AN IMPROVED ALGEBRAIC MODEL FOR BY-PASS TRANSITION FOR CALCULATION OF TRANSITIONAL FLOW IN PIPE AND PARALLEL-PLATE CHANNELS

by

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Modified algebraic intermittency model developed by E. Dick and S. Kubacki was used to describe laminar-turbulent transition. In this work a modification of this model was made for simulating internal flows in pipes and parallel-plate channel. In particular, constants present in this model were modified. These modified constants are the same for different flows in pipes and parallel-plate channels. In this work, a dependence of friction factor on Reynolds number and turbulence intensity were determined as well as the localization of laminar breakdown and fully developed flow. Obtained results were compared with theoretical and experimental data presented in the literature.

Key words: *laminar-turbulent transition, k-ω turbulence model, intermittency factor, CFD, OpenFOAM*

Introduction

The transition from laminar to turbulent flow is a classic problem of fluid mechanics. The linear theory of hydrodynamic stability, that provides for fluid viscosity, predicts unconditional stability for internal flows in conditions of infinitesimal disturbances in large range of the Reynolds number [1, 2].

In practice, the laminar-turbulent transition is observed in the aforementioned cases even when the Reynolds number exceeds 2000 (Re > 2000). The reasons for this inconsistency are most likely changes in the velocity profile and significant amplitude of disturbances in the inlet cross-section. The laminar-turbulent transition region is distinguished by higher values of friction factor and heat transfer coefficient compared to those in the region of fully developed laminar or turbulent flow [3-6].

Rup *et al*. in [3] analyses the statistical steady turbulent exchange of momentum and heat in the range of small values of Reynolds number (5000 < Re < 50000) implemented in the inlet region of a plastic pipe. Research described in [3] includes changes in turbulence intensity in the inlet cross-section and changes in the turbulent Prandtl number, Pr_t, in the flow area.

Abraham *et al*. in [4] the transition model developed by Menter *et al*. [7, 8] based on local variables for the description of the intermittent flow in the pipe was modified. This modified model of the intermittent transition is used to describe the flow in the pipe. Two different

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forms of flow were demonstrated on the basis of numerical calculations [4], namely the fully developed intermittency and the fully developed turbulent flow [4].

Based on the numerical calculations from [4] local values of the friction factor as well as the location of laminar flow breakdown and fully developed turbulent flow were determined. All locations were found for the assumed values of the Reynolds number and the turbulence intensity, *Tu*.

Minkowycz *et al*. in [5], the modified Menter *et al*. [7, 8] model of the intermittency was used to perform numerical simulations of the flow in the channel between two parallel plates. This model of intermittency is similar to the model formulated in [4] and contains the system of seven PDE. This system of seven equations includes one mass balance equation, two equations of momentum (RANS), two equations representing the SST turbulence model, and two intermittency equations formulated by Menter [7, 8]. Based on the obtained numerical calculations results, the initial conditions assumed in the inlet cross-section has significant impact on the developing flow in the initial section of the channel. In particular, homogenous and parabolic velocity profile was assumed in the inlet cross-section and two values of the turbulence intensity (Tu = 1% and Tu = 5%) were taken into account.

Abraham *et al*. in [6], the intensity of heat exchange in the laminar-turbulent transition was analyzed in the initial section of the flow conduit. In particular, a straight section of pipe, a flat duct and a straight pipe section containing a conical diffuser were taken into account. The mathematical model of flow considered in [6] contained in addition to the modified Menter model [4, 5] the differential equation resulting from the energy balance for the fluid. As a result of numerical calculations carried out in [6], original dependences of the Nusselt number from the Reynolds number in the laminar-turbulent transition region of the flow conduit were determined.

This work modifies the algebraic model of intermittency developed by Kubacki and Dick [9] for the purpose of numerical simulations of internal flows performed in flat channels and pipes. The modified algebraic model of intermittency does not contain additional differential equations to determine the intermittency factor, γ. In the presented work, the intermittency factor is determined from the algebraic equation similar to the work [9]. The results of numerical calculations were compared with the corresponding results of calculations with different model found in the works [4-6] and other experimental data available in the literature [10, 11].

Nering and Rup in [12] a comparison with measurement and analysis of velocity profiles for improved model were presented. Results obtained with modified model show high degree of compliance with experimental data.

Numerical model

The formulated numerical model for describing statistically steady fluid-flow in the initial section of a pipe or a flat channel contains the system of PDE. These equations result from the mass and momentum balance described in the k-ω turbulence model [13] and the algebraic intermittency model dependence [9]. Numerical model takes into account the most important physical phenomena occurring in the considered flow region. In particular, it enables an accurate description of the flow in the laminar, transitional and turbulent range. The system of time-averaged equations resulting from mass and momentum balance takes the form:

$$\frac{\partial u_j}{\partial x_j} = 0 \tag{1}$$

$$\rho\left[\frac{\partial}{\partial x_j}\left(u_i u_j\right)\right]=-\frac{\partial p}{\partial x_i}+\frac{\partial}{\partial x_j}\left(t_{ij}+\rho\tau_{ij}\right) \qquad i,j=1,2,3 \tag{2}$$

The tensor components t_{ij} and τ_{ij} are determined from the dependence of:

$$t_{ij}=2\rho\nu S_{ij},\quad \rho\tau_{ij}=2\rho\nu_T S_{ij}-\frac{2}{3}\rho k\delta_{ij} \tag{3}$$

$$S_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)-\frac{1}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij} \tag{4}$$

The equations of the adopted k-ω turbulence model [14] modified in [9] containing the γ intermittency factor are:

$$\rho\frac{\partial}{\partial x_j}\left(u_j k\right)=\rho\gamma\nu_s S^2-\rho\beta^* k\omega+\rho\frac{\partial}{\partial x_j}\left[\left(\nu+\sigma^*\frac{k}{\omega}\right)\frac{\partial k}{\partial x_j}\right] \tag{5}$$

$$\rho\frac{\partial}{\partial x_j}\left(u_j\omega\right)=\rho\alpha\frac{\omega}{k}\nu_s S^2-\rho\beta\omega^2+\rho\frac{\partial}{\partial x_j}\left[\left(\nu+\sigma\frac{k}{\omega}\right)\frac{\partial\omega}{\partial x_j}\right]+\rho\frac{\sigma_d}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial\omega}{\partial x_j} \tag{6}$$

where turbulent viscosity, ν_T, consist of two variables (small-scale turbulent viscosity – ν_S and large-scale turbulent viscosity – ν_l) [9]:

$$\nu_T=\nu_s+\nu_l \tag{7}$$

$$\nu_s=\frac{k_s}{\tilde{\omega}},\quad \tilde{\omega}=\max\left[\omega, C_{\lim}\frac{\sqrt{2S_{ij}S_{ij}}}{a_1}\right],\quad k_s=f_{ss}k,\quad f_{ss}=\exp\left[-\left(\frac{C_{SS}\nu\Omega}{k}\right)^2\right] \tag{8}$$

$$\nu_l=\frac{k_l}{\omega},\quad \omega=\max\left[\omega, C_{\lim}\frac{\sqrt{2S_{ij}S_{ij}}}{a_2}\right],\quad k_l=k-k_s \tag{9}$$

where $C_{\lim}$ is standard k-omega model constant, and Ω is the magnitude of rotation rate tensor. The intermittency factor appearing in eq. (5) has the form [9]:

$$\gamma=\min\left(\frac{\varsigma_T}{A_T},1\right),\quad \varsigma_T=\max\left(\frac{\sqrt{k}y}{\nu}-C_T,0\right) \tag{10}$$

Detailed description of the constants found in the discussed models can be found in works [9].

Solution of the problem

The system of PDE (1), (2), and (5), (6) was solved using the OpenFOAM framework, which uses the finite volume method. OpenFOAM is an open source software which allows access to the source code, and thus facilitates the implementation of subordinate models. The integral region included the inlet section of the straight pipe and the flat channel with infinite width. Homogeneous velocity profile was assumed in the inlet cross-section of the pipe and the flat channel. Additionally, in the inlet cross-section, the turbulence intensity was taken into account, Tu_0 = 3%-5%. This range of turbulence intensity was chosen due to the availability of validation data (measured and other numerical results). In the numerical calculation process, the Reynolds number was also changeable in the range of $1000 \leq \mathrm{Re} \leq 80000$. This range of relatively low Reynolds number is present mainly in pipe heat exchangers used in heating and air-conditioning. When designing heat exchangers, the localization of laminar turbulent transition becomes an important element. In the region of the transition it can be observed different values of friction factor and heat transfer coefficient.

Localization of laminar-turbulent breakdown with constant Reynolds number distinctly depends on turbulence intensity on the channel inlet, Tu_0. Assumed values of Tu_0 allow to compare the results of numerical calculations obtained by the work with values determined using other models and available in the literature [4-6].

According to the idea of the algebraic model of the intermittency [9], the γ factor equals 0 for laminar flow, and $\gamma = 1$ for the fully developed turbulent flow. Therefore, in the calculations, γ in the inlet cross-section was assumed to be zero.

The length of the flow conduit (pipe or flat channel) was assumed taking into account the fully developed turbulent flow occurring after the transition (intermittent) form in the considered range of the Reynolds number. The actual length of the analyzed channels was equal to 160 *D* for the pipe and 160 *H* for the parallel-plate channel, where *D* was the diameter of the pipe and *H* was the distance between plates.

The system of differential eqs. (1), (2) and (5), (6) were solved, using finite volumes method. The largest density of division was assumed in sub-areas of flow, in which the highest gradients of determined values occurred. The smallest thickness of the volume element bordering with the pipe wall was $\Delta y_p = 2.39 \cdot 10^{-5}$ m, while the corresponding thickness of the volume element at the wall of the flat channel was equal to $\Delta y_f = 2.52 \cdot 10^{-5}$ m. For the listed minimum values of elements Δy_p and Δy_f, and taking into account asymptotic values of stress τ_w in both flat channel and pipe, the relevant dimensionless parameter assumed in the performed numerical calculations was $y^+ \sim 1.0$. The growth of element height (in *y*-direction) was assumed to be linear. Calculation model was presented in fig. 1. Flow through parallel-plate channel was assumed as 2-D (infinite width of the channel), flow in pipe was assumed as 3-D.

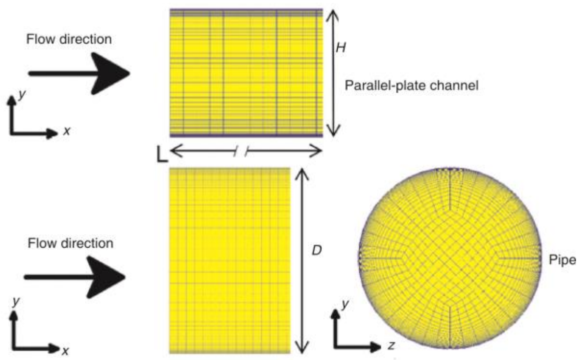

Figure 1. Calculation model for parallel-plate channel and pipe flow

The algebraic model of intermittency adopted in this work was developed in [9] to simulate transition flow in turbomachines. Due to significantly higher pressure gradients and different inlet conditions in the considered internal flows compared to the corresponding pressure gradients occurring in the space between blades of gas turbines for which an algebraic model was developed in [9], it was necessary to modify the discussed model. It should also be noted that the calculated asymptotic values of the flow in the flat duct and the pipe using the model developed in [9] differed by more than 20% from the relevant values, calculated using other models available in the literature. In addition, the location of the laminar-turbulent transition was largely displaced compared to corresponding experimental data available in the literature.

Due to the different flow characteristics (internal flows) analyzed in this work, it was necessary to modify the model developed in the paper [9]. Results were validated using the available, corresponding results of experimental studies. Due to the validation of the values of the four model constants were modified [9]. Modified values of these constants are: C_T = 15.5, A_T = 10.0, C_{SS} = 6.8, and a_2 = 0.55. Values of these constants are identical in the calculations of flows performed in pipes and flat ducts. The numerical values of the C_T, A_T, C_{SS}, a_2 model constants determined in this work are in line with recommendations formulated in the literature [13, 14]. The difference between results obtained using the original and modified constants of the model were presented in fig. 2. In this figure, the wall shear stress in reference to wall shear stress on pipe end, τ_{end}, was shown. Magnitude, L, is the length coordinate of the channel.

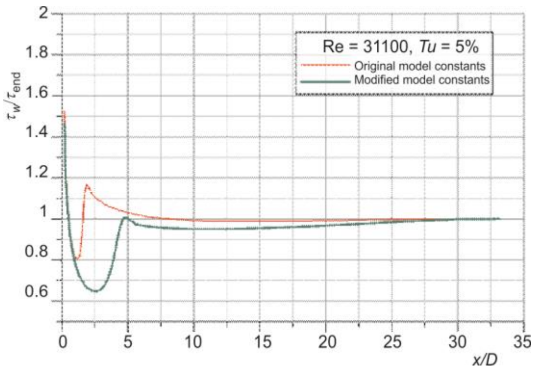

Figure 2. Comparison of results from model with original and modified constants

Result analysis

Based on the numerical calculations performed with the described model, the dependence of the friction factor on the Reynolds number was determined in the pipe and flat duct. The discussed dependence of the friction factor in the case of flow in the smooth pipe is shown in fig. 3. Obtained results shown in fig. 3. resulted assuming the values of the turbulence intensity Tu_0 = 5% in the inlet cross-section. The results of numerical calculations obtained in [4] using the complex model of intermittency were also presented in fig. 3 (curve 2). The results of the own calculations presented in fig. 3 and the corresponding results from the work [4] were obtained assuming the uniform velocity profile in the pipe inlet cross-section and the same turbulence intensity Tu_0 = 5%. Figure 3 also presents the friction factor known from the literature with the Reynolds number applicable in the area of laminar flow (curve 1) and fully developed turbulent flow (curve 3) in hydraulically smooth pipes.

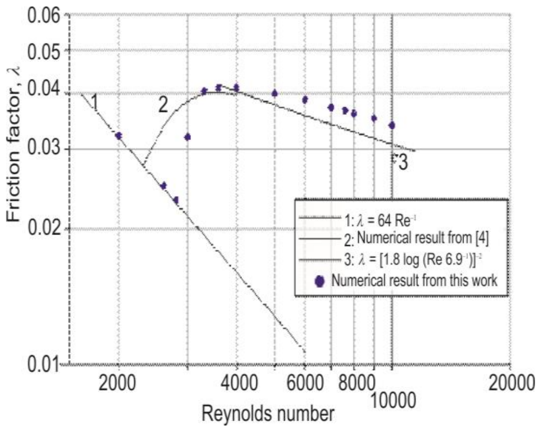

Figure 3. Variation of fully developed friction factor, λ, for pipe flow

Changes of non-dimensional local shear stress on the pipe walls and the flat flow channel were also determined using the modified algebraic intermittency model. Figure 4 presents the discussed dependence

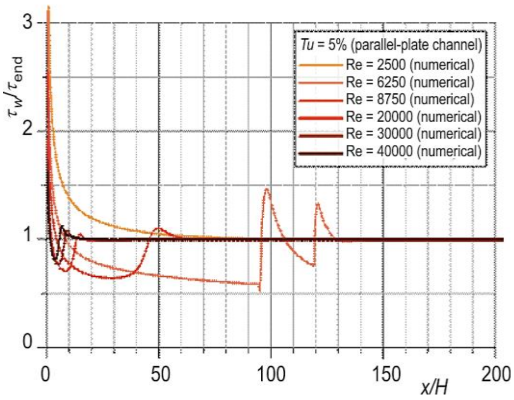

Figure 4. Relative wall shear stress for parallel-plate channel and turbulence intensity 5%
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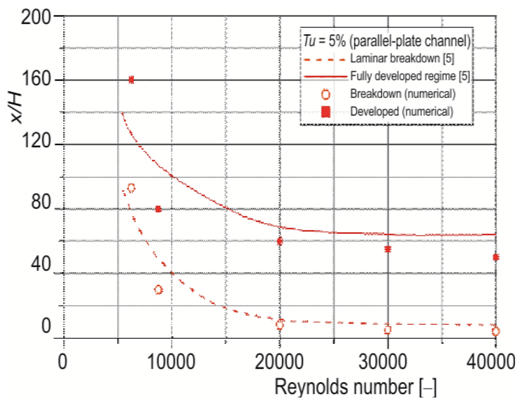

Figure 5. Locations of laminar breakdown and fully developed regime for parallel-plate channel and turbulence intensity 5%

of local shear stress from the Reynolds number on the walls of the flat flow channel. For small Reynolds numbers (Re ~ 6000) an instability of wall shear stress calculation in laminar-turbulent transition area was observed.

Based on the analysis of local shear stress, the location of the breakdown of the laminar flow and the appropriate location of the fully developed turbulent flow were determined, fig. 5. These points were determined according to [5]. Figure 5 presents the results of own calculations using the discrete system of points. Circular points represent the location of the interruption of the laminar flow form [5]. Square points describe the locations of the occurrence of the fully developed turbulent flow. In fig. 5, the relevant results of calculations obtained in the work [5] were also presented. The results presented in figs. 4 and 5 were obtained assuming the uniform profile of velocity of the fluid in the inlet cross-section of the channel and with the assumption of the same value of turbulence intensity Tu_0 = 5%. Comparison of the results of numerical calculations presented in fig. 5 obtained with the use of two different models of intermittency shows high degree of their compliance.

Figure 6 presents changes of local shear stress along the pipe wall in the flow described by the Reynolds numbers Re_1 = 11430, fig. 6(a) and Re_2 = 31100, fig. 6(b). Continuous lines indicate the results of numerical calculations obtained with the assumption of the homogeneous velocity profile in the pipe inlet cross-section. The results of experimental studies available in the literature are also presented in fig. 6 [10] with dotted lines. The results of experimental studies quoted in [10] were obtained under flow conditions described with the same Reynolds numbers Re_1 = 11430 and Re_2 = 31100. From the description of the experiment carried out in [10], it follows that the inlet cross-section of the pipe was rounded to reduce the disturbance in flow, but the turbulence intensity, Tu_0, value was not measured. Taking into account the fact of reducing disturbance in the flow inlet made in [10], three curves representing the results of numerical calculations for assumed three values of turbulence intensity are provided:

$$Tu_0^{(1)} = 3.5\%,\ Tu_0^{(2)} = 4.0\%,\ Tu_0^{(3)} = 4.5\%$$

Comparison of the local shear stress curves presented in fig. 6 shows the best compliance of the obtained numerical calculations for the assumed turbulence intensity $Tu_0^{(2)} = 4.0\%$ with the reported experimental results which were presented with Stanton number [10].

Figure 7 presents velocity profiles in selected cross-sections for the flow described above in the pipe, Re_1 = 11430 and Re_2 = 31100. The results of experimental research, characteristic for the fully formed turbulent flow, found in [11] are also presented in fig. 7. The velocity profiles were determined using the universal velocity distribution found by Reichardt and defined as in [11] with *R* as dimensionless radius [11]. Due to the use of algebraic model, only some discrepancies can be observed. These differences could be explained by overestimation of shear stress calculated by the model.

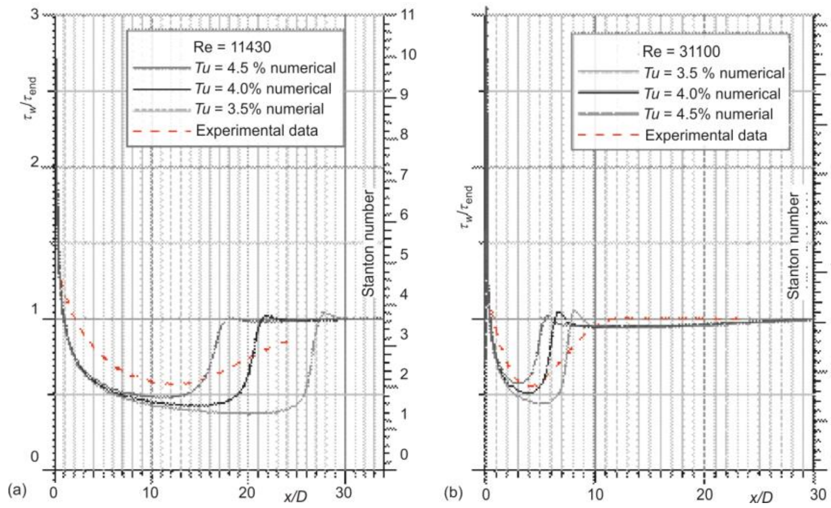

Figure 6. Comparison of numerical result and measurement data of relative wall shear stress in pipe for Reynolds number Re_1 = 11430 (a) and Re_2 = 31100 (b)

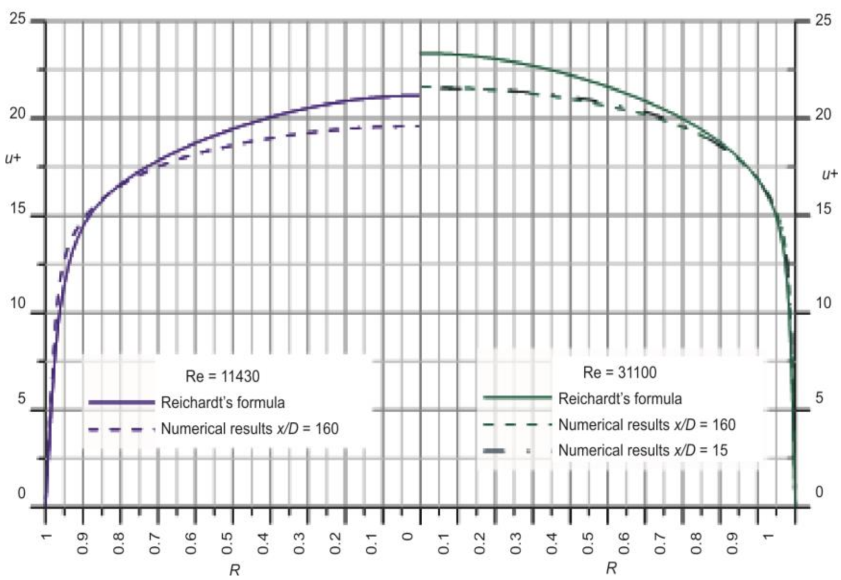

Figure 7. Velocity profiles and their comparison for Reynolds number (a) Re = 11430 and (b) Re = 31100

Concluding remarks

To describe the laminar-turbulent transition in the flowing fluid in the channel, an algebraic model for by-pass transition developed in the work [9] was adapted.

The original model [9] was intended for numerical simulation of flow in turbomachines (external flows). This work modifies the discussed model for the purpose of numerical simulations of internal flows performed in flat pipes or ducts. In particular, by validating the results of numerical calculations, values of the four constants of the original model [9] were modified using appropriate results of experimental studies found in the literature. Modified values of these constants are: $C_T = 15.5$, $A_T = 10.0$, $C_{SS} = 6.8$, and $a_2 = 0.55$. The largest change of model constants was made for C_{SS}, which could be a demarcation between internal and external flows.

It is worth noting that the modified algebraic model of the intermittency formulated in this paper is described by means of only two additional PDE related to the k-ω turbulence model [13]. The modified algebraic model of the intermittency enables precise numerical simulation of flows in the transitional area that is between laminar flow, intermittent and fully turbulent flow, both in pipes and flat channels.

In this work, the dependence of the friction factor on the Reynolds number and the turbulence intensity, Tu_0, was determined in practical applications. The aforementioned flow ranges were applied in hydraulically smooth pipes and flat ducts. For the selected values of the Reynolds number and the assumed values of the turbulence intensity Tu_0 in the inlet cross-section, the location of the breakdown of the laminar flow and the appropriate location of the fully developed turbulent flow were determined. Comparative analysis of the results of numerical calculations obtained in the work and the corresponding results of experiments and other numerical studies available in the literature show high degree of their compliance. The asymptotic values of shear stress on the walls of the pipe or flat channel obtained from numerical calculations and the profile of the dimensionless velocity component at the large distance from the inlet cross-section are very similar to the corresponding results of experimental research found in the literature.

For non-isothermal flows (*i. e*. occurring in heat exchangers) the coefficient of heat transfer in the area of laminar-turbulent transition have much higher values than the corresponding coefficients characteristic for laminar or turbulent flow. The effect of the increased value of the friction factor and the heat transfer coefficient is visible especially for small Reynolds numbers. So the improved model described in this work can be used for simulating flow in different types of heat exchangers where Reynolds nubers are relatively small (Re < 50000).

Nomenclature

a_1 – k-ω model constant (= 0.3)
a_2, A_T, C_{ss}, C_T – modified k-ω model constants
D – pipe diameter [m]
D_h – hydraulic diameter [m]
H – distance between parallel plates [m]
k – turbulent kinetic energy [m^2s^{-2}]
R – dimensionless radius
Re – Reynolds number ($=uD_h/\nu$)
S_{ij} – components of shear rate tensor [Pa]
S – magnitude of shear rate tensor ($= 2S_{ij}S_{ij})^{1/2}$
Tu – turbulence intensity
u – mean velocity [ms^{-1}]
x – co-ordinate parallel to flow direction [m]
x_{fd} – fully developed turbulent flow localization [m]
x_{trans} – laminar breakdown localization [m]
y – co-ordinate perpendicular to flow direction [m]

Δy – distance to the wall [m]

Greek symbols

α – k-ω model constant (= 0.52)
β, σ_d – coefficients defined in [9]
β^* – k-ω model constant (= 0.09)
γ – intermittency factor
δ_{ij} – Kronecker delta
λ – friction factor
ν – kinematic viscosity, [m^2s^{-1}]
ρ – fluid density [kgm^{-3}]
σ^* – k-ω model constant (= 0.6)
τ_{end} – wall shear stress at channel end [Pa]
τ_w – local wall shear stress [Pa]
ω – specific dissipation rate [s^{-1}]
Ω – magnitude of rotation rate tensor [s^{-1}]

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