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Finite element analysis of reinforced concrete elements subjected to torsion

Analiza skręcania elementów żelbetowych metodą elementów skończonych

Abstract

This paper presents FEM techniques used for modelling concrete elements subjected to torsion. It compares the results from a 3D numerical analysis and a numerical homogenization method analysis. Finally, the results are compared to the reported experimental data.

Keywords: beam, torsion, homogenisation, FEM

Streszczenie

W artykule przedstawiono sposoby modelowania skręcanych elementów żelbetowych za pomocą Metody Elementów Skończonych. Zaprezentowano porównanie wyników otrzymanych w trójwymiarowej analizie numerycznej oraz w analizie numerycznej bazującej na teorii homogenizacji. Wyniki zestawiono z wynikami eksperymentalnymi znanymi z literatury.

Słowa kluczowe: belka, skręcanie, homogenizacja, MES

1. Introduction

In a basic case, reinforced concrete structures are subjected simultaneously to axial forces (N), shear forces (V), bending moments (M) and twisting moments (T). In engineering practice most of these states can be analysed independently, or by taking into account the effect of a secondary value on the principal value, such as considering the effect of shearing in bending analyses, or in an interactive manner, as is the case in analysing eccentrically compressed sections or in analysing the combined effect of torsion and compression or the effect of torsion and shear on the cross-section capacity. In common practice a maximum of two internal forces are considered in such analyses.

In the design of sections subjected to combined bending and torsion, the strength analysis is carried out separately for bending and torsion, as if the cross-section was subjected to pure bending or pure torsion at one time. The procedures prescribed by the standards do not address the combined effect of bending and torsion and assign the effect of bending to the longitudinal reinforcement and the effect of torsion to the transverse reinforcement on exclusive basis.

Although very practical, this approach, which separates the bending and torsion effects does not represent the actual behaviour of beam elements in reinforced concrete structures. This concerns, in particular, the outermost beams, such as spandrel beams in column-and-slab structures for which the torsional moment has a considerable effect.

Experimental studies of elements subjected to combined bending and torsion showed that with an increasing share of the bending moment in relation to the torsional moment, bending starts to govern the behaviour of the reinforced concrete element under loading. Bending has a predominant effect not only on the strength and deflections but also on the stress and strain state of the element. The situation is not so clear in the case of increasing the share of torsion and, as a result, it is common practice to design additional reinforcement to resist the entire torsion with no participation of the designed bending reinforcement.

However, before examining the interaction of two or more internal forces, it is important to investigate the behaviour of a section subjected to pure torsion so as to eliminate misrepresentation of results due to the effect of bending.

The following part of this paper presents a numerical approach to the problem of pure torsion using non-linear models of concrete (Concrete Plastic Damage and Menétrey plasticity model with softening behaviour). The results obtained in this way are compared with the experimental data and with the provisions of the relevant standards.

2. Numerical analysis

Two numerical modelling approaches are compared in this paper. The behaviour of the reinforced concrete beam subjected to torsion in the experimental study whose results are reported in [3] is represented in each case. The original dimensions were given in Imperial units. They have been converted to SI and rounded. The rounding takes into account the average dimensions of all elements subjected to the experimental test and does not affect the results of numerical calculations.

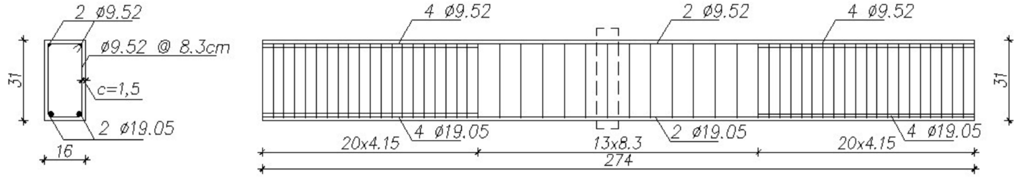

Fig. 1. Schematic representation of beam No. B11 – longitudinal and transverse sections

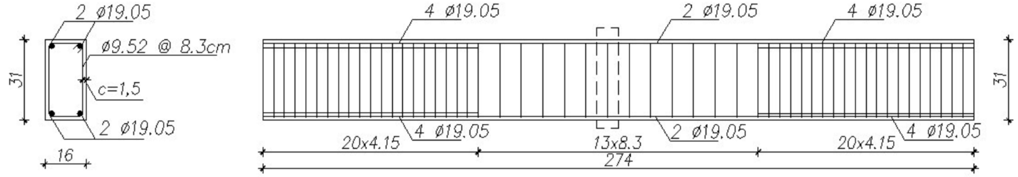

Fig. 2. Schematic representation of beam No. B21 – longitudinal and transverse sections

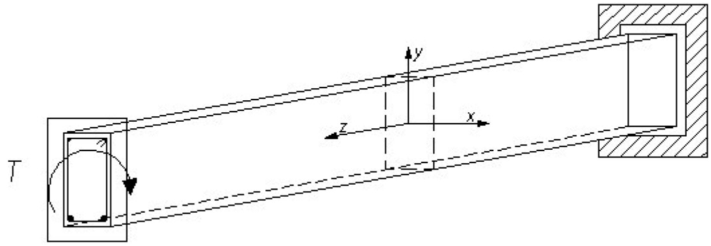

Fig. 3. Boundary conditions

In the first approach, a numerical model is created to represent all the geometric parameters of the beam under analysis, including the boundary conditions and torsion is applied by force control, i.e. the application of a pair of forces. In the second approach, the model is created for a section of a beam subjected to torque without restraint, i.e. with the warping free boundary conditions, and the effect of torsion is obtained by deformation-controlled action.

The ZSoil®.PC v2018 software was used in the analysis, with the Concrete Plastic Damage constitutive model described in [5] and, for comparison, the Menétrey plasticity model with softening behaviour given in [4].

The first approach seems the obvious choice and is available to any user of advanced FEM-based programs. On the other hand, for obvious reasons it is also non-economical due to the labour-intensive preparation of data and time-consuming calculations and processing of results.

In the second approach, representation of the beam behaviour is limited to the middle portion where the torsional moment is constant. The analysed portion of the beam has a length equal to the distance between the stirrups and includes a centrally located stirrup. Periodic boundary conditions are imposed on the cross-sectional surfaces to enforce the same nodal displacement in the *x*, *y* and z directions. Moreover, six independent boundary conditions are imposed in the analysed beam section to constrain translation and rotation. The torsion effect is obtained by applying a macro-strain field in *x*-*y* and *x*-*z* directions.

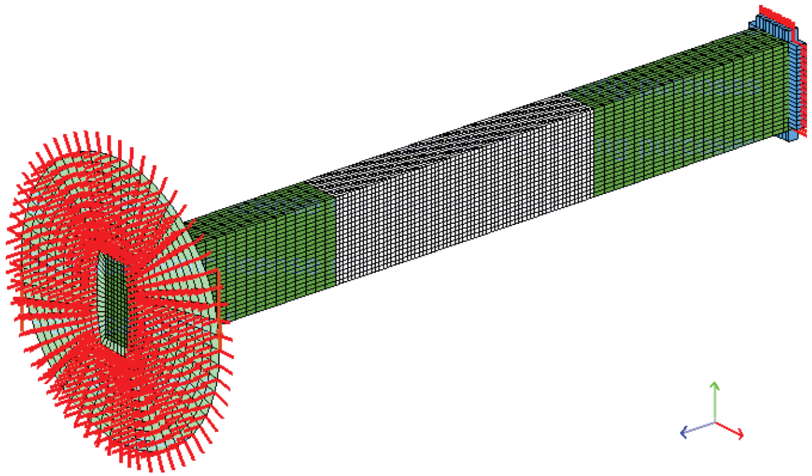

Fig. 4. 3D model of beam in ZSoil

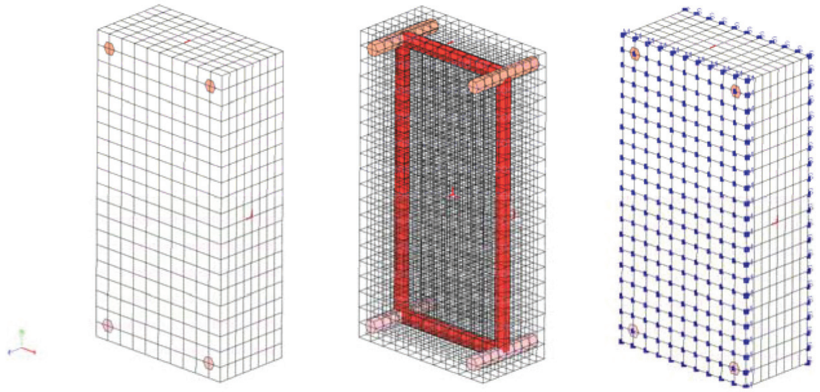

Fig. 5. Beam model obtained with the homogenisation approach – a continuum model, system of bars, periodic boundary conditions

The boundary conditions no longer play a role in this approach and the computation time is reduced.

In numerical analyses of objects, as in the first approach, considerable perturbations can occur at the shear zones due to stress concentrations. In order to avoid their dominating effect a number of measures must be implemented, including, without limitation, use of different material properties or different material models. In the analysed case, the measures used at supports include application of an elastic constitutive model for concrete, an increased amount of longitudinal reinforcement, and smaller distances between stirrups (as described in [3]). However, perturbations were still found at the joint between continuous elements made of Elastic and Concrete Plastic Damage materials, waning away no sooner than about the beam midspan. Application of load was yet another challenge. Thus, in order to avoid the concentrated load effect the load was applied through a membrane. The natural consequence of this approach is the long time required for creating the model and for carrying out the calculations.

The second approach is based on homogenisation theory. Similarly to the homogenisation theory described in [6], it considers a 3D element that is loaded, in the general case, with all the six internal force components $\mathbf{\Sigma}=\left\{N, M_x, M_y, M_z, Q_x=0, Q_y=0\right\}^T$, (i.e. longitudinal force, two bending moment components, torsional moment and two transverse force components) generating generalised strains (macro-strains) that describe the kinematics $\mathbf{E}=\left\{E_0, K_x, K_y, \Phi_z, \Gamma_x, \Gamma_y\right\}^T$. In the analysed case, in which the analysed element is subjected solely to pure torsion, the vector of strain control macro-strains $\mathbf{E}$ takes the form $\mathbf{E}=\left\{0,0,0,\Phi_z,0,0\right\}^T$. In the analysed linear element a repeatable 3D unit (periodic cell) is distinguished of finite length which in the numerical solution stage does not go to zero at the boundary, which differentiates the current solution from the cross sectional analysis of the beam presented in [7].

The total strain field comprises two parts: macro-strains used for deformation-controlled action and strains caused by perturbation of the displacement field $\mathbf{u}^p(\mathbf{x})$ on which the periodic boundary conditions are imposed.

$$\boldsymbol{\varepsilon}(\mathbf{x})=\boldsymbol{\varepsilon}^E(\mathbf{x})+\boldsymbol{\varepsilon}^p(\mathbf{x})=\mathbf{L}_E(\mathbf{x})\mathbf{E}+B\mathbf{u}^p(\mathbf{x}), \tag{1}$$

$$\mathbf{u}(\mathbf{x})=\mathbf{u}^E(\mathbf{x})+\mathbf{u}^p(\mathbf{x})=\mathbf{C}_E(\mathbf{x})\mathbf{E}+\mathbf{u}^p(\mathbf{x}). \tag{2}$$

where:

$$\boldsymbol{\varepsilon}^E=\begin{Bmatrix}\varepsilon_{xx}^E\\ \varepsilon_{yy}^E\\ \gamma_{xy}^E\\ \varepsilon_{zz}^E\\ \gamma_{xz}^E\\ \gamma_{yz}^E\end{Bmatrix}=\begin{bmatrix}0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\1&y&-x&0&0&0\\0&0&0&-y&1&0\\0&0&0&x&0&1\end{bmatrix}\cdot\begin{Bmatrix}0\\0\\0\\\Phi_z\\0\\0\end{Bmatrix}=\mathbf{L}_E(\mathbf{x})\mathbf{E}, \tag{3}$$

$$\mathbf{u}^E=\begin{bmatrix}0&0&\frac{1}{2}z^2&-yz&z&0\\0&-\frac{1}{2}z^2&0&xz&0&z\\z&yz&-xz&0&0&0\end{bmatrix}\cdot\begin{Bmatrix}0\\0\\0\\\Phi_z\\0\\0\end{Bmatrix}=\mathbf{C}_\mathbf{E}(\mathbf{x})\mathbf{E} \tag{4}$$

and B is the differential operator matrix obtained from the Cauchy-Riemann equations.

This approach is a particular case of application of the method described in [6].

In the considered case of pure torsion $\gamma_{xz}^{E} = -y\varphi_z$ and $\gamma_{yz}^{E} = x\varphi_z$.

In this method, the numerical analysis considers a completely consistent 3D stress/ strain state for which geometric and constitutive equations hold at any point. Control over the cross-sectional values is maintained. This approach enables cross-sectional analysis without the need to consider the effects of concentrated forces or boundary conditions.

It is sufficiently general to be applied to the analysis of any cross-sectional shape with longitudinal and transverse reinforcement if needed. Moreover, any constitutive model can be used to describe the mechanical behaviour of materials. Prestress and the effect of non-uniform strains caused by shrinkage or thermal effects can be introduced depending on the needs and capabilities of the constitutive model.

The three-dimensional approach with fully three-dimensional analysis of the strain and stress fields enables the use of realistic and sophisticated material models, such as those which describe damage at the micro-structure level, softening or plasticity. In such models one must not leave out certain components of the strain or stress field as is the case in the so-called engineer's methods based on elastic behaviour of the material used due to their simplicity and universality. Moreover, they must not be used for modelling the behaviour of structures composed of one-dimensional (1D) linear elements. In more advanced constitutive models, different couplings between the strain and stress field components can be observed, increasing the accuracy of representation of the actual behaviour of the analysed structure.

The above approach can be used with any FEM program that enables defining initial (imposed) strains and is provided with the periodic boundary conditions option.

3. Standard procedures

The standard procedures contained in Eurocode 2 [8] require a torsion check for reinforced concrete members when the structure's stability is defined by the torsional resistance of its members. However, as mentioned, there are no guidelines on including the interaction of bending with torsion and in when torsion results from the strain compatibility conditions (as in statically indeterminate structures) the provisions are limited to recommending the use of reinforcement for the crack width control (minimum longitudinal reinforcement, transverse reinforcement and additional bars over the beam height).

All the design procedures are based on the relationships in thin-walled box sections in which equilibrium is satisfied by closed shear flow. Since reinforced concrete members have, as a rule, a solid cross-section, in the design they are represented by thin-walled components.

As far as the torsional resistance of concrete members is concerned, EC-2 [8] distinguishes the torsional resistance identified with the torsional cracking moment $T_{Rd.c}$ limited by the stresses generated in the wall that exceed the tensile strength of the concrete $\tau_{t.i} = f_{ctd}$ and the torsional moment resistance $T_{Rd.\max}$ defined by the diagonal compression failure. This value depends on the freely chosen value of angle θ. Similarly to shear, the important parameter is *cot*θ which in Poland can take any value in the range between 1 and 2 (1 and 2.5 in Europe).

This gives 100% difference of capacity between the limit values. However, in most cases it is governed by the $T_{Rd.c}$ capacity. The required quantity of additional steel is determined from the condition of equilibrium of vertical forces in the section wall where the sum of forces caused by torsion and forces caused by the action of transverse forces is equal to the capacity of stirrups.

$$\frac{T_{Ed}}{2A_k} \cdot h_k + \alpha_v \cdot V_{Ed} = \frac{A_{swt} \cdot f_{yd}}{s} \cdot h_k \cdot cot\ \theta \tag{5}$$

where α_v – factor depending on the number of stirrup legs ($\alpha_v = 0.5$ and $\alpha_v = 0.25$ for two and four-legged stirrups respectively). The amount of reinforcing steel determined in this way depends directly on the value of $cot\theta$.

The following two interaction requirements are used to check the capacity of a section subjected to torsion:

$$\frac{T_{Ed}}{T_{Rd.\max}} + \frac{V_{Ed}}{V_{Rd.\max}} \leq 1 \tag{6}$$

$$\frac{T_{Ed}}{T_{Rd.c}} + \frac{V_{Ed}}{V_{Rd.c}} \leq 1 \tag{7}$$

which take into account the combined action of shear force and torsional moment.

In the case of more complex cross-sections, in particular if they are sensitive to deplanation, the standards prescribe strut-and-tie or beam-truss modelling.

The behaviour of a section subjected to pure torsion and in response to interaction taking into account all the relevant factors might be of interest in this context.

As shown, the standard procedures present a very simplified description of the problem and do not allow more complicated cases to be taken into account.

4. Comparison with the experimental data

In this section the results obtained from numerical modelling of the homogenized model with macro-strain control are compared with the experimental data obtained by McMullen and Warwaruk [3] for solid cross-sections subjected to torsion. The comparative analysis has been carried out for beams No. B11 and No. B21.

The strength of concrete, according to [3] was measured on 15 × 30 cm cylindrical specimens.

The graphs show the relationship between the unit torsion angle and torsional moment for a full-length beam model with force-controlled action and beam modelled on the basis of homogenization assumptions. In both cases, the Concrete Plastic Damage model was chosen, the parameters of which are given below.

Table 1. Material data of beams B11 and B21 based on [3]

Beam	f_{cm} **[MPa]**	f_{ctm} **[MPa]**	**Bottom reinforcement**		**Top reinforcement**		**Transverse reinforcement**	
			A_s **[cm²]**	f_y **[MPa]**	A_s **[cm²]**	f_y **[MPa]**	A_s **[cm²]**	f_y **[MPa]**
B11	35.78	3.41	5.70	323.4	1.42	365.4	1.42	379.2
B21	39.64	2.87	5.70	323.4	5.70	323.4	1.42	370.2

Table 2. Parameters for the Concrete Plastic Damage model of material

Parameter	**B11**	**B21**	**Description**
Compressive/ tensile strength			
f_c	27.78 MPa	31.64 MPa	f_{cm} – 8 MPa, according to EC-2
f_t	1.80 MPa	1.9 MPa	$f_{ctk0.05}$
E	22.5 GPa	22.5 GPa	taking into account the type of aggregate
ν	0.2	0.2	
f_{co}/f_c	0.4	0.4	initial uniaxial compressive strength
f_{cbo}/f_{co}	1.16	1.16	initial biaxial compressive strength
Damage in compression			
$\sigma_{c,D}/f_c$	0.95	0.95	activation stress level
$\bar{\sigma}_c/f_c$	1	1	ref. stress level for damage
D_c	0.55	0.55	damage at ref. stress level
G_c	3.33 kN/m	3.33 kN/m	fracture energy
Damage in tension			
$\tilde{\sigma}_t/f_t$	0.5	0.5	ref. stress level for damage
$\tilde{D}_t$	0.5	0.5	damage at ref. stress level
G_t	0.066 kN/m	0.066 kN/m	fracture energy
s_0	0.2	0.2	stiffness recovery factor
Dilatancy			
type	variable	variable	
α_{po}	0.35	0.35	tensile dilatancy parameter
α_p	0.35	0.35	dilatancy parameter (compr.)
$\underline{\sigma}_{c.dil}/f_c$	0.95	0.95	activation stress level (compr.)
α_d	2	2	appex smoothing parameter

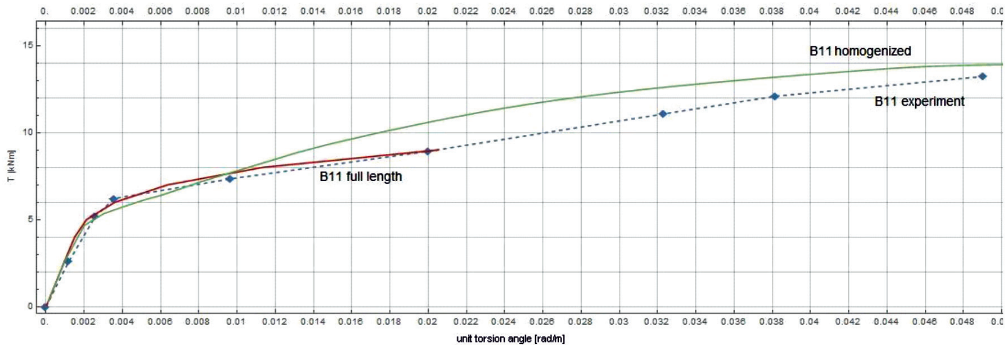

Fig. 6. Torsional moment – unit torsion angle relationship for beam No. B11

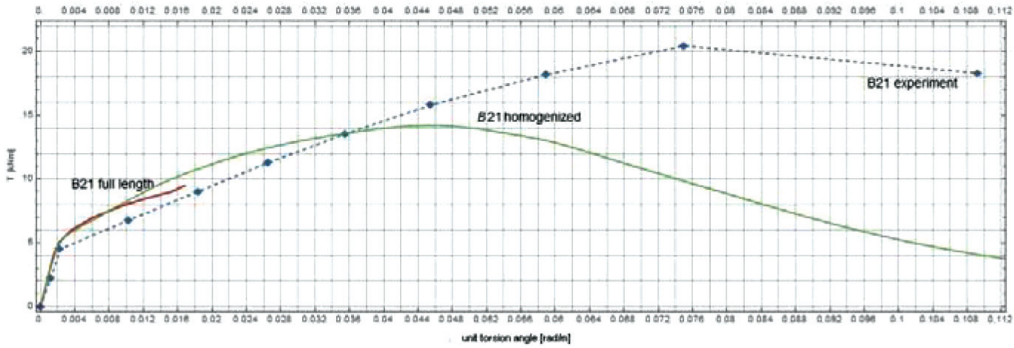

Fig. 7. Torsional moment – unit torsion angle relationship for beam No. B21

The graph shows the relationship between the unit torsion angle and torsional moment for beam in the homogenization approach with reinforcement and for plain concrete cross-section. Also in this case, the Concrete Plastic Damage model was used with the parameters given above.

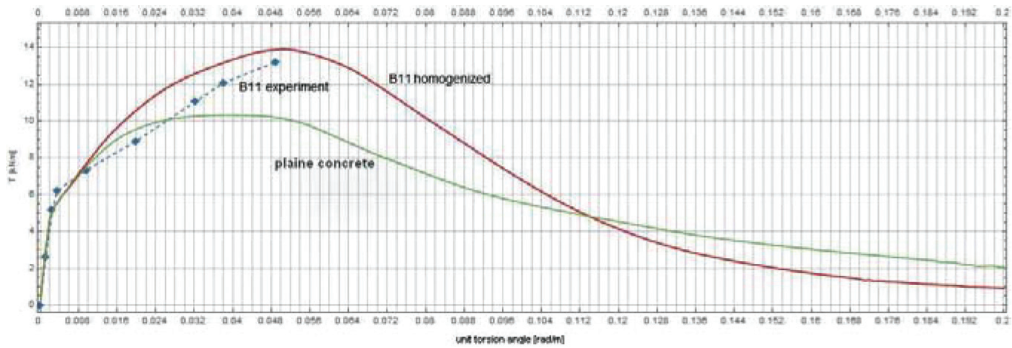

Fig. 8. Torsional moment – unit torsion angle relationship for beam No. B11 with reinforcement and for plain concrete cross-section

Moreover, the beam behaviour is presented in the homogenisation approach for elastic plastic material with softening behaviour (M-W) for w_r = 0.001 m and dilatancy angle of $\Psi_c = 7$

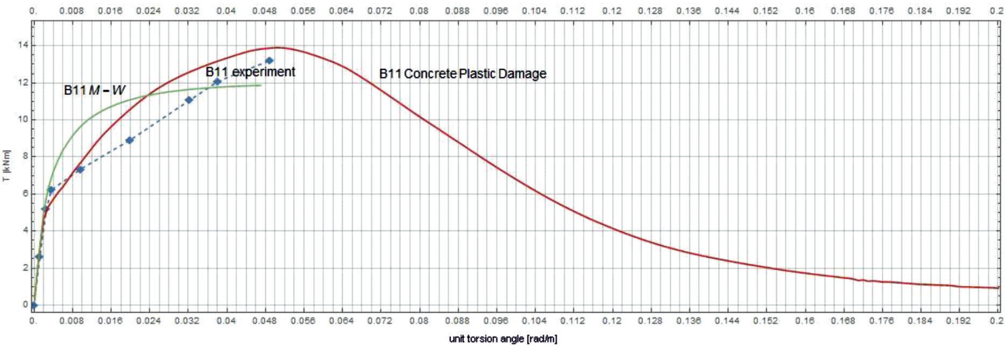

Fig. 9. Torsional moment – unit torsion angle relationship for beam No. B11 for Concrete Plastic Damage and M-W material models

5. Final conclusions

It has been demonstrated that numerical analyses can be limited to analysing a section of a linear element with a user-defined periodic boundary condition imposed on its walls. This approach significantly reduces the calculation time. Moreover, it enables the use of a simple deformation-controlled procedure. The results obtained in this way exhibit a satisfactory consistency with the experimental data.

Note is made of the problem of selecting a material model for the continuum. While more accurate results in the pre-failure stress-strain state of cross-sections can be obtained with complex descriptions of the concrete, these require determination of numerous parameters, not always discernible in the straightforward, engineer's approaches.

Agreement of the results of numerical simulation with the experimental data obtained in simple loading cases gives grounds for further research on the behaviour of beams subjected to combined loading.

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