SYMMETRICAL
MASONRY ARCHES

INCLUDING

NATURAL STONE, PLAIN-CONCRETE, AND
REINFORCED-CONCRETE ARCHES

FOR THE USE OF TECHNICAL SCHOOLS, ENGINEERS, AND
COMPUTERS IN DESIGNING ARCHES ACCORDING
TO THE ELASTIC THEORY

BY

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PREFACE.

THE object of this book is to present in a simple form the method to be employed in the designing of masonry arches according to the *elastic theory*.

The entire subject of arches has been fully treated in the author's Treatise on Arches, in which formulas for special cases and conditions are given. Considering the fact that masonry arches are constructed of materials and under conditions which are more or less uncertain in character, the use of comprehensive or rigid formulas is not necessary or warranted. Consequently the formulas and methods here presented are somewhat approximate, but quite accurate enough for the purpose for which they are intended.

The greater portion of the book is taken up with the solution of examples, giving each step in detail so as to be easily followed by the undergraduate or the engineer who has not the time to review the theory of arches in a comprehensive manner.

The first and second examples have been solved by a somewhat longer method than necessary. This method was used in order to show clearly the several processes and checks.

In the third example will be found the simplest solution of the formulas for the horizontal thrusts and bending moments at the supports presented up to this time.

The numerical and graphical work has been given with such discrepancies as may be expected unless extraordinary care is exercised and many decimal places used. The discrepancies are of no practical importance, as the results are much nearer being exact than any masonry structure can be built, so as to fulfil the conditions upon which the calculations are based.

For the benefit of those who desire to follow precedents and as an aid in making preliminary calculations and estimates, the general data for over five hundred arch bridges have been given in tabular form with references to periodicals, etc., where more extended descriptions can be found Without any doubt many errors exist in this table, which is quite incomplete in some particulars. The data have been derived from many sources and in some cases supplied from drawings by scaling and in others by calculations.

M. A. H.

TERRE HAUTE, July 1906.

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NOMENCLATURE.

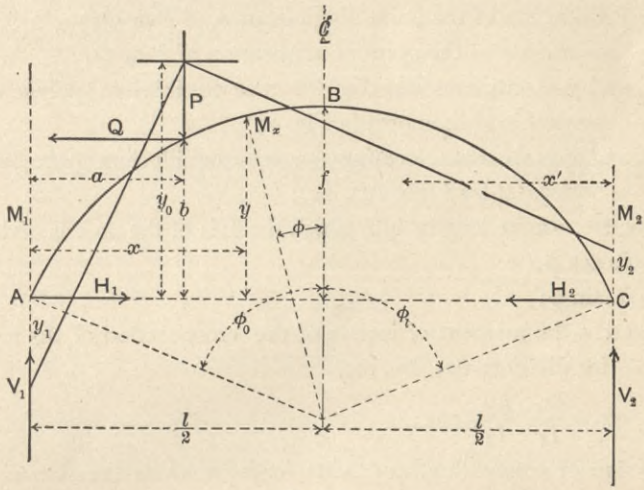

FIG. *a*.

H_1 = the horizontal thrust at the left support for any loading in general and in special formulas for vertical loads only;

h_1 = the horizontal thrust at the left support for horizontal loads only;

H_t = the horizontal thrust at the left support for changes of temperature;

H_a = the horizontal thrust at the left support produced by the axial stress;

M_1 = the moment at the left support;

M_2 = the moment at the right support;

M_x = the moment at any point having the coordinates x and y;

V_1 = the vertical reaction at the left support;

V_2 = the vertical reaction at the right support;

l = the span of the arch axis;

f = the rise of the arch axis;

x and y = the coordinates of any point of the arch axis;

$\phi_0 = -\phi_0$ = one half the total central angle subtended by the axis of the arch;

ϕ = the angular distance to the left of the crown of any point having the coordinates x and y;

P = any vertical load;

Q = any horizontal load;

a = the abscissa of the point of application of P or Q;

b = the ordinate of the point of application of P or Q;

y_1, y_0, and y_2 = ordinates locating the true equilibrium polygon for a vertical load as shown in Fig. a;

x_1, x_0, and x_2 = abscissas locating the true equilibrium polygon for a horizontal load (see Art. 22);

δs_1, δs_2, etc. = finite lengths into which the axis of the arch is divided;

$\delta x = \delta s \cos \phi$;

$\delta y = \delta s \sin \phi$;

I_1, I_2, etc. = the moment of inertia of the cross-section of the arch rib for divisions δs_1, δs_2, etc.

Δ_1, Δ_2, etc. $= \frac{\delta s_1}{I_1}, \frac{\delta s_2}{I_2}$, etc.;

Σ = sign of summation, and when without limits the sum is to be taken from 0 to l;

$\overset{x}{\Sigma}$ = sum from 0 to x;

E = the modulus of elasticity;

F = area of arch rib at any section;

e = coefficient of linear expansion for one degree;

t° = number of degrees change of temperature;

p = unit stress in extreme fibers of arch rib;

p' = unit stress in steel reinforcement in reinforced-concrete ribs.

$$y_1=\frac{M_1}{H_1}, \qquad y_2=\frac{M_2}{H_2}, \qquad y_0=\frac{M_1}{H_1}+\frac{V_1}{H_1}a;$$

$$x_1=\frac{M_1}{V_1}, \qquad x_2=\frac{M_2}{V_2}, \qquad x_0=\left(b-\frac{M_1}{h_1}\right)\frac{h_1}{V_1};$$

$N_x = V_x \sin \phi + H_x \cos \phi$ = axial or normal stress;

$T_x = V_x \cos \phi - H_x \sin \phi$ = radial shear;

$$M_x=M_1+V_1x-H_1y-\overset{x}{\Sigma}P(x-a)+\overset{x}{\Sigma}Q(y-b); \quad x>a. \quad . \quad . \quad . \qquad \text{I}$$

$$V_1=\frac{M_2-M_1}{l}+R_1+\overset{x}{\Sigma}Q\frac{b}{l}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{II}$$

$$\left.\begin{aligned} &M_x=M_1+V_1x-h_1y+Q(y-b), \text{ horizontal load;} \\ &\frac{M_x}{H_1}=\frac{M_1}{H_1}\frac{l-x}{l}+\frac{M_2}{H_1}\frac{x}{l}-y+\frac{m_x}{H_1}, \text{ vertical load.} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \qquad \text{III}$$

SYMMETRICAL MASONRY ARCHES.

CHAPTER I.

FUNDAMENTAL FORMULAS FOR THE ELASTIC ARCH.

1. **Angular Distortion Produced by Bending.**—Let Fig. 1 represent an elastic arch which has been distorted so that the angle ϕ has become $\phi - \delta\phi$ at a section having the co-

FIG. 1a. FIG. 1b.

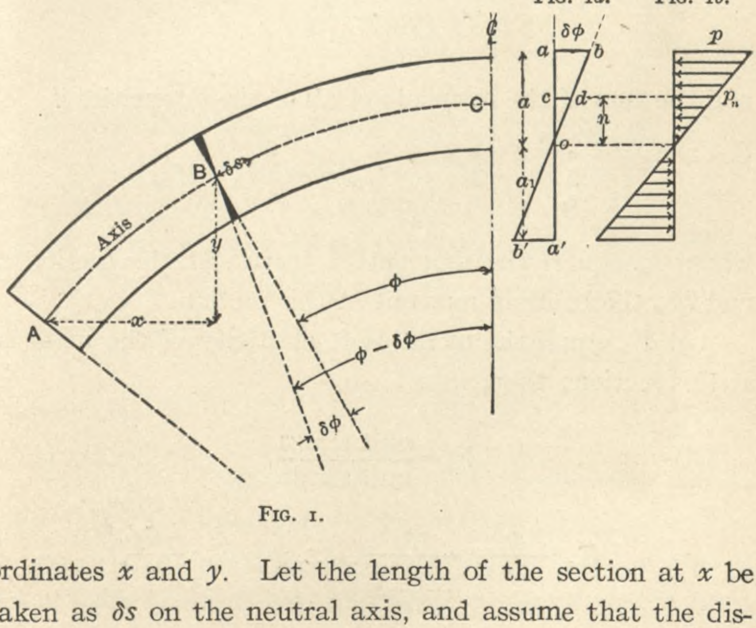

FIG. 1.

ordinates x and y. Let the length of the section at x be taken as δs on the neutral axis, and assume that the distortion is confined to this section and produced by bending alone. Then, according to the common theory of flexure, the distortion of the fibers can be represented by Fig. 1*a*, and the forces producing the distortions by Fig. 1*b*.

In Fig. 1*a*, if *cd* represents the distortion of a fiber distant n from the neutral axis, $cd = +n(-\delta\phi)$, $\delta\phi$ and $\tan \delta\phi$ being assumed equal for very small angles.

In Fig. 1*b*, the intensity of the stress producing the distortion *cd* is p_n, which may be taken in terms of the intensity p upon the outer fiber, or

$$p_n = \frac{np}{a}.$$

The moment of p_n about O upon the neutral axis of the arch is

$$np_n = \frac{n^2p}{a},$$

and the sum of the moments of all of the intensities is

$$\sum_{a_1}^{a} np_n = \sum_{a_1}^{a} \frac{n^2p}{a} = \frac{p}{a}\sum_{a_1}^{a} n^2 = \frac{p}{a} I_x = M_x,$$

where I_x equals the moment of inertia of the section x, and M_x the bending moment at this section.

Let E_x equal the modulus of elasticity of the material at this section; then, since

$$E_x = \frac{\text{unit stress}}{\text{unit strain}},$$

$$E_x = \frac{p}{\frac{ab}{\delta s}} = \frac{p}{\frac{-a\delta\phi}{\delta s}}. \quad \therefore\ p = E_x a \frac{-\delta\phi}{\delta s}.$$

Hooke's Law: stress proportional to strain.

This expression is not exactly correct, as it assumes the length of all fibers before distortion to be δs, while actually each fiber has a different length. Usually the depth of an arch rib is quite small in comparison with its radius of curvature, so that the error is very small.

Substituting this value of p in the expression $M_x = \frac{p}{a} I_x$ and solving for $\delta\phi$,

$$\delta\phi = -\frac{M_x ds}{E_x I_x}.$$

This represents the change in the angle ϕ due to the distortion at the section x alone. If the effect of the distortion at all sections from A to B, Fig. 1, be represented by $\Delta\phi$, then

$$\Delta\phi = -\sum_0^x \frac{M_x \delta s}{E_x I_x}.$$

If ϕ_0 is the total central angle upon the *left* of the crown and $-\phi_l$ that upon the *right*, then $\phi_0 - \phi_l$ is the total central angle. The change in this central angle due to the distortions of all sections between 0 and l (where l is the total span subtending the central angle $\phi_0 - \phi_l$) becomes

$$\Delta\phi_0 = \Delta\phi_l - \sum_0^l \frac{M_x \delta s}{E_x I_x}.$$

2. Changes in the Coordinates x and y Produced by Bending only.—Let the distortion at the section x be the same as in the previous article, and assume the point A free to move; then, after the distortion, it would be in some position as C, Fig. 2. x will be *increased* by δx and y will be *decreased* by δy.

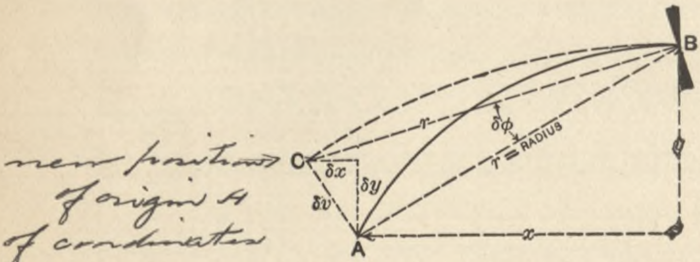

FIG. 2.

From Fig. 2,

$$\delta y : \delta v :: x : r, \quad \text{or} \quad r\delta y = x\delta v = xr\delta\phi.$$

$$\delta x : \delta v :: y : r, \quad \text{or} \quad r\delta x = y\delta v = yr\delta\phi.$$

$$\therefore \ \delta y = x\delta\phi \quad \text{and} \quad \delta x = y\delta\phi.$$

Substituting the value of $\delta\phi$ from Art. 1,

$$\delta y = \frac{M_x \delta s}{E_x I_x} x \quad \text{and} \quad \delta x = \frac{M_x \delta s}{E_x I_x} y.$$

The total change in x and y due to the distortion of all sections between A and B is

$$\Delta x = \sum_0^x \frac{M_x y \delta s}{E_x I_x} \quad \text{and} \quad \Delta y = \sum_0^x \frac{M_x x \delta s}{E_x I_x}.$$

If now x is assumed to equal l, we may write for the total effect of the distortion at all sections upon the span l

$$\Delta l = + \sum_0^l \frac{M_x y \delta s}{E_x I_x}.$$

If y is assumed as positive when measured upward and $+C$ is the value of y when $x=l$, then, noting that y decreases under the particular distortion assumed,

$$\Delta C = -\sum_0^l \frac{M_x x \delta s}{E_x I_x}.$$

3. Changes in x and y Produced by a Direct or Axial Stress. —A direct or axial stress is one producing a uniform intensity at the section being considered; consequently the distortion of each fiber will be the same over the entire section (the modulus of elasticity E_x being assumed constant for the section).

If N_x is the magnitude of the stress and F_x the area of the section, $\frac{N_x}{F_x}=p_0$ is the unit stress or intensity upon the section x. In Fig. 3 let a portion of the arch rib δs in length be acted upon by the direct stress N_x, and suppose this stress produces a uniform *shortening* of the fibers ab; then

$$E_x = \frac{p_0}{\frac{ab}{\delta s}}, \quad \text{or} \quad ab = \frac{p_0 \delta s}{E_x}.$$

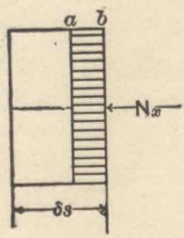

FIG. 3.

If $\sum_0^x ab$ for all sections between x and 0 be represented by Δs, then

$$\Delta s = \sum_0^x \frac{p_0 \delta s}{E_x}.$$

If $x=l$, and since this distortion is in effect a decreasing of the length of the arch axis,

$$\Delta s = -\sum_0^l \frac{p_0 \delta s}{E_x}.$$

In a similar manner

$$\Delta x = -\sum_0^x \frac{p_0 \delta x}{E_x} \quad \text{and} \quad \Delta y = -\sum_0^x \frac{p_0 \delta y}{E_x}.$$

Also

$$\Delta l = -\sum_0^l \frac{p_0 \delta x}{E_x},$$

and

$$\Delta C = -\sum_0^l \frac{p_0 \delta y}{E_x}.$$

4. Changes in s, x, and y Produced by a Rise of Temperature. —Let e = the coefficient of expansion for a change of 1° in temperature;

$t°$ = the number of degrees of change in temperature;

δs = the length in which a uniform change of temperature takes place. Then

$$\Delta s = et° \sum_0^x \delta s,$$

$$\Delta x = et° \sum_0^x \delta x,$$

and

$$\Delta y = et° \sum_0^x \delta y.$$

If $x = l$, then

$$\Delta s = et° \sum_0^l \delta s,$$

$$\Delta l = et° \sum_0^l \delta x,$$

and

$$\Delta c = et° \sum_0^l \delta y.$$

5. The Combination of Bending, Axial Thrust, and Temperature Effects. — Combining the formulas deduced in the previous articles,

$$\Delta\phi_0 = \Delta\phi_l - \sum_0^l \frac{M_x \delta s}{E_x I_x},$$

$$\Delta l = \sum_0^l \frac{M_x y \delta s}{E_x I_x} - \sum_0^l \frac{p_0 \delta x}{E_x} + \sum_0^l e t^\circ \delta x,$$

$$\Delta c = -\sum_0^l \frac{M_x x \delta s}{E_x I_x} - \sum_0^l \frac{p_0 \delta y}{E_x} + \sum_0^l e t^\circ \delta y.$$

In comparing the above equations with those given in the author's "Treatise on Arches," it is seen that the signs of the terms containing M_x are of opposite character. If we had assumed the upper fiber extended by the bending, the signs would have been in agreement. The actual sign of the term depends upon M_x, so the disagreement is of no importance as long as the terms are consistently of opposite signs.

6. Neglecting the Axial Stress and Assuming the Modulus of Elasticity as Constant.—* The effect of the axial stress is quite small excepting in arches which are very flat. For fixed arches having a ratio of rise to span of $^1/_{10}$ the effect of the axial stress is to reduce the magnitude of the horizontal thrust about 30%, while for a ratio of $^2/_{10}$ this percentage drops to about 10%. Formulas which include the effect of the axial stress become somewhat complex, and as its effect can be found with sufficient accuracy for

* See "A Treatise on Arches," by Malverd A. Howe. John Wiley & Sons, New York.

practical purposes by another method, we will omit the term containing p_0 in the formulas which follow.

Usually the modulus of elasticity of the material in an arch rib is uniform, so that it will be unnecessary to consider E_x as a variable in our formulas. We will designate the uniform value by E.

The formulas now become

$$\Delta\phi_0 - \Delta\phi_l = \frac{-1}{E}\sum_0^l M_x \Delta,$$

$$\Delta l = \frac{1}{E}\sum_0^l M_x y\Delta + \sum_0^l et^\circ \delta x,$$

$$\Delta c = -\frac{1}{E}\sum_0^l M_x x\Delta + \sum_0^l et^\circ \delta y,$$

where Δ in the second member of each equation $= \frac{\delta s}{I_x}$.

CHAPTER II.

SYMMETRICAL ARCHES FIXED AT THE ENDS.

7. Conditions which must be Satisfied.—(*a*) The total central angle must remain unchanged, or $\Delta\phi - \Delta\phi_l = 0$;

(*b*) The length of the span must remain constant, or $\Delta l = 0$; and

(*c*) The relative elevations of the supports must remain unchanged, or $\Delta c = 0$.

Expressing these conditions in the form of equations, we have from Art. 6

$$\sum_0^l M_x \Delta = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

$$\sum_0^l M_x y \Delta + et^\circ E \sum_0^l \delta x = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

and

$$-\sum_0^l M_x x \Delta + et^\circ E \sum_0^l \delta y = 0. \quad . \quad . \quad . \quad . \quad . \quad (c)$$

From (I),

$$M_x = M_1 + V_1 x - H_1 y - \sum_0^{x>a} P(x-a) + \sum_0^{x>a} Q(y-b).$$

We have three equations (*a*), (*b*), and (*c*), containing in M_x the three unknowns M_1, V_1, and H_1, and consequently their values can be determined under the assumptions made.

8. Determination of the Horizontal Thrust H_1 Produced by Vertical and Horizontal Loads and Changes of Temperature.—Let two equal vertical loads P and two equal horizontal loads Q be placed upon two points equally distant from the crown. (These may be the vertical and horizontal components of inclined loads.) Then

$$V_1 = P,$$

and

$$M_x = M_1 - H_1 y + \left\{ m_x = Px - \sum_0^{x>a} P(x-a) + \sum_0^{x>a} Q(y-b) \right\},$$

where m_x = *the common moment for symmetrical loads on a simple beam supported at the ends.*

Substituting the value of M_x in (a) and (b), we obtain

$$M_1 \sum_0^l \Delta - H_1 \sum_0^l y\Delta + \sum_0^l m_x \Delta = 0$$

and

$$M_1 \sum_0^l y\Delta - H_1 \sum_0^l y^2\Delta + \sum_0^l m_x y\Delta + et^\circ E \sum_0^l \delta x = 0.$$

Multiplying the first equation by $\sum_0^l y\Delta$ and the second by $\sum_0^l \Delta$, eliminating M_1, and solving for H_1, we obtain

$$H_1 = \frac{et^\circ E \Sigma \delta x + \Sigma m_x y\Delta - \Sigma m_x \Delta \dfrac{\Sigma y\Delta}{\Sigma \Delta}}{\Sigma y^2 \Delta - \dfrac{(\Sigma y\Delta)^2}{\Sigma \Delta}}, \quad . \quad . \quad . \quad (1)$$

which is the general expression for the horizontal thrust produced by two equal and symmetrically placed loads and changes of temperature.

Hereafter all summations between the limits l and o will be designated simply by Σ, as in the equation for H_1 above.

9. The Horizontal Thrust Produced by a Single Vertical Load Placed at any Point upon the Arch.—In this case $m_x =$ *the common moment due to two equal and symmetrically placed loads*, or

$$Px - \overset{x>a}{\Sigma} P(x-a).$$

Since the loads are equal and symmetrically placed, the value of H_1 for one load must be just *one half* that for both loads; hence

$$H_1 = \frac{1}{2} \frac{\Sigma m_x y\Delta - \Sigma m_x \Delta \frac{\Sigma y\Delta}{\Sigma\Delta}}{\Sigma y^2\Delta - \frac{(\Sigma y\Delta)^2}{\Sigma\Delta}}, \quad . \quad . \quad . \quad . \quad (2)$$

where

$$m_x = Px - \overset{x>a}{\Sigma} P(x-a),$$

or

$$H_1 = \frac{1}{2} \frac{\Sigma m_x \Delta \left\{ y - \frac{\Sigma y\Delta}{\Sigma\Delta} \right\}}{\Sigma y\Delta \left\{ y - \frac{\Sigma y\Delta}{\Sigma\Delta} \right\}}. \quad . \quad . \quad . \quad . \quad (2a)$$

10. The Horizontal Thrust Produced by a Single Horizontal Load Placed at any Point upon the Arch. — In this case $m_x =$ *the common moment due to two equal and symmetrically placed loads*, or

$$m_x = \overset{x>a}{\Sigma} Q(y-b).$$

Let h_1 = the horizontal thrust at the left support due to the load upon the left of the crown, and

h_2 = the horizontal thrust at the left support due to the load upon the right of the crown. Then

$$H_1 = h_1 + h_2;$$

but

$$Q = h_1 - h_2;$$

hence

$$2h_1 = H_1 + Q$$

and

$$h_1 = \tfrac{1}{2}H_1 + \tfrac{1}{2}Q.$$

Therefore

$$h_1 = \frac{1}{2}\left\{ Q + \frac{\Sigma m_x y\Delta - \Sigma m_x \Delta \dfrac{\Sigma y\Delta}{\Sigma \Delta}}{\Sigma y^2\Delta - \dfrac{(\Sigma y\Delta)^2}{\Sigma\Delta}} \right\}, \quad . \quad . \quad . \quad (3)$$

where $m_x = \overset{x>a}{\Sigma} Q(y-b)$.

Also,

$$h_1 = \frac{1}{2}\left\{ Q + \frac{\Sigma m_x \Delta \left(y - \dfrac{\Sigma y\Delta}{\Sigma\Delta}\right)}{\Sigma y\Delta\left(y - \dfrac{\Sigma y\Delta}{\Sigma\Delta}\right)} \right\}. \quad . \quad . \quad . \quad (3a)$$

11. The Horizontal Thrust Produced by a Change of Temperature.—We have directly from eq. (1), since $\Sigma\delta x = l$,

$$H_t = \frac{et^\circ El}{\Sigma y^2\Delta - \dfrac{(\Sigma y\Delta)^2}{\Sigma\Delta}}; \quad . \quad . \quad . \quad . \quad . \quad (4)$$

also,

$$H_t = \frac{et^\circ El}{\Sigma y\Delta\left(y - \dfrac{\Sigma y\Delta}{\Sigma\Delta}\right)} \quad . \quad . \quad . \quad . \quad . \quad (4a)$$

12. Determination of the Bending Moment at the Left Support Produced by any Single Load and Changes of Temperature.— From (III),

$$M_x = M_1\frac{l-x}{l} + M_2\frac{x}{l} - H_1 y + m_x,$$

where

$$m_x = P\frac{l-a}{l}x + \frac{Qb}{l}x - P(x-a) + Q(y-b) \quad x > a.$$

Taking the two conditions that the angle at the center shall remain unchanged and that the relative elevations of the supports remain constant, we have from (*a*) and (*c*)

$$\Sigma M_x \Delta = 0$$

and

$$-\Sigma M_x x \Delta + et^\circ E \Sigma \delta y = 0.$$

Substituting the above value of M_x in these two equations, neglecting the temperature term for the present, we have

$$M_1 \Sigma\frac{l-x}{l}\Delta + M_2 \Sigma\frac{x}{l}\Delta - H_1 \Sigma y\Delta + \Sigma m_x \Delta = 0,$$

$$-M_1 \Sigma\frac{l-x}{l}x\Delta - M_2 \Sigma\frac{x^2}{l}\Delta + H_1 \Sigma xy\Delta - \Sigma m_x x\Delta = 0.$$

Multiplying the first equation by $\Sigma\frac{x^2}{l}\Delta$ and the second by $\Sigma\frac{x}{l}\Delta$, they become

$$M_1 \Sigma\frac{l-x}{l}\Delta \Sigma\frac{x^2}{l}\Delta + M_2 \Sigma\frac{x}{l}\Delta \Sigma\frac{x^2}{l}\Delta - H_1 \Sigma y\Delta \Sigma\frac{x^2}{l}\Delta$$
$$+ \Sigma m_x \Delta \Sigma\frac{x^2}{l}\Delta = 0,$$

$$-M_1\Sigma\frac{l-x}{l}x\Delta\Sigma\frac{x}{l}\Delta-M_2\Sigma\frac{x}{l}\Delta\Sigma\frac{x^2}{l}\Delta+H_1\Sigma xy\Delta\Sigma\frac{x}{l}\Delta$$
$$-\Sigma m_x x\Delta\Sigma\frac{x}{l}\Delta=0.$$

Eliminating M_2 by adding these equations, we obtain

$$M_1=H_1\frac{\Sigma y\Delta\left(x-\frac{\Sigma x^2\Delta}{\Sigma x\Delta}\right)}{\Sigma\Delta\left(\frac{\Sigma x\Delta}{\Sigma\Delta}-\frac{\Sigma x^2\Delta}{\Sigma x\Delta}\right)}-\frac{+\Sigma m_x\Delta\left(x-\frac{\Sigma x^2\Delta}{\Sigma x\Delta}\right)}{\Sigma\Delta\left(\frac{\Sigma x\Delta}{\Sigma\Delta}-\frac{\Sigma x^2\Delta}{\Sigma x\Delta}\right)}. \quad (5)$$

Since the arch is symmetrical in every particular, $\frac{\Sigma x\Delta}{\Sigma\Delta}=\frac{l}{2}$ and $\Sigma y\Delta x=\frac{l}{2}\Sigma y\Delta$. Therefore we have

$$M_1=H_1\frac{\Sigma y\Delta}{\Sigma\Delta}-\frac{\Sigma m_x\Delta\left(x-\frac{\Sigma x^2\Delta}{\Sigma x\Delta}\right)}{\Sigma\Delta\left(\frac{l}{2}-\frac{\Sigma x^2\Delta}{\Sigma x\Delta}\right)}. \quad . \quad . \quad (5a)$$

For changes of temperature, from (a)

$$\Sigma M_x\Delta=0.$$

From (III),

$$M_x=M_1-H_t y.$$

Then

$$M_1\Sigma\Delta-H_t\Sigma y\Delta=0,$$

or

$$M_1=H_t\frac{\Sigma y\Delta}{\Sigma\Delta}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5b)$$

13. Formulas which Apply for Vertical Loads only.

$$H_1 = \frac{1}{2}\frac{\Sigma m_x \Delta\left(y - \frac{\Sigma y\Delta}{\Sigma\Delta}\right)}{\Sigma y\Delta\left(y - \frac{\Sigma y\Delta}{\Sigma\Delta}\right)} = \frac{1}{2}\frac{\Sigma y\Delta\left(m_x - \frac{\Sigma m_x\Delta}{\Sigma\Delta}\right)}{\Sigma y\Delta\left(y - \frac{\Sigma y\Delta}{\Sigma\Delta}\right)}, \quad (2a)$$

where m_x for each load considered has the following value:

$$m_x = Px - \overset{x>a}{\Sigma} P(x-a).$$

$$M_1 = H_1\frac{\Sigma y\Delta}{\Sigma\Delta} - \frac{\Sigma m_x\Delta\left(x - \frac{\Sigma x^2\Delta}{\Sigma x\Delta}\right)}{\Sigma\Delta\left(\frac{l}{2} - \frac{\Sigma x^2\Delta}{\Sigma x\Delta}\right)}, \quad . \ . \ . \ (5a)$$

where

$$m_x = R_1 x - \overset{x>a}{\Sigma} P(x-a),$$

or the common moment for loads on a simple beam supported at the ends.

$$M_x = M_1\frac{l-x}{l} + M_2\frac{x}{l} - H_1 y + m_x, \quad . \ . \ . \ . \ (III)$$

where

$$m_x = R_1 x - \overset{x>a}{\Sigma} P(x-a) \ . \ . \ . \ ;$$

$$V_1 = \frac{M_2 - M_1}{l} + R_1,$$

where $R = \Sigma P\frac{l-a}{l}$ = the common reaction for loads on a simple beam supported at the ends.

For symmetrical loading

$$M_1 = H_1\frac{\Sigma y\Delta}{\Sigma\Delta} - \frac{\Sigma m_x\Delta}{\Sigma\Delta} \quad . \ . \ . \ . \ (5aa)$$

14. A Graphical Determination of m_x in (2a) for Vertical Loads.—The equation $m_x = Px - \overset{x>a}{\Sigma} P(x-a)$ may be represented graphically as follows: Lay off a load line $2P$ in length, and with a pole distance of P construct the ordinary equilibrium polygon as indicated in Fig. 4. Since the

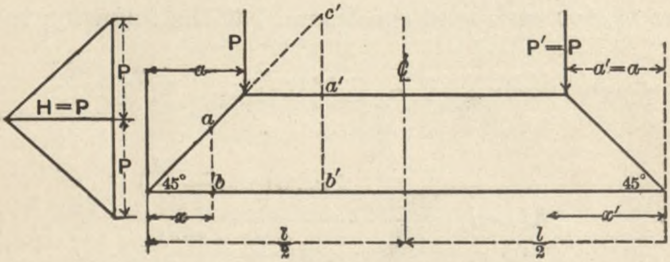

FIG. 4.

loads are equal and symmetrically placed, the reactions are equal and each equal to P. Then in the equilibrium polygon the ordinate ab, before any load is reached, equals x. The moment $m_x = H(ab) = Px$; hence the ordinate ab represents the true value of m_x for $P =$ unity.

The ordinate $a'b'$ between the loads equals a and $m_x = H(a'b') = Pa = R_1x - P(x-a)$, and as before the ordinate $a'b'$ represents the true value of m_x when $P =$ unity.

From the above construction it is evident that when H_1 is desired for any single load the graphical construction is quite unnecessary, as m_x always equals Px or Pa on the left of the center. Since the equilibrium polygon is symmetrical for each value of m_x on the left, there will be a corresponding value upon the right.

In case the values of m_x are desired for a combination of loads, the method of procedure is essentially the same as outlined for one load. Lay off a load line equal to *twice* the loads for which m_x is wanted. Opposite the center of this load line take a pole at any convenient distance H, and construct an equilibrium polygon in the usual manner. The value of m_x at any point equals the ordinate of the equilibrium polygon at that point multiplied by the assumed H. In most cases it is more satisfactory to compute the values of m_x.

15. A Graphical Determination of Some of the Factors in the Equation for H_1 for Vertical Loads.—The expression (2*a*) in Art. 13 may be written

$$H_1 = \frac{\Sigma y\Delta\left(m_x - \frac{\Sigma m_x\Delta}{\Sigma\Delta}\right)}{2\Sigma y\Delta\left(y - \frac{\Sigma y\Delta}{\Sigma\Delta}\right)}.$$

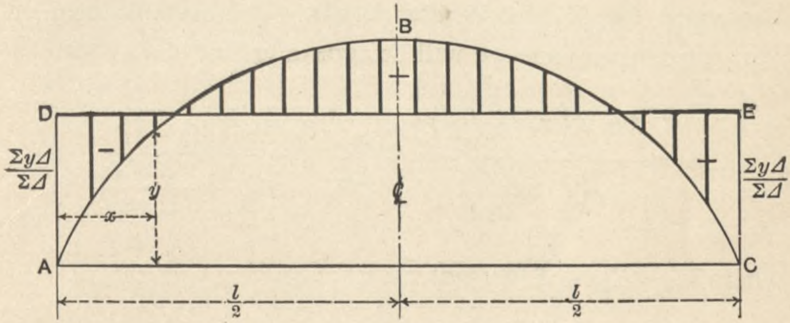

FIG. 5.

Let ABC, Fig. 5, represent the axis of the arch. Compute $\frac{\Sigma y\Delta}{\Sigma\Delta}$ and lay off its value upward from A and C. Then draw DE. The heavy ordinates will be the values of $y - \frac{\Sigma y\Delta}{\Sigma\Delta}$.

In like manner let $A'B'C'$ represent the equilibrium polygon where the ordinates are $\frac{m_x}{H}$. Draw $D'E'$ as indicated in Fig. 6. Then the heavy ordinates represent $\frac{1}{H}\left(m_x - \frac{\Sigma m_x \Delta}{\Sigma \Delta}\right)$.

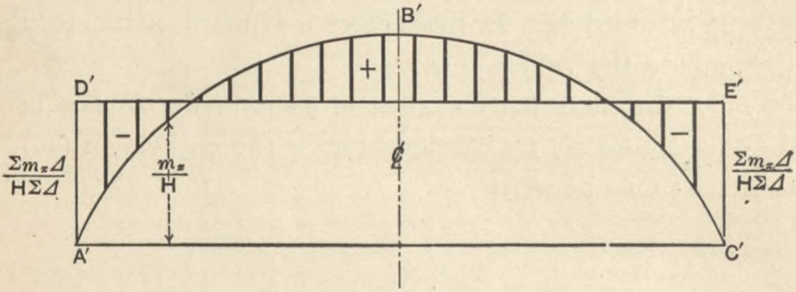

FIG. 6.

16. A Graphical Representation of the Second Term in the Expression for M_1 for Vertical Loads.—The second term of (5a) for convenience we will designate as m_1, or

$$m_1 = \frac{\Sigma m_x \Delta\left(x - \frac{\Sigma x^2 \Delta}{\Sigma x \Delta}\right)}{\Sigma \Delta\left(\frac{l}{2} - \frac{\Sigma x^2 \Delta}{\Sigma x \Delta}\right)} = \frac{\Sigma m_x \Delta}{\Sigma \Delta} - \frac{\Sigma m_x \Delta (x - \frac{1}{2}l)}{\Sigma \Delta\left(\frac{1}{2}l - \frac{\Sigma x^2 \Delta}{\Sigma x \Delta}\right)},$$

where $m_x = R_1 x - \overset{x>a}{\Sigma} P(x-a)$.

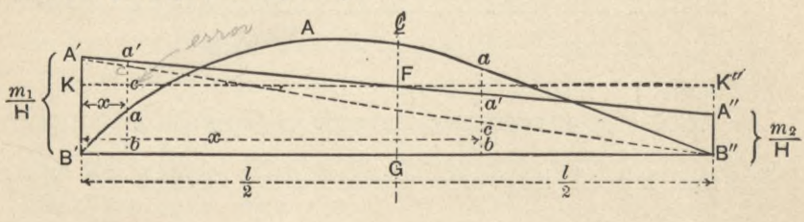

FIG. 7.

Let the common equilibrium polygon for the given loads be represented by $B'AB''$ in Fig. 7.

We will now prove that when the line $A'A''$ is drawn, so that $\Sigma(aa')\Delta = 0$ and $\Sigma(aa')x\Delta = 0$, the distance $A'B' = \frac{m_1}{H}$. When $\Sigma(aa')\Delta = 0$ it at once follows that $\Sigma(ab)\Delta = \Sigma(a'b)\Delta$. From Fig. 7,

$$aa' = a'b - ab = a'c + cb - ab,$$

$$a'c = \frac{m_2}{H}\frac{x}{l} \quad \text{and} \quad cb = \frac{m_1}{H}\frac{l-x}{l}.$$

Hence, since $ab = \frac{m_x}{H}$,

$$aa' = \frac{m_2}{H}\frac{x}{l} + \frac{m_1}{H}\frac{l-x}{l} - \frac{m_x}{H};$$

multiplying through by Δ,

$$aa'\Delta = \frac{m_2}{H}\frac{x}{l}\Delta + \frac{m_1}{H}\frac{l-x}{l}\Delta - \frac{m_x}{H}\Delta;$$

also,

$$(aa')x\Delta = \frac{m_2}{H}\frac{x^2}{l}\Delta + \frac{m_1}{H}\frac{(l-x)x}{l} - \frac{m_x x}{H}\Delta.$$

Making $\Sigma(aa')\Delta = 0$ and $\Sigma(aa')x\Delta = 0$ and eliminating $\frac{m_2}{H}$ between the resulting equations, we obtain

$$m_1 = \frac{\Sigma m_x x\Delta - \Sigma m_x \Delta \frac{\Sigma x^2\Delta}{\Sigma x\Delta}}{\Sigma x(l-x)\Delta - \Sigma(l-x)\Delta\frac{\Sigma x^2\Delta}{\Sigma x\Delta}} l.$$

This readily reduces to

$$m_1 = \frac{\Sigma m_x \Delta\left(x - \frac{\Sigma x^2 \Delta}{\Sigma x \Delta}\right)}{\Sigma \Delta\left(\frac{l}{2} - \frac{\Sigma x^2 \Delta}{\Sigma x \Delta}\right)},$$

the second term in the expression for M_1 in Art. 13.

From the above demonstration it at once follows that

$$m_2 = \frac{\Sigma m_x \Delta\left(l - x - \frac{\Sigma x^2 \Delta}{\Sigma x \Delta}\right)}{\Sigma \Delta\left(\frac{l}{2} - \frac{\Sigma x^2 \Delta}{\Sigma x \Delta}\right)}.$$

In Fig. 7

$$FG = \frac{m_1 + m_2}{2H} = \frac{\Sigma m_x \Delta}{\Sigma \Delta},$$

and

$$A'K = A''K = \frac{\Sigma m_x \Delta (x - \frac{1}{2}l)}{\Sigma \Delta\left(\frac{1}{2}l - \frac{\Sigma x^2 \Delta}{\Sigma x \Delta}\right)}.$$

17. A Graphical Representation of M_x for Vertical Loads only.—From (III),

$$\frac{M_x}{H_1} = \frac{M_1}{H_1}\frac{l-x}{l} + \frac{M_2}{H_1}\frac{x}{l} - y + \frac{m_x}{H_1}.$$

In Fig. 8 let ABC be the axis of the arch and $A'bC'$ the equilibrium polygon for a single load drawn with a pole distance of H_1 and located so that $A'A'' = \frac{m_1}{H_1}$ and $C'C'' = \frac{m_2}{H_1}$. Then

$$AA' = A'A'' - AA'' = \frac{m_1}{H_1} - \frac{\Sigma y \Delta}{\Sigma \Delta},$$

or

$$H_1(AA') = m_1 - H_1\frac{\Sigma y\Delta}{\Sigma\Delta} = -M_1.$$

In like manner $H(CC') = +M_2$.

Let $\frac{M_1}{H_1} = y_1$ and $\frac{M_2}{H_1} = y_2$. Then

$$\frac{M_x}{H_1} = y_1\frac{l-x}{l} + y_2\frac{x}{l} - y + \left(\frac{m_x}{H_1} = be\right)$$

$$= -df + ef - ad + be = -ab.$$

Therefore $M_x = H_1(ab)$, or the bending moment at any

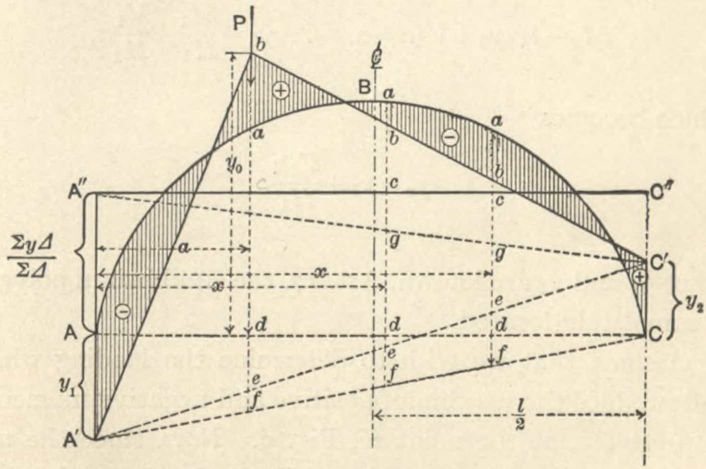

FIG. 8.

point equals the ordinate between the axis of the arch and the true equilibrium polygon.

Usually the ordinate ab is so small that no very accurate results can be obtained from a drawing. From the above demonstration it is evident that

$$ab = ac - cb = \left\{ y - \frac{\Sigma y\Delta}{\Sigma\Delta} \right\} - \left\{ \frac{m_x}{H_1} - \frac{m_1}{H_1}\frac{l-x}{l} - \frac{m_2}{H}\frac{x}{l} \right\},$$

quantities which can be quite accurately determined from a large-scale drawing. However, more satisfactory results will always be obtained by algebraic methods, using graphics merely as a check.

18. The Loads Producing a Maximum M_x and the Ordinates Locating the True Equilibrium Polygon for a Single Vertical Load.—In Fig. 8 take moments of all the forces upon the left of b about b, or

$$M_1 - H_1 y_0 + V_1 a = 0. \quad \therefore \; y_0 = \frac{M_1}{H_1} + \frac{V_1}{H_1} a,$$

which becomes

$$y_0 = y_1 + \frac{V_1 a}{H_1}.$$

Since y_1 and y_2 are known, Art. 17, the equilibrium polygon is completely located.

Assume that we wish to determine the loading which will produce the maximum positive and negative moments, respectively, at the point K, Fig. 9. Now, since the moment is proportional to the ordinate between the arch axis and the equilibrium polygon, it is evident that the moment will be zero for any load which has its equilibrium polygon passing through K. As shown in Fig. 9, the shaded portion of the span loaded will cause one kind of moment and the unshaded portion loaded will produce the opposite kind. In case the moving load is a uniform load these two moments will be greatest at this point.

For arch ribs which do not have too great a variation in section from the crown the absolute maximum moment

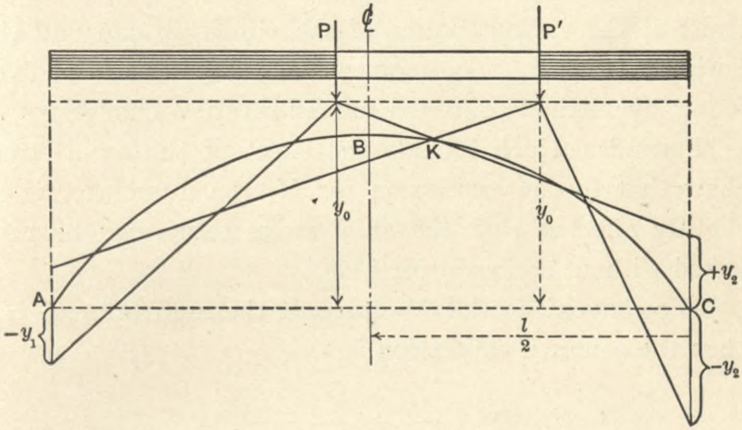

FIG. 9.

between the crown and the support is between 0.25 and 0.35, the span for uniform moving loads, while the greatest moment of all is at the support.*

It also appears from examples solved in detail that sensibly the same loading can be used in both cases. The division of the loads is indicated by the sign of M_1, the moment at the support. Loads which produce *positive* moments at the left support will produce *negative* moments at about the three-quarter point of the span.

19. The Effect of the Axial Stress for Vertical Loads only.— The effect of the axial or direct stress is to *shorten* the arch rib, Art. 3, and may be considered, with a close degree of

* "A Treatise on Arches," by Malverd A. Howe. John Wiley & Sons, New York.

approximation, to a certain drop of temperature. Consequently, if we can determine the horizontal thrust produced by axial stress due to any particular loading, we can compute the resulting stresses in the arch rib. We are not concerned with the actual magnitudes of the axial stress at the various points of the rib if we can find the horizontal thrust, The moments and stresses will at once follow by methods outlined for temperature changes.

Formulas which include the effect of the axial stress show that in the expression for H_1 the numerator is so slightly affected that the axial stress terms can be neglected without serious error.*

For convenience let N represent the numerator of H_1; then the common expression is

$$H_1 = \frac{N}{2\Sigma y\Delta\left(y - \frac{\Sigma y\Delta}{\Sigma\Delta}\right)}.$$

With the effect of the axial stress included this becomes

$$H_1' = \frac{N}{2\Sigma y\Delta\left(y - \frac{\Sigma y\Delta}{\Sigma\Delta}\right) + 2\Sigma\frac{\delta x}{F}\cos\phi}.$$

Let H_a = the horizontal thrust due to the axial stress; then

$$H_a = H_1 - H_1', \quad \text{or} \quad \frac{H_a}{H_1} = 1 - \frac{H_1'}{H_1}.$$

$$\therefore\ H_a = H_1\left(1 - \frac{2\Sigma y\Delta\left(y - \frac{\Sigma y\Delta}{y\Delta}\right)}{2\Sigma y\Delta\left(y - \frac{\Sigma y\Delta}{\Sigma\Delta}\right) + 2\Sigma\frac{\delta x}{F}\cos\phi}\right). \qquad (6)$$

* "A Treatise on Arches," by Malverd A. Howe. John Wiley & Sons, New York.

This value of H_a, which is quickly obtained, is to be treated as the horizontal thrust due to a drop of temperature which would produce a thrust of equal magnitude.

20. Loads which Produce Maximum Values of T_x or Radial Shear.

$$T_x = (V_1 - \Sigma P)\cos\phi - H_1 \sin\phi.$$

For loads upon the right of B, Fig. 10,

$$T_x = V_1 \cos\phi - H_x \sin\phi.$$

If S_1 is normal to the radius passing through B, it is evident from the figure that $T_x = 0$, since $V_1 \cos\phi = H_1 \sin\phi$. Hence all loads upon the right of P'' will produce one kind

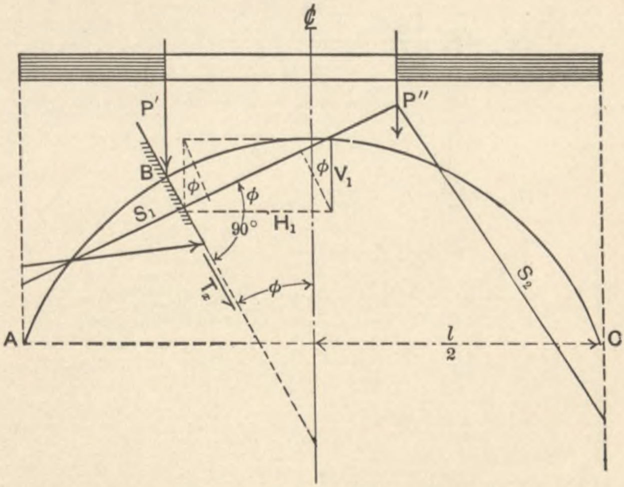

FIG. 10.

of shear and those upon the left the opposite kind until P' is reached. Since $V_1 - \Sigma P$ results in a downward force, the loads upon the left of B produce the same kind of shear as those upon the right of P''. The fields of loading producing the same kind of shear are clearly shown in Fig. 10.

21. Formulas which Apply for Horizontal Loads only.— From Art. 10,

$$h_1 = \frac{1}{2}\left\{ Q + \frac{\Sigma m_x \Delta\left(y - \frac{\Sigma y\Delta}{\Sigma \Delta}\right)}{\Sigma y\Delta\left(y - \frac{\Sigma y\Delta}{\Sigma \Delta}\right)} \right\}, \quad . \quad . \quad . \quad . \quad (3a)$$

where

$$m_x = \overset{x}{\Sigma} Q(y-b). \quad y > b.$$

From Art. 12,

$$M_1 = h_1 \frac{\Sigma y\Delta}{\Sigma \Delta} - \frac{\Sigma m_x \Delta\left(x - \frac{\Sigma x^2\Delta}{\Sigma x\Delta}\right)}{\Sigma \Delta\left(\frac{l}{2} - \frac{\Sigma x^2\Delta}{\Sigma x\Delta}\right)}, \quad . \quad . \quad . \quad . \quad (5c)$$

where

$$m_x = Q\frac{b}{l} + \overset{x}{\Sigma} Q(y-b). \qquad y > b.$$

$$M_x = M_1 \frac{l-x}{l} + M_2 \frac{x}{l} - h_1 y + Q\frac{b}{l}x + \overset{x>a}{\Sigma} Q(y-b),$$

$$V_1 = \frac{M_2 - M_1}{l} + Q\frac{b}{l}.$$

The above formulas are for a single horizontal load which produces a thrust at the left support. In practice the reverse may be the case, but the solution of the equations presents no difficulties if care is taken to give m_x its proper sign. Of course, when there is a *thrust* at the *left* support there will be a *pull* at the *right* support.

22. A Graphical Representation of M_x for a Single Horizontal Load.—From (III),

$$M_x = M_1 + V_1 x - h_1 y + Q(y - b). \qquad y > b,$$

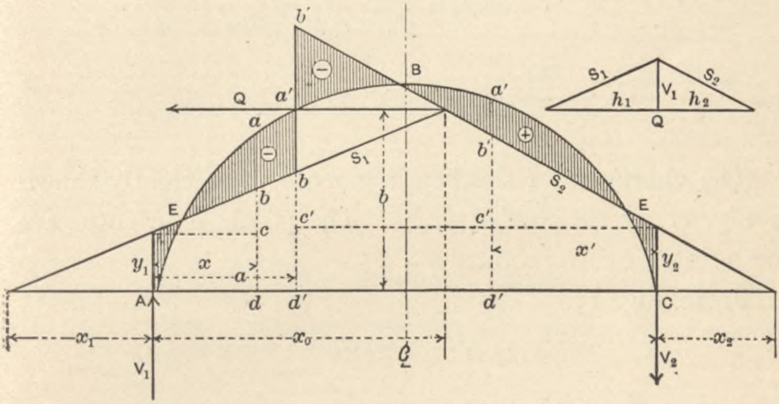

FIG. 11.

For all points between $x = 0$ and $x = a$

$$M_x = M_1 + V_1 x - h_1 y,$$

or

$$\frac{M_x}{h_1} = \frac{M_1}{h_1} + V_1 \frac{x}{h_1} - y.$$

Let the equilibrium polygon be constructed as shown in Fig. 11, where $y_1 = \frac{M_1}{h_1}$, $y_2 = \frac{M_2}{h_2}$, $x_1 = \frac{M_1}{V_1}$, and $x_2 = \frac{M_2}{V_2}$.

On the left of Q, $cd = y_1 = \frac{M_1}{h_1}$, $bc = \frac{V_1 x}{h_1}$; hence

$$\frac{M_x}{h_1} = ab = cd + cb - y.$$

For points upon the right of Q we can write

$$-M_x = M_2 + V_2 x' - h_2 y,$$

or

$$-\frac{M_x}{h_2}=\frac{M_2}{h_2}+\frac{V_2x'}{h_2}-y.$$

$$c'd'=y_2=\frac{M_2}{h_2}, \quad b'c'=\frac{V_2x'}{h_2}.$$

$$\therefore -\frac{M_x}{h_2}=b'a'=c'd'+b'c'-y.$$

The character of the bending moment is clearly shown in Fig. 11 by the shaded areas. The points E, E', etc., are the points of zero moment.

From Fig. 11,

$$x_1:y_1::x_1+x_0:b,$$

or

$$x_0=(b-y_1)\frac{x_1}{y_1}=\left(b-\frac{M_1}{h_1}\right)\frac{h_1}{V_1}.$$

If x_0 is computed, it will check the previous work for locating the equilibrium polygon.

Read

23. A Graphical Representation of M_x for Two Equal and Symmetrical Horizontal Loads.—In (3*a*) m_x for the left

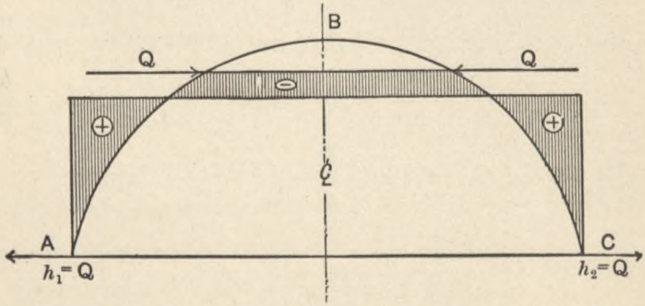

FIG. 12.

load (Fig. 12) will evidently equal m_x for the load upon the right, but will be opposite in character; therefore

$h_1 = Q = h_2$ in magnitude. h_1 and h_2 will be opposite in direction.

Also, from (5*c*),

$$M_1 = h_1 \frac{\Sigma y \Delta}{\Sigma \Delta} = Q \frac{\Sigma y \Delta}{\Sigma \Delta}.$$

From (III),

$$M_x = M_1 + V_1 x - h_1 y + \overset{x>a}{\Sigma} Q(y - b),$$

which becomes, since $V_1 = 0$,

$$M_x = Q\left\{\frac{\Sigma y \Delta}{\Sigma \Delta} - y + y - b\right\} = Q\left(\frac{\Sigma y \Delta}{\Sigma \Delta} - b\right)$$

for all points between the loads, and

$$M_x = Q\left\{\frac{\Sigma y \Delta}{\Sigma \Delta} - y\right\}$$

for all points between the support and a load. This is shown by the shaded ordinates in Fig. 12.

24. Arch Ribs for which Δ is Constant.—Since $\Delta = \frac{\delta s}{I}$, it is evident that if we so divide the axis in parts of δs_1, δs_2, etc., in length, that the quotient of each δs by the moment of inertia of the section of the rib for this distance is constant for all sections, the value of Δ will be constant. Under this assumption the formulas to be given later can be applied to—

1° Arch ribs of *constant cross-section when the axis* is divided in equal parts, each δs in length.

2° *Parabolic* arch ribs for which $EI \cos \phi$ *is constant* when the *span* is divided into equal parts each δx in length.

3° *Any* arch rib for which $\frac{\delta s}{I}$ is *constant* when the *axis* is divided into spaces δs, δs_1, δs_2, etc., so that the *moment of inertia* (usually taken at the center of each division) for each division *bears a constant* ratio to the *length of the division* δs.

25. Formulas for H_1 and M_1 for Vertical Loads when Δ is Constant.—Remembering that $\frac{\Sigma \Delta}{\Delta} = n$, the number of divisions, we have at once from (2*a*), Art. 13,

$$H_1 = \frac{1}{2} \frac{\Sigma m_x\left(y - \frac{\Sigma y}{n}\right)}{\Sigma y\left(y - \frac{\Sigma y}{n}\right)} = \frac{1}{2} \frac{\Sigma m_x(y - y_a)}{\Sigma y(y - y_a)}$$

$$= \frac{1}{2} \frac{\Sigma y\left(m_x - \frac{\Sigma m_x}{n}\right)}{\Sigma y(y - y_a)}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2b)$$

where $m_x = Px - \overset{x}{\Sigma} P(x - a), \quad x > a,$ and

$$*\, M_1 = H_1 y_a - \frac{\Sigma m_x\left(x - \frac{\Sigma x^2}{\Sigma x}\right)}{n\left(\frac{l}{2} - \frac{\Sigma x^2}{\Sigma x}\right)}.$$

Also

$$\left.\begin{matrix} M_1 \\ M_2 \end{matrix}\right\} = H_1 y_a - \left[\frac{\Sigma m_x}{n} \pm \frac{\Sigma m_x\left(x - \frac{l}{2}\right)}{n\left(\frac{l}{2} - \frac{\Sigma x^2}{\Sigma x}\right)} \right]. \quad (5d)$$

---

* When the *span* is divided into n parts δx each and $x = \frac{\delta x}{2}, \frac{3}{2}\delta x, \frac{5}{2}\delta x$, etc.,

$$\frac{\Sigma x^2}{\Sigma x} = \frac{n(4n^2 - 1)}{12}(\delta x)^2 \quad \text{and} \quad n\left(\frac{l}{2} - \frac{\Sigma x^2}{\Sigma x}\right) = -\frac{n-1}{6}\delta x.$$

where $m_x = R_1 x - \overset{x>a}{\Sigma} P(x-a)$ and $\Sigma x = \frac{1}{2}nl$. (See Art. 88.) For any symmetrical loading

$$M_1 = H_1 y_a - \frac{\Sigma m_x}{n}. \quad . \quad . \quad . \quad . \quad (5dd)$$

$$H_1 = \tfrac{1}{2}(\text{total load}).$$

26. Formulas for h_1 and M_1 for Horizontal Loads when Δ is Constant.—From Art. 21,

$$h_1 = \frac{1}{2}\left\{Q + \frac{\Sigma m_x (y - y_a)}{\Sigma y (y - y_a)}\right\}, \quad . \quad . \quad . \quad . \quad (3b)$$

where $m_x = \overset{x}{\Sigma} Q(y-b), \quad y > b.$

$$M_1 = h_1 y_a - \frac{\Sigma m_x\left(x - \frac{\Sigma x^2}{\Sigma x}\right)}{n\left(\frac{l}{2} - \frac{\Sigma x^2}{\Sigma x}\right)}, \quad . \quad . \quad . \quad . \quad (5e)$$

where $m_x = Q\frac{b}{l} + \overset{x}{\Sigma} Q(y-b), \quad y > b$, and

$\Sigma x = \frac{1}{2}nl.$

For any symmetrical loading

$$M_1 = h_1 y_a. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5ee)$$

$$h_1 = \tfrac{1}{2}\ (\text{total load}).$$

27. Formulas for H_t and M_1 for Changes of Temperature when Δ is Constant.—From Art. 11,

$$H_t = \frac{et^\circ EI}{\Delta \Sigma y (y - y_a)}.$$

From Art. 12,

$$M_1 = H_t y_a.$$

28. Effect of the Axial Stress when Δ is a Constant.—From Art. 19,

$$H_a = H_1\left(1 - \frac{\Sigma y(y-y_a)}{\Sigma y(y-y_a) + \Sigma\frac{\delta x \cos\phi}{F\Delta}}\right),$$

where H_1 is the horizontal thrust obtained from formulas which neglect the effect of the axial stress.

29. Determination of N_x, the Normal or Axial Stress, and T_x the Radial Shear at any Point.—In Fig. 13 let S_x be the

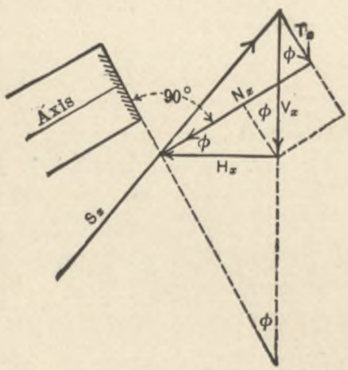

FIG. 13.

stress in the equilibrium polygon in position and magnitude; then we have at once

$$N_x = V_x \sin\phi + H_x \cos\phi,$$

where $V_x = V_1 - \overset{x>a}{\Sigma} P$ and $H_x = H_1 - \overset{x>a}{\Sigma} Q$.

Also, $$T_x = V_x \cos\phi - H_x \sin\phi,$$

V_x and H_x having the values given above.

30. A Graphical Determination of N_x and T_x for Vertical Loads.—In Fig. 14 let S_2 be the side of the equilibrium

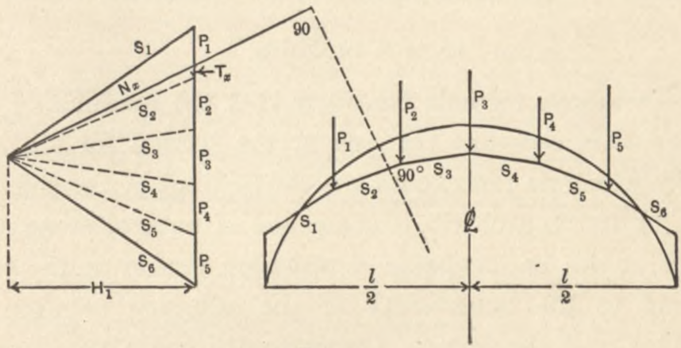

FIG. 14.

polygon cut by the section where N_x and T_x are desired. From the pole in the force diagram draw a line normal to the section, and at the upper extremity of S_2 drop a perpendicular upon this line, forming a right triangle with S_2 as the hypothenuse. The two legs of the triangle will be the magnitudes of N_x and T_x, as indicated in the figure.

31. Fiber Stresses for any Section.—(*a*) In the case of a steel rib, to which the formulas given above probably more nearly apply than for ribs of any other material, the formula based upon the common theory of flexure may be used. This formula may be written

$$p=\frac{N_x}{F}\pm\frac{M_x z}{I}=\frac{N_x}{F}\pm M_x\frac{1}{S},$$

where p = the stress in the outer fiber;

N_x = the axial stress or the normal component of the resultant stress upon the section being considered;

F = the area of the section;

M_x = the bending moment at the section;

z = distance of outer fiber from the neutral axis;

I = the moment of inertia of the section;

$S = \frac{I}{z}$ = the "section modulus."

The above formula considers that the modulus of elasticity E is constant throughout the section for all intensities which do not exceed the elastic limit of the steel.

(*b*) If the arch rib is composed of natural stone voussoirs, it will be incapable of resisting tension at the joints owing to the uncertainty of the adhesion between the mortar and the stone. Consequently the above formula applies only when the resultant pressure upon any joint lies within the middle third of the joint; that is, the entire joint or section will be in compression.

In case the resultant does not lie within the middle third but does lie within the section we may yet have a perfectly stable structure. Suppose that the resultant cuts the section outside the middle third but not outside the stone, as in Fig. 15.

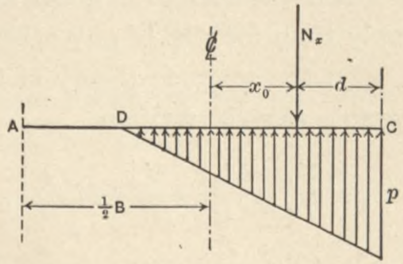

FIG. 15.

Let d = distance from edge C. The pressure may be assumed to be uniformly varying from C towards A, so that N_x will pass through the center of gravity of the intensities;

then

$$N_x = \frac{pCD}{2} = \frac{3pd}{2} = \frac{3}{2}p(\tfrac{1}{2}B - x_0),$$

or

$$p = \frac{2N_x}{3d}.$$

As long as p is so small that there is no danger of the stone being crushed the arch is stable. It is a recognized fact that this condition exists in a large number of arches now standing.

(*c*) Arch ribs constructed of plain concrete are capable of resisting a limited amount of tension, but it is better to treat them the same as if of natural stone. The ring may crack entirely through and yet be perfectly stable. Small rods of steel distributed laterally and circumferentially near the surfaces of the rib will prevent a considerable number of small cracks which might be produced by change of shape after removing the false work or changes of temperature.

(*d*) Reinforced-concrete ribs have circumferential steel rods or bars placed a short distance from the upper and lower surfaces of the rib to resist any tension which may occur. Even in this case the best designers limit the equilibrium polygons for dead and live load to nearly the middle third of the ring, so that there will be no tensile stresses.

The actual distribution of stress on a section of reinforced concrete is at present unknown. Many experiments have been made upon beams reinforced at the bottom, and various formulas advanced to aid in designing such beams, all giving fairly rational results. The elastic theory of the arch assumes that the linear arch is the neutral axis of the material arch, and any great departure from the assumed form will affect the stresses; hence, since the experiments upon beams indicate that the neutral axis shifts for different loadings, it is evident that great refinement either in the calculation of stresses or the distribution of stress over a section is entirely out of place.

In the Melan system of reinforcement steel ribs are used spaced about 3 feet on centers. Here the steel may be assumed to resist the bending moments, and the concrete the direct compression. The concrete also prevents the steel ribs from buckling. It is questionable if the above assumption actually obtains. It is well on the side of safety, however.

One of the simplest methods in use merely replaces the steel reinforcement by an equivalent area of concrete and then employs the formula given above.

If the modulus of steel is E_s and that of concrete E_c, then the equivalent area of concrete will be $\frac{E_s}{E_c}=n$ times the actual area of the steel. The fiber stress in the steel will actually be n times the fiber stress found for concrete in the position the steel occupies.

If a equals the area of the steel and A the area of the concrete, then

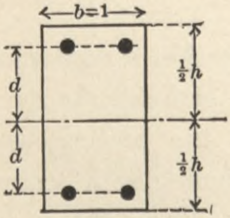

FIG. 16.

$$F=A+na,$$

$$I=\frac{h^3}{12}+nad^2,$$

and hence

$$p=\frac{N_x}{A+na}\pm M_x\frac{h}{\frac{h^3}{6}+2nad^2}.$$

This formula assumes that the concrete resists tensile stresses which it is not capable of doing to any great extent, its tensile strength being somewhere near one tenth the compressive strength.

The above formula may be applied until the maximum safe tensile strength of the concrete or steel is reached, and then the method employed for stone arches when the resultant pressure lies within the ring until the safe compressive strength is reached.

All of the methods are quite approximate for reasons given above, and since the modulus of elasticity of concrete is not constant.

32. Reliability of the Elastic Theory when Applied to Steel Ribs.—There is but little doubt that the theory is correct for solid steel ribs having a depth which is comparatively small when compared with the radius of curvature, when the loading is applied at isolated points through vertical posts which are unbraced in the plane of the rib. The modulus of elasticity of steel is quite constant and it is capable of resisting both tension and compression. The deformation of steel either under direct stress or bending follows very closely that found by theory. In truth the theory is probably as exact for steel arch ribs as the common theory of flexure is for steel beams.

33. Reliability of the Elastic Theory when Applied to Ribs Composed of Natural Stone Voussoirs.—Here we have a material which cannot be trusted in tension; this is especially true of the joints between the voussoirs. In direct compression the modulus of elasticity is not constant but varies with the load, and then not according to any very definite law. However, within narrow limits it may be considered as constant without serious error. Such being the case we may apply the elastic theory with confidence as long as the *equilibrium polygon lies within the middle third* of the ring, or when every section or joint is subjected to compressive stresses. We may also consider the theory as applicable when the polygon *lies within the ring*, provided the compression is not sufficient to crush the stone.

In case the equilibrium polygon passes without the ring at any joint, theoretically a free arch ring would fail. In practice this condition often obtains in stone bridges, yet they do not collapse or show serious signs of failure. It is true that some joints open slightly, but this appears to have little if any detrimental effect. This apparently proves that the elastic theory cannot be applied under such conditions. It is no fault in theory, but a failure to carry out in practice the assumptions made in applying the theory or basing the application of the theory upon wrong assumptions. For example, the elastic theory assumes a free rib capable of changing shape under various loads, while in practice the great majority of stone bridges have the ring securely clamped beneath the solid spandrel walls and by a mass of concrete backing of varying thickness. Such a structure may be said to become more and more stable under an increasing uniform loading, until the safe crushing strength of the arch stone is reached. This backing exerts a great passive force preventing any upward movement of the arch ring. It is evident, then, that if the ring is stable under the elastic theory assuming a free ring, it will be quite safe when clamped as explained above, and furthermore it does not necessarily follow, because the equilibrium polygon lies without the ring proper at some joint, that the arch will fail, for the spandrel masonry will prevent a change in shape of the rib to any great extent. The question at once presents itself: What does happen? Probably the masonry readjusts itself until equilibrium exists, the arch joints are compressed unequally, and the friction of the spandrel masonry aids very materially in reducing the opening or compression of the joints at the extrados—in fact introducing an effective tension or compression, as the case may be.

Again, in bridges having a considerable depth of side wall above the crown a large portion, if not all, of the ring under the walls might be removed in many cases without complete failure, the wall masonry forming an arch in itself. In conclusion, for the dead and live loads the arch ring which is safe when assumed to be a free ring will be safe under the usual construction of the spandrels, or if the loads are transmitted to the ring through verticals as in steel structures. All arch rings should be so designed, using a factor of safety of *ten* for the crushing strength of the stone.

Provision for the stresses produced by changes in temperature was entirely neglected by the old builders, and for that matter by practically all modern builders. A temperature change of but $\pm 40°$ F., according to the elastic theory, produces a very wide range of stress both of tension and compression. These are a maximum at the supports. If any considerable change of temperature actually occurs and the elastic theory can be correctly applied, the arch ring, if free, should collapse. As stone arch bridges have stood for thousands of years without failure, we must conclude that either the stone does not change in temperature through anything like the range of change in the air, or the arch ring adjusts itself with the aid of the spandrel masonry so as to resist the temperature stresses without excessive unit stresses, or the theory does not apply. Probably all three conclusions are more or less true. Even in the Northern States it is doubtful if any of the stonework, excepting possibly the more exposed surfaces, has a change of temperature of a great range,— ±20° F., say. The ring without any doubt adjusts itself to suit new conditions.

To show how small a change would be required in the mortar joints alone to provide for a change of 40° F., take a free rib of granite having a span of 60 ft. and a rise of 8 ft. (measurements taken for the axis). The length of the rib axis is 62.8 ft. The coefficient of expansion for 1° F. is 0.0000038. Then the total change in length of the rib is 0.0095 ft.; if there are 42 joints, the change in each joint would be 0.0002 ft. .0024 in., which is too small to be readily detected. Of course the joints do not all distort the same. Again, assume the rib under masonry spandrel walls, and let there be an increase of 40° F. in temperature, and also assume that the rib cannot rise; then the entire temperature effect must be used up in compressing the ring. The change in length per unit is $40(.0000038) = 0.00015$. If the $E = 6800000$, the stress per square inch is a little over 1000 lbs. This might be increased to 10000 lbs. without the granite being crushed, even with the dead- and live-load stresses added.

Considering our ignorance of the actual temperature changes and the behavior of the stone under these changes, it is useless to attempt any theoretical treatment until our knowledge of the subject has been very much increased. The temperature stresses appear to be able to take care of themselves as long as the rib is stable for the dead and live loads.

34. Reliability of the Elastic Theory when Applied to Plain Concrete Ribs.—Here we have a material which is fully as variable in its physical qualities as natural stone. Generally we have no joints to consider and no masonry spandrel backing, but we do have monolithic spandrel side walls clamping the rib, in many instances. As concrete resists tensile stresses but indifferently, it is not safe to permit more than *one tenth* its safe compressive strength in designing. As this amounts to about 50 lbs. per square inch, it may as well be neglected entirely, and the rib designed for the dead and live loads so that no tension can exist at any section.

The effects of changes of temperature are as uncertain as in stone arches. Having no joints, the ring cannot readily adjust itself, and hence probably resists some tension. As the modulus of elasticity is much less than for natural stone, and the coefficient of expansion but some 60% greater, the theoretical stresses are very much smaller. For free rings no tension exceeding 50 lbs. per square inch should be allowed under any conditions, unless the concrete is reinforced with steel to prevent cracking. At present there appears to be no rational way of determining the amount of steel required so all that can be done is to experiment and follow previous builders where they have been successful. If the rib should crack through, it would not necessarily mean failure, as then the behavior would follow that of a voussoir ring.

35. Reliability of the Elastic Theory when Applied to Reinforced Concrete Ribs.—Concrete when reinforced with steel is very much more reliable than concrete without the steel. The principal difficulty experienced is the location of the neutral axis of any particular section. The location without any doubt shifts about under the action of different loads. As the elastic theory assumes the arch axis to pass through the neutral axis of each section of the rib, it is evident that we must assume the axis to lie at the center of gravity of the section and treat the material according to the common theory of flexure.

While a reinforced rib will safely resist tension by virtue of the steel, yet the best designers so proportion the arch rib that it is never subjected to tension under dead and live loads. For temperature stresses the compression in the concrete must not exceed about 800 pounds per square inch, including the effect of the dead and live loads. Under this assumption the concrete may crack on the tension side, and the steel resist all of the tension.

Even when considering the difficulties briefly mentioned above and our almost absolute ignorance of the actual distribution of stress over a reinforced section, we are compelled to accept the elastic theory as our best guide in designing reinforced-concrete ribs.

36. Reliability of the Elastic Theory: Summary.—For steel ribs it is without doubt quite reliable. For natural stone, concrete, and reinforced concrete the theory can be used with confidence as long as no tensile stresses occur in the rib. When tensile stresses obtain the theory applied under the usual assumptions is but an approximation.

37. Depth of the Arch Rib.—This must be assumed from the best data available, and then calculations made to see if it will answer under all conditions of loading and changes of temperature. If found necessary, the rib can be modified somewhat without making new calculations by changing the moments of inertia of all sections in the same ratio. The dead- and live-load stresses will remain sensibly unchanged, the change in weight of the rib being very small in comparison with the total dead load. The temperature and axial thrust stresses will be slightly modified. The question of the necessity of a new calculation must be decided by the designer according to his best judgment. In Table II are given the data for a large number of arch ribs to aid in assuming the dimensions of a proposed design.

The articles immediately following give the principal empirical formulas for the dimensions of arch rings, etc.

38. Empirical Formulas for the Thickness of the Ring at the Crown in Stone Arches. — Many formulas have been advanced for the depth of the arch ring at the crown. These are usually based upon the dimensions of arches constructed, and hence they merely indicate that an arch built like one which has been standing some time will probably stand also.

NOMENCLATURE.

t_0 = depth of arch ring at the crown, in feet;
R = radius of curvature of intrados at the crown, in feet;
l = clear span of arch, in feet;
f = clear rise of arch, in feet.

*Trautwine's Formulas.**—The following formulas apply to circular and elliptical arches:

For first-class cut stone:

$$t_0 = 0.25\sqrt{R + 0.5l} + 0.2.$$

For second-class work:

$$t_0 = 0.281\sqrt{R + 0.5l} + 0.225.$$

For brickwork or fair rubble:

$$t_0 = 0.333\sqrt{R + 0.5l} + 0.267.$$

Low's Formula: †

$$t_0 = 0.125\sqrt{10(l - f) + 2H},$$

where H = the surcharge above the extrados at the crown.

Rankine's Formulas:

$$t_0 = \sqrt{0.12R} \text{ for a single arch;}$$

$$t_0 = \sqrt{0.17R} \text{ for an arch in a series.}$$

Perronnet's Formula for circular or elliptical arches:‡

$$t_0 = 1 + 0.035l.$$

Dejardin's Formulas for circular arches:‡

For $\frac{f}{l} = \frac{1}{2}$. . . . . $t_0 = 1 + 0.10R.$

For $\frac{f}{l} = \frac{1}{6}$. . . . . $t_0 = 1 + 0.05R.$

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* Trautwine's "Engineer's Pocket-book."

† Engineering News, June 15, 1905.

‡ From paper by E. Sherman Gould, Van Nostrand's Mag., vol. xxix, p. 450. 1883.

For $\frac{f}{l}=\frac{1}{8}$ .... $t_0=1+0.035R.$

For $\frac{f}{l}=\frac{1}{10}$ ...... $t_0=1+0.02R.$

Dejardin's Formula for elliptical and basket-handled arches:*

For $\frac{f}{l}=\frac{1}{3}$ ...... $t_0=1+0.07R.$

*Croizette-Desnoyer's Formulas:**

For $\frac{f}{l}>\frac{1}{6}$ .... $t_0=0.50+0.28\sqrt{2R}.$

For $\frac{f}{l}=\frac{1}{6}$ .... $t_0=0.50+0.26\sqrt{2R}.$

For $\frac{f}{l}=\frac{1}{12}$ ...... $t_0=0.50+0.20\sqrt{2R}.$

For elliptical arches use R for circle having same rise and span.

*German and Russian Practice:**

$$t_0=1+0.035l+0.02H,$$

where H = the surcharge over the extrados at the crown, including the moving load if any.

Austrian Specifications for large arches of brick and stone: †

f/l between $\frac{1}{2}$ and $\frac{2}{5}$.

For $l=$ 30 metres.... $t_0=1.1$ m.

For $l=$ 40 " .... $t_0=1.4$ "

For $l=$ 65 " ... $t_0=2.2$ "

* From paper by E. Sherman Gould, Van Nostrand's Mag., vol. xxix, p. 450. 1883.

† "A Treatise on Arches," by Malverd A. Howe. Wiley.

For $l =$ 80 metres . . . . $t_0 = 2.7$ m
For $l = 100$ " . . . . $t_0 = 3.4$ "
For $l = 120$ " . . . . $t_0 = 4.1$ "

39. Thickness of Arch Ring of Stone at the Support.—For semicircular stone arches it is generally assumed that the masonry for 30° from the spring line is self-supporting and consequently has no arch action. If this is so, then the maximum angle which a stone arch ring can be considered to subtend is 60° each way from the crown. If the loading is so arranged that the equilibrium polygon follows the axis of the ring, then the pressures will vary directly as the secant of the angle ϕ; consequently the ring thickness 60° from the crown should be $t_s = t_0 \sec 60° = 2t_0$.

* *Croizette-Desnoyer's Formulas for segmental arches:*

For $\frac{f}{l} = \frac{1}{6}$ . . . . $t_s = 1.40t_0$.

For $\frac{f}{l} = \frac{1}{8}$ . . . . $t_s = 1.24t_0$.

For $\frac{f}{l} = \frac{1}{10}$ . . . . $t_s = 1.15t_0$.

For $\frac{f}{l} = \frac{1}{12}$ . . . . $t_s = 1.10t_0$.

For basket-handled arches:

when $\frac{f}{l} = \frac{1}{3}$ . . . . . $t_s = 1.80t_0$;

$\frac{f}{l} = \frac{1}{4}$ . . . . . $t_s = 1.60t_0$;

$\frac{f}{l} = \frac{1}{5}$ . . . . . $t_s = 1.40t_0$.

* Van Nostrand's Engineering Magazine, vol. xxix, p. 454.

40. Thickness of Abutment. — Trautwine's rule for all kinds of stone arches is best explained by means of a diagram, Fig. 17. This form of abutment, according to

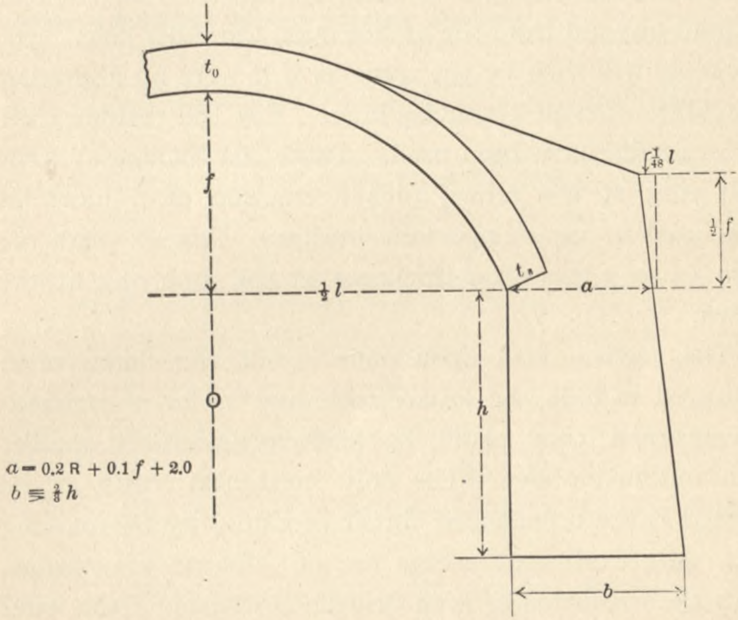

FIG. 17.

Trautwine, is sufficiently strong to take the thrust due to the dead load before the back filling of earth is in place.

Rankine states that in existing structures the thickness a varies from $\frac{1}{3}$ to $\frac{1}{5}$ the radius of the intrados at the crown.

Baker, in "A Treatise on Masonry Construction," gives a formula, said to represent *German* and *Russian* practice, which has the form

$$a = 1 + 0.04(5l + 4h),$$

where h is the distance from the spring line down to the top of the foundation.

41. Thickness of Piers.—In a series of arches it is customary to use several narrow piers and then introduce a much heavier pier, called an abutment pier. This should be of sufficient strength to resist the thrust from one side without any aid from the arches upon the other side. The thickness will then be the same as if it were an abutment in reality without earth backing. For the regular piers various rules have been used. Twice the thickness of the arch ring at the crown plus a fraction of a foot has been used in very important bridges. Usually piers are from $2\frac{1}{2}$ to 3 times the thickness of the arch ring at the crown.

The vertical load upon piers is not very large when measured in tons per square foot, and as far as strength is concerned they could be made considerably smaller than outlined above. The only horizontal thrust to be resisted is the unbalanced thrust produced by the moving load, unless adjacent arches are of different dimensions. With the exception of high bridges the effect of the wind is of no moment.

42. Remarks concerning Empirical Formulas.—The formulas given in the previous articles are based, for the most part, upon actual structures and will without doubt lead to safe structures if the equivalents in materials and workmanship are held throughout. Apparently the formulas apply to all kinds of stone, as no mention is made of the quality of the materials (excepting Trautwine's formulas) used. Unquestionably the arch rings were constructed of average materials, probably no better if as good as those used now; hence the formulas will be of service in assuming dimensions which can be relied upon as being safe for *structures quite similar to those upon which the formulas are based.*

43. **Albula Railroad Practice*** (gage 1 m.).—The following dimensions were used in the construction of a great number of arches on the Albula Railroad.

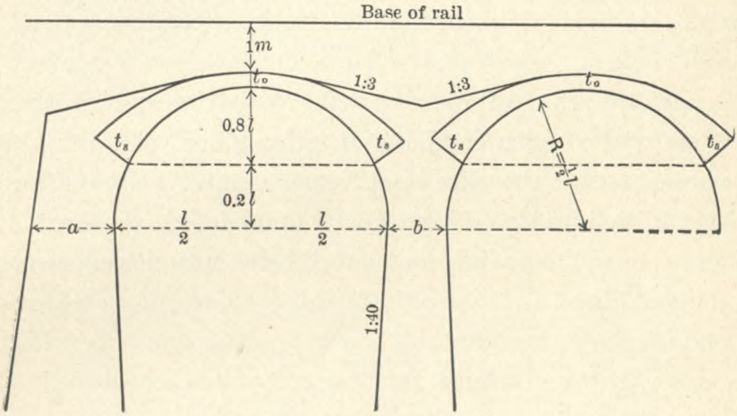

FIG. 18.

Span. l	= 6	8	10	12	15	20	25
Key. . . . . t_0	= 0.55	0.60	0.70	0.75	0.80	0.90	1.00
Spring. . . . . t_s	= 0.80	0.90	1.00	1.10	1.20	1.35	1.50
Pier. . . . . b	= 1.20	1.35	1.50	1.70	2.10	2.70	3.60
Abutment. . . . . . . a	= 1.70	1.90	2.10	2.80	3.50	4.20	5.30

Twenty-six viaducts were built of the spans given below:

Span. l	= 10	11	12	14	15	16	20	22	25	27	30	42
Number of spans.	= 33	3	7	1	16	14	15	1	1	1	1	1

(All dimensions are in metres.)

44. **The Dead Load.**—Very little is required in the way of discussion in reference to the dead load for steel ribs. The floor and all supports, and even the lateral systems,

* The Engineer, 1904.

can all be designed and the actual weights computed. There remains, then, only the weight of the rib proper to be estimated. The weight of the assumed rib will be sufficiently close for all purposes, as a large error in the weight of the rib will be comparatively small for the entire load. The weight above the rib is usually transmitted to the rib through verticals extending up to the roadway.

In the case of masonry ribs with the spandrels completely filled with earth, sand, gravel, etc., the actual load supported by the rib is not very definite. If the filling is put in in horizontal layers well compacted, the load upon the ring will certainly not exceed the actual weight of the material, and it is very doubtful if such filling creates any considerable horizontal thrust against the rib. If perfectly dry and clean sand or gravel is employed, then there may be horizontal forces acting against the rib. These will be very small, however, for segmental arches. This thrust can be found according to the theory of earth pressure.*

The consideration of the horizontal thrust of the spandrel filling is a refinement not warranted in works of this class. The weight of the spandrel filling with pavement, arch rib, etc., should be considered as divided into vertical loads, the horizontal projection of δs being the measure of each division. For computations the load may be assumed to act at the center of the projection of δs.

In case the spandrels are partially filled with concrete its weight may be taken as divided into vertical forces.

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* "Retaining-walls for Earth," by Malverd A. Howe. John Wiley & Sons, New York.

This is probably not as near the truth as when the fill is made of sand or gravel, but the assumption is on the safe side. Overloading the haunches will cause an upward movement at the crown, and overloading the crown causes the haunches to rise; but when the spandrel filling is partially concrete the *passive* resistance to an upward movement is very much in excess of its weight; so also is that of sand or gravel. The arch rib, then, in this type of bridge is anything but a free member, and consequently any great refinement in its design is time wasted. If we can assure ourselves that the rib is safe by adding a few inches to the thickness of the ring, the very small percentage of extra cost need not be considered at all.

When the roadway is supported by longitudinal walls resting upon the rib, the problem is at least as complex as before, for there is no way of knowing how the weights transmitted by the walls are distributed. The only recourse is to treat the material as in the case of sand or gravel filling.

The use of lateral walls or columns to support the roadway places the problem in a shape to be carefully considered theoretically. The actual magnitudes of the loads can be computed and the points of application to the rib are definitely fixed. For long spans this is unquestionably the best and most economical type which can be built. There is an exception to this in very flat arches where the ring occupies the greater portion of the vertical projection of the bridge.

45. Dead-load Equilibrium Polygon Following the Axis of the Arch Rib.—It is assumed that the rib has been dimensioned and that the fill over the crown is known. Compute the weight of the shaded portion in Fig. 19 and call it P_1. Lay off the vertical line DE, and P_1 from D. Draw DO horizontal and CO parallel to the tangent to the arch axis at b. Then from O draw lines parallel to the tangents at c, d, e, etc.; then these lines will cut off on DE the loads P_2, P_3, etc., for which an equilibrium polygon will pass

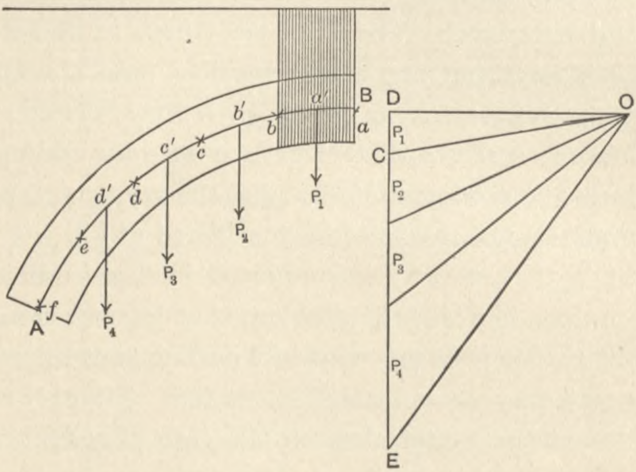


FIG. 19.

through the points a, b, c, d, etc., and DO will be the horizontal thrust for this loading. A check calculation will show that this is the true horizontal thrust according to the elastic theory, *neglecting the effect of the axial stress.*

By a similar construction the polygon may be made to pass through the points where the loads are applied to the axis. In either case the bending moments due to the dead load are sensibly zero. This assumes that the loads are reasonably close together.

Filled spandrels can usually be made so that the above conditions are fulfilled by selecting proper filling materials.

CHAPTER III.

EXAMPLES SHOWING THE APPLICATION OF THE FORMULAS, ETC.

46. **Preliminary.** — In the examples which follow, the computations will be given in detail, with suggestions as to methods and checks. In some cases it will be found that the algebraic work necessary to get the data into shape for applying the arch theory requires as much time as the computation of H_1 for each load respectively. Some of this work will be found quite unnecessary by many. It is given in one case for the benefit of the few who may use the example as a guide for their first arch calculation.

47. **First Example: Data.** — Let us assume that the design shall be for a single-track railway bridge with an arch ring of Quincy, Mass., granite, and that the axis of the ring has a span of 60 ft. and a rise of 8 ft. Let the spandrel filling be cinders, sand, or gravel, in such proportions that the total dead load will have its equilibrium polygon following the axis of the ring. Since this is to be a railway bridge, there should be at least 3 ft. of fill between the base of the rail and the arch ring at the crown. This will distribute the moving load which may be assumed at 5000 lbs. per foot of span. If the ties are 8 ft. long, we may assume that the fill will distribute 5000 lbs. over at

least 13 ft. under the ties, or that the moving load will be about 400 lbs. per square foot. 30 lbs. per square foot will cover the weight of the track.

48. Subdivision of the Arch Axis.—This should not be decided upon until the shape of the arch ring is determined. In this case let the ring be of uniform depth throughout; then, in order that Δ may be constant, the *axis* should be divided into *equal parts*. In all summation formulas it is well known that the smaller the divisions are made the more accurate will be the results. In this

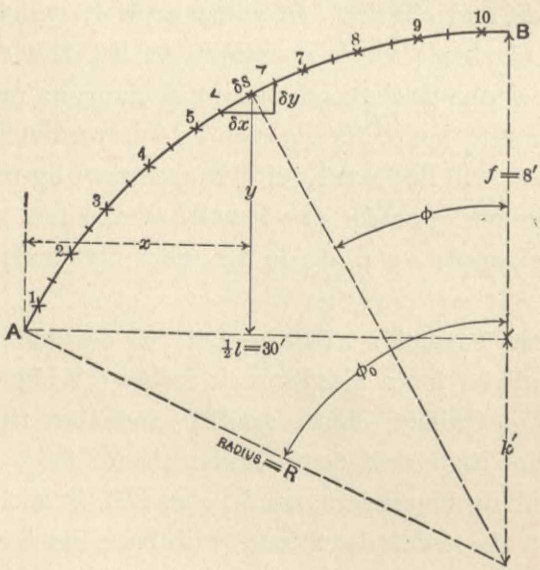


FIG. 20.

particular case δs might be replaced by ds, and the problem solved, as far as H_1, M_1, etc., are concerned, by means of integration.

Ordinarily *twenty* divisions will give results sufficiently accurate for practical purposes. This number will be used.

From Fig. 20,

$$\tfrac{1}{2}l = 30 = \sqrt{R^2 - (R-8)^2}. \quad \therefore\ R = 60.25 \text{ ft., and } k' = 52.25 \text{ ft.}$$

$$\sin \phi_0 = \frac{\frac{1}{2}l}{R} = \frac{30}{60.25} \quad \therefore\ \phi_0 = 29^\circ\ 51'.76.$$

$$\text{Arc } AB = \frac{2\pi R}{360} 29.8616 = 31.40 \text{ ft.}$$

Hence $\delta s = 3.14$ ft., and the angle at the center for each division is $2^\circ.98616$.

49 Computation of x and y.—The values of x and y are computed for the center points of the divisions made above, as shown in detail in Table A.

TABLE A.

Point.	ϕ.	sin ϕ.	cos ϕ.	R sin ϕ.	R cos ϕ.	x $30 - R \sin \phi$.	y $R \cos \phi - 52.25$.
1	28° 22'.172	0.47515	0.87991	28.628	53.015	1.372	0.765
2	25 22 .996	.42867	.90346	25.827	54.433	4.173	2.183
3	22 23 .820	.38102	.92457	22.956	55.705	7.044	3.455
4	19 24 .644	.33234	.94316	20.023	56.625	9.977	4.575
5	16 25 .468	.28275	.95919	17.036	57.791	12.964	5.541
6	13 26 .292	.23239	.97262	14.001	58.600	15.999	6.350
7	10 27 .116	.18141	.98335	10.930	59.247	19.070	7.000
8	7 27 .940	.12993	.99152	7.828	59.739	22.172	7.489
9	4 28 .764	.07810	.99694	4.705	60.065	25.295	7.815
10	1 29 .588	.02606	.99966	1.570	60.229	28.430	7.979
C	0 0	0	1.00000	0	60.25	30.000	8.000

In this particular case we are probably not warranted in using three decimal places in the values of x and y, although the labor is but a very little greater than if but two were used. This is assuming that multiplications are performed by machine or a multiplication-table. After a little practice Crelle's "Rechentafeln" will be found quite satisfactory for all multiplications and many divisions.

50. Computation of H_1 for Unit Loads.—Table B gives in detail the calculations for H_1 corresponding to a unit load at each point respectively. Since the arch and the loading are symmetrical, the summations have been made from $x=0$ to $x=\frac{1}{2}l$.

In column 2 the positive and negative values of $y-y_a$ should sum up the same. As the fourth decimal place has been neglected, the sums differ by 3 in the third decimal place. The method of Art. 14 has been employed in computing m_x, which requires the use of but *ten* multipliers (the values of x) and *fifty-five* multiplications in the complete determination of H_1 for each load.

For the first load each value of $y-y_a$ is multiplied by the first value of x, and therefore, since $\Sigma(y-y_a)=0$, $\Sigma m_x(y-y_a)$ should be zero, and consequently the value of $H_1=0$ for this load. Using the figures shown in the table, $H_1=.000035$ for $P_1=$unity, which is zero for all practical purposes.

The true values of $\Sigma y(y-y_a)$ and $\Sigma m_x(y-y_a)$ are *twice* the numerical values given in the table, but since one expression is in the denominator and the other in the numerator the common factor zero has been neglected.

The method employed in Table B is considerably longer than necessary, but has been used on account of its clearness and because all sums are taken between the same limits.

TABLE B.

COMPUTATION OF H_1 FOR UNIT LOADS.

Point.	1	2	3	4		5		6	
				$P_1 = 1 = P_1'$.		$P_2 = 1 = P_2'$.		$P_3 = 1 = P_3'$.	
	Values of y.	$y - y_a$.	$y(y - y_a)$.	m_x.	$m_x(y - y_a)$.	m_x.	$m_x(y - y_a)$.	m_x.	$m_x(y - y_a)$.
0									
1	.765	− 4.550	− 3.481	1.372	− 6.243	1.372	− 6.243	1.372	− 6.243
2	2.183	− 3.132	− 6.828	1.372	− 4.297	4.173	−13.070	4.173	−13.070
3	3.455	− 1.860	− 6.426	1.372	− 2.552	4.173	− 7.762	7.044	−13.102
4	4.575	− .740	− 3.386	1.372	− 1.015	4.173	− 3.088	7.044	− 5.213
5	5.541	.226	1.252	1.372	.310	4.173	.943	7.044	1.592
6	6.350	1.035	6.572	1.372	1.420	4.173	4.319	7.044	7.291
7	7.000	1.685	11.795	1.372	2.312	4.173	7.032	7.044	11.869
8	7.489	2.174	16.281	1.372	2.983	4.173	9.072	7.044	15.314
9	7.815	2.500	19.538	1.372	3.430	4.173	10.433	7.044	17.610
10	7.979	2.665	21.267	1.372	3.656	4.173	11.121	7.044	18.772
	53.152 Σy	−10.282 +10.285	−20.121 +76.705	13.720 Σm_x	−14.107 +14.111	38.929 Σm_x	−30.163 +42.920	61.897 Σm_x	−37.628 +72.448
	$y_a = \frac{\Sigma y}{10}$ $= 5.315$		+56.584 $\Sigma y(y - y_a)$		+ .004 $\Sigma m_x(y - y_a)$		+12.757 $\Sigma m_x(y - y_a)$		+34 820 $\Sigma m_x(y - y_a)$
$\Sigma m_x \div 10$ see p 60				1.372		3.893		6.190	
$H_1 = \frac{1}{2} \frac{\Sigma m_x(y - y_a)}{\Sigma y(y - y_a)} =$					0.000035		0.1125		0.3075

TABLE B.—COMPUTATION OF H_1 FOR UNIT LOADS—(*Continued*).

Point.	7		8		9		10	
	$P_4=1=P_4'$.		$P_5=1=P_5'$.		$P_6=1=P_6'$.		$P_7=1=P_7'$.	
	m_x.	$m_x(y-y_a)$.	m_x.	$m_x(y-y_a)$.	m_x.	$m_x(y-y_a)$.	m_x.	$m_x(y-y_a)$.
0								
1	1.372	− 6.243	1.372	− 6.243	1.372	− 6.243	1.372	− 6.243
2	4.173	−13.070	4.173	−13.070	4.173	−13.070	4.173	−13.070
3	7.044	−13.102	7.044	−13.102	7.044	−13.102	7.044	−13.102
4	9.977	− 7.383	9.977	− 7.383	9.977	− 7.383	9.977	− 7.383
5	9.977	2.255	12.964	2.930	12.964	2.930	12.964	2.930
6	9.977	10.326	12.964	13.418	15.991	16.551	15.991	16.551
7	9.977	16.811	12.964	21.844	15.991	26.945	19.070	32.133
8	9.977	21.690	12.964	28.184	15.991	34.764	19.070	41.458
9	9.977	24.943	12.964	32.410	15.991	39.978	19.070	47.675
10	9.977	26.589	12.964	34.549	15.991	42.616	19.070	50.822
	82.428 Σm_x	−39.798 +102.614	100.35 Σm_x	−39.798 +133.335	115.485 Σm_x	−39.798 +163.784	127.801 Σm_x	−39.798 +191.569
		+62.816 $\Sigma m_x(y-y_a)$		+93.537 $\Sigma m_x(y-y_a)$		+123.986 $\Sigma m_x(y-y_a)$		+151.771 $\Sigma m_x(y-y_a)$
$\frac{\Sigma m_x}{10}$	8.243		10.03		11.55		12.78	
H_1		.555		.8265		1.0955		1.341

TABLE B.—COMPUTATION OF H_1 FOR UNIT LOADS—(*Concluded*).

Point.	11		12		13		14	
	$P_8 = 1 = P_8'$.		$P_9 = 1 = P_9'$.		$P_{10} = 1 = P_{10}'$.		P = unity.	
	m_x.	$m_x(y-y_a)$.	m_x.	$m_x(y-y_a)$.	m_x.	$m_x(y-y_a)$.	Load.	H_1.
0								
1	1.372	− 6.243	1.372	− 6.243	1.372	− 6.243	P_1 or P_1'	0
2	4.173	−13.070	4.173	−13.070	4.173	−13.070	P_2 " P_2'	0.1125
3	7.044	−13.102	7.044	−13.102	7.044	−13.102	P_3 " P_3'	0.3075
4	9.977	− 7.383	9.977	− 7.383	9.977	− 7.383	P_4 " P_4'	0.555
5	12.964	2.930	12.964	2.930	12.964	2.930	P_5 " P_5'	0.8265
6	15.991	16.551	15.991	16.551	15.991	16.551	P_6 " P_6'	1.0955
7	19.070	32.133	19.070	32.133	19.070	32.133	P_7 " P_7'	1.341
8	22.172	48.202	22.172	48.202	22.172	48.202	P_8 " P_8'	1.5425
9	22.172	55.430	25.295	63.238	25.295	63.238	P_9 " P_9'	1.685
10	22.172	59.088	25.295	67.411	28.430	75.766	P_{10} " P_{10}'	1.7585
	137.107 Σm_x	−39.798 +214.334	143.333 Σm_x	−39.798 +230.465	146.488 Σm_x	−39.798 +238.820		
		+174.536 $\Sigma m_x(y-y_a)$		+190.667 $\Sigma m_x(y-y_a)$		+199.022 $\Sigma m_x(y-y_a)$		
$\frac{\Sigma m_x}{10}$	13.71		14.33		14.65			
H_1		1.5425		1.685		1.7585		

Table B also contains the values of $\Sigma m_x \div n$, which will be used in computing the values of M_1. Having the values of H_1 for unit loads, its value for any other load is simply the product of the load by the values given in Table B.

51. Computation of M_1, V_1, y_1, y_2, and y_0 for Unit Loads.—The formula for M_1 is, Art. 25,

$$M_1 = H_1 y_a - \frac{\Sigma m_x x - \Sigma m_x \frac{\Sigma x^2}{\Sigma x}}{n\left(\frac{1}{2}l - \frac{\Sigma x^2}{\Sigma x}\right)},$$

in which H_1 and y_a are known from Table B. $\Sigma x = \frac{1}{2}nl = \frac{1}{2}(20)60 = 600$. There remains to be found the value of m_x at each point for each load and also the value of Σx^2. Of course Σx^2 can be found by squaring each value of x, but this is rather tiresome, as there are twenty different values. The following method will be found shorter and easier and at the same time a portion of the work in computing m_x will be done.

Taking any symmetrical values of x, that is, the values of x for points 1 and 1′, say,

$$\begin{aligned} x^2 + x_1^2 &= x^2 + (l-x)^2 = x^2 + l^2 - 2lx + x^2 \\ &= l^2 - 2x(l-x) \\ &= l^2 - 2x(\tfrac{1}{2}l - x) - lx. \end{aligned}$$

Then, for all points,

$$\Sigma(x^2 + x_1^2) = \frac{nl^2}{2} - 2\sum_0^{\frac{1}{2}l} x(\tfrac{1}{2}l - x) - l\sum_0^{\frac{1}{2}l} x = \Sigma x^2,$$

$$\frac{nl^2}{2} = \tfrac{1}{2}(20)(60)^2 = 36000,$$

$$-2\sum_0^{\frac{1}{2}l} x(\tfrac{1}{2}l-x) = -2(1498.987) = -2997.974, \quad \text{(Table C.)}$$

$$-l\sum_0^{l/2} x = -(60)(146.48) = -8788.800. \quad \text{(Table C.)}$$

$$\therefore \ \Sigma x^2 = 24213.23 \quad \text{and} \quad \frac{\Sigma x^2}{\Sigma x} = 40.3554.$$

The denominator in the equation for M_1 now becomes

$$20(30-40.3554) = -207.1075.$$

The next step is the determination of m_x at each point for each load. This can be done by constructing an equilibrium polygon for each load and scaling the proper ordinates, which leads to $10 \times 20 = 200$ separate quantities and then 200 multiplications when $m_x x$ is found.

$\Sigma m_x x$ can be found as follows:

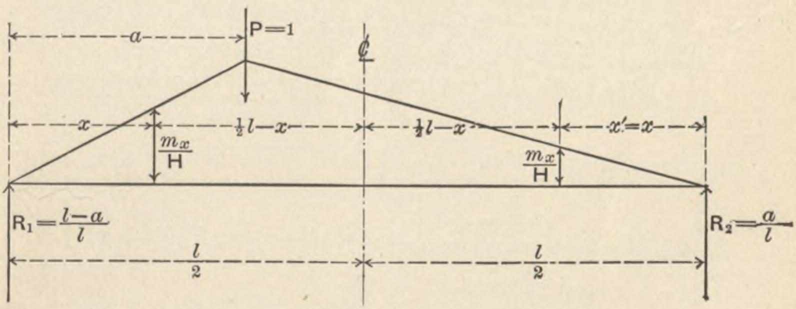


FIG. 21.

From Fig. 21 for load unity,

$$R_1 = \frac{l-a}{l}, \qquad R_2 = \frac{a}{l}.$$

From $x=0$ to $x=a$, From $x=a$ to $x=l$,

$$m_x = R_1 x = \frac{l-a}{l}x. \qquad m_x = R_2(l-x) = \frac{a}{l}(l-x).$$

Now

$$m_x x = m_x[\tfrac{1}{2}l - (\tfrac{1}{2}l - x)] = \tfrac{1}{2}m_x l - m_x(\tfrac{1}{2}l - x).$$

Therefore

$$\Sigma m_x x = \tfrac{1}{2}l\Sigma m_x - \Sigma m_x(\tfrac{1}{2}l - x).$$

The value of $\frac{1}{2}\Sigma m_x$ is given in Table B for two equal and symmetrical loads. This value is equal to Σm_x for a single load. This quickly disposes of $\frac{1}{2}l\Sigma m_x$.

The value of $\Sigma m_x(\frac{1}{2}l - x)$ can be found quite easily by remembering that for $m_x(\frac{1}{2}l - x)$ upon the left there will be an $m_x'(\frac{1}{2}l - x)$ upon the right but *opposite in sign until* $x = a$. For $x < a$ and $x = l - x$,

$$\begin{aligned}(m_x + m_x')(\tfrac{1}{2}l - x) &= R_1 x(\tfrac{1}{2}l - x) - R_2 x(\tfrac{1}{2}l - x) \quad (x < a)\\ &= (R_1 - R_2)x(\tfrac{1}{2}l - x);\end{aligned}$$

hence

$$\sum_{x=0}^{x=a}(m_x + m_x')(\tfrac{1}{2}l - x) = (R_1 - R_2)\sum_{x=0}^{x=a} x(\tfrac{1}{2}l - x).$$

For $x = a$ to $x = l - a$,

$$\sum_{x=a}^{x=l-a} m_x(\tfrac{1}{2}l - x) = \sum_{x=a}^{x=\frac{1}{2}l-a}\{R_2(l - 2x)(\tfrac{1}{2}l - x) = 2R_2(\tfrac{1}{2}l - x)^2\}.$$

Then

$$\Sigma m_x(\tfrac{1}{2}l - x) = (R_1 - R_2)\sum_{x=0}^{x=a} x(\tfrac{1}{2}l - x) + 2R_2\sum_{x=a}^{x=\frac{1}{2}l}(\tfrac{1}{2}l - x)^2.$$

With the above explanations, Table C becomes very simple and gives us all of the coefficients required in treating vertical loads. In col. 21 the values of M_1 give also the values of M_2 by merely numbering the points 1′, 2′,

TABLE C.

COMPUTATION OF M_1, V_1, AND y_1.

Point.	1	2	3	4	5	6	7	8
	R_1.	R_2.	x.	$\frac{1}{2}l-x$.	$x(\frac{1}{2}l-x)$.	$(\frac{1}{2}l-x)^2$.	$\sum_0^a x(\frac{1}{2}l-x)$.	R_1-R_2.
1	.977	.023	1.37	28.63	39.223	819.667	39.223	.954
2	.931	.069	4.17	25.83	107.711	667.189	146.934	.862
3	.883	.117	7.04	22.96	161.638	527.162	308.572	.766
4	.834	.166	9.98	20.02	199.800	400.800	508.372	.668
5	.784	.216	12.96	17.04	220.838	290.362	729.210	.568
6	.733	.267	16.00	14.00	224.000	196.000	953.210	.466
7	.682	.318	19.07	10.93	208.435	119.465	1161.645	.364
8	.631	.369	22.17	7.83	173.591	61.309	1335.236	.262
9	.578	.422	25.29	4.71	119.116	22.184	1454.352	.156
10	.526	.474	28.43	1.57	44.635	2.465	1498.987	.052

Point.	9	10	11	12	13	14	15	16
	Column 7 times Column 8.	$\sum_a^{\frac{1}{2}l}(\frac{1}{2}l-x)^2$.	$2R_2$.	Column 10 times Column 11.	Column 9 plus Column 12, $\Sigma m_x(\frac{1}{2}l-x)$.	$\Sigma m_x \frac{1}{2}l$.	$H_1 y_a$.	Σm_x. Table B.
1	37.419	2286.936	.046	105.199	142.618	411.60	.000	13.720
2	126.657	1619.747	.138	223.525	350.182	1167.87	.598	38.929
3	236.366	1092.585	.234	255.655	492.031	1856.91	1.634	61.897
4	339.592	691.785	.332	229.673	569.265	2472.84	2.950	82.428
5	414.191	401.423	.432	173.415	587.606	3010.50	4.393	100.350
6	444.196	205.423	.534	109.696	553.892	3464.55	5.823	115.485
7	422.839	85.958	.636	54.669	477.508	3834.03	7.127	127.801
8	349.832	24.649	.738	18.191	368.023	4113.21	8.198	137.107
9	226.879	2.465	.844	2.080	228.959	4299.99	8.956	143.333
10	77.947	....	.948	....	77.947	4394.64	9.346	146.488

TABLE C.—COMPUTATION OF M_1, V_1, AND y_1—(*Concluded*).

Point.	17	18	19	20	21	22	23	24
	$\frac{\Sigma m_x x}{-207.1075}$.	$\Sigma m_x x$. Col. (14 − 13).	$\Sigma m_x \frac{\Sigma x^2}{\frac{n}{2} lD}$.	$-m_1$. Col. (19 − 17).	M_1. Col. (15 − 20).	y_1.	V_1.	y_0.
1	− 1.299	268.982	− 2.673	1.374	− 1.374		1.000	
2	− 3.948	817.688	− 7.583	3.635	− 3.037	− 26.996	.987	9.588
3	− 6.590	1364.879	− 12.058	5.468	− 3.834	− 12.468	.962	
4	− 9.191	1903.575	− 16.057	6.866	− 3.916	− 7.056	.926	9.595
5	− 11.699	2422.894	− 19.548	7.849	− 3.456	− 4.181	.879	
6	− 14.054	2910.658	− 22.496	8.442	− 2.619	− 2.391	.822	9.614
7	− 16.207	3356.522	− 24.896	8.689	− 1.562	− 1.165	.759	
8	− 18.083	3745.187	− 26.708	8.625	− .427	− .277	.690	9.639
9	− 19.657	4071.031	− 27.921	8.264	.692	.411	.615	
10	− 20.843	4316.693	− 28.536	7.693	1.653	.940	.539	9.654
10′	− 21.595	4472.587	− 28.536	6.941	2.405	1.368	.461	
9′	− 21.868	4528.949	− 27.921	6.053	2.903	1.723	.385	
8′	− 21.637	4481.233	− 26.708	5.071	3.127	2.027	.310	
7′	− 20.818	4311.538	− 24.896	4.078	3.049	2.274	.241	
6′	− 19.403	4018.442	− 22.496	3.093	2.730	2.492	.178	
5′	− 17.373	3598.106	− 19.548	2.175	2.218	2.684	.121	
4′	− 14.689	3042.105	− 16.057	1.368	1.582	2.850	.074	
3′	− 11.342	2348.941	− 12.058	.716	.918	2.985	.038	
2′	− 7.330	1518.052	− 7.583	.253	.345	3.067	.013	
1′	− 2.676	554.218	− 2.673	.003	.003	....	.000	

$$M_1 = H_1 y_a - \frac{\Sigma m_x \left(x - \frac{\Sigma x^2}{\Sigma x}\right)}{n\left(\frac{1}{2} l - \frac{\Sigma x^2}{\Sigma x}\right)} = H_1 y_a - \frac{\Sigma m_x}{-207.1075} + \Sigma m_x \frac{\Sigma x^2}{\frac{n}{2} lD}.$$

$$D = -207.1075.$$

$$y_1 = \frac{M_1}{H_1}; \qquad y_0 = y_1 + \frac{V_1 a}{H_1};$$

$$V_1 = \frac{M_2 - M_1}{L} + R_1.$$

3′, etc. The quantities in cols. 22 and 23 reversed give y_2 and V_2 respectively. Col. 24 shows how nearly constant y_0 is in this case.

For mathematical accuracy the value of V_1 for a load at point 1 should be unity as given in Table C, which evidently is not the actual condition. When the first point is quite near the support, however, the value of V_1 approaches unity very nearly.

In col. 22 the value of y_1 for point 1 is not given, since it is not possible to obtain its value directly from the formula $M_1 \div H_1 = y_1$, as H_1 is zero. The same is true for point 1′. This will be the condition whenever graphical or algebraic summation methods are used. This difficulty does not occur in integration formulas. Fortunately, the peculiarity of the summation methods is of no practical importance if δs is not assumed too great. The defect is quite marked where ribs have a much greater depth at the springing than at the crown, and δs is so taken that everywhere $\delta s \div I$ is constant.

52. Depth of Ring and the Dead Load.—An examination of Table II shows that a number of railway bridges have been constructed with spans of about 60 feet with arch rings 3 feet deep. Let this be assumed as the depth of the ring.

The load at point 10 can be found as follows: Divide the vertical projection of the arch as shown in Fig. 22, and carefully scale the distances *ab*, *bc*, and *de*. Then the weight of the ring at point 10 is $[(bc)(de) = (3.00)(3.14)]170 = 1601$ pounds, taking granite at 170 lbs. per cubic foot. Assume the fill to be made of material weighing 95 lbs. per cubic foot, then the weight at 10 is $(3.02)(3.14)95 = 905$ lbs., say. The weight from the track is $(3.14)30 = 94$ lbs.

The total dead load at 10 now is $1601 + 905 + 94 = 2600$ pounds. In order that the equilibrium polygon shall pass

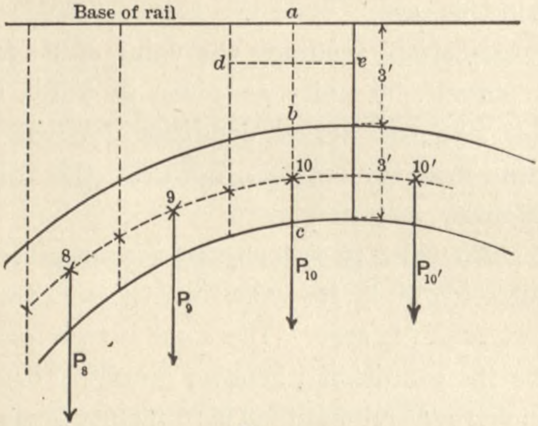


FIG. 22.

through 10 and 9, Fig. 23, the pole distance must be

$$H = \frac{2600}{\tan 2^\circ - 59'.2} = \frac{2600}{0.052} = 50000.$$

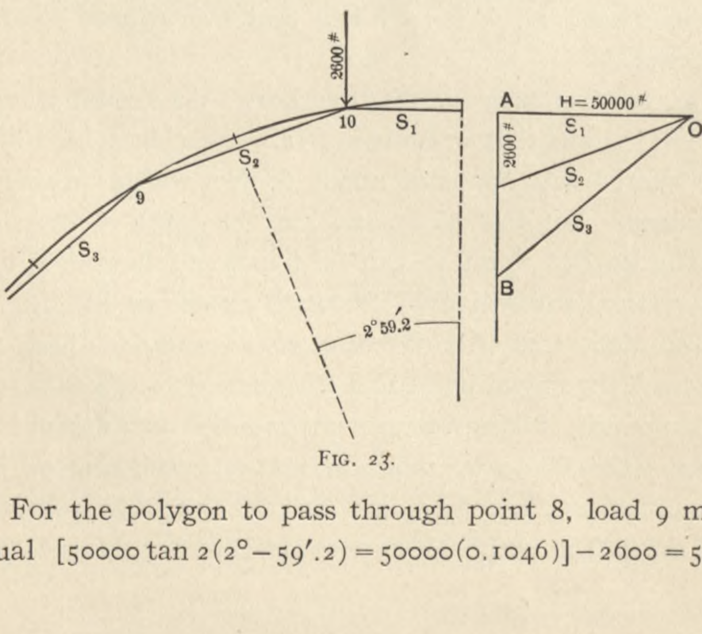


FIG. 23.

For the polygon to pass through point 8, load 9 must equal $[50000 \tan 2(2^\circ - 59'.2) = 50000(0.1046)] - 2600 = 5250$

$-2600 = 2650$ pounds. In like manner all loads may be computed, or obtained by drawing strings parallel to the chords connecting the points of division as indicated in Fig. 23.

COMPUTATION OF DEAD LOAD.

Point.	ϕ.	tan ϕ.	50000 tan ϕ.	Dead Load P.	Unit H_1, Table B.	Computed H_1.
1	29° 51′.760	0.574	28700	3350	0	0
2	26 52 .584	0.507	25350	3200	0.113	362
3	23 53 .408	0.443	22150	3050	0.308	939
4	20 54 .232	0.382	19100	2950	0.555	1637
5	17 55 .056	0.323	16150	2800	0.827	2316
6	14 55 .880	0.267	13350	2800	1.096	3069
7	11 56 .704	0.211	10550	2700	1.341	3621
8	8 57 .528	0.157	7850	2600	1.543	4012
9	5 58 .352	0.105	5250	2650	1.685	4465
10	2 59 .176	0.052	2600	2600	1.759	4573
				28700		24994

The above table gives the computations necessary for obtaining the proper dead loads and also the corresponding values of H_1. The value of H_1 for the entire dead load is in round figures $2(25000) = 50000$.

The next step will be the separating of the above dead loads into parts, the ring, filling, and track. The ring and track are fixed, so that their combined weight taken from the total will leave the weight of fill required. The tabular statement on page 68 shows the process in detail.

No great degree of accuracy has been attempted in this table, as a hard rain may change the weight of the fill a considerable amount. The last column gives the average weight per cubic foot of the fill which is necessary to just fulfill the requirement that the equilibrium polygon coincides with the arch axis. It will be noticed that the weight of the arch ring is very nearly uniform for each section.

The lack of uniformity in the variation of the values given is due to inaccuracies of scaling *ab*, *bc*, and *de* from a drawing.

FINAL DEAD LOADS.

Point.	Fig. 22.			170 lbs. per cu. ft. Ring.	30 lbs. per. sq. ft. Track.	Ring and Track.	Fill.	Area of Fill.	Average Weight of Fill per Cubic Foot.
	ab.	*bc.*	*de.*						
1	10.10	3.40	2.77	1601	83	1684	1666	28.5	59
2	8.72	3.28	2.83	1578	85	1663	1537	24.7	62
3	7.45	3.25	2.91	1608	87	1695	1355	21.7	63
4	6.36	3.16	2.96	1590	89	1679	1271	18.8	68
5	5.42	3.13	3.01	1602	90	1692	1108	16.3	68
6	4.62	3.08	3.05	1597	92	1689	1111	14.1	79
7	4.00	3.05	3.09	1602	93	1695	1005	12.4	81
8	3.52	3.00	3.11	1586	93	1679	921	10.9	85
9	3.20	3.00	3.13	1596	94	1690	960	10.0	96
10	3.02	3.00	3.14	1601	94	1695	905	9.5	95

53. Live Load and Loads Producing Maximum Moments.—The live load is 400 lbs. per linear foot and hence the load at each point is obtained by multiplying *de*, Fig. 22, by 400. These products are given in Table D.

In order to select the loads which produce maximum moments draw the equilibrium polygons for a load unity at each point respectively, as shown on Plate I. One-half of the polygons are shown. These reversed will be the polygons for loads upon the right of the crown.

By inspection we see that loads 1–8 inclusive produce negative moments at the left support, and the remaining loads produce positive moments.

At the crown loads 1–7 and 7′–1′ inclusive produce negative moments, and loads 8–8′ inclusive positive moments.

For point 6′, between $\frac{1}{3}$ and $\frac{1}{4}$ point of the span, loads 1–8′ produce negative moments, and loads 7′–1′ positive moments.

TABLE D.

LIVE LOADS.

400 POUNDS PER FOOT.

Pt.	Load.	$\overset{x}{\Sigma}$ Loads.	H_1.	$\overset{x}{\Sigma} H_1$	M_1.	$\overset{x}{\Sigma} M_1$.	V_1.	$\overset{x}{\Sigma} V_1$.
0								
1	1108	1108	00.0	00.0	−1522.4	− 1522.4	1108.0	1108.0
2	1132	2240	127.4	127.4	−3437.9	− 4960.3	1117.3	2225.3
3	1164	3404	357.9	485.3	−4462.8	− 9423.1	1119.8	3345.1
4	1184	4588	657.1	1142.4	−4636.6	−14059.7	1096.4	4441.5
5	1204	5792	995.1	2137.5	−4161.0	−18220.7	1058.3	5499.8
6	1220	7012	1336.5	3474.0	−3195.2	−21415.9	1002.8	6502.6
7	1236	8248	1657.5	5131.5	−1930.6	−23346.5	938.1	7440.7
8	1244	9492	1918.9	7050.4	− 531.2	−23877.7	858.4	8299.1
9	1252	10744	2109.6	9160.0	866.4	−23011.3	770.0	9069.1
10	1256	12000	2208.7	11368.7	2076.2	−20935.1	677.0	9746.1
10′	1256	13256	2208.7	13577.4	3020.7	−17914.4	579.0	10325.1
9′	1252	14508	2109.6	15687.0	3634.6	−14279.8	482.0	10807.1
8′	1244	15752	1918.9	17605.9	3890.0	−10389.8	385.6	11192.7
7′	1236	16988	1657.5	19263.4	3768.6	− 6621.2	297.9	11490.6
6′	1220	18208	1336.5	20599.9	3330.6	− 3290.6	217.2	11707.8
5′	1204	19412	995.1	21595.0	2670.6	− 620.0	145.7	11853.5
4′	1184	20596	657.1	22252.1	1873.1	1253.1	87.6	11941.1
3′	1164	21760	357.9	22610.0	1068.6	2321.7	44.2	11985.3
2′	1132	22892	127.4	22737.4	390.5	2712.2	14.7	12000.0
1′	1108	24000	00.0	22737.4	3.4	2715.6	0.0	12000.0
0′								

If the ring is safe at these three points or even at the spring line and points 6 and 6′, it will be safe at all other points.

54. M_1, V_1, and H_1 for Live Loads.—These values are obtained by multiplying the values for a load unity given in Table C by the live load. The results are given in Table D. For convenience these values are summed from 0 to x, as shown.

55. Maximum Moments at Point 0 Produced by the Live Load.—For loads 1–8 inclusive

$$M_1 = -23878 \text{ (Table D)}.$$

For loads 9-1′ inclusive

$$M_1 = 2716 - (-23878) = +26594.$$

For a full load

$$M_1 = -23878 + 26594 = +2716.$$

For a load up to the crown

$$M_1 = -20935 \text{ (Table D)}.$$

For load 10′-1′ inclusive

$$M_1 = +23651 \text{ (Table D)}.$$

Evidently loading one half the span does not produce maximum moments at point 0. The difference between the moment for a load extending from one support up to the crown and the maximum moments will not make any serious difference in the fiber stresses, as the dead load contributes a large portion of these stresses. If temperature effects are considered, the live-load effect becomes almost insignificant.

56. Maximum Moments at the Crown Produced by the Live Load.—For loads 1-7 inclusive

$$M_1 = -23347 \text{ (Table D)},$$
$$V_1 = 7441 \quad \text{" \quad "}$$
$$H_1 = 5132 \quad \text{" \quad "}$$

$$M_x = M_1 + V_1 x - H_1 y - \overset{x}{\Sigma} P(x-a). \quad x=30 \text{ and } y=8.$$

$\overset{x}{\Sigma} P(x-a)$ can be found graphically by means of the ordinary equilibrium polygon. In this instance we will compute its value as shown in the following table.

$\overset{x}{\Sigma}P(x-a)$.

Point	P.	$\frac{1}{2}l-a$.	$P(x-a)$.
1	1108	28.63	31722
2	1132	25.83	29239
3	1164	22.96	26725
4	1184	20.02	23703
5	1204	17.04	20516
6	1220	14.00	17080
7	1236	10.93	13509
	Table D	Table C	162494 $\overset{x}{\Sigma}P(x-a)$

$$V_1x = 7441(30) = 223230,$$

$$H_1y = 5132(8) = 41056,$$

$$\overset{x}{\Sigma}P(x-a) = 162494;$$

$$M_x = -23347 + 223230 - 41056 - 162494 = -3667.$$

If this is the moment for loads 1–7, then for loads 1–7 and 7′–1′ inclusive, $M_x = 2(-3667) = -7334$.

If our coefficients are absolutely correct, the moment for loads 7′–1′ inclusive should be the same as for loads 1–7 inclusive, as assumed. For loads 7′–1′ inclusive

$$M_x = M_1 + V_1x - H_1y,$$

$$M_1 = +13105 \text{ (Table D)},$$

$$V_1x = 807(30) = 24210,$$

$$H_1y = 5132(8) = 41056,$$

and

$$M_x = +13105 + 24210 - 41056 = -3741.$$

This is $3741 - 3667 = 74$ larger than obtained by above method, an error of about 2%.

Considering that the summation method leads necessarily to approximate results, it will be more consistent when possible to always use the formula for M_x in which $\overset{x}{\Sigma} P(x-a)$ does not appear.

57. Moment at the Crown Produced by Live Loads 1–10 Inclusive.—From Table D,

$$M_2 = +23651, \quad V_2x = 2254(30) = 67620,$$
$$H_1y = 11369(8) = 90952.$$
$$\therefore M_x = +23651 + 67620 - 90952 = +319.$$

For a load over all,

$$M_x = 2(319) = +638.$$

58. Moment at the Crown Produced by Loads 8–8′ Inclusive.—We will first compute the moment for loads 10′, 9′, and 8′ by the formula

$$M_x = M_1 + V_1x - H_1y.$$

From Table D,

$$M_1 = +10545, \quad V_1x = 1447(30) = 43410,$$
$$H_1y = 6237(8) = 49896.$$
$$\therefore M_x = +10545 + 43410 - 49896 = +4059.$$

Check:

From Art. 44, $M_x = -3741$ for loads 1–7 inclusive.
" " 45, $M_x = +\ 319$ " " 1–10 "
$\therefore M_x = +4060$ " " 8–10 "

or practically the same as found above.

The above computations show that the moment at the crown produced by a load covering the half-span is hardly *one tenth* the maximum moment.

59. Maximum Moment at Point 6′ Produced by Live Loads 1–8′ Inclusive.—Use the formula

$$M_x = M_2 + V_2 x' - H_1 y.$$

From Table D,

$$M_2 = +2715 - (-23347) = +26062,$$
$$V_2 x' = 4559(16) = 72944,$$
$$H_1 y = 17606(6.35) = 111798.$$
$$\therefore\ M_x = +26062 + 72944 - 111798 = -12792.$$

60. Maximum Moment at Point 6′ Produced by Live Loads 7′–1′ Inclusive.—From Table D,

$$M_1 = +13105, \quad V_1 x = 807(44) = 35508,$$
$$H_1 y = 5132(6.35) = 32588,$$
$$\overset{x}{\Sigma} P(x-a) = 1236(3.07) = 3795,$$
$$M_x = M_1 + V_1 x - H_1 y - \overset{x}{\Sigma} P(x-a)$$
$$= +13105 + 35508 - 32588 - 3795 = +12230.$$

61. Moment at Point 6′ Produced by Live Loads 1–10 Inclusive.

$$M_x = M_2 + V_2 x' - H_1 y.$$

From Table D,

$$M_2 = +23651, \quad V_2 x' = 2254(16) = 36064,$$
$$H_1 y = 11369(6.35) = 72193.$$
$$\therefore\ M_x = +23651 + 36064 - 72193 = -12478,$$

which is about $2\frac{1}{2}\%$ *less* than the maximum moment as found in Art. 59.

62. Moments at all Points Produced by Live Loads 1–8′ Inclusive Determined Graphically.—The constructions are given on Plate II. Lay off a load line in the usual way and scale off V_1 downward. Horizontally opposite this point, at a distance H_1, take a pole and draw the strings S_1, S_2, S_3, etc. The equilibrium polygon can now be drawn. As check upon the correctness of the polygon the *common closing line*, when transferred to the force polygon, should cut off the value of R_1, the common reaction, on the load line. (In this particular case the check was not perfect, but so close that it was deemed unnecessary to draw a new polygon. The effect will appear later.) The closing line is AB.

Following the methods of Arts. 16 and 17, scale each ordinate of the equilibrium polygon and find the mean ordinate $= \Sigma B'C' \div 20$. At the center of the span scale upward this distance, and through the point just found draw CD parallel to the string S_0 in the force diagram, and scale the ordinates $A'B'$. Then M_x at any point equals the difference between the ordinate $A'B'$ for that point and the corresponding value of $y - y_a$ multiplied by H_1.

The values of $M_x \div H_1$ can be found, also, by drawing the arch axis so that the y_a line coincides with the line CD of the equilibrium polygon and scaling the ordinates indicated in the shaded area.

The line CD can also be located by making $AC = m_1$ and $BD = m_2$, where

$$m_1 = \frac{\Sigma M_1 - \Sigma H_1 y_a}{H_1} \quad \text{and} \quad m_2 = \frac{\Sigma M_2 - \Sigma H_2 y_a}{H_2}.$$

The computation of M_x in detail is given in Table E.

TABLE E.

LIVE LOADS, 1-8′ INCLUSIVE.

Point.	$A'B'$. See Plate II.	$y-y_a$.	$\frac{M_x}{H_1}=(y-y_a)-A'B'$.	M_x.
0	−5.905	−5.315	.590	−10387
1	−5.09	−4.550	.54	− 9506
2	−3.48	−3.132	.348	− 6127
3	−2.01	−1.860	.150	− 2641
4	− .74	− .740	.000	− 0000
5	.40	.226	− .174	+ 3066
6	1.32	1.035	− .285	+ 5017
7	2.06	1.685	− .375	+ 6602
8	2.60	2.174	− .426	+ 7509
9	2.88	2.500	− .380	+ 6690
10	2.96	2.665	− .295	+ 5194
10′	2.80	2.665	− .135	+ 2377
9′	2.43	2.500	.070	− 1232
8′	1.83	2.174	.344	− 6056
7′	1.05	1.685	.635	−11180
6′	.27	1.035	.765	−13468
5′	− .50	.226	.726	−12781
4′	−1.28	− .740	.540	− 9506
3′	−2.05	−1.860	.190	− 3345
2′	−2.78	−3.132	− .352	+ 6198
1′	−3.52	−4.550	−1.030	+18134
0′	−3.834	−5.315	−1.481	+26075

The point of maximum moment is at 6′, as stated above, and $M_x=-13468$. From Art. 59, by computation, $M_x=-12792$, showing a difference of 676 or an error of about 5%, corresponding to an ordinate of 0.033 feet. The scale employed was 3 feet to the inch, hence 0.033 feet corresponds to 0.011 of an inch on the drawing. This shows that the greatest care must be employed when graphical methods are applied and all possible checks applied.

63. Maximum Moment at Point 6 Produced by Loads 7′–1′ Inclusive. Graphical Determinations.—Plate II shows the construction, and Table F the computation of M_x in detail. Here again there is a difference in the results obtained by the two methods. From Art. 60, $M_x = +12230$, while by graphics $M_x = +11520$, a difference of 710, or about 6%.

TABLE F.

LIVE LOADS, 7′–1′ INCLUSIVE.

Point.	$A'B'$. See Plate II.	$y-y_a$.	$\frac{M_x}{H_1} = (y-y_a)-A'B'$.	M_x.
0	−2.761	−5.315	−2.554	+13105
1	−2.59	−4.550	−1.960	+10058
2	−2.16	−3.132	− .972	+ 4987
3	−1.71	−1.860	− .150	+ 770
4	−1.26	− .740	.520	− 2668
5	− .80	.226	1.026	− 5262
6	− .33	1.035	1.365	− 7009
7	.15	1.685	1.535	− 7882
8	.60	2.174	1.574	− 8077
9	1.10	2.500	1.400	− 7184
10	1.55	2.665	1.115	− 5722
10′	2.05	2.665	.615	− 3158
9′	2.53	2.500	− .030	+ 154
8′	3.00	2.174	− .826	+ 4238
7′	3.50	1.685	−1.815	+ 9318
6′	3.28	1.035	−2.245	+11520
5′	2.30	.226	−2.074	+10643
4′	.65	− .740	−1.390	+ 7133
3′	−1.62	−1.860	− .240	+ 1231
2′	−4.45	−3.132	1.318	− 6763
1′	−7.90	−4.550	3.350	−17188
0′	−9.864	−5.135	4.549	−23341

64. Fiber Stresses Produced by Dead and Live Loads.—From Art. 31,

$$p = \frac{N_x}{F} \pm \frac{M_x z}{I}.$$

For this problem, $N_x = (V_1 - \sum^{x} P) \sin \phi + H_1 \cos \phi.$

$$F = 3 \text{ sq. ft.}, \quad z = 1.5 \text{ ft.}, \quad I = \frac{1}{12} bh^3 = \frac{3^3}{12} = \frac{9}{4}.$$

Then

$$\frac{z}{I} = \frac{1.5 \times 4}{9} = \frac{2}{3}$$

and

$$p = \tfrac{1}{3}N_x \pm \tfrac{2}{3}M_x.$$

Point 0. *Dead Load.*

From Art. 52, ½ the total load $= V_1 = 28700$, say 29000, and $H_1 = 50000$; then

$$N_x = 29000(0.498) + 50000(0.867) = 14442 + 43350 = 57792.$$

$\therefore\ p = \tfrac{1}{3}(57792) + \tfrac{2}{3}(0) = 19264$, say 19300 comp. for both the upper and lower extreme fibers.

Live Loads.

From Art. 55 . . . . $M_x = M_1 = -23878$ for loads 1–8 incl.
" " " . . . . $M_x = M_1 = +26594$ for loads 9–1′ "
$N_x = 8299(0.498) + 7050(0.867) = 10245$ for loads 1–8 "
$N_x = 3701(0.498) + 15687(0.867) = 15443$ for loads 9–1′ "

Then

$p = \tfrac{1}{3}(10245) - \tfrac{2}{3}(23878) = 3415 - 15919 = -12500$ tension in *upper fiber* and $3415 + 15919 = +19300$ compression in the *lower fiber* for loads 1–8 inclusive.

For loads 9–1′ inclusive

$p = \tfrac{1}{3}(15443) + \tfrac{2}{3}(26594) = 5143 + 17729 = 22900$ compression in the *upper fiber* and

$p = 5143 - 17729 = 12600$ tension in the *lower fiber.*

Combined Stresses.

Combining the above results we have for the maximum fiber stresses produced by the dead and live loads the following:

Load.	Upper Fiber.	Lower Fiber.
Dead Load...........	19300 compression	19300 compression
L.L. 1–8.............	12500 tension	19300 "
L.L. 9–1'.............	22900 compression	12600 tension
Maximum compression..	42200	38600
Maximum tension......	0	0

These intensities are pounds per square foot.

For pounds per square inch we have, 293 and 268 as the maximum compression in the upper and lower fibers respectively.

Considering that granite has an ultimate crushing strength of from 13000 to 17000 pounds per square inch, the above fiber stresses are of little consequence if the mortar joints have an equal strength, or even one fourth the strength of the granite. The fiber stresses at other points are obtained in the manner followed for point 0. A tabulated statement for points 0, 6', and the crown is given below:

FIBER STRESSES.

Load.	N_x.	M_x.	$\frac{1}{3}N_x$.	$\frac{2}{3}M_x$.	p. Upper.	p. Lower.	Point.
Dead load..........	57792	0	19300	0	+19300	+19300	0
L.L. 1–8...........	10245	−23878	3415	−15919	−12500	19300	0
L.L. 9–1'..........	15443	+26594	5148	+17729	+22900	−12600	0
Max. compression...	....	....	....	....	42200	38600	0
Max. tension........	....	....	....	....	0	0	0
Dead load..........	50000	0	16666	0	+16666	+16666	Crown
L.L. 1–7 and 7'–1'...	10264	− 7482	3421	− 4988	− 1567	+ 8409	"
L.L. 8–8'..........	12474	+ 8118	4158	+ 5412	+ 9570	− 1254	"
Max. compression...	....	....	....	....	26200	25100	"
Max. tension........	....	....	....	....	0	0	"
Dead load..........	50500	0	16833	0	+16833	+16833	6'
L.L. 1–8'..........	18170	−12792	6057	− 8528	− 2471	+14585	6'
L.L. 7'–0'..........	5093	+12230	1698	+ 8154	+ 9852	− 6546	6'
Max. compression...	....	....	....	....	26700	31400	6'
Max. tension........	....	....	....	....	0	0	6'

In this table all stresses are given in pounds per *square foot.*

From the above table we see that there is no tension at the three points considered, and that the maximum compression is well within the safe strength of the material assumed. Also, that the greatest fiber stress is at the supports.

65. Effect of Temperature Changes.—Our knowledge of the effect of changes of temperature upon stone arches is very meager. The coefficients of expansion for different stones are known, but how long it takes for a stone bridge to become of uniform temperature we do not know. Probably all portions of the arch ring are never of the same temperature. The range of the average temperature is probably small. (See Arts. 33 and 34.)

In this case we will assume that the temperature changes 40° above or below the temperature of the arch when built. This is without doubt an excessive range. The horizontal thrust is (Art. 27)

$$H_t = \frac{et^\circ lE}{\Sigma \Delta y(y-y_a)} = \frac{et^\circ lE}{1.4(113.168)},$$

where $\Delta = \delta s \div I = 3.14 \div 2.25 = 1.4.$

For Quincy granite

$$e = 0.00000381,$$

$$E = 6776000.$$

Then

$$H_t = \frac{(0.00000381)(40)(60)(6776000)(144)}{158.4} = 56100.$$

From Art. 27,

$$M_1 = H_t y_a = 56100(5.315) = 298200,$$

$$M_x = M_1 - H_t y = H_t(y_a - y).$$

The above values of H_t and M_1 will have signs depending upon whether the change of temperature is an increase or a decrease.

For *falling* temperature the *upper fibers* at the support are in *tension*, and at the *crown* in *compression*.

The following table gives the fiber stresses at the support and the crown.

FIBER STRESSES DUE TO CHANGES OF TEMPERATURE.

POINT O.

Temperature.	N_x.	M_x.	$\frac{1}{3}N_x$.	$\frac{2}{3}M_x$.	p. Upper.	p. Lower.
−40°	−48638	298200	−16213	198800	−215000	+182000
+40°	+48638	298200	+16213	198800	+215000	−182000
CROWN.						
−40°	−56100	149600	−18700	99700	+ 81000	−118400
+40°	+56100	149600	+18700	99700	− 81000	+118400

Combining the above values with those obtained for the dead load and live load we have

	Upper Fibers.	Lower Fibers.
For point o:		
Maximum compression........	257200	220600
" tension............	208200	175300
For the crown:		
Maximum compression........	107200	143500
" tension............	65900	103000

The above values correspond to a maximum compression of 1786 pounds per square inch and a maximum tension of 1446 pounds per square inch. In compression the factor of safety is from 8 to 10, but in tension the ultimate strength of the joints is exceeded. As a large number of railway bridges have been built upon practically the dimensions we assumed and no indications of failure having been found, we must conclude that the range of temperature change assumed in this example is very much too great. Furthermore, it requires a drop in temperature of only *four* degrees to completely balance the compression produced by the dead load in the upper fibers at the support. Without question, then, our assumptions about the effect of temperature changes are not correct. Until we know more about the subject it is useless to make calculations according to the ordinary assumptions. (See Art. 33.)

66. Effect of the Axial or Direct Stress.—In all of the work above, the effect of the direct compression or tension has been neglected. If the rib is subjected to a uniform stress, it will be shortened or lengthened according to the character of the stress. All vertical loads produce direct stresses which in effect shorten the rib.

As explained in Art. 19, the horizontal thrust produced by this shortening, when found, will be treated the same as the thrust for a change of temperature.

From Art. 19,

$$H_a = H_1\left(1 - \frac{\Sigma y(y-y_a)}{\Sigma y(y-y_a) + \Sigma \dfrac{\delta x \cos\phi}{F\Delta}}\right),$$

in which all quantities are known from previous calculations, with the exception of the last term in the denominator. The computation of this term is given in detail below.

Point.	δx.	cos ϕ.	δx cos ϕ.
1	2.77	0.880	2.44
2	2.83	.903	2.56
3	2.90	.925	2.68
4	2.96	.943	2.79
5	3.01	.959	2.89
6	3.05	.973	2.97
7	3.08	.983	3.03
8	3.11	.992	3.09
9	3.13	.997	3.12
10	3.14	.999	3.14
			28.71

$\Sigma \delta x \cos \phi = 2(28.71) = 57.42, \quad F = 3, \quad \Delta = 1.4, \quad F\Delta = 4.2,$

$$\frac{\Sigma \delta x \cos \phi}{F\Delta} = \frac{57.42}{4.2} = 13.67, \quad \Sigma y(y - y_a) = 113.168.$$

Then

$$\frac{113.168}{126.84} = 0.892. \quad \therefore \; H_a = 0.108 H_1 = 11\% \, H_1, \text{ say.}$$

The value of H_1 for the dead load is 50000; then the corresponding axial stress produces a thrust, opposite in character, of 5500. The horizontal thrust produced by a drop of 40° in temperature is 56100; therefore the effect of the axial stress equals $\frac{5500}{56100} = .091$ of the stresses due to this drop of temperature. At joint zero the upper fiber stress due to $-40°$ is 215000 tension. $215000(0.091) =$ 19600 tension.

FIBER STRESSES DUE TO THE AXIAL STRESS.

	H_1.	H_a.	$\frac{H_a}{H_t}$.	Point o.		Crown.	
				Upper.	Lower.	Upper.	Lower.
Dead load........ ..	50000	5500	0.091	−19600	+16600	+7400	−10800
L.L. 1–8...........	7050	776	0.014	− 3000	+ 2500	+1100	− 1700
" 9–1′..........	15687	1726	0.031	− 6700	+ 5600	+ 2500	− 3700
" 1–7 and 7′–1′...	10263	1128	0.020	− 4300	+ 3600	+ 1600	− 2400
" 8–8′..........	12474	1372	0.024	− 5200	+ 4400	+ 1900	− 2900
" 1–8′..........	17606	1936	0.034	− 7300	+ 6200	+ 2800	− 4000
" 7′–1′.........	5132	565	0.001	− 220	+ 182	+ 81	− 118
−40°..............	51600	5676	0.110	+24000	−20000	−8900	+13000

Combining these stresses with the dead- and live-load stresses previously obtained, we have (see Art. 64):

FINAL STRESSES, INCLUDING EFFECT OF AXIAL STRESS.

	Loads.	Upper Fibers.	Maximums.
Point o.	Dead load....	+19300−19600= − 300	Max. comp.=12200
	L.L. 1–8.........	−12500− 3000= −15500	" ten. =18500
	" 9–1′..... ..	+22900− 6700= +15200	
		Lower Fibers.	
	Dead load.........	+19300+16600= +35900	Max. comp.=57700
	L.L. 1–8........ ...	+19300+ 2500= +21800	" ten. 0
	" 9–1′.........	−12600+ 5600= − 7000	
		Upper Fibers.	
Crown.	Dead load........ ..	+16666+ 7400= +24100	Max. comp.=35600
	LL. 1–7 and 7′–1′...	− 1567+ 1600= + 33	" ten. 0
	' 8–8′..........	+ 9570+ 1900= +11470	
		Lower Fibers.	
	Dead load...... .	+16666−10800= + 5900	Max. comp.=11900
	L L. 1–7 and 7′–1′..	+ 8409− 2400= + 6000	" ten. 0
	" 8–8′...	− 1254− 2900= − 4200	
	Temperature.		
	∓40°	∓215000±24000= ∓191000	Upper fibers at o.
	∓40°	±182000∓20000= ±162000	Lower " " o.
	∓40°	± 81000∓ 8900= ± 72100	Upper " " crown
	±40°	∓118400±13000= ∓105400	Lower " " "

These stresses show that the effect of the axial stress is considerable, and also that the fiber stresses at the support are reversed in one case so that the upper fibers are in tension about 128 lbs. per square inch. As this tension is not large and exists for but a short distance, the ring may be considered safe. This assumes that no temperature effects are considered. The maximum compression is 400 lbs. per square inch in the lower fibers at the support.

The effect of the axial stress is to *lower* the equilibrium polygon at the *support* and *raise* it at the *crown*, or it *increases* the *compression* and *decreases* the *tension* in the lower fibers and *decreases* the *compression* and *increases* the *tension* in the upper fibers at the support. While at the crown the reverse is true.

If this arch ring had been assumed free, then the above tension could not have been allowed (see Arts. 31 and 33).

67. A Check upon the Effect of the Axial Stress for Dead Loads.—To show how nearly the results of the above method of considering the axial stress agrees with those obtained by direct calculation, we will briefly compute H_1 and M_1 for the dead load, and also the fiber stresses at the support (see upper table on page 85).

$$M_1 = \Sigma H_1 y_a - \Sigma m_1 = 237049 - 266020 = -29000,$$

$$N_x = 29000(0.498) + 44600(0.867) = 14442 + 38668 = 53100.$$

$$\therefore\ p = \tfrac{1}{3}(53100) \pm \tfrac{2}{3}(29000) = 17700 \pm 19300$$

$$= 1600 \text{ tension in upper fibers}$$

$$= 37000 \text{ compression in lower fibers.}$$

From Art. 66 the corresponding stresses are 300 tension and 35900 compression, the results in the table being about 1100 too large numerically. This equals a stress of less than 8 pounds per square inch and for the compression a relative error of 3%+.

COMPUTATION OF H_1 and M_1 WHEN AXIAL STRESS IS CONSIDERED.

Point.	Common H_1, Art. 52.	True H_1.	True $H_1 y_a$.	m_1 for Load Unity, Table C.	Dead Load, Art. 52.	m_1 for Dead Load.
1	0			1.377	3350	4613
2	362			3.888	3200	12442
3	939			6.184	3050	18861
4	1637			8.234	2950	23669
5	2316			10.024	2800	28067
6	3069	Common $H_1(1-0.108)$, Art. 66	$y_a = 5.315$, Table B.	11.535	2800	32298
7	3621			12.767	2700	34471
8	4012			13.696	2600	35610
9	4465			14.317	2650	37940
10	4573			14.634	2600	38048
	24994 2			Symmetrical values combined	28700 say 29000	266020
	49988 say 50000	44600	237049		½ D.L. or V_1	Σm_1

Evidently the method employed in Art. 66 is quite accurate enough for practical purposes.

68. Effect of Making Spandrel Filling of Uniform Material Weighing 100 Pounds per Cubic Foot.

COMPUTATION OF H_1.

Point.	Ring and Track.	Fill 100 Lbs. per Cu. Ft.	Total Dead Load.	Common H_1, Load Unity.	Common H_1.	H_1 with Effect of Axial Stress.
1	1684	2850	4500	0.0	0	
2	1663	2470	4100	0.1125	461	
3	1695	2170	3900	3075	1199	
4	1679	1880	3600	5550	1998	
5	1692	1630	3300	8265	2727	
6	1689	1410	3100	1.0955	3396	$54200(1-0.108)=48300$
7	1695	1240	2900	1.3410	3889	
8	1679	1090	2800	1.5425	4319	
9	1690	1000	2700	1.6850	4550	
10	1695	950	2600	1.7585	4572	
			33500 2	Table B	27111 2	
			67000		54200	

COMPUTATION OF m_1.

Point.	m_1, Load Unity. Table C.	Dead Load.	m_1 for Dead Load.
1	1.377	4500	6197
2	3.888	4100	15941
3	6.184	3900	24129
4	8.234	3600	29642
5	10.024	3300	33079
6	11.535	3100	35759
7	12.767	2900	37024
8	13.696	2800	38349
9	14.317	2700	38656
10	14.634	2600	38048
	Table C		296824

$$M_1 = \Sigma H_1 y_a - \Sigma m_1 = 48300(5.315) - 296824$$
$$= 256715 - 296824 = -40100,$$
$$y_1 = \frac{M_1}{H_1} = \frac{-40100}{48300} = -0.83 \text{ ft.},$$
$$N_x = 33500(0.498) + 48300(0.867) = 16683 + 41876$$
$$= 58600.$$
$$\therefore\ p = \tfrac{1}{3}(58600) \pm \tfrac{2}{3}(40100) = 19500 \pm 26700$$
$= 7200$ tension in upper fibers

$= 46200$ compression in bottom fibers.

This shows that the fill over the haunches and near the supports is too heavy for the load upon the crown. The original loading could be made less to an advantage.

At the crown the moment is

$$M_x = M_1 + V_1 x - H_1 y - \overset{x}{\Sigma} P(x-a)$$
$$= -40100 + 1005000 - 386400 - 566400 = +12100,$$
$$N_x = H_1 \text{ sensibly} = 48300.$$
$$\therefore\ p = \tfrac{1}{3}(48300) \pm \tfrac{2}{3}(12100) = 16100 \pm 8100$$
$= 24200$ compression in upper fibers

$= 8000$ " " lower fibers.

The equilibrium polygon is $M_x \div H_1 = 12100 \div 48300 = 0.25$ ft. *above* the neutral axis.

Combining these stresses with the live-load stresses of Art. 66, we have

$-7200+15200=8000$	comp. in upper fibers	at support
$-7200-15500=22700$	tension in upper fibers	
$+46200+21800=68000$	comp. in lower fibers	
$+46200-7000=39020$	" " " "	
$+24200+11470=35670$	comp. in upper fibers	at crown.
$+8000+6000=14000$	" " lower fibers	

If the above tension is considered more than allowable, then the spandrel filling should be made lighter. Since the maximum compression is very much less than the allowable stress for granite, the ring will unquestionably adjust itself by increasing this compression, and not resist much tension, if any (see Art. 31.)

69. The Radial Shear.—From Art. 29,

$$T_x = V_x \cos\phi - H_x \sin\phi.$$

For point zero, or the support, this becomes

$$T_x = V_1(0.867) - H_1(0.498).$$

For dead load (see Art. 52)

$$T_x = 28700(0.867) - 50000(0.498) = 24880 - 24900 = 0.$$

For live load over all (see Table D)

$$T_x = 12000(0.867) - 22737(0.498) = 10400 - 11320 = -920.$$

At the crown ϕ is zero, hence

$$T_x = V_x = V_1 - \sum^x P.$$

For the dead load $T_x = 0$.

For a live load 10′–1′ inclusive $T_x = V_1 = 2254$.

In like manner any other point may be considered. When equilibrium polygons are drawn a glance is sufficient to determine if there is danger of slipping at the joints. Usually the radial shear requires but little attention in *stone arches.*

70. Second Example. Data.—For this example we will take a reinforced-concrete rib of the Thacher type.* Clear span 50 ft. and rise 10 ft. The thickness at the crown is taken as 12 inches, and at the spring line 4 feet 6 inches. Plate IV gives all data concerning dimension and reinforcement. The dead weight of the entire structure is assumed at 140 pounds per cubic foot, and the live load 112 pounds per square foot. The first step in the solution of a problem of this type is to obtain all the data shown in Plate IV either by algebraic or graphical methods. In the present instance many of the data were obtained from a carefully constructed drawing as indicated in the figure. The modulus of elasticity of the concrete is assumed to be $\frac{1}{20}$ that of steel, and hence the area of the steel is equivalent to twenty times that area in concrete.

71. Subdivision of the Arch Axis. — Contrary to the usual custom we will not attempt to so divide the arch ring that $\delta s \div I$ will be constant, but simply divide the span into twenty equal parts and determine all quantities necessary for points at the centers of these divisions. This is clearly shown in Plate IV.

The moment of inertia at each point is found as shown

* Essentially the arch taken by Professor Cain in "Theory of Concrete Arches and of Vaulted Structures."

in Table I, page 90. Prof. Cain in his book referred to above gives a very complete exposition of the method for dividing the axis so that $\delta s \div I$ shall be constant.

72. Computation of H_1 for Unit Loads.—The process is precisely that followed in the first example, only we use the general formula (Art. 13)

$$2H_1 = \frac{\Sigma m_x \Delta\left(y - \frac{\Sigma y\Delta}{\Sigma\Delta}\right)}{\Sigma y\Delta\left(y - \frac{\Sigma y\Delta}{\Sigma\Delta}\right)} = \frac{\Sigma m_x B}{C}.$$

Tables I and II give the work in detail (see pp. 90, 91).

73. Computation of M_1.—The general formula in this case is (Art. 13)

$$M_1 = H_1\frac{\Sigma y\Delta}{\Sigma\Delta} - \frac{\Sigma m_x\Delta\left(x - \frac{\Sigma x^2\Delta}{\Sigma x\Delta}\right)}{\Sigma\Delta\left(\frac{1}{2}l - \frac{\Sigma x^2\Delta}{\Sigma x\Delta}\right)}.$$

H_1 and $\frac{\Sigma y\Delta}{\Sigma\Delta}$ have been found in Tables I and II, so there remains simply the multiplication of the two factors. The determination of the second term we will take up in detail, as it is well to know a few checks and short methods.

Designating this term by m_1,

$$m_1 = \frac{\Sigma m_x\Delta\left(x - \frac{\Sigma x^2\Delta}{\Sigma x\Delta}\right)}{\Sigma\Delta\left(\frac{1}{2}l - \frac{\Sigma x^2\Delta}{\Sigma x\Delta}\right)} = \frac{\Sigma m_x\Delta\left(x - \frac{\Sigma x^2\Delta}{\frac{1}{2}l\Sigma\Delta}\right)}{\Sigma x\Delta(x - \frac{1}{2}l)\frac{2}{l}}.$$

$\Sigma x\Delta = x\Delta_1 + 3x\Delta_2 + 5x\Delta_3 + 7x\Delta_3 + 9x\Delta_2 + 11x\Delta_1$

To find $\Sigma x^2\Delta$, let $x = \frac{\delta x}{2}z$; then $x^2 = \left(\frac{\delta x}{2}\right)^2 z^2$, where

$= 12x\Delta_1 + 12x\Delta_2 + 12x\Delta_3$

$= 6x(\Delta_1 + \Delta_1 + \Delta_3 + \Delta_3 + \Delta_2 + \Delta_2)$

$= \frac{1}{2}l\Sigma\Delta$

TABLE I.—COMPUTATION OF Δ.

	1	2	3	4	5	6	7	8	9	10	11	12
Point No.	h.	h^3.	$\frac{h^3}{12}=I_c$.	$\frac{h}{2}$.	$\frac{h}{2}-0.17=h'$.	$(h')^2$.	$\frac{2}{10}(h')^2=I_s$.	$\frac{h^3}{12}+\frac{2}{10}(h')^2=I_x$.	δs.	$\frac{\delta s}{I_x}=\Delta$.	y.	$y\Delta$.
1	3.80	54.87	4.57	1.90	1.73	2.99	0.598	5.17	3.41	0.66	1.10	0.726
2	2.80	21.95	1.83	1.40	1.23	1.51	0.302	2.13	3.21	1.51	3.05	4.606
3	2.08	9.00	0.75	1.04	0.87	0.77	0.154	0.90	3.08	3.42	4.66	15.937
4	1.54	3.65	0.30	0.77	0.60	0.36	0.072	0.37	2.92	7.89	5.95	96.946
5	1.38	2.63	0.22	0.69	0.52	0.27	0.054	0.27	2.81	10.41	6.98	72.662
6	1.30	2.20	0.18	0.65	0.48	0.23	0.046	0.23	2.75	11.96	7.77	92.929
7	1.19	1.69	0.14	0.59	0.42	0.18	0.036	0.18	2.74	15.22	8.30	126.326
8	1.10	1.33	0.11	0.55	0.38	0.14	0.828	0.14	2.70	19.28	8.71	167.929
9	1.03	1.09	0.09	0.51	0.34	0.12	0.024	0.11	2.68	24.36	8.98	218.753
10	1.00	1.00	0.08	0.50	0.33	0.11	0.022	0.10	2.68	26.80	9.12	244.416
										121.51		991.23
										2		2
										243.02 $\Sigma\Delta$		1982.46 $\Sigma y\Delta$

$$\frac{\Sigma y\Delta}{\Sigma\Delta}=\frac{1982.46}{243.02}=8.1576$$

TABLE II.—COMPUTATION OF H_1.

	1	2	3	4	5	6	7	8	9
					$P_1=1=P_1'$.		$P_2=1=P_2'$.		
Point No.	Δ.	$y-\frac{\Sigma y\Delta}{\Sigma\Delta}$ or $y-8.1576$.	$\Delta(y-8.1576)$ B.	$y\Delta(y-8.1576)$ C.	m_x.	m_xB.	m_x.	m_xB.	x.
1	0.66	−7.0576	− 4.658	− 5.1238	1.34	Not necessary to compute, as sum = zero.	1.34	− 6.242	1.34
2	1.51	−5.1076	− 7.712	− 23.5256	1.34		4.02	− 31.002	4.02
3	3.42	−3.4976	−11.962	− 55.7412	1.34		4.02	− 48.087	6.70
4	7.89	−2.2076	−17.418	−103.6379	1.34		4.02	− 70.020	9.38
5	10.41	−1.1776	−12.259	− 85.5667	1.34		4.02	− 49.281	12.06
6	11.96	−0.3876	− 4.636	− 36.0192	1.34		4.02	− 18.637	14.74
7	15.22	+0.1424	+ 2.167	+ 17.9888	1.34		4.02	+ 8.711	17.42
8	19.28	+0.5524	+10.650	+ 92.7639	1.34		4.02	+ 42.813	20.10
9	24.36	+0.8224	+20.034	+179.9024	1.34		4.02	+ 80.537	22.78
10	26.80	+0.9624	+25.792	+235.2259	1.34		4.02	+103.684	25.46
	121.51		−58.645	−309.6144				−223.269	
			+58.643	+525.8810				+235.745	
			.002 ΣB	+216.2666 ΣC		0 Σm_xB		+ 12.476 Σm_xB	
					$2H_1=0$		0.0576		

TABLE II.—COMPUTATION OF H_1—*Concluded.*

Point No.	10	11	12	13	14	15	16	17	x.
	$P_3 = 1 = P_3'$.		$P_4 = 1 = P_4'$.		$P_5 = 1 = P_5'$.		$P_6 = 1 = P_6'$.		
	m_x.	m_xB.	m_x.	m_xB.	m_x.	m_xB.	m_x.	m_xB.	
1	1.34	− 6.242	1.34	− 6.242	1.34	− 6.242	1.34	− 6.242	1.34
2	4.02	− 31.002	4.02	− 31.002	4.02	− 31.002	4.02	− 31.002	4.02
3	6.70	− 80.145	6.70	− 80.145	6.70	− 80.145	6.70	− 80.145	6.70
4	6.70	−116.701	9.38	−163.381	9.38	−163.381	9.38	−163.381	9.38
5	6.70	− 82.135	9.38	−114.98	12.06	−147.844	12.06	−147.844	12.06
6	6.70	− 31.061	9.38	− 43.486	12.06	− 55.910	14.74	− 68.335	14.74
7	6.70	+ 14.519	9.38	+ 20.326	12.06	+ 26.134	14.74	+ 31.942	17.42
8	6.70	+ 71.355	9.38	+ 99.897	12.06	+128.439	14.74	+156.981	20.10
9	6.70	+134.228	9.38	+187.919	12.06	+241.610	14.74	+295.301	22.78
10	6.70	+172.806	9.38	+241.929	12.06	+311.052	14.74	+380.174	25.46
		+392.908		+550.071		+707.235		+864.398	
		−347.286		−439.245		−484.524		−496.949	
		+ 45.622 Σm_xB		+110.826 Σm_xB		+222.711 Σm_xB		+367.449 Σm_xB	
	$2H_1 = 0.2109$		0.5123		1.0298		1.6990		

Point No.	18	19	20	21	22	23	24	25
	$P_7 = 1 = P_7'$.		$P_8 = 1 = P_8'$.		$P_9 = 1 = P_9'$.		$P_{10} = 1 = P_{10}'$.	
	m_x.	m_xB.	m_x.	m_xB.	m_x.	m_xB.	m_x.	m_xB.
1		−496.949		−496.949		−496.949		−496.949
2								
3								
4								
5								
6								
7	17.42	+ 37.749	17.42	+ 37.749	17.42	+ 37.749	17.42	+ 37.749
8	17.42	+ 185.523	20.10	+ 214.065	20.10	+ 214.065	20.10	+ 214.065
9	17.42	+ 348.992	20.10	+402.683	22.78	+ 456.374	22.78	+ 456.374
10	17.42	+ 449.297	20.10	+ 518.419	22.78	+ 587.542	25.46	+ 656.664
		+1021.561		+1172.916		+1295.730		+1364.852
		− 496.949		− 496.949		− 496.949		− 496.949
		+ 524.612 Σm_xB		+ 675.967 Σm_xB		+ 798.781 Σm_xB		+ 867.903 Σm_xB
	$2H_1 = 2.4257$		3.1256		3.6935		4.0131	

$\frac{\partial x}{2}=1.34$, or one half of one of the twenty divisions into which we divided the span of the axis. The first five columns of Table III give the complete determination of $\frac{\Sigma x^2 \Delta}{\Sigma x \Delta}=30.373$.

$$\Sigma \Delta\left(\tfrac{1}{2}l-\frac{\Sigma x^2 \Delta}{\Sigma x \Delta}\right)=243.02(26.8-30.373)=-868.304.$$

We now have

$$m_1=\frac{\Sigma m_x \Delta(x-30.373)}{-868.304}.$$

Cols. 6, 7, 8, and 9 give the deduction of $\Delta(x-30.373)$, and in col. 9 the algebraic sum is found to be -868.308, which should equal the denominator when all work is correct. In this case the difference is 4 in the third decimal place (see cols. 10, 11, and 12).

The next step is the computation of

$$\frac{\Sigma m_x \Delta(x-30.373)}{-868.304}=\frac{\Sigma \frac{m_x}{1.34}\Delta(x-30.373)}{-648}.$$

This may be written

$$-m_1=R_1\sum_{x=0}^{x=a}\frac{\Delta(x-30.373)}{648\times 1.34}x+R_2\sum_{x=0}^{x'<l-a}\frac{\Delta(x-30.373)}{648\times 1.34}x',$$

since

$$m_x=R_1 x \quad \text{for } x=0 \text{ to } x=a$$

and

$$m_x=R_2 x' \quad \text{for } x=a \text{ to } x=l, \quad x'=l-x.$$

TABLE III.—COMPUTATION OF M_1.

	1	2	3	4	5	6	7	8	9
Point.	$\frac{x}{1.34}$ z.	$\frac{x^2}{(1.34)^2}$ z^2.	Δ.	z^2.	$z^2\Delta$.	x.	$x-\frac{\Sigma x^2\Delta}{\Sigma x\Delta}$ or $x-30.373$.	Δ.	$\Delta\left(x-\frac{\Sigma x^2\Delta}{\Sigma x\Delta}\right)$ or $\Delta(x-30.373)$.
1	1	1	0.66	1522	1004.52	1.34	−29.033	0.66	− 19.162
2	3	9	1.51	1378	2080.78	4.02	−26.353	1.51	− 39.793
3	5	25	3.42	1250	4275.00	6.70	−23.673	3.42	− 80.961
4	7	49	7.89	1138	8978.82	9.38	−20.993	7.89	− 165.635
5	9	81	10.41	1042	10847.22	12.06	−18.313	10.41	− 190.638
6	11	121	11.96	962	11505.52	14.74	−15.633	11.96	− 186.971
7	13	169	15.22	898	13667.56	17.42	−12.953	15.22	− 197.145
8	15	225	19.28	850	16388.00	20.10	−10.273	19.28	− 198.063
9	17	289	24.36	818	19926.48	22.78	− 7.593	24.36	− 184.965
10	19	361	26.80	802	21493.60	25.46	− 4.913	26.80	− 131.668
10′	21	441				28.14	− 2.233	26.80	− 59.844
9′	23	529	121.51		110167.50	30.82	+ 0.447	24.36	+ 10.889
8′	25	625	2	Symmetrical values combined		33.50	+ 3.127	19.28	+ 60.289
7′	27	729				36.18	+ 5.807	15.22	+ 88.383
6′	29	841	243.02		$\Sigma z^2\Delta$	38.86	+ 8.487	11.96	+ 101.505
5′	31	961	$\Sigma\Delta$			41.54	+11.167	10.41	+ 116.248
4′	33	1089				44.22	+13.847	7.89	+ 109.253
3′	35	1225				46.90	+16.527	3.42	+ 56.522
2′	37	1369				49.58	+19.207	1.51	+ 29.003
1′	39	1521				52.26	+21.887	0.66	+ 14.445
								243.02 $\Sigma\Delta$	+ 586.537
									−1454.845
									− 868.308 $\Sigma\Delta\left(x-\frac{\Sigma x^2\Delta}{\Sigma x\Delta}\right)$

$$\frac{\Sigma x^2\Delta}{\Sigma x\Delta}=\frac{\Sigma z^2\Delta}{\frac{1}{2}l\Sigma\Delta}(\partial x)^2=\frac{\Sigma z^2\Delta}{20\Sigma\Delta}1.34$$

or $$\frac{110167.50}{20(243.02)}1.34=30.373$$

TABLE III.—COMPUTATION OF M_1—(*Continued*).

Point.	10	11	12	13	14	15	16	17	18	19
	$x-\frac{1}{2}l$ or $x-26.8$.	$x\Delta$.	$x\Delta(x-\frac{1}{2}l)$ or $x\Delta(x-26.8)$.	$\frac{\left(x-\frac{\Sigma x^2\Delta}{\Sigma x\Delta}\right)\Delta}{868.304\div 1.34}$ or $\frac{\Delta(x-30.373)}{648}$.	$x\div 1.34$.	$(l-x)\div 1.34$.	R_1.	R_2.	$\frac{\Delta(x-30.373)}{648\times 1.34}x$.	$\frac{\Delta(x-30.373)x'}{648\times 1.34}$.
1	− 25.46	0.88	855.711	−0.0296	1	39	.975	.025	− 0.0296	− 1.1544
2	− 22.78	6.07	1567.264	− .0614	3	37	.925	.075	− .1842	− 2.2718
3	− 20.10	22.91	2763.549	− .125	5	35	.875	.125	− .625	− 4.375
4	− 17.42	74.01	4788.584	− .256	7	33	.825	.175	− 1.792	− 8.448
5	− 14.74	125.54	4523.559	− .294	9	31	.775	.225	− 2.646	− 9.114
6	− 12.06	176.29	3479.069	− .289	11	29	.725	.275	− 3.179	− 8.381
7	− 9.38	265.13	2678.271	− .304	13	27	.675	.325	− 3.952	− 8.208
8	− 6.70	387.53	1730.945	− .306	15	25	.625	.375	− 4.590	− 7.650
9	− 4.02	554.92	787.357	− .285	17	23	.575	.425	− 4.845	− 6.555
10	− 1.34	682.33	96.234	− .203	19	21	.525	.475	− 3.857	− 4.263
10′	+ 1.34	754.15	23270.543	− .0924	21	19	.475	.525	− 1.9404	− 1.7556
9′	+ 4.02	750.78		+ .0168	23	17	.425	.575	+ 0.3864	+ 0.2856
8′	+ 6.70	645.88	$\Sigma x\Delta\left(x-\frac{l}{2}\right)$	+ .0934	25	15	.375	.625	+ 2.335	+ 1.401
7′	+ 9.38	550.66		+ .136	27	13	.325	.675	+ 3.672	+ 1.768
6′	+ 12.06	464.77		+ .157	29	11	.275	.725	+ 4.553	+ 1.727
5′	+ 14.74	432.43		+ .179	31	9	.225	.775	+ 5.549	+ 1.611
4′	+ 17.42	348.90		+ .169	33	7	.175	.825	+ 5.577	+ 1.183
3′	+ 20.10	160.40		+ .0872	35	5	.125	.875	+ 3.052	+ 0.436
2′	+ 22.78	74.87		+ .0447	37	3	.075	.925	+ 1.6539	+ 0.1341
1′	+ 25.46	34.49		+0.0223	39	1	.025	.975	+ 0.8697	+ 0.0223
	−134.00		$\frac{\Sigma x\Delta\left(x-\frac{l}{2}\right)}{\frac{1}{2}l}=868.304$	+0.9054					−27.6402	+ 8.5680
	+134.00		See Col 9.	−2.2454					+27.6480	−62.1758
	0.00 $\Sigma(x-26.8)$		Denominator of m_1.	−1.3400					+ .0078 $\Sigma=0$	−53.6078 $\Sigma=L$

TABLE III.—COMPUTATION OF M_1—(*Concluded*).

Point.	20	21	22	23	24	25	26	27
	$\sum_0^a \frac{\Delta(x-30.373)x}{648\times 1.34}$ See Col. 18.	$\sum_{x'=0}^{x<(l-a)} \frac{\Delta(x-30.373)}{648\times 1.34}x'$. See Col. 19.	R_1 "A".	R_2 "B".	$-m_1$.	$H_1\frac{\Sigma y\Delta}{\Sigma\Delta}$ or $8.1576H_1$.	M_1 Unit Loading.	M_1 For Symmertical Loading.
1	− 0.0296	−52.4534	− 0.0289	−1.311	− 1.339	0	− 1.339	− 1.339
2	− 0.2138	−50.1816	− 0.198	−3.764	− 3.962	0.235	− 3.727	− 3.536
3	− 0.8388	−45.8066	− 0.734	−5.726	− 6.460	0.860	− 5.600	− 4.918
4	− 2.6308	−37.3586	− 2.170	−6.538	− 8.708	2.089	− 6.619	− 5.016
5	− 5.2768	−28.2446	− 4.090	−6.355	−10.445	4.200	− 6.245	− 3.177
6	− 8.4558	−19.8636	− 6.130	−5.463	−11.593	6.930	− 4.663	+ 0.131
7	−12.4078	−11.6556	− 8.375	−3.788	−12.163	9.894	− 2.269	+ 4.171
8	−16.9978	− 4.0056	−10.624	−1.502	−12.126	12.749	+ 0.623	+ 8.328
9	−21.8428	+ 2.5494	−12.560	+1.084	−11.476	15.065	+ 3.589	+11.833
10	−25.6998	+ 6.8124	−13.492	+3.236	−10.256	16.368	+ 6.112	+13.849
10′	−27.6402	+ 8.5680	−13.129	+4.498	− 8.631	16.368	+ 7.737	
9′	−27.2538	+ 8.2824	−11.583	+4.762	− 6.821	15.065	+ 8.244	
8′	−24.9188	+ 6.8814	− 9.345	+4.301	− 5.044	12.749	+ 7.705	
7′	−21.2468	+ 5.1134	− 6.905	+3.451	− 3.454	9.894	+ 6.440	
6′	−16.6938	+ 3.3864	− 4.591	+2.455	− 2.136	6.930	+ 4.794	
5′	−11.1448	+ 1.7754	− 2.508	+1.376	− 1.132	4.200	+ 3.068	
4′	− 5.5678	+ 0.5924	− 0.974	+0.489	− 0.486	2.089	+ 1.603	
3′	− 2.5158	+ 0.1564	− 0.315	+0.137	− 0.178	0.860	+ 0.682	
2′	− 0.8619	+ 0.0223	− 0.0646	+0.0206	− 0.044	0.235	+ 0.191	
1′	− 0					0	0	
							−30.462 +50.788	
	"A."	"B."					+20.326 ΣM_1	

Dividing both numerator and denominator by 1.34, x becomes z and the denominator 648. Cols. 14 to 26 inclusive show the solution of the above equation in detail. As checks col. 18 should sum zero and col. 19 have an algebraic sum equal to the span, in this case 53.6. The error in each case is 0.0074.

Cols. 26 and 27 give the values of M_1 for unit loads.

74. Values of V_1, y_1, y_2, y_0, etc., for Unit Loads.—These quantities are quickly determined as shown in Table IV, which also contains for convenience in future calculations the values of $\sum_0^x$ of H_1, V_1, V_2, M_1, and M_2.

75. Values of H_1 and M_1 for the Dead Load.—Since the span is divided into equal parts, the dead load at each point equals $140 \times 2.68 \times$ the ordinate from the intrados to the roadway, nearly; so it is unnecessary to carry the common factor 375.2 through the work. Column 2 of Table V, page 98, contains the ordinates which must be multiplied by 375.2 in order to obtain the dead load assumed at each point.

Tables V and VI give the values of H_1 and M_1 as found by considering each load separately, and also by considering the loading as a whole. For M_1 we have -40.289 and -40.301. For H_1 we have 46.504 and 46.502, in both cases close agreement.

76. Location of the Equilibrium Polygon for the Dead Load.—Knowing H_1, V_1, and M_1 we can graphically locate the polygon. The algebraic determination of $M_x \div H_1$, however, is more accurate and requires hardly any more time. From Arts. 16 and 17,

$$\frac{M_x}{H_1} = \left(y - \frac{\Sigma y\Delta}{\Sigma\Delta}\right) - \left(m_x - \frac{\Sigma m_x\Delta}{\Sigma\Delta}\right)\frac{1}{H_1}.$$

TABLE IV.—VALUES OF V_1, y_1, y_0, ETC.

Point No.	1	2	3	4	5	6	7	8
	$y_1=\frac{M_1}{H_1}$.	$V_1=\frac{M_2-M_1}{L}+R_1$.	$y_0=y_1+\frac{V_1a}{H_1}$.	$\sum_0^x V_1$.	$\sum_0^x V_2$.	$\sum_0^x H_1$.	$\sum_0^x M_1$.	$\sum_0^x M_2$.
1	∞	1.000	....	1.000	0	0	− 1.339	0
2	−129.41	.998	9.86	1.998	.002	.0288	− 5.066	+ 0.191
3	− 53.13	.992	9.93	2.990	.010	.1342	−10.666	+ 0.873
4	− 25.84	.978	10.02	3.968	.032	.3903	−17.285	+ 2.476
5	− 12.13	.948	10.07	4.916	.084	.9052	−23.530	+ 5.544
6	− 5.49	.901	10.15	5.817	.183	1.7547	−28.193	+10.338
7	− 1.87	.837	10.15	6.654	.346	2.9675	−30.462	+16.778
8	+ 0.39	757	10.13	7.411	.589	4.5303	−29.839	+24.483
9	+ 1.94	.662	10.11	8.073	.927	6.3770	−26.250	+32.727
10	+ 3.04	.555	10.08	8.628	1.372	8.3835	−20.138	+40.464
10′	+ 3.85	.445		9.073	1.927	10.3900	−12.401	+46.576
9′	+ 4.46	.338		9.411	2.589	12.2367	− 4.157	+50.165
8′	+ 4.93	.243		9.654	3.346	13.7995	+ 3.548	+50.788
7′	+ 5.31	.163		9.817	4.183	15.0123	+ 9.998	+48.519
6′	+ 5.64	.099		9.916	5.084	15.8618	+14.782	+43.856
5′	+ 5.96	.052		9.968	6.032	16.3767	+17.850	+37.611
4′	+ 6.26	.022		9.990	7.010	16.6328	+19.453	+30.992
3′	+ 6.47	.008		9.998	8.002	16.7382	+20.135	+25.392
2′	+ 6.63	.002		10.000	9.000	16.7670	+20.326	+21.665
1′	——	0		10.000	10.000	16.7670	+20.326	+20.326

TABLE V.—H_1 AND M_1 FOR DEAD LOAD.

Point No.	1	2	3	4	5	6	7	8	
	H_1. $P=P'=1$. Table II.	P. Dead Load.	H_1. Dead Load. (1)×(2).	M_1. $P=P'=1$. Table III.	M_1. Dead Load. (2)×(4).	m_x. Table VI, Col. 6.	Δ. Table I, Col. 10.	$m_x\Delta$.	Check calculations for M_1.
1+1′	0	11.50	....	− 1.339	− 15.399	69.04	0.66	45.57	
2+2′	0.0576	9.55	.5501	− 3.536	− 33.769	176.29	1.51	266.20	
3+3′	0.2109	7.30	1.5396	− 4.918	− 35.901	257.95	3.42	882.19	
4+4′	0.5123	5.63	2.8842	− 5.016	− 28.240	320.05	7.89	2525.19	
5+5′	1.0298	4.40	4.5311	− 3.177	− 13.979	367.05	10.41	3820.99	
6+6′	1.6990	3.55	6.0315	+ 0.131	+ 0.465	402.27	11.96	4811.15	
7+7′	2.4257	2.93	7.1073	+ 4.171	+ 12.221	427.97	15.22	6513.70	
8+8′	3.1256	2.45	7.6577	+ 8.328	+ 20.404	445.82	19.28	8595.40	
9+9′	3.6935	2.18	8.0518	+11.833	+ 25.796	457.10	24.36	11134.96	
10+10′	4.0131	2.03	8.1466	+13.849	+ 28.113	462.54	26.80	12396.07	
	16.7675	51.52	46.5019	−30.462	−127.288		121.51	50991.42	
		2		+50.788	+ 86.999		2	2	
		103.04		+20.326	− 40.289		243.02	101982.84	

For symmetrical loading $M_1=H_1\frac{\Sigma y\Delta}{\Sigma\Delta}-\frac{\Sigma m_x\Delta}{\Sigma\Delta}=379.345-419.646=-40.301$ for dead load.

TABLE VI.—CHECK CALCULATIONS—H_1 FOR DEAD LOAD.

Point No.	1	2	3	4	5	6	7	8
	P Dead Load.	$\frac{x}{1.34}$ z.	$P(x-a)\frac{1}{1.34}$.	$R_1=\frac{\Sigma P}{2}$. $R_1\times\frac{x}{1.34}$.	$\frac{m_x}{1.34}$. (4)−(3).	m_x. 1.34(5).	$\Delta(y-8.1576)$. Table II, Col. 3.	$m_x\Delta(y-8.1576)$. (6)×(7).
1+1′	11.50	1	0	51.52	51.52	69.04	− 4.658	− 321.588
2+2′	9.55	3	23.00	154.56	131.56	176.29	− 7.712	− 1359.548
3+3′	7.30	5	65.10	257.60	192.50	257.95	−11.962	− 3085.082
4+4′	5.63	7	121.80	360.64	238.84	320.05	−17.418	− 5574.631
5+5′	4.40	9	189.76	463.68	273.92	367.05	−12.259	− 4499.666
6+6′	3.55	11	266.52	566.72	300.20	402.27	− 4.636	− 1864.924
7+7′	2.93	13	350.38	669.76	319.38	427.97	+ 2.167	+ 927.411
8+8′	2.45	15	440.10	772.80	332.70	445.82	+10.650	+ 4747.983
9+9′	2.18	17	534.72	875.84	341.12	457.10	+20.034	+ 9157.541
10+10′	2.03	19	633.70	978.88	345.18	462.54	+25.792	+11929.832
								−16705.439
								+26762.767
								+10057.328

$$H_1=\frac{\Sigma m_x\Delta(y-8.1576)}{\Sigma y\Delta(y-8.1576)}=\frac{10057.328}{216.2666}=46.504.$$

$46.5\times140\times2.68=17400=H_1$ for dead load.

TABLE VII.—LOCATION OF EQUILIBRIUM POLYGON FOR DEAD LOAD.

Point No.	1	2	3	4	5	6	7	8
	m_x. Table VI.	$\frac{\Sigma m_x \Delta}{\Sigma \Delta}$.	$m_x - \frac{\Sigma m_x \Delta}{\Sigma \Delta}$.	$\frac{m_x - \frac{\Sigma m_x \Delta}{\Sigma \Delta}}{H_1}$.	$y - \frac{\Sigma y \Delta}{\Sigma \Delta}$. Table II, Col. 2.	$\frac{M_x}{H_1}$.	Above or below Axis of Arch.	Remarks.
0	0	419.65	−419.65	−9.000	−8.158	0.844	below	The moment M_x at any point equals the ordinate in Col. 6 multiplied by H_1.
1	69.04	419.65	−350.61	−7.540	−7.058	0.482	"	
2	176.29	419.65	−243.36	−5.234	−5.108	0.126	"	
3	257.95	419.65	−161.70	−3.477	−3.498	0.021	above	
4	320.05	419.65	− 99.60	−2.144	−2.208	0.064	"	
5	367.05	419.65	− 52.60	−1.131	−1.178	0.047	"	
6	402.27	419.65	− 17.38	−0.373	−0.388	0.015	"	
7	427.97	419.65	+ 8.32	+0.179	+0.142	0.037	"	
8	445.82	419.65	+ 26.17	+0.563	+0.552	0.011	"	
9	457.10	419.65	+ 37.45	+0.805	+0.822	0.017	below	
10	462.54	419.65	+ 42.89	+0.923	+0.962	0.039	"	

$$\frac{M_x}{H_1} = \left(y - \frac{\Sigma y \Delta}{\Sigma \Delta}\right) - \left(m_x - \frac{\Sigma m_x \Delta}{\Sigma \Delta}\right)\frac{1}{H_1} = (y - 8.158) - (m_x - 419.65)\frac{1}{46.5}.$$

Table VII gives the values of $\frac{M_x}{H_1}$ in col. 6, showing that the polygon nearly coincides with the arch axis.

77. Maximum Fiber Stresses Produced by the Dead Load at Point 1.

Moment of inertia $=5.17=I$.

Area of section in equivalent concrete $=3.80+0.20$
$=4.00$ sq. ft. $=F$.

Dist. outermost fiber of concrete from neut. axis $=1.90=z$.

" c.g. of steel above or below neutral axis $=1.73=z'$.

$x=1.34$, $y=1.10$. $\sin\phi=0.618$, $\cos\phi=0.786$.

$$M_x=M_1+V_1x-H_1y,$$

$$N_x=V_1\sin\phi+H_1\cos\phi,$$

$$T_x=V_1\cos\phi-H_1\sin\phi,$$

$$M_1=-40.289(375.2)=-15116,$$

$$H_1=+46.502(375.2)=17448,$$

$$V_1=51.520(375.2)=19330,$$

$$H_1y=17448(1.10)=19193,$$

$$V_1x=19330(1.34)=25902.$$

Then

$$M_x=-15116+25902-19193=-8400.$$

From Table VII we have

$$M_x=17448(-0.482)=-8200,$$

a difference of 200 pounds per square foot.

$$N_x=19330(0.618)+17448(0.786)=25660,$$

$$p=\frac{N_x}{4}\pm 0.367(M_x)=\frac{25660}{4}\pm 0.367(-8400).$$

$\therefore$ $p=9500$ comp. in the lower fibers of concrete

and 3300 comp. in the upper fibers of concrete

The unit stresses in the steel are as follows:

$$p' = \left\{\frac{N_x}{4} \pm 0.334 M_x\right\} 20 = (6415 \pm 2800) 20.$$

$$\therefore\ p' = 184300 \text{ comp. in lower steel}$$

and 72300 comp. in upper steel.

The above unit stresses are pounds per square foot. Reducing them to pounds per square inch,

the maximum compression in the concrete is 66 lbs.
and " " " " " steel is 1280 lbs.

These values are quite insignificant when compared with the ultimate strengths of the materials.

78. Maximum Fiber Stresses Produced by the Live Load at Point 1.—From Plate III we see that loads 1–7 inclusive produce one kind of stress and loads 8–1′ inclusive the opposite kind.

A live load of 112 pounds per square foot of roadway is equivalent to about 300 pounds for each division of the span. For loads 1–7 inclusive the fiber stresses are obtained as follows (see Table IV):

$$M_1 = -30.462(300) = -9139$$
$$H_1 = 2.9675(300) = 890$$
$$V_1 = 6.654(300) = 1996$$
$$M_x = -9139 + 2680 - 979 = -7438$$

$$V_1 \sin\phi = 2000(0.618) = 1236$$
$$H_1 \cos\phi = 890(0.786) = 700$$
$$\therefore\ N_x = 1936$$

$$p = \frac{1936}{4} \pm 7438(0.367)$$

$= 3214$ compression in lower fibers of concrete

and 2246 tension in upper fibers of concrete.

For the steel we obtain

$$p' = \left\{ \frac{1936}{4} \pm 7438(0.334) \right\} 20 = 59400 \text{ comp. in lower steel}$$

and 40000 tension in upper steel.

For loads 8–1′ inclusive we have (see Table IV)

$$M_1 = 50.788(300) = +15236$$
$$H_1 = 13.7995(300) = 4140$$
$$V_1 = 3.346(300) = 1000$$
$$M_x = +15236 + 1340 - 4550 = +13330$$

$$V_1 \sin \phi = 1000(0.618) = 620$$
$$H_1 \cos \phi = 4140(0.786) = 3250$$
$$\therefore N_x = 3870$$

$$p = \frac{3870}{4} \pm 13330(0.367)$$

$= 5860$ compression in upper fibers of concrete

and 3920 tension in lower fibers of concrete.

For the steel,

$p' = [970 \pm 13330(0.334)]20 = 108400$ comp. in the upper steel

and 69600 tens. in the lower steel.

79. Maximum Fiber Stresses Produced by the Dead Load at Point 7.

Moment or inertia $= 0.18$ (see Table I).

Area of section in equiv. conc. $=1.19+0.20=1.39$ sq. ft.

$$z=0.60, \quad z'=0.43, \quad x=17.42, \quad y=8.3.$$

$\sin\phi=0.208, \qquad \cos\phi=0.978. \qquad M_1=-15116$

$M_x=M_1+V_1x-H_1y-\overset{x}{\Sigma}P(x-a), \qquad H_1=17448$

$N_x=(V_1-\overset{x}{\Sigma}P)\sin\phi+H_1\cos\phi, \qquad V_1=19330,$

$T_x=(V_1-\overset{x}{\Sigma}P)\cos\phi-H_1\sin\phi, \qquad \overset{x}{\Sigma}P=15732,$

$V_1x=336728, \qquad H_1y=144818. \qquad \overset{x}{\Sigma}P(x-a)=176241,$

$$\therefore\ M_x=+553.$$

From Table VII,

$$M_x=17448(+0.037)=+646.$$

This indicates a large percentage of error. The error is of no consequence as it amounts to less than 3 pounds per square inch fiber stress. In order that col. 6 of Table VII should be correct much greater accuracy would be required in the previous work. For practical purposes, however, col. 6 is quite accurate enough.

$$(V_1-\overset{x}{\Sigma}P)\sin\phi=749, \qquad H_1\cos\phi=17064.$$

$$\therefore\ N_x=17813.$$

$$p=\frac{N_x}{1.39}\pm M_x(3\tfrac{1}{3})=12820\pm1843,$$

or $p = 14663$ compression in upper fibers of concrete

and $p = 10977$ compression in lower fibers of concrete.

For the steel,

$$p' = \left\{ \frac{N_x}{1.39} \pm M_x(2.4) \right\} 20 = \{12820 \pm 1327\} 20$$

or $p' = 282940$ compression in upper steel

and $p' = 229860$ compression in lower steel.

80. Maximum Fiber Stresses Produced by the Live Loads at Point 7.—From Plate III we find that loads 1–8 inclusive produce positive moments at this point and loads 9–1″ negative moments.

Considering first, loads 1–8 inclusive: from Table IV,

$$M_1 = -29.839(300) = -8952, \quad (V_1 - \overset{x}{\Sigma} P)\sin\phi = 88,$$

$$H_1 = 4.53(300) = 1359, \quad H_1 \cos\phi = 1329.$$

$$V_1 = 7.411(300) = 2223, \quad \therefore N_x = 1417.$$

$$V_1 x = 2223(17.42) = 38725,$$

$$H_1 y = 1359(8.3) = 11280,$$

$$\overset{x}{\Sigma} P(x-a) = 16884. \quad \therefore M_x = +1609.$$

$$p = \frac{1417}{1.39} \pm 1610(3\tfrac{1}{3}) = 1020 \pm 5367.$$

Then

$p=4347$ tension in the lower fibers of concrete

and $p=6387$ compression in the upper fibers of concrete.

For the steel we have

$$p'=\{1020\pm1610(2.4)\}20=\{1020\pm3864\}20,$$

or $p'=56880$ tension in the lower steel

and $p'=97680$ compression in the upper steel.

Proceeding in a manner similar to that employed above for loads 8–1′ we obtain

$p=\ \ 8040$ compression in lower fibers of concrete

and $p=\ \ 2640$ tension in upper fibers of concrete,

$p'=145280$ compression in lower steel

and $p'=\ \ 37280$ tension in upper steel.

81. Maximum Fiber Stresses Produced at Points 1 and 7 by the Dead and Live Loads.—Tabulating the above results and combining those producing maximums we have the results given in the table at top of page 107.

The maximum stress in the concrete is 146 pounds compression per square inch and in the steel 2650 pounds compression per square inch, values considerably below the allowable. There is no tension at these points.

82. Temperature Stresses.—For a change of temperature of $\pm40°$ F. the horizontal thrust is 6500 when $E=1500000$ and $e=0.000006$.

MAXIMUM FIBER STRESSES.

(POUNDS PER SQUARE FOOT.)

Loads, etc.	Concrete.		Steel.		Point.
	Upper.	Lower.	Upper.	Lower.	
Dead load	+ 3300	+ 9500	+ 72300	+184300	1
Live load 1–7	− 2240	+ 3214	− 40000	+ 59400	1
" " 8–1′	+ 5860	− 3920	+108400	− 69600	1
Maximum compression	9160	12714	180700	243700	1
" tension	0	0	0	0	1
Dead load	+14663	+10977	+282940	+229860	7
Live load 1–8	+ 6384	− 4347	+ 97680	− 56880	7
" " 9–1′	− 2640	+ 8040	− 37280	+145280	7
Maximum compression	21047	19017	380620	375140	7
" tension	0	0	0	0	7

$$M_x=6500\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)=6500(y-8.1576),$$

$$N_x=H\cos\phi=6500\cos\phi.$$

For point 1,

$$M_x=6500(7.06) \quad =45900,$$

$$N_x=6500(0.786)= \ 5100.$$

For a drop of 40° F.,

$$p=-\frac{5100}{4}\mp 45900(0.367)=-1275\mp 16850,$$

or $\quad p=18125$ tension in upper fibers of concrete

and $\quad p=15575$ compression in lower fibers of concrete.

For the steel,

$$p'=332100 \text{ tension in upper steel}$$

and $\quad p' = 281100$ compression in lower steel.

For point 7,

$$M_x = 6500(0.142) = 923,$$

$$N_x = 6500(0.978) = 6400.$$

For a drop of 40° F.,

$$p = \frac{-923}{1.39} \mp 6400(3\tfrac{1}{3}) = -664 \mp 21333,$$

or $\quad p = 22000$ tension in upper fibers of concrete

and $\quad p = 20700$ compression in lower fibers of concrete.

For the steel,

$$p' = [-623 \mp 6400(2.4)]20 = -12460 \mp 307200$$

or $\quad p' = 319700$ tension in upper steel

and $\quad p' = 294700$ compression in lower steel.

A rise of 40° F. will reverse the above stresses.

83. Maximum Stresses Produced by Dead Load, Live Load, and Changes of Temperature.—Combining the stresses of Art. 81 and 82 we have:

Point 1:

$p = 27285$ compression, 17065 tension } upper fibers of concrete;

$p = 28289$ compression, 10000 tension } lower fibers of concrete;

$p' = 512800$ compression, 300000 tension } upper steel;

$p' = 524800$ compression, 166400 tension } lower steel.

Point 7:

$p = 43000$ compression, 10000 tension } upper fibers of concrete;

$p = 39700$ compression, 14100 tension } lower fibers of concrete;

$p' = 700300$ compression, 74400 tension } upper steel;

$p' = 669800$ compression, 121700 tension } lower steel.

The allowable compression in the concrete, when temperature is considered, may be assumed at $800 \times 144 = 115200$ pounds per square foot, and the tension at 11500 pounds per square foot.

In compression the maximum stresses are considerably less than the allowable, while in tension they are much larger. Yet if the tensile strength of concrete is taken as one tenth the compressive strength, the above stresses are less than the ultimate strength of the material. If it should happen that a maximum change of temperature and a maximum live load should occur at the same time, the concrete would probably crack, but the steel and the compression concrete have ample margin to cover this contingency. It is quite improbable that a range of $\pm 40°$ F. ever occurs, so the two sections may be considered safe. The crown should be examined in an actual case. Although the live-load moment will be small, the temperature moment will be considerably larger than at point 7.

84. The Axial Stress.—Thus far the effect of the axial stress has been neglected. Proceeding in the manner followed in example 1, the value of H_a is found to be about 6.7% of H_1. The effect is seen to be somewhat less than in the previous example. As the rise of the span increases the effect grows less.

85. Assumption that Steel Resists Entire Bending Moment Due to Changes of Temperature at Point 1.

Max. comp. in upper steel due to D.L. + L.L. $=1255$ lbs. per sq. in.

Max. comp. in lower steel due to D.L. + L.L. $=1700$ lbs. per sq. in.

Moment due to $\pm 40^\circ = \pm 45900$ ft.-lbs.

Area of steel $=\frac{1}{2}[\frac{3}{4}(2\frac{5}{8}-\frac{3}{4})]=0.70$ sq. in.

Dist. c. c. steel $=3.80-0.34=3.46$ ft.

Total stress in steel $=\dfrac{45900}{3.46}=13300$ lbs.

Stress per sq. in. $=\dfrac{13300}{0.70}=19000$ lbs.

Max. comp. $=19000+180+1700=20880$ lbs. per sq. in.

Max. tension $=-19000-180+1225=17960$ lbs. per sq. in.

All well within the elastic limit of the steel.

This shows that even if the ring should crack entirely through at point 1, the steel would safely carry the maximum temperature moment even when combined with the dead- and live-load stresses.

A brief calculation for point 7 and the crown shows that the steel is here stressed well within the elastic limit.

86. Third Example. — In this example we will take the data used in the second example and show how the computations of H_1 and M_1 can be quite rapidly made.

87. The Computation of H_1.—The equation used in the former calculations was

$$H_1 = \frac{\Sigma m_x \Delta \left(y - \frac{\Sigma y \Delta}{\Sigma \Delta}\right)}{2 \Sigma y \Delta \left(y - \frac{\Sigma y \Delta}{\Sigma \Delta}\right)},$$

where m_x = the common moment for equal and symmetrically placed loads. Assuming unit loads, the following values of m_x may be written:

Between the load and the left support

$$m_x = R_1 x = x = \frac{\delta x}{2} z,$$

where δx is the length of the division into which the span is divided, or $l = n\delta x$.

Between the first load and the center of the span

$$m_x = R_1 x - (x-a) = a = k\frac{\delta x}{2}.$$

Then

$$\tfrac{1}{2}\Sigma m_x \Delta\left(y - \frac{\Sigma y\Delta}{\Sigma\Delta}\right) = \left(\sum_{x=0}^{x=a} z\left(y - \frac{\Sigma y\Delta}{\Sigma\Delta}\right)\Delta + k\sum_{x=a}^{x=\frac{l}{2}}\left(y - \frac{\Sigma y\Delta}{\Sigma\Delta}\right)\Delta\right)\frac{\delta x}{2} = D\frac{\delta x}{2},$$

an expression which is very quickly handled numerically.

Although the general data, such as the values of x, y, I, δs, etc., are given in the second example, we will repeat some of it for convenience.

GENERAL DATA.

	x.	y.	I.	δs.	Δ.	$y\Delta$.	$y-\frac{\Sigma y\Delta}{\Sigma\Delta}$.	B. $\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)$
1	1.34	1.10	5.17	3.41	0.66	0.726	−7.0576	− 4.658
2	4.02	3.05	2.13	3.21	1.51	4.606	−5.1076	− 7.712
3	6.70	4.66	0.90	3.08	3.42	15.937	−3.4976	−11.962
4	9.38	5.95	0.37	2.92	7.89	46.946	−2.2076	−17.418
5	12.06	6.98	0.27	2.81	10.41	72.662	−1.1776	−12.259
6	14.74	7.77	0.23	2.75	11.96	92.929	−0.3876	− 4.636
7	17.42	8.30	0.18	2.74	15.22	126.326	+0.1424	+ 2.167
8	20.10	8.71	0.14	2.70	19.28	167.929	+0.5524	+10.650
9	22.78	8.98	0.11	2.68	24.36	218.753	+0.8224	+20.034
10	25.46	9.12	0.10	2.68	26.80	244.416	+0.9624	+25.792
					121.51	991.23		−58.645
					2	2		+58.643
					243.02 $\Sigma\Delta$	1982.46 $\Sigma y\Delta$		.002

The values of B in the last column when multiplied by y give the denominator of the expression for H_1.

COMPUTATIONS FOR H_1.

(UNIT LOADS.)

Point.	C. $y\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)$	z.	k.	zB.	$\sum_{x=0}^{x=a} zB$.	$\sum_{x=a}^{x=\frac{l}{2}} B$.	$k\sum_{x=a}^{x=\frac{l}{2}} B$.	D.	$H_1=\frac{D}{\Sigma C}\frac{\delta x}{2}$.
1	− 5.1238	1	1	− 4.658	− 4.658	+ 4.656	4.656	0	0
2	− 23.5256	3	3	− 23.136	− 27.794	+12.368	37.104	9.310	0.0288
3	− 55.7412	5	5	− 59.810	− 87.604	+24.330	121.650	34.046	0.1055
4	−103.6379	7	7	−121.926	−209.530	+41.748	292.236	82.706	0.2562
5	− 85.5667	9	9	−110.331	−319.861	+54.007	486.063	166.202	0.5149
6	− 36.0192	11	11	− 50.996	−370.857	+58.643	645.073	274.216	0.8495
7	+ 17.9888	13	13	+ 28.171	−342.686	+56.476	734.188	391.502	1.2129
8	+ 92.7639	15	15	+159.750	−182.936	+45.826	687.390	404.454	1.5628
9	+179.9024	17	17	+340.578	+157.642	+25.792	438.464	596.106	1.8468
10	+235.2259	19	19	+490.048	+647.690	0		647.680	2.0066
	−309.6144								8.3840
	+525.8810								2
	+216.2666								16.7680
	2								
	+432.5332 ΣC								

The values of H_1 are identical with those obtained in the second example. The number of operations is very much reduced and the multiplications simplified. This method is shorter than any algebraic or graphical method advanced up to this time. (See pages 90 and 91.)

88. The Computation of M_1 and M_2.—In this case we will employ the formula

$$M_1 = H_1 \frac{\Sigma y \Delta}{\Sigma \Delta} - \left(\frac{\Sigma m_x \Delta}{\Sigma \Delta} + \frac{\Sigma m_x \left(x - \frac{l}{2} \right) \Delta}{\Sigma \Delta \left(\frac{l}{2} - \frac{\Sigma x^2 \Delta}{\Sigma x \Delta} \right)} \right),$$

$$\text{———} \quad H_1 \frac{\Sigma y \Delta}{\Sigma \Delta}. \quad \text{———}$$

This expression contains only known quantities and requires but one division and ten multiplications.

$$\text{———} \quad \frac{\Sigma m_x \Delta}{\Sigma \Delta}. \quad \text{———}$$

As before let $x = \frac{\delta x}{2} z$ and $a = \frac{\delta x}{2} k$.

For all points upon the left of the load

$$m_x = R_1 x, \qquad R_1 = \frac{l-a}{l} = \frac{2n-k}{2n}.$$

$$\therefore \ m_x \Delta = \Delta \frac{2n-k}{2n} \cdot z \frac{\delta x}{2}.$$

Upon the right of the load, between $x' = a$ and $x' = 0$

$$m_x = R_2 x', \qquad R_2 = \frac{a}{l} = \frac{k}{2n}.$$

$$\therefore \ m_x \Delta = \Delta \frac{k}{2n} z \frac{\delta x}{2}.$$

Since Δ has symmetrical values,

$$\sum_{0}^{x=a} m_x \Delta = \left\{ \sum_{x=0}^{x=a} z \Delta \right\} \frac{\delta x}{2},$$

represents the summation of $m_x \Delta$ from $x = 0$ to $x = a$ and $x = l - a$ to $x = l$.

Upon the right of the load and until $x=l-a$, $m_x=R_2x'$, and for the two values of m_x corresponding to symmetrical values of Δ this becomes

$$R_2x'+R_2(l-x')=R_2l=k\frac{\delta x}{2}.$$

$$\therefore \sum_{x=a}^{x=l-a} m_x\Delta=\left\{k\sum_{x=a}^{x=l/2}\Delta\right\}\frac{\delta x}{2},$$

and therefore

$$\Sigma m_x\Delta=\left\{\sum_{x=0}^{x=a} z\Delta+k\sum_{x=a}^{x=l/2}\Delta\right\}\frac{\delta x}{2}.$$

$\Sigma\Delta$, the denominator of the expression $\frac{\Sigma m_x\Delta}{\Sigma\Delta}$, is already known, hence the value of the expression is quickly determined.

—— $\Sigma m_x\Delta(x-\frac{1}{2}l)$, ——

$(x-\frac{1}{2}l)=(z-n)\frac{\delta x}{2}$, where the values are evidently symmetrical about the center of the span but *contrary in sign.*

Until $x=a$

$$m_x=\frac{2n-k}{2n}z\frac{\delta x}{2}.$$

Between $x=l-a$ and $x=l$

$$m_x=\frac{k}{2n}z\frac{\delta x}{2}.$$

Then for the symmetrical values of $(z-n)$ which have contrary signs we have for the two values of m_x

$$\left\{\frac{2n-k}{2n}-\frac{k}{2n}\right\}z\frac{\delta x}{2}=\frac{n-k}{n}z\frac{\delta x}{2}.$$

For $x=a$ to $x=0$ and $x'=a$ to $x'=0$

$$\Sigma m_x \Delta (x - \tfrac{1}{2}l) = \frac{1}{n}\left(\frac{\delta x}{2}\right)^2 (n-k) \sum_{x=0}^{x=a} (z-n)z\Delta.$$

From $x=a$ to $x=l-a$ or $x'=a$

$$m_x = R_2 x' \quad \text{and} \quad m_x = R_2(l-x').$$

For symmetrical points

$$R_2(l-x') - R_2 x' = 2R_2(\tfrac{1}{2}l - x') = \frac{k}{n}(n-z)\frac{\delta x}{2}.$$

Summing the symmetrical values,

$$\Sigma m_x \Delta (x - \tfrac{1}{2}l) = -\frac{k}{n}\left(\frac{\delta x}{2}\right)^2 \sum_{x=a}^{x=l/2} (z-n)^2.$$

∴ For the total summation

$$\Sigma m_x \Delta (x - \tfrac{1}{2}l) = \left[(n-k) \sum_{x=0}^{x=a} (z-n)z\Delta - k \sum_{x=a}^{x=l/2} (z-n)^2 \Delta \right] \left(\frac{\delta x}{2}\right)^2 \frac{1}{n}.$$

This expression is somewhat long but very easy to use

$$\text{———} \quad \Sigma\Delta\left(\frac{l}{2} - \frac{\Sigma x^2 \Delta}{\Sigma x \Delta}\right), \quad \text{———}$$

$$\Sigma x^2 \Delta = \left(\frac{\delta x}{2}\right)^2 \Sigma z^2 \Delta,$$

$$\Sigma x \Delta = \frac{\delta x}{2} \Sigma z \Delta = n \frac{\delta x}{2} \Sigma \Delta, \quad \tfrac{1}{2}l = n\frac{\delta x}{2}.$$

$$\therefore \ \frac{l}{2} - \frac{\Sigma x^2 \Delta}{\Sigma x \Delta} = \left(n - \frac{\Sigma z^2 \Delta}{n \Sigma \Delta}\right)\frac{\delta x}{2},$$

and the denominator becomes

$$\left(n-\frac{\Sigma z^2\Delta}{n\Sigma\Delta}\right)\frac{\delta x}{2}\Sigma\Delta.$$

The expression for M_1 now becomes

$$\left.\begin{matrix}M_1\\M_2\end{matrix}\right\}=H_1\frac{\Sigma y\Delta}{\Sigma\Delta}-\left[\left\{\sum_{x=0}^{x=a}z\Delta+k\sum_{x=a}^{x=l/2}\Delta\right\}\frac{\delta x}{2\Sigma\Delta}\right.$$

$$\left.\pm\left\{(n-k)\sum_{x=0}^{x=a}(z-n)z\Delta-k\sum_{x=a}^{x=l/2}(z-n)^2\Delta\right\}\frac{\delta x}{2n\left(n-\frac{\Sigma z^2\Delta}{n\Sigma\Delta}\right)\Sigma\Delta}\right]$$

COMPUTATIONS FOR $\frac{\Sigma m_x\Delta}{\Sigma\Delta}$.

Point.	1	2	3	4	5	6	7	8
	z.	$z\Delta$.	$\sum_{x=0}^{x=a} z\Delta$.	$\sum_{x=a}^{x=\frac{1}{2}l}\Delta$.	k.	$k\sum_{x=a}^{x=\frac{1}{2}l}\Delta$.	$\frac{\Sigma m_x\Delta}{1.34}$.	$\frac{\Sigma m_x\Delta}{\Sigma\Delta}$.
1	1	0.66	0.66	120.85	1	120.85	121.51	0.662
2	3	4.53	5.19	119.34	3	358.02	363.21	2.003
3	5	17.10	22.29	115.92	5	579.60	601.89	3.318
4	7	55.23	77.52	108.03	7	756.21	833.73	4.597
5	9	93.69	171.21	97.62	9	878.58	1049.79	5.788
6	11	131.56	302.77	85.66	11	942.26	1245.03	6.865
7	13	197.86	500.63	70.44	13	915.72	1416.35	7.810
8	15	289.20	789.83	51.16	15	767.40	1557.23	8.586
9	17	414.12	1203.95	26.80	17	455.60	1659.55	9.150
10	19	509.20	1713.15	0	19	0	1713.15	9.446
							Cols. (3+6)	Col. 7 $\left(\frac{\delta x}{2\Sigma\Delta}\right)$

$$\frac{\delta x}{2\Sigma\Delta}=\frac{2.68}{2(243.02)}=0.0005514.$$

Col. 8 represents the sum of the moments for each load multiplied by the corresponding value of Δ, divided by $\Sigma\Delta$. By ordinary methods the determination of $\Sigma m_x\Delta$ for one load only requires the scaling of 20 ordinates, 10 additions, and 10 multiplications.

COMPUTATION OF $\dfrac{\Sigma m_x \Delta(x-\frac{1}{2}l)}{\Sigma\Delta\left(\dfrac{l}{2}-\dfrac{\Sigma x^2\Delta}{\Sigma x\Delta}\right)}$.

	1	2	3	4	5	6	7	8
	$n-k$.	k.	$z-n$.	z.	$(z-n)z$.	$z(z-n)\Delta$.	$\sum_{x=0}^{x=a}\Delta(z-n)z$.	$(n-k)\sum_{x=0}^{x=a}\Delta(z-n)z$.
1	19	1	−19	1	−19	− 12.54	− 12.54	− 238.26
2	17	3	−17	3	−51	− 77.01	− 89.55	− 1522.35
3	15	5	−15	5	−75	− 256.50	− 346.05	− 5190.75
4	13	7	−13	7	−91	− 717.99	−1064.04	−13832.52
5	11	9	−11	9	−99	−1030.59	−2094.63	−23040.93
6	9	11	− 9	11	−99	−1184.04	−3278.67	−29508.03
7	7	13	− 7	13	−91	−1385.02	−4663.69	−32645.83
8	5	15	− 5	15	−75	−1446.00	−6109.69	−30548.45
9	3	17	− 3	17	−51	−1242.36	−7352.05	−22056.15
10	1	19	− 1	19	−19	− 509.20	−7861.25	− 7861.25

	9	10	11	12	13	14
	$-(z-n)^2$.	$-\Delta(z-n)^2$.	$-\sum_{x=a}^{x=l/2}\Delta(z-n)^2$.	$-k\sum_{x=a}^{x=l/2}\Delta(z-n)^2$.	Cols. (8+12).	Col. 13 Multiplied by −0.0001034.
1	−361	− 238.26	−6241.49	− 6241.49	− 6479.75	+0.670
2	−289	− 436.39	−5805.10	−17415.30	−18937.65	+1.958
3	−225	− 769.50	−5035.60	−25178.00	−30368.75	+3.140
4	−169	−1333.41	−3702.19	−25915.33	−39747.85	+4.110
5	−121	−1259.61	−2442.58	−21983.22	−45024.15	+4.656
6	− 81	− 968.76	−1473.82	−16212.02	−45720.25	+4.728
7	− 49	− 745.78	− 728.04	− 9464.52	−42110.35	+4.354
8	− 25	− 482.00	− 246.04	− 3690.60	−34238.05	+3.540
9	− 9	− 219.24	− 26.80	− 455.60	−22511.75	+2.328
10	− 1	− 26.80	0	0	− 7861.25	+0.813

$\dfrac{\delta x}{2n\left(n-\dfrac{\Sigma z^2\Delta}{n\Sigma\Delta}\right)\Sigma\Delta}=-0.0001034.$ See Table III, page 93, of the second example.

Col. 14 is the complete value of $\dfrac{\Sigma m_x\Delta(x-\frac{1}{2}l)}{\Sigma\Delta\left(\frac{1}{2}l-\dfrac{\Sigma x^2\Delta}{\Sigma x\Delta}\right)}$ for each load, 1 to 10 respectively.

Note that cols. 1, 2, 3, 4, 5, and 9 will remain the same as long as $n=20$ regardless of the span. The formation of the remaining columns requires but 50 multiplications and 30 additions.

FINAL VALUES OF M_1 AND M_2.

	1	2	3	4	5	6	7
	$H_1\frac{\Sigma y\Delta}{\Sigma\Delta}$.	m_1: $+\frac{\Sigma m_x\Delta}{\Sigma\Delta} \pm$	$\frac{\Sigma m_x\Delta(x-\frac{1}{2}l)}{\Sigma\Delta\left(\frac{1}{2}l-\frac{\Sigma x^2\Delta}{\Sigma x\Delta}\right)}$.	m_1.		M_1.	M_2.
1	+ 0	+0.662	±0.670	1.332	0	−1.332	0
2	+ 0.235	+2.003	1.958	3.961	0.045	−3.726	+0.190
3	+ 0.860	+3.318	3.140	6.458	0.178	−5.598	+0.682
4	+ 2.089	+4.597	4.110	8.707	0.487	−6.619	+1.602
5	+ 4.200	+5.788	4.656	10.444	1.132	−6.244	+3.068
6	+ 6.930	+6.865	4.728	11.593	2.137	−4.663	+4.793
7	+ 9.894	+7.810	4.354	12.164	3.456	−2.270	+6.438
8	+12.749	+8.586	3.540	12.126	5.046	+0.623	+7.703
9	+15.065	+9.150	2.328	11.478	6.822	+3.587	+8.243
10	+16.368	+9.446	±0.813	10.259	8.633	+6.109	+7.735

Combining the values found we obtain the values of M_1 and M_2 for each load 1 to 10 respectively; for loads 1′ to 10′ M_1 and M_2 simply change places. Compare cols. 6 and 7 with col. 26, page 95.

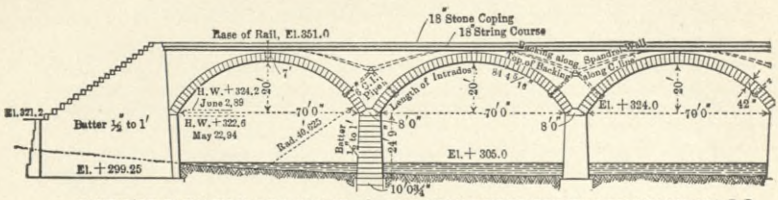
The values of V_1, y_1, y_2, etc., can now be found as in the second example.

The above calculations require but little more time than some of the graphical constructions in common use which only give the equilibrium polygon for one set of loads. Here we can quickly determine the effect of each load and then use those producing maximum results

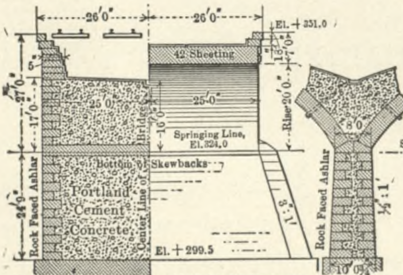
CHAPTER IV.

TYPICAL ARCHES.

A FEW typical bridges will be illustrated in this chapter which will clearly show that, as ordinarily constructed, the arch ring proper is heavily reinforced either by masonry or concrete backing or masonry side walls. Since this masonry does not readily follow the arch ring if it sinks, the actual dead load is never more than the dead weight of the material above the ring; and since the passive resistance of this masonry against moving upward is large in case the arch ring has such a tendency, it is evident that any ring which is stable under the elastic theory must be stable in the structures as built. Furthermore, experience teaches that temperature stresses may be ignored in stone arches well backed, as is customary. A recording thermometer placed in the ring of a reinforced-concrete bridge having earth filling indicated that the total range of temperature change did not exceed about 20° F. in some ten or twelve months. All rings without backing should be designed to resist a change of temperature of about ±35° F. In rings like that of the Luxemburg bridge full account of temperature must be considered, the range approaching that for steel.

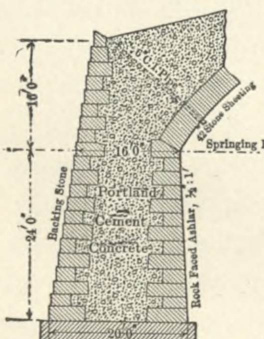


PART SIDE ELEVATION OF 3,820 FT. STONE ARCH BRIDGE FOR THE PENNSYLVANIA R.R. AT ROCKVILLE, PA.



HALF CROSS SECTION THROUGH PIER. HALF CROSS SECTION THROUGH KEY. LONGITUDINAL SECTION
SECTIONS SHOWING REGULAR PIER MASONRY, ROCKVILLE BRIDGE.

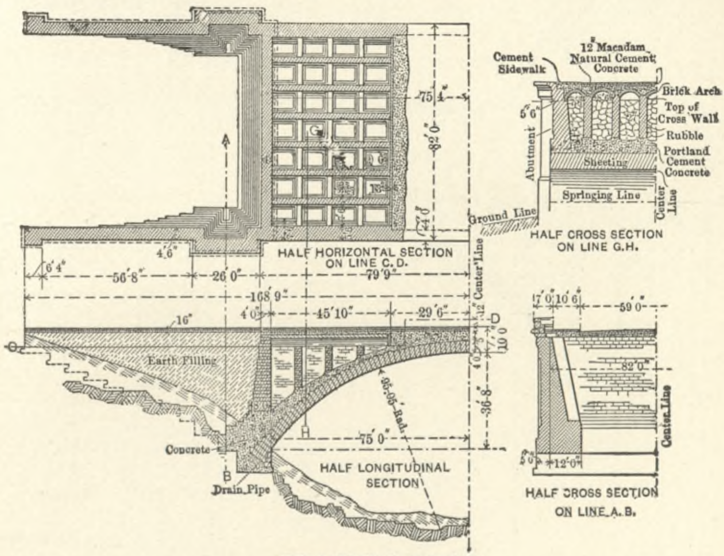
TRANSVERSE SECTION SHOWING ABUTMENT PIER MASONRY, ROCKVILLE BRIDGE.



TRANSVERSE SECTION SHOWING ABUTMENT MASONRY, ROCKVILLE BRIDGE.

89. The Rockville Stone Arch Bridge.—This is typical of a large number of arches recently constructed by the Pennsylvania R.R. The arch ring is backed with Portland-cement concrete to such an extent that it is increased in thickness nearly three times near the spring line. (Eng. News, May 10, 1900.)

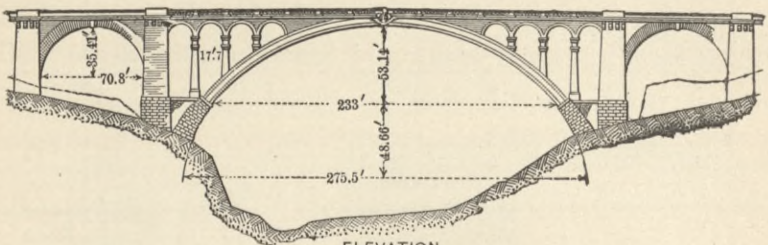
90. The Bellefield Stone Arch Bridge, Pittsburg, Pa.—In this bridge the outside spandrels are of solid masonry. Inside there are six longitudinal walls reinforced with three lateral walls. The lateral walls do not support any vertical load, as they stop at the springing of the arches between the longitudinal walls. The arch ring is securely held by a backing of concrete and the spandrel walls. (Eng. Record, June 9, 1900.)



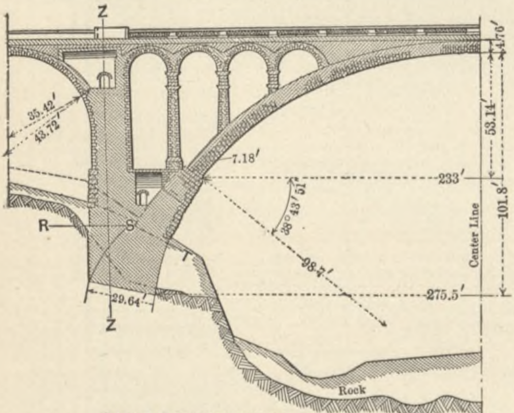
BELLEFIELD BRIDGE

91. The Luxemburg Stone Arch Viaduct.—This bridge is an excellent illustration of spandrels pierced with lateral arched openings. The elastic theory can be applied with confidence in bridges of this type. (Eng. Record, Oct. 12, 1901.)

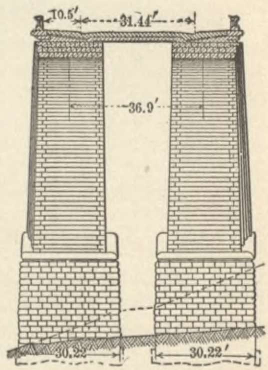
92. Approaches to the Thebes Bridge.—The approaches are composed of eleven *plain-concrete* arches having a span of 65 feet, and one with a span of 100 feet. The proportions of the 65-foot arches are clearly shown above. Note that the spandrel side walls cover nearly all of the arch ring at the supports, and at the crown two fifths of the ring, practically preventing distortion under moving loads. (R. R. Gazette, Jan. 9, 1903.)



ELEVATION.

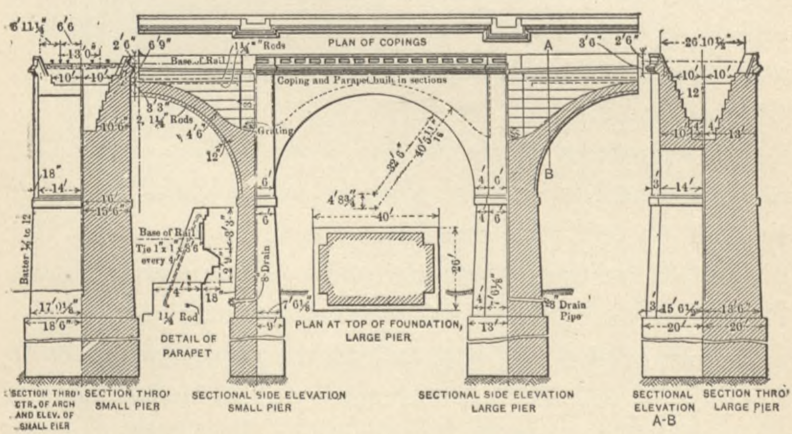


HALF LONGITUDINAL SECTION.



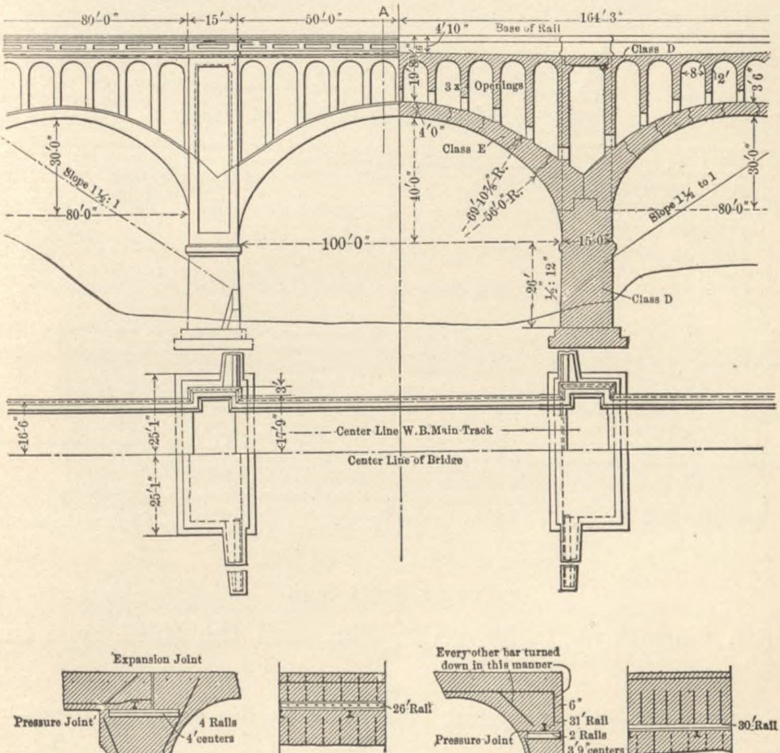
SECTION THROUGH CROWN.

LUXEMBURG BRIDGE



DETAILS OF ARCHES IN APPROACHES: THEBES BRIDGE

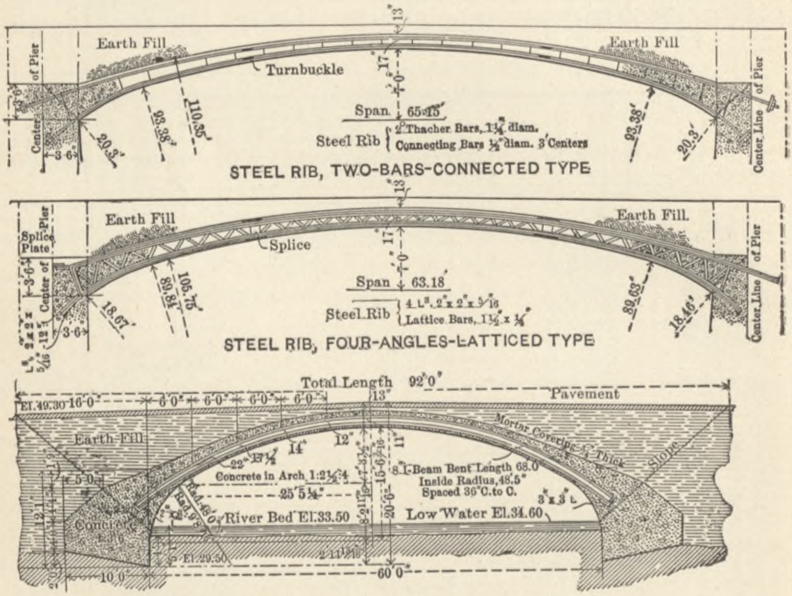
93. Vermillion River Plain-concrete Arch Bridge. — This bridge is composed of three spans. The entire loading above the ring is supported by lateral walls which makes the application of the elastic theory quite rational.



CONCRETE ARCH BRIDGE OVER SALT FORK OF THE VERMILLION RIVER

The ring was designed without reinforcement, but when built the entire concrete work was reinforced with steel bars. This reinforcement is shown in the Railroad Gazette, Oct. 27, 1905.

94. Steel Reinforcement in the Form of Ribs.— Where the steel reinforcement has been concentrated in concrete rings the three types shown below have been successfully used. The steel is usually assumed to take the entire bending moment. The two upper types were used



STEEL RIB, TWO-BARS-CONNECTED TYPE

STEEL RIB, FOUR-ANGLES-LATTICED TYPE

STEEL RIB, ROLLED I BEAM TYPE

TYPES OF STEEL RIBS

in a viaduct at Jacksonville, Fla., and the third type in St. Louis, Mo.

95. **Steel Reinforcement other than in the Form of Ribs.**— The present method of reinforcement appears to follow the idea of thoroughly distributing the steel in layers a few inches from the upper and lower surfaces of the arch ring. This is accomplished by the use of small rods, wire netting, expanded metal, etc. The majority of reinforced concrete bridges in the United States are reinforced with rolled rods, some plain and some "deformed."

96. **Area of Steel Reinforcement.** — The amount of steel is seldom less than 0.6% or greater than 1% of the

area of the ring at the crown, regardless of the type of reinforcement employed.

97. **Abstracts from Specifications.** — The following abstracts from specifications will indicate some of the methods employed and assumptions made in the construction of reinforced-concrete bridges.

Conditions of Calculation.—Modulus of elasticity of concrete, 1,400,000 lbs.; modulus of elasticity of steel, 28,000,000 lbs.; maximum stress per square inch on steel, 10,000 lbs.; maximum compression per square inch on concrete, 500 lbs.; maximum shear per square inch on concrete, 100 lbs.; maximum tension per square inch on concrete, 50 lbs. The above to be exclusive of temperature stresses. The steel ribs, under a stress not exceeding their elastic limit, must be capable of taking the entire bending moment of the arch without aid from the concrete, and have a flange area of not less than one one-hundred-and-fiftieth part of the total area of the arch at crown.

Portland-cement Concrete.—The concrete shall be composed of clean hard broken stone, or gravel with irregular surface, clean sharp sand, and cement, mixed in the proportions hereafter specified. Whenever the amount of work to be done is sufficient to justify it, approved mixing-machines shall be used. The ingredients shall be placed in the machine in a dry state, and in the volumes specified, and be thoroughly mixed, after which clean water shall be added and the mixing continued until the wet mixture is thorough and the mass uniform. No more water shall be used than the concrete will bear without quaking in ramming. The mixing must be made as rapidly as possible, and the batch deposited in the work without delay. If the mixing is done by hand, the cement and sand shall first be thoroughly mixed dry in the proportions specified. The stone, previously drenched with water, shall then be deposited on this mixture. Clean water shall be added and the mass be thoroughly mixed and turned over until each stone is covered with mortar, and the batch shall be deposited without delay, and be thoroughly rammed until all voids are filled. The grades of concrete to be used are as follows: For the arches between skewbacks, one part Portland cement, two parts sand, and four parts broken stone or gravel that will pass through a one-and-one-quarter-inch ring. For the piers, one part Portland cement, three parts sand, and six parts broken stone that will pass through a two-inch ring. For the foundations, abutments, and spandrels, one part Portland cement, four parts sand, and eight parts broken stone or gravel that will pass through a two-inch ring.

Concrete Facing.—Concrete facing will be used and shall be composed of one part Portland cement and two and one-half parts sand, and shall have a thickness of at least one inch on all arch soffits, arch faces, abutments, piers, spandrels, or other exposed surfaces. There must be no definite plane or surface of demarkation between the facing and the concrete backing. The facing and backing must be deposited in the same layer, and be well rammed in place at the same time. If the arch faces, quoins, or other exposed surfaces are marked to represent masonry, such division-marks shall be made by triangular strips two inches wide and one inch deep fastened to the casing in perfectly straight and parallel lines, and all projecting corners will be beveled to correspond.

Connections.—In connecting concrete already set with new concrete the surface shall be cleaned and roughened, and mopped with a mortar composed of one part Portland cement and one part sand, to cement the parts together.

*Arches.**—The concrete for the arches shall be started simultaneously from both ends of the arch, and be built in longitudinal sections wide enough to inclose at least two steel ribs, and of sufficient width to constitute a day's work. The concrete shall be deposited in layers, each layer being well rammed in place before the previously deposited layer has had time to partially set. The work shall proceed continuously day and night if necessary to complete each longitudinal section. These sections while being built shall be held in place by substantial timber forms, normal to the centering and parallel to each other, and these forms shall be removed when the section has set sufficiently to admit of it. The sections shall be connected as specified above, and also by steel clamps or rib connections built into the concrete.

Steel Ribs.—Steel ribs shall be imbedded in the concrete of the arch. They shall be spaced at equal distances apart, and be of the number shown on plans. Each rib shall consist of two flat bars of the sizes marked on plans. The bars shall be in lengths of about 30 ft., thoroughly spliced together, and extending into the abutments

* The arch rings are also made in the form of voussoirs so as to symmetrically and uniformly load the falsework to prevent its unsymmetrical or excessive distortion.

as shown. Through the center of each bar shall be driven a line of rivets spaced 8 inches c. to c. with heads projecting about $\frac{7}{8}$ inch from each face of bar, except through splice-plates, where ordinary heads will be used. The bars shall be in pairs with their centers placed two inches within the inner and outer lines of the arch respectively as shown. All steel must be free from paint and oil, and all scale and rust must be removed before imbedding in the concrete. The tensile strength, limit of elasticity, and ductility shall be determined from a standard test-piece cut from the finished material and turned or planed parallel. The area of cross-section shall not be less than $\frac{1}{2}$ square inch. The elongation shall be measured after breaking on an original length of 8 inches. Each melt shall be tested for tension and bending. Test-pieces from finished material prepared as above described shall have an ultimate strength of from 60,000 to 68,000 lbs. per square inch, an elastic limit of not less than one-half of the ultimate, shall elongate not less than 20% in 8 inches, and show a reduction of area at point of fracture of not less than 40%. It must bend cold 180 degrees around a curve whose diameter is equal to the thickness of piece tested without crack or flaw on convex side of bend. In tension tests the fracture must be entirely silky. (Engineering Record, Aug. 3, 1901.)

APPENDIX.

TABLE I.

PHYSICAL PROPERTIES OF STONE AND CONCRETE.

BUILDING STONES.

C = Ultimate crushing strength in pounds per square inch. Test Specimens· Cubes.
R = Cross-bending fiber stress in pounds per square inch.
S = Ultimate shearing strength in pounds per square inch.
e = Expansion per degree F. Determined Wet.
E = Young's modulus in pounds per square inch. Compression.
r = Limits between which E was determined.
W = Weight in pounds per cubic foot.

Compiled from Tests of Metals and Other Materials, etc. made at Watertown Arsenal, Mass.

GRANITE.

Color	Name, Source, etc.	C 10^2.	R 10^2.	S 10^2.	e 10^{-8}.	E 10^5.	r 10^2.	W
Dark.	Braddock, near Little Rock, Ark.	242	12	24	324	71		
Light.	" " " " "	196	16	21	341	71	1–50	156
"	Exeter, Tulare Co., Cal.	226	19	24	461			
Red.	Platte Cañon, Colorado	146	.....	.....	.....	.....	.....	164
	Branford, Conn.	.....	12	.....	398	57	1–50	161
	" "	157	.....	18	398	83	10–30	162
	Pine Mt., Quarry, Lithonia, Ga.	277						
	Stone " " " "	209	26	.....	375			
	Troy, N. H.	.....	22	.....	337	48	1–50	
	" "	262	.....	22	337	45	10–30	165
Black.	Maine	210						
	Mt. Waldo Quarry, Frankfort, Me	322						
Light.	White Rock Mt., Millbridge, Me	199	20	28	400	98	1–50	
	Cape Ann, Rockport, Mass.	242	24	25	.....	65	10–30	
	" " " "	203						
	" " " "	173						
	Pigeon Hill, Rockport, Mass.	197	24	15	.....	67	10–20	162
	Quincy, Mass.	130	20	.....	381	68	10–30	
Pink.	Milford, Mass.	238	.....	26	418	77	10–30	163
"	" "	190	17	18	.....	51	10–20	162
	" "	.....	15	.....	417	53	1–50	163
	" "	90	21	24	408	76	10–30	
	" "	252						
Pink.	Ortonville, Minn.	204						
Mottled.	Rockville, Stearns Co., Minn.	181	13	19	397	94	1–50	
Red.	Sioux Falls, Minn.	182	12	21	389	60	1–50	
Light.	Korah Station, Va.	234	16	27	431			
"	Broad Rock Quarry, Chesterfield Co., Va.	154	17	21	402			
	Snoqualmie District, Wash.	198						

TABLE I.—PHYSICAL PROPERTIES OF STONE AND CONCRETE—(*Continued*).

SANDSTONE.

Color, Name, Source, etc.		*C* 10^2.	*R* 10^2.	*S* 10^2.	*e* 10^{-8}.	*E* 10^5.	*r* 10^2.	*W*
	Near Ft. Smith, Sebastian Co., Ark.	128	15	18	620	35	1–40	
Blue.	Cabin Creek, Johnson Co., Ark.	185	17	25	613	39	1–50	
"	Jasper, Ala.	159	19	22				
	Piedmont Quarry, near Oakland, Cal.	110	11	16	436			
Light Red.	St. Vrain Cañon, Col., laminated.	115	.....	.....	.....	.....	.....	150
Red.	Manitou, Col.	110	.....	.....	.....	.....	.....	140
Gray.	Ft. Collins, Col., laminated.	117	.....	.....	.....	.....	.....	140
"	Trinidad, Col.	100	.....	.....	.....	.....	.....	145
Brown.	Portland (Middlesex Quarry), Conn.	109						
	" " " "	103						
	" " " "	83						
	" " " "	43						
Brown.	Portland, Conn., 1st quality	136						
"	" " 2d "	139						
"	" " 3d "	150						
"	" " Bridge stone	96						
"	Cromwell, Conn.	169	19	15	549	77	1–50	
Red.	Brainard Quarry Co., Conn.	62	.....	.....	526			
"	" " " "	107	.....	14				
	Near Redfield, Bourbon Co., Kan.	80	21	19	516			
	Carreyville, Ky.	106						
Red.	Potomac Red Sandstone Co., Maryland	130	23	23	501	39	10–20	
"	" " " " "	190						
	Kibbe, East Longmeadow, Mass.	.....	10	.....	577	11	1–20	134
	" " " "	.....	.....	.....	.....	22	1–25	
	" " " "	127						
	" " " "	104	.....	12	577	18	10–20	133
Soft Saulsbury	" " " "	85						
Hard "	" " " "	140						
Red Brown.	" " " "	114						
	Maynard, East Longmeadow, Mass.	94	.....	12	567	19	10–20	134
	" " " "	.....	8	.....	561	13	1–20	133
	Worcester " " "	.....	11	.....	517	19	1–20	135
Brown.	" " " "	98	.....	12	517	24	10–20	137
Light Drab.	Frontinac, Minn.	101						
" "	Mautorville, Minn.	88						
" Yellow	Mankato, Minn.	96						
Salmon.	Kettle River, Pine Co., Minn.	125	9	16	686			
	Ohio.	58	.....	.....	622			
	"	87	.....	9				
	Chitwood, Oregon.	63						
	Cooper Quarry, Douglas Co., Oregon	.....	7	.....	177	22	1–30	
Blue.	Coquille River, Coos Co., Oregon.	75	.....	13				
	Cooper Quarry, Douglas Co., Oregon	152	.....	18	177	28	10–20	160

LIMESTONE.

	Beaver, Carrol Co., Ark.	206	27	20	471	67	1–50	
	Batesville, Ark.	95						
	Rockwood, Ala.	60	7	10	58			
Blue.	Bedford, Ind.	108	14	10	389	73	1–30	
Buff.	" "	90	21	12	.....	72	1–30	
	Indiana.	60	.....	11	407	52	2–30	
Buff.	Oolitic, Bedford, Ind.	41	.....	.....	375	36	2–10	
"	" " "	71	.....	11				
	Johnson, Co. Iowa.	47						
Drab.	Iowa State Quarry.	72						
"	" " " (1843).	36						
Blue.	Hutchinson, Iowa.	241						
Mottled.	Crowley's, Iowa.	78						
Cream.	Cedar Valley, Iowa.	50						

TABLE I.—PHYSICAL PROPERTIES OF STONE AND CONCRETE—(*Continued*).

LIMESTONE.

Color, Name, Source, etc.		*C* 10^2.	*R* 10^2.	*S* 10^2.	*e* 10^{-8}.	*E* 10^5.	*r* 10^2.	*W*
Light Buff.	Ft. Riley, Kansas	.....	5	10	300			
	Junction City, Kansas	32						
	Mt. Vernon, Ky	.....	13	.....	464	40	1–20	
	Bowling Green, Ky.	60	11	12	89	93	1–20	
	" " " Oolitic	62						
	Hopkinsville, Ky., Oolitic	78						
	Bowling Green, Ky., Oolitic	150						
	Spring Ledge, Mt. Vernon, Ky.	76	.....	17	464	32	10–20	139
	Kelly Island, Lake Erie	122						
Pink.	Kasota, Minn.	108						
Gray.	Wasioja, Dodge Co., Minn.	50	3	12	217			
Buff.	" " " "	40	.....	11				
	Carthage, Mo., Quality No. 1.	137						
	" " " " 2.	162						
	Yammerthal Flint, Buffalo, N. Y.	237	17	21				
	Isle La Motte, Vt.	146	16	22	219	147	1–40	

MARBLE.

Color, Name, Source, etc.		*C* 10^2.	*R* 10^2.	*S* 10^2.	*e* 10^{-8}.	*E* 10^5.	*r* 10^2.	*W*
Fossiliferous.	St. Joe, Searcy Co, Ark	103	11	.....	219	82	1–40	
Chocolate.	" " " " "	123	16	.....	.....	107	1–30	
	Marble Hill, Ga	115	.....	13	202	91	10–30	169
	" " "	.....	10	.....	193	55	1–40	168
	Tate, Ga., Creole Quarry	.....	8	.....	.....	41	1–40	169
	" " " "	135	.....	14	.....	69	10–30	170
	" " Cherokee Quarry	.....	4	.....	441	40	1–40	168
	" " " "	126	.....	12	441	91	10–30	168
	" " Etowah Quarry	.....	18					
	" " " "	141	.....	14	.....	78	10–30	170
	" " Kennesaw Quarry	96	13	12	.....	76	10–30	168
	Lee Co., Mass	160	16	.....	562	67	10–20	
	" " "	181	.....	21	454			
Drab.	Faribault, Minn.	178						
	Tuckahoe, N. Y	162	.....	15	441	136	10–30	178
White.	Richville, Dekalb, St. L. Co., N. Y.	125	8	.....	634	120	1–40	
	Vermont	90	10	.....	361	66	10–30	
	"	120	.....	19				
White.	Rutland, Vt.	119	7	10	312	45	1–40	168
Blue.	" "	139	21	12	.....	65 }		
"	" "	.....	.....	.....	.....	78 }	1–40	
	" "	.....	.....	.....	.....	75 }		
Mt. Dark.	" "	128	18	15	433	93	1–40	
	Sutherland Falls, Vt.	162	23	16	550	126	1–40	
	" " "	113						
	" " "	102						
	Roche Harbor, San Juan Co., Wash.	90						
	Snoqualmie District Wash.	47						

CONCRETE.

The physical properties of concrete depend upon so many variable factors that it is useless to attempt to give more than approximate values.

Mr. Edwin Thacher, C.E., has deduced formulas, based upon a large number of experiments made at the Watertown Arsenal, showing the effect of age and composition upon the ultimate strength. Values according to these formulas are given below. For tests of concrete of all kinds reference is made to *Tests of Metals and other Materials, etc., made at the Watertown Arsenal, Mass.*

ULTIMATE STRENGTH IN POUNDS PER SQUARE INCH.

Mixture.	Age.				Remarks.
	7 Days.	1 Month.	3 Months.	6 Months.	
1-1-3	1600	2750	3360	4300	
1-2-4	1400	2400	2900	3700	$S = 1800 - 200x$; 7 days
1-2½-5	1300	2225	2670	3400	$S = 3100 - 350x$; 1 month
1-3-6	1200	2050	2440	3100	$S = 3820 - 460x$; 3 months
1-3½-7	1100	1875	2210	2800	$S = 4900 - 600x$; 6 months
1-4-8	1000	1700	1980	2500	
1-5-10	800	1350	1520	1900	x = parts of sand to one part cement.
1-6-12	600	1000	1060	1300	S = ultimate strength for 12-inch cubes.

E = MODULUS OF ELASTICITY IN THOUSANDS OF POUNDS PER SQUARE INCH.

(Compiled by E. Thacher.)

Mixture.	Age 7 Days.			Age 1 Month.			Age 3 Months.			Age 6 Months.		
	100 to 600.	100 to 1000.	1000 to 2000.	100 to 600.	100 to 1000.	1000 to 2000.	100 to 600.	100 to 1000.	1000 to 2000.	100 to 600.	100 to 1000.	1000 to 2000.
1-1-3	2450	2050	1380	2830	2580	1910	3500	3140	2120	3850	3580	2700
1-2-4	2580	2050	1350	2660	2450	1460	3670	3160	2160	3670	3570	2580
1-2½-5	2220	1800	......	2550	2300	1350	3320	2900	1980	3630	3540	2220
1-3-6	1860	1540	......	2440	2130	1220	2970	2650	1800	3600	3500	1860
1-4-8	......	......	......	2100	1800	......	2530	2220	......	3020	2840	
1-5-10	......	......	......	1740	1460	......	2100	1780	......	2420	2200	
1-6-12	......	......	......	1380	1140	......	1640	1360	......	1820	1520	

These values are means of tests made upon 12-inch cubes made with four brands of cement respectively. A statement of the data upon which the above tables are based is given in an article by Mr. E. Thacher, entitled Effect of Age and Composition on the Strength and Modulus of Elasticity of Concrete, *Cement*, May 1902.

EXPANSION OF CONCRETE.

Prof. Pence gives 0.0000055 as the coefficient of expansion for one degree F. for 1–2–4 concrete composed of Lehigh Portland cement and limestone. With the limestone replaced with gravel the coefficient becomes 0.0000054. This makes the coefficient of expansion of concrete and steel essentially the same.

WEIGHT OF CONCRETE.

The weight of concrete will vary somewhat according to the materials used and the methods of mixing. In "Tests of Metals, etc.," for 1898, the weights of a large number of 12-inch cubes are given, the proportions varying from 1–1 to 1–4, with 33 and 40 per cent of the stone used as mortar. The mortar was made "dry," "plastic," and "wet." The weights per cubic foot ranged from 138.9 to 143.7 pounds. For all ordinary purposes 140 pounds per cubic foot may be used. Some specifications state that concrete shall be taken at 150 pounds per cubic foot.

WEIGHT OF FILLING MATERIAL.

This will vary according to the kind of material and the method of depositing it. For average conditions, when the spandrels are filled with sand or gravel, 100 pounds per cubic foot may be assumed. For gravel deposited in thin layers and rolled, some specifications state that the fill shall be taken as weighing 120 pounds per cubic foot.

TABLE II.

DATA FOR ABOUT 500 ARCH BRIDGES ARRANGED ACCORDING TO SPAN.

MASONRY ARCHES.

Number.	Name.	Place.	Over.	Date.	Engineer.	No. of Spans.
1	Taff Vale Viad.	Near Quaker's Yard, S. Wales	Taff R.		Brunel	6
2	Queretaro	Near Queretaro, Mexico	Valley	1726–35	Antonio Avana	74
3	Malaunay	Near Rouen, France	Malaunay Val.	1840–44	Locke	8
4	Magnolia St.	Elizabeth, N. J., U. S. A.	Magnolia St.	1894	Brown	1
5	L. Juniata No. 8	Penn., U. S. A.	L. Juniata R.		Brown	3
6		Morpeth, England	Wansbeck R.	1831	Telford	3
7	Mass. Ave.	Washington, D. C., U. S. A.	Rock Crk.	1900–1	Douglas	1
8			Cree R.		Rennie	1
						2
						2
9	Chateau Thierry	France	Marne.	1786	Perronet	1
10	Charles	Nuremburg, Bavaria		1728		?
11	Starrucca Viad	Nr. Lanesboro', Pa., U.S.A.	Starrucca Crk.	1847	Adams	17
12	Pont Neuf	Paris, France	Seine R.	1578–1604	Cerceau and Marchand	12
13	Enz	Wildbad, Germany	Enz R.	1886	Leibbrand	1
14	Pont au Change	Paris, France	Seine R.	1639–47		7
15						
16	Court St.	Rochester, N. Y., U.S.A.	Genesee R.	1893	McClintock	8
17		Rochester, N. Y., U. S.A.				7
18		Bet. Norwood-Bromley, England	Lon. Croydon Ry.		Gibbs	1
19		Dôle, France	Doubs R.	1760–64?	Gueret	7?
20		England	Mouse R.	1822	Telford	3
21		Gien, France	Loire, R. Val.	1888–9	Lethier	15
						70
22	Dinan Viad.	Dinan, France	Rance R.	1845–50	Fessard	10
23	Guétin	Bet. Digoin and Mainsbray, France	Valley	1890–98		18
24	Stura	N. of Turin, Italy	Stura R.		Bella	5
25		Montalierie, Italy	Po R.	1849	Barbavara	7
26	Digoin	Digoin, France	Valley	Changed 1890–98		11
27	Roquefavour	Vic. of Marseilles, France	Arc R. and Val.	1841–47	de Montricher	15
						13
						51
28	Strasbourgh Sta.	Paris, France	Station Platform			3
29	Croydon	Near Croydon, England	Lon. & C. Ry.		Gibbs	1
30	Abattoir St.	Paris, France				
31		Nemours, France	Loing R.	1805	Perronet	3
32		Stirling, Scotland	Forth R.	1400†		
33		Moret, France	Loing R.	1771	Perronet	3
34		Moulins, France	Allier R.	1758–60	Regemortes	13
35	Park St.	Hartford, Conn., U.S.A.		1898	Graves	1
36	Pathhead	Pathhead, Scotland, U.S.A.	Tyne R	1830	Telford	5
37	Mill Creek	W. of Bird-in-Hand, Pa.,	Mill Creek	1889–90	Brown	4
38	L. Juniata No. 13	1 mi. W. of Tyrone, Pa., U.S.A.	L. Juniata R.	1892–93	Brown	3

Span.	Rise.	Thickness of Arch Ring at Crown t_0.	At Springing, t_s.	Curve.	Radius at Crown.	$\frac{t_0}{R}$.	Width, Face to Face at Crown.	Thickness of Piers at Springing.	Material.	Class of Bridge.	Reference.	Number.
50.0	25.0			C	25.0		14.0		Blue Grit	Ry.	A. March, 1850	1
50.0										Aqued't	B. June 2, 1888	2
50.0	25.0	3.1	3.1	C	25.0	.124	27.7	8.8		Ry.	C. 1851–2	3
50.0	7.4	2.8	2.8	C	38.5	.073	50.0			Ry.	D.	4
50.0	12.5									Ry.	E.	5
*50.0	15.0			E	48.0		31.5	8.0			F. 1852, p. 290	6
50.0	25.0	3.0	7.0	C2	25.0	.150	200.		Brick	H.W.	B. Dec. 25, 1902	7
50.0	6.6	1.3		C							S.	8
46.0	5.9	to						8.0 &				
39.0	4.9	1.0						7.6				
51.0	17.0	4.0		E							Woodbury, 1858	9
51.1	16.0	4.3		E	37.2	.115	19.2	13.8		H.W.	F. 1852, p. 278	10
51.0	†20.0	2.5	2.5	C	26.3	.095	24.8	7.0	Sandstone	Ry.	B. Sept. 1, 1888	11
*51.1	21.9	2.3	3.6	C	25.8	.089	72.5			H.W.	G. 1891, p. 887	12
51.2	10.7	1.6	3.9	C?	35.9	.045				H.W.	G. 1891, p. 918	13
51.2 to	25.6	5.3		C	25.6	.207		16.0		H.W.	F. 1852, p. 276	14
35.2	17.6											15
*52.0	20.5			C	26.7		?64.0	6.0		H.W.	B. Feb. 2, 1893	16
*52.0	10.0	2.5		C	38.8	.064			Limestone	Aqued't	H.	17
52.0	†12.0	2.3	2.3	C	34.2	.067	†30.0		Brick	H.W.	C. 1855–6	18
52.0	17.5	3.8		E							I.	19
52.0	26.0			C2				8.5		H.W.	F. 1852, p. 197	20
52.5	26.2	3.0	3.3	C2	26.2	.114	17.7	3.3		Ry.	G. VI, 1893	21
52.5 to	17.5											
42.6	14.2											
*52.5	26.2			C2	26.2		22.1	†13.1		H.W.	G. 1888, p. 363	22
52.5	23.0	3.6	3.6	3C	30.7	.117	31.9	10.0		Canal	G. 1899	23
52.5	5.1	3.0		E	102.7	.042	35.1	9.8	Brick(?)	H.W.	F. 1852, p. 296	24
52.5	13.6	3.0		C	32.1	.093	29.5	7.5	Brick	Ry.	F. 1852, p. 296	25
52.5	23.0	3.9	3.9	3C	30.7	.127	31.8	9.8		Canal	F. 1852, p. 286	26
52.5	26.3			C2	26.3					Canal	A. 1855, p. 65	27
49.2	24.6	3.3			24.6	.134						
16.4	8.2	3.4			8.2							
*52.7	5.0	3.0		E	71.6	.041	42.6	8.2	Mill'e Grit	H.W.	F. 1852, p. 296	28
53.0	12.0	3.0	3.0	E			?20.0		Brick	H.W.	C. 1855–6	29
53.0	5.1	3.0		C	71.3	.042				Ry.	I.	30
53.0	3.2	3.2		C	99.1	.032		6.4	Freestone	H.W.?	F. 1852, p. 286	31
53.0	10.3	2.8		C	39.2	.071					I.	32
53.3	6.1	4.3		C	61.2	.072			Freestone	H.W.?	J.	33
53.9	21.3	3.2		E			34.0	12.8		H.W.	F. 1852, p. 276	34
54.0	7.3	3.3	4.3				70.0		Brick	H.W.	B. Jan. 12, 1899	35
54.0 &	8.0						24.0	8.0 &		H.W.	F. 1852, p. 195	36
50.0	25.0							4.0				
54.0	13.5	2.7	2.7	C	33.7	.081	32.0			Ry.	E.	37
54.0	13.5	3.2	3.2	C	33.7	.095	49.4			Ry.	E.	38

* Maximum.

† About.

REMARKS.—1. Piers 14′ octagon. On curve. Skew 40°. 1320′ Radius. 2. Max. H. = 95′ Av. = 75′–80′. 3. Found. upon piles. 4. 4 tracks. Intrados to base of rail, 5′.6. 5. Middle Div. Penn. Ry. 7. Skew 17°. Cost $132,000. 11. 2 tracks. Max. H. = 110′. N. Y., L. E., & W. 12. Repaired 1886. 13. Middle 3d joints at crown and springing filled with lead. 18. Ribbed skew. 20. H. = 134.5′. Piers hollow. 21. Approaches to metal spans. 22. Max. H. = 130′.5. Nat. Route No. 176. 23. Max. H. = 39′.4. 24. Route Turin-Milan. 25. Turin-Genoa. 27. 3 tiers of arches. Max. H. = 271′.0. 29. Ribbed skew. 35. Stone facing. 36. Max. H. = 75′.0. The 54′.0 spans are under roadway. 37. Three tracks—1° curve. 38. 4 tracks. Mid. Div. Penn. Ry. ribbed skew.

TABLE II.—DATA FOR ABOUT 500 ARCH BRIDGES

MASONRY

Number.	Name.	Place.	Over.	Date.	Engineer.	No. of Spans.
39	Kennet	Near Caversham, England	Kennet R.			3
40	Monocacy Viad.	Monocacy, Penn., U.S.A.			Fisk	
41	Nashawtuc	Concord, Mass., U.S.A.	Sudbury R.	1883	Wheeler	1
42	Ouctoine		Rieneros R.	1770-90	Garipuy	3
43	Big Conestoga	E. of Lancaster, Pa.,U.S.A.			Brown	5
44	Peas	Bet. Berwick and Edinburgh, Scotland	Deep Dingle		Henderson	4
45	Oder	Kunnersdorf, Saxony	Oder R.			7
46	Bachthal	Dermitz, Saxony	Bach R.	1844-5		11
47	Dauphin		Romanche R.	1842-4	Potie	1
48	Carmes	France	Beauvoir Ravine	1843-47	Cunit	3
49		Löbau, Saxony	Spree R.?	1845-46		7
50		Königstein, Saxony				2
51	Spreethal	Saxony	Spree R.	1845-46		32
52		Neuneck, Germany	Glatt R.	1886	Leibbrand	15
53	No. 28	26.5 m. Pittsburg, Pa., U.S.A.	Raccoon Crk.	1887-88		1 1
54	Washington	New York, N. Y., U.S.A.	Harlem R.	1886-89	Hutton	1
55	Nôtre Dame	Paris, France	Seine R.	1507	Jaconde	6
56		Chateau Thierry, France	Marne R.	1765	Pérronet	1
57		Pontlieu, France	Huisine R.?	1773	Voglie	2
58	Gravant	France	Yonne R.	1760	Adwine	3 3
59	Zempoala Aq.	7 m. south of Huauchinango, Mexico	Valley	1553-70	Tembleque	68
60	Monford	England	Severn R.	1790-92	Telford	1
61	Johnstown	Johnstown, Penn., U.S.A.	Conemaugh R.	1888	Brown	2 6
62		Llanrwst, Wales	Conway R.	1634-36	Inigo Jones	1
63	Carrolton Viad.	Nr. Baltimore, Md., U.S.A.	Patapsco R.	1833-35	Latrobe	3
64	Jamaica St.	Glasgow, Scotland	Clyde R.	1833-36	Telford	8 1 2 2
65	Brives	France	Loire R.	1772	Grangent	2 5
66	Tournelle	Paris, France	Seine R.	1630-56	Marie	6
67	Marie	Paris, France	Seine R.	1635-58	Marie	5
68	Aelius	Rome, Italy	Tiber R.	136	Hadrian	3
69	Sèvres	Near Paris, France	Seine R.	1820	Beaupre	9
70	Rahway Ave.	Elizabeth, N. J., U.S.A.	Rahway Ave.		Brown	2
71	Washington	New York, N. Y., U.S.A.	Harlem R.	1886-89	Hutton	1
72	Ingersheim	France	Tech R.	1773	Clinchamp	6 7
73	Trenton	Near Trenton, N. J., U.S.A.	Delaware R.	1902	Brown	18
74	L. S. & M. S. Ry.	Elyria, Ohio, U.S.A.	W. Br. Black R.			3

* Maximum.

REMARKS.—39. Skew. S. E. Ry. Co. 40. Chesapeake & Ohio Canal. 41. Granite from Fitchburg, Mass. 43. Penn. Ry. 44. Max. H. = 124'.0. 45. Löbau-Zitlau, H. = 62.3. 46. Pile found. H. = 59, L. = 725. 49. Saxony-Silesia, H. = 95'.0. 50. Prague-Dresden, H. = 33.6. 51. Saxony-Silesia, H. = 66.9. 52. Sheet-lead "Hinges," 3. 53. 2 tracks. Rail 27'.5 above key. 54. Approach to metal spans. 56. See No. 9. 59. Waterway 8½"×12". H. = 124'.0. On two tangents. Max. span is highest. 61. Skew = 55°. Ribbed. Stood through Johnstown Flood.

ARRANGED ACCORDING TO SPAN—(*Continued*).

ARCHES.

Span.	Rise.	Thickness of Arch Ring at Crown t_0.	At Springing, t_s.	Curve.	Radius at Crown.	t_0/R	Width Face to Face at Crown.	Thickness of Piers at Springing.	Material.	Class of Bridge.	Reference.	Number.
*54.0	11.0	2.6					?24.0		Brick	Ry.	K. Dec. 20, 1895	39
54.0	9.0	2.5		*E*	50.0	.050				Canal	I.	40
54.0	6.0	1.2	1.2	*C*	63.75	.019	25.0		Granite	H.W.	Wm. Wheeler	41
54.5	17.1	2.1		*E*				12.8			F. 1852, p. 280	42
*54.5	27.3			C_2	27.3					Ry.	E.	43
*55.0										H.W.	L.	44
										Ry.	F. 1852, p. 229	45
*55.8				*E*				7.4		Ry.	F. 1852, p. 226	46
55.8	27.9			C_2	27.88			9.2			F. 1852, p. 292	47
55.8	27.9	3.0		C_2	27.88	.108	19.7				F. 1852, p. 292	48
55.8	27.9	3.1		C_2	27.9	.111	19.7	4.9		Ry.	F. 1852, p. 222	49
55.8	27.9			C_2	27.9			9.5	Brick	Ry.	F. 1852, p. 225	50
37.9	18.9			*E*								
*55.8												
*55.8	27.9			C_2	27.9			15.8	Stone	Ry.	F. 1852, p. 228	51
c-c55.8	c-c9.8	†1.0	†2.0				13.1			H.W.	G. 1891-1, p. 920	52
56.0	28.0	3.0		C_2	28.0	.108	107.6			Ry.	D.	53
56.0	14.0	2.0	2.0	*E*	55.8	.036	80.8	†13.2		H.W.	Washington Bridge by Hutton	54
56.7	28.4	5.3		C_2	28.4	.187	77.4	12.8		H.W.	F. 1852, p. 274	55
to31.2	15.6											
57.5	19.2	4.0		*E*	41.5	.096	35.2	14.4		H.W.	F. 1852, p. 280	56
51.1	17.1	3.7		*E*			35.2					
57.5	21.3	3.8		*E*	42.6	.089		12.8			F. 1852, p. 282	57
57.5 to				*E*	45.8	.093		12.8			F. 1852, p. 280	58
53.9	21.3	4.3										
*58.0	29.0			C_2	29.0		4.7			Aqued't	B. July 7, 1888, p. 2	59
											F. 1852, p. 284	60
58.0	22.5	3.0		*E*			24.0	11.0				
50.0	20.0											
58.0	14.5	2.7	2.7	*C*	36.2	.075	48.0	6.0	Sandstone	Ry.	B. July 20, 1889	61
40.0	14.5	2.7	2.7	*C*	21.0	.135	48.0				E.	
*58.0	17.0	1.5		*P*			14.0	10.0		H.W.	L.	62
58.3	29.2	2.5	2.5	C_2	29.2	.085		10.0	Granite	Ry.	I. , F. 1852, p. 237	63
58.5	10.8	2.5		*C*	43.7	.057	40.0			H.W.	F. 1852, p. 290	64
57.8	10.5											
55.5	9.7											
52.0	8.3										F. 1852, p. 280	65
*58.6	26.6	3.2		*E*	45.8	.069	29.1	11.7				
58.7 to	29.8	5.4		C_2	29.8	.281	53.3	12.8		H.W.	F. 1852, p. 276	66
44.8	22.4									H.W.	F. 1852, p. 276	67
58.7 to	29.8	4.3		C_2	29.3	.147	77.7	11.7				
44.8	22.4											
59.0	29.5			C_2	29.5		†33.5	†23.0	Travert'e	H.W.	M. Feb. 18, 1899	68
59.0	29.5	3.3		C_2	29.5	.112	42.6	12.1		H.W.	A. April, 1847	69
16.4	8.2										F. 1852, p. 288	
59.2	9.5	3.2	3.2	*C*	50.9	.063	62.2			Ry.	D.	70
59.7	29.8	2.0	4.5	C_2	29.8	.067	80.8			H.W.	See No. 54	71
59.7 to	11.7	3.2		*E*				8.5			F. 1852, p. 282	72
50.1												
60.0	12.0	3.3	3.3	*C*	43.5	.076	52.0	8.0		Ry.	B. Jan. 30, 1902	73
60.0	30.0	2.0		C_2	30.0	.067	22.2		Berea sandstone	Ry.	N. June 8, 1899	74

† About.

62. On 4½° curve. Arches Cyl. H. = 66'.0. 64. 1st Bridge by Mylne, 1768-72 called New Jamaica St. Bridge. Old Bromielaw Bridge. 68. Originally 8 arches, 5 now buried. To give access to St. Ange Castle. 69. Paris-Versailles. 70. Five tracks. Ribbed arch Skew 45° 44'. Pa. Ry. N. Y. Div. 73. Two abut. piers, 22'.0. Skew 71° 30'. 74. Twin arches 4'.2 apart. Two tracks. L. S. & M. S. Ry.

TABLE II.—DATA FOR ABOUT 500 ARCH BRIDGES ARRANGED ACCORDING TO SPAN—(*Continued*).

MASONRY ARCHES.

Number.	Name.	Place.	Over.	Date.	Engineer.	No. of Spans.
75		Minneapolis, Minn. U.S.A.	Mississippi R.			1
						2
						2
76	Dee	Val. Llangallen, Wales	Dee R.			19
77	S. approach Voyne Viad.	Drogheda Ireland	Boyne R.	1851–67	Macneill	13
78	Muddy Crk.	Addystone O. U.S.A.	Muddy Crk.	†1895	Kittredge	1
						2
79	W. Jersey St.	Elizabeth N. J. U.S.A.	W. Jersey St.	189 –	Brown	1
80	Kennet	Near Caversham. England	Kennet R.	†1840	Brunel	1
						2
81	Staines	Staines, England	Thames R.	1796	Sanby	1
						2
82	Ballater	Ballater, Scotland	Dee R.	1809	Telford	5
83	Stirling	Stirling, Scotland	Forth R.	1829–32	Stevenson	1
						2
						2
84	Richmond	Richmond, England	Thames R.	1774–77	Payne & Couse	5
85	Alness	Alness, Scotland		1816	Telford	1
86		Warfield, England		1846	Grainger	21
87	Anker	England				1
						19
88	Dutton Viad.	England	Weaver Val.		Stephenson	20
89	Holy Cross (old)	Feldkirch, Austria	Ill. R.	13th cen. ?		1
90	Kingston	Kingston England	Thames R.	1825–28	Lapidge	5
						2
						2
91		Saumur, France	Arm of Loire R.	1756–64	Voglio & Cessart	12
92	Conemaugh	W. of Ben's Crk, Penn., U.S.A.	Conemaugh R.	1896	Brown	1
93	Ben's Creek	Lilly-Portage, Pa., U.S.A.	Conemaugh R.	1896	Brown	1
94	L. Juniata No. 7	1 m. E. Schoenberger's Penn., U.S.A.	L. Juniata R.	1889	Brown	3
95	Big Chiques	Penn., U.S.A.	Big Chiques Crk.	1884	Brown	2
96	Big Viaduct	Viad. Sta., Penn., U.S.A.	L. Conemaugh R.	1889	Brown	2
97	L. Conemaugh No. 6	E. of L. Conemaugh, Pa., U.S.A.	L. Conemaugh R.	1889–90	Brown	3
98	Chestnut St.	Philadelphia, Pa., U.S.A.	Schuylkill R.	1861–66	Kneass	
99		Bewdley England	Severn R.	1797–9	Telford	1
						2
100	Congleton Viad.	Congleton England	Dane R. & H.W.	1839–	Stevenson	42
101		Ratisbon, Germany	Danube R.	1135		15
102	Tweed	Berwick England	Tweed R.	1847–50	Stevenson	28
103		Charmes, France	Moselle R.	1740		10
104		Kew England	Thames R.			5
105	Görlitz	Near Görlitz, Silesia	Neisse R. & Val.	1844–47		6
						6
						18
						1
106		Dôle France	Doubs R.	1760–64	Gueret	7
107	W. Grand St.	Elizabeth, N. J U.S.A.	W. Grand St.	189–	Brown	1

Span.	Rise.	Thickness of Arch Ring at Crown t_0.	At Springing, t_s.	Curve.	Radius at Crown.	$\frac{t_0}{R}$.	Width, Face to Face at Crown.	Thickness of Piers at Springing.	Material.	Class of Bridge.	Reference.	Number.
60.0	15.0			C			40.0	8.0		H.W.	N. Nov. 23, 1895	75
57.0	14.3											
54.0	13.5						27.8	13.1	Brick	Ry.	F. 1852, p. 156	76
60.0	30.0			C_2	30.0					Ry.	A. July 1851, p. 384	77
60.0	30.0			C_2	30.0							
60.0	12.0	3.0	3.0	C	43.5	.069	30.0	10.0	Berea	Ry.	Blue	78
55.0	12.0	3.0	3.0	C	37.5	.080			sandstone	Ry.	D.	79
60.0	9.5	3.5	3.5	C	52.1	.067	51.0		Brick			80
60.0		3.0	4.5				?36.0		Brick	Ry.	K. Dec. 20, 1895	
18.0		1.5	1.5							H.W.	K. Sept. 13, 1895	81
60.0								8.0				
52.0									Granite	H.W.	C. 1855–56	82
60.0									Greenst'e	H.W.	C. 1855–56	83
60.0	13.5	2.8	3.5	C	40.0	.070	?32.8	9.0	from near			
58.0	12.5	2.8	3.5		38.6	.073			Stirling			
53.5	10.3	2.8	3.5		39.8	.073				H.W.	K. July 12, 1895	84
*60.0											F. 1852, p. 288	85
60.0	20.0			C			25.0		Brick	Ry.	F. 1852, p. 169	86
60.0	30.0								BrickRing		F. 1852, p. 188	87
60.0							31.0					
30.0										Ry.	A. 1837, 8, p. 125	88
60.0							30.0			H.W.	O. June 1898	89
60.0	30.0			C_2	30.0		?11.0			H.W.	A. Dec. 1842	90
60.0	19.0			E			?25.0	†10.0	Brick			
56.0	18.3							9.3	faced with			
52.0	16.5							8.7	stone		K. July 26, 1895	
*60.0	21.0	4.8		E	55.4	.087	44.7	12.8		H.W.	L. F, 1852, p. 276	91
60.0	20.0	3.0	3.0	C	32.5	.092	83.5			Ry.	D.	92
60.0	20.0	3.0	3.0	C	32.5	.092				Ry.	E.	93
60.0	15.0	3.0	3.0	C	37.5	.080	36.6	7.0		Ry.	E.	94
60.0	15.0	2.8	2.8	C	37.5	.075	11.0	†12.0	BrickRing	Ry.	E.	95
60.0	30.0	2.7	2.7	C_2	30.0	.090	36.5	8.0		Ry.	E.	96
60.0	20.0	3.0	3.0	C	32.5	.092	†41.0	7.0		Ry.	E.	97
									Brick	H.W.	I.	98
60.0	18.0	2.5		C	34.0	.074					I. L. F. 1852 p. 284	99
60.0	18.0	2.0		C	34.0	.059	28.0	8.0				
52.0	16.0								Brick	Ry.	A. 1839, p. 444	100
60.8	20.0			C			31.0			H.W.	F. 1852, p. 274	101
60.8	30.4	3.2		C_2	30.4	.105	25.6	20.3	Brick	Ry.	F. 1852, p. 155	102
61.5	30.8			C_2	30.8			5.2		H.W.	F. 1852, p. 276	103
61.8 to	30.9	4.3		C_2	30.9	.139		17.1				
34.1	17.1										F. 1852, p. 282	104
61.8 to	30.9	2.7		C_2	30.9	.087		8.5				
38.4	19.2								Red Gran.	Ry.	F. 1852, p. 215	105
61.8	30.9			C_2	30.9							
41.2	20.6				20.6							
30.9	15.5				15.5							
24.8	12.4				12.4							106
61.8 to	19.2	4.3							11.5 to		F. 1852, p. 280	
51.2				E	44.7	.096			10.7	Ry.	D.	107
62.0	9.0	3.6	3.6	C	57.9	.062	51.0					

* Maximum.

† About.

REMARKS.—76. H.=147'.6. Intrados to rail=6'.1. Shrewsbury-Chester, stone facing. 78. Approach to metal spans Av. H.=90'. 78. Big 4 Ry. 79. Pa. Ry., N. Y. Div. Skew 60°. Ribbed 80 Great Western Ry. Co. 81. Closed 1797 on account of poor foundation. 84. Clear headway above L.W.=25'.0. 86. Leeds-Thirsk. H.=90'.0. 87. 816' long. 88. Grand Junc. Ry. H=73'.0. 89. Replaced. 90. Found. upon blue clay. Kingston-Hamptonwick. 91. A. 1856, p. 376. Found. upon piles. 92. Pa. Ry. Pitts. Div. Four tracks. 93. Pa. Ry. Pitts. Div. 94. Ribbed. Skew 45°. Three tracks. Pa. Ry. 95. Pa. Ry. Phila. Div. 96. On 2° curve. Replaced 80' arch, destroyed May 31, 1889, Johnstown Flood. 97. On 5° 33' curve. Skew 57° 54'. Ribbed. Pa. Ry. Three tracks. 100. Manchester-Birmingham Ry. 101. J. 1896, p. 126, gives span=53-33 and C_2. 102. Stone facing. H.=124'.6. 105. H.=115'.3. 328'.0 on curve. Berlin-Breslau. 107. Pa. Ry., N. Y. Div. Skew 57° 41'. Ribbed. Four tracks

TABLE II.—DATA FOR ABOUT 500 ARCH BRIDGES ARRANGED ACCORDING TO SPAN—(*Continued*).

MASONRY ARCHES.

Number.	Name.	Place.	Over.	Date.	Engineer.	No. of Spans.	Span.	Rise.	Thickness of Arch Ring at Crown t_0.	At Springing, t_s.	Curve.	Radius at Crown.	$\frac{t_0}{R}$.	Width, Face to Face at Crown.	Thickness of Piers at Springing.	Material.	Class of Bridge.	Reference.	Number.
108	Holy Cross (new)	Feldkirch, Austria	Ill R.	1898		1	62.3	15.6									H.W.	O. June 1898	108
109	St. Angelo	Rome, Italy	Tiber R.	135	Hadrian		*62.3							?50.8			H.W.	L. Q.	109
110	Barton Aq.	Worsley, England	Irwell R.	1760–	Brinkley	3	*63.0	31.5			*C2*	31.5						R. Dec. 1888	110
111		Athlone, Ireland	Shannon R.	1844	Rhodes	3	63.0				*E*			?42.0		Limestone	H.W.	A. 1844, p. 444	111
112		Mirepoix, France	Lers R.	1776–90	Garipuy	7	63.9	10.7	5.3		*C*	55.4	.096	†25.6	11.7			F. 1852, p. 282	112
113		Frouard, France	Moselle R.	1788	Le Creuix	7	63.9	19.2	5.3		*E*	55.7	.092		12.8			F. 1852, p. 284	113
114		Ferté, France	Marne R.		Pitrou	1	63.9	23.5	3.7		*E*	59.7	.062	22.4				F. 1852, p. 276	114
115		Montélimar, France	Roubion R.	1806	Voglie	3	64.0	21.3	4.3		*E*	50.6	.084		19.4			F. 1852, p. 286	115
116	Actius	Rome, Italy		138	M. Rusticus	7	64.0 to	32.0	4.8		*C2*	32.0	.150	50.8	24.5		H.W.	F. 1852, p. 274	116
							25.4	12.7				12.7							
117	Neuf	Paris, France	Seine R.	1578–1604	Cerceau & Marchand	12	64.0 to	32.0	4.5		*C2*	32.0	.141	72.5	13.8		H.W.	F. 1852, p. 276	117
							45.3	22.7				22.7							
118	Rock River	Watertown, Wis., U.S.A.	Rock R.	1902–03	Loweth	4	64.0	16.5	3.0	3.0		39.3	.076	28.3	8.0	Ring sandstone	Ry.	B. Mar. 26, 1903	118
119	Coldstream	Coldstream, Scotland	Tweed R.	1771–	Smeaton	5	64.0											L.	119
120	L. Conemaugh No. 3	Summerhill, Penn., U.S.A.	L. Con. R. H. W.	1887	Brown	3	64.0	16.0	2.7	2.7	*C*	40.0	.067	23.0			Ry.	E.	120
121	L. Conemaugh No. 2	Penn., U.S.A.	L. Conemaugh R.		Brown	1	64.0	16.0			*C*	40.0					Ry.	E.	121
122	Stockport Viad.	Stockport, England				22	65.0	32.5	2.8		*C2*	32.5	.086	32.0		Brick	Ry.	F. 1852, p. 158. J	122
123		Carlisle, Scotland	Eden R.		Smirke	5	65.0	21.0	3.8	7.3	*E*			†36.0			H.W.	C. 1855–6	123
124	Rivanna Aq.	U.S.A.			Ellet	5?	65.0	15.0	2.8	2.8	*C*	42.7	.066		7.0		Canal	Pub. W'ks, U. S., '41	124
125	Teviot-Tweed	Near Kelso, Scotland	Teviot R.	1794–95	Elliot	3	*65.0	17.0			*C*			†23.0			H.W.	L.	125
126	Houghton River				Haskoll		65.0	32.5	2.8	2.8	*C2*	32.5	.086				H.W.	I.	126
127	Conon	England	Conon R.	1809	Telford	1	65.0	21.8	3.0		*C*	36.4	.082	20.0	8.0 to 6.5	Freestone		L. J. F. 1852, p. 286	127
						2	55.0												
						2	45.0												
128	Boberthal	Near Bunzlau, Silesia	Bober R. & Val.			5	65.6				*E*						Ry.	F. 1852, p. 212	128
						30	less												
129	Cher	France	Cher R.		Beaudemoulin	6	65.6	21.9	3.3		*E*	47.6	.069		8.5		Ry.	F. 1852, p. 294	129
130	Scrivia	Italy	Scrivia R.	1850	Ferraris	3	65.6	13.1	3.9		*C*	47.5	.082	29.5	13.1	Brick ring	Ry.	F. 1852, p. 296	130
131	Cinq-Mars	France	Loire R.	1845–46	Bailloud	19	65.6	21.6	3.9		*E*	47.7	.082	28.9	11.5		Ry.	F. 1852, p. 294	131
132		Val-Benoist, Belgium		1832		5	65.6	8.8	3.3		*C*	65.5	.054				Ry.	J.	132
133		Furand	Furand, R.	1834	Montluisant	1	65.6	32.8	3.3		*C2*	32.8	.101	26.2		Freestone	H.W.	J. F. 1852, p. 290	133
134		Auzon, France	Vienne R.	1846–47	Beaudemoulin	5	65.6	21.9	3.3		*E*	47.5	.069		8.5		Ry.	J. F. 1852, p. 294	134
135	Chante-Perdrix	France			Lamothe	9	65.6	32.8						26.2			Ry.	G. 1st Tri., 1901	135
136	Landwasser V.		Landwasser R.	1901		5	65.6	32.8	3.0	4.4	*C2*	32.8	.091	8.5	11.5	Limestone	Ry.	Engineer, April, '04	136
137	Raritan River	New Brunswick, N. J., U.S.A.	Raritan R.	1902–3	Brown	10	66.0	33.0			*C2*	33.0		55.0	8.0		Ry.	N. Oct. 10, 1903	137
						2	56.0	28.0	3.2			28.0	.114		to				
						8	51.0	25.5				25.5			11.0				
						1	72.0	24.0	3.3			39.0	.084						
138	Kew	Kew, England	Thames R.	1789	Paine	1	66 0							24.0			H.W.	K. June 14, 1895	138
						2	55.0												
						4	45.8												
139	Bow	Stratford, England	Lea R.	1835–39	Walker & Burgess	1	66.0	13.8	2.5	4.0	*E2*	81.0	.031	42.5		Granite ring	H.W.	A. Oct. 1837. p. 14; A. April 1839. S	139
140		Near York, England	Ouse R.		J. & B. Greene	3	66.0	19.3	3.5		*E*			29.6	10.0	Brick interior	Ry.	S.	140
141	Montignac	France	Vézère R.	1766–72	Tardif	3	66.1 to 42.6	21.3			*C*				17.1			F. 1852, p. 280	141
142		Lancaster, England				5	66.9	33.5			*C2*	33.5			13.1		Canal	F. 1852, p. 286	142
143	Brig o'Balgownie	Old Aberdeen, Scotland	Don R.	1281	Bishop Cheyne	1	67.0				*P*						H.W.	L.	143
144		Horbury, England	Aire R.	1775	Clinchamp	5	67.9 to 55.4	12.6	6.4		*E*	57.5	.111		11.7			F. 1852, p. 282	144
145	Bellecour	Lyon, France		1789–1810		5	68.2	24.4	2.7		*E*	53.3	.050		18.6			F. 1852, p. 284	145
146	Viad. d'Arles	Near d'Arles, France	Valley			31	68.9	23.0			*E*						Ry.	F. 1852, p. 129	146

* Maximum

† About.

REMARKS.—108. Replaced No. 89. 110. Removed for Manch. Ship. Canal. 111. Gravel foundation. Coffer-dams used. H.=98′.5. 116. See Nos. 68 and 109. 117. Repaired 1849–51. *E* arches built under the circular. See No. 12. 118. C. M. & St. P. Ry. 120. Pitts. Div. Pa. Ry. Skew 60°. Ribbed. On piles. 121. Pitts. Div. Pa. Ry. Stone parapet. 122. Manchester-Birmingham. H.=105′.9. 123. Intrados has five centres. 124. James River and Kanawha Canal. 128. Berlin-Breslau. Intrados of each at same elevation; 75 ft. high. 129. Tours-Bordeaux. Skew 34° 30′. 130. Turin-Genoa. 131. Tours-Nantes. 134. Tours-Bordeaux. 135. Cost 1,017,300 f. 136. Thusis-Engadine. 137. Penn. Ry. 139. Replaced old bridge of 1100–1118. Slight skew. Foundation on gravel. 140. Great North of England Ry. On piles. 142. Bottom of canal to intrados=8′.5. 146. Avignon-Marseilles. H.=27′.9. Pile foundation.

TABLE II.—DATA FOR ABOUT 500 ARCH BRIDGES ARRANGED ACCORDING TO SPAN—(*Continued*).

MASONRY ARCHES.

Number.	Name.	Place.	Over.	Date.	Engineer.	No. of Spans.	Span.	Rise.	Thickness of Arch Ring at Crown t_0.	At Springing, t_s.	Curve.	Radius at Crown.	$\frac{t_0}{R}$.	Width, Face to Face at Crown.	Thickness of Piers at Springing.	Material.	Class of Bridge.	Reference.	Number.
147	Central Ave.	Indianapolis, Ind., U.S.A.	Fall Crk.	1899		2	69.0	14.0	2.0	2.0				50.0	6.0	Limestone	H.W. and	H. W. Klausman	147
						1	69.0	15.0	2.0	2.0				50.0	6.0	Ring Oö-	E. Ry.		
148	Bord		Oeil R. (?)	1764	Leclerc	1	69.2	25.6			E					[litic		F. 1852, p. 280	148
149	Karlsbürcke	Prague, Austria	Moldau	1357–1507		16	69.5											*P*. 1896, p. 126	149
150	Potarch	Wales(?)	Dee R.	1813	Telford	1	70.0	25.8	2.5		C	36.6	.068	20.0	10.0			F. 1852, p. 288	150
151	Lockwood	Nr. Huddersfield, England	Sheffield R. Val.	1846–49	Hawkshaw	2	60.0												
						1	70.0	8.0	2.6	2.6				28.0	4.5	Sandstone	Ry.	C. 1850	151
						1	45.0											A. April, 1851, p. 215	
						32	30.0	15.0	†1.5	†1.5		15.0	.100						
152	Wellesley	Limerick, Ireland	Shannon R.	1827	Nimmo	5	70.0	9.0	†2.0	†3.6	E			43.0	10.0		H.W.	C. 1855–6. S, Q.	152
153	Wharncliffe Viad.	Brent-Knoll, Enlgand	Brent R.	1836–37	Brunel	8	70.0	17.5	3.0		E			35.0		Brick, stone fac'g	Ry.	A. 1837–8, p. 126; F. 1852, p. 181	153
154	Crum Elbow	Hyde-Pk.-on-Hudson, N. Y., U.S.A.	Crum Elbow Crk.	1898	Morris	1	70.0	7.0	2.5	2.5							H.W.	B. Feb. 16, 1899	154
155			Lea R.		Braithwaite	1	70.0	17.5	3.8		E			30.0		Brick ring	Ry.	S.	155
156	Wissahickon	Pa., U.S.A.		1881–2	Buchholz	5	70.0	23.0	3.0					28.0	9.5	Talcose sl.	Ry.	B. May 2, 1902	156
157	Shock's Mills	Shock's Mills, Pa., U.S.A.	Susquehanna R.	1903–	Brown	28	70.0	20.0	3.5	3.5	C	40.6	.086	28.0	8.0		Ry.	T. March 11, 1904	157
158	Rockville	Rockville, Penn., U.S.A.	Susquehanna R.	1901	Brown	48	70.0	20.0	3.5	3.5	C	40.6	.086	50.0	8.0	Stone ring	Ry.	B. March 10, 1900 T. Oct. 25, 1901	158
159	Mazares	France(?)	Lère R.	1787	Pertichamp	3	70.3	35.2			C_2	35.2			16.0			F. 1852, p. 284	159
160		Pavia, Italy	Ticino R.	14th Cent.	Under Visconti	7	70.4	64.0	3.9		P	85.3	.046		18.7	Brick	H.W.	F. 1852, p. 276	160
161		Swatara, Penn., U.S.A.			Osborn	6	70.0	25.0	3.5		C	37.0	.093	22.5	8.5	Brick	Ry.	I.	161
162	Homps		Aude R.	1785	Ducros	3	70.2	9.4	4.3		C	68 2	.063		11.7	Freestone		J. F. 1852, p. 284	162
163	Spoleto Aq.	Spoleto, Italy	Valley(?)	741	Theodelapius	10	70.3	35.2			C_2	35.2			11.7	Brick	Aqued't	J. Q.	163
164	Colorado St.	St. Paul, Minn., U.S.A.		1889	Rundlett	1	70.5	11.0	3.5	†4.6	C	62.0	.056	†50.0		Limestone	H.W.	N. Nov. 23, 1889	164
165		Helmsdale, Scotland(?)	Helmsdale R. (?)	1816	Telford	2	70.7	25.0			C			21.0	14.0			F. 1852, p. 288	165
166		Luxemburg, Germany	Petrusse R.	1899–1903	Sejourne	2	70.8	35.4	2.9								H.W.	N. Oct. 12, 1901 N. Mar. 1, 1902	166
167	North	Edinburgh, Scotland	Waverley Ry. Sta.	1763†		3	71.0	35.5			C_2	35.5		54.5	13.5	Craigleith stone	H.W.	K. Oct. 1899, pp. 423, 491	167
168	North Loch	Nr. Edinburgh, Scotland	N. Loch Valley		Mylne	3	*72.0	36.0	2.8		C_2	36.0	.078	42.3			H.W.	I.	168
169	Schuylkill	Philadelphia, Pa., U.S.A.	Schuylkill R.			4	72.0	16.5	2.0		C	47.5	.042				Ry.	F. 1852, p. 292	169
170	Black Rock Tunnel	Penn., U.S.A.		1836	Robinson	4	72.0	16.5	2.8	2.8	C	47.5	.059	18.3	8.0		Ry.	I.	170
171	London (old)	London, England	Thames R.	1176–1209 1758	Peter of Colechurch	19	9 to 20							40†	25′ to [34′		H.W.	L.	171
172	Brunswick	N. Brunswick, N. J., U.S.A.	Raritan R.	1902	Brown	1	72.0												
						1	72.0	36.0	3.3	3.3	C	36.0	.092	55.0			Penn. Ry. 4 tracks	B. Jan. 30, 1902, p. 86	172
						11	66.0	33.0	3.3	3.3	C	33.0	.100	55.0	9.0				
						1	56.0	28.0	3.2	3.2	C	28.0	.114	55.0					
						8	51.0	25.5	3.0	3.0	C	25.5	.118	55.0	8.0				
173	Aulne	France			Arnoux	12	72.2	31.1			C_2	31.1		26.6			Ry.	G. 1st Tri. 1901	173
174	Boston Ave.	Medford, Mass., U.S.A.	Mystic R.	1900	Bailey	1	72.2	15.5	2.5	2.5		49.8	.050	56.0		Granite	H.W.	Wm. G. Taylor	174
175	Zeniec	Austria					72.2		2.6	4.3							Ry.	B. Dec. 7, 1893	175
176	Schmiedtobel	Near Klösterle, Austria	Schmiedtobel	1882?	Huss	3	72.2	36.1	4.1	7.5	C_2	36.1	.114				Ry.	G. 1888, XVI, p. 575	176
						2	39.4	19.7				19.7			[17.1				
177	Po	Near Valenza, Italy	Po R.	1850	Rovere	21	72.2	11.2	3.8		C	63.9	.059	32.8	9.84&	Brick	Ry.	F. 1852, p. 296	177
178		Dresden, Saxony	Elbe R.	1179–1260	Fotius	18	72.5	36.3	6.4		C_2	36.3	.180	37.3	29.9		H.W.	F. 1852, p. 274	178
179	Teviot-Tweed	Kelso, Scotland	Tweed R.	1799–1803	Rennie	5	73.0	21.0			E			†26.0			H.W.	L.	179
180	Conon Viad.	Conon, Scotland	Conon R.		Mitchell	5	73.0		3.0	4.0							Ry.	K. Sept. 20, 1867, pp. 206, 257	180
181	Staines	Staines, England	Thames R.	1832	Rennie	1	74.0	9.3	3.0	6.0		78.0	.038	34.0	8.3	Granite	H.W.	C. S. K. Sept. 13, 1895, p. 336	181
						2	66.0	8.3											
182		Fucecchio, Italy	Arno R.	1869		5	74.0											Jour. F. Inst., Feb. 1870, p. 101	182

* Maximum.

† About.

REMARKS.—147. Cost about $39,000. Pile foundation 3′ center to center. 149. Partially destroyed, flood 1890. 151. Huddersfield & Sheffield Ry. 70′ and 45′ arches on skew and ribbed. H. = 122′.0. 152. "Bell-mouthed" type. 153. Great Western Ry. of England. 156. Cost $375,000. 157. Penn. Ry., two tracks. 158. Penn. Ry., four tracks, 6° curve at one end. 160. Replaced by another bridge. Covered. Roof supported by marble columns 161. Lebanon Valley Ry. 163. Now in use. Piers of stone. H. = 292′.. at springing. 164 Skew 60°. Pile foundation. 166. Four lateral arches in each spandrel. 167. Replaced by steel, 1898–9. 168. H. = 65′.0 to top of parapet. 169. To Mt. Carbon, W. Va. 170. Phila. & Reading Ry. 171. Replaced in 1824–31. 172. 72′ span. Stepped Skew 63° 10′. Backing of masonry 24′ to 28′ above springing, then earth fill. 173. Cost 2,165,000 f. 174. Cost $17,300. Skew 16°. 175. Austrian State Ry. 176. Austrian State Ry. 177. Alessandria to Lake Maggiore. 179. Below Elliot's Bridge. 180. Highland Ry. one track.

TABLE II.—DATA FOR ABOUT 500 ARCH BRIDGES

MASONRY

Number.	Name.	Place.	Over.	Date.	Engineer.	No. of Spans.
183		Scotland	Earn R.	1781–1821	Rennie	3
184	Albany St.	N. Brunswick, N. J., U.S.A.	Raritan R.	1892	Dean and West-brook	7
185	Whitadder	Allantown England(?)	Whitadder R.	1842	Stevenson	2
186	Westminster (old)	Westminster, England	Thames R.	1738–50	Labelye	15
187		St. Maxence, France	Oise R.	1774–85	Perronet	3
188		Navilly, France	Doubs R.	1780	Gauthey	5
189		Roanne, France	Loire R.	1789–1809	Vareigne and Vimar	7
190		Compiègne, France	Oise R.	1783	Lahite	3
191		Semur, France	Armançon R.	1780	Dumorey	1
192	Pont Royal	Paris, France	Seine R.	1685	Mansard	5
193	Cestius	Rome, Italy	Tiber R.	1st c. B.C.	Under Cestius	1
194		Perth, Scotland	Tay R.	1760–71	Smeaton	9
195	Hyde Park	Readville, Mass., U.S.A.	Hyde Park Ave.	1897–98	Curtis	1
196		Italy	Taro R.	1816–20	Cocconcelli	20
197		Orleans, France	Loire R.			
198	Crown St. or Hutchenson	Glasgow, Scotland	Clyde R.	1829–33	Stevenson	1 2
199	Molle	Near Rome, Italy	Tiber R.	†100 B.C.	Scaurus	2
200	Fabricius	Rome, Italy	Tiber R.	†62 B.C.	Fabricius	1
201		Scotland	Avon R.	1820	Telford	1 1
202	Annan	Near Johnstown, Scotland	Annan R.	1820	Telford	1
203	High Bridge	New York, N. Y., U.S.A.	Harlem R.	1837–42	Jervis	8
204	Conewago	W. of Conew'o, Pa., U.S.A.	Conewago Crk.	1891–92	Brown	7 3
205	Schuylkill Falls	Philadelphia, Pa., U.S.A.	E. Pk. Drive	1890	Nichols	1
206	Conemaugh	Viad. Station, Pa., U.S.A.	Conemaugh R.	1833	Penn. Ry.	1
207	Posen Viad.	Posen, Germany				
208	Vittorio	Turin, Italy	Po R.	1810	Pertinchamp	5
209	Painsville Viad.	Near Painsville, O., U.S.A.				4
210		Trilport, France	Marne R.	1758–64	Peronnet	3
211	Pont du Gard	14 m. from Nismes, France	Gardon R.	Bet. 27 B.C.–14 A.D. 1st tier 2d tier top tier	Under Agrippa	{1 3 2} 12 36
212		Prague Austria	Moldau R.			8
213		York, England	Ouse R.	16th cent.		3
214		Near Montlouis, France	Loire R.	1845	Morandière	12
215		Tablonica Austria				
216		Baiersbonn, Germany	Forbach R.	1890	Leibbrand	1
217	Oise	Near Pontoise, France	Oise R.	1843	De Breville and Couche	3

* Maximum.

REMARKS.—183. On piles. 184. Skew. 186. First use of modern caisson. Replaced by cast-iron bridge. 187. Radial joints in spandrels. 193. Continuation of Fabricius Bridge. 195. Skew 61° and 77° 52′. N. Y., N. H. & H. Ry. 200. 13′ arch in pier. 201. Glasgow–Carlisle. 203. H. = 100′.0. Parapet 116′ above water. 204. Phila. Div. Pa. Ry. 6° curve. Two tracks. 205. Phila. & Reading Ry. 206. Pitts. Div. Pa. Ry. Destroyed by Johnstown Flood, 1889. 208. Commenced by French 1810. Completed by King Victor

ARRANGED ACCORDING TO SPAN—(*Continued*).

ARCHES.

Span.	Rise.	Thickness of Arch Ring at Crown t_0.	At Spring-ing, t_s.	Curve.	Radius at Crown.	$\frac{t_0}{R}$.	Width, Face to Face at Crown.	Thickness of Piers at Springing.	Material.	Class of Bridge.	Reference.	Number.
75.0	18.5	2.7		*E*			40.9	18.6		H.W.	S. 1839	183
*75.0	15.0	2.4	2.4	*C*	54.4	.044	?35.0	10.0	Brick stone fac'g	H.W.	B. April 16, 1892, p. 373	184
75.0	11.5	2.5	3.0				†30.0	10.0	Soft red sandstone	H.W.	A. March, 1844, p. 128	185
*76.0	38.0	7.6	14.0	*C2*	38.0	.200	46.9	18.1–12.8	Portland stone	H.W.	K. 1895, p. 306; L. F.	186
76.0	6.4	4.8		*C*	118.2	.041	76.7	9.6		H.W.	S. F. 1852, p. 282	187
76.7	25.6	4.3		*E*	53.3	.080		16.0			F. 1852, p. 284	188
76.7	26.6	3.2		*E*	63.9	.050	35.1	13.3			F. 1852, p. 284	189
76.7 to 70.3	25.6 19.3	4.3 3.2		*E*	53.3	.080	32.0	12.8		H.W.	F. 1852, p. 278	190
76.8	38.4	3.2		*C2*	38.4	.084					F. 1852, p. 282	191
76.8 to 68.2	25.7 24.6	4.8		*E*			55.4	14.9		H.W.	F. 1852, p. 276	192
76.8	38.4	4.8		*C2*	38.4	.125	30.9			H.W.	Q. F. 1852, p. 274	193
*77.0							†26.0			H.W.	L.	194
78.0	14.3	3.0	3.0	*C*	60.3	.049	165.0			Ry.	T. Aug. 12, 1898	195
78.7	21.6	1.2		*C*	46.6	.026		15.1	Brick		F. 1852, p. 288	196
79.0	26.3	4.0		*E*						Ry.	I.	197
79.0 74.5 65.0	13.4 11.8 8.7	3.5 3.5 3.5	4.5	*C*	64.9 64.7 65.0	.054 .054 .054	38.0		Sandstone	H.W.	C. 1855–6	198
79.3 to 51.0							†29.0			H.W.	Q. Cresy's Enc. C. E.	199
80.0 79.5	40.0 39.8	6.0 6.0		*C2*	40.0 39.8	.150 .151	51.2	†32.0	Peperino, tufa and traverti'e	H.W.	F. 1852, p. 274 M. No. 1207	200
80.0	20.0			*C*	50.0		27.0				F. 1852, p. 288	201
80.0	20.0	3.0		*C*	50.0	.060	20.0				F. 1852, p. 288	202
80.0 50.0	40.0 25.0	2.5	2.5	*C2*	40.0 25.0	.062	†21.0			Aqued't	Johnson's Ency.	203
80.0	40.0	3.5	3.5	*C2*	40.0	.088	25.0	12.0		Ry.	E.	204
80.0	26.0	3.0	3.0	*C*	43.8	.069	30.0			Ry	B. May 24, 1894; Jan. 24, 1891	205
80.0	40.0	3.0	3.5	*C2*	40.0	.075			Sandstone	Ry.	Q. Am. Sup.	206
80.0	16.0	4.7		*C*	58.0	.081			Brick		I.	207
*80.0				*E*					Granite		U.	208
80.0	40.0	3.0	3.0	*C2*	40.0	.075	90.0	10.0		Ry.	B. May 2, 1902	209
80.4 to 76.7	28.8 27.7	4.8		*C*	42.0	.114	32.0	16.0		H.W.	Q. F. 1852, p. 280	210
80.5 63.0 51.0 15.8	40.3 31.5 25.5 7.9	5.3 5.0 5.0 2.6		*C2*	40.3 31.5 25.5 7.9	.124 .159 .196	20.8 15.0 11.8		Freestone	Aqued't and H.W.	Johnson's Ency. P. Oct. 1896, p. 122	211
*80.9				*C*					Granite	Ry.	K. May 10, 1878	212
*81.0	26.3			*P*						H.W.	Q.	213
81.2	23.3	4.4		*E*	78.5	.056	28.2	10.7		Ry.	K. Dec. 22, 1871	214
82.0		3.6	5.2							Ry.	B. Dec. 7, 1893	215
82.0	9.8	2.0	2.6				?21.7			H.W.	G. 1st T. 1901	216
82.0	11.7	4.6		*C*	77.6	.059	25.4	10.1		Ry.	F. 1852, p. 294	217

† About.

Emmanuel. 209. Lake Shore & M. S. R.R. 210. First bridge entirely designed by Peronnet. 211. Fifth century, ends destroyed. Repaired 1743 and piers prolonged for new bridge. H. = 160′.0. 212. Between Karlin and Bubua. Viaduct has 87 arches. 214. Orleans-Tours. Damaged in War 1870–71. 215. Austrian State Ry. 216. Three-lead hinges. Cost 18,260 f. 217. Skew 76°. Ch. de fer du Nord.

TABLE II.—DATA FOR ABOUT 500 ARCH BRIDGES ARRANGED ACCORDING TO SPAN—(*Continued*).

MASONRY ARCHES.

Number.	Name.	Place.	Over.	Date.	Engineer.	No. of Spans.	Span.	Rise.	Thickness of Arch Ring at Crown t_0.	At Springing, t_s.	Curve.	Radius at Crown.	$\frac{t_0}{R}$.	Width, Face to Face at Crown.	Thickness of Piers at Springing.	Material.	Class of Bridge.	Reference.	Number
218	Crueize Viad.	Near Marvejois, France	Crueize R.			6	*82.0	†41.0	†4.2	†8.2	3C	41.0	.102	?26.2	16.4		Ry.	R. March, 1891	218
219	Stulz Viad.		Stulz Gorge				82.0	41.0	3.3	4.9				8.5			Ry.	Engineer, April 8, '04, pp. 357, 366	219
220	Mussy Viad.	Mussy, France	Mussy R.	1892–6	Geoffroy, Morris and Pouthier	18	82.2	41.1	4.6		C2	41.1	.112	26.5	16.4–23.0	Granite	Ry.	K. April 30, 1897, p. 575	220
221	Pont Royal	Paris, France	Seine R.	1685	Mansard	5	*82.3							†60.0			H.W.	L.	221
222		Moret, France	Loing R.	1771	Peronnet	3	83.1		4.3		C	153.4	.028	41.6	8.0			F. 1852, p. 280	222
223		Elkader, Iowa, U.S.A.	Turkey R.	1888	Tschirgi	2	84.0	27.9	3.0	4.0	C	45.5	.066	30.0		Limestone	H.W.	B. April 11, 1891	223
224	Cart	Paisley, Scotland	Cart R.	1839	Locke		85.0	18.0									Ry.	A. 1839, p. 313	224
225	Big Walnut	U.S.A.		1902	Graham	3	85.0		3.3	3.3	C			32.0	10.0		Ry.	T. Aug. 22, 1902	225
226		Sisteron, France	Durance R.	1500		1	85.2	57.5	2.7		E							F. 1852, p. 282	226
227	Cognet	Hautes Alpes, France	Drac R.	1605		2	*85.3	42.6	4.6		C2	42.6	.108	11.8			H.W.	F. 1852, p. 276	227
228		Maligny	Serin R.		Werbruge	1	85.3	42.6	3.0		C2	42.6	.070	21.3		Freestone	H.W.	J. F. 1852, p. 282	228
229		Darlaston, England					86.0	13.5	3.5		C	75.2	.047	?26.5			H.W.	C. 1855–56	229
230		Coatsville, Pa., U.S.A.	W. B. Brandywine	1902	Brown	2 7	86.0 78.0	43.0 39.0				43.0 39.0					Ry.	T. Nov. 21, 1902, p. 898	230
231		Blois, France	Loire R.	1723	Gabriel	11	86.3 to 54.3	30.9	6.9		E	57.5	.120		22.4–15.9		H.W.	F. 1852, p. 276 P. 1886, p. 134	231
232		Bordeaux, France	Garonne R.	1813–22	Deschamps	17	86.9 to 65.9	28.9	3.9 3.9		E			49.2	13.8	Brick and stone	H.W.	J. F. 1852, p. 288	232
233	Lea Cut	Lea Cut, England	Lea Cut R.				87.0	16.0	3.8					30.0		Brick, stone trim		C. 1855–56	233
234	Salarius	Narses, Italy	Teverone R.	Rebuilt 6th cent.		1	87.8	43.9			C2	43.9		27.8			H.W.	M. Feb. 18, 1899, p. 19346	234
235	Fouchards	Samur, France	Thouet R.		Trudaine or Voglie	3	88.4	7.4			C	84.2			10.1			F. 1852, p. 282	235
236	Pont de Pierre	Grenoble, Franee	Isère	1839	Picot	1 2	88.6 75.4	22.1 20.7	3.9 3.5		E	114.8	.034	32.8	16.4			F. 1852, p. 292	236
237		La Voulte, France	Allier R.			3	*89.5	28.8			E				12.8			F. 1852, p. 282	237
238		Albois(?), France	Aveyron R.	1770	Boesnier	3	*89.5	33.0	3.7		E	52.2	.070	38.4	13.9			F. 1852, p. 280	238
239	Dee Viad.	Bet. Rhos-y-Medre and Chirk, Wales(?)	Dee R.	†1849		19	*90.0									Fine stone	Ry.	A. Oct. '48, p. 317	239
240	Dunkeld	Dunkeld, Scotland	Tay R.	1809	Telford	7	90.0 to 22.0	30.0	3.0			48.7	.062	27.0	16.0–14.0		H.W.	L. F. 1852, p. 286	240
241	Dean	Near Edinburgh, Scotland			Telford		90.0	30.0	3.0		C	48.8	.061					I.	241
242	Licking Aq.		Licking R.		Fisk		90.0	15.0	2.8		C	76.0	.037			Aqued't		I.	242
243		Castellane, France	Verdon R.	1404		1	90.3	29.9	4.3			73.5	.058				H.W.	F. 1852, p. 274	243
244		Romans, France	Isère			4	91.1 to 70.2	23.9	4.3		C	55.1	.077	19.9	28.8			F. 1852, p. 282	244
245	Enz	Near Hofen, Germany	Enz R.	1885	Leibbrand	1	c-c91.9	c-c9.2	3.3	4.9	C	119.4	.041	10.7		Sandstone	H.W.	K. 1892, p. 560; G. 1891, p. 920	245
246	Jena	Paris, France	Seine R.	1806–12	Lamandé	5	*91.8	10.8	4.7	†8.0	C	102.0	.046	46.4	9.8	Freestone	H.W.	C. 1855–56; H. F. 1852, p. 286	246
247		Stonleigh, Enlgand	Avon R(?)	1781–1821	Rennie	1	92.0	13.0	†4.6		C	87.9	.052	16.0			H.W.	S.	247
248	Alcantara	Toledo, Spain	Tagus R.	997	Romans(?)	3	*93.0	46.5									H.W.	P. 1896, p. 130	248
249	Louis XVI(?)	France		1791	Perronet		94.0	9.8	3.7									Woodbury, 1858	249
250	Spey	Fochabers, Scotland	Spey R.		Burn	4	*95.0							21.5			H.W.	L.	250
251	Trinity	Florence, Italy	Arno R.	1569	Ammanati	3	95.8 to 87.6	16.0 14.8	3.2	3.2	E			33.8	26.3	White marble	H.W.	A. April, 1847, p. 104	251
252		Pontoise, France	Oise R.(?)	1772	Peronnet	3	95.9	7.1	5.3		C	165.6	.032	41.5	9.8			F. 1852, p. 280	252
253	St. Edme	Nogent-on-Seine, France	Seine R.	1766–69	Peronnet	1	95.9	28.8	5.3		E	79.9	.066	32.0			H.W.(?)	F. 1852, p. 280; V. July 17, 1897	253
254	Vecchio	Florence, Italy	Arno R.	1177	Gaddi	3	*95.9	19.2	5.3		P	85.2	.062	105.0	23.5	Freestone	H.W.	J. F. 1852, p. 276; P. 1896, p. 129	254
255		Neuville, France	Ain R.	1775	Aubry	2	96.0	26.6	4.3		E	71.1	.060		19.2			F. 1852, p. 282	255

* Maximum.

† About.

REMARKS.—218. H. = 207'.6. Midland Ry. 219. Thusis-Engadine. 220. B. Nov. 8, 1894, p. 388. Paris, Lyon. 224. Glasgow & Paisley Joint Ry. 225. B. & O. Ry., Newark Div., two tracks. 230. Over Wilmington & Northern Ry. and deep ravine. 234. Blown up in 1867. 239. Shrewsbury & Chester Ry. 242. Chesapeake & Ohio Canal. 245. Three lead "hinges." 247. On piles. 254. Covered. Rebuilt about 1350.

TABLE II.—DATA FOR ABOUT 500 ARCH BRIDGES ARRANGED ACCORDING TO SPAN—(*Continued*).

MASONRY ARCHES.

Number.	Name.	Place.	Over.	Date.	Engineer.	No. of Spans.	Span.	Rise.	Thickness of Arch Ring at Crown t_0.	At Springing, t_s.	Curve.	Radius at Crown.	$\frac{t_0}{R}$.	Width, Face to Face at Crown.	Thickness of Piers at Springing.	Material.	Class of Bridge.	Reference.	Number.
256	Dean	Edinburgh, Scotland		1831	Telford	4	96.0	16.7	3.0			77.5	.030	41.0			H.W.	F. 1852, p. 192	256
							90.0	30.0	2.3			48.7	.062						
257		Drôme, France		1774	Bouchet	3	*96.0	27.7	6.4		*E*	74.5	.086		17.1			F. 1852, p. 282	257
258	Fleischbrücke	Nuremberg, Bavaria		1599	Carln	1	97.0	13.0	4.0		*P*	80.0	.050	53.3	—		H.W.	P. 1896, p. 132; F. 1852, p. 276	258
259	Imnau	Near Imnau	Eyach R.	1896	Liebbrand	1	c-c98.4	c-c9.8	1.5	1.6	*C*	128.4	.012	?13.0		Beton	H.W.	G. '98, 2d Tri.	259
260		Charrey, France	Saône R.	1888	Mocquery	5	*98.4	12.3	3.8	4.9	*C*	104.5	.036	18.2	11.5		H.W.	G. VI, 1896, p. 737	260
261		Near Chalonnes, France	Loire R.	1864–5	Morandiere	17	98.4	†24.6			*E*			?26.2		Limestone	Ry.	K. Oct. 18, 1867	261
262	Rialto	Venice, Italy	Grand Canal	1588–91	Ant. da Ponte	1	98.5	23.0			*P*			64.0		Marble	H.W.	Q. P. 1896, p. 122; F. 1852, p. 276	262
263	Margherita	Rome, Italy	Tiber R.	1891	Vescovali	3	*99.0	16.5	†5.0	†6.0	5*C*			?67.5		Rezzato & travertine stone	H.W.	R. June, 1892, p. 260	263
264		Carbonne, France	Garonne R.	1770	Saget	3	99.1	40.5	3.7		*E*	73.5	.050	25.6	22.6			F. 1852, p. 280	264
265	Pont du Jour	Paris, France	Seine R.	1864	Bassompierre	5	99.2	31.2	5.3		*E*			101.7		Stone from Château Haudon	H.W.	K. Feb. 8 & Jan. 25, 1867	265
						31	15.8	7.9			*C*	7.9		29.5			Ry.		
266	Alcantara Aq.	Near Lisbon, Portugal		1731–75		35	*100.0	88.0			*P*						Aqued't	P. 1896, p. 137	266
267	Bishop Aukland	England	Wear R.	1388			100.0	22.0	1.8	1.8							H.W.	I.	267
268	Etherow River		Etherow R.		Hoskoll	4	*100.0	25.0	4.0	4.0	*C*	62.5	.064				Ry.	I.	268
269	Blackfriars (old)	London England	Thames R.	1760–70	Robt. Mylne	9	*100.0	43.0	5.0		*E*			?45.0			H.W.	L. P. 1896, p. 136; F. 1852, p. 280	269
270	Alcantara	Alcantara, Portugal	Tagus R.	100†	Trajan	6	*100.0	50.0			C_2	50.0				Granite	H.W.	L.	270
271	Wellington	Leeds, England	Aire R.	1816–19	Jno. Rennie	1	100.0	15.0	4.0	7.0	*C*	90.8	.043			Brown sandstone	H.W.	A. 1844, p. 128 and 246	271
272	Rutherglen	Bet. Glasgow and Rutherglen, Scotland	Clyde R.	1895	Crouch & Hogg	1	100.0	12.6	4.0	4.0	*C*	97.6	.041						272
						2	90.0	11.7	4.0	4.0		91.4	.044	†50.0	13.5	Granite	H.W.	Engineer, Aug. 23, 1895, p. 182	
273		Minneapolis, Minn., U.S.A.	Mississippi R.	1882–93	Smith	4	100.0	39.7	3.0					28.0	7.0–14.0	Limestone	Ry.	Jour. West. Soc. Eng.	273
						15	80.0	40.0	2.7			40.0	.067					Vol. 8, 1903, p. 421	
						1	71.4	15.0	2.7										
						1	42.9	13.0	2.5										
						1	40.0	5.3	2.7										
274	Elster Viad.	Bet. Reichenbach and Plauen, Saxony	Elster R. & V. (two tiers)	1846–50	Wilke	18	100.3–23.2				C_2			26.1		Brick mostly	Ry.	F. 1852, p. 209	274
275	Göltzsch	Bet. Reichenbach and Plauen, Saxony	Göltzsch R. & V. (four tiers)	1846–5–	Wilke(?)	1	100.3		3.7		*C*			?26.1		Brick mostly	Ry.	F. 1852, p. 199	275
						1	92.9	46.4	3.7		C_2	46.8	.080					Q. Am. Sup.	
						29	46.8	23.4	1.5		C_2	23.4	.064			Rings of brick			
						23	44.6		1.5		*C*	23.4	.064						
						16	41.8		1.5		*C*	23.4	.064						
						10	39.0		1.5		*C*	23.4	.064						
276	Lempde		Alagnon	1785	Mauriset	1	101.2	32.0			*E*							F. 1852, p. 284	276
277		Rouen, France	Seine R.		Lamandé	5	*101.7	13.7	4.5		*C*	95.7	.047				H.W.	H.	277
278	Montlyon	France	Durance R.	1805	Delbergue-Cormon	1	101.7	32.0			*E*	77.8						F. 1852, p. 286	278
279	Pont au Double	Paris, France	Seine R.	1847	De Lagalisserie	1	101.8	9.8	5.3		*C*	136.2	.039	52.2		Millstone grit	H.W.	J. F. 1852, p. 296	279
280	Guillotière	Lyons, France	Rhone R.	1265	Ass. des frères Dupont	18	102.3–26.2	38.4	2.1			62.9	.034		34.1		H.W.	F. 1852, p. 274	280
281	P. de la Concorde	Paris, France	Seine R.	1787–92	Perronet	1	102.3	9.8	3.7		*C*	148.0	.025	51.1	9.6	Freestone	H.W.	J. F. 1852, p. 284	281
						2	92.7	8.7	3.4			127.5	.027						
						2	83.1	6.4	3.2			138.2	.023						
282		Munich, Bavaria	Isar R.(?)	1814	Wiebeking	3	102.3	17.1	4.3		*C*	82.5	.050	42.6	9.6	Freestone	H.W.	J. F. 1852, p. 288	282
283	Gère	Vienna, Austria		1781	Vimar	1	102.7	28.2	5.2		*C*	57.4	.090	35.6				F. 1852, p. 284	283
284	Avignon	Avignon, France	Rhone R.	1177–87	Benezet	21	*102.9	51.5	2.4		C_2	51.5	.047	15.4	22.8	Freestone	H.W.	J. L. P. 1896, F. 1852, p. 274	284

* Maximum.

† About.

REMARKS.—256. 96′.0 arches are under sidewalks. 257. 90′.0 arches are under roadway. 259. Three granite "hinges." 261. Granite piers on concrete foundation. Two tracks. 265. Parapets, etc., Jura marble. 266. H. = 230′.0. Highest single tier of stone arches in the world. 269. Replaced by cast iron, 1865. 270. H. = 210′.0. 271. Coffer-dams employed. 273. Minneapolis Union Ry. Two tracks. 274. Saxony-Bavaria. 275. Saxony-Bavaria. H. = 264′.0. 284. In ruins.

TABLE II.—DATA FOR ABOUT 500 ARCH BRIDGES ARRANGED ACCORDING TO SPAN—(*Continued*).

MASONRY ARCHES.

Number.	Name.	Place.	Over.	Date.	Engineer.	No. of Spans.	Span.	Rise.	Thickness of Arch Ring at Crown t_0.	At Springing t_s.	Curve.	Radius at Crown.	$\frac{t_0}{R}$.	Width, Face to Face at Crown.	Thickness of Piers at Springing.	Material.	Class of Bridge.	Reference.	Number.
285		Port de Piles, France	Creuse R.	1846–47	Bayeux	1	103.8	40.5	4.3	4.3	*E*	70.8	.060	30.0	19.2	Freestone	Ry.	C. 1851–52; J. F. 1852 p. 276	285
						2	98.4											F. 1852, p. 294	
286	Herault	Route of Nice, France	Herault R.(?)		Grangent	1	104.4	15.4	2.7		*C*	90.5	.029	22.4				F. 1852, p. 280	286
287		Prague, Bohemia	Moldau R.	1878	Reiter	1 2 2 2	105.0 99.8 94.5 89.3	16.2 15.3 14.5 13.7	4.0	5.3				39.8	13.1	Granite	H.W.	K. May 10, 1878, p. 359	287
288		Marbach, Germany	Murr R.	1887	Leibbrand	1	105.0	10.2	3.9	4.9	*C*	140.2	.028	18.4			H.W.	G. 1891, 1, p. 922	288
289	Wissahickon	Philadelphia, Pa., U.S.A.	Wissahickon Crk.	1897	Gen. Thayer	1	105.0	11.0	3.0	4.5	*C*(?)	118.1	.025	?35.0		Cousho-hocken stone	H.W.	B. Sept. 9, 1897, p. 162	289
290	Potomac Aq.	Washington, D. C., U.S.A.	Potomac R.			7	*105.0										Aqued't	A. 1837, 8, p. 148	290
291	Ponthaut	Germany	Bonne R.	1793		1	106.3	53.1	5.7		C_2	53.1	.108	29.5				F. 1852, p. 284	291
292		Orleans France	Loire R.	1750–60	Hupeau	9	106.5–98.0	28.8 16.0	6.9 5.8			83.9	.083	49.0	19.2–18.1		H.W.	L. F. 1852, p. 276	292
293		Hartford, Conn., U. S. A.	Connecticut R.	1903	Graves	1	108.0	27.0			*E*				†18.0	Granite	H.W.	N. Dec. 26, 1903	293
294		Baiersbronn, Germany	Murg. R.	1889	Leibbrand	1	108.2	10.8	2.0	2.6				21.7?			H.W.	G. 1st Tri., 1901	294
295		Wurtemberg, Germany	Nagold R.	1882		1	108.8	10.8	3.3	5.3				†16.4			H.W.	G. 1891, 1, p. 903	295
296		Winstone, England	Tees R.	1762	Robinson		108.8											L.	296
297		Sault, France	Rhone R. Br.	1825–27	Montluisant	1 2	111.5 91.8	31.9 29.5	4.6 4.3		*E* *E*	114.2 81.0	.057 .037	23.0	22.2			F. 1852, p. 288	297
298	Lodi St.	Elyria, Ohio, U.S.A.	W. Br. Black R.	1894	Jackson and Bunce	1	112.0	19.5	3.5	4.3				38.0		Elyria sandstone	H.W.	C. H. Snow, City Engineer, Elyria, O.	298
299		Toulouse, France	Garonne R.	1543–1632	Souffron	7	113.0–44.8	38.4	3.7			76.8	.049	64.0	26.6	Brick	H.W.	F. 1852, p. 276	299
300	2d Worochta	Worochta, Austria	Pruth R.	1892–93	Huss	12	113.5–32.8	56.8	4.3	6.7	C_2	56.8	.076	14.7			Ry.	B. Dec. 7, 1893, p. 448	300
301		St. Esprit, France	Rhone R.	1265–1309	Ass. des frères Dupont	19	114.1–81.0	44.8	5.9		*C*	70.4	.084	17.6	27.8		H.W.	F. 1852, p. 274	301
302		Nantes, France	Loire R.				115.2	34.4	6.4									Q.	302
303		Mantes, France	Seine R.	1757–65	Hupeau	3 21	*115.4	34.0	6.4		*E*						H.W.	I.	303
304	Grand-Maître	Fontainebleau, France	Fontainebleau V.	1869	Belgrand	1 Many	115.8 †42.5	19.3								Beton	Aqued't	K. Oct. 1869, p. 275	304
305	Cresheim	Fairmont Park, Philadelphia, Penn., U.S.A.	Cresheim Crk.	1892	Webster	1	116.0	21.2	3.5	4.5				10.0		Buff sandstone	Sewer	B. Aug. 31, 1893, p. 170	305
306	Napoleon	Paris, France					116.0	14.8	4.0								Ry.	I.	306
307	Tongueland	Near Kirkendbright, Scotland	Dee R.	1806	Telford	1 6	118.0 Small	38.0	3.6		*C*	64.8	.056	24.0			H.W.	L. F. 1852, p. 286	307
308		Hartford, Conn., U.S.A.	Connecticut, R.	1904–	Graves	1 1 1 2 1 2	68.0 74.0 81.0 108.0 115.0 119.0	21.1 22.9 25.1 27.0 28.8 29.8						102.0	15.0–40.0	Granite	H.W. and El. Ry.	T. Feb. 19, 1904, p. 123 N. Dec. 26, 1903 N. Dec. 31, 1904, p. 765	308
309	Waterloo (new)	London, England	Thames R.	1817	Rennie	9	120.0	34.6	4.5	10.0	*E*			44.0	20.0	Granite	H.W.	S. K. Feb. 22, 1895, p. 236; F. 1852	309
310	Devil's Br.	Near Lucca, Italy	Serchio R.	1000†		1	120.5	60.3	†4.5		C_2	60.3	.074	12.0		Limestone sandstone	H.W.	C.	310
311	Têtes	France	Durance R.	1732	Hanriana	1	123.6	61.8	4.7		C_2	61.8	.065	15.9			H.W.	F. 1852, p. 278	311
312		Bourbonnais, France			Vaudray		124.0	6.9	2.7	3.6	*C*	255.7	.010			Granite	Ry.	I.	312
313	Vingeanne Val.	Near Oisilly, France	Vingeanne Val.			7	127.0	46.0			*E*			?14.0			Ry.	B. Dec. 7, 1893	313
314		Rumilly, France	Cheran R.	1785	Garella	1	127.6	63.8	5.3		C_2	63.8	.086	23.5		Freestone	H.W.	J. F. 1852, p. 284	314
315	Maidenhead	Maidenhead, England	Thames R.	1832–38	Brunel	6	128.0–21.0	24.3	5.3	7.5	*E*	169.0	.031	?36.0		Brick	Ry.	P. W'ks, G.B., '46; K. Oct. 25, 1895	315

* Maximum.

† About.

REMARKS.—285. Tours-Bordeaux. Two tracks. 288. Three lead "hinges." 289. Skew 69° 26′. Ten 4′ ribs. 293. See No. 308. 294. Three lead "hinges." Cost 23,890 f. 299. Stone trimmings. 300. Austrian State Ry. 301. Small arches in piers. 304. Paris water-supply. 308. Pneumatic foundations. Cost (est.) $1,600,000. 310. Four small side arches. 313. E. Ry. of France. 315. Great W. Ry.

TABLE II.—DATA FOR ABOUT 500 ARCH BRIDGES ARRANGED ACCORDING TO SPAN—(*Continued*).

MASONRY ARCHES.

Number.	Name.	Place.	Over.	Date.	Engineer.	No. of Spans.	Span.	Rise.	Thickness of Arch Ring at Crown t_0.	At Springing, t_s.	Curve.	Radius at Crown.	$\frac{t_0}{R}$.	Width, Face to Face at Crown.	Thickness of Piers at Springing.	Material.	Class of Bridge.	Reference.	Number.
316		Neuilly, France	Seine R.	1768–74	Perronet	5	*128.2	32.0	5.3		*E*	250.0	.021	47.9	14.0	Freestone	H.W.	L. J. P. 1896; F. 1852, p. 280	316
317		Mantes, France	Seine R.	1757–65	Hupeau under Perronet	1 2	128.2 115.4	38.5 34.9	6.4		*C*	89.5	.071	35.6	25.6		H.W.	L. V. July 17, 1897 F. 1852, p. 276	317
318	Echo Br.	Newton Upper Falls, Mass. U.S.A.	Charles R.	1876	Fitzgerald	1 1 4	129.0 34.0 37.0	42.3 18.5	5.0	6.0	 *C* *C*2	67.5 18.5	.074	18.0		Granite	H.W. Aqued't	Boston Water W'ks Fitzgerald A. Fteley	318
319		Elyria, Ohio	Black R.			1	37.0	14.0						27.0				See No. 298.	319
320		Aberdeen, Scotland	Den Burn Rill	1801+	Telford	1	129.0	29.0			*C*			43.0		Granite	H.W.	L. p. 477	320
321		Wan Hsien, China				1	130.0	65.0			*C*2						H.W.	B. June 19, 1902	321
322	North Ave.	Baltimore, Md., U.S.A.	Gorge-Jones' F'lls	1893–95	Smith	3	130.0	26.0	5.0	8.4	*E*			100.0	16.0	BrickRing	H.W. & E. R.R.	B. July 6, 1893, p. 7	322
323	1st Worochta	Worochta, Austria	Pruth R.	1892–93	Huss	1 6 1	131.2 36.1 26.2	32.8	4.6	7.2				14.7			Ry.	B. Dec. 7, 1893, p. 448	323
324	Boucicault	Verjux, France	Saône R.	1888–90	Jozon	5	131.2	16.4	3.4	†4.9	*E*	177.0	.019	?26.0		Villebois stone	H.W.	B. May 18, 1893 G. 1892, p. 445; P. 1892, p. 50	324
325	Moret Viad.	Moret, France	Loing Val.	1847–49		2 30	131.2 32.8	16.4 16.4	2.6			139.4 16.4	.019	29.5	8.2		Ry.	F. 1852, p. 117	325
326		Scrivia, Italy	Scrivia R.	1850+	Ranco	1	131.2	43.7	5.9		*E*	86.9	.068			Brick	Ry., Turin to Genoa	F. 1852, p. 296	326
327	St. Martin	Toledo, Spain	Tagus R.	1203		5	*132.0				*P*						H.W.	P. 1896, p. 130	327
328		Villeneuve, France	Lot R.	1732		4	*132.6	66.3	5.3		*C*2	66.3	.080	19.6			H.W.	F. 1852, p. 276	328
329	Vizille	Near Grenoble, France	Romanche R.	1766	Bouchet	1	133.8	†38.2	7.7		*E*	115.0	.067	32.3				F. 1852, p. 280	329
330	Waldi-Tobel	Near Bludenz, Austria	Gorge	1884	Huss	1	134.5	42.6	5.6	10.2						—	Ry.	G. 1888, p. 575	330
331		Verdun, France	Doubs R.	1895–97	Jozon	1 2	134.5 126.3	30.1 27.9	3.9 3.9		*E*2			19.7	13.1	Limestone	H.W.	G. 1897, 4°, p. 179	331
332	Castalet				Sejourné		134.5											B. Feb. 27, 1902	332
333	Albula R. Viad.	Sales	Al. R. Gorge	1903			137.8							8.5			Ry.	Engineer, April 8 and Mar. 4, 1904, pp. 228 and 355	333
334	Br. *C*33	Bellows Falls, Vt., U.S.A.	Connecticut R.	1899	Cheever	2	98.4 140.0	20.0	4.0	4.0	*C*	132.6	.030	27.0			Ry.	B. June 21, 1900, p. 402, and Blues	334
335	St. Sauveur	France				1	140.0											P. 1896, p. 140	335
336	Pont-y-tu-prydd	Nr. Newbridge, S. Wales.	Taff R.	1755	Edwards	1	140.0	35.0	1.5		*C*			15.8		Freestone	H.W.	Q. L.	336
337	Alma	Paris, France	Seine R.	1855	Darcel	1	141.4	28.2	4.9						16.4	Millstone grit	H.W.	Q. J.	337
338		Near Narni, Italy		Bet. 27 B.C.–14 A.D		2 1 1 1 1	126.0 142.0 135.0 114.0 75.0	25.2	4.9								H.W.	A. 1856, p. 376 L.	338
339	Putney Road	Putney, England	Thames R.	†1882	Bazalgette	1 2 2	144.0 129.0 112.0	19.3 16.3 13.0	4.5 4.3 4.2	5.5 5.3 5.2		144.0 136.0 127.0	.031 .032 .033	47.0	18.0 and 19.0	Granite ring	H.W.	K. May 17, 1895 K. July 23, 1886, p. 85	339
340	Outer Maximilian	Munich, Bavaria	Isar R.	1904		2	144.3							71.5	20.9	Limestone	H.W.	B. Oct. 27, 1904	340
341	Verone	Near Vieux-Château, Italy	Adige R.	1354	Under Scala	1 1 1	146.0 87.4 33.0	35.8 21.3 17.1	5.3		*C*	90.5	.059	22.4	36.2 and 22.4	Freestone	H.W.	J. F. 1852, p. 274	341
342		Moulins, France	Allier R.	1705–1710	Mansard	1 2	147.1 115.1								36.2			F. 1852, p. 276	342
343	Pont-du-Cèret	Near Perpignan, France	Tech R.	1336		1	147.6	73.8	4.6	13.1	*C*2	73.8	.062	12.8			H.W.	B. Dec. 7, 1893 F. 1852, p. 274	343
344		Turin, Italy	Dora Riparia R.	1834	Mosca	1	148.0	18.0	4.9		*C*	160.0	.031	40.0		Granite	H.W.	I. U. 1846, p. 27; F. 1852, p. 290	344

* Maximum.

† About.

REMARKS.—316. In design R. = 160′.0. 317. Destroyed in War 1870. 318. Sudbury Aqueduct for Boston. H. = 79′.0. 321. Slightly pointed. 322. Skew 55°. Ribbed. 323. Austrian State Ry. 324. Radius at spr. = 75′.5. Paris-Lyon. Two tracks. Approach to metal spans crossing river. Curve about 0° 52′. 330. H. = 165′.0. Slight curve. 331. Extrados arc of circle 144′.3 and 150′.9 radius. 333. Thusis-Engandine. H. = 282′. 335. H. = 215′.0. 337. Rubble, grouted. Foundation on piles. 338. Probably the most magnificent bridge built by the Romans in Italy. 340. Three metal hinges. Failed by hinges slipping, June 27, 1904. 342. Failed 1710. 343. Mostly brick. Stone ring.

TABLE II.—DATA FOR ABOUT 500 ARCH BRIDGES ARRANGED ACCORDING TO SPAN—(*Continued*).

MASONRY ARCHES.

Number.	Name.	Place.	Over.	Date.	Engineer.	No. of Spans.
345		Nr. Kleinwolmsdorff, Sax.	Roeder R.			1
346	Bellefield	Pittsburgh, Penn., U.S.A.	St. Pierre Hollow	1896–	Rust	1
347	Claix	Near Grenoble, France	Drac R.	1611		1
348		Elyria, Ohio, U.S.A.	Black R.	1886	Kinney	1
349	Gloucester	Gloucester, England	Severn R.	1827	Telford	1
350		Berne, Switzerland	Aar R.	†1204		
351	Vieille Brioude	Brioude, France	Allier R.	1454	Greiner and Estone	1
352	London	London, England	Thames R.	1821–30	Rennie	1 2 2
353		Near Tournon	Doux R.	1545		1
354	Jamma	Jamma, Austria	Pruth R.	1892–3	Huss	1 5
355	Main St	Wheeling, W. Va., U.S.A.	Wheeling Crk.	1892	Hoge & White	1
356	Tyne Wear Viad. Victoria	Near Newcastle, Eng. Sunderland Low Lambton	Tyne R. Wear R. Wear R & Val.			1 1 2 6
357		Gignac, France	Herault R.	1777–93	Garipuy	1 2
358		Near Lavaur, France	Agout R.	1775	Saget	1
359	Nydeck	Berne, Switzerland	Aar R.	1840–44	Müller	3
360	Antoinette				Sejourné	
361	Ballochmoyle	Near Ballochmoyle, Scot.	Ayr R.		Millar	1 6
362	Vieille Brioude	Brioude, France	Allier R.		Romans	1
363		Near Coppel, Germany	Schwaendenholz Ravine & Brook	1901		1
364	Gour Noir	4 k. from Uzerche, France	Vézère R.	1888–9	Daigrement	1
365		Turin, Italy	Dora Riparia R.	1833	Hartley	1
366	Grosvenor	Chester, England	Dee R.	1832–3	Hartley	1
367	Lavaur	Near Lavaur, France	Agout R.	1888	Sejourné	
368		Bogenhausen, Bavaria	Isar R.	1901–02	Fischer, Archt.	1
369		Germany	Gutach R.	1901		1
370		Jaremcze, Austria	Pruth R.	1892–3	Huss	1 5 2
371	Cabin John	Washington, D. C., U.S.A.	Cabin John Crk.	1857–64	Meigs	1
372		Italy	Adda R.	1903		1
373	Trezzo	Near Trezzo, Italy	Adda R.	1380	Under Barnabo Visconti	1
374		Luxemburg, Germany	Petrusse R	1899–03	Sejourné	1
375	Plauen	Plauen, Saxony	Valley	1905	Leibold	1 1

Span.	Rise.	Thickness of Arch Ring at Crown t_0.	At Springing, t_s.	Curve.	Radius at Crown.	$\frac{t_0}{R}$.	Width, Face to Face at Crown.	Thickness of Piers at Springing.	Material.	Class of Bridge.	Reference.	Number.
148.6	49.5	5.6		C	80.1	.070	†26.0			Ry.	F. 1852, p. 294	345
150.0	36.6	4.0	6.0				82.0		Gray sandstone	H.W.	B. June 22, 1899, p. 391	346
150.2	54.4	3.2		C	82.0	.039	20.2			H.W.	H, F. 1852, p. 276	347
150.0	27.0	3.8	4.5	C	117.6	.032	28.0		Sandstone	H.W.	B. May 31, 1890	348
150.0	54.0	4.5		E	152.4	.029	27.4			H.W.	H, F. 1852, p. 290	349
150.5										H.W.	B. Dec. 19, 1895	350
150.9	75.5	4.3		C2	75.5	.057	24.7			H.W.	A. 1844, p. 247; F. 1852, p. 274	351
152.0	37.7	4.8	10.0	E2	162.0	.029	56.1	24.0 –22.0	Granite	H.W.	A. 1847, p. 106	352
140.0		4.6	9.0								F. 1852, p. 290	
130.0		4.5	8.5								P. 1896, p. 128	
156.7	65.0	2.8		C	78.9	.035	16.0			H.W.	J. F. 1852, p. 276	353
157.4		5.6	8.5	C			14.7			Ry.	B. Dec. 7, 1893, p. 448	354
29.5												
159.0	28.4	4.5	6.0	C	125.4	.036	48.0			H.W.	Blues	355
159.9	79.9	4.6	4.6	C2	79.9	.058	25.8	21.5	Soft	Ry.	F. 1852, p. 178	
144.0	72.0	4.6	4.6	C2	72.0	.064		23.8&	Sandstone		A. 1837–38, p. 57	356
99.8	49.9	4.6	4.6	C2	49.9	.092		21.5	1½ miles from Br.		C. 1855–56	
20.2	10.1											
160.0	44.0	6.5		E	117.7	.055		25.6	Freestone	H.W.	H. J. F. 1852, p. 284	357
83.1	41.6	6.5		C2	41.6	.156						
160.5	65.0	6.3		E	103.4	.095	38.4			H.W.	H. F. 1852, p. 282	358
*160.7							39.8			H.W.	O. Nov. '97, p. 322	359
											B. Dec. 19, 1895	
											F. 1852, p. 294	
164.0											B. Feb. 27, 1902	360
180.0	90.0	6.0?	6.0?	C2	90.0	.067	28.0			Ry.	C. 1851–52	361
50.0	25.0	4.5			25.0	.180						
183.7	†60.0	5.3		C	91.8	.058	16.0		Volcanic rock	H.W.	A. June, 44, p. 247	362
187.0	55.8	5.9	8.5				14.4		Sandstone	Ry.	B. Dec. 26, 1901, p. 487	363
196.8	52.8	5.6	13.8	C	118.1	.047			Granite ring	Ry.	G. 1892, p. 545	364
200.0	42.0	4.0		C	140.0	.028				H.W.	H. p. 225	365
200.0	42.0	4.5	7.0	C	140.0	.032	35.5		Sandstone	H.W.	A. G. June, 1891;	366
											B. Oct.'91, Dec. 7, '93	367
										Ry.	R. 1889, p. 584	
201.7 c.–c.	90.2 c.–c.						60.6		Limestone	H.W.	N. Oct. 4, 1902	368
											B. Sept. 18, 1902	
209.9	21.4	3.4										
210.0	52.5	6.6	9.2				13.7		Sandstone	Ry.	B. Jan. 18, 1902	369
											B. Dec. 26, 1901	
213.0	59.0	6.9	10.2				14.7			Ry.	B. Dec. 7, 1893, p. 447	370
39.4												
26.2												
220.0	57.3	4.2	6.2	C	134.3	.031	20.3		Granite ring	Aq. and H.W.	K. April 19, 1867; N. July 29, 1899	371
†230.0		4.9	7.2	3C	246.0	.020	17.3		Granite	Ry.	N. Oct. 17, 1903	372
251.0	87.8	4.0	4.0	C?	133.6	.030				H.W.	B. Dec. 7, 1893; C. 1855	373
277.7	101.7	4.7	7.1				18.1			H.W.	B. Feb. 27, 1902	374
295.3	56.4	4.9	6.6		344.5	.044	52.5		Hard slate	H.W.	B. Aug. 17, 1905, p. 156	375
43.3	15.6											

* Maximum.

† About.

REMARKS.—345. Saxony-Silesia. Cut-stone ring. 348. Rock foundation. 350. First stone bridge over Aar near Nydeck castle. 353. Rock foundation. 354. Austrian State Ry. 356. Durham Junc. Ry. H. = 151′ about. 358. H. says five arches. 361. Glasgow and S. W. Ry. 362. Fell 1822. See No. 351. 363. On curve $R = 2660'$. Clear H. = 124′.5. 364. Limoges-Brive.

366. F. 1852, p. 290. Lead in ring joints ⅓ span from abutment. 367. Rough stone in cement. 368. Three metal hinges backed with granite. Five lateral arches in each spandrel. 369. Lateral arches. Max. H. = 111′.5. 372. Three-hinged for D. L. Fixed for L.L. 373. Destroyed 1416. 374. Twin arches 19′.4 apart. 375. Longest stone arch in the world.

TABLE II.—DATA FOR ABOUT 500 ARCH BRIDGES ARRANGED ACCORDING TO SPAN—(*Continued*).

PLAIN CONCRETE ARCHES.

Number.	Name.	Place.	Over.	Date.	Engineer.	No. of Spans.	Span.	Rise.	Thickness at Crown.	At Springing.	Curve.	Radius at Crown.	Thickness at Crown $= t_0$.	$\frac{t_0}{R}$.	Width, Face to Face at Crown.	Thickness of Piers at Springing.	Class of Bridge.	Reference.	Number
1	Fern St.	W. Hartford, Conn., U.S.A	Trout Brook	1902–3	Crawford	1	26.0	5.0	2.0	2.0		19.5	2.0	.103					1
						2	10.0	3.0	1.3	1.3		5.5	1.3	.236	†30.0	4.0	H.W.	N. April 25, 1903	
2	Casey R.	Las Marias, Porto Rico	Casey R.	1899(?)	Buel	1	30.0	15.0	2.5		*C*2	15.0	2.5	.167			H.W.	Cem., Jan. 1902	2
3	Bridge No. 41	Sharpsville, Penn., U.S A.	Pine Run	1900	Geer	1	30.0	15.0	2.5	2.5	*C*2	15.0	2.5	.167	75.5		Ry.	T. Nov. 16, 1900	3
4		Cheltenham, Mo., U.S.A.	Des Peres R.	1904(?)	Purdon	3	36.0	18.0			*C*2	18.0			87.0	5.2	Ry.	B. Nov. 3, 1904	4
5		Bet. Manati and Aales, Porto Rico	Quebrada R.(?)	1899(?)	Buel	1	39.4	9.8	1.6				1.6				H.W.	Cem., Jan. 1902, p. 328	5
6		Mansfield, Ohio, U.S.A.		1904(?)	Keith	1	40.0	7.5	.7	.8			.7		40.0		H.W.	N. Feb. 18, 1905	6
7		Bet. Santiago and El Caney, Cuba	San Juan R.	1902	Rockenbach	2	40.0	11.5	1.5			30.0	1.5	.050	20.0	8.0	H.W.	B. Jun. 13, 1903, p. 549	7
8	Cannington Viad.	Cannington, England		1900–02	Pain	10	50.0	16.0	2.5	2.5	*E*		2.5		16.0		Light Ry.	N. Oct. 21, 1905	8
9	Ewarton Br.	Jamaica, W. I.	Ravine	1881–82	Bell	4	50.0	22.2	2.0	3.0		25.2	2.0	.079	16.0	6.0	Ry.	B. July 27, 1893	9
10	Lochnanuamh Viad.	Scotland		1899†	Simpson and Wilson	8	50.0										Ry.	B. Feb. 9, 1899, p. 85	10
11		Scotland	Arnabol Burn	1899†	Simpson and Wilson	6	50.0										Ry.	B. Feb 9, 1899, p. 85	11
12	Finnan Viad.	Scotland	Finnan Valley	1899†	Simpson and Wilson	21	50.0	25.0	2.5		*C*2	25.0	2.5	.100		6.0	Ry.	B. Feb. 9, 1899, p. 85	12
13		Washington, D.C., U.S.A.	Broad Branch	1901	Douglas		50.3	†7.0	†1.8	†6.0			†1.8		26.0		H.W.		13
14		Northampton, Pa., U.S.A.	Hokendauqua Crk. and Highway	1900	Thompson	1	51.8	13.5	3.5			31.5	3.5	.111	42.0		Ry.	N. Jun. 8, 1901, p. 541	14
						1	34.0	11.3	2.8			28.0	2.8	.100					
15		Adjuntas, Porto Rico	Small stream	1899(?)	Buel	1	55.0	11.0	1.5		*E*		1.5				H.W.	Cem., Jan. 1902	15
16		Salt River, Ariz., U.S.A.	Dam spillway	1905–		3	59.0		†1.5				1.5		10.3	6.5	H.W.	N. Oct. 14, 1905	16
17	Bridge No. 242	W. of Cincinnati, O., U. S. A.	Tanner's Crk.	1903–4	Kittridge	1	60.0	26.0	2.7		*C*		2.7		†33.0	9.3	Ry.	N. Mar. 5, 1904, p. 292	17
18		Thebes, Ill., U. S.A.	Bank of Mississippi R.	1902	Noble and Mojeski	2	40.0	20.0	2.3		*C*2	20.0	2.3	.115					18
						11	65.0	32.5	3.3		*C*2	32.5	3.3	.102	28.0	12.0	Ry.	T. Jan. 9,'03, p. 21; B. Nov. 20, 1902	
19		Concord, Mass., U.S.A.	Assabet R.	1901	Worcester	1	66.0	11.0			*E*				†35.0		H.W.	Municip. Engineering, March, 1902	19
20	Bridge No. 163	W. of Cincinnati, O., U.S.A.	Tanner's Crk.	1903–04	Kittridge	3	68.0	17.0	3.5	6.0	5*C*	64.0	3.5	.055	33.0	12.5	Ry.	N. Mar. 5, 1904	20
21		Ehingen, Wurtemberg	Danube R.	1898		2	69.0		2.3	3.0	*C*		2.3		24.6	6.6	H.W.	B. Jan. 9, 1902, p. 35	21
						1	66.0	7.2											
22	Ashtabula Br.	Ashtabula, Ohio, U.S.A.	Ashtabula R.	1904	Beckwith	2	74.0	37.0	†6.5		*C*2	37.0	6.5	.176	145.0		Ry.	T. Jan. 27, 1905	22
23		Near Rechtenstein, Wurtemberg	Danube R.	1893	Braun	2	74.4	8.2	2.1				2.1					Y. 1898	23
24		Plano, Ill., U.S.A.	Big Rock Crk.	1903–4	Breckenridge	1	75.0		3.0			43.0	3.0	.070	44.0		Ry.	N. Jan. 2, '04, p. 18	24
25		Near San Leandro, Cal., U.S.A.	S. Leandro Crk.	1901	County Surveyor	1	81.3	26.0	3.0	15′–20′	5*C*	61.5	3.0	.048	†50.0		H.W.	B. Aug. 27, 1903, p. 174	25
26	St. Ana Viad.	Riverside, Cal., U.S.A.	Santa Ana R.	1902–04	Hawgood	8	86.0	43.0	3.5		*C*	43.5	3.5	.081			Ry.	N. Sept. 9, 1905, p. 284	26
						2	38.5												
27	Morar Viad.	Scotland	Morar R. & H.W.	1898–9	Simpson and Wilson	1	90.0	24.0	3.0				3.0				Ry.	B. Feb. 9, 1899, p. 85	27
						1	50.0												
						2	20.0												
28		Near Imnau, Germany	Eyach R.	1896	Leibbrand		98.4	9.8	1.5	1.6			1.5		8.2		H.W.	G. 2 Tri., 1898	28
29		Pittsburg, Penn., U.S.A.	Silver Lake	1905	Brown	1	100.0	50.0	4.0	4.0	*C*2	50.0	4.0	.080	54.0	12.0	Ry.	N. May 6, 1905, p. 528	29
						5	80.0	40.0	3.6	3.6		40.0	3.5	.088					
30		Near Tarvis, Austria	Schlitza R.	†1903			100.0	10.0	2.3	2.3							H.W.	Engineer, April 22, 1904, p. 424	30
31		Thebes, Ill., U.S A.	Bank of Mississippi R.	1902	Noble and Mojeski	1	100.0	50.0	4.5		*C*2	50.0	4.5	.090	28.0		Ry.	T. Jan. 9,'02, p. 21; B. Nov. 20, 1902	31
32		Near Mechanicsville, N. Y., U.S.A.	Anthony Kill			2	100.0										El. Ry.	B. Nov. 5, 1903, p. 408	32
						1	50.0												

* Maximum.

† About.

REMARKS.—1. Cost $4050. 3. Skew, 15° 0′. Penn. Ry. 4. 1 : 6 "chats." St. L. & S. F. Ry. 6. Three cast-iron hinges. 7. Contract price, $31,000. 9. On curve, 1980′ *R*. Jamaica Govt. Rys. 12. On curve 1200′ *R*. *L* = 1248′; H. = 100′. 13. Pebble-faced. Cost $4159.17. 14. Three tracks. C. R.R. of N. J. Ex. metal used in radial planes. 15. 1600′ above sea-level. 16. Very flat arches; about 12″ fill over key. 17. "Big 4" Ry., Chicago Div. 18. Approach to Thebes Bridge. 20. "Big 4" Ry., Chicago Div. 21. Cost $21,000. 22. L. S. & M. S. Ry. Four tracks. 23. Three lead "hinges." 24. C. B. & Q. Ry. Two tracks. 25. Skew, 10° Cost $25,840. 26. One track. S. P., L. A. & S. L. Ry. 27. Mallaig Ex. of W. Highland Ry. 28. Three granite "hinges." 29. Penn. Ry. Four tracks, 5° curve. 30. Three steel "hinges." 31. Approach to Thebes Bridge.

TABLE II.—DATA FOR ABOUT 500 ARCH BRIDGES ARRANGED ACCORDING TO SPAN—(*Continued*).

PLAIN CONCRETE ARCHES.

Number.	Name.	Place.	Over.	Date.	Engineer.	No. of Spans.	Span.	Rise.	Thickness at Crown.	At Springing.	Curve.	Radius R at Crown.	Thickness at Crown $= t_0$.	$\frac{t_0}{R}$.	Width, Face to Face at Crown.	Thickness of Piers at Springing.	Class of Bridge.	Reference.	Number.
33	Danville Arch	2 miles from Danville, Ill., U.S.A.	Vermillion R.	1905	"Big 4"	1	100.0	40.0	4.0		*C*	51.3	4.0	.088	42.0	15.0	Ry.	N. Mar. 3, 1906, p. 338	33
						2	80.0	30.0	3.6			41.7	3.6	.086					
34		Near Mittenberg, Germany	Main R.	1898–99	Fleischman and Bosch	2	112.0	16.4	2.5	2.8			2.5		23.0	10.2	H.W.	B. July 25, 1901, p. 61	34
								17.5											
						2	107.3	14.8	2.5	2.8			2.5						
								18.3											
						2	102.3	13.8	2.3	2.6			2.3						
								19.0											
35	Grand Maître	Fontainebleu Forest, Fra'e	Valley	1869	Belgrand		*115.8	†19.3	1.3				†1.3				Aqued't	K. Oct. '69, p. 275	35
36		Kirchheim, Wurtemberg	Neckaar R.	1898†		4	124.6	19.0	2.6	3.0			2.6		†18.0		H.W.	B. Mar. 9, 1900	36
37	16th St.	Washington, D. C., U.S.A.	Piney Branch	1905	Douglas		125.0	39.0	5.0		Par		5.0		25.0		H.W.	B. Nov. 16, 1905	37
38	Borrowdale	Scotland	Bor'dale Burn	1898–99	Simpson and Wilson	1	127.5	22.5	4.0				4.0				Ry.	B. Feb. 9, 1899, p. 85	38
						2	20.0												
39	Coulouvrenière	Geneva, Switzerland	Rhone R.	1895	Butticaz	2	131.2	18.2	3.0	3.0	*C*	127.3	3.0	.024			H.W.	Y. 1898	39
40	Big Muddy	Near Grand Tower, Ill., U.S.A.	Big Muddy R.	1901–03	Parkhurst	3	140.0	30.0	7.0	†20.0	*E*	167.0	7.0	.042	50.6		Ry.	B. Nov. 12, 1903, p. 423	40
41	Inzigkofen	Inzigkofen, Wurtemberg	Danube R.	1896	Leibbrand	1	141.0	14.4	2.3	2.6			2.3		†12.5			B. April 22, 1897	41
42	Vauxhall	London, England	Thames R.	1899	Binnie	1	144.6	†18.6	3.9	3.9			3.9						42
						2	130.6	†20.0	3.9	3.9			3.9		†84.0		H.W.	N. Feb. 25, 1899	
43	Conn. Ave. Br.	Washington, D. C., U.S.A.	Rock Creek	1889–1906	Morison & Biddle	5	150.0	75.0	5.0		C_2	75.0	5.0	.067	52.0	20.0	H.W.	N. July 8, 1905, p. 30	43
					Douglas	2	82.0	41.0	3.3			41.0	3.3	.080				B. June 1, 1905	
44		Munderkingen, Wurtemberg	Danube R.	1893	Leibbrand	1	164.0	16.4	3.3	3.6			3.3		†26.2		H.W.	G. 3 Tri., 1897, p. 356	44
45		Near Oviédo, Spain	Nalon R.	Proposed			165.0	18.8	3.7	3.7			3.7		†17.0		H.W.	B. Sept. 26, 1901	45
46		Neckarhausen, Germany	Neckar R.	1903†	Leibbrand	1	165.0	13.5	2.8	3.7			2.8		15.8		H.W.	Engineer, Dec. 30, 1904, p. 650	46
47		Ulm, Germany	Ry. Yards	1905†		1	215.0								†46.0		H.W.	B. March 15, 1906	47

* Maximum.

† About.

REMARKS.—34. Three lead "hinges." 35. Paris water-supply from Vanne. 36. Three lead "hinges." 38. Mallaig Ex. of W. Highland Ry. 39. Three "hinges." 40. Two tracks, Ill. Cent. Ry. 41. Three cast-iron "hinges." 42. Three "hinges." 44. Three steel "hinges." 45. Three "hinges." 46. Three cast-iron and steel "hinges." 47. Three "hinges;" centre to centre of hinges 187′.0; rise centre to centre of hinges 18′.7; cost $45,000.

TABLE II.—DATA FOR ABOUT 500 ARCH BRIDGES ARRANGED ACCORDING TO SPAN—(*Continued*).

REINFORCED CONCRETE ARCHES.

Number.	Name.	Place.	Over.	Date.	Engineer.	No. of Spans.
1		Elkhart, Ind., U.S.A.	La Rue H. W.	1903		1
2		Vulcanite, N. J., U.S.A.	Highway	1905–6	Osgood	1
3		Rock Rapids, Ia., U.S.A.	Ravine	1894	M. A. C. Co.	1
4	Ridgewood Ave.	Ridgewood, N. J., U.S.A.	Br. of Saddle C.	1897	M. A. C. Co.	1
5		Marion Co., Ind., U.S.A.		1899		1
6		Waldwick, N. J., U.S.A.	Stream	1898?	M. A. C. Co.	1
7		Mahwah, N. J., U.S.A.	Stream	1898	M. A. C. Co.	1
8		Crystal Lake, N. J., U.S.A.	Stream	1898	M. A. C. Co.	1
9		Delaware Co., Penn., U.S.A	Stream	1905?		1
10		Wayne Township, N. J., U.S.A.	Stream	1896	M. A. C. Co.	1
11	Linwood Ave.	Ridgewood, N. J., U.S.A.	Saddle R.	1895	M. A. C. Co.	1
12		W. Edwardsville, Kan., U.S.A.	Mission Creek	1904?	Walter	1
13		Indian Creek, Ill., U.S.A.	Indian Creek	1903	Smith	3
14		Ocnomowoc, Wis., U.S.A.	Lake	1899	Hall	1
15	S. Jefferson St.	Battle Creek, Mich., U.S.A.	Kalamazoo R.	1899	Reiseger	2
16	McKinley Arch	St. Louis, Mo., U.S.A.	Des Peres R.	1902	Phillips	1
17		Sorsogan, Philippines	Stream	1905	Stevens	1
18	San Miguel	Manila, Philippines	Estero S. Miguel	1905	White & Co.	1
19		Albion, Mich., U.S.A.		1898?	Keepers & Thacher	3
20	Como Park	St. Paul, Minn., U.S.A.	Rapid Transit Ry.	1904	Wilson	1
21	Como Park	St. Paul, Minn., U.S.A.	Rapid Transit Ry.	1904	Wilson	1
22	Mount St.	Atlantic Highlands, N. J., U.S.A.	Grand Ave.	1895–96	M. A. C. Co.	1
23	Salem St.	Carbondale, Penn., U.S.A.	Lackawanna R.	1896	M. A. C. Co.	1
24	Florida Keys Viad.	Florida Keys, U.S.A.	Salt Water	1905–		
25	Lamington Br.	Marysborough, Queensland	Mary R.	1896	Brady	11
26		Louisville, Ky., U.S.A.	Beargrass Creek	1897	Keepers & Thacher	1
27		Decatur Township, Ind., U.S.A.	Goose Creek		Nelson	1
28	Mich. Cent. Ry.	Detroit, Mich., U.S.A.	Southern Bvd.	1895–6	Keepers & Thacher	1
29		Hyde Park, N. Y., U.S.A.	Crum Elbow C.	1897	M. A. C. Co.	1
						1
30		Plainwell, Mich., U.S.A.	Kalamazoo R.	1903	Courtwright	7
31	Arch St.	Paterson, N. J., U.S.A.	Passaic R.	1903	Schwiers	3
32	Jackson St.	Newark, N. J., U.S.A.	Jackson St.	1904	Osgood	1
33	Sixth Ave.	Carbondale, Penn., U.S.A.	Lackawanna R.	1896	M. A. C. Co.	
34	Goat Island	Niagara Falls, N. Y., U.S.A.	Niagara R.	1900–1	Buck (State) Waldo (Con.)	1
						2
35		Clifton, N. J., U.S.A.	Passaic R.	1903	Schwiers	5
36		Route Neutra, Hungary		1892		6
37		Route Nymphenburg, Wurtemberg				1
38	Castle Eichorn	Mähren, Austria	Ravine	1898	Venier	
39	Eighth Ave.	Carbondale, Penn., U.S.A.	Lackawanna R.	1896	M. A. C. Co.	
40	Franklin Br.	St. Louis, Mo., U.S.A.	Des Peres R.	1897–98	Dean	
41	Montgomery St.	Jersey City, N. J., U.S.A.	Street	1895–96	M. A. C. Co.	

Span.	Rise.	Thickness at Crown.	At Springing.	Curve.	Radius *R* at Crown.	Kind of Steel.	Per Cent Steel at the Crown.	Width, Face to Face at Crown	Thickness of Piers at Springing.	Class of Bridge.	Reference.	Number.
30.0	9.0	3.3	6.4			⅞″ J‡	.55			Ry.	B. July 14 1904	1
30.0	†9.0					¾″ O		32.5		Ry.	N. Sept. 9, 1905	2
30.0	6.6	0.5	2.5	3*C*	39.0		1.02	17.5		H.W.	X.	3
30.0	3.0	0.9	1.8				.80	26.0		H.W.	X.	4
32.0	3.2									H.W.	Cement, Sept. 1900	5
32.0	3.2	0.9	2.0				.85	24.0		H.W.	X.	6
32.0	3.2	0.9	2.0				.93	22.0		H.W.	X.	7
32.0	6.4	0.9	3.3				.80	26.0		H.W.	X.	8
33.0	16.0	0.5	0.9	*E*		O				Int'r.R.R.	N. Dec. 2, 1905	9
35.0	3.5	0.8	2.0				.82	25.0		H.W.	X.	10
40.0	8.0	1.0					.66	20.0		H.W.	X.	11
40.0	†20.0	2.2	10.0		21.0	⅞″ J	.70	38.0		Ry.	T. Dec. 8, 1905	12
40.0	20.0	2.5		*C*2	20.0	1¼″ J		32.3		Ry.	T. March 11, 1904	13
42.0	6.7	0.5						42.0		H.W.	B. Oct. 19, 1899	14
42.0								66.0		H.W.	T. Sept. 24, 1900	15
45.0				*E*		¾″ J		45.0		H.W.	B. June 11, 1903	16
45.0	6.0			*E*		Var.		†35.0		H.W.	N. Oct. 21, 1905	17
45.9				5*C*	43.0	+		34.0		H.W.	N. July 8, 1905	18
46.7	6.3							†28.0		H.W.	B. Sept. 21, 1899	19
50.0	12.5	0.8	2.5				†.75	†17.0		Foot-bri'e	N. Dec. 3, 1904	20
50.0	12.5	0.8	2.5				†.99	†17.0		Foot-bri'e	B. April 6, 1905	21
50.0	11.0	0.8	3.0	*C*	33.9		1.11	25.0		H.W.	N. August 22, 1896	22
50.0	8.3	0.8	2.8				1.25	55.0		H.W.	X.	23
50.0	25.0	2.0		*C*2	25.0	⅞″ J	.76	15.0		Ry.	B. Oct. 19, 1905	24
50.0	4.0	1.7	5.7				.71	22.7		H.W.	N. Nov. 17, 1900	25
											B. Feb. 14, 1901	
50.0	11.2	1.0	5.0	3*C*	41.7	3″×⅝″		†60.0		H.W.	K. & T. Blues	26
50.0										H.W.	Cement, Nov. 1901	27
50.3	9.5	1.5	7.5	*C*	38.2		3.75	109.9		Ry.	N. Sept. 28, 1895	28
											T. March 3, 1899	
53.0	7.5	0.8	2.5				1.23	17.0		H.W.	B. Nov. 10, '98; X.	29
26.0	7.5	0.7	1.7				1.00					
54.0	8.0							†24.0	6.0	H.W.	B. May 12, 1904	30
54.2	2.4	1.7	4.2				3.60	†45.5		H.W.	N. Sept. 10, 1904	31
54.3	10.5	2.5		*C*	48.4	1¼″ O	1.37	†32.0		Ry.	N. August 6, 1904	32
54.6	5.5	0.9	3.0				1.12	48.0		H.W.	X.	33
55.0	10.0						.63	†40.0	8.0	H.W.	B. Dec. 6, 1900	34
50.5	9.0						.66					
†55.0	3.0	2.0					2.40	†30.0		H.W.	N. Sept. 10, 1904	35
55.8	3.7	0.8						†19.7		H.W.	G. 1st Tri., 1904	36
56.7	5.9	1.0						†32.8		H.W.	G. 1st Tri., 1904	37
57.6	19.7	1.9	2.2	3*C*?	32.8			21.3		H.W.	Öst. Monat.Baud'st	38
58.7	6.0	1.0	3.2				1.02	49.0		H.W.	X.	39
60.0	15.5	0.9		3*C*	48.0		1.40	32.0		H.W.	X.	40
61.2	12.0	1.0	3.6				1.02	83.3		H.W.	X.	41

* Maximum.

† About. ‡ J=Johnson bars. O=Round bars.

REMARKS.—1. L. S. & M. S. Ry. 2. C. R. R. of N. J.; two tracks. 3. Six 4″ 7.5-lb I beams, 36″ centre to centre. 4. Nine 5″ 9.75-lb. I beams; 36″ centre to centre. 5. Melan type. 6. Nine 5″ 9.75-lb. I beams; 34″ centre to centre. 7. Nine 5″ 9.75-lb. I beams; 31″ centre to centre. 8. Nine 5″ 9.75-lb. I beams; 36″ centre to centre. 9. Skew. Phila. R. T. Co. 10. Nine 5″ 9.75-lb. I beams; 35″ centre to centre. 11. Seven 5″ 9.75-lb. I beams 36″ centre to centre. 12. U. P. Ry.; two tracks. 13. C. I. & St. L. Ry., Short Line; two tracks. 14. Flat bars and expanded metal. 15. Melan type; twenty-one 6″ I beams. 17. Various sizes and shapes of bars. 19. Thacher type. 20. 5″ 9.75-lb. I beams; 38″ centre to centre (?). 21. Four angles, 2″×2″×¼″; 38″ centre to centre (?). 22. Eight 6″ 12.25-lb. I beams; 36″ centre to centre. 23. Nineteen 7″ 15-lb. I beams; 36″ centre to centre. 24. Arches vary in span but all same type. 25. 41.25-lb. rails; 2′ centre to centre. 27. Melan type. 28. Fcu, 4″×4″×½″ angles; 22½″ centre to centre. 29. Five 7″ 15-lb. I beams, 36″ centre to centre; fivr 5″ 9.75-lb. I beams; 36″ centre to centre. 30. 4″–6″ channels; 1.9′ centre to centre. 32. Ce R. R. of N. J., two tracks. 33. Sixteen 7″ 15-lb. I beams; 36″ centre to centre. 34. Thacher type. 35. See No. 31. 36. Wünch type. 37. Monier type. 38. Melan type; four I beams. 39. Sixteen 7″ 15-lb. I beams, 36″ centre to centre. 40. Eleven 8″ 18-lb. I beams; 36″ centre to centre. 41. Twenty-one 7″ 15-lb. I beams 36″ centre to centre; two elevated tracks.

TABLE II.—DATA FOR ABOUT 500 ARCH BRIDGES ARRANGED ACCORDING TO SPAN—(*Continued*).

REINFORCED CONCRETE ARCHES.

Number.	Name.	Place.	Over.	Date.	Engineer.	No. of Spans.
42		Troy, N. Y., U.S.A.	Wynant's Kill	1897	Kenney	1
43		Route Ebhausen, Wurtemberg		1891		1
44		Vigneux, France		1900		1
45		Italy	Dora R.	1902		2
46	Herkimer Viad.	Herkimer, N. Y., U.S.A.	W. Canada Crk.	1902–3	Osborn E. Co.	3
						7
47		Jacksonville, Fla., U.S.A.	McCoy's C. & R.R.	1903–4		11
48		Auch, France	Gers R.	1899		1
49		Military Road, San Juan, Ponce, Porto Rico	Guayo R.	1900–01	Thacher	3
50	Bloomfield Ave.	Newark, N. J., U.S.A.	Park drive	1904	Reynolds	1
51		Cincinnati, Ohio, U.S.A.	Park drive	1894–95	M. A. C. Co.	1
52		Trinidad, Col., U.S.A.	Purgatorie R.	1905	Hibbard	2
53		Copenhagen, Denmark	Railway	1879		1
54		Route Painpardu, Belgium		1899		1
55		La Salle, Ill., U.S.A.	Gorge	1905	Strauss	1
56	Cedar R.	Waterloo, Ia., U.S.A.	Cedar R.	1902–3	Z	7
57	Meridian St.	Indianapolis, Ind., U.S.A.	Fall Creek	1900	Jeup	3
58	Illinois St.	Indianapolis, Ind., U.S.A.	Fall Creek	1900	Jeup	3
59	Wealthy Ave.	Grand Rapids, Mich., U.S.A.	Grand R.	190	Anderson	1
60		Wabash, Ind., U.S.A.	Creek	1905		2
61	Hamilton St.	Hartford, Conn., U.S.A.	Park R.	1898	M. A. C. Co.	1
62		Hyde Park, N. Y., U.S.A.	Crum Elbow C.	1897	M. A. C. Co.	1
63		Polasky, Cal., U.S.A.	S. Joaquin R.	1905	Leonard	10
64		Route Bade, Austria		1900		1
65	Rock Creek	Washington, D. C., U.S.A.	Rock Creek	1901–	Beach	
66	Soissons	Soissons, France	L'Aisne	1902	Riboud	1
						1
						1
67		Halder	Lenne R.	1904		1
						2
68	De l'Empéreur	Sarajero, Bosnie		1897		1
69	Fabriano Viad.	Italy		1905		2
						2
70		Rt. Payerbach, Austria		1900		1
71	Seeley St	Brooklyn, N. Y., U.S.A.	Prospect Ave.	1903–4	Foot	1
72		Austria	Bialka R.?	1894		1
73		Gr'd Rapids, Mich., U.S.A	Grand R.	1903–4	Anderson	1
						2
						2
74	Main St.	Dayton, Ohio, U.S.A.	Great Miami R.	1902–3	Turner	7
75	West St.	Paterson, N. J., U.S.A.	Passaic R.	1897–8	M. A. C. Co.	1
						2
76		Yorktown, Ind., U.S.A.	Stream	1905?	Luten	
77		Papigus, Italy	Nera R.			
78	N. Sixth Ave.	Des Moines, Ia., U.S.A.	Des Moines R.	1901–2?	Z	3

Span.	Rise.	Thickness at Crown.	At Springing.	Curve.	Radius R at Crown.	Kind of Steel.	Per Cent Steel at the Crown.	Width, Face to Face at Crown.	Thickness of Piers at Springing.	Class of Bridge.	Reference.	Number.
65.0	8.5	1.0					1.23	27.0		H.W.	X.	42
65.6	8.2	0.7						†13.1		H.W.	G. 1st Tri., 1904	43
65.6	14.8	1.6						†13.1		H.W.	G. 1st Tri., 1904	44
65.6	6.6							†46.6		H.W.	G. 1st Tri., 1904	45
66.0	14.0	1.8	4.5		46.5	1¼" T‡	.96		8.0	E. Ry.	N. Feb. 22, 1903, p. 240	46
62.0	12.0	1.8	4.5		46.0		.96					
*68.0	7.0	1.5						52.0	†7.0	H.W. & E. Ry.	T. July 3, 1903, p. 428	47
68.9	6.6	1.0								H.W.	G. 1st Tri., 1904	48
70.0	2–7.0							20.0		H.W.	N. Aug. 3, 1901, p. 98	49
	1–7.5											
70.0	8.5							†65.0		H.W.	N. Aug. 12, 1905, p.	50
70.0	10.0	1.3	4.0	C	106.3		†.80	32.5		H.W.	M. A. C. Co., B. Oct. 3, 1895	51
70.0	7.0	1.2	3.0	3C		1⅛" T	2.67	†65.0	7.0	H.W.	N. Feb. 10, 1906	52
71.7	8.5	0.9	1.2	3C	77.2					Foot-bri'e	B. July 21, 1898	53
71.8	9.2	1.3						†39.4		H.W.	G. 1st Tri., 1904	54
72.0	7.5	†2.0	†2.0			1×1" Ts		†5.0		Foot-bri'e	B. Sept. 21, 1905	55
72.0	7.2	1.2	2.7			2½"×⅝"		†46.0		H.W.	N. Feb. 13, 1904	56
74.0	9.5	1.3	1.8	3C			1.3	†70.0	8.0	H.W.	B. April 11, 1901	57
74.0	9.5	1.3	1.8	3C			1.3	†60.0	8.0	H.W.	B. April 11, 1901	58
75.0	14.0			3C				†39.0		H.W.	B. Mar. 22, 1906, p. 321	59
75.0	18.0	1.5	3.3	Par		1¼"×1¾" 1"×3"		†32.0		H.W.	B. Mar. 15, 1906 N. Dec. 2, 1905	60
75.0	7.5	1.3	4.5				1.17	49.3		H.W.	X.	61
75.0	14.7	1.3	†1.9	5C	97.5		.69	20.0		H.W.	X.	62
75.0	†11.0	1.5			62.5	¾" J	1.12	19.5	5.0	H.W.	N. Feb. 24, 1906	63
77.4	7.7							†39.4		H.W.	G. 1st Tri., 1904	64
80.0	15.0	1.5					.90	27.0		H.W.	B. Aug. 14, 1902	65
80.7	7.9	1.0	9.0			O		45.9	4.9	H.W. & Ry.	G. 1st Tri., 1904 G. 3d Tri., 1903, p. 47	66
80.3	8.2	1.0	9.0									
79.6	7.9	1.0	9.0									
82.0		1.6	2.2							H.W.	G. 4th Tri., 1905, p. 295	67
60.7		1.6	1.6									
83.2	8.3	1.0	†9.0					†37.6		H.W.	G. 1st Tri., 1904	68
84.9	26.9	2.0	3.3					†32.0		H.W.	N. Dec. 9, 1905, p. 645	69
30.2	13.1											
85.3	5.9	1.5	4.8					†18.0		H.W.	G. 1st Tri., 1904	70
85.3	8.5	†3.5	†10.0			1¼" J	†.76	53.1		H.W.	B. Dec. 31, 1903	71
86.3	20.6	1.1	†1.5							H.W.	G. 1st Tri., 1904	72
87.0						1¼" T		64.3	9.0	H.W.	B. Dec. 1, 1904, p. 489	73
83.0	8.0	1.6	2.8				1.16					
79.0	11.0	1.5	3.0				1.22					
*88.0								54.0	8.8	H.W.	B. May 19, 1904	74
89.0	9.5	1.3	5.5				1.37	50.8	8.5	H.W.	X.	75
88.5	9.5	1.3	5.5				1.37				B. March 16, 1899	
95.0	11.1					¾" O		†18.0			B. May 11, 1905	76
95.2		2.3	3.3								G. 4th Tri., 1905	77
100.0	27.7	1.8	4.0		68.8			42.7	10.0	H.W.	Cement, July, 1902	78
	23.0	1.9	4.2		79.2							
	20.0	1.9	4.3		93.6							

* Maximum.

† About. ‡ T = Thacher bars. O = Round bars. J = Johnson bars.

REMARKS.—42. Nine 8" 18-lb. I beams; 36" centre to centre. 43. Monier type. 44. Piketty type. 45. Hennebique type. 46. U. & M. V. Ry. Two tracks. 47. Melan ribs and Thacher bars. 48. Bonna type. 49. Thacher type. 50. Melan type. Two E. Ry. tracks. 51. Eleven 9" 21-lb. I beams; 36" centre to centre. 53. Five 18.8 lb. (per foot) rails. 54. Hennebique type. 55. Two ribs. In Deer Park. 56. Thacher type. 57. 10" 25-lb. I beams; 36" centre to centre. 58. 10" 25-lb. I beams; 36" centre to centre. 60. Kahn bars. 61. Seventeen 9" 21-lb. I beams; 36" centre to centre. 62. Seven 9" 18-lb. I beams. 63. Spandrel wall tied to ring. 64. Hennebique type. 65. Four 3"×3"×6-lb. angles; 33" centre to centre. 66. Hennebique type. 67. Large arch has three "hinges." 68. Wünch type. 69. Total length = 354'. 70. Melan type. 71. Skew. 72. Monier type. 74. Four angles, 2½"×2½"×$\frac{5}{16}$". 36" centre to centre. 75. Seventeen 10" 25-lb. I beams. 35¼" centre to centre. 77. Five ribs; four angles latticed. 78. Melan type; four angles latticed.

TABLE II.—DATA FOR ABOUT 500 ARCH BRIDGES ARRANGED ACCORDING TO SPAN—(*Continued*).

REINFORCED CONCRETE ARCHES.

Number.	Name.	Place.	Over.	Date.	Engineer.	No. of Spans.	Span.	Rise.	Thickness at Crown.	At Springing.	Curve.	Radius R at Crown.	Kind of Steel.	Per Cent Steel at the Crown.	Width, Face to Face at Crown.	Thickness of Piers at Springing.	Class of Bridge.	Reference.	Number.
79	Icy Glen	Stockbridge, Mass., U.S.A.	Housatonic R.	1895	M. A. C. Co.	1	100.0	10.0	0.8	2.5				2.30	7.5		Foot-bri'e	X. B. Nov. 7, '95	79
80	François-Joseph	Buda-Pesth(?), Austria-Hungary	Danube R.?	1900		1	108.2	14.4	1.6	†2.2					†45.9		H.W.	G. 1st Tri., 1904	80
81		Laibach, Austria	Laibach R.	1900–1	Melan	1	108.3	14.6	1.7			123.0			†50.0		H.W.	B. July 16, 1903	81
82	Green Island	Niagara F'lls, N.Y., U.S.A.	Niagara R.	1900–1	Buck (Con.) Waldo (State)	1	110.0	11.5	3.2	5.9			6"×¾"	.63	40.0	13.5	H.W.	B. Dec. 6, 1900 N. Feb. 16, 1901, p. 146	82
						2	103.5	10.0	3.3	6.3				.66					
83	Third Street	Dayton, Ohio, U.S.A.	Great Miami R.	1904–	Turner	1	110.0	14.3							†62.0				
						2	100.0	13.3	2.1					.69		11.0	H.W.	T. Mar. 4, 1904, p. 154	83
						2	90.0	11.3	2.1					.69		10.0			
						2	80.0	9.7	1.6					.91		9.0			
84	Wayne St.	Peru, Ind., U.S.A.	Wabash R.	1905	Luten	1	100								32?		H.W.	B. Mar. 29, 1906, p. 347	84
						2	95						¾ O‡						
						2	85												
						2	75												
85		Portugal	Pena R.	1901		5	114.8	14.4							11.8		Tramway	G. 1st Tri., 1904	85
86	Lake Park	Milwaukee, Wis., U.S.A.	Ravine	1905	Turneaure		118.0	18.0	5.0	5.0			"×3"K		14.0		H.W.	N. Nov. 25, 1905	86
87		Yellowstone Nat. Park, U.S.A.	Yellowstone R.	1903	Chittenden	1	120.0	15.0	2.0	4.0				†1.39	17.5		H.W.	B. Jan. 14, 1904, p. 25	87
88	Jacaquas R.	Military Road, San Juan-Ponce, Porto Rico	Jacaquas R.	1900–1	Jackson	1	120.0	12.0	2.3		3C	226.0	4"×¾"	.63	20.0	12.0	H.W.	N. Aug. 3, 1901 B. Aug. 1, 1901, p. 66	88
						2	100.0	11.3	1.9			167.2		.80					
89	Washington Ave., So.	Lansing, Mich., U.S.A.	Grand R.	1902	Collar	1	120.0								54.0		H.W.	Cement, Mar. 1902	89
90	Y-Bridge	Zanesville, Ohio, U.S.A.	Muskingum R.	1900–2	Landor	4	122.0	14.5 11.5	2.5				3"& 5" ×¾"		43.0		H.W. El. Ry.	N. Mar. 1, 1902, p. 194	90
						3	81.0	14.5 10.9	1.5										
						1	99.0	6.3											
91		Route Wildegg, Switz		1890		1	122.0	11.4	0.6	†0.8					†12.8		H.W.	G. 1st Tri., 1904	91
92	Kansas Ave.	Topeka, Kan., U.S.A.	Kansas R.	1896–98	M. A. C. Co.	1	125.0	18.9	1.8	6.0				1.58	36.0		H.W.	M. A. C. Co. B. April 2, 1896	92
						2	110.0	16.3	1.8	6.0				1.58					
						2	97.5	14.6	1.7	5.0				1.73					
93	Park Ave.	Newark, N. J., U.S.A.	Park	1905	Reynolds	1	132.0	16.2							†72.0		H.W.	N. Aug. 12, 1905	93
94	Schwimmschul-brücke	Steyr	Stream	1897†			138.4	9.4	2.0	2.3					19.7		H.W.	Z. Oe. Ing. u. Arch. Ver., Dec. 23, '98	94
95		Playa-del-Rey, Cal., U.S.A.		1906	De Palo	1	146.0	18.0	2.0				4 angles					N. Mar. 31, 1906	95
96		Route Waidhofen, Austria				1	144.3										H.W.	G. 1st Tri., 1904	96
97	St. Pierre Hollow	Schenley Park, Pittsburg, Penn., U.S.A.	St. Pierre Hollow	Proposed	Keepers and Thacher	1	150.0	29.8	2.0	8.0				.94	84.0		H.W.	K. & T. Blues	97
98		Chatellerault, France	Vienne R.	1900		1	164.0	15.8	1.8	†3.0							H.W.	G. 1st Tri., 1904	98
						2	131.2	13.2	1.4	†2.6					†19.0				
99		Route Bormida, Italy		1902		1	167.3	16.7	1.9								H.W.	B. April 10, 1902	99
100		Decize, France	Loire R.			2	183.7	15.3	1.6						34.4		H.W.	G. 4th Tri., 1905	100
101	Gruenwald	Munich, Bavaria	Isar R.	1904	Mörsch	2	230.0	42.0	2.5	2.5			1.1"O	.19	26.3		H.W.	B. Feb. 20, 1905	101

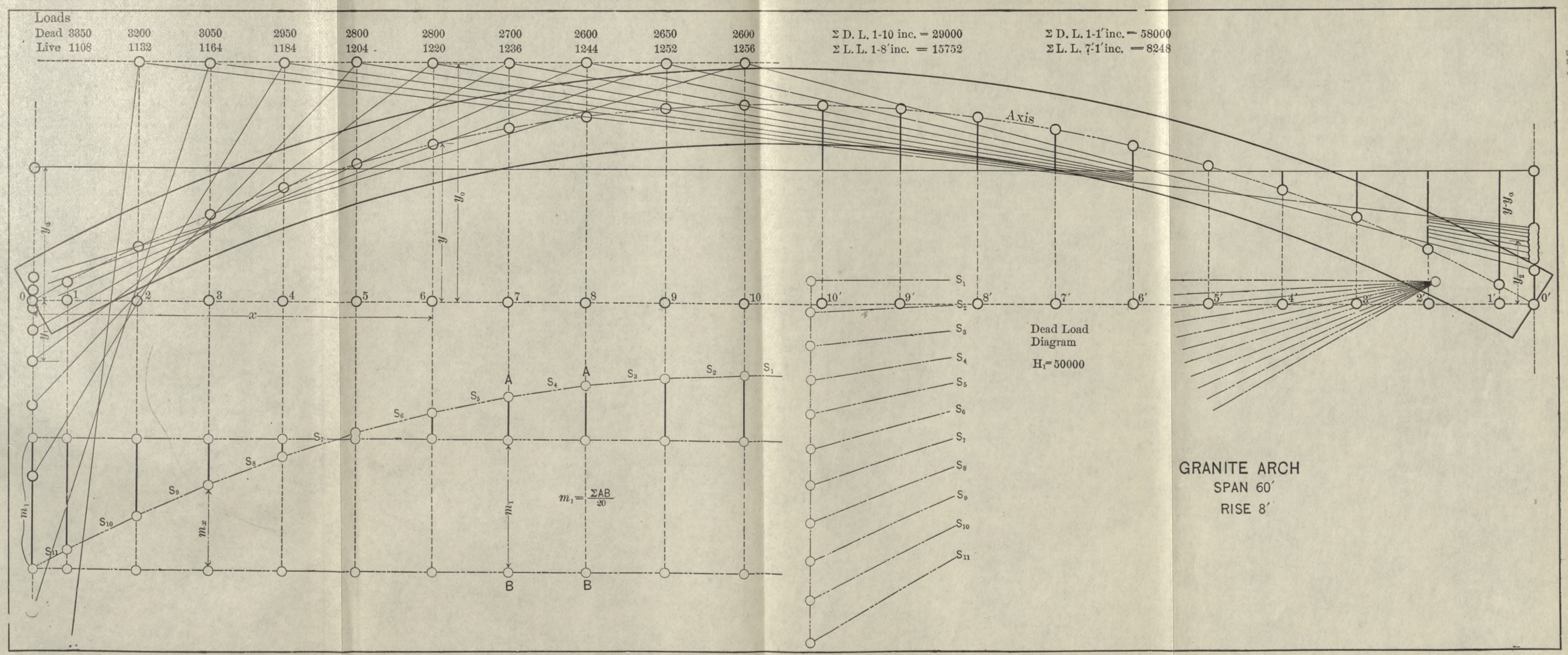
* Maximum.

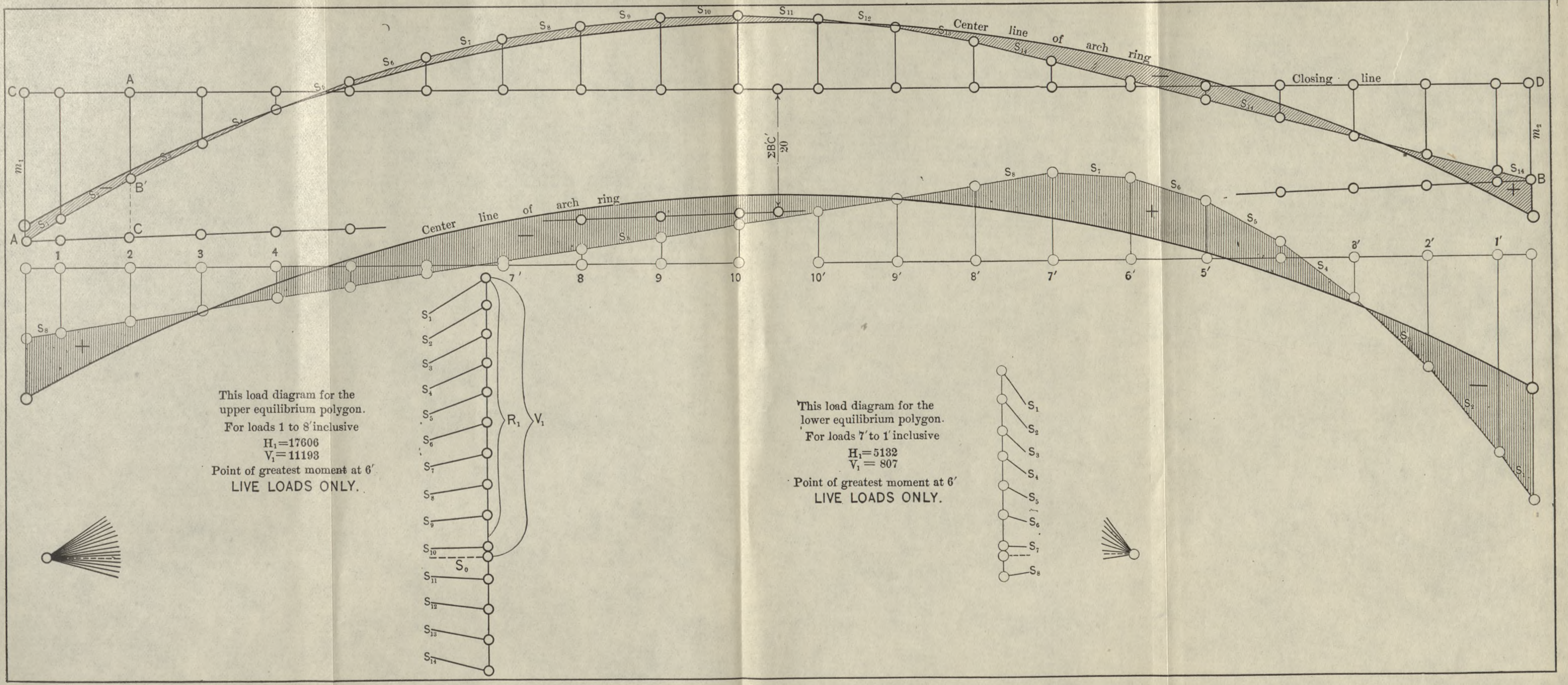
† About. ‡ O = Round bars. K = Khan bars.

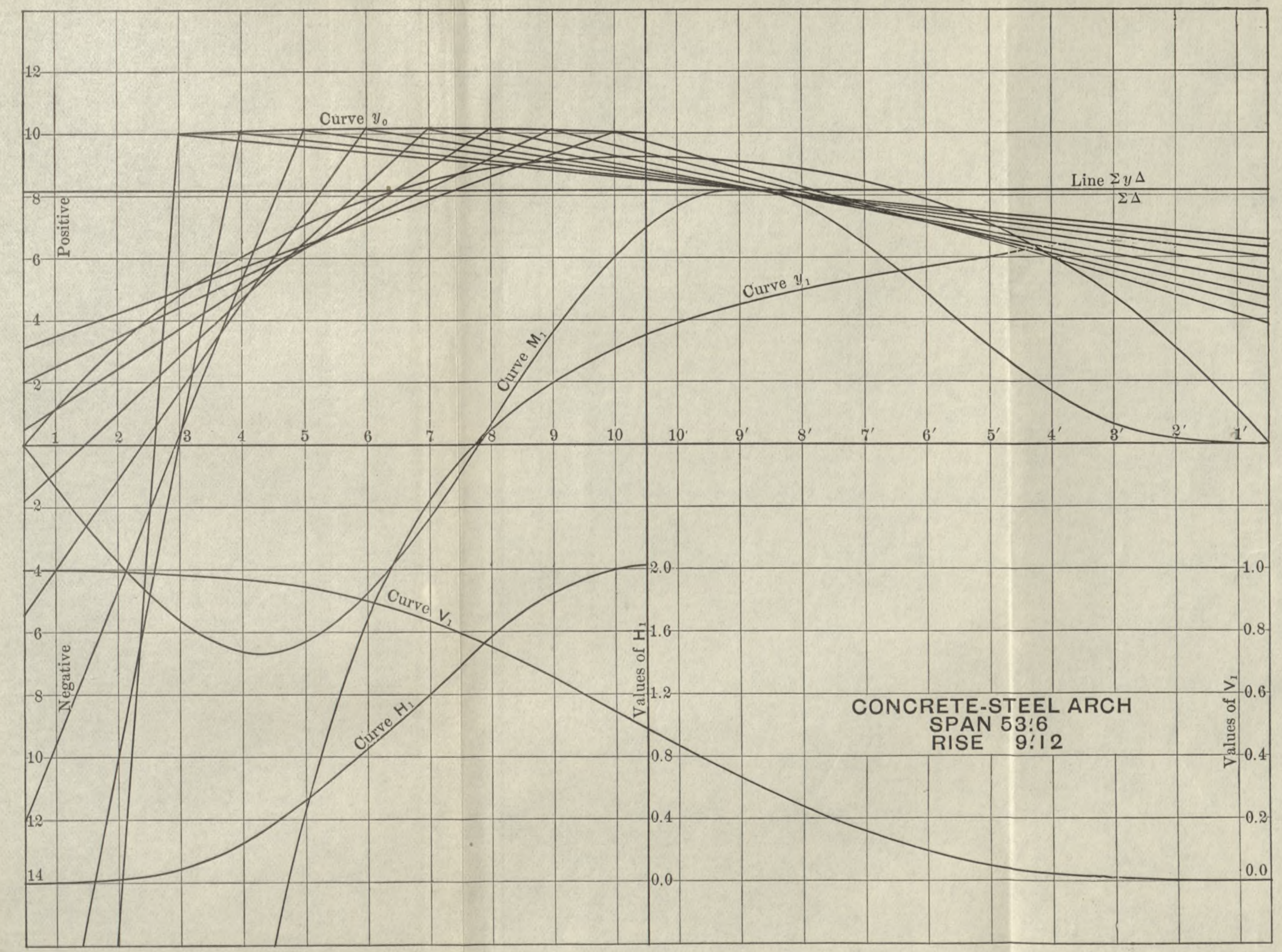
REMARKS.—79. Four 7" 15-lb. I beams; 28" centre to centre. 80. Three "hinges." Lattice ribs. 81. Three "hinges." Fourteen lattice ribs. 83. Four angles, 2½"×2½"×5/16". 34" centre to centre. 85. Hennebique type. 87. Four 2½"×3"×½" angles; 30" centre to centre. 88. Thacher type. 89. Melan type. 90. Thacher type. In plan, Y-shaped. 91. Monier type. 92. Four angles, 3"×3"×7.2 lbs. Twelve ribs. 93. Four angles 3"×3"×½"; 36" centre to centre. Thacher bars. 94. Melan type. 95. Foot-bridge 96. Monier type. 97. Twenty-seven ribs. Four angles, 3"×3"×7.2 lbs. 98. Hennebique type. 101. Three steel "hinges."

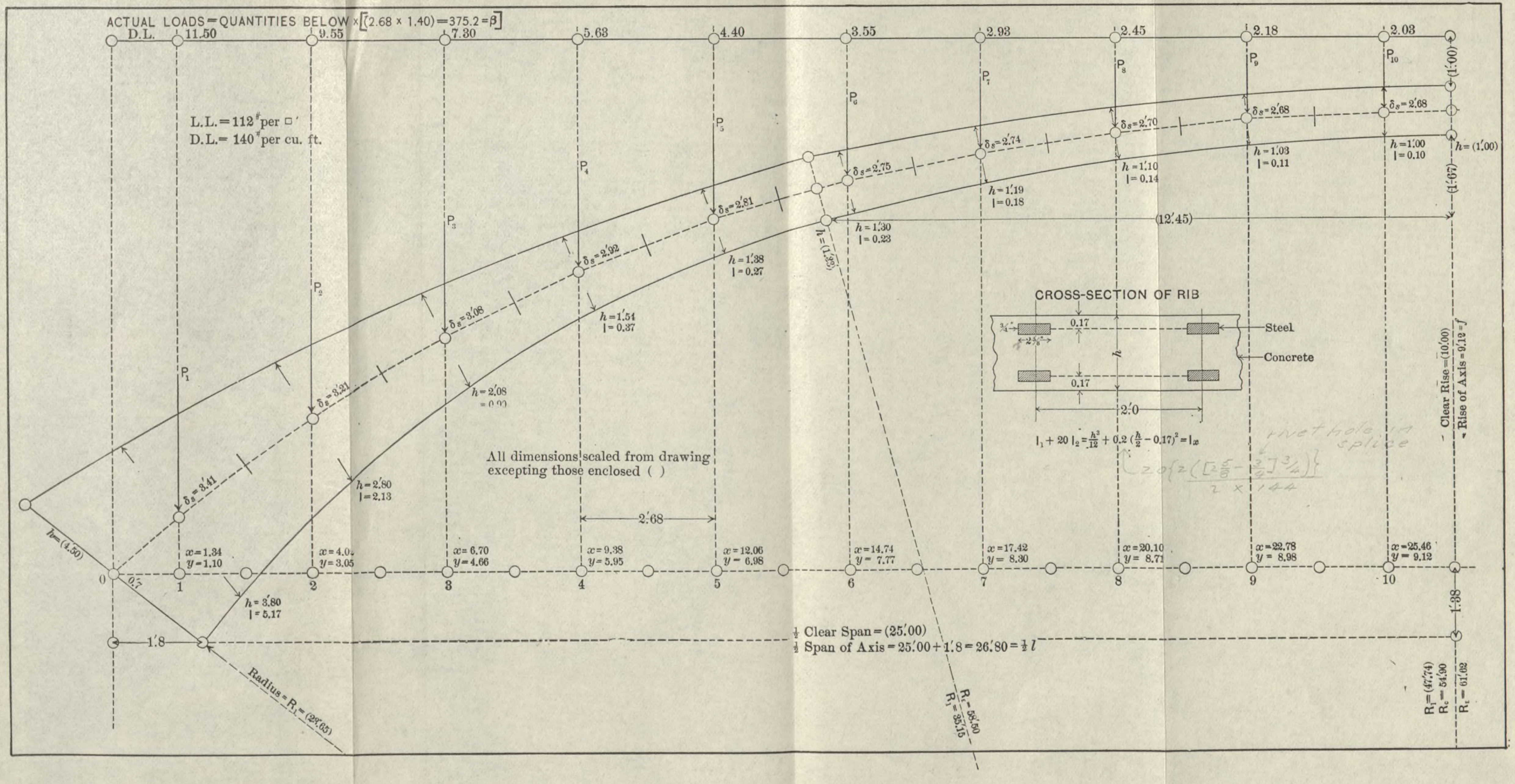
KEY TO REFERENCES.

Symbol.	Name of Periodical or Book, etc.	Address.
A	Civil Engineers' and Architects' Journal . . .	
B	Engineering News and Am. Ry. Journal. . .	New York City, U. S. A.
C	Weale's Bridges.	
D	Penn. Ry. Co.'s Blues.	Philadelphia, Pa., U. S. A.
E	Wm. H. Brown, Ch. Eng'r Penn. Ry. Co. . .	Philadelphia, Pa., U. S. A.
F	Construction des Viaducs, Tony Fontenay. .	Paris, France.
G	Annales des Ponts et Chausses.	Paris, France.
H	Mahan's Civil Engineering.	John Wiley & Sons, New York City, U. S. A.
I	Masonry Construction by Baker.	John Wiley & Sons, New York City, U. S. A.
J	Spon's Dictionary of Engineering.	
K	Engineering.	London, England.
L	Edinburgh Encyclopædia, 9th ed	
M	Scientific American Supplement.	New York City, U. S. A.
N	Engineering Record.	New York City, U. S. A.
O	Engineering Magazine.	New York City, U. S. A.
P	Journal of the Association of Engineering Societie`.	
Q	Encyclopædia Britannica, 9th ed.	
R	Railway and Engineering Journal.	New York City, U. S. A.
S	Cresy' Bridges.	John Wiley & Sons, New York City, U. S. A.
T	Railway Gazette.	New York City, U. S. A.
U	Murray's Handbook of Northern Italy. . . .	
V	Le Genie Civil.	Paris, France.
W	Messrs. Keepers & Thacher	Paterson, N. J., U. S. A.
X	The Melan Arch Construction Co.	New York City, U. S. A.
Y	Transactions of Am. Soc. C. E.	New York City, U. S. A.
Z	Concrete-steel Engineering Co.	New York City, U. S. A.


Loads
Dead 3350 3200 3050 2950 2800 2800 2700 2600 2650 2600
Live 1108 1132 1164 1184 1204 1220 1236 1244 1252 1256
Σ D. L. 1-10 inc. = 29000
Σ L. L. 1-8′ inc. = 15752
Σ D. L. 1-1′ inc. = 58000
Σ L. L. 7′-1′ inc. = 8248
Axis
Dead Load Diagram
H₁ = 50000
GRANITE ARCH
SPAN 60′
RISE 8′


Center line of arch ring
Closing line
Center line of arch ring
This load diagram for the upper equilibrium polygon.
For loads 1 to 8′ inclusive
H_1=17606
V_1=11193
Point of greatest moment at 6′
LIVE LOADS ONLY.
This load diagram for the lower equilibrium polygon.
For loads 7′ to 1′ inclusive
H_1=5132
V_1= 807
Point of greatest moment at 6′
LIVE LOADS ONLY.


Curve y_0
Line $\frac{\Sigma y \Delta}{\Sigma \Delta}$
Curve y_1
Curve M_1
Curve V_1
Curve H_1
Positive
Negative
Values of H_1
Values of V_1
CONCRETE-STEEL ARCH
SPAN 53:6
RISE 9:12


ACTUAL LOADS = QUANTITIES BELOW × [(2.68 × 1.40) = 375.2 = β]
D.L.
11.50
9.55
7.30
5.63
4.40
3.55
2.93
2.45
2.18
2.03
L.L. = 112# per □'
D.L. = 140# per cu. ft.
CROSS-SECTION OF RIB
Steel
Concrete
All dimensions scaled from drawing excepting those enclosed ( )
½ Clear Span = (25.00)
½ Span of Axis = 25.00 + 1.8 = 26.80 = ½ l
Radius = R₁ = (28.05)
Clear Rise = (10.00)
Rise of Axis = 9.12 = f
(12.45)

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