4. Detecting of signals in half-strips

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Introduction

In the classical Wiener signal theory (see [1]) a signal g is a function of the continuous time parameter t and a filter Φ is a device ("a box") transforming an input signal into a certain output signal. The energy of a signal g is proportional to $\int_\gamma |g(z)|^2|dz|$.

Without entering into the physical nature of a stationary filter Φ, we consider it as a translation invariant linear operator on the corresponding L^2 space.

Methods

Let $H^p_\sigma(\mathbb{C}_+)$, $\sigma \geq 0$, $1 \leq p < \infty$ be the space of analytic functions in the half-plane $\mathbb{C}_+ = \{z: Re\ z > 0\}$ for which

$$\|f\| := \sup_{|\varphi|<\frac{\pi}{2}} \left\{ \int_{-\infty}^{+\infty} |f(re^{i\varphi})| e^{-pr\sigma|\sin\varphi|}\, dr \right\} < +\infty$$

For the case $\sigma = 0$ the space $H^p_\sigma(\mathbb{C}_+)$ is the (classical) Hardy space.

A function G is called cyclic in $H^p_\sigma(\mathbb{C}_+)$, $\sigma \geq 0$, $1 \leq p < \infty$, if $G \in H^p_\sigma(\mathbb{C}_+)$ and the system $\{G(z): \tau \leq 0\}$ is complete in $H^p_\sigma(\mathbb{C}_+)$.

Results

Theorem 1. Let $G \in H^2_\sigma(\mathbb{C}_+)$, $\sigma > 0$, $G \not\equiv 0$. Then G is cyclic in $H^2_\sigma(\mathbb{C}_+)$ if and only if the function G is zero-free in $\mathbb{C}_+$, the singular boundary function of G is an identical constant and

$$\lim_{x\to+\infty} \left(\frac{\ln|G(x)|}{x} + \frac{2\sigma}{\pi} \ln x \right) = +\infty.$$

Theorem 2. Let f be an unknown filter at the half-strip $D_\sigma = \{z: Re z < 0, |Im\, z| < \sigma\}$ and let g be a test signal at the Hardy space at the $\mathbb{C}\backslash D_\sigma$. Then $\int_{-\infty}^{+\infty} f(w+\tau) g(w)\, dw = 0$ for all $\tau \leq 0$ implies $f \equiv 0$ if and only if

$$G(z) = \int_{\partial D_\sigma} g(w) e^{wz}\, dw$$

is cyclic in $H^2_\sigma(\mathbb{C}_+)$.

Also, we discuss about applications in the signal theory, zeta function theory, convolution equations and other applications.

Conclusions. We describe all test signals for identification of the unknown filter in the half-strip.

References

1. Nikolskii N.K. Operators, Functions, and Systems: An Easy Reading. Hardy, Hankel, and Toeplitz. AMS, 2002.