120 |
| 26 | - 68548 | 126 | -85834 | 126 | -03097 | 126 | -2033 | 126 | -37552 | 125 |
| 27 | -68836 | 131 | -86122 | 131 | .03384 | 131 | -20623 | 131 | - 37839 | 130 |
| 28 | -69124 | 136 | - 86410 | 136 | -03672 | 136 | -20910 | 136 | - 38125 | 135 |
| 29 | -69412 | 140 | - 86698 | 140 | -03959 | 40 | - 2119 | 140 | $\cdot 38412$ | 139 |
| 30 | -69701 | 144 | -86986 | 144 | - 04247 | 144 | - 2148 | 14 | 38699 | 143 |
| 31 | 2.69989 | 148 | $2 \cdot 87273$ | 48 | . 045 | 148 | -2177 | 148 | $\cdot 38985$ | 14 |
| 32 | -70277 | 153 | $\cdot 87561$ | 153 | -0482 | 153 | - 22059 | 153 | - 39272 | 152 |
| 33 | $\cdot 70565$ | 158 | -87849 | 158 | -05109 | 158 | - 22346 | 158 | -39559 | 157 |
| 34 | -70853 | 163 | -88137 | 163 | -05396 | 163 | - 22633 | 163 | - 39845 | 162 |
| 35 | -71142 | 168 | -88425 | 168 | -05684 | 168 | - 22920 | 168 | -40132 | 167 |
| 36 | $\cdot 71430$ | 173 | -88712 | 173 | -0597 | 173 | - 23207 | 173 | -40418 | 172 |
| 37 | $\cdot 71718$ | 178 | -89000 | 178 | -06258 | 178 | - 23494 | 178 | -40705 | 177 |
| 38 | - 72006 | 183 | -89288 | 183 | -06546 | 183 | - 23782 | 183 |  | 182 |
| 39 | -72294 | 188 | -89576 | 188 | -06833 | 188 | - 24069 | 188 | - 41278 | 187 |
| 40 | - 72583 | 192 | -89864 | 192 | -07121 | 192 | - 24356 | 192 | - 41565 | 191 |
| 4 I | $2 \cdot 72871$ | 197 | -9015 | 97 | 3.07408 | 97 | 3. 24643 | 197 | 3.41851 | 196 |
| 42 | -73159 | 202 | -90439 | 202 | -07695 | 201 | - 24930 | 201 | -42138 | 200 |
| 43 | -73447 | 207 | -90727 | 207 | -07983 | 206 | - 25217 | 206 | - 42425 | 205 |
| 44 | -73735 | 212 | -91015 | 212 | -08270 | 211 | - 25504 | 211 | -42711 | 210 |
| 45 | -74024 | 217 | -91303 | 217 | -08558 | 1 | - 25791 | 216 | -42998 | 215 |
| 46 | $\cdot 743$ | 221 | -91590 | 221 | -08845 | 220 | - 2607 | 220 | - 43284 | 219 |
| 47 | -74600 | 226 | -91878 | 226 | -09132 | 225 | - 26365 | 225 | $\cdot 43571$ | 224 |
| 48 | $\cdot 74888$ | 230 | -92166 | 230 | -09420 | 229 | - 26652 | 229 | -43858 | 228 |
| 49 | 75176 | 235 | -92454 | 235 | -09707 | 234 | - 26939 | 234 | -44144 | 233 |
| 50 | -75465 | 240 | -92742 | 240 | -09995 | 239 | - 27226 | 239 | -4443 | 238 |
| 51 | 2.75753 | 245 | 2.93029 | 245 | 3-10282 | 244 | 3.27513 | 244 | 3.44717 | 243 |
| 52 | -76041 | 250 | -93317 | 250 | -10569 | 249 | - 27800 | 249 | - 45004 | 248 |
| 53 | - 76329 | 255 | -93605 | 255 | -10856 | 254 | - 28086 | 254 | -45290 | 253 |
| 54 | -76617 | 260 | -93892 | 260 | -11144 | 259 | - 28373 | 259 | - 45577 | 258 |
| 55 | $\cdot 76905$ | 264 | -94180 | 264 | -11431 | 263 | - 28660 | 263 | $\because 45863$ | 262 |
| 56 | -77193 | 269 | -94468 | 269 | -11718 | 268 | - 28947 | 268 | - 46150 | 267 |
| 57 | - 77481 | 274 | -94755 | 274 | -2006 | 273 | - 29234 | 273 | -46436 | 272 |
| 58 | - 77769 | 279 | -95043 | 279 | -12293 | 278 | - 29521 | 278 | $\cdot{ }^{4} 6723$ | 277 |
| 59 | - 78057 | 284 | -95331 | 284 | - 12580 | 283 | - 29808 | 283 | -47009 | 282 286 |
| 60 | $\cdot 78346$ | 288 | -95619 | 288 | ${ }^{1} 12868$ | 287 | $\cdot 30095$ | 287 | -47296 | 286 |


| Min. | $20^{\circ}$ | $\begin{array}{\|l\|l} \text { Parts } \\ \text { for } \\ \prime \end{array}$ | $21^{\circ}$ | $\begin{gathered} \text { Parts } \\ \text { for } \\ \text { II } \end{gathered}$ | $22^{\circ}$ | $\begin{aligned} & \text { Parts } \\ & \text { for } \end{aligned}$ | $23^{\circ}$ | $\begin{aligned} & \text { Parts } \\ & \text { for } \\ & \prime \prime \end{aligned}$ | $24^{\circ}$ | $\begin{gathered} \text { Parts } \\ \text { for } \\ \prime \prime \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\bigcirc$ | 3*47296 | $\bigcirc$ | 3.64471 | $\bigcirc$ | 3.81618 | - | 3.98735 | $\bigcirc$ | -15824 |  |
| 1 | -47582 | 5 | -64757 | 5 | -81904 | 5 | -99020 | 5 | 16109 |  |
| 2 | . 47869 | 10 | - 65043 | 10 | -82189 | 10 | -99305 | 10 | -16393 |  |
| 3 | -48155 | 15 | - 65329 | 15 | -82475 | 15 | -99590 | 15 | -16678 | 14 |
| 4 | -48441 | 20 | -65615 | 20 | -82760 | 20 | -99875 | 20 | -16962 | 19 |
| 5 | -48728 | 25 | -65901 | 25 | -83046 | 25 | -00160 | 25 | - 17247 | 24 |
| 6 | -49014 | 30 | -6618 | 30 | -8333 | 30 | -00445 | 30 | -17531 | 29 |
| 7 | -4930 | 35 | - 6647 | 35 | -8361 | 35 | -00730 | 35 | -17816 | 34 |
| 8 | -49587 | 39 | -66759 | 39 | -8 | 39 | -01015 | 39 | -18100 | 8 |
| 9 | - 49873 | 44 | - 67045 | 44 | - 84188 | 44 | -01300 | 44 | -18385 | 43 |
| 0 | -50 | 48 | -67331 | 48 | -84473 | 48 | -01585 | 48 |  | 47 |
| 11 | 3.50446 | 53 | 5.67617 | 53 | 3.84758 | 53 | 4-01870 | 53 | -18953 | 52 |
| 12 | -50732 | 58 | $\cdot 67903$ | 58 | -85044 | 58 | -02155 | 58 | -19238 | 57 |
| 13 | -51019 | 62 | -68189 | 62 | -85329 | 62 | -02440 | 62 | -19522 |  |
| 14 | $\cdot 51305$ | 67 | -68475 | 67 | -85615 | 67 | -02725 | 67 | -19807 | 66 |
| 15 | -51591 | 72 | -687 | 72 | -85900 | 72 | -03010 | 72 | - 20091 | 71 |
| 16 | - 51878 | 77 | -69046 | 77 | -86185 | 77 | -03294 | 77 | -20375 | 76 |
| 17 | -52164 | 81 | -69332 | 81 | -86471 | 81 | -03579 | 81 | - 20 | 80 |
| 18 | - 52450 | 86 | -69718 | 86 | - 86756 | 86 | -03864 | 86 | - 20944 | 85 |
| 19 | - 52736 | 91 | -70004 | 91 | -87042 | 91 | -04149 | 91 | - 21229 | 90 |
| 20 | - 53023 | 95 | -7019 | 95 | -87327 | 95 | - 04434 | 95 | - 215 | 94 |
| 21 | 3.53309 | 100 | 3.70476 | 100 | 3.87612 | 100 | 4.04719 | 100 | - 21797 | 99 |
| 22 | 5359 | 105 | -7076 | 105 | -87898 | 105 | -05004 | 105 | -2082 | 104 |
| 23 | -53882 | 10 | $\cdot 71047$ | 10 | -88183 | 110 | -05289 | 110 | -223010 | 109 |
| 24 | - 54168 | 115 | 71333 | 115 | -8846 | 115 | -05574 | 115 | - 226 | 114 |
| 25 | -54454 | 120 | $\cdot 71619$ | 120 | -88754 | 120 | -05859 | 120 | 229 | 19 |
| 26 | -54741 | 125 | -71905 | 125 | -89039 | 125 | .06143 | 125 | -2321 | 124 |
| 27 | - 5502 | 130 | 72191 | 130 | -89324 | 130 | -0642 | 130 | $\stackrel{2350}{ }$ | 129 |
| 28 | - 55313 | 135 | - 72476 | 135 | -89609 | 135 | -0671 | 135 | - 2378 | 134 |
| 29 | 55600 | 139 | 72762 | 139 | -8989 | 139 | -06998 | 139 | - 24072 | 138 |
| 30 | -5588 | 143 | 73048 | 143 | -9018 | 143 | -07283 | 143 | - 2435 | $14^{2}$ |
| 31 | 3.56172 | 147 | 3.7333 | 147 | 3.9046 | 147 | 4.07568 | 147 | - 24640 | 146 |
| 32 | -56458 | 152 | 73619 | 152 | -9075 | 152 | -07853 | 152 | -24924 | 151 |
| 33 | - 56745 | 157 | -73905 | 157 | .91036 | 157 | -08137 | 157 | - 25209 | 156 |
| 34 | -57031 | 162 | $\cdot 74191$ | 162 | -91321 | 16 r | -08422 | 16 r | - 25493 | 160 |
| 35 | -57317 | 167 | - 74477 | 167 | -916 | 166 | -08707 | 166 | - 25777 | 165 |
| 36 | - 57603 | 172 | $\cdot 74762$ | 172 | -9189 | 171 | -08992 | 171 | - 2606 | 170 |
| 37 |  | 177 | - 75048 | 177 | -9217 | 176 | -0927 | 176 | 26345 | 175 |
|  | 58176 | 182 | - 75334 | 182 | -9246 | 181 | -09561 | 181 | - 26630 | 180 |
| 39 40 | -58462 | 187 | -75619 | 187 | -92 | 186 | $\cdot 09846$ | 186 | - 26914 | 185 |
| 40 | 58 | 191 | $\cdot 75905$ | 19 I | -93 | 190 | -1013 | 19 | -2719 | 189 |
| 4 I | 3.59034 | 196 | 3.76191 | 196 | 3.933 | 195 | 4*1041 | 195 | . 27482 | 94 |
| 42 | -59320 | 200 | $\cdot 76476$ | 200 | -93602 | 199 | -10700 | 199 | - 27766 | 198 |
| 43 | - 59607 | 205 | $\cdot 76762$ | 205 | -9388 | 204 | - 10985 | 204 | - 2805 | 203 |
| 44 | - 59893 | 210 | -77048 | 210 | -94173 | 209 | -11270 | 209 | -28334 | 208 |
| 45 | -60179 | 215 | - 77334 | 215 | -94458 | 214 | -11555 | 214 | -28519 | 213 |
| 46 | -60465 | 219 | -77619 | 219 | -9474 | 218 | -11839 | 218 | -28903 | 217 |
| 47 | -6075 ${ }^{2}$ | 224 | - 77905 | 224 | -95028 | 223 | -12124 | 223 | - 2918 | 222 |
| 48 | -61038 | 228 | $\cdot 78191$ | 228 | -95314 | 227 | - 1240 | 227 | - 29471 | 226 |
| 49 | -61324 | 233 | $\cdot{ }^{78} 476$ | 233 | -95599 | 232 | -12693 | 232 | - 29755 | 231 |
| 50 |  | 238 | $\cdot{ }^{8}$ | 238 | -9588 | 237 | -12978 | 237 | - 3003 | 236 |
| 51 | $3 \cdot 61896$ | 243 | 3.79048 | 243 | 3.96169 | 42 | 4.13263 | 242 | - 30323 | 241 |
| 52 | -62182 | 248 | -79333 | 248 | -96454 | 247 | -13547 | 247 | - 30607 | 246 |
| 53 | -62468 | 253 | -79619 | 253 | -96739 | 252 | -13832 | 252 | - 30891 | 251 |
| 54 | -62754 | 258 | -79904 | 258 | -97024 | 257 | -14116 | 257 | - 31175 | 256 |
| 55 | . 63041 | 262 | -80190 | 262 | - 97310 | 261 | - 14401 | 261 | - 31459 | 260 |
| 56 | -63327 | 267 | - 80476 | 267 | - 97595 | 266 | - 14686 | 266 | - 31743 | 265 |
| 57 | .63613 | 272 | -80761 | 272 | -97880 | 27 r | 14970 | 271 | - 32027 | 270 |
| 58 | . 63889 | 277 | -81047 | 277 | -98165 | 276 | -15255 | ${ }^{2} 76$ | - 32311 | 275 |
| 59 | . 64185 | 282 | .81332 | 282 | -98450 | 281 | -15539 | 281 | - 32595 | 280 |
| 60 | -6447r | 286 | -81618 | 286 | -98735 | 285 | -15824 | 285 | -32879 | 284 |


| Min. | . $25^{\circ}$ | Parts <br> for | $26^{\circ}$ | Parts for | $27^{\circ}$ | $\begin{aligned} & \text { Parts } \\ & \text { for } \end{aligned}$ | 8 $28^{\circ}$ | Parts for | $29^{\circ}$ | Parts <br> for |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\bigcirc$ | 4.32879 | 0 | 4.49901 | 0 | 4.66890 | 0 | $4 \cdot 83843$ | 0 | $5 \cdot 00760$ | $\bigcirc$ |
| 1 | -33163 | 5 | - 50184 | 5 | -67173 | 5 | -84125 | 5 | - 01042 | 5 |
| 2 | - 33447 | 9 | - 50468 | 9 | -67456 |  | - 84407 | 9 | .01323 | 9 |
| 3 | -33730 | 14 | $\cdot 50751$ | 14 | -67738 | 14 | -84690 | 14 | . 01605 | 14 |
| 4 | - 34015 | 19 | - 51035 | 19 | -6802 1 | 19 | -84972 | 19 | -01886 | 19 |
| 5 | - 34298 | 24 | - 51318 | 24 | -68304 | 24 | -85254 | 24 | -02168 | 24 |
| 6 | -34582 | 29 | - 51601 | 29 | -68587 | 29 | - 85536 | 29 | -02450 | 29 |
| 7 | - 34866 | 34 | - 51885 | 34 | -68870 | 34 | -85818 | 34 | -02731 | 34 |
| 8 | - 35150 | 38 | - 52168 | 38 | . 69152 | 38 | -86101 | 38 | . 03013 | 38 |
| 9 | - 35434 | 43 | -52452 | 43 | -69435 | 43 | -86383 | 43 | . 03294 | 43 |
| 10 | - 35718 | 47 | - 52735 | 47 | -69718 | 47 | -86665 | 47 | -03576 | 47 |
| II | 4*36002 | 52 | 4.53018 | 52 | 4*70001 | 52 | $4 \cdot 86947$ | 51 | .03857 | 51 |
| 12 | - 36286 | 57 | - 53302 | 57 | - 70283 | 57 | -87229 | 56 | -04139 | 56 |
| 13 | - 36569 | 61 | $\cdot 53585$ | 61 | - 70566 | 61 | -87511 | 61 | -04420 | 61 |
| 14 | - 36853 | 66 | - 53868 | 66 | - 70849 | 66 | -87793 | 66 | -04702 | 66 |
| 15 | - 37137 | 71 | -54152 | 71 | -71113 | 71 | -88075 | 71 | -04983 | 71 |
| 16 | -37421 | 76 | - 54435 | 76 | -71414 | 76 | -88358 | 76 | -05265 | 76 |
| 17 | - 37705 | 80 | -54718 | 80 | - 71697 | 80 | -88640 | 80 | - 05546 | 80 |
| 18 | - 37988 | 85 | -55001 | 85 | - 71980 | 85 | -88922 | 85 | -05828 | 85 |
| 19 | - 38272 | 90 | - 55285 | 90 | - 72262 | 90 | -89204 | 90 | -06109 | 90 |
| 20 | - 38556 | 94 | - 55568 | 94 | -72545 | 94 | - 89486 | 94 | - 06391 | 94 |
| 21 | 4.38840 | 99 | 4.55851 | 98 | 4.72828 | 98 | 4-89768 | 98 | 5.06672 | 98 |
| 22 | -39123 | 104 | -56134 | 103 | -73110 | 103 | -90050 | 103 | -06954 | 103 |
| 23 | - 39407 | 109 | -56418 | 108 | - 73393 | 108 | . 90332 | 108 | -07235 | 108 |
| 24 | -39691 | 114 | - 56701 | I13 | -73675 | 113 | -90614 | 113 | -07516 | 113 |
| 25 | - 39975 | II9 | -56984 | 118 | -73958 | I18 | -90896 | 118 | -07797 | 118 |
| 26 | - 40258 | 124 | - 57267 | 123 | -7424 | 123 | -91178 | 123 | - 08079 | 123 |
| 27 | - 40542 | 129 | -57550 | 128 | -74523 | 128 | -91460 | 128 | -08360 | 128 |
| 28 | - 40826 | 134 | - 57834 | 133 | -74806 | 133 | -91742 | 133 | -0864I | 133 |
| 29 | -41109 | 138 | -58117 | 137 | -75088 | 137 | -92024 | 137 | -08923 | 137 |
| 30 | -41393 | 142 | -58400 | 141 | -75371 | 14 I | -92306 | 14 I | -09204 | 141 |
| 31 | $4 \cdot 41677$ | 146 | 4.58683 | 145 | 4.75653 | 145 | 4.92588 | 145 | 5.09485 | 145 |
| 32 | - 41960 | 151 | - 58966 | 150 | -75936 | 150 | -92870 | 150 | - 09766 | 150 |
| 33 | - 42244 | 156 | - 59249 | 155 | - 76218 | 155 | -93152 | 155 | -10048 | 155 |
| 34 | - 42528 | 160 | - 59532 | 159 | - 76501 | 159 | -93434 | 159 | -10329 | 159 |
| 35 | - 42812 | 165 | -59816 | 164 | - 76783 | 164 | -93615 | 164 | -10610 | 164 |
| 36 | - 43095 | 170 | -60099 | 169 | -77066 | 169 | -93997 | 169 | - 1089 r | 169 |
| 37 | - 43379 | 175 | -60382 | 174 | - 77348 | 174 | -94279 | 174 | - 11172 | 174 |
| 38 | - 43663 | 180 | - 60665 | 179 | -77631 | 179 | -9456r | 179 | - 11454 | 179 |
| 39 | - 43946 | 185 | - 60948 | 184 | - 77913 | 184 | -94843 | 184 | - 11735 | 184 |
| 40 | - 44230 | 189 | -61231 | 188 | $\cdot 78196$ | 188 | -95125 | 188 | -12016 | 188 |
| 4 I | 4.445 14 | 194 | 4.61514 | 1934 | 4*78478 | 593 | 4*95407 | 193 | -12297 | 193 |
| 42 | - 44797 | 198 | -61797 | 197 | $\cdot 78761$ | 197 | -95689 | 197 | - 12578 | 197 |
| 43 | * 45080 | 203 | -62080 | 202 | - 79043 | 202 | - 95970 | 202 | - 12859 | 202 |
| 44 | - 45363 | 208 | - 62363 | 207 | - 79326 | 207 | - 96252 | 207 | -13140 | 207 |
| 45 | - 45647 | 213 | - 62646 | 212 | - 79608 | 212 | - 96534 | 212 | - 13421 | 212 |
| 46 | - 45931 | 217 | -62929 | 216 | -79890 | 216 | -968r6 | 216 | -13703 | 216 |
| 47 | - 46214 | 222 | . 63212 | 221 | -80173 | 221 | -97098 | 221 | - 13984 | 221 |
| 48 | - 46498 | 226 | -63495 | 225 | -80455 | 225 | *97379 | 225 | - 14265 | 225 |
| 49 | - 4678 I | 231 | -63778 | 230 | -80738 | 230 | *97661 | 230 | -14546 | 230 |
| 50 | - 47065 | 236 | -64061 | 235 | -81020 | 235 | -97943 | 235 | -14827 | 235 |
| 51 | 4.47349 | 2414 | $4 \cdot 64344$ | $240+$ | 4.81302 | 240 | 4*98225 | 239 | - 15108 | 239 |
| 52 | . 47632 | 246 | -64627 | 245 | -81585 | 245 | -98506 | 244 | - 15389 | 244 |
| 53 | . 47916 | 251 | -64910 | 250 | -81867 | 250 | -98788 | 249 | -15670 | 249 |
| 54 | -48199 | 256 | -65193 | 255 | -82149 | 255 | -99070 | 254 | -15951 | 254 |
| 55 | - 48483 | 260 | -65475 | 259 | -8243I | 259 | -9935 | 258 | - 16232 | 258 |
| 56 | -48767 | 265 | -65758 | 264 | -82714 | 264 | -90633 | 263 | -16513 | 263 |
| 57 | - 49050 | 270 | -66041 | 269 | - 82996 | 269 | *99915 | 268 | -16794 | 268 |
| 58 | - 49334 | 275 | - 66324 | 274 | -83278 | 2745 | . 00197 | 273 | - 17075 | 273 |
| 59 | * $49^{6 \times 1} 7$ | 280 | -66607 | 279 | -8356r | 279 | -00478 | 278 | - 17356 | 278 |
| 60 | - 4990 r | 284 | -66890 | 283 | $\cdot 83843$ | 283 | $\cdot .00760$ | 282 | -17637 | 282 |


| Min. | $30^{\circ}$ | $\begin{aligned} & \text { Parts } \\ & \text { for } \\ & \text { /" } \end{aligned}$ | $31^{\circ}$ | $\begin{aligned} & \text { Parts } \\ & \text { for } \end{aligned}$ | $32^{\circ}$ | Parts for | $33^{\circ}$ | $\begin{gathered} \text { Parts } \\ \text { for } \\ \text { // } \end{gathered}$ | $34^{\circ}$ | $\begin{aligned} & \text { Parts } \\ & \text { for } \\ & \text { / } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - | 5.17638 | $\bigcirc$ | 5.34476 | - | 5.51274 | $\bigcirc$ | $5 \cdot 68030$ | $\bigcirc$ | -84744 |  |
| I | -17919 | 5 | - 34757 | 5 | - 51654 | 5 | . 68309 | 5 | -85022 |  |
| 2 | - 18200 | 9 | - 35038 | 9 | - 51834 | 9 | - 68588 | 9 | -85300 |  |
| 3 | -1848r | 14 | - 35318 | 14 | -52114 | 14 | -68867 | 14 | -85578 | 14 |
| 4 | - 18762 | 19 | - 35598 | 19 | - 52394 | 19 | - 69146 | 19 | - 85856 | 19 |
| 5 | -19043 | 23 | - 35878 | 23 | - 52673 | 23 | - 69425 | 23 | -86134 | 23 |
| 6 | -19324 | 28 | - 36158 | 28 | - 52952 | 28 | -69704 | 28 | -86412 | 28 |
| 7 | -19605 | 33 | - 36438 | 33 | - 53232 | 32 | -69983 | 32 | - 86690 | 32 |
| 8 | - 19886 | 37 | - 36718 | 37 | - 53512 | 36 | -70262 | 36 | -86968 | 36 |
| 9 | - 20167 | 42 | - 36999 | 42 | 53791 | 41 | -70541 | 41 | -87246 | 41 |
| 1 | - 2044 | 47 | - 37280 | 47 | - 54070 | 46 | -70820 | 46 | -87524 | 46 |
| 11 | 5.20729 | 51 | 5.37560 | 51 | - 54350 | 50 | -71098 | 50 | $5 \cdot 87802$ | 50 |
| 12 | - 21010 | 56 | - 37840 | 56 | - 54630 | 55 | - 71376 | 55 | -88080 | 55 |
| 13 | - 21290 | 61 | - 38120 | 61 | - 54909 | 60 | - 71655 | 60 | -88358 | 60 |
| 14 | - 21570 | 65 | - 38400 | 65 | - 55188 | 65 | 71934 | 65 | - 88636 | 65 |
| 15 | - 21851 | 70 | - 38680 | 70 | - 55468 | 70 | -72213 | 70 | -88914 | 70 |
| 16 | -22132 | 75 | - 38960 | 75 | - 55748 | 75 | -72492 | 75 | -89192 | 75 |
| 17 | - 22413 | 79 | - 39240 | 79 | - 56027 | 79 | - 72771 | 79 | -89470 | 79 |
| 18 | - 22694 | 84 | - 39520 | 84 | - 56306 | 84 | -73050 | 84 | -89748 | 84 |
| 19 | - 22975 | 89 | - 39800 | 89 | - 56585 | 89 | - 73328 | 89 | -90026 | 89 |
| 20 | - 23256 | 93 | -4 | 93 | - 56864 | 93 | -73606 | 93 | -90304 | 93 |
| 21 | 5.23537 | 98 | 5.40360 | 98 | 5.57144 | 97 | -73885 | 97 | 5.90582 | 97 |
| 22 | - 23818 | 103 | -40640. | 103 | -57424 | 102 | -74164 | 102 | -90860 | 102 |
| 23 | - 24098 | 108 | -40920 | 108 | - 57703 | 106 | -74443 | 106 | $\cdot 91138$ | 106 |
| 24 | - 24378 | 112 | -4120 | 112 | - 57982 | III | -74722 | 111 | -91416 | III |
| 25 | - 24659 | 117 | 41481 | 117 | -58261 | 116 | -75000 | 116 | -91694 | 16 |
| 26 | - 24940 | 122 | $\cdot 41762$ | 122 | $\cdot 58540$ | 121 | - 75278 | 121 | -91972 | 121 |
| 27 | - 2522 I | 126 | -42042 | 126 | - 58820 | 125 | - 75557 | 125 | -92250 | 125 |
| 28 | - 25502 | 131 | -42322 | 131 | - 59100 | I30 | -75836 | 130 | -92528 | 130 |
| 29 | - 25782 | 136 | -42601 | 136 | - 59379 | 135 | -76114 | 135 | -92806 | 135 |
| 30 |  | 140 | -42880 | 140 | - 5 | 139 | -76392 | 139 | 93084 | 139 |
| 31 | 5:26343 | 145 | 5.43160 | 145 | 5.59937 | 144 | 5.76671 | 144 | 5.93361 | 144 |
| 32 | - 26624 | 150 | -43440 | 150 | -602 16 | 149 | 76950 | 148 | . 93638 | 148 |
| 33 | - 26905 | 154 | $\cdot 43720$ | 154 | - 60496 | 153 | -77228 | 152 | -93916 | 152 |
| 34 | - 27186 | 159 | - 4400 | 159 | - 60776 | 158 | - 77506 | 157 | -94194 | 157 |
| 35 | - 27466 | 164 | - 4428 | 164 | - 61055 | 163 | $\cdot 77885$ | 162 | -94472 | 162 |
| 36 37 | - 27746 | 168 <br> 173 | -44560 | 168 | . 61334 | 167 | $\cdot 78064$ | 166 | -94750 | 166 |
| 38 | -28308 | 178 | - 445120 | 178 | . 61892 | 177 | $\begin{array}{r}\cdot 78342 \\ \cdot 78620 \\ \hline 78\end{array}$ | 171 | -95028 | 171 176 |
| 39 | - 28588 | 183 | - 45400 | 183 | -62171 | 182 | $\cdot 78899$ | 181 | -95583 | 176 181 |
| 40 | - 28868 | 187 | . 45680 | 187 | -62450 | 186 | -79178 | 185 | -98560 | 185 |
| 41 | 5.29149 | 192 | 5.45960 | 192 | 5.62729 | 191 | 5•79456 | 190 | 5.96r38 | 190 |
| 42 | - 29430 | 196 | - 46240 | 196 | . 63008 | 195 | -79734 | 194 | 96416 | 194 |
| 43 | - 29710 | 201 | - 46520 | 201 | -63287 | 200 | -80013 | 199 | -96694 | 199 |
| 44 | - 29990 | 206 | -46800 | 206 | -63566 | 205 | -80292 | 204 | - 96972 | 204 |
| 45 | -30271 | 10 | -47079 | 210 | -63845 | 209 | -80570 | 208 | -97249 | 208 |
| 46 | -30552 | 215 | -47i5 ${ }^{8}$ | 215 | -64124 | 214 | -80848 | 213 | -97526 | 213 |
| 47 | - 30832 | 220 | - 47638 | 219 | - 64404 | 218 | -81126 | 217 | -97804 | 217 |
| 48 | - 31112 | 224 | -47918 | 223 | - 64684 | 222 | -81404 | 221 | -98082 | 221 |
| 49 | -31393 | 229 | -48198 | 228 | - 64963 | 227 | $\cdot 81683$ | 22 | -98359 | 226 |
| 50 | - 31674 | 234 | - 48478 | 233 | -65242 | 232 | -819 | 231 | -9863 | 231 |
| 51 | 5.31954 | 238 | 5.48758 | 237 | 5.65521 | 236 | 5-82240 | 235 | 5.98914 | 235 |
| 52 | - 32234 | 243 | -49038 | 242 | -65800 | 241 | -82518 | 240 | -99192 | 240 |
| 53 | -32514 | 248 | -49317 | 247 | -66079 | 246 | -82796 | 245 | -99469 | 245 |
| 54 | - 32794 | 252 | -49596 | 251 | . 66358 | 250 | - 83074 | 249 | - 99746 | 249 |
| 55 | - 33075 | 257 | -49876 | 256 | -66637 | 255 | -83352 | 254 | 6.00024 | 254 |
| 56 | - 33356 | 262 | - 50156 | 261 | . 66916 | 260 | -83630 | 259 | -00302 | 259 |
| 57 | - 33636 | 266 | -50436 | 265 | -67194 | 264 | -83909 | 263 | -00579 | 263 268 |
| 58 | - 35916 | 271 | -50716 | 270 | -67472 | 269 | -84188 | 268 | -00856 | 268 |
| 59 | - 34196 | 276 | - 50995 | 275 | . 67751 | 274 | -84466 | ${ }^{2} 73$ | - 01134 | 273 278 |
| 60 | - 34476 | 281 | -51274 | 280 | - 68030 | 279 | -84744 | 278 | -01412 | 278 |


| Min. | $35^{\circ}$ | $\begin{aligned} & \text { Parts } \\ & \text { for } \\ & \prime \prime \end{aligned}$ | $36^{\circ}$ | $\begin{gathered} \text { Parts } \\ \text { for } \\ \text { /" } \end{gathered}$ | $37^{\circ}$ | $\begin{aligned} & \text { Parts } \\ & \text { for } \\ & \text { /" } \end{aligned}$ | $38^{\circ}$ | $\begin{aligned} & \text { Parts } \\ & \text { for } \\ & \text { /" } \end{aligned}$ | - $39^{\circ}$ | $\begin{aligned} & \text { Parts } \\ & \text { for } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 6.01412 | 0 | $6 \cdot 18034$ | $\bigcirc$ | 6.34610 | 0 | $6 \cdot 51136$ | 0 | 6.67614 | 0 |
| 1 | -01689 | 5 | -18311 | 5 | - 34886 | 5 | 51411 | 5 | -67888 | 5 |
| 2 | -01966 | 9 | -18588 | 9 | -35162 | 9 | -51686 | 9 | -68162 | 9 |
| 3 | -02244 | 14 | -18864 | 14 | - 35437 | 14 | -51961 | 14 | -68436 | 14 |
| 4 | . 02522 | 18 | -19140 | 18 | - 35712 | 18 | -52236 | 18 | -68710 | 18 |
| 5 | - 02799 | 23 | -19417 | 23 | - 35988 | 23 | -52511 | 23 | -68984 | 23 |
| 6 | -03076 | 28 | -19694 | 28 | - 36264 | 28 | - 52786 | 28 | -69258 | 27 |
|  | . 03353 | 32 | - 19970 | 32 | - 36540 | 32 | . 53061 | 32 | -69532 | 31 |
| 8 | . 03630 | 37 | - 20246 | 37 | -36816 | 37 | - 53336 | 37 | -69806 | 36 |
| 9 | -03908 | 41 | - 20523 | 41 | - 37092 | 41 | . 53611 | 4 I | -70081 | 40 |
| 10 | -04186 | 46 | -20800 | 46 | - 37368 | 46 | - 53886 | 46 | - 70356 | 45 |
| 11 | $6 \cdot 04463$ | 50 | $6 \cdot 21076$ | 50 | 6.37643 | 50 | 6.54161 | 50 | $6 \cdot 70630$ | 50 |
| 12 | -04740 | 55 | - 21352 | 55 | - 37918 | 55 | - 54436 | 55 | 70904 | 55 |
| 13 | -05017 | 59 | - 21629 | 59 | -38194 | 59 | -54711 | 59 | - 71178 | 59 |
| 14 | -05294 | 64 | - 21906 | 64 | - 38470 | 64 | - 54986 | 64 | - 71452 | 64 |
| 15 | . 05571 | 69 | - 222182 | 69 | $\cdot 38746$ | 68 | - 55261 | 68 | - 71726 | 68 |
| 16 | -05848 | 74 | - 22458 | 74 | - 39022 | 73 | - 55536 | 73 | - 72000 | 73 |
| 17 | .06126 | 78 | - 22735 | 78 | - 39297 | 77 | -55810 | 77 | - 72274 | 77 |
| 18 | - 06404 | 83 | - 23012 | 83 | - 39572 | 82 | -56084 | 82 | - 72548 | 82 |
| 19 | - 06681 | 88 | - 23288 | 88 | - 39848 | 87 | - 56359 | 87 | - 72822 | 87 |
| 20 | -06958 | 92 | - 23564 | 92 | - 40124 | 91 | - 56634 | 91 | -73096 | 91 |
| 21 | 6.07235 | 96 | $6 \cdot 2384 \mathrm{I}$ | 96 | 6.40399 | 95 | 6.56909 | 95 | 6.73369 | 95 |
| 22 | -07512 | IOI | -24118 | IOI | - 40674 | 100 | . 57184 | 100 | -73642 | 100 |
| 23 | -07789 | 105 | - 24394 | 105 | - 40950 | 104 | - 57459 | 104 | -73916 | 104 |
| 24 | -08066 | 110 | - 24670 | 110 | - 41226 | 109 | - 57734 | 109 | - 74190 | 109 |
| 25 | - 08343 | 115 | - 24946 | 115 | - 41502 | II4 | -58008 | 114 | - 74464 | 114 |
| 26 | - 08620 | 120 | - 25222 | 120 | - 41778 | I19 | -58282 | 119 | -74738 | 119 |
| 27 | -08897 | 124 | - 25499 | 124 | - 42053 | 123 | - 58557 | 123 | - 75012 | 123 |
| 28 | -09174 | 129 | - 25776 | 129 | . 42328 | 128 | -58832 | 128 | -75286 | 128 |
| 29 | - 09451 | 134 | - 26052 | 134 | - 42604 | 133 | -59107 | 133 | -75560 | 133 |
| 30 | - 09728 | 138 | - 26328 | 138 | - 42880 | 137 | - 59382 | 137 | - 75834 | 137 |
| 31 | $6 \cdot 10005$ | 143 | 6-26604 | 143 | 6.43155 | 142 | 6.59656 | 142 | 6-76108 | 142 |
| 32 | -10282 | 147 | - 26880 | 147 | - 43430 | 146 | - 59930 | 146 | - 76382 | 146 |
| 33 | - 10559 | 151 | - 27156 | 151 | - 43705 | 150 | - 60205 | 150 | - 76655 | 150 |
| 34 | - 10836 | 156 | - 27432 | 156 | - 43980 | 155 | -60480 | 155 | - 76928 | 155 |
| 35 | -11113 | 161 | - 27709 | 16 I | - 44256 | 160 | -60754 | 160 | - 77202 | 160 |
| 36 | - 11390 | 165 | - 27986 | 165 | - 44532 | 164 | -61028 | 164 | - 77476 | 164 |
| 37 | -11667 | 170 | - 28262 | 170 | - 44807 | 169 | -61303 | 169 | - 77750 | 169 |
| 38 | - 11944 | 175 | - 28538 | 175 | - 45082 | 174 | -61578 | 174 | - 78024 | 173 |
| 39 | - 12221 | 180 | - 28814 | 180 | - 45357 | 179 | -61852 | 179 18 | -78297 | 178 |
| 40 | - 12498 | 184 | - 29090 | 184 | - 45632 | 183 | -62126 | 183 | $\cdot 78570$ | 182 |
| 4 I | 6.12775 | 189 | 6*29366 | 189 | 6.45908 | 188 | $6 \cdot 62401$ | 188 | 6-78844 | 187 |
| 42 | -13052 | 193 | - 29642 | 193 | - 46184 | 192 | -62676 | 192 | -79119 | 191 |
| 43 | -13329 | 198 | - 29918 | 198 | -46459 | 197 | - 62950 | 197 | - 79392 | 196 |
| 44 | -13606 | 203 | - 30194 | 203 | - 46734 | 202 | -63224 | 202 | -79666 | 201 |
| 45 | - 13883 | 207 | - 30470 | 207 | - 47009 | 206 | -63499 | 206 | - 79939 | 205 |
| 46 | -14160 | 212 | - 30746 | 212 | - 47284 | 211 | -63774 | 211 | -80212 | 210 |
| 47 | ${ }^{\prime} 14437$ | 216 | -31022 | 216 | -47559 | 215 | -64048 | 215 | - 80486 | 214 |
| 48 | -14714 | 220 | - 31298 | 220 | - 47834 | 219 | -64322 | 219 | -80760 | 218 |
| 49 | - 14990 | 225 | - 31574 | 225 | . 48110 | 224 | -64597 | 224 | - 8:033 | 223 |
| 50 | -15266 | 230 | - 31850 | 230 | -48386 | 229 | -64872 | 229 | -81306 | 228 |
| 51 | 6-15543 | 234 | $6 \cdot 32126$ | 234 | $6 \cdot 4866$ I | 233 | 6.65146 | 233 | -81580 | 232 |
| 52 | -15820 | 239 | $\cdot 32402$ | 239 | - 48936 | 238 | - 65420 | 238 | -81854 | 237 |
| 53 | -16097 | 244 | - 32678 | 244 | -49211 | 243 | - 65694 | 243 | -82127 | 242 |
| 54 | -16374 | 248 | - 32954 | 248 | -49486 | 247 | - 65968 | 247 | - 82400 | 246 |
| 55 | -1665 | 253 | - 33230 | 253 | - 49761 | 252 | - 66242 | 252 | -82673 | 251 |
| 56 | - 16928 | 258 | - 33506 | 257 | -50036 | 256 | - 66516 | 256 | - 82946 | 255 |
| 5.7 | - 17204 | 262 | $\cdot 33782$ | 261 | . 50311 | 260 | -66791 | 260 | -83220 | 259 |
| 58 | - 17480 | 267 | - 34058 | 266 | - 50586 | 265 | - 67066 | 265 | -83494 | 264 |
| 59 | -17757 | 272 | - 34334 | 271 | $\cdot 50861$ | 270 | - 67340 | 270 | -83767 | 269 |
| 60 | ${ }^{\cdot} 18034$ | 277 | - 34610 | 276 | $\cdot 51136$ | 275 | -67614 | 275 | -84040 | 274 |


| Min. | $40^{\circ}$ | Parts for | $41^{\circ}$ | Parts for | $42^{\circ}$ | Parts for | $43^{\circ}$ | Parts for | 4.0 | Parts for |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\bigcirc$ | 6.84040 | 0 | 7.00414 | $\bigcirc$ | 7.16736 | $\bigcirc$ | 7-33002 | $\bigcirc$ | 7*49214 | $\bigcirc$ |
| 1 | -84314 | 5 | -00687 | 5 | - 17008 | 5 | - 33273 | 5 | -49483 | 5 |
| 2 | -84588 | 9 | -00960 | 9 | - 17280 | 9 | - 33544 | 9 | -49752 | 9 |
| 3 | -84861 | 14 | . 01232 | 14 | -17551 | 14 | -33815 | 14 | - 50022 | 14 |
| 4 | -85134 | 18 | . 01504 | 18 | -17822 | 18 | - 34086 | 18 | - 50292 | 18 |
| 5 | -85407 | 23 | -01777 | 23 | -18094 | 23 | - 34356 | 23 | $\cdot 50562$ | 23 |
| 6 | -85680 | 27 | - 02050 | 27 | -18366 | 27 | - 34626 | 27 | -50832 | 27 |
| 7 | -85953 | 31 | -02322 | 31 | -18637 | 31 | - 34897 | 31 | -51101 | 31 |
| 8 | -86226 | 36 | -02594 | 36 | -18908 | 36 | - 35168 | 36 | - 51370 | 36 |
| 9 | - 86500 | 40 | - 02866 | 40 | -19179 | 40 | - 35438 | 40 | - 51640 | 40 |
| 10 | -86774 | 45 | . 03138 | 45 | -19450 | 45 | - 35708 | 45 | -51910 | 45 |
| 11 | $6 \cdot 87047$ | 50 | 7.03411 | 50 | 7-19722 | 50 | 7.35979 | 50 | 7.52179 | 50 |
| 12 | - 87320 | 55 | -03684 | 55 | -19994 | 55 | - 36250 | 54 | - 52448 | 54 |
| 13 | - 87593 | 59 | . 03956 | 59 | - 20265 | 59 | - 36520 | 59 | -52718 | 59 |
| 14 | -87866 | 64 | . 04228 | 64 | - 20536 | 64 | - 36790 | 63 | - 52988 | 63 |
| 15 | -88139 | 68 | - 04500 | 68 | - 20808 | 68 | -37c60 | 68 | - 53257 | 68 |
| 16 | -88412 | 73 | . 04772 | 73 | - 21080 | 73 | - 37330 | 72 | -53526 | 72 |
| 17 | -88685 | 77 | . 05044 | 77 | - 21351 | 77 | -3-60r | 77 | - 53796 | 77 |
| 18 | -88958 | 82 | .05316 | 82 | -21622 | 81 | -37872 | 81 | - 54066 | 81 |
| 19 | -89231 | 87 | . 05589 | 87 | - 21893 | 86 | -38142 | 86 | - 54335 | 86 |
| 20 | -89504 | 9 I | -05862 | 91 | -22164 | 90 | $\cdot 38412$ | 90 | - 54604 | 90 |
| 21 | 6-89777 | 95 | 7.06134 | 95 | $7 \cdot 22435$ | 94 | $7 \cdot 38683$ | 95 | 7.54873 | 95 |
| 22 | -90050 | $100 \cdot$ | - 06406 | 100 | - 22706 | 99 | - 38954 | 99 | . 55142 | 99 |
| 23 | -90323 | 104 | - 06678 | 104 | - 22978 | 103 | -39224 | 104 | - 55412 | 104 |
| 24 | - 90596 | 109 | - 06950 | 109 | -23250 | 108 | -39494 | 108 | - 55682 | 108 |
| 25 | -90869 | 113 | -07222 | 113 | -2352 1 | 112 | - 39764 | 113 | -55951 | 113 |
| 26 | -91142 | 118 | - 07494 | 118 | - 23792 | 117 | - 40034 | 117 | -56220 | 117 |
| 27 | -91415 | 122 | -07766 | 122 | - 24063 | 12 I | - 40304 | 122 | -56489 | 122 |
| 28 | -91688 | 127 | -08038 | 127 | - 24334 | 126 | - 40574 | 126 | -56758 | 126 |
| 29 | -91961 | 132 | -08310 | 132 | - 24605 | 131 | -40844 | 131 | -57028 | 131 |
| 30 | -92234 | 136 | -08582 | 136 | - 24876 | 135 | -41114 | 135 | - 57298 | 135 |
| 31 | 6.92507 | 141 | 7-08854 | 141 | $7 \cdot 25147$ | 140 | 7.41385 | 140 | 7-57567 | 140 |
| 32 | -92780 | 146 | -09126 | 146 | -25418 | 145 | - 41656 | I44 | - 57836 | 144 |
| 33 | -93053 | 150 | -09398 | 150 | - 25689 | 149 | -41926 | 149 | -58105 | 149 |
| 34 | -93326 | 155 | - 09670 | 155 | - 25960 | 154 | -42196 | 153 | - 58374 | 153 |
| 35 | -93599 | 160 | -09942 | 159 | - 26231 | 158 | - 42466 | 158 | - 58643 | 158 |
| 36 | -93872 | 164 | -10214 | 163 | - 26502 | 162 | - 42736 | 162 | -58912 | 162 |
| 37 | -94145 | 169 | -10486 | 168 | - 26773 | 167 | - 43006 | 167 | -59181 | 167 |
| 38 | -94418 | 173 | -10758 | 172 | - 27044 | 171 | - 43276 | 171 | - 59450 | 171 |
| 39 | -94690 | 178 | - 11030 | 177 | - 27315 | 176 | - 43546 | 176 | . 59719 | ${ }^{1} 75$ |
| 40 | -94962 | 182 | -11302 | 181 | - 27586 | 180 | -43816 | 180 | - 59988 | 179 |
| 41 | 6.95235 | 187 | 7*11574 | 186 | 7*27857 | 185 | $7 \cdot 44086$ | 185 | $7 \cdot 60257$ | 184 |
| 42 | -95508 | 191 | -11846 | 190 | -28128 | 189 | - 44356 | 189 | - 60526 | 188 |
| 43 | -95781 | 196 | -12117 | 195 | -28399 | 194 | - 44626 | 194 | -60795 | 193 |
| 44 | -96054 | 200 | - 12388 | 199 | - 28670 | 198 | - 44896 | 198 | -61064 | 197 |
| 45 | - 96326 | 204 | -12660 | 203 | - 28941 | 202 | - 45166 | 203 | -61333 | 202 |
| 46 | -96598 | 2 C 9 | -12932 | 208 | - 29212 | 207 | - 45436 | 207 | -61602 | 206 |
| 47 | -96871 | 213 | -13204 | 212 | - 29483 | 211 | - 45706 | 212 | -61871 | 211 |
| 48 | -97144 | 217 | -13476 | 216 | - 29754 | 215 | - 45976 | 216 | -62140 | 215 |
| 49 | -97417 | 222 | -13748 | 221 | - 30025 | 220 | -46246 | 221 | -624c9 | 220 |
| 50 | -97690 | 227 | -14020 | 226 | - 30296 | 225 | - 46516 | 225 | -62678 | 224 |
| 5 I | 6.97962 | 231 | 7.14291 | 230 | $7 \cdot 30566$ | 229 | $7 \cdot 46786$ | 230 | 7-62947 | 229 |
| 52 | -98234 | 236 | -14562 | 235 | - 30836 | 234 | - 47056 | 234 | -63216 | 233 |
| 53 | -98507 | 24 I | -14834 | 240 | -31107 | 239 | - 47325 | 239 | -63485 | 238 |
| 54 | -98780 | 245 | -15106 | 244 | -31378 | 243 | - 47594 | 243 | -637i4 | 242 |
| 55 | -99052 | 250 | -15378 | 249 | - 31649 | 248 | - 47864 | 248 | - 64023 | 247 |
| 56 | -99324 | 254 | -15650 | 253 | - 31920 | 252 | - 48134 | 252 | -64292 | 251 |
| 57 | -99597 | 258 | -15921 | 257 | -32191 | $2 ; 6$ | - 48404 | 257 | -6456r | 256 |
| 58 | -99870 | 263 | -16192 | 262 | - 32462 | 26 r | - 48674 | 261 | -64830 | 260 |
| 59 | 7.00142 | 268 | -16464 | 267 | -32732 | 266 | -48944 | 266 | -65098 | 265 |
| 60 | -00414 | 273 | - 167736 | 272 | $\cdot 33002$ | 271 | -49214 | 270 | -65366 | 269 |


| Min. | $45^{\circ}$ | $\begin{aligned} & \text { Parts } \\ & \text { for } \end{aligned}$ | 4 $46^{\circ}$ | $\begin{aligned} & \text { Parts } \\ & \text { for } \end{aligned}$ | $47^{\circ}$ | Parts <br> for | $48^{\circ}$ | Parts for | $49^{\circ}$ | Part: <br> for |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\bigcirc$ | 7.65366 | $\bigcirc$ | 7.81462 | - | 7-97498 | $\bigcirc$ | 8•13474 | $\bigcirc$ | 8-29386 | 0 |
| 1 | - 65635 | 5 | -81730 | 5 | -97765 | 4 | -13739 | 4 | - 29651 | 4 |
| 2 | - 65904 | 9 | -81998 | 9 | -98032 | 8 | -14004 | 8 | - 29916 | 8 |
| 3 | - 66173 | 14 | - 82266 | 14 | -98299 | 13 | - 14270 | 13 | -30181 | 13 |
| 4 | - 66442 | 18 | -82534 | 18 | -98566 | 17 | -14536 | 17 | - 30446 | 17 |
| 5 | -66711 | 23 | -82801 | 23 | -98832 | 22 | -14802 | 22 | - 30710 | 22 |
| 6 | -66980 | 27 | -83068 | 27 | -99098 | 26 | -15068 | 26 | - 30974 | 26 |
| 7 | -67248 | 31 | . 83336 | 31 | -99365 | 30 | -15333 | 30 | - 31239 | 30 |
| 8 | . 67516 | 36 | -83604 | 36 | -99632 | 35 | -15598 | 35 | - 31504 | 35 |
| 9 | - 67785 | 40 | -83872 | 40 | -99899 | 39 | -15864 | 39 | - 31768 | 39 |
| 10 | -68054 | 45 | -84140 | 45 | $8 \cdot 00166$ | 44 | -16130 | 44 | - 32032 | 44 |
| 1 I | 7.68322 | 50 | 7.84407 | 50 | 8.00432 | 49 | 8-16396 | 49 | 8. 32297 | 49 |
| 12 | -68590 | 54 | -84674 | 54 | -00698 | 53 | - 16,662 | 53 | - 32562 | 53 |
| 13 | -68859 | 59 | - 84942 | 59 | -00965 | 58 | - 16927 | 58 | - 32826 | 58 |
| 14 | -69128 | 63 | -85210 | 63 | - 01232 | 62 | -17192 | 62 | - 33090 | 62 |
| ${ }^{15}$ | - 69396 | 67 | - 85477 | 67 | -01498 | 66 | -17458 | 66 | - 33355 | 66 |
| 16 | -69664 | 71 | -85744 | 71 | -01764 | 70 | - 17724 | 70 | - 33620 | 70 |
| 17 | -69933 | 76 | - 86012 | 76 | -02031 | 75 | - 17989 | 75 | - 33884 | 75 |
| 18 | - 70202 | 80 | -86280 | 80 | -02298 | 79 | -18254 | 79 | -34148 | 79 |
| 19 | - 70470 | 85 | -86547 | 85 | - 02564 | 84 | -18519 | 84 | - 34413 | 84 |
| 20 | - 70738 | 89 | -86814 | 89 | -02830 | 88 | - 18784 | 88 | - 34678 | 88 |
| 2 I | 7*71007 | 94 | $7 \cdot 87082$ | 94 | 8.03096 | 93 | 8.19050 | 93 | 8. 34942 | 93 |
| 22 | -71276 | 98 | - 87350 | 98 | . 03362 | 97 | -19316 | 97 | -35206 | 97 |
| 23 | -71544 | 103 | -87617 | 103 | -03629 | 102 | -19581 | 102 | - 35470 | 102 |
| 24 | -71812 | 107 | -87884 | 107 | -03896 | 106 | - 19846 | 106 | - 35734 | 106 |
| 25 | -72080 | 112 | -8815 1 | 112 | - 04162 | III | - 20111 | III | - 35998 | III |
| 26 | - 72348 | IIf | -88418 | 116 | -04428 | 115 | - 20376 | 115 | - 36262 | 115 |
| 27 | -72617 | 121 | -88686 | 121 | -04694 | 120 | - 20642 | 120 | - 36527 | 120 |
| 28 | - 72886 | 125 | - 88954 | 125 | -04960 | 124 | - 20908 | 124 | - 36792 | 124 |
| 29 | - 73154 | 130 | -89221 | 130 | -05227 | 129 | - 21173 | 128 | -37056 | 128 |
| 30 | -73422 | 134 | -89488 | 134 | -05494 | 133 | - 21438 | 132 | - 37320 | 132 |
| 31 | 7-73690 | 139 | 7-89755 | 139 | 8-05760 | 138 | 8. 21703 | 137 | 8. 37584 | 137 |
| 32 | - 73958 | 143 | -90022 | 143 | -06026 | 142 | - 21968 | 141 | $\cdot 37848$ | 141 |
| 33 | -74226 | 148 | -90289 | 148 | -06292 | 147 | - 22233 | 146 | -38112 | 146 |
| 34 | -74494 | 152 | -90556 | 152 | -06558 | 151 | - 22498 | 150 | -38376 | 150 |
| 35 | - 74763 | 157 | -90824 | 157 | -06824 | 156 | - 22763 | 155 | - 38640 | 155 |
| 36 | -75032 | 161 | -91092 | 161 | -07090 | 160 | - 23028 | 159 | -38904 | 159 |
| 37 | -75300 | r66 | -91359 | I66 | -07359 | 165 | - 23294 | 164 | -39168 | 164 |
| 38 | - 75568 | 170 | -91626 | 170 | -07622 | 169 | - 23560 | 168 | -39432 | 168 |
| 39 | - 75836 | 175 | -91893 | 174 | -07889 | 173 | - 23825 | 172 | - 39696 | 172 |
| 40 | -76104 | I79 | -92160 | 178 | -08156 | 177 | $\cdot 24090$ | 176 | - 39960 | 176 |
| 417 | 7.76372 | 184 | 7*92427 | 183 | - 08422 | 182 | 8-24355 | 181 | 8.40224 | 181 |
| 42 | - 76640 | 188 | -92694 | 187 | - 08688 | 186 | - 24620 | 185 | - 40488 | 185 |
| 43 | -76908 | 193 | -92961 | 192 | -08954 | 191 | - 24885 | 190 | -4075 2 | 190 |
| 44 | - 77176 | 197 | -93228 | 196 | -09220 | 195 | - 25150 | 194 | -41cr6 | 194 |
| 45 | -77444 | 202 | -93495 | 2 CI | -09486 | 200 | - 25415 | 199 | -41280 | 199 |
| 46 | -77712 | 206 | -93762 | 205 | - 09752 | 204 | - 25680 | 203 | - 41544 | 203 |
| 47 | -77980 | 211 | -94029 | 210 | - 10018 | 209 | - 25944 | 208 | -41808 | 208 |
| 48 | -78248 | 215 | -94296 | 214 | - 10284 | 213 | - 26208 | 212 | - 42072 | 212 |
| 49 | - 78516 | 219 | -94563 | 218 | - 10550 | 217 | - 26473 | 216 | -42336 | 216 |
| 50 | -78784 | 223 | -94830 | 222 | -10816 | 221 | - 26738 | 220 | -42600 | 220 |
| 517 | 7-79052 | 2287 | 7*95097 | 227 | 8*11081 | 226 | 8. 27003 | 2258 | $8 \cdot 42863$ | 224 |
| 52 | -79320 | 232 | . 95364 | 231 | - 11346 | 230 | - 27268 | 229 | . 43126 | 228 |
| 53 | -79588 | 237 | - 95631 | 236 | -11612 | 235 | - 27533 | 234 | . 43390 | 233 |
| 54 | - 79856 | 241 | -95898 | 240 | -11878 | 239 | - 27798 | 238 | - 43654 | 237 |
| 55 | -80124 | 246 | - 96165 | 245 | -12144 | 244 | - 28063 | 243 | -43918 | 242 |
| 56 | -80392 | 250 | -96432 | 249 | - 12410 | 248 | - 28328 | 247 | - 44182 | 246 |
| 57 | -80659 | 255 | -96698 | 254 | - 12676 | 253 | - 28593 | 252 | - 44446 | 251 |
| 58 | -80926 | 259 | -96964 | 258 | - 12942 | 257 | - 28858 | 257 | - 44710 | 256 |
| 59 | -81194 | 264 | -97231 | 263 | -13208 | 262 | - 29122 | 261 | - 44973 | 260 |
| 60 | -81462 | 268 | -97498 | 267 | - 13474 | 266 | - 29386 | 265 | - 45236 | 264 |


| Min. | $50^{\circ}$ | Parts <br> for | $51^{\circ}$ | Parts <br> for | $52^{\circ}$ | $\begin{gathered} \text { Parts } \\ \text { for } \\ / / \end{gathered}$ | $53^{\circ}$ | Parts for | $54^{\circ}$ | Parts for |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 8.45236 | $\bigcirc$ | $8 \cdot 61022$ | $\bigcirc$ | 8-76742 | $\bigcirc$ | 8.92396 | $\bigcirc$ | 9.07982 | $\bigcirc$ |
| 1 | . 45500 | 4 | -61285 | 4 | - 77004 | 4 | - 92656 | 4 | -08241 | 4 |
| 2 | - 45764 | 8 | -61548 | 8 | -77266 | 8 | -92916 | 8 | - 08500 | 8 |
| 3 | - 46028 | 13 | -61810 | 13 | - 77527 | 13 | -93176 | 12 | - 08759 | 12 |
| 4 | - 46292 | 17 | -62072 | 17 | - 77788 | 17 | - 93436 | 16 | -09018 | 16 |
| 5 | - 46555 | 22 | -62335 | 22 | - 78049 | 22 | -93697 | 2 I | -09277 | 2 I |
| 6 | - 46818 | 26 | . 62598 | 26 | - 78310 | 26 | - 93958 | 25 | - 09536 | 25 |
| 7 | - 47082 | 30 | -62860 | 30 | - 78572 | 30 | -94218 | 29 | - 09795 | 29 |
| 8 | - 47346 | 35 | -63122 | 35 | $\cdot 78834$ | 35 | -94478 | 34 | -10054 | 34 |
| 9 | -47609 | 39 | . 63384 | 39 | - 79095 | 39 | -94738 | 38 | -10313 | 38 |
| 10 | $\cdot 47872$ | 44 | . 63646 | 44 | - 79356 | 44 | -94998 | 43 | -10572 | 43 |
| II | 8.48135 | 49 | 8.63909 | 48 | 8.79617 | 48 | 8-95258 | 47 | $9 \cdot 10831$ | 47 |
| 12 | - 48398 | 53 | -64172 | 52 | -79878 | 52 | -95518 | 51 | - 11090 | 51 |
| 13 | - 48662 | 58 | -64434 | 57 | -80139 | 57 | -95778 | 56 | - II349 | 56 |
| 14 | - 48926 | 62 | -64696 | 61 | - 80400 | 61 | - 96038 | 60 | -11608 | 60 |
| 15 | - 49189 | 66 | -64958 | 65 | - 80662 | 65 | - 96298 | 64 | - 1867 | 64 |
| 16 | -49452 | 70 | -65220 | 69 | -80924 | 69 | - 96558 | 68 | -12126 | 68 |
| 17 | -49716 | 75 | . 65483 | 74 | -81185 | 74 | -96818 | 73 | -12385 | 73 |
| 18 | - 49980 | 79 | - 65746 | 78 | -81446 | 78 | -97078 | 77 | - 12644 | 77 |
| 19 | -50243 | 84 | - 66008 | 83 | -81707 | 83 | -97338 | 82 | -12902 | 82 |
| 20 | - 50506 | 88 | -66270 | 87 | -81968 | 87 | -97598 | 86 | -13160 | 86 |
| 21 | 8.50769 | 93 | 8.66532 | 92 | 8.82229 | 92 | 8.97858 | 91 | $9^{*} 13419$ | 9 I |
| 22 | -51032 | 97 | -66794 | 96 | -82490 | 96 | -98118 | 95 | -13678 | 95 |
| 23 | -51295 | 102 | -67056 | IOI | -82751 | 101 | -98378 | 100 | -13937 | 100 |
| 24 | - 51558 | 106 | -67318 | 105 | -83012 | 105 | -98638 | 104 | -14196 | 104 |
| 25 | - 51822 | III | -67580 | 110 | -83273 | 110 | -98898 | 109 | - 14455 | 109 |
| 26 | -52086 | 115 | -67842 | 114 | -83534 | 114 | -99158 | II4 | -14714 | 113 |
| 27 | - 52349 | 120 | -68104 | II9 | -83795 | 119 | -99418 | I18 | - 14972 | 118 |
| 28 | -52612 | 124 | -68366 | 123 | -84056 | 123 | -99678 | 122 | -15230 | 122 |
| 29 | - 52875 | 128 | -68628 | 127 | -84317 | 127 | - 99937 | 126 | -15489 | 126 |
| 30 | -53138 | 132 | -68890 | 131 | -84578 | 131 | 9*00196 | 130 | -15748 | 130 |
| 31 | 8.53401 | I36 | 8.69152 | 135 | 8.84839 | 135 | 9.00456 | 134 | $9 \cdot 16007$ | 134 |
| 32 | -53664 | 140 | -694r4 | 139 | -85100 | 139 | -00716 | 138 | - 16266 | 138 |
| 33 | -53927 | 145 | - $6967{ }^{\circ}$ | 144 | . 85360 | 144 | -00976 | 143 | -16524 | 143 |
| 34 | -54190 | 149 | - 69938 | 148 | -85620 | 148 | -01236 | 147 | -16782 | 147 |
| 35 | - 54453 | 154 | - 70200 | 153 | -85881 | 153 | - or496 | 152 | -17041 | 152 |
| 36 | -54716 | 158 | - 70462 | 157 | -86142 | 157 | - or 756 | 156 | - 17300 | 156 |
| 37 | - 54979 | 163 | - 70724 | 162 | - 86403 | 162 | - 02015 | 161 | - 17558 | 161 |
| 38 | - 55242 | 167 | - 70986 | 166 | - 86664 | 166 | -02274 | 165 | - 17816 | 165 |
| 39 | - 55505 | ${ }^{1} 71$ | -71248 | ${ }^{1} 70$ | -86925 | 170 | -02534 | 169 | - 18075 | 169 |
| 40 | - 55768 | ${ }^{1} 75$ | - 71510 | 174 | -87186 | 174 | -02794 | 173 | -18334 | 173 |
| 4 I | 8.56031 | 180 | 8.71772 | $\pm 79$ | 8-87446 | 178 | 9.03053 | 177 | $9^{*} 18592$ | 177 |
| 42 | - 56294 | 184 | - 72034 | 183 | -87706 | 182 | . 03312 | 181 | - 18850 | 181 |
| 43 | - 56557 | 189 | - 72295 | 188 | - 87967 | 187 | -03572 | 186 | -19108 | 186 |
| 44 | - 56820 | 193 | - 72556 | 192 | - 88228 | 191 | -03832 | 190 | - 19366 | 190 |
| 45 | $\cdot 57082$ | 198 | -728r8 | 197 | -88489 | 196 | -04091 | 195 | - 19625 | 195 |
| 46 | - 57344 | 202 | -73080 | 201 | -88750 | 200 | - 04350 | 199 | - 19884 | 199 |
| 47 | - 57607 | 207 | - 73342 | 206 | -89010 | 205 | -04610 | 204 | - 20142 | 204 |
| 48 | $\cdot 57870$ | 211 | - 73604 | 210 | -89270 | 209 | -04870 | 208 | - 20400 | 208 |
| 49 | -58133 | 215 | - 73865 | 214 | -89531 | 213 | -05129 | 212 | - 20658 | 212 |
| 50 | -58396 | 219 | -74126 | 218 | -89792 | 217 | .05388 | 216 | - 20916 | 216 |
| 51 | 8. 58659 | 223 | 8-74388 | 222 | 8.90052 | 22 I | 9.05648 | 220 | 9*2II74 | 220 |
| 52 | -58922 | 227 | - 74650 | 226 | -90312 | 225 | -059>8 | 224 | - 21432 | 224 |
| 53 | - 59184 | 232 | -74912 | 231 | -90573 | 230 | -06rti 7 | 229 | - 21690 | 229 |
| 54 | - 59446 | 236 | -75174 | 235 | -90834 | 234 | - 06426 | 233 | - 21948 | 233 |
| 55 | - 59709 | 241 | - 75435 | 240 | -91094 | 239 | -06685 | 238 | - 22207 | 238 |
| 56 | - 59972 | 245 | - 75696 | 244 | -91354 | 243 | -06944 | 242 | - 22466 | 242 |
| 57 | -60235 | 250 | -75958 | 249 | -916r5 | 248 | -07203 | 247 | - 22724 | 247 |
| 58 | -60498 | 255 | - 76220 | 254 | -91876 | 253 | -07462 | 252 | - 22982 | 251 |
| 59 | -60760 | 259 | -76481 | 258 | -92136 | 257 | -07722 | 256 | - 23240 | 255 |
| 60 | -61022 | 263 | - 76742 | 262 | *92396 | 261 | -07982 | 260 | - 23498 | 259 |


| Min. | $55^{\circ}$ | Part <br> for | $56^{\circ}$ | $\begin{aligned} & \text { Parts } \\ & \text { for } \\ & \hline \prime \end{aligned}$ | $57^{\circ}$ | $\begin{aligned} & \text { Parts } \\ & \text { for } \end{aligned}$ | $58^{\circ}$ | $\begin{aligned} & \text { r'arts } \\ & \text { for } \end{aligned}$ | 8 $59^{\circ}$ | Parts <br> for |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 9*23498 | $\bigcirc$ | $9 \cdot 38944$ | $\bigcirc$ | 9.54318 | $\bigcirc$ | 9.69620 | $\bigcirc$ | $9 \cdot 84848$ | $\bigcirc$ |
| 1 | - 23756 | 4 | - 39200 | 4 | - 54573 | 4 | -69874 | 4 | -85101 | 4 |
| 2 | - 24014 | 8 | - 39456 | 8 | - 54828 | 8 | -70128 | 8 | -85354 | 8 |
| 3 | - 24272 | 12 | -39713 | 12 | - 55084 | 12 | -70382 | 12 | -85607 | 12 |
| 4 | - 24530 | 16 | - 39970 | 16 | - 55340 | 16 | -70636 | 16 | -85860 | 16 |
| 5 | - 24788 | 2 I | -40227 | 2 I | - 55596 | 21 | - 70891 | 21 | -86113 | 21 |
| 6 | - 25046 | 25 | - 40484 | 25 | - 55852 | 25 | -71146 | 25 | -86366 | 25 |
| 7 | - 25303 | 29 | - 40741 | 29 | - 56107 | 29 | $\cdot 71400$ | 29 | -86619 | 29 |
| 8 | - 25560 | 34 | -40998 | 34 | - 56362 | 34 | -71654 | 34 | -86872 | 34 |
|  | - 25818 | 38 | -41254 | 38 | - 56617 | 38 | - 71908 | 38 | -87125 | 38 |
| 10 | - 26076 | 43 | -41517 | 43 | - 56872 | 42 | - 72162 | 42 | -87378 | 42 |
| II | 9.26334 | 47 | 9.41767 | 47 | 9.57128 | 47 | 9*72416 | 47 | 9.87631 | 46 |
| 12 | - 26592 | 51 | -42024 | 51 | $\cdot 57384$ | 51 | -72670 | 51 | - 87884 | 50 |
| 13 | - 26850 | 56 | - 42281 | 55 | - 57639 | 55 | - 72925 | 55 | -88137 | 54 |
| 14 | - 27108 | 60 | - 42538 | 59 | $\cdot 57894$ | 59 | - 73180 | 59 | -88390 | 58 |
| IS | - 27366 | 64 | - 42794 | 63 | - 58150 | 63 | - 73434 | 63 | -88643 | 62 |
| 16 | - 27624 | 68 | - 43050 | 67 | - 58406 | 67 | - 73688 | 67 | -88896 | 66 |
| 17 | - 2788 I | 73 | - 43307 | 72 | -5866r | 72 | - 73942 | 72 | -89149 | 71 |
| 18 | - 28138 | 77 | -43564 | 76 | -58916 | 76 | -74196 | 76 | -89402 | 75 |
| 19 | - 28396 | 82 | - 43820 | 81 | -59171 | 81 | - 74450 | 81 | -89654 | 80 |
| 20 | - 28654 | 86 | - 44076 | 85 | - 59426 | 85 | - 74704 | 85 | -89906 | 84 |
| 2 I | $9^{* 28912}$ | 90 | $9 * 44332$ | 89 | 9.59681 | 89 | 9*74958 | 89 | 9.90159 | 89 |
| 22 | - 29170 | 94 | - 44588 | 93 | - 59936 | 93 | -75212 | 93 | -90412 | 92 |
| 23 | - 29427 | 99 | - 44845 | 98 | -60192 | 97 | - 75466 | 97 | -90665 | 96 |
| 24 | - 29684 | 103 | - 45102 | 102 | -60448 | IOI | - 75720 | 101 | -90918 | 100 |
| 25 | - 29942 | 108 | - 45358 | 107 | -60703 | 106 | -75974 | 106 | -91170 | 105 |
| 26 | - 30200 | II2 | - 45614 | III | -60958 | IIO | -76228 | I 10 | -91422 | 109 |
| 27 | - 30457 | 117 | - 45870 | 116 | -61213 | 115 | - 76481 | 115 | -91675 | 114 |
| 28 | - 30714 | 121 | -46126 | 120 | -61468 | 119 | - 76734 | II9 | -91928 | 118 |
| 29 | - 30972 | 125 | - 46383 | 124 | -61723 | 123 | - 76988 | 123 | -92181 | 122 |
| 30 | - 31230 | 129 | - 46640 | 128 | -61978 | 127 | - 77242 | 127 | -92434 | 126 |
| 31 | $9 \cdot 31487$ | 133 | $9 \cdot 46896$ | 132 | 9.62233 | 131 | 9*77496 | 131 | 9.92686 | 130 |
| 32 | -31744 | 137 | - 4715 | 136 | -62488 | 135 | - 77750 | 135 | -92938 | 134 |
| 33 | -32001 | 142 | - 47408 | 141 | -62743 | 140 | - 78004 | 139 | -93191 | 138 |
| 34 | - 32258 | 146 | - 47664 | 145 | -62998 | 144 | - 78258 | 143 | -93444 | 142 |
| 35 | - 32516 | 151 | - 47920 | 150 | . 63253 | 149 | - 78512 | 148 | -93696 | 147 |
| 36 | - 32774 | 155 | - 48176 | 154 | -63508 | 153 | - 78766 | 152 | -93948 | 151 |
| 37 | - 33031 | 160 | - 48432 | 159 | . 63763 | 158 | -79019 | 157 | -94200 | 156 |
| 38 | - 33288 | 164 | - 48688 | 163 | -64018 | 162 | -79272 | 161 | -94452 | 160 |
| 39 | . 33545 | 168 | - 48944 | 167 | -64272 | 166 | -79526 | 165 | *94705 | 164 |
| 40 | - 33802 | 172 | - 49200 | 171 | -64526 | 170 | -79780 | 169 | -94958 | 168 |
| 419 | 9*34059 | 1769 | 9*49456 | 175 | 9.64781 | 174 | 9.80033 | 173 | 9*95210 | 172 |
| 42 | -34316 | 180 | -49712 | 179 | . 65036 | 178 | -80286 | 177 | - 95462 | 176 |
| 43 | - 34574 | 185 | - 49968 | 184 | -65291 | 183 | -80540 | 182 | -95714 | 180 |
| 44 | - 34832 | I 89 | - 50224 | 188 | -65546 | 187 | -80794 | 186 | -95966 | 184 |
| 45 | - 35089 | 194 | - 50480 | 193 | -65801 | 192 | -81047 | 191 | -96219 | 189 |
| 46 | - 35346 | $19^{9}$ | - 50736 | 197 | - 66056 | 196 | . 81300 | 595 | -96472 | 193 |
| 47 | - 35603 | 203 | - 50992 | 202 | -66310 | 201 | -81554 | 200 | -96724 | 198 |
| 48 | - 35860 | 207 | - 51248 | 206 | -66564 | 205 | -81808 | 204 | -96976 | 202 |
| 49 | -36117 | 211 | - 51504 | 210 | -668r9 | 209 | -8206r | 208 | -97228 | 206 |
| 50 | - 36374 | 215 | - 51760 | 214 | -67074 | 213 | . 82314 | 212 | -97480 | 210 |
| 519 | 9*36631 | 2199 | 9.52016 | 218 | 9.67329 | 2179 | 9.82568 | 216 | $9^{*} 97732$ | 214 |
| 52 | - 36888 | 223 | -52272 | 222 | -67584 | 221 | -82822 | 220 | -97984 | 218 |
| 53 | - 37145 | 227 | -52528 | 226 | - 67838 | 225 | -83075 | 224 | -98236 | 222 |
| 54 | - 37402 | 231 | - 52784 | 230 | -68092 | 229 | -83328 | 228 | -98488 | 226 |
| 55 | - 37659 | 236 | - 53039 | 235 | -68347 | 234 | -8358r | 233 | -98740 | 231 |
| 56 | - 37916 | 240 | - 53294 | 239 | - 68602 | 238 | -83834 | 237 | -98992 | 235 |
| 57 | -38173 | 245 | -53550 | 244 | -68856 | 243 | -84087 | 242 | -99244 | 240 |
| 58 | - 38430 | 249 | - 53806 | 248 | - 698110 | 247 | - 84340 | 246 | -99496 | 244 |
| 59 | - 38687 | 253 | - 54062 | 252 | - 69365 | 251 | -84594 | 250 | -99748 | 248 |
| 60 | - 38944 | 257 | - 54318 | 256 | -69620 | 255 | -84848 | 254 I | 10.00000 | 252 |

Table L.-Tables showing the length in feet of a degree, minute, and second of latitude and longitude, for every ten minutes of the quadrant. Based on the Ordnance Geodetical Tables, compression $\frac{1}{294}$. By Robert C. Carrington, F.R.G.S., F.A.S.L.

| Latitude. |  |  |  | LONGITUDE. |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Latitude. | Length in Feet of a |  |  | Latitude | Length in Feet of a |  |  |
|  | Degree. | Minute. | Second. |  | Degree. | Minute. | Second. |
| $0^{\circ} 0^{\prime}$ | $362755^{*} 6$ | $6045 \cdot 93$ | $100 \cdot 71$ | $0^{\circ} 0^{\prime}$ | 365233.7 | $6087 \cdot 23$ | 101*454 |
| 10 | $362755 \cdot 6$ | $6045 \cdot 93$ | 100'77 | 10 | $365232 \cdot 1$ | $6087 \cdot 20$ | 101.453 |
| 20 | 362755*7 | $6045 \cdot 93$ | $100 \cdot 77$ | 20 | 365227.5 | $6087 \cdot 13$ | 101. 452 |
| 30 | $362755^{\circ} 9$ | $6045 \cdot 93$ | 100.77 | 30 | 365219*9 | $6087 \cdot 00$ | 101.450 |
| 40 | $3^{62} 755^{6} \cdot 1$ | $6045 \cdot 93$ | $100 \cdot 77$ | 40 | $3^{65209}{ }^{\prime \prime}$ | 6086-82 | 101. 447 |
| 50 | $362756 \cdot 4$ | 6045.94 | $100 \cdot 77$ | 50 | 365195*3 | $6086 \cdot 59$ | IOI. 443 |
| $1^{\circ} 0^{\prime}$ | 362756.7 | 6045*94 | $100 \cdot 77$ | $1^{\circ} 0^{\prime}$ | 365178.4 | 6086.31 | 101.438 |
| 10 | $362757^{\circ} \mathrm{I}$ | 6045.95 | $100 \cdot 77$ | 10 | 365158.5 | 6085.98 | 101.433 |
| 20 | $362757 \cdot 6$ | 6045*96 | $100 \cdot 77$ | 20 | 365135.5 | 6085.59 | 101.427 |
| 30 | $362758^{1}$ | 6045.97 | $100 \cdot 77$ | 30 | 365109.4 | $6085 \cdot 16$ | 101.419 |
| 40 | $362758 \cdot 7$ | $6045 \cdot 98$ | $100 \cdot 77$ | 40 | 365080.2 | $6084 \cdot 67$ | 101.411 |
| 50 | 362759.4 | 6045.99 | 100.77 | 50 | $365048 \cdot 0$ | 6084.13 | 101. 402 |
| $2^{\circ} \mathrm{o}^{\prime}$ | $362760 \cdot 1$ | 6046.00 | 100*77 | $2^{\circ} 0^{\prime}$ | $365012 \cdot 7$ | $6083 \cdot 54$ | 101.392 |
| 10 | $362760 \cdot 9$ | 6046.01 | $100 \cdot 77$ | 10 | $364974 \cdot 3$ | $6082 \cdot 91$ | 101.381 |
| 20 | 362761.7 | $6046 \cdot 03$ | 100'77 | 20 | $364932 \cdot 9$ | $6082 \cdot 22$ | 101.370 |
| 30 | $362762 \cdot 6$ | $6046 \cdot 04$ | $100 \cdot 77$ | 30 | $364888 \cdot 4$ | $608 \mathrm{I} \cdot 47$ | 101.358 |
| 40 | $362763 \cdot 6$ | 6046.06 | 100'77 | 40 | $364840 \cdot 8$ | $6080 \cdot 68$ | 101*345 |
| 50 | $362764 \cdot 6$ | 6046.08 | 100.77 | 50 | $364790 \cdot 2$ | 6079•84 | 101.331 |
| $3^{\circ} 0^{\prime}$ | $362765^{\circ} 7$ | 6046.09 | $100 \cdot 77$ | $3^{\circ} 0^{\prime}$ | 364736.5 | $6078 \cdot 94$ | 101.316 |
| 10 | $362766 \cdot 9$ | $6046 \cdot 11$ | $100 \cdot 77$ | 10 | $364679 \cdot 8$ | $6078 \cdot 00$ | 101.300 |
| 20 | $362768 \cdot 1$ | $6046 \cdot 13$ | $100 \cdot 77$ | 20 | 364619.9 | $6077 \cdot 00$ | 101. 283 |
| 30 | $362769^{\circ} 4$ | $6046 \cdot 16$ | $100 \cdot 77$ | 30 | $364557 \cdot 0$ | $6075 \cdot 95$ | 101.266 |
| 40 | $362770 \cdot 7$ | 6046-18 | $100 \cdot 77$ | 40 | $36449 \mathrm{I}^{\text {I }}$ | $6074 \cdot 85$ | 101. 248 |
| 50 | $362772^{\cdot 1}$ | 6046.20 | $100 \cdot 77$ | 50 | $364422 \cdot 1$ | $6073 \cdot 70$ | 101. 228 |
| $4^{\circ} \quad \mathrm{o}^{\prime}$ | $362773 \cdot 6$ | $6046 \cdot 23$ | $100 \cdot 77$ | $4^{\circ} 0^{\prime}$ | $364350 \cdot 0$ | $6072 \cdot 50$ | 101.208 |
| 10 | 362775*1 | $6046 \cdot 25$ | $100 \cdot 77$ | 10 | 364274.9 | $6071 \cdot 25$ | IOI.187 |
| 20 | $362776 \cdot 7$ | 6046.28 | 100. 77 | 20 | $364196 \cdot 7$ | 6069.95 | 101'166 |
| 30 | $362778 \cdot 3$ | 6046.30 | $100 \cdot 77$ | 30 | $364115 \cdot 4$ | $6068 \cdot 59$ | 101.143 |
| 40 | $362780 \cdot 0$ | 6046.33 | 100.77 | 40 | $364031^{\prime \prime}$ | $6067 \cdot 19$ | 101.120 |
| 50 | $362781 \cdot 8$ | $6046 \cdot 36$ | $100 \cdot 77$ | 50 | $363943 \cdot 7$ | $6065 \cdot 73$ | 10r*096 |


| Latitude. | LATITUDE. |  |  | LONGITUDE. |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Length in Feet of a |  |  | Latitude. | Length in Feet of a |  |  |
|  | Degree. | Minute. | Second. |  | Degree. | Minute. | Second. |
| $5^{\circ}$ | 362783.6 | $6046 \cdot 39$ | 100'77 | $5^{\circ} 0^{\prime}$ | $363853 \cdot 2$ | $6064 \cdot 22$ | 101.070 |
|  | 362785.5 | $6046 \cdot 42$ | 100'77 | 10 | $363759^{\circ} 7$ | $6062 \cdot 66$ | IOI* 044 |
|  | 362787.5 | $6046 \cdot 46$ | $100 \cdot 77$ | 20 | $363663^{*} 2$ | $6061 \cdot 05$ | 101-018 |
|  | 362789.5 | $6046 \cdot 49$ | 100'77 | 30 | 363563.5 | $6059 * 39$ | ICO* 990 |
|  | $362791 \cdot 6$ | $6046 \cdot 53$ | $100 \cdot 78$ | 40 | $363460 \cdot 9$ | $6057 \cdot 68$ | $100 * 961$ |
|  | $362793 \cdot 7$ | $6046 \cdot 56$ | $100 \cdot 78$ | 50 | $363355^{\circ} \mathrm{I}$ | $6055^{\prime} 9^{2}$ | 100.932 |
| $\begin{array}{cc}6^{\circ} & \\ & 1 \\ & 2 \\ & 3 \\ & 4 \\ & \\ & 5\end{array}$ | $362795 * 9$ | $6046 \cdot 60$ | $100 \cdot 78$ | $6^{\circ} 0^{\prime}$ | $363246^{\prime} 3$ | $6054 \cdot 11$ | 100*902 |
|  | $362798 \cdot 2$ | $6046 \cdot 64$ | $100 \cdot 78$ | 10 | $363134^{\prime} 5$ | $6052 \cdot 24$ | $100 \cdot 871$ |
|  | $362800 \cdot 5$ | $6046 \cdot 68$ | $100 \cdot 78$ | 20 | 363019.6 | $6050 \cdot 33$ | $100 \cdot 839$ |
|  | $362802 \cdot 9$ | $6046 \cdot 72$ | $100 \cdot 78$ | 30 | $362901 \cdot 7$ | $6048 \cdot 36$ | $100 \cdot 806$ |
|  | $362805 \cdot 4$ | $6046 \cdot 76$ | $100 \cdot 78$ | 40 | $362780 \cdot 7$ | $6046 \cdot 35$ | $100 \cdot 772$ |
|  | 362807.9 | $6046 \cdot 80$ | $100 \cdot 78$ | 50 | $362656 \cdot 6$ | 6044.28 | $100 \cdot 738$ |
| $\begin{array}{rrr}70 & 0 \\ 10 \\ & 20 \\ 30 \\ 30 \\ 40 \\ 40\end{array}$ | 362810.4 | $6046 \cdot 84$ | $100 \cdot 78$ | $7^{\circ} 0^{\prime}$ | 362529.5 | $6042 \cdot 16$ | 100:703 |
|  | $362813^{\circ} 1$ | $6046 \cdot 89$ | $100 \cdot 78$ | 10 | $362399^{\circ} 4$ | 6039.99 | 100.667 |
|  | 362815.2 | $6046 \cdot 93$ | $100 \cdot 78$ | 20 | $362266 \cdot 2$ | $6037 \cdot 77$ | 100.630 |
|  | 362818.5 | $6046 \cdot 98$ | $100 \cdot 78$ | 30 | $362130 \cdot 0$ | $6035 \cdot 50$ | 100.592 |
|  | 362821.3 | $6047 \cdot 02$ | $100 \cdot 78$ | 40 | $361990 \cdot 7$ | $6033 \cdot 18$ | 100. 553 |
|  | $362824^{\circ} 2$ | $6047 \cdot 07$ | $100 \cdot 78$ | 50 | 361848.4 | $6030 \cdot 81$ | 100:513 |
| $\begin{array}{rrr}8 & 0 \\ & 10 \\ 20 \\ & 3 \\ & \\ 40 \\ & 50\end{array}$ | $362827^{\circ} \mathrm{I}$ | $6047 \cdot 12$ | $100 \cdot 79$ | $8^{\circ} 0^{\prime}$ | $361703^{\circ} 0$ | $6028 \cdot 38$ | $100 \cdot 473$ |
|  | $362830^{\circ} \mathrm{I}$ | $6047 \cdot 17$ | $100 \cdot 79$ | 10 | 361554*6 | $6025 \cdot 91$ | 100.432 |
|  | $362833 \cdot 2$ | $6047 \cdot 22$ | $100 \cdot 79$ | 20 | $361403 \cdot 2$ | $6023 \cdot 39$ | 100'390 |
|  | $362836 \cdot 3$ | $6047 \cdot 27$ | 100.79 | 30 | $361248 \cdot 7$ | 6020•81 | 100.347 |
|  | $362839^{\circ} 4$ | $6047 \cdot 32$ | 200•79 | 40 | $361091 \cdot 2$ | 6018.19 | 100:303 |
|  | $362842 \cdot 7$ | $6047 \cdot 3^{8}$ | 100:79 | 50 | $360930 \cdot 6$ | $6015 \cdot 51$ | $100 \cdot 258$ |
| $\begin{array}{rrr}\mathbf{9}^{\circ} & 0 \\ & 10 \\ & 20 \\ 30 \\ 30 \\ 40\end{array}$ | $362846^{\circ} 0$ | $6047 \cdot 43$ | 100.79 | $9^{\circ}{ }^{\prime}$ | $360767^{\circ}$ | $6012 \cdot 78$ | - 213 |
|  | -362849*3 | $6047 \cdot 49$ | 100.79 | 10 | - 4 | $6010 \cdot 01$ | $\cdot 167$ |
|  | $362852^{\circ} 7$ | $6047 \cdot 55$ | 100. 79 | 20 | $360430 \cdot 7$ | $6007 \cdot 18$ | $100 \cdot 120$ |
|  | $362856 \cdot 2$ | 6047-60 | 100.79 | 30 | $36025^{\circ} \cdot 0$ | 6004•30 | 100.072 |
|  | $362859 \cdot 7$ | $6047 \cdot 66$ | 100'79 | 40 | $360082 \cdot 3$ | $6001 \cdot 37$ | 100*023 |
|  | $362863 \cdot 3$ | $6047 \cdot{ }^{2}$ | 100.80 | 50 | 359903*5 | $5998 \cdot 39$ | 99.973 |
| $10^{\circ} \circ^{\prime}$ | $362866 \cdot 9$ | $6047 \cdot 78$ | $100 \cdot 80$ | $10^{\circ} \circ^{\prime}$ | $359721 \cdot 7$ | $5995 * 36$ | $99^{*} 923$ |
| 10 | $362870 \cdot 7$ | $6047 \cdot 85$ | $100 \cdot 80$ | 10 | $359536 \cdot 7$ | 5992-28 | $99 \cdot 871$ |
| 20 | $362874^{\circ} 4$ | $6047 \cdot 91$ | $100 \cdot 80$ | 20 | $359349^{*} \mathrm{I}$ | $5989 \cdot 15$ | 99.819 |
| 30 | $362878 \cdot 2$ | 6047*97 | 100.80 | 30 | $359158 \cdot 3$ | 5985*97 | 99766 |
| 40 | $362882 \cdot 1$ | $6048 \cdot 04$ | $100 \cdot 80$ | 40 | $35^{8} 964.4$ | 5982.74 | $99^{*} 712$ |
| 50 | $362886 \cdot 1$ | $6048 \cdot 10$ | 100.80 | 50 | $35^{8} 767.5$ | 5979.46 | 99.658 |


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| $11^{\circ} \circ^{\prime}$ | $362890 \cdot 1$ | 6048.17 | 100.80 | $11^{\circ} 0^{\prime}$ | $358567 \cdot 6$ | 5976•13 | 99.602 |
| 10 | $362894{ }^{\circ} \mathrm{I}$ | $6048 \cdot 23$ | $100 \cdot 80$ | 10 | 358364.7 | 5972.75 | $99 \cdot 546$ |
| 20 | $362898 \cdot 2$ | 6048.30 | $100 \cdot 80$ | 20 | $358158 \cdot 7$ | 5969.31 | 99.489 |
| 30 | $362902 \cdot 4$ | $6048 \cdot 37$ | 100.81 | 30 | $357949 \cdot 8$ | $5965 \cdot 83$ | 99.43 I |
| 40 | $362906 \cdot 6$ | $6048 \cdot 44$ | $100 \cdot 81$ | 40 | $357737 \cdot 8$ | $5962 \cdot 30$ | 99.372 |
| 50 | 362910.9 | 6048.52 | $100 \cdot 81$ | 50 | $357522 \cdot 8$ | $5958 \cdot 71$ | 99.312 |
| $12^{\circ} 0^{\prime}$ | $362915 \cdot 2$ | 6048.59 | 100.81 | $12^{\circ} 0^{\prime}$ | $357304 \cdot 8$ | 5955.08 | 99.25 I |
| 10 | 3629196 | $6048 \cdot 66$ | $100 \cdot 81$ | 10 | $357083 \cdot 9$ | 5951.40 | $99^{*} 190$ |
| 20 | 362924.1 | $6048 \cdot 74$ | $100 \cdot 81$ | 20 | 356859.9 | $5947 \cdot 67$ | $99 \cdot 128$ |
| 30 | $362928 \cdot 6$ | 6048.81 | 100.81 | 30 | $356632 \cdot 9$ | $5943 \cdot 88$ | $99 \cdot 065$ |
| 40 | $362933 \cdot 2$ | $6048 \cdot 89$ | $100 \cdot 8 \mathrm{I}$ | 40 | 356402.9 | $5940 \cdot 05$ | 99.001 |
| 50 | $362937 \cdot 8$ | $6048 \cdot 96$ | $100 \cdot 82$ | 50 | $356169^{\circ} 9$ | $5936 \cdot 17$ | $98 \cdot 936$ |
| $13^{\circ} \circ^{\prime}$ | $362942 \cdot 5$ | 6049.04 | $100 \cdot 82$ | $13^{\circ} 0^{\prime}$ | 355933*9 | $5932 \cdot 23$ | 98.871 |
| 10 | $362947 \cdot 2$ | 6049.12 | $100 \cdot 82$ | 10 | 355694*9 | $5928 \cdot 25$ | $98 \cdot 804$ |
| 20 | 362952.0 | 6049.20 | $100 \cdot 82$ | 20 | $355452 \cdot 9$ | 5924.22 | $98 \cdot 737$ |
| 30 | $362956^{\circ} 9$ | $6049 \cdot 28$ | $100 \cdot 82$ | 30 | 355207*9 | $5920 \cdot 13$ | $98 \cdot 669$ |
| 40 | $362961 \cdot 8$ | $6049 \cdot 36$ | $100 \cdot 82$ | 40 | $354959{ }^{\circ} 9$ | 5916.00 | $98 \cdot 600$ |
| 50 | $362 y 66 \cdot 8$ | 6049.45 | $100 \cdot 82$ | 50 | $354700^{\circ}$ | $59 \mathrm{IT} \cdot 82$ | 98.530 |
| $14^{\circ} 0^{\prime}$ | $362971 \cdot 8$ | 6049.53 | $100 \cdot 83$ | $14^{\circ}{ }^{\prime}$ | $354455^{\circ} \mathrm{I}$ | 5907.59 | 98.460 |
| 10 | $362976 \cdot 9$ | $6049 \cdot 62$ | $100 \cdot 83$ | 10 | 354198.1 | 5903.30 | $98 \cdot 388$ |
| 20 | $362982 \cdot 0$ | 6049 70 | $100 \cdot 83$ | 20 | $353938 \cdot 2$ | 5898.97 | $98 \cdot 316$ |
| 30 | $362987 \cdot 2$ | 6049'79 | $100 \cdot 83$ | 30 | 353675:3 | 5894.59 | 98.243 |
| 40 | 362992.4 | $6049 \cdot 87$ | 100.8; | 40 | 353409.4 | $5890 \cdot 16$ | $98 \cdot 169$ |
| 50 | 362997.7 | $6049 \cdot 96$ | $100 \cdot 83$ | 50 | $353140 \cdot 6$ | 5885.68 | $98 \cdot 095$ |
| $15^{\circ}{ }^{\prime}$ | $363003 \cdot 1$ | $6050 \cdot 05$ | $100 \cdot 83$ | $15^{\circ}{ }^{\prime}$ | $352868 \cdot 8$ | $588 \mathrm{I} \cdot 15$ | 98-019 |
| Io | $363008 \cdot 5$ | $6050 \cdot 14$ | $100 \cdot 84$ | 10 | $352594 \cdot 1$ | $5876 \cdot 57$ | $97 \cdot 943$ |
| 20 | 363013.9 | $6050 \cdot 23$ | $100 \cdot 84$ | 20 | $352316 \cdot 3$ | 5871.94 | 97-866 |
| 30 | 363019.4 | 6050.32 | 100.84 | 30 | 352035*6 | $5867 \cdot 26$ | $97 \cdot 788$ |
| 40 | $363025^{\circ}$ | 6050.42 | $100 \cdot 84$ | 40 | $351751 \times 9$ | $5862 \cdot 53$ | 97.709 |
| 50 | $363030 \cdot 6$ | 6050.51 | $100 \cdot 84$ | 50 | $351465 \cdot 3$ | $5857 \cdot 76$ | 97*629 |
| $16^{\circ} \circ^{\prime}$ | $363036 \cdot 3$ | 6050.6r | $100 \cdot 84$ | $16^{\circ} \circ^{\prime}$ | $351175^{\circ} 7$ | $5852 \cdot 93$ | 97.549 |
| 10 | $363042 \cdot 0$ | 6050.70 | $100 \cdot 84$ | 10 | $350883^{\circ} \mathrm{I}$ | $5848 \cdot 05$ | $97 \cdot 468$ |
| 20 | $363047 \cdot 8$ | 6050.80 | $100 \cdot 85$ | 20 | $350587 \cdot 6$ | $5843 \cdot 13$ | 97.386 |
| 30 | 363053.6 | 6050•89 | $100 \cdot 85$ | 30 | $350289 \cdot 1$ | $5838 \cdot 15$ | 97.303 |
| 40 | 363059*3 | 6050.99 | 100.85 | 40 | $349987 \cdot 7$ | $5833 \cdot 13$ | 97*219 |
| 50 | $363065 \cdot 4$ | $6051 \cdot 09$ | $100 \cdot 8 ;$ | 50 | $349683 \cdot 4$ | $5828 \cdot 06$ | 97*134 |


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|  | Degree. | Minute. | Second. |  | Degree. | Minute. | Second. |
| $17^{\circ} 0^{\prime}$ | 363071.4 | $6051 \cdot 19$ | $100 \cdot 85$ | $17^{\circ}{ }^{\prime}$ | $349376 \cdot 0$ | 5822.93 | $97 \cdot 049$ |
| 10 | $363077 \cdot 4$ | $6051 \cdot 29$ | $100 \cdot 85$ | 10 | $349065 \cdot 8$ | 5817.76 | $96 \cdot 963$ |
| 20 | 363083.5 | $6051 \cdot 39$ | $100 \cdot 86$ | 20 | $348752 \cdot 6$ | 5812.54 | $96 \cdot 876$ |
| 30 | $363089^{\circ} 7$ | 6051.50 | 100.86 | $30^{\circ}$ | 348436.5 | 5807.28 | $96 \cdot 788$ |
| 40 | 363095*9 | $6051 \cdot 60$ | $100 \cdot 86$ | 40 | 348117.4 | 5801.96 | $96 \cdot 699$ |
| 50 | $363102 \cdot 1$ | $605 \mathrm{I} \cdot 70$ | 100.86 | 50 | $347795 \cdot 4$ | 5796.59 | $96 \cdot 610$ |
| $18^{\circ}{ }^{\prime}$ | 363108.4 | $605 \mathrm{I} \cdot 8 \mathrm{I}$ | $100 \cdot 86$ | $18^{\circ} \circ^{\prime}$ | $347470 \cdot 5$ | $5791 \cdot 18$ | 96.520 |
| 10 | 363114.8 | 6051.91 | $100 \cdot 87$ | 10 | $347142 \cdot 6$ | $5785 \cdot 71$ | $96 \cdot 429$ |
| 20 | 363121.2 | 6052.02 | $100 \cdot 87$ | 20 | 3468 II $\cdot 8$ | $5780 \cdot 20$ | $96 \cdot 337$ |
| 30 | $363127 \cdot 6$ | $6052 \cdot 13$ | $100 \cdot 87$ | 30 | $346478 \cdot 1$ | 5774.64 | $96 \cdot 244$ |
| 40 | $363134^{\circ} \mathrm{I}$ | $6052 \cdot 24$ | $100 \cdot 87$ | 40 | 346141-5 | 5769.03 | $96 \cdot 150$ |
| 50 | $363140 \cdot 7$ | $6052 \cdot 35$ | 100.87 | 50 | 345801.9 | 5763.37 | $96 \cdot 056$ |
| $19^{\circ} 0^{\prime}$ | $363147 \cdot 3$ | 6052.46 | $100 \cdot 87$ | $19^{\circ} \circ^{\prime}$ | 345459.5 | 5757.66 | $95 \cdot 961$ |
| 10 | 363153.9 | 6052.57 | 100.88 | 10 | $345114^{\circ} \mathrm{I}$ | 5751*90 | $95 \cdot 865$ |
| 20 | $363160 \cdot 6$ | $6052 \cdot 68$ | $100 \cdot 88$ | 20 | $344765 \cdot 8$ | $5746 \cdot 10$ | $95 \cdot 768$ |
| 30 | $363167^{\circ} 4$ | $6052 \cdot 79$ | 100.88 | 30 | 344414.6 | $5740 \cdot 24$ | $95 \cdot 671$ |
| 40 | $363174{ }^{\circ} 2$ | $6052 \cdot 90$ | $100 \cdot 88$ | 40 | $344060 \cdot 6$ | 5734.34 | $95^{\circ} 572$ |
| 50 | 363181.0 | 6053.02 | 100.88 | 50 | $343703 \cdot 6$ | $5728 \cdot 39$ | 95.473 |
| $20^{\circ}{ }^{\prime}$ | $363187^{\circ} 9$ | $6053 \cdot 13$ | 100.89 | $20^{\circ} 0^{\prime}$ | $343343 \cdot 7$ | $5722 \cdot 40$ | $95 \cdot 373$ |
| 10 | $363194 \cdot 8$ | $6053 \cdot 25$ | $100 \cdot 89$ | 10 | $342980^{\circ} 9$ | $5716 \cdot 35$ | $95 \cdot 272$ |
| 20 | $363201 \cdot 8$ | $6053 \cdot 36$ | 100.89 | 20 | $342615 \cdot 2$ | $5710 \cdot 25$ | $95 \cdot 171$ |
| 30 | $363208 \cdot 8$ | 6053.48 | $100 \cdot 89$ | 30 | $342246^{\circ} 7$ | 5704.11 | 95.069 |
| 40 | $363215^{\circ} 9$ | $6053 \cdot 60$ | $100 \cdot 89$ | 40 | $341875^{\circ} 2$ | 5697*92 | 94.965 |
| 50 | $363223^{\circ} \mathrm{I}$ | $6053 \cdot 72$ | $100 \cdot 90$ | 50 | $341500 \cdot 9$ | 5691.68 | 94.861 |
| $21^{\circ} \circ^{\prime}$ | $363230 \cdot 2$ | $6053 \cdot 84$ | 100.90 | $21^{\circ}{ }^{\prime}$ | $341123^{\circ} 7$ | $5685 \cdot 40$ | 94*756 |
| 10 | 363237.5 | $6053 \cdot 96$ | 100.90 | 10 | $340743 \cdot 6$ | 5679.06 | 94.651 |
| 20 | $363244^{\circ} 7$ | 6054.08 | $100 \cdot 90$ | 20 | $340360 \cdot 6$ | $5672 \cdot 68$ | 94.545 |
| 30 | $363252^{\prime}$ I | 6054.20 | $100 \cdot 90$ | 30 | 339974*8 | 5666.25 | 94.438 |
| 40 | $363259^{\circ} 4$ | 6054.32 | $100 \cdot 91$ | 40 | $339586 \cdot 1$ | 5659.77 | 94.330 |
| 50 | 363266.8 | 6054.45 | $100 \cdot 91$ | 50 | 339194*5 | 5653.24 | 94.221 |
| $22^{\circ} \circ^{\prime}$ | $363274{ }^{\prime} 3$ | $6054 \cdot 57$ | 100.91 | $22^{\circ} \circ^{\prime}$ | $338800 \cdot 1$ | $5646 \cdot 67$ | $94^{\text {\% III }}$ |
| 10 | $36328 \mathrm{r} \cdot 8$ | 6054.70 | 100.91 | 10 | $338402 \cdot 8$ | $5640 \cdot 05$ | 94.001 |
| 20 | $363289 \cdot 3$ | 6054.82 | $100 \cdot 91$ | 20 | $338002 \cdot 7$ | $5633 \cdot 38$ | 93.890 |
| 30 | 363296.9 | 6054.95 | $100 \cdot 92$ | 30 | $3375999^{\circ} 7$ | $5626 \cdot 66$ | 93.778 |
| 40 | 363304.6 | 6055.08 | 100.92 | 40 | $337193{ }^{\circ} 9$ | 5619.90 | $93 \cdot 665$ |
| 50 | $363312 \cdot 2$ | $6055^{* 20}$ | 100.92 | 50 | $336785 \cdot 2$ | 5613.09 | 93.551 |


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|  | Degree. | Minute. | Second. |  | Degree. | Minute. | Second. |
| $23^{\circ}{ }^{\prime}$ | $363320 \cdot 0$ | $6055 \cdot 33$ | 100.92 | $23^{\circ}{ }^{\prime}$ | $336373 \cdot 6$ | 5606.23 | 93.437 |
|  | $363327^{\circ} 7$ | 6055*46 | $100 \cdot 92$ | Io | 335959.3 | 5599.32 | $93 \cdot 322$ |
| 20 | $363335 \cdot 5$ | $6055 \cdot 59$ | $100 \cdot 93$ | 20 | $335542 \cdot 1$ | $5592 \cdot 37$ | 93. 206 |
| 30 | 363343.4 | 6055.72 | $100 \cdot 93$ | 30 | 335122.0 | $5585 \cdot 37$ | 93.089 |
| 40 | $363351 \cdot 3$ | 605586 | $100 \cdot 93$ | 40 | $334699 \cdot 2$ | $5578 \cdot 32$ | $92 \cdot 972$ |
| 50 | $363359^{\circ} 2$ | 6055.99 | 100.93 | 50 | 334273.5 | $5571 \cdot 23$ | $92 \cdot 854$ |
| $24^{\circ}{ }^{\prime}$ | $363367^{\circ} 2$ | $6056 \cdot 12$ | 100.94 | $24^{\circ} \circ^{\prime}$ | $333845^{\circ} \mathrm{O}$ | 5564.08 | $92 \cdot 735$ |
| 10 | $3^{6} 3375{ }^{\circ} 2$ | $6056 \cdot 25$ | 100.94 | 10 | $333413^{\circ} 7$ | 5556.89 | $92 \cdot 615$ |
| 20 | $363383 \cdot 3$ | 6056.39 | 100.94 | 20 | 332979.5 | $5549 \cdot 66$ | 92.494 |
| 30 | 363391.4 | 6056.52 | 100.94 | 30 | $332542 \cdot 6$ | $5542 \cdot 38$ | $92 \cdot 373$ |
| 40 | 363399.6 | 6056.66 | 100.94 | 40 | $332102 \cdot 8$ | $5535 \cdot 05$ | 92.251 |
| 50 | $363407 \cdot 8$ | 6056.80 | 100.95 | 50 | $331660 \cdot 3$ | $5527 \cdot 67$ | ç2'128 |
| $25^{\circ}{ }^{\prime}$ | $363416 \cdot 0$ | 6056.93 | 100.95 | $25^{\circ}{ }^{\prime}$ | 331214*9 | 5;20.25 | 92-004 |
| 10 | $363424 \cdot 3$ | 6057.07 | $100 \cdot 95$ | 10 | $330766 \cdot 7$ | $5512 \cdot 78$ | 91.879 |
| 20 | $363432 \cdot 6$ | $6057 \cdot 21$ | 100.95 | 20 | $330315 \cdot 8$ | 5505.26 | 91.754 |
| 30 | $363440^{\circ} 9$ | 6057.35 | 100.96 | 30 | 329862.0 | 5497* 70 | $91 \cdot 628$ |
| 40 | $363449^{\circ} 3$ | $6057 \cdot 49$ | $100 \cdot 96$ | 40 | 329405*5 | 5490.09 | $91 \cdot 502$ |
| 50 | $363457 \cdot 7$ | $6057 \cdot 63$ | 100*96 | 50 | $328946 \cdot 2$ | 5482.44 | $91 \cdot 374$ |
| $26^{\circ} 0^{\prime}$ | $363466 \cdot 2$ | $6057 \cdot 77$ | 100*96 | $26^{\circ}{ }^{\prime}$ | 328484* I | 5474.74 | 9r.245 |
| 10 | 363474*7 | $6057 \cdot 91$ | 100.97 | 10 | 328019.2 | 5466.99 | 9r*i16 |
| 20 | $363483^{\prime} 3$ | 6058.06 | $100 \cdot 97$ | 20 | $327551 \times 6$ | 5459.19 | 90.987 |
| 30 | $363491 \cdot 9$ | 6058.20 | 100.97 | 30 | $327081 \cdot 2$ | 5451.35 | 90.856 |
| 40 | $363500 \cdot 5$ | 6058.34 | 100.97 | 40 | 326608* | $5443 \cdot 47$ | 90.724 |
| 50 | $363509 \cdot 2$ | $6058 \cdot 49$ | 100.97 | 50 | $326132^{\circ} \mathrm{I}$ | $5435 \cdot 54$ | 90.592 |
| $27^{\circ} 0^{\prime}$ | $363517 \cdot 9$ | $6058 \cdot 63$ | $100 \cdot 98$ | $27^{\circ}{ }^{\prime}$ | 325653.4 | 5427.56 | 90.459 |
| 10 | $363526 \cdot 6$ | 6058.78 | $100 \cdot 98$ | 10 | $325171 \cdot 9$ | 5419.53 | 90.326 |
| 20 | 363535.4 | 6058.92 | $100 \cdot 98$ | 20 | $324687 \cdot 7$ | $5411 \cdot 46$ | 90.191 |
| 30 | 363544.2 | 6059.07 | 100.98 | 30 | $324200 \cdot 8$ | $5403 \cdot 35$ | 90.056 |
| 40 | 363553.0 | 6059.22 | 100*99 | 40 | $323711^{\prime} 2$ | 5395 19 | 89.920 |
| 50 | 363561'9 | 6059.37 | 100*99 | 50 | $323218 \cdot 8$ | $5386 \cdot 98$ | 89.783 |
| $28^{\circ}{ }^{\prime}$ | $363570 \cdot 8$ | 6059.51 | 100•99 | $28^{\circ}{ }^{\prime}$ | 322723.6 | $5378 \cdot 73$ | 89.645 |
| 10 | $363579 \cdot 8$ | 6059.66 | 100*99 | 10 | 322225.7 | $5370 \cdot 43$ | 89.507 |
| 20 | $363588 \cdot 8$ | 605:31 | 101.00 | 20 | $321725 \cdot 1$ | 5362.09 | 89.368 |
| 30 | $363597 \cdot 8$ | $6059 \cdot 96$ | 101.00 | 30 | $32122 \mathrm{I}^{\circ} 8$ | $5353 \cdot 70$ | 89.228 |
| 40 | $363606 \cdot 8$ | $6060 \cdot 11$ | 101.00 | 40 | $320715 \cdot 8$ | 5345.26 | 89.088 |
| 50 | 363615.9 | $6060 \cdot 27$ | 101*00 | 50 | $320207^{1} 1$ | $5336 \cdot 78$ | 88.946 |


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|  | Degree. | Minute. | Second. |  | Degree. | Minute. | Second. |
| $29^{\circ}{ }^{\circ}$ | $363625^{\circ}$ | 6060*42 | ror-or | $29^{\circ}{ }^{\prime}$ | 319695.6 | $5328 \cdot 26$ | $88 \cdot 804$ |
| Io | $363634^{\circ}$ | 6960.57 | ror.or | 10 | $31918 \mathrm{r}^{-5}$ | $5319 \cdot 69$ | 88.66 r |
| 20 | 363643.4 | $6060 \cdot 72$ | ror $\cdot \mathrm{or}$ | 20 | 318664.6 | $53 \mathrm{II} \cdot 08$ | 88.518 |
| 30 | 363652.6 | 6060.88 | ror $\cdot \mathrm{or}$ | 30 | $318145^{\circ} \mathrm{I}$ | $5302 \cdot 42$ | $88 \cdot 374$ |
| 40 | $36366 \mathrm{r} \cdot 9$ | $606 \mathrm{r} \cdot \mathrm{O}_{3}$ | ror'02 | 40 | 317622.8 | $5293 \cdot 7 \mathrm{r}$ | 88.229 |
| 50 | $36367 \mathrm{~F} \cdot 2$ | $606 \mathrm{r} \cdot 19$ | 101.02 | 50 | $317097 \cdot 9$ | 5284.97 | 88.083 |
| $30^{\circ} \circ^{\prime}$ | $363680 \cdot 5$ | $6061 \cdot 34$ | 101.02 | $30^{\circ}{ }^{\prime}$ | $316570 \cdot 3$ | $5276 \cdot 17$ | $87 \cdot 936$ |
| 10 | 363689.9 | $606 \mathrm{r} \cdot 50$ | ror $\cdot 03$ | 10 | $316040^{\circ} 0$ | $5267 \cdot 33$ | 87.789 |
| 20 | 363699.3 | $606 \mathrm{I} \cdot 66$ | $101 \cdot 03$ | 20 | $315507^{\circ}$ | 5258.45 | 87.64r |
| 30 | $363708 \cdot 7$ | $606 \mathrm{r} \cdot 8 \mathrm{I}$ | ror-03 | 30 | $31497{ }^{1} \cdot 4$ | 5249.52 | $87 \cdot 492$ |
| 40 | $363778 \cdot \mathrm{I}$ | $606 \mathrm{r} \cdot 97$ | 101.03 | 40 | $314433 \cdot 1$ | $5240 \cdot 55$ | $87 \cdot 343$ |
| 50 | $363727 \cdot 6$ | $6062 \cdot 13$ | 101.04 | 50 | $313892 \cdot 1$ | $5231 \cdot 54$ | $87 \cdot 192$ |
| $31^{\circ} \circ^{\prime}$ | $363737 \cdot \mathrm{r}$ | 6062.29 | ror.04 | $31^{\circ} \circ^{\prime}$ | $313348 \cdot 5$ | $5222 \cdot 48$ | $87 \cdot 04 \mathrm{I}$ |
| Io | $363746 \cdot 7$ | $6062 \cdot 45$ | ror.04 | 10 | $312802 \cdot 2$ | 5213.57 | 86.889 |
| 20 | $363756 \cdot 2$ | 6062 . 60 | 101-04 | 20 | $312253 \cdot 3$ | 5204.22 | $86 \cdot 737$ |
| 30 | $363765 \cdot 8$ | $6062 \cdot 76$ | ror.05 | 30 | $311700^{\circ} 7$ | $5195 \cdot 03$ | $86 \cdot 584$ |
| 40 | 363775.4 | $6062 \cdot 92$ | 101-05 | 40 | $311147 \%$ | 5185.79 | 86.430 |
| 50 | $363785 \cdot \mathrm{r}$ | 6063.09 | ror*05 | 50 | $310590 \cdot 7$ | 5176.51 | $86 \cdot 275$ |
| $32^{\circ} \circ^{\prime}$ | $363794 \cdot 8$ | $6063 \cdot 25$ | 100.05 | $32^{\circ}{ }^{\prime}$ | $31003{ }^{1} \cdot 2$ | $5^{167}$-19 | $86 \cdot 119$ |
| Io | 363804.5 | 6063.4 r | 100-06 | 10 | 309469 - 1 | 5157.82 | $85 \cdot 963$ |
| 20 | 363814.2 | $6063 \cdot 57$ | rorec 6 | 20 | $308904 \cdot 4$ | $5148 \cdot 41$ | 85.807 |
| 30 | $363824 \cdot 0$ | $6063 \cdot 73$ | 100-06 | 30 | $308337 \cdot 1$ | 5138.95 | 85.649 |
| 40 | $363833 \cdot 8$ | 6063.90 | 101.07 | 40 | 307767.2 | 5129.45 | 85.49 r |
| 50 | $363843 \cdot 6$ | 6064.06 | ror-07 | 50 | 307194*6 | 5119.91 | 85.332 |
| $33^{\circ} 0^{\prime}$ | 363853.5 | 6064.23 | 101.07 | $33^{\circ} \circ^{\prime}$ | 306619•5 | $5110 \cdot 33$ | $85 \cdot 172$ |
| 10 | 363863.4 | 6064.39 | ror-07 | Io | $306041^{\prime} 7$ | $5100 \cdot 70$ | 85 -011 |
| 20 | $363873 \cdot 3$ | 6064.56 | roi•08 | 20 | $305461 \cdot 4$ | 5091.02 | 84.850 |
| 30 | $363883 \cdot 2$ | 6064.72 | 101-08 | 30 | $304878 \cdot 5$ | 5081.31 | 84.688 |
| 40 | $363893 \cdot 1$ | 6064.89 | 101.08 | 40 | $304293 \cdot 0$ | 5071.55 | 84.526 |
| 50 | $363903 \cdot 1$ | $6065 \cdot 05$ | 101-08 | 50 | 303704.9 | 5061.75 | 84.362 |
| $34^{\circ} 0^{\prime}$ | $363913 \cdot 1$ | 6065.22 | 101*09 | $34^{\circ}{ }^{\circ}$ | 303114.2 | $5051 \cdot 90$ | $84 \cdot 198$ |
| 10 | $363923 \cdot 1$ | $6065 \cdot 39$ | ror*09 | 10 | 302521.0 | $5042 \cdot 02$ | 84.034 |
| 20 | $363933 \cdot 2$ | $6065 \cdot 55$ | ror 09 | 20 | 301925.2 | 5032.09 | $83 \cdot 868$ |
| 30 | 363943.2 | $6065 \cdot 72$ | 101-10 | 30 | 301326.8 | $5022 \cdot 11$ | $83 \cdot 702$ |
| 40 | $363953 \cdot 3$ | $6065 \cdot 89$ | ror'to | 40 | $300725^{\circ} 9$ | 5012.10 | 83.535 |
| 50 | 363963.4 | 6066.06 | 101'10 | 50 | 300122.4 | $5002 \cdot 04$ | $83 \cdot 367$ |


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| $35^{\circ}{ }^{\prime}$ | $363973 \cdot 6$ | $6066 \cdot 23$ | 101.10 | $35^{\circ}{ }^{\prime}$ | 299516.4 | 4991*94 | $83 \cdot 199$ |
| 10 | $363983 \cdot 7$ | $6066 \cdot 40$ | ror'il | 10 | 298907.8 | $498 \mathrm{I} \cdot 80$ | 83.030 |
| 20 | 363993.9 | $6066 \cdot 57$ | ror'il | 20 | $298296 \cdot 8$ | $497 \mathrm{I} \cdot 61$ | 82.860 |
| 30 | $364004^{\circ} \mathrm{I}$ | $6066 \cdot 74$ | ror'il | 30 | 297683. 1 | $496 \mathrm{I} \cdot 38$ | 82.690 |
| 40 | $364014^{\circ} 3$ | $6066 \cdot 91$ | 101. 12 | 40 | $297067^{\circ} \mathrm{O}$ | 4951*12 | 82.519 |
| 50 | 364024.6 | $6067 \cdot 08$ | 101.12 | 50 | 296448.4 | 4940*81 | $82 \cdot 347$ |
| $36^{\circ} 0^{\prime}$ | $364034^{\circ} 9$ | $6067 \cdot 25$ | 101.12 | $36^{\circ}{ }^{\prime}$ | 295827.2 | $4930 \cdot 45$ | $82 \cdot 174$ |
| 10 | $364045^{\text {. }}$ I | $6067 \cdot 42$ | 101.12 | 10 | 295203.5 | $4920 \cdot 06$ | $82 \cdot 001$ |
| 20 | $364055^{\circ} 4$ | $6067 \cdot 59$ | 101'13 | 20 | $294577 \cdot 3$ | $4909 \cdot 62$ | $8 \mathrm{I} \cdot 827$ |
| 30 | $364065 \cdot 8$ | $6067 \cdot 76$ | 101.13 | 30 | $293948 \cdot 7$ | $4899 \cdot 15$ | 81.652 |
| 40 | $364076 \cdot 1$ | $6067 \cdot 94$ | 101.13 | 40 | $293317{ }^{\circ} 5$ | $4888 \cdot 63$ | 81.477 |
| 50 | $364086 \cdot 4$ | $6068 \cdot \mathrm{II}$ | 101'14 | 50 | $292683 \cdot 8$ | $4878 \cdot 06$ | 81.301 |
| $37^{\circ}{ }^{\prime}$ | $364096 \cdot 8$ | $6068 \cdot 28$ | 101.14 | $37^{\circ}{ }^{\prime}$ | $292047^{\circ} 7$ | $4867 \cdot 46$ | 8I. 124 |
| 10 | $364107 \cdot 2$ | $6068 \cdot 45$ | Ior.14 | 10 | 291409*0 | $4856 \cdot 82$ | $80 \cdot 947$ |
| 20 | $364117^{\prime} 6$ | $6068 \cdot 63$ | 101'14 | 20 | 29076 $7^{\circ} 9$ | $4846 \cdot 13$ | $80 \cdot 769$ |
| 30 | $364128 \cdot 1$ | 6068.80 | 101.15 | 30 | 290124.4 | $4835 \cdot 41$ | $80 \cdot 590$ |
| 40 | $364138 \cdot 5$ | 6068-98 | 101.15 | 40 | 289418.3 | $4824 \cdot 64$ | $80 \cdot 4 \mathrm{II}$ |
| 50 | $364149^{\circ} 0$ | $6069 \cdot 15$ | IOI'15 | 50 | 288829.8 | $4813 \cdot 83$ | $80 \cdot 23 \mathrm{I}$ |
| $38^{\circ} 0^{\prime}$ | $364159{ }^{\circ} 5$ | $6069 \cdot 33$ | 101'16 | $38^{\circ}{ }^{\prime}$ | 288178.9 | $4802 \cdot 98$ | $80 \cdot 050$ |
| 10 | $364170 \cdot 0$ | 6069.50 | 101.16 | 10 | 287525.5 | $4792 \cdot 09$ | 79.868 |
| 20 | $364180 \cdot 5$ | $6069 \cdot 68$ | 101•16 | 20 | 286869 7 | $478 \mathrm{I} \cdot 16$ | 79.686 |
| 30 | $36419{ }^{1} \cdot 0$ | $6069 \cdot 85$ | 101:16 | 30 | 286211.4 | $4770 \cdot 19$ | 79.503 |
| 40 | 364201.5 | 6070.03 | 101.17 | 40 | 285550.7 | $4759^{\circ} 18$ | 79.320 |
| 50 | $364212 \cdot 1$ | $6070 \cdot 20$ | 101'17 | 50 | 284887.0 | $4748 \cdot 13$ | 79*136 |
| $39^{\circ} \circ^{\prime}$ | $364222 \cdot 6$ | $6070 \cdot 38$ | 101'17 | $39^{\circ} \circ^{\prime}$ | $284222{ }^{\circ}$ | $4737 \cdot 03$ | 78.951 |
| 10 | $364233 \cdot 2$ | $6070 \cdot 55$ | 101.18 | 10 | 283554* 0 | $4725^{\circ} 90$ | $78 \cdot 765$ |
| 20 | $364243 \cdot 8$ | 6070.73 | 101.18 | 20 | $242883 \cdot 7$ | $4714 \cdot 73$ | $78 \cdot 579$ |
| 30 | 364254.4 | $6070 \cdot 91$ | 101-18 | 30 | $282210{ }^{\circ} 9$ | $4703 \cdot 52$ | $78 \cdot 392$ |
| 40 | $364265^{-1}$ | 6071.09 | 101.18 | 40 | 281535-8 | $4692 \cdot 26$ | $78 \cdot 204$ |
| 50 | $364275 \cdot 7$ | $6071 \cdot 27$ | 101'19 | 50 | $280858 \cdot 2$ | $4680 \cdot 97$ | $78 \cdot 016$ |
| $40^{\circ} \circ^{\prime}$ | $364286 \cdot 3$ | 6071.44 | ror'19 | $40^{\circ} \circ^{\prime}$ | $280178 \cdot 2$ | 4669.64 | $77 \cdot 827$ |
| 10 | 364297* | $6071 \cdot 62$ | 101.19 | 10 | $279495 \cdot 9$ | $4658 \cdot 27$ | 77.638 |
| 20 | $364307 \cdot 7$ | $6071 \cdot 80$ | 101. 20 | 20 | 278811.2 | $4646 \cdot 85$ | $77 \cdot 448$ |
| 30 | $364318 \cdot 3$ | $6071 \cdot 97$ | 101. 20 | 30 | 278124.1 | $4635 \cdot 40$ | $77 \cdot 257$ |
| 40 | 364329.0 | $6072 \cdot 15$ | 101. 20 | 40 | $277434^{\circ} 7$ | 4623.91 | $77 \cdot 065$ |
| 50 | $364339 \cdot 7$ | $6072 \cdot 33$ | 101. 21 | 50 | $276742 \cdot 9$ | $4612 \cdot 38$ | $76 \cdot 873$ |


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| $41^{\circ} \mathrm{o}^{\prime}$ | $364350 \cdot 4$ | $6072 \cdot 51$ | 101.2I | $41^{\circ}{ }^{\prime}$ | $276048 \cdot 7$ | $4600 \cdot 8 \mathrm{I}$ | 76•680 |
| 10 | $36436 \mathrm{I}^{\prime} \mathrm{I}$ | $6072 \cdot 69$ | 101. 21 | 10 | $275352 \cdot 2$ | 4589'20 | $76 \cdot 480$ |
| 20 | $364377^{\prime 9}$ | $6072 \cdot 87$ | 101.21 | 20 | 274653.4 | $4577 \cdot 56$ | $76 \cdot 293$ |
| 30 | $364382 \cdot 6$ | 6073.04 | 101. 22 | 30 | $27395{ }^{\circ} \cdot 2$ | $4565 \cdot 87$ | $76 \cdot 098$ |
| 40 | $364393 \cdot 4$ | $6073 \cdot 22$ | 101. 22 | 40 | $273248 \cdot 7$ | $4554 \cdot 75$ | $75 \cdot 902$ |
| 50 | 364404* 1 | $6073 \cdot 40$ | 101. 22 | 50 | $272542 \cdot 9$ | $4542 \cdot 38$ | $75 \cdot 706$ |
| $42^{\circ} 0^{\prime}$ | 364414.9 | $6073 \cdot 58$ | 101. 23 | $42^{\circ} \mathrm{o}^{\prime}$ | 271834.7 | $4530 \cdot 58$ | $75 \cdot 509$ |
| 10 | $364425 \cdot 6$ | $6073 \cdot 76$ | 101.23 | 10 | $271124^{\circ} 3$ | $4518 \cdot 74$ | $75 \cdot 312$ |
| 20 | $364436 \cdot 4$ | 6073.94 | 101. 23 | 20 | 270411.5 | $4506 \cdot 86$ | $75 \cdot 114$ |
| 30 | $364447 \cdot 2$ | $6074 \cdot 12$ | 101. 24 | 30 | $269696 \cdot 4$ | 4494.94 | $74 \cdot 916$ |
| 40 | $364458 \cdot 0$ | 6074*30 | 101. 24 | 40 | $268979{ }^{\prime} \mathrm{I}$ | $4482 \cdot 99$ | 74*717 |
| 50 | $364468 \cdot 8$ | $6074 \cdot 48$ | ror. 24 | 50 | 268259.5 | $4470^{\circ} 99$ | 74.517 |
| $43^{\circ} \mathrm{o}^{\prime}$ | $364479 \cdot 6$ | $6074 \cdot 66$ | 101. 24 | $43^{\circ} \mathrm{o}^{\prime}$ | $267537 \cdot 5$ | $4458 \cdot 96$ | 74.316 |
| 10 | $364490 \cdot 4$ | $6074 \cdot 84$ | 101. 25 | 10 | 266813*3 | $4446 \cdot 89$ | $74^{* 115}$ |
| 20 | $364501 \cdot 2$ | $6075 \cdot 02$ | 101. 25 | 20 | $266086 \cdot 8$ | $4434 \cdot 78$ | 73.913 |
| 30 | $364512 \cdot 0$ | 6075.20 | 101. 25 | 30 | $265358 \cdot 1$ | $4422 \cdot 64$ | $73^{\cdot 711}$ |
| 40 | $364522 \cdot 8$ | $6075 \cdot 38$ | IOI. 26 | 40 | $264627^{\prime}$ I | $4410 \cdot 45$ | 73.508 |
| 50 | $364533 \cdot 6$ | $6075 \cdot 56$ | 101. 26 | 50 | $263893 \cdot 8$ | $4398 \cdot 23$ | $73 \cdot 304$ |
| $44^{\circ}{ }^{\prime}$ | $364544^{\circ} 4$ | 6075.74 | 101. 26 | $44^{\circ}{ }^{\prime}$ | 263158.3 | $4385 \cdot 97$ | $73 \cdot 100$ |
| 10 | $364555 \cdot 2$ | 6075.92 | 101.27 | 10 | $262420 \cdot 5$ | $4373 \cdot 68$ | 72.895 |
| 20 | 364566.1 | $6076 \cdot 10$ | 101.27 | 20 | $261680 \cdot 6$ | $436 \mathrm{r} \cdot 34$ | $72 \cdot 689$ |
| 30 | $364576 \cdot 9$ | $6076 \cdot 28$ | 101. 27 | 30 | $260938 \cdot 4$ | $4348 \cdot 97$ | 72.483 |
| 40 | $364587 \cdot 7$ | $6076 \cdot 46$ | 101. 27 | 40 | 260193.9 | $4336 \cdot 57$ | $72 \cdot 276$ |
| 50 | $364598 \cdot 5$ | $6076 \cdot 64$ | 101.28 | 50 | $259447 \cdot 3$ | $4324 \cdot 12$ | 72.069 |
| $45^{\circ} \mathrm{o}^{\prime}$ | 364609.4 | 6076•82 | 101.28 | $45^{\circ}{ }^{\prime}$ | $258698 \cdot 4$ | 4311.64 | $7 \mathrm{~F} \cdot 86 \mathrm{I}$ |
| 10 | $364620 \cdot 2$ | 6077.00 | 10, $\cdot 28$ | 10 | $257947 \cdot 3$ | $4^{29} 99^{\prime} 12$ | $7 \mathrm{~F} \cdot 652$ |
| 20 | $364631^{\circ} \mathrm{O}$ | $6077 \cdot 18$ | 101. 29 | 20 | $257194^{\circ} 1$ | $4286 \cdot 57$ | 71.443 |
| 30 | $364641 \cdot 9$ | $6077 \cdot 37$ | 101. 29 | 30 | $256438 \cdot 6$ | $4273 \cdot 98$ | 71.233 |
| 40 | $364652 \cdot 7$ | $6077 \cdot 55$ | 101.29 | 40 | 255681* | 4261.35 | $71 \cdot 022$ |
| 50 | $364663 \cdot 5$ | $6077 \cdot 73$ | 1or. 30 | 50 | $254921 \cdot 2$ | $4248 \cdot 69$ | 70.811 |
| $46^{\circ}{ }^{\prime}$ | $364674{ }^{\circ} 4$ | 6077.91 | $101 \cdot 30$ | $46^{\circ}{ }^{\prime}$ | 254159.2 | 4235 *99 | $70 \cdot 600$ |
| 10 | $364685^{\circ} 2$ | $6078 \cdot 09$ | 101.30 | 10 | $253395{ }^{\circ} \mathrm{O}$ | 4223.25 | $70 \cdot 388$ |
| 20 | $364696 \cdot 0$ | $6078 \cdot 27$ | 101*30 | 20 | $252628 \cdot 7$ | $4210 \cdot 48$ | $70 \cdot 175$ |
| 30 | $364706 \cdot 8$ | $6078 \cdot 45$ | 101*31 | 30 | $251860 \cdot 2$ | $4197 \cdot 67$ | $69 \cdot 961$ |
| 40 | $364717 \cdot 7$ | $6078 \cdot 63$ | 101.31 | 40 | $251089 \cdot 6$ | $4184 \cdot 83$ | 69*747 |
| 50 | 364728.5 | $6078 \cdot 81$ | 101.31 | 50 | $250316 \cdot 8$ | $417 \mathrm{~T} \cdot 95$ | 69.532 |


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| $47^{\circ}{ }^{\circ}$ | $364739^{\circ} 3$ | $6078 \cdot 99$ | 101. 32 | $47^{\circ}{ }^{\prime}$ | 24954 ${ }^{\circ} 9$ | 4159.03 | $69^{* 317}$ |
| 10 | $364750 \cdot 1$ | $6079{ }^{\circ} 17$ | 101.32 | 10 | $248764^{\circ} 9$ | 4146.08 | $69^{\circ} \mathrm{IOI}$ |
| 20 | $364760 \cdot 9$ | $6079 \cdot 35$ | 101* 32 | 20 | 247985*8 | $4133^{\circ} 10$ | $68 \cdot 885$ |
| 30 | $364771 \cdot 7$ | 6079.53 | ror 33 | 30 | 247204.5 | $4120 \cdot 08$ | 68.668 |
| 40 | $364782 \cdot 5$ | $6079 \cdot 71$ | 101.33 | 40 | $246421 \cdot 2$ | 4107.02 | 68.450 |
| 50 | $364793 * 3$ | $6079 \cdot 89$ | IOI. 33 | 50 | $245635^{\circ} 8$ | $4093 \cdot 93$ | 68.232 |
| $48^{\circ} 0^{\prime}$ | 364804* 1 | $6080 \cdot 07$ | 101.33 | $48^{\circ} \bigcirc^{\prime}$ | $244848 \cdot 2$ | $4080 \cdot 80$ | 68.013 |
| 10 | 364814*9 | 6080:25 | 101.34 | 10 | $244058 \cdot 5$ | $4067 \cdot 64$ | $67 \cdot 794$ |
| 20 | $364825 \cdot 6$ | $6080 \cdot 43$ | 101•34 | 20 | $243266 \cdot 8$ | 4054.45 | 67.574 |
| 30 | $364836 \cdot 4$ | $6080 \cdot 61$ | IOI•34 | 30 | 242473* | $404 \mathrm{I} \cdot 22$ | $67 \cdot 353$ |
| 40 | $364847^{\circ} \mathrm{I}$ | $6080 \cdot 79$ | IOI. 35 | 40 | 241677* | $4027 \cdot 95$ | $67 \cdot 132$ |
| 50 | 364857.9 | $6080 \cdot 97$ | 101*35 | 50 | $240879^{\circ} 2$ | 4014.65 | $66 \cdot 911$ |
| $49^{\circ} 0^{\prime}$ | $364868 \cdot 6$ | $608 \mathrm{I} \cdot 14$ | 101*35 | $49^{\circ} \mathrm{o}^{\prime}$ | $240079^{\circ} 2$ | 4001•32 | $66 \cdot 689$ |
| 10 | 364879.4 | $608 \mathrm{I} \cdot 32$ | IOI $\cdot 36$ | 10 | $239277^{\circ} \mathrm{I}$ | $3987 \cdot 95$ | 66.466 |
| 20 | $364890 \cdot 1$ | $608 \mathrm{I} \cdot 50$ | IOI* 36 | 20 | 238473.1 | 3974*55 | $66 \cdot 242$ |
| 30 | $364900 \cdot 8$ | $608 \mathrm{r} \cdot 68$ | IOI• 36 | 30 | $237667^{\circ} 0$ | 3961'12 | $66 \cdot 018$ |
| 40 | 364911.5 | $608 \mathrm{I} \cdot 86$ | 101. 36 | 40 | 236858.9 | $3947 \cdot 65$ | $65 \cdot 794$ |
| 50 | 364922.2 | $6082 \cdot 04$ | IOI* 37 | 50 | $236048 \cdot 7$ | $3934 * 15$ | 65.569 |
| $50^{\circ}{ }^{\prime}$ | $364932 \cdot 9$ | $6082 \cdot 22$ | 101* 37 | $50^{\circ}{ }^{\prime}$ | $235236 \cdot 5$ | $3920 \cdot 6 \mathrm{~J}$ | $65 \cdot 343$ |
| 10 | $364943 \cdot 6$ | $6082 \cdot 39$ | IOI'37 | 10 | 234422*3 | 3907.04 | $65 \cdot 117$ |
| 20 | $364954^{\prime 2}$ | $6082 \cdot 57$ | IOI. 38 | 20 | $233606^{\circ} \mathrm{I}$ | 3893.44 | $64 \cdot 890$ |
| 30 | 364964*9 | $6082 \cdot 75$ | 101.38 | 30 | $232787^{\circ} 9$ | $3879 \cdot 80$ | 64.663 |
| 40 | $364975 \cdot 5$ | $6082 \cdot 93$ | IOI. 38 | 40 | $231967 \cdot 8$ | 3866.13 | $64 \cdot 435$ |
| 50 | $364986 \cdot 2$ | $6083 \cdot 10$ | 1OI $\cdot 3^{8}$ | 50 | $231145^{\circ} 7$ | 3852.43 | $64 \cdot 207$ |
| $51^{\circ} \mathrm{o}^{\prime}$ | $364996 \cdot 8$ | $6083 \cdot 28$ | 101.39 | $51^{\circ} \mathrm{o}^{\prime}$ | $230325 \cdot 4$ | 3838.69 | 63.978 |
| 10 | 365007.4 | $6083 \cdot 46$ | 101.39 | 10 | $229495 \cdot 3$ | 3824.92 | $63 \cdot 749$ |
| 20 | 365018.0 | $6083 \cdot 63$ | IOI•39 | 20 | $228667^{\circ} 2$ | 3811*12 | 63.519 |
| 30 | $365028 \cdot 6$ | $6083 \cdot 8 \mathrm{I}$ | IOI.40 | 30 | 227837.2 | $3797 \cdot 29$ | $63 \cdot 288$ |
| 40 | $365039^{\circ} \mathrm{r}$ | $6083 \cdot 99$ | 101.40 | 40 | 22700; 3 | 3783.42 | 63.057 |
| 50 | $365049 \cdot 7$ | $6084 \cdot 16$ | IOR. 40 | 50 | 226171.4 | 3769.52 | $62 \cdot 825$ |
| $52^{\circ} 0^{\prime}$ | $365060 \cdot 2$ | $6084 \cdot 34$ | 1OI.4I | $52^{\circ} \circ^{\prime}$ | $225335^{\circ} 5$ | 3755*59 | 62.593 |
| 10 | $365070 \cdot 7$ | $6084 \cdot 51$ | 101.41 | 10 | 224497*7 | $3741 \cdot 63$ | $62 \cdot 360$ |
| 20 | $36 ; 081 \cdot 2$ | $6084 \cdot 69$ | IOI.41 | 20 | $22365^{\circ} \cdot 1$ | 3727.64 | $62 \cdot 127$ |
| 30 | $365091 \cdot 7$ | $6084 \cdot 86$ | IOI.4I | 30 | 222816.5 | $3713 \cdot 61$ | $6 \mathrm{r} \cdot 893$ |
| 40 | $365102 \cdot 2$ | $6085 \cdot 04$ | IOI'42 | 40 | $221973^{\circ} 0$ | 3699.55 | 6r.659 |
| 50 | 365112.7 | $6085 \cdot 2 \mathrm{I}$ | IOI $\cdot 42$ | 50 | $2211127^{\circ} 6$ | $3685 \cdot 46$ | 6I*424 |


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|  | Degree. | Minute. | Sec ond. |  | Degree. | Minute. | Second. |
| $53^{\circ}{ }^{\prime}$ | 365123.1 | $6085 \cdot 39$ | 101.42 | $53^{\circ} \circ^{\prime}$ | $220280 \cdot 3$ | $3671 \cdot 34$ | $6 \mathrm{~F} \cdot 189$ |
| 10 | 365133.6 | $6085 \cdot 56$ | 101.43 | 10 | $21943{ }^{\circ} \cdot 1$ | $3657 \cdot 19$ | $60 \cdot 953$ |
| 20 | $365144^{\circ} \mathrm{O}$ | 6085.73 | 101.43 | 20 | $218580{ }^{\circ} \mathrm{O}$ | $3643 \cdot 00$ | $60 \cdot 717$ |
| 30 | 365154.4 | $6085^{\circ} 91$ | Ior.43 | 30 | 217727.1 | $3628 \cdot 79$ | $60 \cdot 480$ |
| 40 | $365164{ }^{\circ} 7$ | 6086.08 | 101.43 | 40 | $216872 \cdot 3$ | 3614.54 | $60 \cdot 242$ |
| 50 | $365175 \cdot 1$ | $6086 \cdot 25$ | 101.44 | 50 | 216015'7 | $3600 \cdot 26$ | 60.004 |
| $54^{\circ} \mathrm{o}^{\prime}$ | $365185 \cdot 4$ | $6086 \cdot 42$ | IOT.44 | $54^{\circ}{ }^{\prime}$ | 215157.2 | 3585.95 | $59 \cdot 766$ |
| 10 | $365195 \cdot 7$ | 6086.60 | 101.44 | 10 | 214296.9 | $3571 \cdot 62$ | 59.527 |
| 20 | $365206 \cdot 1$ | $6086 \cdot 77$ | 101. 45 | 20 | 213434.7 | $3557 \cdot 25$ | 59.287 |
| 30 | $365216 \cdot 3$ | 6086.94 | 101.45 | 30 | $212570 \cdot 7$ | $3542 \cdot 85$ | 59.047 |
| 40 | $365226 \cdot 6$ | $6087 \cdot 11$ | 101.45 | 40 | 211704.9 | 3528.42 | 58.807 |
| 50 | $365236 \cdot 8$ | $6087 \cdot 28$ | 101.45 | 50 | $210837^{\prime} 3$ | 3513.96 | $58 \cdot 566$ |
| $55^{\circ} 0^{\prime}$ | $365247^{\circ} \circ$ | 6087.45 | 101.46 | $55^{\circ}{ }^{\prime}$ | 209968.0 | 3499.47 | $58 \cdot 324$ |
| 10 | $365257^{\circ} \mathrm{Z}$ | $6087 \cdot 62$ | 101.46 | 10 | 209096.8 | 3484.95 | $58 \cdot 082$ |
| 20 | $365267 \cdot 4$ | $6087 \cdot 79$ | ror 46 | 20 | 208223.8 | $3470 \cdot 40$ | $57 \cdot 840$ |
| 30 | $365277 \cdot 6$ | 6088.96 | 101. 47 | 30 | $207349^{\circ} 0$ | $3455 \cdot 82$ | 57.597 |
| 40 | $365287 \cdot 7$ | $6088 \cdot 13$ | 10147 | 40 | 206472.5 | 3441.2r | 57-353 |
| 50 | $365297 \cdot 8$ | $6088 \cdot 30$ | $101 \cdot 47$ | 50 | 205594.2 | $3426 \cdot 57$ | 57.109 |
| $56^{\circ}{ }^{\prime}$ | $365307 \cdot 9$ | $6088 \cdot 47$ | ror-47 | $56^{\circ}{ }^{\prime}$ | 204714.0 | 3411.90 | $56 \cdot 865$ |
| 10 | 365318.0 | $6088 \cdot 63$ | Ior 48 | 10 | $203832 \cdot 2$ | 3397. 20 | $56 \cdot 620$ |
| 20 | 365328.0 | $6088 \cdot 80$ | Ior. 48 | 20 | $202948 \cdot 6$ | 3382.48 | $56 \cdot 375$ |
| 30 | 365338.0 | $6088 \cdot 97$ | Ior:48 | 30 | $202063 \cdot 3$ | $3367 \cdot 72$ | $56 \cdot 129$ |
| 40 | $365348^{\circ} 0$ | $6089 \cdot 13$ | ror 49 | 40 | $201176 \cdot 2$ | $3352 \cdot 94$ | $55 \cdot 882$ |
| 50 | 365358.0 | $6089 \cdot 30$ | ror-49 | 50 | 200287.4 | 3338.12 | $55 \cdot 635$ |
| $57^{\circ} \circ^{\prime}$ | $565367 \cdot 9$ | 6089.47 | 10r*49 | $57^{\circ}{ }^{\prime}$ | 199396.9 | $3323 \cdot 28$ | $55 \cdot 388$ |
| 10 | $365377 \cdot 8$ | $6089 \cdot 63$ | TOL 49 | 10 | 198504.7 | $3308 \cdot 41$ | 55.140 |
| 20 | 365387'7 | $6089 \cdot 80$ | 101. 50 | 20 | 197610.8 | $3293 \cdot 51$ | 54.892 |
| 30 | $365397 \cdot 6$ | 6089.96 | IOI*50 | 30 | 196715*2 | $3278 \cdot 59$ | 54.643 |
| 40 | $365407 \cdot 4$ | $6090 \cdot 12$ | ror ${ }^{50}$ | 40 | 195817.9 | 3263.63 | 54*394 |
| 50 | $365417 \cdot 2$ | 6090.29 | 101.50 | 50 | 194919 ${ }^{\circ}$ | $3248 \cdot 65$ | 54.144 |
| $58^{\circ}{ }^{\prime}$ | 365427.0 | $6090 \cdot 45$ | 101.51 | $58^{\circ}{ }^{\prime}$ | 194018.3 | $3233 \cdot 64$ | $53 \cdot 643$ |
| 10 | $365436 \cdot 8$ | 6090.6r | 101. 51 | 10 | 193116.0 | $3218 \cdot 60$ | $53 \cdot 643$ |
| 20 | $365446 \cdot 5$ | $6090 \cdot 78$ | 10. 51 | 20 | 192212.I | 3203.54 | 53.392 |
| 30 | $365456 \cdot 2$ | $6090 \cdot 94$ | IOI. 52 | 30 | 191306.5 | 3188.44 | 53.141 |
| 40 | $365465 \cdot 9$ | $609 \mathrm{I} \cdot 11$ | 101. 52 | 40 | 190399*3 | 3173.32 | 52.889 |
| 50 | $365475 \cdot 5$ | $609 \mathrm{I} \cdot 26$ | 101.52 | 50 | 189490.4 | 3158.17 | 52.636 |


| Latitude. |  |  |  | LONGITUDE. |  |  |  |
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| $59^{\circ}{ }^{\prime}$ | $365485^{\circ} \mathrm{I}$ | $6091 \cdot 42$ | 101-52 | $59^{\circ} \circ^{\prime}$ | 188579*9 | 314300 | $52 \cdot 383$ |
|  | $365494^{\circ} 7$ | $609 \mathrm{r} \cdot 5^{8}$ | Ior 53 | 10 | $187667 \cdot 8$ | 3127-80 | $52 \cdot 130$ |
| 20 | $365504 \cdot 3$ | 609r.74 | ror. 53 | 20 | 186754 ${ }^{\text {r }}$ | 3112.57 | $51 \cdot 876$ |
| 30 | $365513 \cdot 8$ | 6091.90 | 101.53 | 30 | 185838.8 | 3097.31 | $51 \cdot 622$ |
| 40 | $365523 \cdot 3$ | 6092.06 | 101.53 | 40 | 184921.9 | 3082.03 | 51-367 |
| 50 | $365532 \cdot 8$ | $6092 \cdot 21$ | 101. 54 | 50 | 184003.4 | 3066•72 | $51 \cdot 112$ |
| $60^{\circ} \circ^{\prime}$ | $365542 \cdot 2$ | $6092 \cdot 37$ | 101.54 | $60^{\circ}{ }^{\prime}$ | 183083.3 | 3051.59 | 50.856 |
| 10 | $365551 \cdot 6$ | $6092 \cdot 53$ | 101.54 | 10 | 182161.6 | 3036.03 | 50.600 |
| 20 | 365561.0 | $6092 \cdot 68$ | 101. 54 | 20 | 181238.4 | $3020 \cdot 64$ | 50.344 |
| 30 | $365570 \cdot 3$ | $6092 \cdot 84$ | 101.55 | 30 | 180313.7 | 3005.23 | 50.087 |
| 40 | $365579 \cdot 6$ | 6092*99 | 101. 55 | 40 | $179387^{\circ} 4$ | 2989.79 | $49 \cdot 830$ |
| 50 | 365588.9 | $6093 \cdot 15$ | 101. 55 | 50 | 178459.5 | 2974*33 | 49.572 |
| $61^{\circ} 0^{\prime}$ | $365598 \cdot 1$ | $6093 \cdot 30$ | 101.56 | $61^{\circ} \circ^{\prime}$ | 177530*1 | 2958.84 | $49^{*} 314$ |
| 10 | $365607 \cdot 3$ | $6093 \cdot 46$ | 101. 56 | 10 | 176599.2 | 2943.32 | 49.055 |
| 20 | 365616.5 | $6093 \cdot 61$ | 101*56 | 20 | 175666.8 | 2927.78 | $48 \cdot 796$ |
| 30 | $365625^{\circ} 7$ | $6093 \cdot 76$ | 101. $5^{6}$ | 30 | $174732 \cdot 8$ | 2912.21 | +8.537 |
| 40 | 365634.8 | $6093 \cdot 91$ | 101. 57 | 40 | 173797.4 | 2896.62 | $48 \cdot 277$ |
| 50 | 365643*9 | 6094.07 | 101.57 | 50 | $172860 \cdot 5$ | 2881.01 | $48 \cdot 017$ |
| $62^{\circ} \circ^{\prime}$ | $365652 \cdot 9$ | 6094.22 | 101.57 | $62^{\circ} \circ^{\prime}$ | 171922.1 | $2865 \cdot 37$ | 47*750 |
| 10 | $365661 \cdot 9$ | 6094*37 | 101*57 | 10 | $170982 \cdot 2$ | 2849.70 | $47 \cdot 495$ |
| 20 | $365670^{\circ} 9$ | 6094.52 | 101.58 | 20 | 170040'9 | $2834^{\circ} 02$ | $47 \cdot 234$ |
| 30 | $365679 \cdot 8$ | 6094.66 | 101 58 | 30 | 169098.1 | $2818 \cdot 30$ | $46 \cdot 972$ |
| 40 | $365688 \cdot 7$ | 6094.81 | Ior. 58 | 40 | 168153.8 | 2802.56 | $46 \cdot 709$ |
| 50 | $365697 \cdot 6$ | 6094.96 | Ior $5^{8}$ | 50 | $167208 \cdot 1$ | 2786.80 | $46 \cdot 447$ |
| $63^{\circ}$ 。 | $365706 \cdot 4$ | 6095.11 | 101.59 | $63^{\circ} \circ^{\prime}$ | 166261.0 | $277{ }^{\circ} \cdot 01$ | $45 \cdot 184$ |
| 10 | 365715.2 | $6095 \cdot 25$ | 101-59 | 10 | 165312.4 | 2755.21 | $45 \cdot 920$ |
| 20 | 365723.9 | $6095 \cdot 40$ | 101.59 | 20 | 164362.5 | $2739 \cdot 38$ | $45 \cdot 656$ |
| 30 | $365732 \cdot 6$ | $6095 \cdot 54$ | 101'59 | 30 | 163411. 1 | 2723.52 | $45 \cdot 392$ |
| 4 C | 365741•3 | $6095 \cdot 69$ | 101.59 | 40 | 162458.4 | 2707.64 | $45 \cdot 127$ |
| 50 | 365749*9 | $6095 \cdot 83$ | 101.60 | 50 | 161504.2 | $2691 \cdot 74$ | $44 \cdot 862$ |
| $64^{\circ}$ 。 | 365758.5 | $6095 \cdot 98$ | 101. 60 | $64^{\circ}{ }^{\prime}$ | $160548 \cdot 6$ | $2675 \cdot 8 \mathrm{I}$ | 44.587 |
| 10 | $365767^{1}$ | $6096 \cdot 12$ | Ior. 60 | 10 | 159591*6 | $2659 \cdot 86$ | $44 \cdot 331$ |
| 20 | $365775 \cdot 6$ | $6096 \cdot 26$ | 101.60 | 20 | $158633^{\circ} 2$ | $2643 \cdot 89$ | $44 \cdot 065$ |
| 30 | 365784.1 | $6096 \cdot 40$ | ror.61 | 30 | $157673^{\circ} 5$ | 2627.90 | $43 \cdot 798$ |
| 40 | $365792 \cdot 6$ | $6096 \cdot 54$ | 101.61 | 40 | 156712.5 | 2611-88 | 43.53 I |
| 50 | $365801 \cdot 0$ | 6096.68 | 101.61 | 50 | 155750'1 | $2595 \cdot 84$ | $43^{\circ} 264$ |


| Latitude. |  |  |  | LONGITUDE. |  |  |  |
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| $65^{\circ}{ }^{\prime}$ | $365809 \cdot 3$ | 6096.82 | 101.61 | $65^{\circ}{ }^{\prime}$ | 154786.3 | 2579*77 | $42 \cdot 996$ |
| 10 | $365817 \times 6$ | $6096 \cdot 96$ | 101.62 | 10 | $15382 \mathrm{I} \cdot 2$ | 2563.69 | $42 \cdot 728$ |
| 20 | 365825.9 | 6097-10 | 101.62 | 20 | 152854.8 | $2547 \cdot 58$ | $42 \cdot 460$ |
| 30 | $365834 \cdot 2$ | $6097 \cdot 24$ | 101.62 | 30 | 151887.2 | 2531.45 | $42 \cdot 191$ |
| 40 | 365842.4 | 6097*37 | 101. 62 | 40 | 150918.2 | $2515 \cdot 30$ | 41.922 |
| 50 | $365850 \cdot 5$ | 6097.51 | 101.63 | 50 | 149947*9 | 2499* 13 | $41 \cdot 652$ |
| $66^{\circ}{ }^{\prime}$ | 365858.6 | $6097 \cdot 64$ | 101.63 | $66^{\circ}{ }^{\prime}$ | $148976 \cdot 3$ | $2482 \cdot 94$ | $41 \cdot 382$ |
| 10 | $365866 \cdot 7$ | $6097 \cdot 78$ | Ior. 63 | 10 | $148003^{\circ} 4$ | $2466 \cdot 72$ | $4 \mathrm{I} \cdot 112$ |
| 20 | 365874.7 | $6097 \cdot 91$ | 101.63 | 20 | $147029^{\circ} 3$ | $2450 \cdot 49$ | 40.84 I |
| 30 | $365882 \cdot 7$ | $6098 \cdot 05$ | 101.63 | 30 | 146053.9 | $2434 \cdot 23$ | 40.570 |
| 40 | $365890 \cdot 7$ | $6098 \cdot 18$ | 101. 64 | 40 | 145977*3 | 2417*96 | 40.299 |
| 50 | $365898 \cdot 6$ | 6098.31 | ror. 64 | 50 | 144099*3 | $2401 \cdot 66$ | 40.028 |
| $67^{\circ} 0^{\prime}$ | $365906 \cdot 4$ | 6098.44 | 101. 64 | $67^{\circ}{ }^{\prime}$ | $143120{ }^{\circ} 2$ | $2385 \cdot 34$ | $39^{*} 756$ |
| 10 | 365914.3 | $6098 \cdot 57$ | 101. 64 | 10 | $142139{ }^{\circ} 8$ | 2369.00 | $39^{\circ} 483$ |
| 20 | $365922 \cdot 0$ | $6098 \cdot 70$ | IOI. 65 | 20 | 141158.2 | 2352.64 | 39.211 |
| 30 | 365929.8 | $6098 \cdot 83$ | 101. 65 | 30 | $140175{ }^{\circ} 4$ | 2336.26 | 38.938 |
| 40 | $365937{ }^{\circ} 4$ | 6098.96 | 101. 65 | 40 | 139191.4 | 2319.86 | $38 \cdot 664$ |
| 50 | 365945 ${ }^{1}$ | 6099.09 | 101. 65 | 50 | $138206 \cdot 1$ | 2303.44 | $38 \cdot 390$ |
| $68^{\circ}{ }^{\prime}$ | $365952 \cdot 7$ | $6099 \cdot 2 \mathrm{I}$ | $101 \cdot 65$ | $68^{\circ} \circ^{\prime}$ | 137219.7 | $2287^{\circ} 00$ | $38 \cdot 116$ |
| 10 | 365960.2 | 6099.34 | 101.66 | Io | $136232^{1} \mathrm{I}$ | $2270 \cdot 54$ | $37 \cdot 842$ |
| 20 | $365967 \cdot 7$ | $6099 \cdot 46$ | 101. 66 | 20 | $135243^{\circ} 3$ | 2254.06 | 37.568 |
| 30 | $365975^{\circ} \cdot$ | $6099 \cdot 59$ | 101.66 | 30 | 134253.4 | 2237.56 | $37 \cdot 293$ |
| 40 | $365982 \cdot 6$ | $6099 \cdot 71$ | 101. 66 | 40 | 133262.3 | 2221.04 | 37.017 |
| 50 | 365989.9 | $6099 \cdot 83$ | 101.66 | 50 | $132270 \cdot 1$ | 2204.50 | $36 \cdot 742$ |
| $69^{\circ}{ }^{\circ}$ | $365997 * 3$ | $6099 \cdot 96$ | 101.67 | $69^{\circ}{ }^{\circ}$ | $131276 \cdot 7$ | 2187.95 | $36 \cdot 466$ |
| 10 | $366004 \cdot 5$ | $6100 \cdot 08$ | IOI. 67 | 10 | 130282.2 | 2171*37 | 36.190 |
| 20 | $36601{ }^{\prime} 7$ | $6100 \cdot 20$ | 101. 67 | 20 | 129286.6 | 2154.78 | 35.913 |
| 30 | $366018 \cdot 9$ | $6100 \cdot 32$ | 101. 67 | 30 | 128289*9 | 2138.17 | $35 \cdot 636$ |
| 40 | $366026 \cdot 1$ | $6100 \cdot 44$ | Ior. 67 | 40 | $127292 \cdot 1$ | 2121.54 | 35*359 |
| 50 | $366033 \cdot 1$ | $6100 \cdot 55$ | Ior. 68 | 50 | 126293. ${ }^{2}$ | 2104.89 | $35 \cdot 082$ |
| $70^{\circ} 0^{\prime}$ | $366040 \cdot 2$ | $6100 \cdot 67$ | 101.68 | $70^{\circ} \circ^{\prime}$ | 125293. ${ }^{2}$ | 2088.22 | $34 \cdot 804$ |
| 10 | $366047 \cdot 2$ | 6100. 79 | IOI.68 | 10 | $124292 \cdot 1$ | 2071*54 | 34.526 |
| 20 | $366054^{1} 1$ | 6100.90 | 101.68 | 20 | 123289*9 | 2054.83 | 34.247 |
| 30 | $366061 \cdot 0$ | 6101•02 | 101.68 | 30 | $122286 \cdot 7$ | $2038 \cdot 11$ | 33.968 |
| 40 | $366067 \cdot 8$ | $6101 \cdot 13$ | 101.69 | 40 | 121282.4 | 2021.37 | 33.690 |
| 50 | $366074 \cdot 6$ | 6ror-24 | 101*69 | 50 | 120277 t | 2004.62 | $33^{\circ} 410$ |


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| $71^{\circ} 0^{\prime}$ | $366081 \cdot 3$ | 6101.36 | 101*69 | $71^{\circ}{ }^{\circ}$ | $119270 \cdot 7$ | $1987 \cdot 85$ | $33^{\circ} 131$ |
| 10 | 366088.0 | 6101.47 | 101. 69 | 10 | 118263.3 | 1971.06 | $32 \cdot 8$ |
| 20 | $36609+6$ | 6101.58 | ror. 69 | 20 | 117254.9 | 1954.25 | $32 \cdot 571$ |
| 30 | $366101 \cdot 2$ | 6ror. 69 | 101. 69 | 30 | 116245.6 | 1937*43 | $32 \cdot 290$ |
| 40 | $366 \mathrm{ro} 7 \cdot 8$ | $6 \mathrm{IOT} \cdot 80$ | 101'70 | 40 | 115235 ${ }^{\circ} 2$ | 1920.59 | 32.009 |
| 50 | $366 \mathrm{rr}^{4} 3$ | $6101 \cdot 91$ | 101.70 | 50 | 114223.8 | $1903 \cdot 73$ | $31 \cdot 729$ |
| $72^{\circ} \circ^{\prime}$ | 366r20.7 | $6102 \cdot 01$ | ror' 70 | $72^{\circ} 0^{\prime}$ | 112311.4 | 1886.86 | $31 \cdot 448$ |
| 10 | $366127^{\prime} 1$ | $6102 \cdot 12$ | 101.70 | 10 | $112198{ }^{\circ} \mathrm{O}$ | 1869.97 | $3 \mathrm{I} \cdot 166$ |
| 20 | . $366133 \cdot 4$ | 6102.22 | ror 70 | 20 | 111183.7 | 1853.06 | 30.884 |
| 30 | $366139^{\circ} 7$ | $6102 \cdot 33$ | ror'7r | 30 | 110168.4 | $1836 \cdot 14$ | 30.602 |
| 40 | 366145*9 | $6102 \cdot 43$ | ror'71 | 40 | 109132.2 | 1819.20 | $30 \cdot 320$ |
| 50 | $366153^{\cdot 1}$ | $6102 \cdot 54$ | 101.71 | 50 | $108135^{\circ}$ | 1802.25 | 30.038 |
| $73^{\circ} \circ^{\prime}$ | $366158 \cdot 2$ | 6102.64 | ror.7r | $73^{\circ} \circ^{\prime}$ | 107116.9 | 1785.28 | $29^{\circ} 755$ |
| 10 | $366164{ }^{\circ} 3$ | 6102.74 | ror.7r | 10 | 106098.0 | $1768 \cdot 30$ | $29^{.472}$ |
| 20 | $366170 \cdot 3$ | $6102 \cdot 84$ | 101.71 | 20 | 105071.9 | 1751.30 | 29.189 |
| 30 | $366176 \cdot 3$ | $6102 \cdot 94$ | 101.72 | 30 | 104057.0 | 1734.28 | 28.905 |
| 40 | $366182 \cdot 2$ | 6103.04 | 101.72 | 40 | 103035.3 | 1717.26 | 28.62 I |
| 50 | 366188. 1 | $6103 \cdot 14$ | 101'72 | 50 | 102012.8 | 1700.21 | 28.337 |
| $74^{\circ} 0^{\prime}$ | $366193 \cdot 9$ | $6103 \cdot 23$ | $101 \cdot 73$ | $74{ }^{\circ} \circ^{\prime}$ | $100989^{\prime} \mathrm{I}$ | $1683 \cdot 15$ | 28.053 |
| 10 | $366199 \cdot 6$ | $6103 \cdot 33$ | 10r. 73 | 10 | 99964*7 | $1666 \cdot 08$ | 27.768 |
| 20 | $366205 \cdot 3$ | $6103 \cdot 42$ | 101'73 | 20 | 98939.5 | 1648.99 | 27.483 |
| 30 | $366211^{\circ}$ | $6103 \cdot 52$ | 101.73 | 30 | 97913.4 | 1631-89 | $27 \cdot 198$ |
| 40 | $366216 \cdot 6$ | 6103.61 | 101•73 | 40 | $96886 \cdot 5$ | 1614.78 | $26 \cdot 913$ |
| 50 | $366222 \cdot 1$ | $6103 \cdot 70$ | 101*73 | 50 | $95858 \cdot 7$ | 1597.65 | $26 \cdot 627$ |
| $75^{\circ} \circ^{\prime}$ | $366227^{\circ} 6$ | $6103 \cdot 79$ | ror'73 | $75^{\circ} 0^{\prime}$ | $94830 \cdot 1$ | 1580.50 | $26 \cdot 342$ |
| 10 | $366233 \cdot 0$ | $6103 \cdot 88$ | ror'73 | 10 | $93800 \cdot 6$ | 1563.34 | 26.056 |
| 20 | $366238 \cdot 4$ | $6103 \cdot 97$ | 101.73 | 20 | $92730^{\circ} 4$ | ${ }^{1546.17}$ | $25^{\circ} 770$ |
| 30 | 366243*7 | 6104.06 | 101'73 | 30 | 91739.4 | 1528.99 | $25 \cdot 4^{83}$ |
| 40 | $366249^{\circ} \mathrm{O}$ | $6104 \cdot 15$ | 101'74 | 40 | $90707^{6}$ | 1511*79 | $25^{\prime} 196$ |
| 50 | $366254^{\circ} \mathrm{C}$ | $6104^{.24}$ | 101*74 | 50 | 89675* | 1494.58 | $24^{\cdot 901}$ |
| $76^{\circ} 0^{\prime}$ | 366259.6 | 6104.32 | 101'74 | $76^{\circ} 0^{\prime}$ | $8864 \mathrm{I}^{6} 6$ | 1477* 36 | 24.623 |
| 10 | $366264 \cdot 4$ | 6104.41 | ror' 74 | 10 | 87607.4 | 1460. 12 | 24.335 |
| 20 | $366269^{\circ} 5$ | 6104.49 | Ior' 74 | 20 | 86572.5 | $1442 \cdot 88$ | $24 \cdot 048$ |
| 30 | 366274.5 | 6104.58 | 101'74 | 30 | $85536 \cdot 9$ | $1425 \cdot 62$ | $23 \cdot 760$ |
| 40 | 366279.4 | 6104.66 | 101. 74 | 40 | $84500 \cdot 5$ | $1408 \cdot 34$ | 23.472 |
| 50 | $366284 \cdot 3$ | 6104. 74 | IOI• 75 | 50 | 83463.4 | 1391.06 | $23^{\circ} 184$ |


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| $77^{\circ}{ }^{\prime}$ | $3^{66289} 9^{\text { }}$ I | $6104 \cdot 82$ | 101'75 | $77^{\circ} \circ^{\prime}$ | $82425^{\circ} 6$ | 1373.76 | 22.896 |
| 10 | $366293 \cdot 8$ | 6104.90 | 10175 | 10 | 81387.0 | 1356.45 | 22.108 |
| 20 | $366298 \cdot 5$ | 6ro4.98 | 101•75 | 20 | $80347 \cdot 8$ | $1339{ }^{\circ} 13$ | 22.319 |
| 30 | $366303 \cdot 1$ | 6105.05 | 101'75 | 30 | 79307*9 | $132 \mathrm{I} \cdot 80$ | 22.030 |
| 40 | $366307 \cdot 7$ | $6105 \cdot 13$ | 101.75 | 40 | $78267 \cdot 3$ | 1304.46 | 21.741 |
| 50 | $366312 \cdot 3$ | $6105 \cdot 21$ | 101.75 | 50 | 77226.0 | 1287.10 | 21.452 |
| $78^{\circ}{ }^{\prime}$ | $366316 \cdot 7$ | $6105 \cdot 28$ | 101.75 | $78^{\circ}{ }^{\prime}$ | $76184^{\circ} \circ$ | 1269*73 | $21^{\prime} 162$ |
| $79^{\circ} 0^{\prime}$ | $366342 \cdot 3$ | 6105*71 | 101.76 | $79^{\circ} \circ^{\prime}$ | $69918 \cdot 8$ | 1165.31 | 19.422 |
| $80^{\circ}{ }^{\prime}$ | $366365 \cdot 8$ | 6106•10 | 101'77 | $80^{\circ}{ }^{\prime}$ | 63631.8 | 1060. 53 | 17.676 |
| $81^{\circ}{ }^{\prime}$ | $366387^{1}$ I | 6106.45 | 101'77 | $81^{\circ}{ }^{\circ}$ | $57325 \cdot 2$ | $955 \cdot 42$ | 15.924 |
| $82^{\circ} \circ^{\prime}$ | $366406 \cdot 3$ | 6106.77 | 101.78 | $82^{\circ}{ }^{\prime}$ | $51000 \cdot 6$ | 850.01 | 14.167 |
| $83^{\circ}{ }^{\prime}$ | $366423^{\circ} 2$ | 6107.05 | 101.78 | $83^{\circ}{ }^{\prime}$ | $44660 \cdot 3$ | $744 \cdot 34$ | 12.406 |
| $84^{\circ}{ }^{\prime}$ | $366438 \cdot 0$ | 6107.30 | 101'79 | $84^{\circ} 0^{\prime}$ | $38306 \cdot 1$ | 638.44 | 10.64 r |
| $85^{\circ}{ }^{\prime}$ | 366450.5 | 6107.51 | 101'79 | $85^{\circ}{ }^{\prime}$ | 31939.9 | $532 \cdot 33$ | 8.872 |
| $86^{\circ}{ }^{\prime}$ | $366460 \cdot 7$ | 6107•68 | ror'79 | $86^{\circ}{ }^{\circ}$ | 25563.9 | 426.07 | $7 \cdot 101$ |
| $87^{\circ}{ }^{\prime}$ | $366468 \cdot 7$ | 6107.81 | 101.80 | $87^{\circ}{ }^{\circ}$ | 19179.8 | 319.66 | 5.328 |
| $88^{\circ}{ }^{\prime}$ | $366474 \cdot 4$ | 6107.91 | 101 ${ }^{\prime} 80$ | $88^{\circ}{ }^{\prime}$ | 12789.9 | $213 \cdot 17$ | 3.553 |
| $89^{\circ}{ }^{\prime}$ | 366477'9 | 6107*97 | 101 ${ }^{\text {8 }}$ 80 | $89^{\circ} \circ^{\prime}$ | $6395{ }^{\circ} 9$ | $106 \cdot 60$ | 1*777 |
| $90^{\circ}{ }^{\prime}$ | $366479^{\circ} \circ$ | $6107 \cdot 98$ | 101-80 | $90^{\circ} \circ^{\prime}$ | $0 \cdot 0$ | $\bigcirc \cdot 0$ | 0.0 |

Table M．－For computing the reduction to the meridian in seconds，or Vers H．A


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|  |  | － |  |

Table N．－Dip Table for calculation of Heights to 30 miles．
Dip in feet $=0.8815 d^{2}$（in miles）．

| Dist． | ${ }_{\text {Dip in }}^{\text {Dipet．}}$ | Dist． | Dip in feet． | Dist． | Dip in feet． | Dist． | Dip in feet． | Dist． | Dip in feet． |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I | 0.9 | $10 \frac{1}{2}$ | 97 | 163 | 240 | 21年 | 398 | 26 | 596 |
| I $\frac{1}{2}$ | 2.0 | II | 107 | 163 | 247 | 212 $\frac{1}{2}$ | 407 | $26 \frac{1}{4}$ | 607 |
| 2 | $3 \cdot 5$ | 119 $\frac{1}{2}$ | 117 | 17 | 255 | $21 \frac{3}{4}$ | 417 | $26 \frac{1}{2}$ | 619 |
| 2 $\frac{1}{2}$ | 5＊5 | 12 | 127 | $17 \frac{1}{4}$ | 262 | 22 | 427 | 26妥 | 631 |
| 3 | $7 \cdot 9$ | 122 $\frac{1}{2}$ | 138 | $17 \frac{1}{2}$ | 270 | $22 \frac{1}{4}$ | 436 | 27 | 643 |
| $3 \frac{1}{2}$ | $10 \cdot 8$ | 13 | 149 | $17 \frac{8}{4}$ | 278 | 22 $\frac{1}{2}$ | 446 | 27－1 | 655 |
| 4 | $14^{\circ} \mathrm{I}$ | 13 ${ }^{\frac{1}{4}}$ | 155 | 18 | 286 | 223 $\frac{3}{4}$ | 456 | $27 \frac{1}{2}$ | 667 |
| $4 \frac{1}{2}$ | 17.8 | 132 $\frac{1}{2}$ | 161 | $18 \frac{1}{4}$ | 294 | 23 | 466 | $27 \frac{3}{4}$ | 679 |
| 5 | $22^{\circ} \mathrm{O}$ | 133 | 167 | 181 $\frac{1}{2}$ | 302 | $23 \frac{1}{4}$ | 476 | 28 | 691 |
| 5 $\frac{1}{2}$ | $26 \cdot 6$ | 14 | 173 | 188 | 310 | 23砍 | 486 | $28 \frac{1}{4}$ | 703 |
| 6 | $31 \cdot 7$ | $14 \frac{1}{4}$ | 179 | 19 | 318 | $23 \frac{3}{4}$ | 497 | $28 \frac{1}{2}$ | 716 |
| $6 \frac{1}{2}$ | $37^{\circ} 2$ | 14 ${ }^{\frac{1}{2}}$ | 185 | $19 \frac{1}{4}$ | 327 | 24 | 507 | $28 \frac{3}{4}$ | 729 |
| 7 | $43^{\circ} \mathrm{I}$ | $14 \frac{3}{4}$ | 192 | $19 \frac{1}{2}$ | 335 | $24 \frac{1}{4}$ | 518 | 29 | 741 |
| $7 \frac{1}{2}$ | 49.6 | 15 | 198 | $19{ }^{\frac{2}{4}}$ | $3+4$ |  | 529 | $29 \frac{1}{4}$ | 754 |
| 8 | 56.4 | $15 \frac{1}{4}$ | 205 | 20 | 353 | $24 \frac{3}{4}$ | 540 | $29 \frac{1}{2}$ | 767 |
| $8 \frac{1}{2}$ | $63 \cdot 7$ | 15 $\frac{1}{2}$ | 212 | $20 \frac{1}{4}$ | 36 r | 25 | 551 | $29 \frac{3}{4}$ | 780 |
| 9 | 71 | 15 ${ }^{\text {星 }}$ | 219 | $20 \frac{1}{2}$ | 370 | $25 \frac{1}{4}$ | 562 | 30 | 793 |
| $9 \frac{1}{2}$ | 79 | 16 | 226 | $20 \frac{8}{4}$ | 379 | $25 \frac{1}{2}$ | 573 |  |  |
| 10 | 88 | 164 | 233 | 21 | 388 | $25 \frac{3}{4}$ | 584 |  |  |

Table O.-Angles subtended by various lengths at different distances.

|  | $\infty$ |  |
| :---: | :---: | :---: |
|  | - |  <br>  |
|  | $\checkmark$ | - $\dot{\circ} \dot{o} \dot{o} \dot{o} \dot{o} \dot{o} \dot{o} \dot{o} \dot{\mathrm{o}} \dot{\mathrm{i}} \dot{\mathrm{i}} \dot{\mathrm{i}} \dot{\mathrm{i}}$ |
|  | 製 |  |
|  | in |  |
|  | 薙 |  |
|  | + | $=\infty$ ¢ |
|  | - ${ }^{\text {m }}$ |  |
|  | m |  |
|  | - | $=\ddagger$ - |
|  | $\sim$ |  <br> - $\dot{\circ} \dot{o} \dot{o} \dot{i} \dot{i} \dot{i} \dot{i} \dot{a} \dot{\operatorname{con}}$ in in in $\dot{j}$ |
|  | $\stackrel{\text { gra }}{\sim}$ |  |
|  | + |  |
|  | + |  |
|  | - |  <br>  |
|  | n* |  <br> - $\dot{o} \dot{-} \dot{i}$ in in in $\dot{i} \dot{i} \dot{\infty} \dot{o} \dot{\sigma} \dot{b} \dot{b}$ |
|  | ** |  |
|  | $\dagger$ |  |
|  |  |  |

Table P.-Table of Distances at which Objects can be Seen at Sea, according to their respective Elevations and the Elevation of the Eye of the Observer.

| $\begin{gathered} \text { Height } \\ \text { in } \\ \text { Feet. } \end{gathered}$ | Distance in English or Statute Miles. | Distance in Geographical or Nautical Miles. | Height in Feet. | Distance in English or Statute Miles. | Distance in Geographical or Nautical Miles. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | 2.958 | 2.565 | 100 | $13 \cdot 228$ | 11.47 |
| 10 | $4 \cdot 184$ | $3 \cdot 628$ | 110 | $13 \cdot 874$ | 12.03 |
| 15 | $5 \cdot 123$ | $4 \cdot 443$ | 120 | 14.490 | - 12.56 |
| 20 | 5'916 | $5 \cdot 130$ | 130 | $15 \cdot 083$ | 13.08 |
| 25 | 6.614 | 5•736 | 140 | 15652 | 13.57 |
| 30 | $7 \cdot 245$ | $6 \cdot 283$ | 150 | 16.201 | $14 \cdot 22$ |
| 35 | $7 \cdot 826$ | $6 \cdot 787$ | 200 | 18.708 | $16 \cdot 22$ |
| $4^{\circ}$ | $8 \cdot 366$ | $7 \cdot 255$ | 250 | 20.916 | $18 \cdot 14$ |
| 45 | $8 \cdot 874$ | $7 \cdot 696$ | 300 | 22.912 | 19.87 |
| 50 | $9 \cdot 354$ | 8.112 | 350 | $24 \cdot 743$ | $21 \cdot 46$ |
| 55 | $9^{\cdot 811}$ | $8 \cdot 509$ | 400 | $26 \cdot 457$ | 22.94 |
| 60 | $10 \cdot 246$ | 8-886 | 450 | $28 \cdot 062$ | $24 \cdot 33$ |
| 65 | 10.665 | $9 \cdot 249$ | 500 | 29.580 | $25 \cdot 65$ |
| 70 | 1 1.067 | 9.598 | 550 | 36.024 | $26 * 90$ |
| 75 | 11.456 | $9 \cdot 935$ | 600 | 32.403 | $28 \cdot 10$ |
| 80 | 11.832 | $10 \cdot 26$ | 650 | $33 \cdot 726$ | 29.25 |
| 85 | 12.196 | 10.57 | 700 | $35^{\circ} 000$ | $30 \cdot 28$ |
| 90 | 12.549 | 10.88 | 800 | 37.416 | $32 \cdot 45$ |
| 95 | 12.893 | I 1 18 | 900 | 39.836 | $34 \cdot 54$ |
|  |  |  | 1000 | $41 \cdot 833$ | $36 \cdot 28$ |

Example.-A tower 150 feet high will be visible to an observer whose eye is elevated 15 feet ahove the water 19 nautical miles; thus, from the Table :

15 feet elevation distance visible $4^{\circ} 44$ nautical miles.

Table Q.-True Depression or Distance of the Sea Horizon.

| Height. | Dep. | Square. | Height. | Dep. | Square. | Height. | Dep. | Square. | Dep. | Square. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ft |  |  | ft . |  |  | ft . | , |  |  |  |
| 1.1 | 1 | 1 | 3293 | 6 r | 372 I | 12966 | 12 I | 14641 | 181 | 32761 |
| $3 \cdot 5$ | 2 | 4 | 3403 | 62 | 3844 | 13183 | 122 | 14884 | 182 | 33124 |
| 8.0 | 3 | 9 | 3513 | 63 | 3969 | 13397 | 123 | 15129 | 183 | 33489 |
| 14.2 | 4 | 16 | 3924 | 64 | 4096 | 13615 | 124 | 15376 | 184 | 33856 |
| $22^{\circ} \mathrm{I}$ | 5 | 25 | 3740 | 65 | 4225 | 13836 | 125 | 15625 | 185 | 34225 |
| 31.9 | 6 | 36 | 3855 | 66 | 4356 | 14061 | 126 | 15876 | 186 | 34596 |
| $43 \cdot 3$ |  | 49 | 3974 | 67 | 4489 | 14282 | 127 | 16129 | 187 | 34969 |
| $56 \cdot 6$ | 8 | 64 | 4093 | 68 | 4624 | 14502 | 128 | 16384 | 188 | 35344 |
| 71*7 | 9 | 81 | 4213 | 69 | 4761 | 14737 | 129 | 1664 r | 189 | 3572 I |
| 88.5 | 10 | 100 | 4337 | 70 | 4900 | 14970 | I30 | 16900 | 190 | 36100 |
| 107 | 11 | 12 I | 4461 | 71 | 5041 | 15197 | 131 | 17561 | 191 | 36481 |
| 127 | 12 | 144 | 4587 | 72 | 5184 | 15429 | 132 | 17424 | 192 | 36864 |
| 149 | 13 | 169 | 4716 | 73 | 5329 | 15664 | 133 | 17689 | 193 | 37249 |
| 174 | 14 | 196 | 4846 | 74 | 5476 | 15901 | 134 | 17956 | 194 | 37636 |
| 199 | 15 | 225 | 4976 | 75 | 5625 | 16139 | 135 | 18225 | 195 | 38025 |
| 226 | 16 | 256 | 5112 | 76 | 5776 | 16380 | I36 | 18496 | 196 | 38416 |
| 256 | 17 | 289 | 5249 | 77 | 5929 | 16622 | 137 | 18769 | 197 | 38809 |
| 287 | 18 | 324 | 5385 | 78 | 6084 | 16866 | 138 | 19044 | 198 | 39204 |
| 319 | 19 | 361 | 5524 | 79 | 6241 | 17111 | 139 | 1932I | 199 | 39601 |
| 354 | 20 | 400 | 5665 | 80 | 6400 | 17362 | 140 | 19600 | 200 | 40000 |
| 390 | 21 | 441 | 5808 | 81 | 6561 | 17608 | 141 | 19881 | 201 | 40401 |
| 428 | 22 | 484 | 5952 | 82 | 6724 | 17860 | 142 | 20164 | 202 | 40804 |
| 468 | 23 | 529 | 6098 | 83 | 6889 | 18111 | 143 | 20449 | 203 | 41209 |
| 510 | 24 | 576 | 6246 | 84 | 7056 | 18366 | 144 | 20736 | 204 | 41616 |
| 550 | 25 | 625 | 6394 | 85 | 7225 | 18622 | 145 | 21025 | 205 | 42025 |
| 598 | 26 | 676 | 6547 | 86 | 7396 | 18878 | 146 | 21316 | 206 | 42436 |
| 645 | 27 | 739 | 6700 | 87 | 7569 | 19140 | 147 | 21609 | 207 | 42849 |
| 694 | 28 | 784 | 6855 | 88 | 7744 | I9401 | 148 | 21904 | 208 | 43264 |
| 744 | 29 | 84 I | 7012 | 89 | 7921 | 19664 | 149 | 22201 | 209 | 43681 |
| 797 | 30 | 900 | 7172 | 90 | 8100 | 19930 | 150 | 22500 | 210 | 44100 |
| 850 | 31 | 961 | 7332 | 91 | 8281 | 20197 | 151 | 22801 | 211 | 44521 |
| 906 | 32 | 1024 | 7492 | 92 | 8464 | 20465 | 152 | 23104 | 212 | 44944 |
| 964 | 33 | 1089 | 7656 | 93 | 8649 | 20736 | 153 | 23429 | 213 | 45369 |
| 1023 | 34 | 1156 | 7824 | 94 | 8836 | 21008 | 154 | 23716 | 214 | 45796 |
| 1084 | 35 | 1225 | 7987 | 95 | 9025 | 21282 | 155 | 24025 | 215 | 46225 |
| 1147 | 36 | 1296 | 8158 | 96 | 9216 | 21558 | 156 | 24336 | 216 | 46656 |
| 1211 | 37 | 1369 | 8330 | 97 | 9409 | 21836 | 157 | 24649 | 217 | 47089 |
| 1278 | 38 | 1444 | 8504 | 98 | 9604 | 22115 | 158 | 24964 | 218 | 47524 |
| 1346 | 39 | 1521 | 8678 | 99 | 9801 | 22397 | 159 | 25281 | 219 | 47961 |
| 1416 | 40 | 1600 | 8852 | 100 | 10000 | 22680 | 160 | 25600 | 220 | 48400 |
| 1487 | 41 | 1681 | 9032 | 101 | 10201 | 22964 | 161 | 2592 I | 221 | 4884 I |
| 1561 | 42 | 1764 | 92 IO | 102 | 10404 | 23251 | 162 | 26244 | 222 | 49284 |
| 1636 | 43 | 1849 | 9393 | 103 | 10609 | 23540 | 163 | 26569 | 223 | 49729 |
| 1713 | 44 | 1936 | 9577 | 104 | 10816 | 23830 | 164 | 26896 | 224 | 50176 |
| 1792 | 45 | 2025 | 9760 | 105 | 11025 | 24121 | 165 | 27225 | 225 | 50625 |
| 1872 | 46 | 2116 | 9951 | 106 | 11236 | 24415 | 166 | 27556 | 226 | 51076 |
| 1954 | 47 | 2209 | 10135 | 107 | 11449 | 24711 | 167 | 27889 | 227 | 51529 |
| 2039 | 48 | 2304 | 10325 | 108 | 11664 | 25008 | 168 | 28224 | 228 | 51984 |
| 2124 | 49 | 2401 | 10518 | 109 | 1188 t | 25307 | 169 | 28561 | 229 | 5244 I |
| 2212 | 50 | 2500 | 10712 | 110 | 12100 | 25608 | 170 | 28900 | 230 | 52900 |
| 2301 | 51 | 2601 | 10908 | III | 12321 | 25911 | 171 | 2924 I | 231 | 53361 |
| 2393 | 52 | 2704 | 11105 | 112 | 12544 | 26215 | 172 | 29584 | 232 | 53824 |
| 2485 | 53 | 2809 | 11304 | 113 | 12769 | 2652 I | 173 | 29929 | 233 | 54289 |
| 2581 | 54 | 2916 | 11506 | 114 | 12996 | 26829 | 174 | 30276 | 234 | 54756 |
| 2677 | 55 | 3025 | 11700 | 115 | 13225 | 27139 | 175 | 30625 | 235 | 55225 |
| 2775 | 56 | 3136 | 11913 | 116 | 13465 | 27451 | 176 | 30976 | 236 | 55696 |
| 2875 | 57 | 3246 | 12120 | 117 | 13689 | 27764 | 177 | 31329 | 237 | 56169 |
| 2977 | 58 | 3364 | 12328 | 118 | 13924 | 28079 | 178 | 31684 | 238 | 56644 |
| 3081 | 59 | 3485 | 12538 | 119 | 14161 | 28396 | - 79 | 3204 I | 239 | 57121 |
| 3186 | 60 | 3600 | 12749 | 120 | 14400 | 28715 | 180 | 32400 | 240 | 57600 |

Table R.-Angles subtended at different distances by a pole ten feet in length.

| Yds. | Angle, |  |  | Yds. | Angle. |  |  | Yds. | Angle. |  |  | Yds. | Angle. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | , | " |  | - | , | " |  | $\bigcirc$ | , | " |  | - | , | " |
| 17 | 1 I | 25 | 20 | 92 | 2 | 05 | 00 | 167 |  | 08 | 46 | 242 | - | 47 | 26 |
| 18 | 10 | 23 | 20 | 93 | 2 | 02 | 50 | 168 | I | 08 | 00 | 243 | - | 47 | 08 |
| 20 | 9 | 00 | 00 | 95 | 2 | 00 | 40 | 170 |  | 07 | 24 | 245 | - | 46 | 48 |
| 22 | 8 | 47 | 48 | 97 | I | 58 | 30 | 172 | I | 06 | 44 | 247 | - | 46 | 28 |
| 23 | 8 | 10 | 18 | 98 | I | 56 | 30 | 173 | 1 | 06 | 04 | 248 | $\bigcirc$ | 46 | 08 |
| 25 | 7 | 37 | 54 | IOI | I | 54 | 30 | 175 |  | 05 | 30 | 250 | - | 45 | 48 |
| 27 | 7 | 09 | 10 | 102 | 1 | 52 | 40 | 177 | I | 04 | 40 | 252 | - | 45 | 30 |
| 28 | 6 | 45 | 48 | 103 | 1 | 50 | 54 | 178 | I | 04 | 16 | 253 | O | 45 | 12 |
| 30 | 6 | 2 I | 36 | 105 | I | 49 | 00 | 180 | 1 | 03 | 40 | 255 | $\bigcirc$ | 44 | 54 |
| 32 | 6 | OI | 32 | 107 | I | 47 | 20 | 182 | 1 | 03 | 04 | 257 | - | 44 | 38 |
| 33 | 5 | 43 | 28 | 108 | I | 45 | 44 | 183 | I | 02 | 30 | 258 | - | 44 | 20 |
| 35 | 5 | 27 | 10 | 110 | I | 44 | 10 | 185 | 1 | OI | 56 | 260 | - | 44 | 02 |
| 37 | 5 | 12 | 20 | 112 | 1 | 42 | 20 | 187 | I | 01 | 22 | 262 | - | 43 | 50 |
| 38 | 4 | 58 | 44 | 113 | I | 41 | 00 | 188 | 1 | -0 | 50 | 263 | - | 43 | 36 |
| 40 | 4 | 46 | 16 | 115 | 1 | 39 | 40 | 190 | I | 00 | 18 | 265 | - | 43 | 14 |
| 42 | 4 | 34 | 50 | 117 | 1 | 38 | 14 | 192 | - | 59 | 46 | 267 | - | 43 | 00 |
| 43 | 4 | 24 | 20 | 118 | 1 | 36 | 50 | 193 | $\bigcirc$ | 59 | 16 | 268 | 0 | 42 | 42 |
| 45 | 4 | 14 | 00 | 120 | 1 | 35 | 30 | 195 | - | 58 | 46 | 270 | - | 42 | 26 |
| 47 | 4 | 05 | 30 | 122 | I | 34 | 10 | 197 | 0 | 58 | 16 | 272 | - | 42 | 11 |
| 48 | 3 | 57 | 00 | 123 | I | 32 | 50 | 198 | - | 57 | 48 | 273 |  | 41 | 55 |
| 50 | 3 | 49 | 06 | 125 | 1 | 31 | 40 | 200 | $\bigcirc$ | 57 | 18 | 275 | 0 | 41 | 40 |
| 52 | 3 | 41 | 42 | 127 | 1 | 30 | 30 | 202 | - | 56 | 48 | 277 |  | 4 I | 25 |
| 53 | 3 | 34 | 50 | 128 | 1 | 29 | 16 | 203 | 0 | 56 | 20 | 278 | - | 41 | 10 |
| 55 |  | 28 | 20 | 130 | 1 | 28 | 10 | 205 | - | 55 | 54 | 280 | 0 | 40 | 55 |
| 57 | 5 | 22 | 10 | 132 | 1 | 27 | 00 | 207 | $\bigcirc$ | 55 | 26 | 382 | - | 40 | 4 t |
| 58 | 3 | 16 | 24 | 133 | 1 | 26 | 00 | 208 | $\bigcirc$ | 55 | 00 | 283 | - | 40 | 26 |
| 60 | 3 | 10 | 50 | 135 | 1 | 24 | 50 | 210 | 0 | 54 | 34 | 285 | - | 40 | 12 |
| 62 | 3 | 05 | 40 | 137 | I | 23 | 50 | 212 | $\bigcirc$ | 54 | 16 | 287 | - | 39 | 58 |
| 63 | 3 | Of | 00 | 138 | 1 | 22 | 50 | 213 | $\bigcirc$ | 54 | 00 | 288 |  | 39 | 44 |
| 65 | 2 | 56 | 14 | 140 | 1 | 25 | 50 | 215 | $\bigcirc$ | 53 | 26 | 290 | 0 | 39 | 30 |
| 67 | 2 | 51 | 40 | 142 | I | 20 | 50 | 217 | 0 | 52 | 54 | 292 |  | 39 | 17 |
| 68 | 2 | 47 | 40 | 143 | I | 20 | 00 | 218 | $\bigcirc$ | 52 | 28 | 293 | 0 | 39 | 04 |
| 70 | 2 | 43 | 40 | 145 | I | 19 | $\bigcirc$ | 220 | 0 | 52 | 04 | 295 |  | 38 | 50 |
| 72 | 2 | 40 | -0 | 147 | 1 | 18 | 08 | 222 | - | 51 | 42 | 297 | 0 | 38 | 37 |
| 73 | 2 | 36 | 14 | 148 | 1 | 17 | 16 | 223 | O | 51 | 20 | 298 |  | 38 | 24 |
| 75 | 2 | 32 | 44 | 150 | 1 | 16 | 22 | 225 | 0 | 50 | 56 | 300 | 0 | 38 | 11 |
| 77 | 2 | 29 | 30 | 152 | I | 15 | 30 | 227 | 0 | 50 | 34 | 302 |  | 37 | 59 |
| 78 | 2 | 26 | 20 | 153 | 1 | 14 | 44 | 228 | - | 50 | 12 | 303 | 0 | 37 | 47 |
| 80 | 2 | 23 | 14 | 155 | 1 | 14 | $\bigcirc 0$ | 230 |  | 49 | 50 | 305 |  | 37 | 34 |
| 82 | 2 | 20 | 16 | 157 | 1 | 13 | 08 | 232 |  | 49 | 28 | 307 |  | 37 | 22 |
| 83 |  | 17 | 30 | 158 | 1 | 12 | 20 | 233 |  | 49 | 06 | 308 |  | 37 | 10 |
| 85 | 2 | 14 | 50 | 160 | 1 | 11 | 40 | 235 | 0 | 48 | 44 | 310 | 0 | 36 | 58 |
| 87 | 2 | 12 | 10 | 162 | 1 | 11 | 00 | 237 | 0 |  | 22 | 312 |  | 36 | 46 |
| 88 | 2 | 09 | 44 | 163 | I | 10 | 10 | 238 |  | $4^{8}$ | 02 | 313 | - | 36 | 34 |
| 90 | 2 | 07 | 20 | 165 | 1 | 09 | 30 | 240 | $\bigcirc$ | 47 | 44 | 315 | $\bigcirc$ | 36 | 23 |

Table S.-For converting Intervals of Time or Longitude into Decimals of a Day.

| Long. | Time, | Decimals of a Day. | Long. | Time. | Decimals of a Day. | Long. | Time. | Decimals of a Day. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - | ${ }^{\text {b }}$ |  | 0 - | m |  | 0 , | m |  |
| 15 | 1 | -0417 | $\bigcirc 15$ | 1 | -0007 | 745 | 31 | -0215 |
| 30 | 2 | . 0833 - | - 30 | 2 | -0014 | 80 | 32 | -0222 |
| 45 | 3 | -1250 | - 45 | 3 | -0021 | 815 | 33 | - 0222 |
| 60 | 4 | -1667 | 10 | 4 | -0028 | 830 | 34 | . 0236 |
| 75 | 5 | 2083 | I 15 | 5 | . 0035 | 845 | 35 | -0243 |
| 90 | 6 | - 2500 | 130 | 6 | -0042 | 90 | 36 | . 0250 |
| 105 | 7 | - 2917 | 145 | 7 | - 0049 | 915 | 37 | . 0257 |
| 120 | 8 | -3333 | 20 | 8 | -0056 | 930 | 38 | - 0264 |
| 135 | 9 | - 3750 | 215 | 9 | - 0062 | 945 | 39 | . 0271 |
| 150 | 10 | $\cdot 4167$ | 230 | 10 | - 2069 | 100 | 40 | . 0278 |
| 165 | II | - 4583 | 245 | II | -0076 | 1015 | 41 | - 0285 |
| 180 | 12 | - 5000 | 30 | 12 | $\cdot .0083$ | 1030 | 42 | - 0292 |
| 195 | 13 | * 5417 | 315 | 13 | -0090 | 1045 | 43 | - 0299 |
| 210 | 14 | - 5833 | 330 | 14 | -0097 | 110 | 44 | -0306 |
| 225 | 15 | -6250 | 345 | 15 | -0104 | 1115 | 45 | .0312 |
| 240 | 16 | -6667 | 40 | 16 | - Orit | 1130 | 46 | .0319 |
| 255 | 17 | $\cdot 7083$ | 415 | 17 | -0118 | if 45 | 47 | . 0326 |
| 270 | 18 | - 7500 | 430 | 18 | . 0125 | 120 | 48 | . 0333 |
| 285 | 19 | ${ }^{*} 7917$ | 445 | 19 | -0132 | $12 \quad 15$ | 49 | . 0340 |
| 300 | 20 | 8333 | 50 | 20 | - 0139 | 1230 | 50 | -0347 |
| 315 | 21 | -8750 | 515 | 21 | . 0146 | 1245 | 51 | -0354 |
| 330 | 22 | '9167 | 530 | 22 | -0153 | 130 | 52 | . 036 r |
| 345 | 23 | -9583 | 545 | 23 | - 0160 | 1315 | 53 | -0368 |
| 360 | 24 | I'0000 | 6 - | 24 | . 0167 | 1330 | 54 | . 0375 |
|  |  |  | 615 | 25 | - 0174 | 1345 | 55 | -0382 |
|  |  |  | 630 | 26 | -0181 | 140 | 56 | -0389 |
|  |  |  | 645 | 27 | -0187 | 1415 | 57 | - 0396 |
|  |  |  | 70 | 28 | - 0194 | 1430 | 58 | -0403 |
|  |  |  | 715 | 29 | . O 201 | 1445 | 59 | - 0410 |
|  |  |  | 730 | 30 | - 0208 | 15 - | 60 | -0417 |

From Shaduelps "Chronometers."

Table T.-Metrical and English Barometers.

| Barometer Scales. |  | Barometer Scales. |  | Barometer Scales. |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Fr. Mill. | Eng. In. | Fr. Mill. | Eng. In. | Fr. Mill. | Eng. In. |
| 640 | $25 \cdot 2$ | 691 | $27 \cdot 2$ | 742 | $29^{\circ} 2$ |
| 643 | $25 \cdot 3$ | 693 | $27 \cdot 3$ | 744 | $29^{*} 3$ |
| 645 | 25.4 | 696 | 27.4 | 747 | 29.4 |
| 648 | $25 \cdot 5$ | 698 | 27.5 | 749 | 29.5 |
| 650 | $25 \cdot 6$ | 701 | 27.6 | 752 | 29.6 |
| 653 | $25 \cdot 7$ | 704 | $27 \cdot 7$ | 754 | 29.7 |
| 655 | $25 \cdot 8$ | 706 | $27 \cdot 8$ | 757 | 29.8 |
| 658 | 25.9 | 709 | 27.9 | 759 | 29.9 |
| 660 | $26 \cdot 0$ | 711 | 28.0 | 762 | 30.0 |
| 663 | $26 \cdot 1$ | 714 | $28 \cdot 1$ | 765 | $30^{\circ} \mathrm{I}$ |
| 665 | $26 \cdot 2$ | 716 | $28 \cdot 2$ | 767 | $30 \cdot 2$ |
| 668 | $25 \cdot 3$ | 719 | $28 \cdot 3$ | 770 | $30 \cdot 3$ |
| 670 | $26 \cdot 4$ | 721 | 28.4 | 772 | $30 \cdot 4$ |
| 673 | $26 \cdot 5$ | 724 | 28.5 | 775 | $30 \cdot 5$ |
| 676 | 26.6 | 726 | $28 \cdot 6$ | 777 | $30 \cdot 6$ |
| 678 | $26 \cdot 7$ | 729 | 28.7 | 780 | $30 \cdot 7$ |
| 681 | $26 \cdot 8$ | 732 | 28.8 | 782 | $30 \cdot 8$ |
| 683 | $26 \cdot 9$ | 734 | 28.9 | 785 | $30 \cdot 9$ |
| 686 | 27.0 | 737 | $29^{\circ} 0$ | 787 | $31^{\circ} 0$ |
| 688 | $27^{1} 1$ | 739 | $29^{\circ} \mathrm{I}$ |  |  |

Table U.-Corresponding 'Thermometers, Fahrenheit, Centigrade, Réaumur.

| F. | C. | R. | F. | C. | R. | F. | C. | K. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - |  | $\bigcirc$ | - | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | - | - |
| $\bigcirc$ | -17.8 | - 14.2 | 41 | $5^{\circ} 0$ | $4^{\circ} 0$ | 81 | $27 \cdot 2$ | $2 \mathrm{~F} \cdot 8$ |
| I | - $17{ }^{\circ} 2$ | -13.8 | 42 | $5 \cdot 6$ | $4 \cdot 4$ | 82 | $27 \cdot 8$ | 22.2 |
| 2 | - $16 \cdot 7$ | -13.3 | 43 | $6 \cdot 1$ | $4 \cdot 9$ | 83 | $28 \cdot 3$ | $22^{\circ} 7$ |
| 3 | - $16 \cdot 1$ | - 12.9 | 44 | $6 \cdot 7$ | $5 \cdot 3$ | 84 | 28.9 | $23^{\circ} 1$ |
| 4 | -15.6 | - 12.4 | 45 | $7 \cdot 2$ | $5 \cdot 8$ | 85 | 29.4 | 23.6 |
| 5 | -15 ${ }^{\circ}$ | - 12.0 | 46 | $7 \cdot 8$ | $6 \cdot 2$ | 86 | $30^{\circ} 0$ | $24^{\circ} 0$ |
| 6 | -14.4 | - 11.6 | 47 | $8 \cdot 3$ | $6 \cdot 7$ | 87 | 30*6 | $24^{\circ} 4$ |
| 7 | -13.9 | - II' 1 | 48 | $8 \cdot 9$ | $7{ }^{\circ} 1$ | 88 | $3 \mathrm{I}^{\circ} \mathrm{I}$ | 24.9 |
| 8 | - 13.3 | - 10.7 | 49 | $9 \cdot 4$ | $7 \cdot 5$ | 89 | $31 \cdot 7$ | $25^{\prime} 3$ |
| 9 | - 12.8 | - 10.2 | 50 | 10\% | $8 \cdot 0$ | 90 | $32 \cdot 2$ | $25^{\circ} 8$ |
| 10 | - 12.2 | - 9.8 | 51 | 10.6 | $8 \cdot 4$ | 91 | 32.8 | $26 \cdot 2$ |
| II | - 11.7 | - $9 \cdot 3$ | 52 | II' I | $8 \cdot 9$ | 92 | $33^{\circ} 3$ | 26.7 |
| 12 | - II'r | - 8.9 | 53 | 11'7 | $9^{*} 3$ | 93 | $33^{\circ} 9$ | $27^{\circ} \mathrm{I}$ |
| 13 | - 10.6 | - 8.4 | 54 | 12.2 | $9 \cdot 8$ | 94 | 34.4 | $27^{\circ} 6$ |
| 14 | - 10.0 | - 8.0 | 55 | 12.8 | $10^{\circ} 2$ | 95 | $35^{\circ} 0$ | 28.0 |
| 15 | - 9.4 | - 7.5 | 56 | 13.3 | 10.7 | 96 | $35^{\circ} 6$ | 28.4 |
| 16 | - 8.9 | - $7 \cdot 1$ | 57 | 13*9 | II ${ }^{\prime}$ | 97 | 36.1 | 28.9 |
| 17 | - $8 \cdot 3$ | - $6 \cdot 7$ | 58 | $14^{\circ} 4$ | II. 6 | 98 | $36 \cdot 7$ | $29^{\circ} 3$ |
| 18 | - $7 \cdot 8$ | - $6 \cdot 2$ | 59 | $15^{\circ} \mathrm{O}$ | 12.0 | 99 | $37 \cdot 2$ | 29.8 |
| 19 | - 7.2 | - $5 \cdot 8$ | 60 | $15 \cdot 6$ | 12.4 | 100 | $37 \cdot 8$ | $30 \cdot 2$ |
| 20 | - $6 \cdot 7$ | - 5.3 | 61 | 16.1 | 12.9 | IOI | $38 \cdot 3$ | $30 \cdot 7$ |
| 21 | - 6.1 | - 4.9 | 62 | $16 \cdot 7$ | 13.3 | 102 | $38 \cdot 9$ | $31^{\circ} \mathrm{I}$ |
| 22 | - 5.6 | - 4.4 | 63 | $17 \cdot 2$ | 13.8 | 103 | $39^{\circ} 4$ | $31 \cdot 6$ |
| 23 | - 5.0 | - $4 \cdot 0$ | 64 | 17.8 | $14^{\prime} 2$ | 104 | $40^{\circ} 0$ | 32.0 |
| 24 | - 4.4 | - 3.6 | 65 | 18.3 | 14.7 | 105 | $40 \cdot 6$ | 32.4 |
| 25 | - 3.9 | - 3.1 | 66 | 18.9 | $15^{\circ} \mathrm{I}$ | 106 | $4 \mathrm{I}^{\prime} \mathrm{I}$ | 32.9 |
| 26 | - 3.3 | - 2.7 | 67 | 19.4 | 15.6 | 107 | 41'7 | $33^{\prime} 3$ |
|  | - 2.8 | - 2.2 | 68 | 20.0 | 16.0 | 108 | $42 \cdot 2$ | $33^{\circ} 8$ |
| 28 | - 2.2 | - 1.8 | 69 | $20 \cdot 6$ | 16.4 | 109 | $42 \cdot 8$ | $34^{\circ} 2$ |
| 29 | - I.7 | - $1 \cdot 3$ | 70 | $2 \mathrm{I}^{1} \mathrm{I}$ | 16.9 | 110 | $43 \cdot 3$ | $34^{\circ} 7$ |
| 30 | - I. 1 | - 0.9 | 71 | 21.7 | 17.3 | III | $43^{\prime} 9$ | $35^{\circ} \mathrm{I}$ |
| 31 | - 0.6 | - 0.4 | 72 | 22.2 | 17.8 | 112 | $44^{\prime} 4$ | $35^{\circ} 5$ |
| 32 | $\bigcirc$ | - | 73 | 22.8 | 18.2 | 113 | $45^{\circ} 0$ | $36^{\circ} \mathrm{O}$ |
| 33 | 0.6 | 0.4 | 74 | $23^{\prime} 3$ | 18.7 | 114 | $45^{\circ} 6$ | 36.4 |
| 34 | I'I | $0 \cdot 9$ | 75 | $23^{\circ} 9$ | $19^{\circ} \mathrm{I}$ | 115 | $46 \cdot 1$ | $36^{\circ} 9$ |
| 35 | 1.7 | I-3 | 76 | $24^{.4}$ | 19.6 | 116 | $46 \cdot 7$ | $37 \cdot 3$ |
| 36 | $2 \cdot 2$ | I-8 | 77 | $25^{\circ} 0$ | 20.0 | 117 | $47^{\circ} 2$ | 37.8 |
| 37 | $2 \cdot 8$ | $2 \cdot 2$ | 78 | $25^{\circ} 6$ | 20.5 | 118 | $47^{\circ} 8$ | $38 \cdot 2$ |
| 38 | $3 \cdot 3$ | $2 \cdot 7$ | 79 | $26 \cdot 1$ | 20.9 | 119 | $48 \cdot 3$ | $38 \cdot 7$ |
| 39 | $3.9$ | $3 \cdot 1$ | 80 | $26 \cdot 7$ | $21 \cdot 3$ | 120 | $48 \cdot 9$ | $39^{\circ} 1$ |
| 40 | $4 \cdot 4$ | $3 \cdot 6$ |  |  |  |  |  |  |

Table V.-Measures used to express depths in Foreign Charts.

| National Measure. |  | Eng. Feet. | Eng. Fathoms. |
| :--- | :--- | :---: | :---: |
| French | Metre | 3.281 | 0.5468 |
|  | Brasse | 5.329 | 0.888 I |
| Spanish | Braza | 5.492 | 0.9153 |
| Swedish | Fomn | 5.843 | 0.974 |
| Danish | Favn | 6.175 | 1.0292 |
| Norwegian | $\#$ | 6.175 | 1.0292 |
| German | Faden | 5.906 | 0.984 |
| Dutch | Vaden | 5.575 | 0.929 |
| Russian | Marine Sashine | 6.000 | 1.000 |
| Portuguese | Braca | 6.004 | $\mathbf{1 . 0 0 0}$ |

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[^0]:    JOHN MURRAY, ALBEMARLE STREET. 1898.

[^1]:    * "Mathematical Instruments." J. F. Heather, M.A. Lockwood and Co., London.

[^2]:    * The up-and-down piece of a sextant is the portion that bears the collar for the telescope.

[^3]:    * From Heather.

[^4]:    * Appendix, Table S.

[^5]:    * " Notes on Management of Chronometers and Measurement of Meridian Distances," by Captain C. Shadwell, C.B. Potter, London, 1861.

[^6]:    * See Appendix F.

[^7]:    * Appendix, Table 0.

[^8]:    * The rule where sines only are involved must be used.

[^9]:    * See following article for application of Conrergency.

[^10]:    * Vide proof of this rule in Appendix C.

[^11]:    * Appendix J.

[^12]:    * Communicated by Captain A. M. Field.

[^13]:    * Appendix R.

[^14]:    * Experience gained from recent extensive sounding operations $a_{i}$ pears to indicate that this angle of slope to great depths does not actually occur, and that a base of ten miles may be pretty safely assumed in the depth given.

[^15]:    Definitions.
    "Pise" of a tide is the height of the high-water level above the low spring datum.
    "Range" is the difference between the height of high and

[^16]:    "The observations may be regarded as forming three groups of three each, the member of each group being separated by

[^17]:    * These are respectively compiled and devised by Lieut. H. B. T. Somerville and Commander Purey Cust, and much facilitate the selection of stars.

[^18]:    * Vide Artificial Forizon, lage 13.

[^19]:    * Some officers now fit their sextants with small electric lights as a luxuy.

[^20]:    * Some observers fiud a small level fitted to the index bar, and another on the arm of the sextant stand, a great convenience in setting a sextant to the right star.

[^21]:    * For proof, see Appendix D.

[^22]:    To the sextant, telegraph, and chronometers, therefore, our Remarks remarks will be confined.

    By the use of the two latter we obtain only the "difference of longitude," or "meridian distance," between two places, neither of which may have its meridian distance from the confined to Diffe-
    rential Longitude and Sextant. primary meridian of Greenwich determined; but by the accumulation of such observations, the absolute longitudes of certain places are from time to time decided.

    These, then, become secondary meridians, on which the Secondary

[^23]:    * This method of Interpolation is not recognised as being as valuable as I believe it to be, and the
    remarks on it must be taken as my private opinion only.-W. J. L. W.

[^24]:    * Ex-Meridian Altitude Tables and New Navigation, by C. Brent, R.N., A. F. Walton, R.N., and G. Williams, R.N. Geo. Philip \& Son, London, 1886.

[^25]:    * Vide "Instructions for Hydrographic Surveyors" for useful hints.

[^26]:    * Page 88.

[^27]:    * See Appendix B.

[^28]:    * This chapter is entirely from information supplied by Captain A. M. Field, R.N., supplemented by notes from Captain W. U. Moore, R.N.

[^29]:    * Observation of Currents in Dardanelles and Bosporus.

