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A method for the solution of the homogeneous inventory-production optimisation problem

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Abstract

The subject of this paper is the inventory-production problem, which is a one of the optimization problems in a decision area in which inventory volume and production volume are considered together. There are many approaches to this problem but for the first time, this problem is modelled by means of a capacitated graph network and a solution to the problem is proposed on the basis of this model which consists of finding the maximum flow with the minimum sum of production and inventory cost. In this article, only a solution for one kind of product for the deterministic inventory-production optimisation problem is presented and for this one kind of product, a maximum flow with a minimum cost for each considered demand period is calculated. The maximum flow with minimum cost is a solution to the homogenous inventory-production optimisation problem. The solution to the one kind of product for the inventory-production problem consist of maximum flow with minimum cost for a total demand from all periods, which has been taken into consideration.

Keywords: inventory-production problem, optimisation, capacitated graph network

1. Introduction

Satisfying consumer demand is one of the main goals of modern logistics management, and for this purpose, methods of forecasting the demand and production of individual products over a specific period of time are developed so that this demand is met whilst incurring minimal costs. Basically, we have two types of costs to consider: production costs and storage costs. These production and storage costs are not constant and fluctuate over the forecast time horizon. In order to determine the optimal production volume and inventory, deterministic and stochastic models have been developed (Baten, 2011; Samanta, 2004; Bayindira, 2007; Bhownick, 2011; Yaul, 2008; Rad, 2011) and methods of optimal control have even been have even been used for this purpose (Baten, 2009; Baten, 2010; El-Gohary, 2008; Al-khenhairi, 2010; Chaudhary, 2013; Bounkhel, 2005; Emamverdi, 2011), among which, one can distinguish the frequently frequently used method of dynamic programming (Olanrele, 2013; Bensoussen, 2011; Read, 1990). This article presents a new approach to solving the problem of optimizing the production and warehouse system. The problem was modelled differently than before because it employed a capacitive network graph. In order to solve the problem, a method for determining the maximum stream in the graph network at a minimal cost was adopted because the order of satisfying the demand in the periods of the considered time horizon needs to be strictly defined. This definition ranges from the nearest period to the most distant, and cannot be accidental. The classical method for determining the maximum stream in the graph at minimum cost is useless in the case of the problem of optimizing the production and warehouse system, so it needs to be specially adopted. This article presents a new approach to solving the problem of optimizing the production and warehouse system, namely the problem was modeled differently than before because it was modeled using a capacitive network graph and for which, in order to solve the problem, the method of determining the maximum stream in the graph network at a minimal cost was adopted, because the order of satisfying the demand in the periods of the considered time horizon must be strictly defined, ranging from the nearest period to the more distant, and cannot be accidental, which makes the method for determining the maximum flux at the minimum cost useless in this case, because this method does not take into account the order of satisfying demand and in some cases it could not fully meet the demand occurring in individual periods over the time horizon under consideration. Section 2 discusses the model of the capacitive graph for the problem of optimisation of the production and warehouse system, while in Section 3, the modified method for solving the problem in question is presented step by step.

2. Modelling the problem of production and warehouse system optimisation

The following notation applies:

N – number of periods
D_n – demand for production in period n; $n = 1, 2, ..., N$
x_n – state variable, stock level at the beginning of the period $n = 1, 2, ..., N$
d_n – decision variable in period n; production volume in the period $n = 1, 2, ..., N$
P_n – production capacity in period n; $n = 1, 2, ..., N$
W_n – storage capacity in period n; $n = 1, 2, ..., N$
c_n – unit cost of production in period n; $n = 1, 2, ..., N$
h_n – unit cost of storage in period n; $n = 1, 2, ..., N$

The mathematical model for optimising the production and warehouse system is as follows:

$$minf(x) = \sum_{n=1}^{N} \left(c_n * d_n + h_n * x_{n+1} \right)$$

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where: $x_{n+1} = x_n + d_n - D_n$
with constraints: $x_n + d_n \leq W_n$, $d_n \leq P_n$
and $x_n + d_n \geq D_n$

The following assumptions were made to construct a mathematical model:

1. The average periodic demand fluctuates.
2. The model takes into account the volume of production and the state of warehouses at the same time.
3. The single goal of optimisation is to minimise total costs.The goal of optimisation is single: it is to minimise total costs.
4. The model is deterministic.
5. Storage shortages are not allowed.
6. Unit production and storage costs fluctuate from period to period.

The above mathematical model can be presented using a network graph with capacity constraints. A one-day production period was modelled as it is shown in Fig. 1 and a 3-day production period was modelled with the demand being satisfied on Day 3 as it is shown in Fig. 2. First, model the one-day period as shown in Figure 1 and then the 3-day production period with satisfied demand on Day 3, as shown in Figure 2.

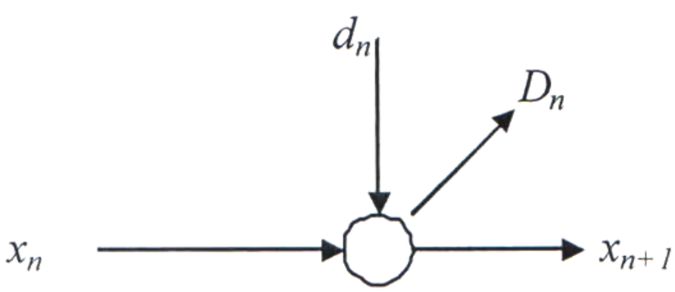

Fig. 1. One day period

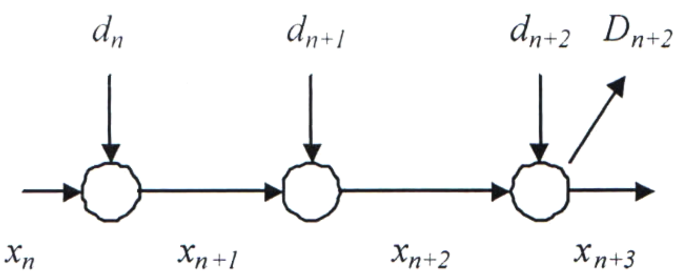

Fig. 2. A 3-day period with demand being satisfied on the third day

Six one-day periods with demand satisfied on days 3 and 6 are presented in Fig. 3 as a graph network with source S and flow T and capacities assigned to the edges in square brackets, which represent production capacity d_i, storage capacity w_i and demand D_i in period i, as well as the initial stock level x_0 at the beginning of period 1, which is less than or equal to the storage capacity $x_0 \leq w_0$.

The problem of optimising the production and storage system is to determine the maximum stream of products flowing in the graph network with the production capacity constraints P_i assigned to edges, the warehouse capacity constraints W_i assigned to vertices, the production costs c_i and worehouse cost h_i and demands D_i which are satisfied at determined days. There are many cases of maximal flow for optimal satysfing demands in determined days in a period, which is taken into consideration, but we are looking for one of them, which has the lowest total costs of the production and the inventory. The problem of optimising the production and storage system is to determine the maximum stream of products flowing in the graph network with the capacity constraints assigned to the edges, which for a particular case is shown in Figure 3, taking into account the total minimum production and storage costs, and therefore the maximum stream at a minimum cost is the solution to the problem optimisation of the production and warehouse system and the method of determining this

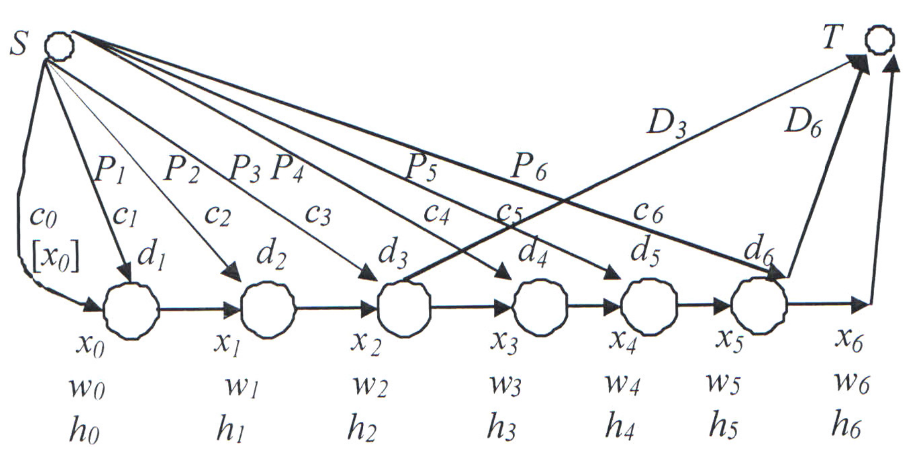

Fig. 3. Time horizon consisting of 6 one-day periods

maximum stream can be adapted to determine the optimal parameters of the production and warehouse system, which are the optimal values of the production of inventory on individual days in the solved horizon. The maximum flow balances the aggregate demand over all one-day periods over the time horizon under consideration.

3. The proposed method

The developed method for solving the problem of optimising the production and warehouse system uses the proposed model for the problem under consideration, which is a graph network with capacity constraints assigned to the edges. The maximum stream at a minimum cost is shaped by the production and maintenance plan for the inventory in order to meet consumer demand over a resolved time horizon. For demand occurring in periods over the time horizon under consideration, the optimal stream is determined at the minimum cost of satisfying it.

The application of the presented method for the case of the problem of the optimisation of the production and warehouse system presented in Fig. 4 is presented step by step in the next part of the article. The complete procedure for determining the optimal parameters of the production and storage system as a result of determining the maximum stream of products flowing through the system at minimum production and storage costs while ensuring complete satisfaction of demand in the time horizon components is presented as Procedure 1.

Procedure 1.

```
Begin
    for each period with D_i demand, ranging from nearest to more distant,
    begin
    repeat
        find the cheapest path from S to T passing through the vertex of
        period i
        add the cost of this path to the total cost of meeting D_i demand
        send the maximum stream along the designated path
        add the designated stream to the total flow for demand D_i
        upgrade edge capacities
    until (demand is fully met in the period)
    sum up total minimum costs determined for demand D_i
    end
Return: maximum stream with minimal cost
End.
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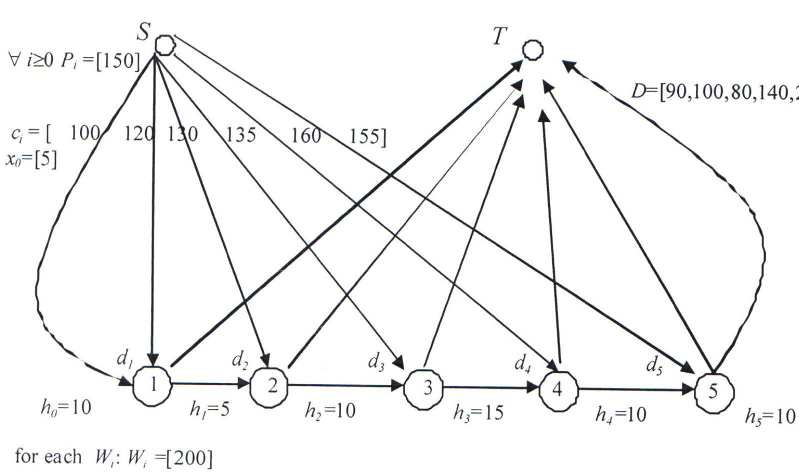

Fig. 4. The case of the problem of optimisation of the production and warehouse system

The proposed method starts with the first cheapest path first found cheapest path S-x_0-1-T with a cost of $c_0 + h_0 = 100 + 10 = 110$ and the maximum possible stream of products is sent along this path, i.e. the initial stock level $x_0 = [5]$. Then it remove it from the graph. The second cheapest next cheapest path is S-1-T with a cost of $c_1 = 120$ and the maximum stream transmitted along this path is 85 and in this way, the 1-T edge is saturated, which means that the total demand $D_1 = 90$ is satisfied. After these two steps, the current state of the problem looks as shown in Fig. 5. The updated capacity value for the first period is now $W_1 = 120\text{-}5\text{-}85 = 110$, while the updated capacity value for the first period is $P_1 = 65$. The demand of the first period is now fully satisfied, and therefore the updated value of the demand of the first period is now $D_1 = 0$.

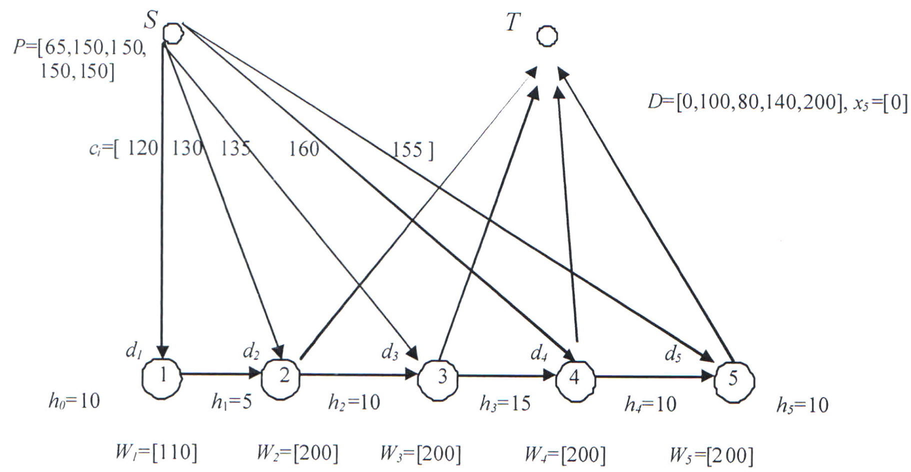

Fig. 5. Optimisation status of the production and warehouse system after two steps

At this point, the next cheapest path is the S-1-2-T path with a cost of $c_1 + h_1 = 120 + 5 = 125$ and a stream of 65 is sent along this path as it is the updated value of production capacity P_1. Then the cheapest path is the S-2-T path with a cost of $c_2 = 130$ and a stream of 35 is sent along this path. The flow along the S-2-T together with flow along the S-1-2-T path now satisfies total consumer demand in period 2. because it is the level of the stream value that with the previously determined stream transmitted along the S-1-2-T path in its entirety satisfies consumer demand from Period 2. The current state of the problem of optimising the production and warehouse system is shown in Fig. 6.

Then the cheapest path is the S-3-T path with a cost of $c_3 = 135$ and the stream of products sent along this path satisfies the total consumer demand $D_3 = 80$ in Period 3. The current state of the production and warehouse system is shown in Fig. 7.

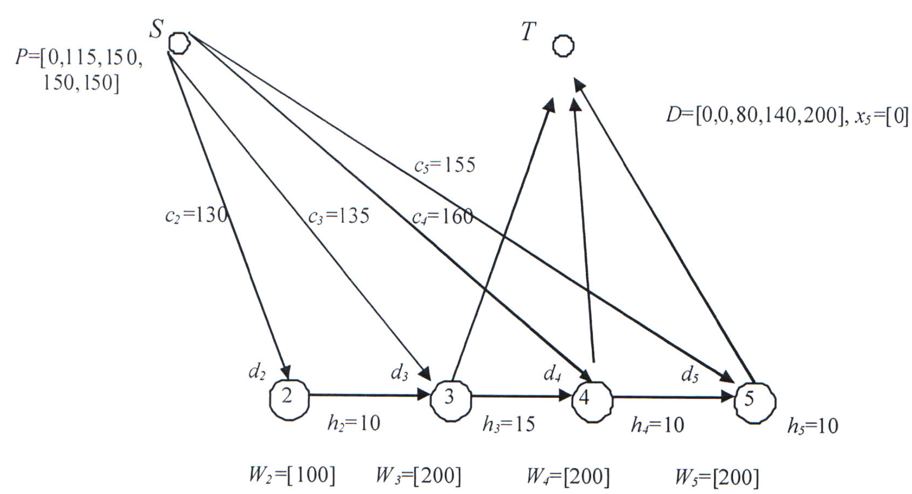

Fig. 6. Optimisation status of the production and warehouse system after four steps

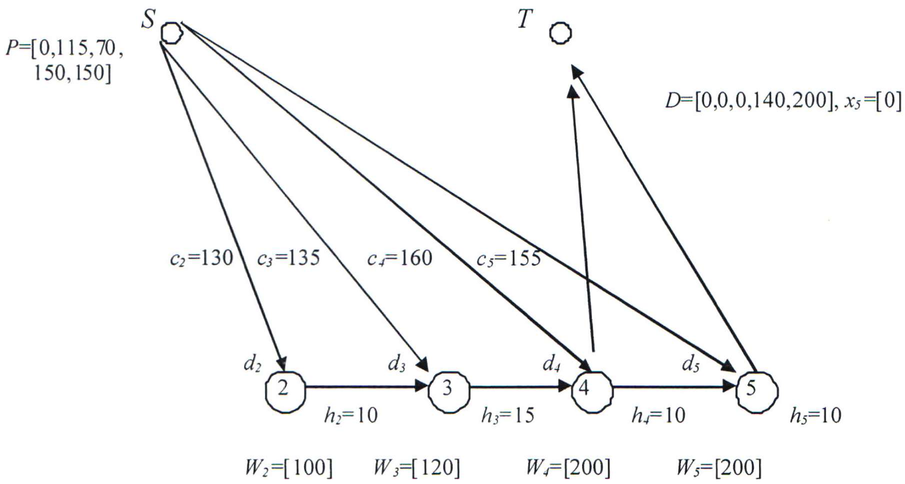

Fig. 7. Optimisation status of the production and warehouse system after five steps

The next cheapest path is the *S*-3-4-*T* path with a cost of $c_3 + h_3 = 135 + 15 = 150$ and a stream of 70 products is sent along this path, as this is the level of production capacity available in Period 3. After this step, the current state of optimisation of the production and warehouse system is shown in Fig. 8.

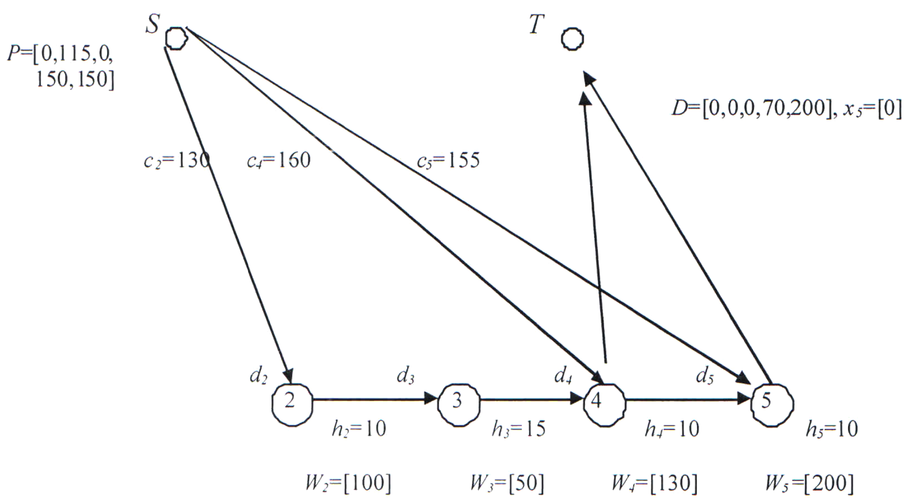

Fig. 8. Optimisation status of the production and warehouse system after six steps

Now, for consumer demand D_4 the cheapest path is *S*-2-3-4-*T* with a cost of $c_2 + h_2 + h_3 = 155$. A stream of 50 products is sent along this path and the current state of optimisation of the production and warehouse system is shown in Fig. 9.

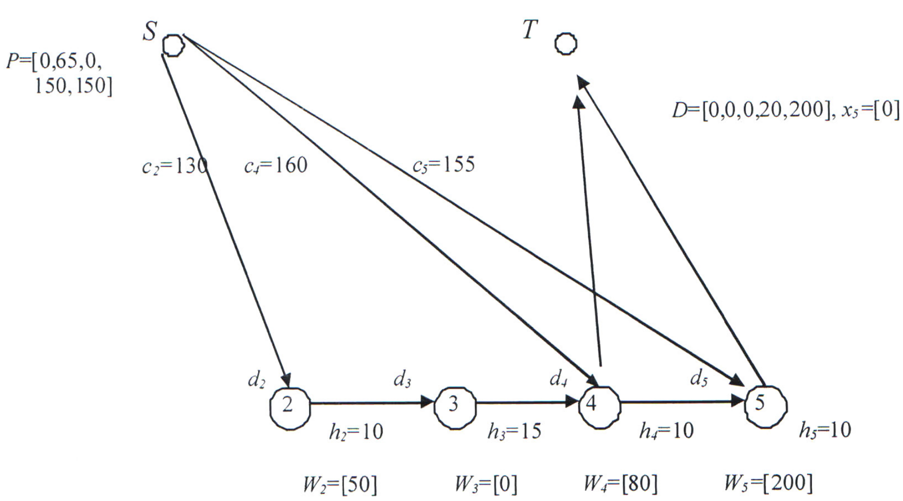

Fig. 9. Optimisation status of the production and warehouse system after seven steps

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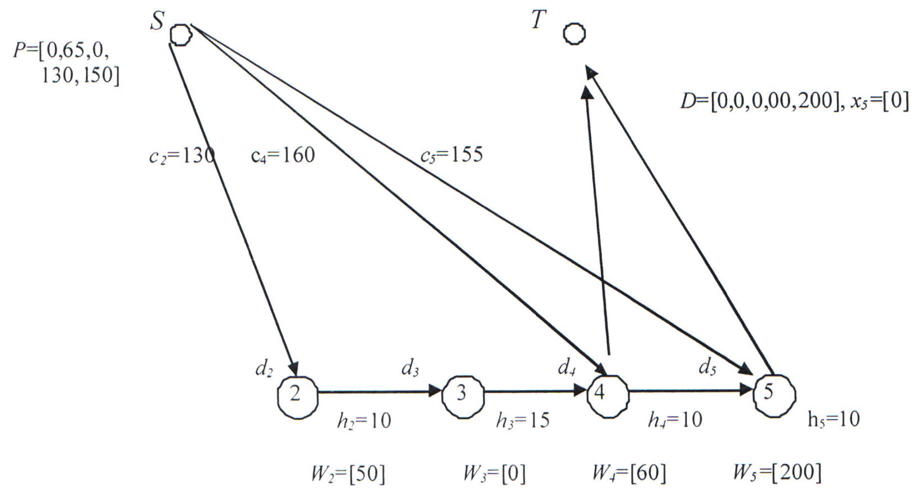

Fig. 10. Optimisation status of the production and warehouse system after eight steps

Currently, the *S*-2-3-4-*T* path cannot be used despite the fact that there are production capacities in the second period P_2 = 65 because it is not possible to store these products in a warehouse in Period 3 due to the lack of storage capacity W_3 = [0] in the third period. For consumer demand D_4, the cheapest path is the *S*-4-*T* path with a cost of c_4 = 160 and a stream of 20. The current state of production and warehouse system optimisation is shown in Fig. 10.

At this point, the cheapest path is the *S*-5-*T* path with a cost of c_5 = 155 and a stream of 150, and this cheapest path is removed from the graph and now there is the another cheapest cheapest path *S*-4-5-*T* in graph with a stream of 50 and a cost of $c_4 + h_4$ = 160 = 10 = 170. Both of these pathways are used to meet consumer demand in Period 5.

In the manner presented above, the total consumer demand from all periods in which it occurred in the considered time horizon was satisfied. The total cost of production and storage was 87,425. The optimal production and inventory levels determined are presented in Table 1.

Table 1. Production plan and inventory levels in the considered time horizon

period	0	1	2	3	4	5
production	0	150	85	150	70	150
warehouse	5	65	50	120	50	0
demand	0	90	100	80	140	200

4. Conclusion

This article presents a new approach for the deterministic optimisation of a homogeneous production and warehouse system. Optimal inventory levels and production volumes for individual periods while maintaining restrictions on production and storage capacities were determined using the adopted method of determining the maximum flow at minimum cost in a weighted graph network. According to this method, the cheapest paths to meet consumer demand are determined, through which, product streams limited by production and storage capacities assigned to the edges of the graph network as capacitive scales are sent. The solution to the problem obtained is deterministic and optimal. The presented method of solving the problem is a new approach and can be extended to other cases of the optimisation of production and storage systems, those that take into account stochastic consumer demand, stochastic volume of production or cases of heterogeneous systems as well as other cases not listed here.

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