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A 2D/3D METHOD OF THE GROUNDWATER FLOW AND STABILITY ANALYSIS OF A SLOPE WITH DEWATERING WELLS

METODA ANALIZY 2D/3D PRZEPLYWU WÓD GRUNTOWYCH I STATECZNOŚCI ZBOCZA ODWADNIANEGO PRZY POMOCY STUDNI ODWADNIAJĄCYCH

Abstract

This paper presents an approximate method for a groundwater flow analysis in the case of a slope with vertical dewatering wells distributed equidistantly. Firstly, a multi-scale problem of flow to a drainage well is solved and flow model to a perforated tube with a filter screen is shown. Secondly, a method of building quasi-3D flow model of a slope with vertical dewatering wells using a technique of overlapped 2D mesh is shown. A short description of the implementation of this 2D/3D flow model for the ZSoil.PC code is given. The results of the described 2D/3D method are compared with a referential 3D analysis giving an acceptable level of agreement. Finally, the 2D/3D method of flow analysis is used in a two-phase (flow + deformation) formulation of a slope stability problem. The comparison of the results of these analyses with fully 3D (referential) analysis, on an example problem provides a very close accurate stability and deformation estimation; however, the computational time used for the 2D/3D analysis is significantly shorter than required by 3D analysis.

Keywords: FEM analysis of groundwater flow, slope stability analysis, dewatering wells

Streszczenie

Przedstawiono przybliżoną metodę analizy przepływu wód gruntowych dla przypadku zbocza ze studniami odwadniającymi rozmieszczonymi w równych odstępach. Na wstępie rozwiązano wieloskalowe zagadnienie przepływu do studni odwadniającej i zaprezentowano sposób budowy zhomogenizowanego modelu filtracji do filtra. Następnie zaprezentowano sposób budowy quasi-3D modelu filtracji w zboczu z pionowymi studniami odwadniającymi z wykorzystaniem techniki nakładających się siatek 2D. Podano krótki opis implementacji modelu filtracji 2D/3D w programie ZSoil.PC. Wyniki opisanej metody 2D/3D porównano z referencyjnym modelem 3D dającą akceptowalną zgodność. Na końcu zastosowano metodę analizy przepływu 2D/3D do zagadnień dwufazowych (przepływ + deformacja) dla problemu stateczności zbocza. Porównanie wyników z referencyjną pełną analizą 3D, dla przykładowego problemu daje bardzo dobrą ocenę stateczności i deformacji, ale czas obliczeniowy używany do analizy 2D/3D jest znacznie krótszy.

Słowa kluczowe: analiza filtracji MES, stateczność zboczy, studnie odwadniające.

1. Introductory remarks and the motivation

Numerical modelling of groundwater flow performed with use of the finite element method may be presented as a separate problem or as a component of a two-phase (flow + deformation) analysis. The first option is often used for the design of water supply (see [6]) or drainage systems. The second option is necessary when attempting to investigate the influence of the presence of water within a soil mass, which is particularly essential in the context of slope stability problems. This concerns both natural and man-made slopes such as an earth dam downstream side slope. It is a widely recognised fact that the introduction of drilled dewatering wells lowers the level of the saturated zone, which is beneficial for the stability of a structure.

An overview of the concerned situation is presented in Fig. 1. where vertical dewatering wells with a filtrating zone (screen filter), located on some depth, are equidistantly distributed in the longitudinal (Z) direction. Even if the filtration properties of the soil are assumed as being constant in the Z -direction (while they may vary in the XY cross section plane), the problem of flow is three-dimensional, but submitted to periodicity constraints. In a recent practice, 3D flow problem set in the domain of one segment (with Z -length a) between drainage wells may be solved within an acceptably short timeframe, thus, for flow analysis exclusively there would not be a requirement for an approximate method. However, this is not the case of slope stability analysis basing on two-phase (flow + deformation) formulation and Terzaghi-Bishop principle. Modelling a continuum with a 3D field of water pressure would require full 3D fields of stresses, and what follows, strains and displacements, even in the case when the plane strain (PS) assumptions are commonly accepted, like while performing stability analysis of man-made slopes. Plane strain assumptions are, in considered case, violated. 3D stability analyses performed with use of the c - ϕ reduction method, which might be formulated as well, require considerably more time and computational power than 2D analysis – see the comparison in the example given in section 6. It is worth noting that with these types of problems; dense meshing in zones where strain localisation is expected (which are *a-priori* unknown), is necessary.

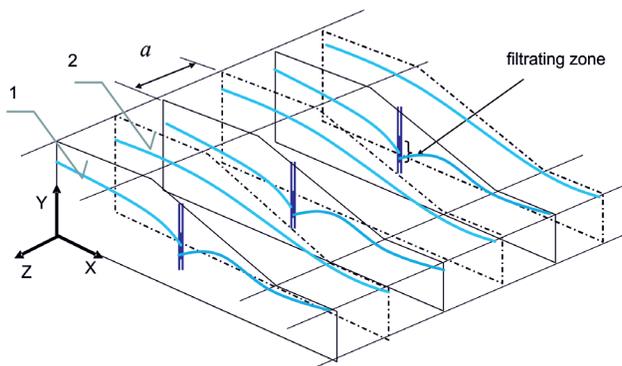


Fig. 1. An overview of a slope with dewatering wells; free surfaces: 1) at the section coming through vertical well, 2) at the section in the mid-distance

The main objective of this article is to perform the analysis of dewatered slopes in the 2D domain of its cross-section, with plane strain assumption and water pressure field being an averaged of these in Z-direction. The developed method is based on an approximate solution of the groundwater flow problem on overlapped meshes describing flow through the slope and flow to the draining wells. An implementation done in custom version of ZSoil.PC code allows to show the robustness and acceptable engineering accuracy of the developed 2D/3D method.

2. Flow to a drainage well as a multi-scale problem

The first task to be solved is to develop a proper model of seepage to a filter screen in order to estimate its efficiency. This can be achieved empirically, based on in situ or laboratory testing, which is beyond the scope of this paper, or numerically with the use of the micro-level analysis of flow and the homogenisation technique presented below. The validity of Darcy's law with full saturation condition is assumed. A gravity term in Darcy's law is neglected while performing the described micro-level analysis.

The scale of the geometry of the openings in the filter screen tube is within the range of 10^{-3} m; the scale of macro-level characteristic dimensions may reach 10^1 m. In these circumstances, the need for a multi-scale analysis is evident as it is extremely difficult to build a single model dealing with the flow within the vicinity of the drain tube (scale mm) and at a large distance from it. Scale separation leads to multi-scale analysis, which consists of two problems concerning:

- ▶ micro level – set in the 3D domain of periodic cell resulting from the geometry of the filter screen. This analysis is used to establish fluxes q for the given pressure head p , see Fig. 2b,
- ▶ macro level – referring to the whole system (2D or 3D), with well modelled as a tube (diameter D , thickness Δ), and permeability coefficient set by equivalence of flow with these established during micro level analysis .

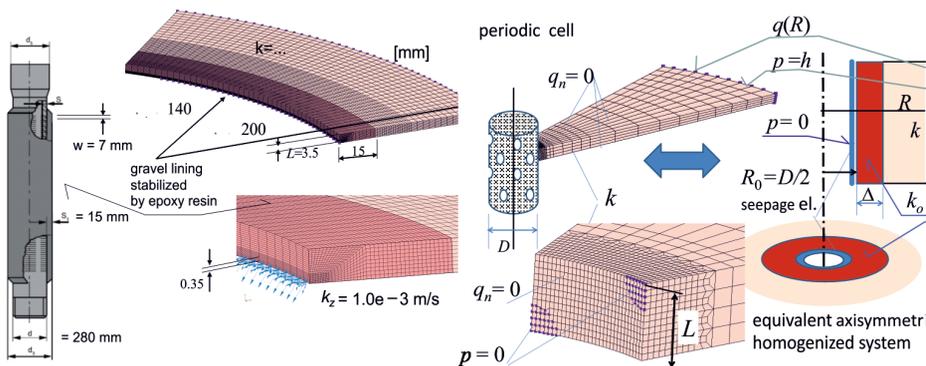


Fig. 2. Multi-scale analysis based on the homogenisation of the flow

Fig. 2a shows an exemplary piece of the filter screen and its surrounding, used in the system which is presented in section 6. In the Fig. 2b, the outlook of following homogenisation procedure is shown. Having given uniform flux $q(R)$ on the outer boundary of the periodic cell from the solution of 3D micro-level problem, where k [length/time] is a permeability coefficient of soils surrounding the well, an axi-symmetric problem is solved in order to found such permeability k_0 of fictitious layer of (arbitrarily set) thickness Δ , which assure the same flux q for zero pressure condition on the inner surface of a tube.

In the auxiliary procedure of identification, one has to solve Laplace differential equation $\nabla^2 p = 0$ for the pressure head distribution in the radial direction with constraints $\frac{\partial}{\partial \theta} = 0$, $\frac{\partial}{\partial y} = 0$, coming from assumed axisymmetry of homogenised model of the filter screen, which takes a form of eq. (1):

$$\frac{d^2 p}{dr^2} + \frac{1}{r} \frac{dp}{dr} = 0, \quad (1)$$

with the following boundary and compatibility conditions (eq. 2),

$$p_1(R_0) = 0, \quad p_2(R) = h, \\ k_0 \left. \frac{dp_1}{dr} \right|_{r=R_0+\Delta} = k \left. \frac{dp_2}{dr} \right|_{r=R_0+\Delta}, \quad p_1(R_0 + \Delta) = p_2(R_0 + \Delta), \quad (2)$$

For given R_0 , R , k , h , this leads to a formula for permeability k_0 of fictitious layer with the thickness Δ :

$$k_0 = \frac{\beta \ln \frac{R_0 + \Delta}{R_0}}{1 + \beta \ln \frac{R}{R_0 + \Delta}} k, \quad \beta = \frac{qR}{kh} = \frac{Q}{2\pi kh} [-]. \quad (3)$$

Flux q [length/time] at the radius R , or total water inflow Q [length²/time] per unit length of a tube can be used in the formula for the dimensionless factor β appearing in eq. (3).

This homogenised model of filter screen will be later used for both 2D/3D as well as 3D macro modelling of dewatering wells.

3. Approximate 2D model of the ground flow in a slope with periodically spaced vertical dewatering wells

In ZSoil.PC FE software system stationary or transient ground water flow problem is formulated basing on the modified version of Darcy law, taking into account not fully saturated zone, proposed by Van Genuchten [3]. Moreover, Richards continuity equation is used for modelling internal retention in not fully saturated zone. Another tool enabling modelling

without any user intervention in computation consists of seepage elements, automatically switching boundary condition from fully free flow to no-flow, depending on the pressure at the boundary. For details, see [2, 4, 5]. All mentioned features are active in the present issue.

The idea is to decompose flow field in the dewatered slope at following parts:

- ▶ **flow along the slope**, for which base 2D FE mesh describing geometry of the slope cross-section is built, preserving its permeability properties $k(X, Y)$ and boundary conditions,
- ▶ **flow towards the well**, performed on a set of overlapped FE meshes, applied in the well influence zone, size of which is related to the distance of wells ($= 2a$) in Z direction (see the Fig. 3). Each of them represents a sector of 3D system. Permeability of materials set in this fictitious media inherits permeability $k(X, Y)$ from the base mesh but those are finally modified on the principle of geometrical equivalence, assuming radial flow direction with the centre at the well. The equivalence must take into consideration a fact, that plane flow is set for unit thickness $g = 1.0$ [length] of the model in perpendicular direction. On the length of filter screen, the tube of diameter D is surrounded by fictitious layer with permeability coming from k_o of homogenised model and current sectorial geometry. Outside the filter screen, on the tube inner surface, a no-flow condition $q_n = 0$ is set. Additionally, on the limits of the well influence zone, the condition of equal pressures with base mesh is set by introducing penalty permeability k_∞ (i.e. equal to a very large value) at the “arms” perpendicular to the base mesh.

It is worth noting, that in a given FE software building a FE model with overlapped meshes must be allowed, like it is in ZSoil.PC.

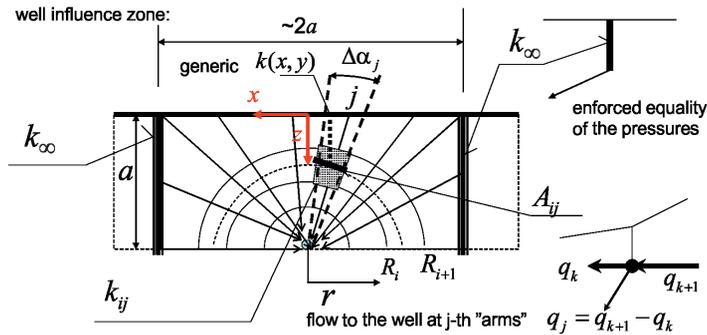


Fig. 3. Details of the equivalent 2D/3D flow model

Number of radii m is set by the user as well as the number of circles n . The permeability at the ij position, considering geometry equals:

$$k_{ij} = k(x, y) \cdot \frac{A_{ij}}{a}, \quad (4)$$

where A_{ij} is averaged width attached to i -th zone at j -th sector of the radial mesh. For regular shape they are given in eq. 5, for the remaining they can be derived analogously

$$F_{ij} = \pi(R_{i+1}^2 - R_i^2) \frac{\Delta \alpha_j}{2\pi}, \quad A_{ij} = \frac{F_{ij}}{R_{i+1} - R_i} = \frac{R_{i+1} + R_i}{2} \Delta \alpha_j, \quad (5)$$

where F_{ij} is an area.

In a custom version of ZSoil.PC code process of building any model as described above is fully automatized and after creating the base 2D FE mesh the user needs to provide only minimal amount of data describing geometry of wells, additional discretization, permeability k_o and thickness Δ of fictitious layer of homogenised well model, according to p. 2. There may be a few wells modelled at single base mesh, with one objection, i.e. that their influence zones do not overlap.

User interface for wells definition is presented in the Fig. 4.

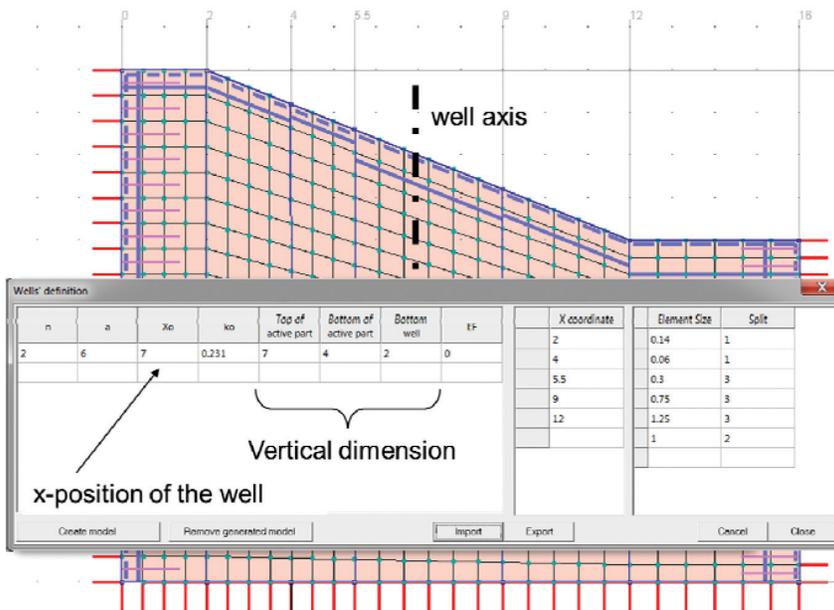


Fig. 4. User interface for wells definition

4. Comparison of results between 2D/3D against 3D flow modelling

The first test example is based on comparison of a result for steady-state flow analysis obtained for full 3D model with results for equivalent 2D/3D model. Draft of the geometry with resulting base FE mesh is presented in Fig. 5. Parameters presented in Fig. 5a were used to generate mesh for 2D/3D model. FE meshes with fluid boundary conditions for 2D/3D method and 3D model are shown in Fig. 6a and 6b respectively. Permeability coefficient is $k = 1$ m/d for whole domain, despite zone of the filter screen ($D = 0.28$ m) where homogenised model is applied with $k_o = 0.231$ m/d at the fictitious layer of thickness $\Delta = 0.06$ m.

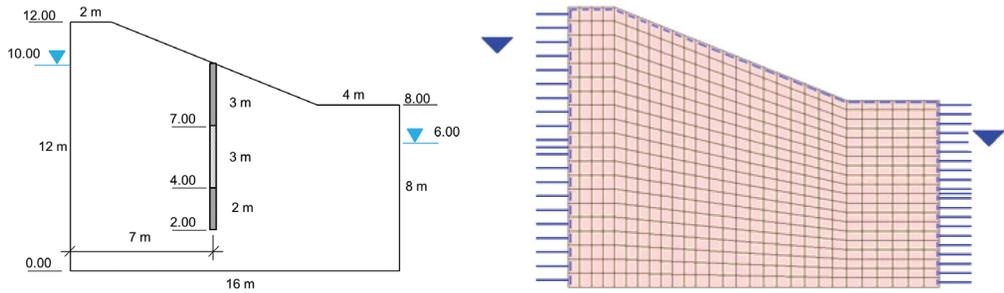


Fig. 5. Geometry of the test model and resulting base FE mesh

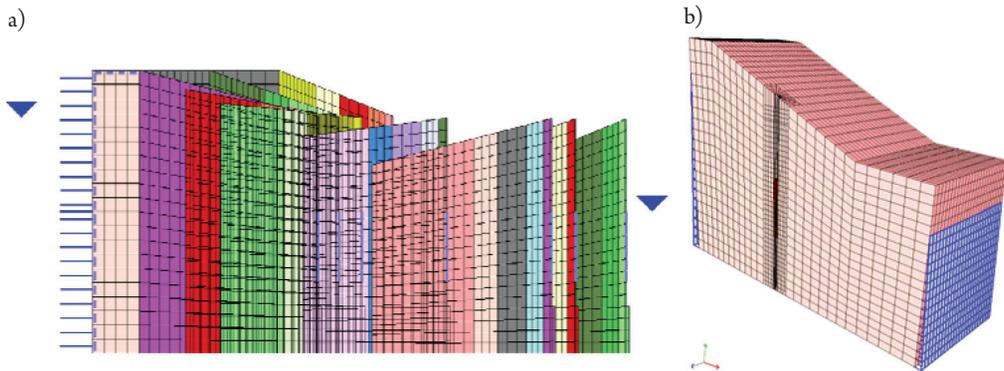


Fig. 6. FE meshes with fluid boundary conditions for: a) 2D/3D model, b) referential 3D model

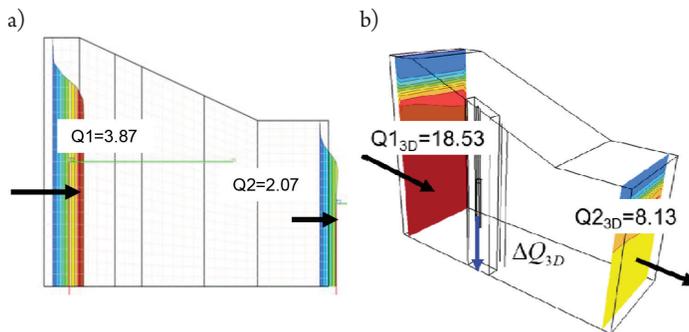


Fig. 7. Fluid velocities distribution in cross sections for: a) 2D/3D model, b) referential 3D model

In order to compare total flow to the dewatering well for both models the integrals of a fluid velocities through cross sections defined on left and right boundary sides are calculated. Fluid velocities distribution in cross sections with resultant integrals for 2D/3D model and 3D model are shown in Fig. 7a and 7b. The assumed distance between wells is $2a=12\text{m}$.

Total flow to dewatering well for 2D/3D model equals:

$$\Delta Q_{2D} = (Q1 - Q2)a = \left(3.87 \left[\frac{m^3}{d \cdot m} \right] - 2.07 \left[\frac{m^3}{d \cdot m} \right] \right) \cdot 6m = 10.79 \left[\frac{m^3}{d} \right] \quad (6)$$

Total flow to dewatering well for 3D model equals:

$$\Delta Q_{3D} = Q_{1_{3D}} - Q_{2_{3D}} = \left(18.53 \left[\frac{m^3}{d} \right] - 8.13 \left[\frac{m^3}{d} \right] \right) = 10.40 \left[\frac{m^3}{d} \right] \quad (7)$$

Difference is:

$$e\% = \frac{10.79 - 10.40}{10.40} = 3.75\% \quad (8)$$

The comparison of the pore pressure distribution for 2D/3D model and referential 3D model is shown in Fig. 8. Differences for both models in total flow to dewatering well (less than 4%) and pore pressure distribution are acceptable for engineering applications. Data preparation is much simpler for 2D/3D model with automatic well generation than for 3D model, for which all details of the well and fine mesh around well have to be generated manually. Additionally, mesh generation when a well position is changed is created automatically for 2D/3D model. The 3D model has to be created nearly from the beginning.

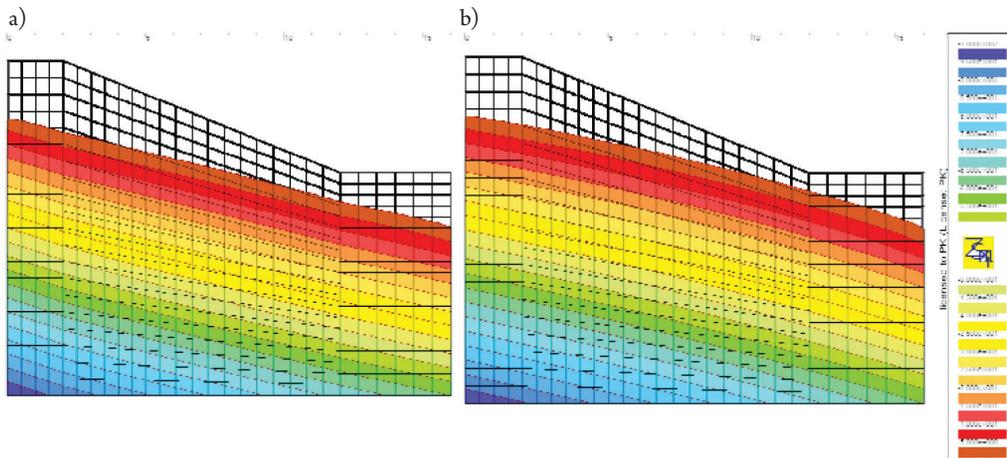


Fig. 8. Pore pressure distribution for: a) 2D/3D model, b) referential 3D model (in the mid-section)

5. Application of the 2D/3D method to two-phase (deformation + flow) analysis

The two-phase analysis for 2D/3D model requires special treatment for material properties and boundary conditions. Deformation + flow analysis is performed on 2D base mesh when perpendicular or radial “arms” have influence on flow analysis only. Young modulus is assumed to be close to zero and displacement degrees of freedom are fixed everywhere in the domain of perpendicular “arm” in order to neglect its influence on mechanical analysis. This sub-model has pressure degrees of freedom only. Meshes of the base model and “arms” can be inconsistent. Continuity of pressure fields between both sub-models is preserved by

means of nodal link option available in ZSoil.PC. Such schema of partial separation of flow and deformation+flow sub-models is presented in Fig. 9.

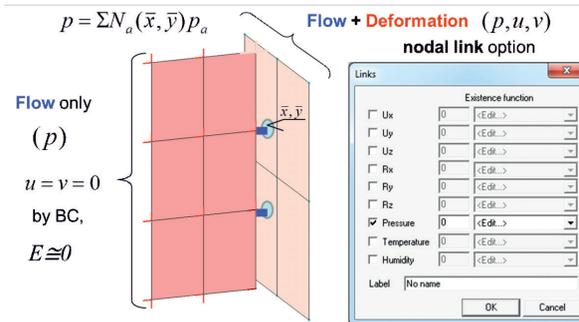


Fig. 9. Partial separation of flow and deformation+flow sub-models for two-phase analysis

6. Application of developed method of flow analysis to the slope stability problem

The effectiveness of 2D/3D model compared with full 3D model is presented on an example of slope stability problem. The analysis of pore pressure distribution influences on results of slope stability was performed by Finite Element and $c-\phi$ reduction method, using ZSoil.PC® code. The applied $c-\phi$ reduction method was firstly introduced into the early version of ZSoil.PC code (circa 1985); its details may be found in ZSoil documentation [8]. More examples of usage of this method in landslide stability analyses can be found in Truty et al. [2], Zheng et al. [7], Ozbay and Cabalar [1]. The soil continuum was treated as an elastic-plastic one, with Mohr-Coulomb yield condition. The influence of an underground water pressure field in saturated or partially saturated zone was taken into account, with enhancements of the flow theory given in Van Genuchten [3], basing on observed water table.

The geometry of the analysed problem with resulting base FE mesh are presented in Fig. 10 and parameters for mesh generation are shown in Fig. 11. FE meshes with fluid boundary conditions for 2D/3D method and 3D model are shown in Fig. 12 and Fig. 13 respectively.

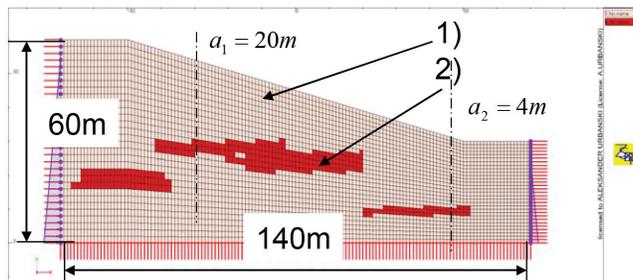


Fig. 10. The geometry of the test model and resulting base FE mesh

n	a	Xo	ko	Top of active part	Bottom of active part	Bottom well	EF
4	20	-30	0.01	35	25	20	0
3	4	45	0.01	25	15	10	0

X coordinate	Element Size	Split
-50	0.14	1
-40	0.06	1
-30	1	2
-20	2	2
-10	3	3
	4	2

n	a	Xo	ko	Top of active part	Bottom of active part	Bottom well	EF
4	20	-30	0.01	35	25	20	0
3	4	45	0.01	25	15	10	0

X coordinate	Element Size	Split
42	0.14	1
44	0.06	1
46	0.3	2
48	0.75	2
50	1	2

Fig. 11. The geometry of the wells and parameters for mesh generation

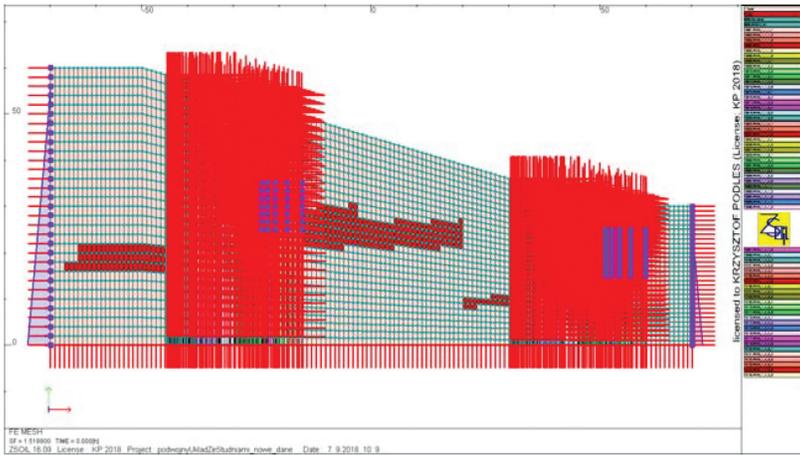


Fig. 12. FE mesh with boundary conditions for 2D/3D model

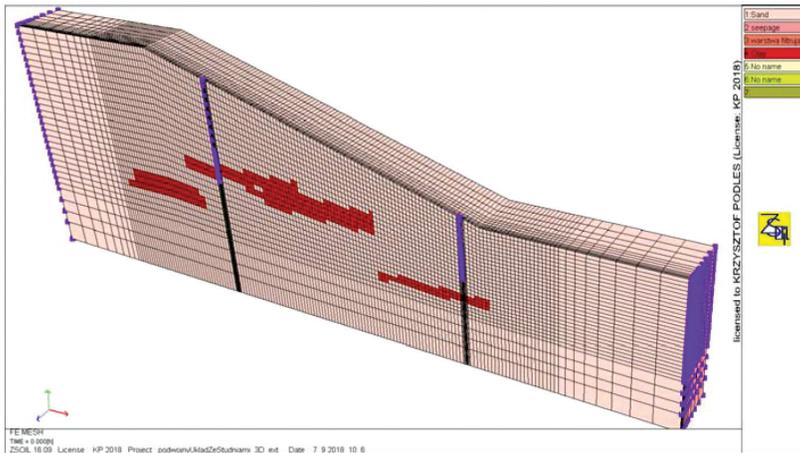


Fig. 13. FE mesh with boundary conditions for referential 3D model

In the analysis following strength and permeability parameters were assumed:

- ▶ material 1: $c_1 = 3.0$ kPa, $\varphi_1 = 30^\circ$, $k_1 = 0.0416$ m/d, (sand),
- ▶ material 2: $c_2 = 20.0$ kPa, $\varphi_2 = 30^\circ$, $k_2 = 0.0000416$ m/d, (clay);

The results of stability analysis are presented in Fig. 14 and 15. It is worth to mention that almost identical deformation (failure pattern) appears in both approaches. The computed safety factors are very close one to another, as well. For 2D/3D model it equals $SF_{2D/3D} = 1.519$, and for referential 3D model $SF_{3D} = 1.509$. It is worth mentioning that elapsed time of computations differs substantially, as the analysis for 2D/3D model was performed in 10 minutes, while for 3D model it takes 10 hours, i.e. ~ 60 times more, for the same mesh density in XY plane.

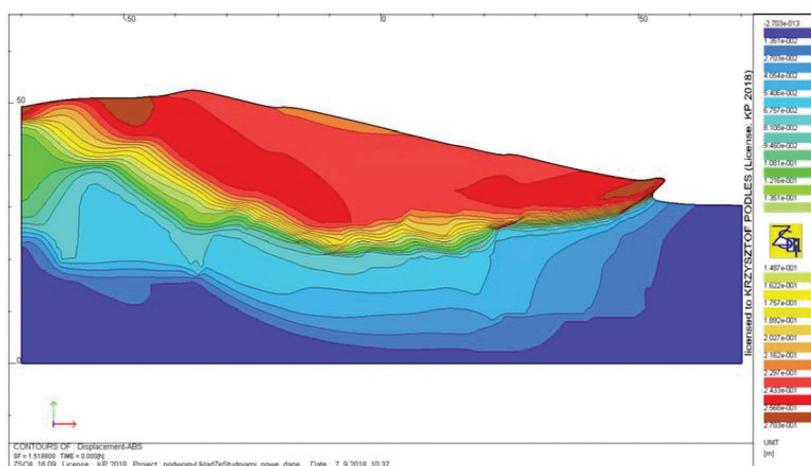


Fig. 14. Distribution of total displacements on deformed mesh for 2D/3D model for safety factor $SF_{2D/3D} = 1.519$

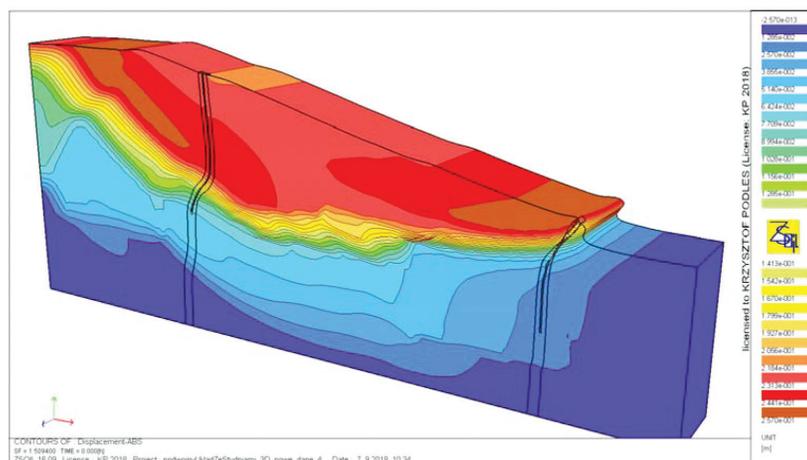


Fig. 15. Distribution of total displacements on deformed mesh for referential 3D model for safety factor $SF_{3D} = 1.509$

7. Conclusions

The presented approximate method of an analysis of the groundwater flow in a slope with vertical dewatering wells; distributed periodically proves to be an efficient tool for engineering problems requiring spatial distribution of the water pressure field.

The main gain is that analysis can be set in 2D domain, instead of 3D modelling, leading to practically very close results remaining within the 5% margin of error. It concerns deformation and stability assessment in case of two-phase analysis, as well as inflow to the wells.

The presented method allows to obtain results in a rapid way. In case of problems with assessment of stability in existing structure, it may be useful tool for prediction of its behaviour in varying condition. In a design process it may serve as a tool to optimize dewatering system in terms of location of the wells or applied drainage capability.

References

- [1] Ozbay, A., Cabalar, A.F., *FEM and LEM stability analyses of the fatal landslides at Çöllolar open-cast lignite mine in Elbistan, Turkey*. "Landslides", 12/2015, 155–163.
- [2] Truty A., Urbański A., Grodecki M., Podleś K., *Computer aided models of landslides and their protection problems*, "Scientific-Technical Papers of Communication Engineers and Technicians of the Republic of Poland" in Krakow, No. 88, issue 88 (144)/2009, 395–419.
- [3] Van Genuchten M.Th., *A closed form equation for predicting the hydraulic conductivity of unsaturated soils*. "Soil Science Society of America Journal", 44/1980, 892–898.
- [4] Urbański A., Grodecki M., Kot A., *Numerical simulations in analysis of anti-flood ambankments*, "Journal of Civil Engineering, Environment and Architecture", 2016.
- [5] Urbański A., Grodecki M., *FE Approach to coupled transient flow and stability analysis of anti-flood ambankments* "3-rd Int. Conference on Advances of Computer Methods in Geotechnical and Geo-environmental Engineering", Moscow 2000.
- [6] Wilk B.K., Urbański A., *The impact of the shape of screen openings on groundwater flow to a deep drilled well*, "Technical Transactions" 11/2018, to appear.
- [7] Zheng Y., Tang X., Zhao S., Deng C., Lei W.3., *Strength reduction and step-loading finite element approaches in geotechnical engineering*. "Journal of Rock Mechanics and Geotechnical Engineering", 1(1)/2009, 21–30.
- [8] Zimmemann Th., Truty A., Urbański A., Podleś K., *Z_Soil.PC 2003 manual. Theory*. Elmepress international & Zace Services Ltd., Lausanne, Switzerland 2005.