technical transactions
No. 2020/042
mathematics
sciendo

An Explanation of the Landauer bound and its ineffectiveness with regard to multivalued logic

Radosław A. Kycia
kycia.radoslaw@gmail.com | http://orcid.org/0000-0002-6390-4627
Faculty of Materials Science and Physics, Cracow University of Technology

Agnieszka Niemczynowcz
niemaga@matman.uwm.edu.pl | http://orcid.org/0000-0002-4370-3326
University of Warmia and Mazury, Faculty of Mathematics and Computer Science

Received: October 13, 2020
Accepted: November 18, 2020

Data Availability Statement: All relevant data are within the paper and its Supporting Information files.

Funding: RK was supported by the GACR grant GA19-06357S and the Masaryk University grant MUNI/A/0885/2019. We would like to acknowledge COST CA18223 action for support.

Competing interests: The authors have declared that no competing interests exist.

Citation: Kycia, R.A., Niemczynowicz, A. (2020). An Explanation of the Landauer bound and its ineffectiveness with regard to multivalued logic. *Technical Transactions, e2020042*. https://doi.org/10.37705/TechTrans/e2020042

Abstract

We discuss, using recent results on the thermodynamics of multivalued logic, the difficulties and pitfalls of how to apply the Landauer's principle to thermodynamic computer memory models. The presentation is based on Szilard's version of Maxwell's demon experiment and use of equilibrium Thermodynamics. Different versions of thermodynamic/mechanical memory are presented – a one-hot encoding version and an implementation based on a reversed Szilard's experiment. The relationship of the Landauer's principle to the Galois connection is explained in detail.

Keywords: Landauer's principle; entropy; multivalued logic; encoding; the second law of thermodynamics; thermodynamic memory implementation; Galois connection

1. Introduction

In thermodynamics, Maxwell's demon paradox was a long unresolved problem until the nineteen-sixties when R. Landauer postulated (Landauer, 1961) the bound for heat Q emitted during the erasure of one bit of information to be no less than the Landauer's bound $k_B T ln(2)$, where k_B is the Boltzmann constant, T is the temperature of the environment in which the memory is embedded, and ln is the natural logarithm. It was then suggested by Bennet (Bennett, 1973, Bennett, 1987) that this bound can be applied to the demon's memory in the cycle to make the second law of thermodynamics applicable to Maxwell's demon experiment and to resolve this long-lasting paradox.

Currently, the Landauer principle is the subject of substantial experimental investigation (Bérut, 2012), including quantum level (Yan et al., 2018), as well as, understood on the level of classical (equilibrium) thermodynamics, where it is equivalent to the second law of thermodynamics (Parrondo, Horowitz, Sagawa, 2015, Bennett, 1973, Bennett, 1987, Bormashenko, 2019) on the grounds of statistical physics (Piechocinska, 2000, Sagawa, 2014) or under theoretical generalisations (Ladyman, Presnell, Short, Groisman, 2017, Kycia, 2019, Sagawa, 2014) and even abstract formulations using category theory (Kycia, 2018, Kycia, 2019) with many potential applications. Therefore, the Landauer's principle is solid stated law.

As it has been pointed out in many places (for an excellent review, see the literature (Hayes, 2001), the ternary system can be seen as a more optimal coding base for numbers than the classical binary system used in contemporary computers. This results from the maximisation of the expression $ln(B)/B$, which is the average information per digit in the system (Reza, 1994) and associates information for a number system of B letters (digits) per element of the alphabet. The extreme value is reached at $B = e$ – the base of the natural logarithm. Since $B = 3$ is closer to e than $B = 2$, it is suggested that the trit-base system is closer to optimal encoding (Hayes, 2001).

However, in nature, we are also accustomed to the systems with higher than three base, e.g., in living organisms or DNA computing, the 'bits' of DNA and RNA consisting of four fundamental chemical components. In addition, in human culture, systems based on $B = 10$, $B = 12$, $B = 16$ or even $B = 60$ are common.

There has been a recent attempt to merge the Landauer principle with non-binary base memory systems. It has been presented (Bormashenko, 2019) that the Landauer principle is applicable for trit memory. The physical bound for trit is $k_B T ln(3)$; however, the correcting factor from the efficiency of coding the $B = 3$ system in the binary system is $\log_2(3)$, which restores the original Landauer's bound. It was also suggested that the Szilard version (Szilard, 1929) of Maxwell's demon may also be used as a memory model for trit, which deserves much more elaboration and extension to multivalued logic.

In this paper, we want to explicitly explain with full details, using the Szilard's approach to Maxwell's demon experiment, that the Landauer's bound is valid for a system with an arbitrary base. The presentation is provided using the quasistatic setup of classical thermodynamics for simplicity. We want to strongly underline that all the classical results on Szilrad's realisation of Maxwell's demon were thoroughly explored in various setups, and we do not claim we present something new. We present a new insight on the use of the different base of logic in memory and its connection with Landauer's principle, an issue which was recently raised in (Bormashenko, 2019). We also present an insight on the efficiency of coding and its connection with Landauer's bound, as well as some practical examples of the realisation of the Galois connection in memory systems that have recently been proposed (Kycia, 2018, Kycia, 2019). We think that this adds a better understanding of the Landauer principle in multivalued logic.

The paper is organised as follows: First, we present the original Szilard's experiment and apply it to bits. We then show, by the second law of thermodynamics, how an extension of this experiment can be applied to deduce

https://doi.org/10.37705/TechTrans/e2020042

Landauer's bound. We will also discuss some potential problems in the naive application of the Landauer principle to multivalued logic. Finally, it is presented how mechanical memory can be implemented and at which stage of this implementation, heat is generated for irreversible operation

2. Setup for bits

The standard Szilard's version of Maxwell's demon for binary computations (Szilard, 1929, Kycia, 2018, Bormashenko, 2019) consists of a box with a single particle of an ideal gas that fulfils the equation of state $pV = k_B T$, which is a one-particle version of the equation $pV = nRT$, where n is the number of moles of gas and $nR = N k_B$, where N is the number of particles. It is in the thermal bath of the box at temperature T. The 'demon' is the additional device that schedules the cycle. This situation is presented in Fig. 1. The cycle consists of the following steps:

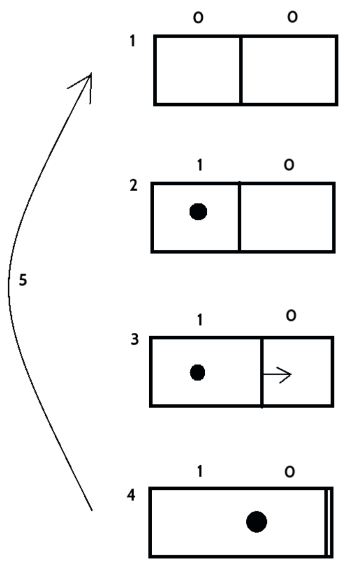

Fig. 1. The Szilard's version of Maxwell's demon

1. The border is put in the middle of the box, splitting the whole volume V into halves. The state of the memory of 'the demon' is erased in the state $[0, 0]$.
2. The particle is localised in, for example, the left part of the box. Here 'the demon', registers the state of the box in its memory by changing its state to $[1, 0]$.
3. In this step, the border becomes movable, and the gas expands from the volume $V/2$ to V isothermally giving the work

$$W = \int_{V/2}^{V} k_B T \frac{dV}{V} = k_B T ln(2).$$

4. When the border reaches the right end of the box, the extraction of the work stops.
5. In order to return to the initial step, the border must be moved to the middle by taking it out from the box. It can be done (assuming no friction) without any work and with no heat generated. In this transition, the knowledge on the position of the particle is lost, and thus, the demon's memory is no longer correlated with its state. Therefore, it can be deleted in order to restore initial conditions for the new cycle.

Since the system operates in the cycle the Second Law of Thermodynamics is valid. This takes the form of Theorem 1 (see below).

Theorem 1 The Second Law of Thermodynamics (Kelvin) (Frankel, 2011)
In the quasi-static cyclic process, a quantity of heat cannot be entirely converted into its mechanical equivalent of work.

However, the system does not return any heat to the environment, and therefore the law is broken. In order to force the system to obey the law, it must return an amount of heat to the environment that is no less than the work done, i.e., $Q \geq k_B T ln(2)$. As pointed out by Bennet (Bennett, 1973, Bennett, 1987), this is precisely the Landauer's bound and can be associated with erasing the demon's memory in transition 5 in Fig. 1.

This experiment can also be seen as the thermodynamic implementation of the computer memory, and we will use it in the next section to describe the details of the calculation of Landauer's bound for systems higher than binary systems. Although a specific thermodynamic system was selected, the final result will not depend on specific characteristics of this particular system, which suggests that the result is universal.

3. One-hot encoding and the Landauer's bound for memory of B-ary system

The $B > 0$ values can be uniquely coded in a binary system using so-called one-hot encoding. This method is currently used in machine learning for encoding categorical values (Raschka, Mirjalili, 2017). Due to this coding, the alphabet is the set of digits/letters of B-system and the resulting information is the unique code of each element of this alphabet in the B bits. For B values (from 0 to $B - 1$), such encoding is as follows:

$$\begin{aligned} b_1 &= 0 \Leftrightarrow 0\underbrace{\ldots}_{B-1}01 \\ b_2 &= 1 \Leftrightarrow 0\underbrace{\ldots}_{B-2}010 \\ b_B &= B-1 \Leftrightarrow 10\underbrace{\ldots}_{B-1}0 \end{aligned} \tag{1}$$

We can also add the value $b_0 = 0\underbrace{\ldots}_{B}0$, which does not belong to the encoding, but we add it since it is used as an undetermined value. The coding is one of the worst, since there is an abundance of digits, and since the numbers with two, three etc. 1's, are not used. The efficiency of this coding (see Reza, 1994), equation (4–2)) is

$$E = \frac{log_2(B)}{log_2(2)} = log_2(B) \tag{2}$$

This quantity puts the bound on how many bits is necessary to encode the information.

Consider now the following experiment with one particle of ideal gas in the box, in a similar manner to Szilard's idea. The cycle consists of the following steps:

1. START: Put $B - 1$ borders equidistantly[1] inside the parallelepiped box of volume V, so that we get B chambers of volume V/B.
2. LOCALISE: Localise the particle in the chamber encoded by a 1 in the sequence, e.g., b_i.
3. EXTRACT: Start to isothermally expand the particle from the chamber of volume V/B to the selected ONE nearest the chamber finishing with the particle in the chamber of volume $2V/B$. This allows the extraction of the work of this minimal expansion

$$W_{min} = \int_{V/B}^{2V/B} k_B T \frac{dV}{V} = k_B T ln(2) \tag{3}$$

4. RESET: Reset the system by taking out all borders from the box that particle can again move freely. Then put the borders into the box equidistantly again and reset the memory of b_i. It is irreversible free expansion with no work and no heat generation or consumption.

The situation for trit is presented in Fig. 2.

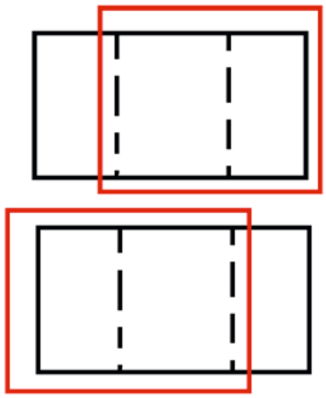

Fig. 2. Minimal expansion for trit. The chambers which take part in the EXTRACT part (closest neighbours) are marked in red

In this approach, everything depends upon which base the memory that stores the information on the particle localisation is constructed.

For binary memory, we have a full one-hot encoding of the numbers $\{b_1, \ldots, b_B\}$. In this case, in the EXTRACT step, we can trace the location of the particles up to the V/B volume, in the sense that if for trit $(B = 3)$ the initial data was $b_1 = 0 = 001_2$,

[1] If we would use non-equal volume splitting into B parts, then averaging of the emitted heat for each configuration must be used - this reflects the fact that the Landauer's principle results from statistical considerations and averaging (Piechocinska, 2000, Sagawa, 2014) on the most fundamental level of statistical physics.

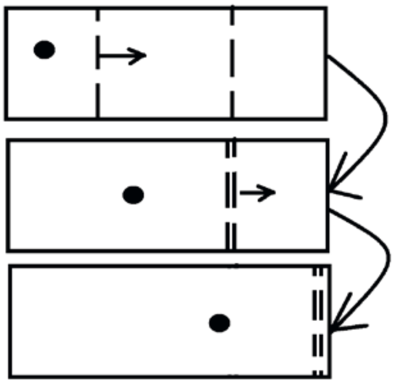

Fig. 3. The case of 'Maximal' expansion where both pistons are shifted to the V boundary and the gas makes a work
$W = k_B T \int_{V/3}^{V} \frac{dV}{V} = k_B T ln(3)$,
which is bigger and needs the same or larger compensation by expelled heat in order to preserve the second law of thermodynamics

then after expansion, it will be coded as $a = 011_2$, which is not allowed by the one-hot encoding range given in (1). Resetting the cycle is then equivalent to the erasure in the binary Szilard experiment described in the previous section, and therefore, generates the Landauer's bound for heat.

For memory based on the system with B states (B-it), the EXTRACT part gives a state which cannot be described by a single number from the B-it system. Therefore, the RESTORE part resets the system and memory, giving the same Landauer's bound for erasing a single B-it for the base B system.

This is an alternative approach to the argument presented in the literature (Bormashenko, 2019). It also explains the minimality of Landauer's bound. The above discussion shows that if the memory cannot store more coarse information about the state of the system, then the Landauer's bound for heat must be calculated concerning the minimal change of the system that cannot be stored in the memory. If we expand the initial chamber to the whole volume V, the work is $W_{max} = k_B T ln(B)$ and this is also a minimal bound for the heat in this experiment (resulting from the second law of thermodynamics), which is higher than the Landauer's bound. To our knowledge, this observation was not made before for $B > 2$, even though it is a simple derivation of the case $B = 2$.

We can also consider more substantial changes in a system using more than two neighbour cells. This will be considered in the next section.

4. More general decompression

Now consider the same experiment as before with the expansion of the single ideal gas particle to the $2 < N \leq B$ neighbour chambers. For trit, there is only one such expansion for $N > 2$, and it is 'maximal'. This is presented in Fig. 3.

In this case, the particle isothermally expanding from the V/B initial volume chamber to the NV/B final volume makes the work

$$W_N = k_B T \int_{V/B}^{NV/B} \frac{dV}{V} = k_B T ln(N) \tag{4}$$

which is independent of B. This work also puts a minimal bound for the heat of erasure of the information by expanding to N chambers and is required by the second law of thermodynamics. Comparing this to minimal decompression (3) we get the increase of work we did due to additional (non-minimal/non-optimal for coding) decompression

$$E_N = \frac{W_N}{W_{min}} = log_2(N) \tag{5}$$

which is also the effectiveness of the coding of the N different values using binary coding. For example, for $N = 3$ (and $B = 3$) we get the result of previous work (Bormashenko, 2019). However, in general, it can be used for $N = 3$ and $B > 3$.

In this context, the correction factor $log_2(3)$ in (Bormashenko, 2019) for efficiency of encoding is coincidental since $B = N = 3$ (for trit) – the equation (2) is then the same as (5). In general, the number of individual chambers B can be larger than the number of chambers which are merged in decompression. This remark will be even more visible when we discuss the implementation of the memory in the next section.

5. Thermodynamical memory

In this section, the thermodynamic realisation of memory is presented. It is provided here to avoid indirect derivation of Landauer's bound from the previous chapter and possible 'circular argument' accusations. The optimal implementation that reaches Landauer's bound and then non-optimal implementations is presented. We also provide a connection of these implementations with the Galois connection associated with memory systems (Kycia, 2018, Kycia, 2019).

5.1. One-hot optimal implementation

This implementation is based on one-hot encoding. First, the single bit is constructed by reversing the construction from the Szilard's description. This situation is presented in Fig. 4. It is a box with a movable border and a single particle of an ideal gas in thermal contact with thermostat (environment) of temperature T. There is an additional source of energy that powers the memory which is not visible in the figure. The representation of a bit is as follows:

- a) zero bit 0;
- c) one bit 1;

and the transitions:

- b) isothermal (reversible) compression from the volume V to $V/2$. The heat
- expelled to the environment is $Q = -k_B T \int_V^{V/2} \frac{dV}{V} = k_B T ln(2)$
- isothermal (reversible) decompression (opposite to b)) from c) to a), which extracts the heat $Q = k_B\, T\, ln(2)$ from the environment and converts it to work.
- transition from c) to a) that results from the adiabatic free decompression that is realised by taking out from the box the border from the middle and putting it to the box along one of the borders of the box. It is free (irreversible) decompression with no work or heat generated or consumed.

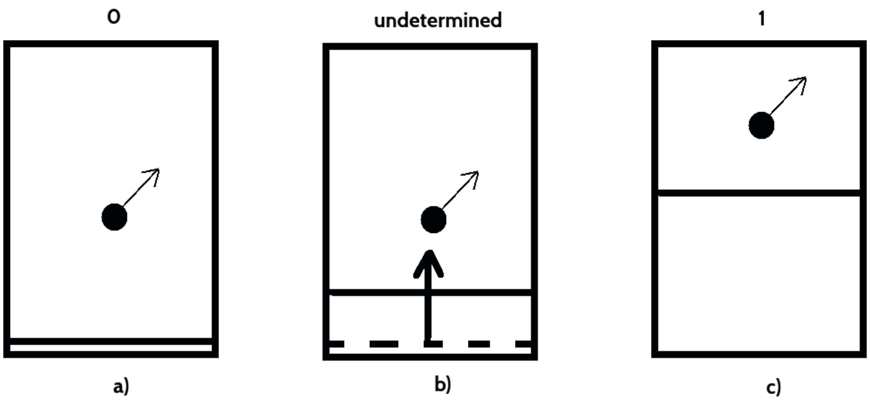

Fig. 4. An implementation of a single bit of a memory. It consists of a box with a movable border and a single particle of a gas in the thermal bath of temperature T. a) state encodes 0, b) represents isothermic compression from the volume V to $V/2$ and represents the undetermined (internal) state of memory, c) represents the state 1

As was pointed out in the literature (Kycia, 2019), the implementation of the memory can be associated with the Galois connection. It is the relationship between two sets[2] that have some ordering relations of elements – they are called pre-ordered sets, or posets for short. The Galois connection consists of two maps between sets in opposite directions, which preserve this ordering. It is a prototype of more general connections between two categories called adjointness (Smith, 2019, Fong, Spivak, 2019). For physical applications, it is probably too weak a notion (it is not isomorphism) to be useful; however, it has an interesting interpretation which sheds some light on its occurrence in this context. It represents the relationship between theories and their

[2] Generally, the Galois connection is a relationship between two ordered categories, which are not necessary sets; however, in this presentation, we restrict ourselves to a less general Galois connection between ordered sets.

https://doi.org/10.37705/TechTrans/e2020042

implementations on the model[3], or alternatively, between the abstraction map (from model to theory) and realisation map (from theory to model). This scenario is evidently present in considerations of implementing Boolean algebra on a physical memory device. There are two levels – logic and a physical system upon which the logic is implemented by labelling specific configurations as is presented hereafter.

Introducing the ordering at the level of bits $0 \leq 1$ we get poset A. At the level of physical implementation of the bit, we have ordering $a \leq b \leq c$ we get poset B. The implementation of binary logic in physical memory can then be presented by the mapping (functor) $f\colon A \to B$, and the mapping (functor) $g\colon B \to A$ presented in Fig. 5. The map fulfils the following Galois connection condition (Kycia, 2018, Smith, 2019, Fong, Spivak, 2019)

$$f(p) \leq q \Leftrightarrow p \leq g(q) \tag{6}$$

for $p \in A$ and $q \in B$. That is f is a left adjoint to g, i.e., $f \dashv g$ and not the otherwise as it can be checked by inspecting all possible pairs p, q in mapping.

In addition, the transitions can be

- $a \to b \to c$ – reversible isothermal compression operation associated with logically reversible (bijective, see Sagawa, 2014) NOT: $0 \to 1$. Heat emitted to the environment $Q = k_B T ln(2)$.
- $c \to b \to a$ – reversible isothermal decompression operation associated with logical reversible NOT: $1 \to 0$. Heat absorbed $Q = k_B T ln(2)$.
- $c \to a$ – irreversible free decompression operation associated with logical irreversible deletion $x \to 0$.

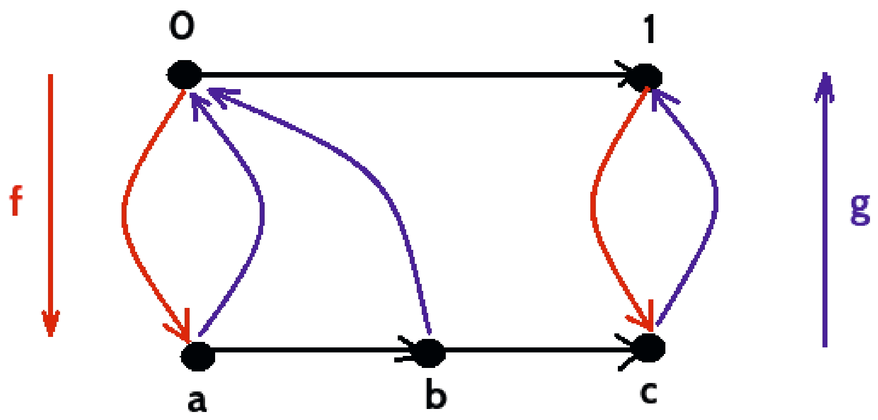

Fig. 5. Mapping between logical states and the states of physical realisation of memory. The arrows between poset states are given by its ordering, i.e., since $0 \leq 1$, $0 \to 1$, which is the usual convention for treating a poset as category on its own (Smith, 2019, Fong, Spivak, 2019)

Table 1. Table 1. System A is implemented on B

Possibilities	B Reversible	B irreversible
A reversible	YES	YES
A irreversible	NO	YES

As it can be noted, if the NOT reversible operation is applied two times, then there is no net heat emitted or absorbed. However, if reversible NOT and then deletion will be performed then the net heat, equal to the Landauer's bound, will be emitted, as it is required by the Landauer principle or the Second Law of Thermodynamics. This observation was generalized in the following Table 1 (see Ladyman, Presnell, Short, Groisman, 2017, Kycia, 2018), which explains which types of operations can be realized between both Galois (Ladauer's) connected systems.

[3] One poset is a set of theories ordered by finer assumptions, and the second poset is a set of models for these theories also ordered by finer details. See the literature for details (Smith, 2019, Fong, Spivak, 2019).

This single-bit memory cell can be composed to implement a one-hot encoding of the state of the extended Szilard minimal expansion experiment presented in the previous section. The deletion of the memory in this experiment is connected with one bit; therefore, the Landauer's bound is achieved.

The Galois connection is represented in this case introducing partial order in one-hot encoding set for $i = 1 \dots N$ that makes poset A. At the level of memory implementation, the ordering is induced by the ordering for a single bit ordering from Fig. 5 because only one bit in the memory is changing.

A final remark on the real implementation of the optimal memory – in Szilard's version of the experiment, only a single bit (a single particle) must be traced, and therefore 0 bit can describe the undetermined state or no particle state. However, when both 0 and 1 bits have some meaning in the experiment, then the single bit must be implemented, as in Fig. 6.

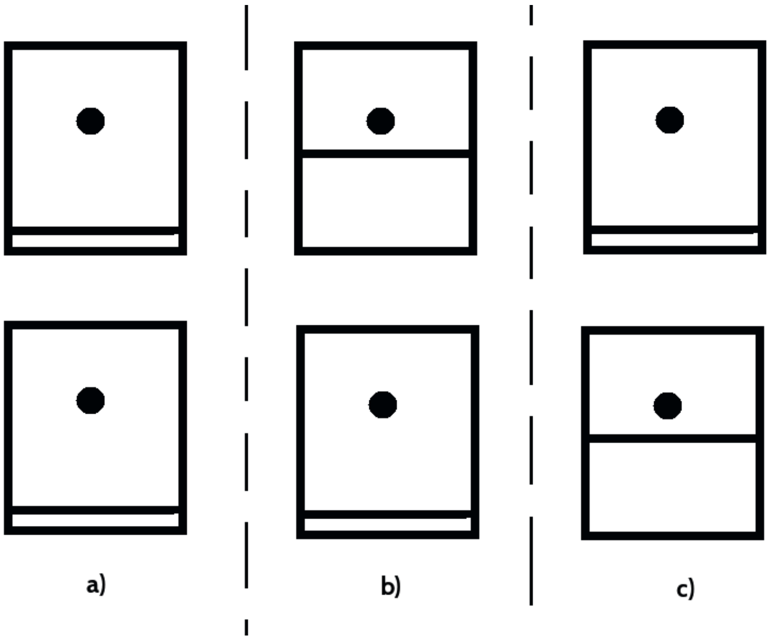

Fig. 6. One-hot encoding of a single bit thermodynamic model: a) represents undefined state; b) represents, say, 1; c) represents 0 bit. Reversible operation $0 \leftrightarrow 1$ can be made by making the pair of isothermal decompression-compression sequence on complementary up-down chambers that pass through a) state. An irreversible operation engages free decompression to a) state and then a transition to the required configuration (0 or 1) by isothermal compression, which generates Landauer's heat bound. The ordering is $b \leq a \leq c$ (in more detail, $a \leq b$, $a \leq c$ and $b \leq c$) which corresponds to the ordering $0 \leq 1$ on a logical level

5.2. Non-optimal vs optimal implementation

We propose the implementation of thermodynamical memory, which during (irreversible) erasure, generates more heat than Landauer's bound. It can be made simple by using the one-hot implementation from the above subsection and using irreversible compression instead of isothermal (reversible) process. This is off-diagonal 'YES' case in Table 1.

Another possibility and this will be an elaboration of the idea noted in (Bormashenko, 2019), is by a reversal of non-minimal expansion Szilard's version of the experiment. In this approach, the memory has some abundance of internal states not used in the representation of bits. This makes that the Landauer's bound is not reached. An example of this situation for the trit is presented in Fig. 7. For the sake of clarity, Galois connection is given in Fig. 8. This idea can be extended to arbitrary base $B > 1$. Consider an irreversible free decompression $31 \rightarrow 21$ and then reversible operation $21 \rightarrow 32$, which corresponds to logical irreversible transition $1 \rightarrow 2$. Irreversible operation $31 \rightarrow 21$ generates no heat, however isothermal compression $21 \rightarrow 32$ from volume $2V/3$ to $V/3$ generates the heat $Q = k_B T ln(2)$, so the Landauer's heat is attained and this implementation is optimal too. It is even less complex than one-hot encoding.

Consider the same transition that engages irreversible free decompression $31 \rightarrow 1$ and then isothermal compression $1 \rightarrow (21 \vee 22) \rightarrow 32$ from volume V to $V/3$, which generates the heat $k_B T ln(3)$. This path engages the states hidden on a deeper level than the layer neighbour to the one giving the representation of digits for trit, and gives the heat greater than the Landauer's bound.

This example shows that the optimal implementation of the memory can be accomplished by reversing Szilard's version of Maxwell's demon using minimal decompression. Besides, the more levels of the tree the transition engages (an example of which is Fig. 7), the larger the bound for the heat expelled. The correction factor is exactly the ratio of the depth of the levels of the tree used to implementation of memory to the optimal implementation (minimal decompression), for trit it is, ; however, for the larger base, the denominator can be larger. For trit (see Bormashenko, 2019), there is only minimal ($log_2(2)$) and maximal ($log_2(3)$) levels to use in transitions, and therefore the ratio is unique. However, for higher base the correction factor can range from to , which, when used uncarefully an be a caveat or pitfall to derivation of Landauer's bound for general-base memory.

The Landauer's bound is always true, however, given that memory can have a higher bound for heat emission for irreversible operations and never reach the Landauer's bound.

The above example also shows that irreversible decompression makes it impossible to restore the work previously stored in the heat of the environment, and this is the main idea behind the Landauer's bound. Reversible processes use this heat to change the state by the reversible isothermal compression-decompression transition between top states. From this viewpoint, the information is encoded in the ability to store and restore energy from the system which is treated as an operation on the information. Therefore, the energy transitions in memory are a fundamental level (physical realisation) upon which the information level is constructed.

6. Summary

We have shown, using Szilard's version of Maxwell's demon experiment, the validity and difficulties in the interpretation of Landauer's bound. The essential ingredient in the implementation is an engagement of the inner states of the memory. The Landauer's bound is always true; however, some memory systems have a higher value of the bound for heat emission during the irreversible operation due to a more complex structure of the internal states of the memory. It is hoped that this will be an additional argument to the statement that all our models are accurate up to some scale, and the second law of thermodynamics governs what happens with these inaccuracies.

The one-hot encoding implementation of memory, as well as the implementations using the reversed Szilard's version of Maxwell's demon experiments, shows the problems and bounds resulting from the second law of thermodynamics for these realisations. It also shows that the second law of thermodynamics is more fundamental than the Landauer principle on the level of classical (equilibrium) thermodynamics – by specific choice of the implementation of memory, one can never reach Landauer's bound even without friction in the system.

We also presented relations to the abstract Galois connection that relate to the implementation of memory with the base system that is stored in it. The appearance of this theoretical construction is expected in every situation when we have two systems and when one can be considered as an abstract theory (Boolean algebra or multivalued logic) and the second is its realisation/implementation (physical implementation of memory).

References

Bennett, C.H. (1973). The logical reversibility of computation. *IBM Journal of Research and Development*, *17*, 525–532.
Bennett, C.H. (1987). Demons, Engines and the Second Law. *Scientific American*, 257.
Bérut, A. at al. (2012). Experimental verification of Landauer's principle linking information and thermodynamics. *Nature*, *483*, 187–189. http://doi.org/10.1038/nature10872
Bormashenko, E. (2019). Generalization of the Landauer Principle for Computing Devices Based on Many-Valued Logic. *Entropy*, *21*(12), 1150. http://doi.org/10.3390/e21121150
Bormashenko, E. (2019). The Landauer Principle: Re-Formulation of the Second Thermodynamics Law or a Step to Great Unification?. *Entropy*, *21*(10), 918. https://doi.org/10.3390/e21100918
Fong, B., Spivak, D.I. (2019). *An Invitation to Applied Category Theory: Seven Sketches in Compositionality*. Cambridge University Press.
Frankel, T. (2011). *Geometry of Physics.* Cambridge University Press.
Hayes, B. (2001). Third base. *American Scientist*, *89*, 490–494.
Kycia, R.A. (2018). Landauer's Principle as a Special Case of Galois Connection. *Entropy*, *20*(12), 971. https://doi.org/10.3390/e20120971
Kycia, R.A. (2020). *Entropy in Themodynamics: from Foliation to Categorization*, Accepted to Communications in Mathematics; arXiv:1908.07583 [math-ph].
Ladyman, J., Presnell, S., Short, A.J., Groisman, B. (2007). The connection between logical and thermodynamic irreversibility. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, *38*, 1, 58–79. https://doi.org/10.1016/j.shpsb.2006.03.007
Landauer, R. (1961). Irreversibility and heat generation in the computing process. *IBM Journal of Research and Development*, *5*, 183–191.
Parrondo, J.M.R., Horowitz, J.M., Sagawa, T. (2015). Thermodynamics of information. *Nature Phys*, *11*, 131–139. https://doi.org/10.1038/nphys3230
Piechocinska, B. (2000). Information erasure. *Phys. Rev. A*, *61*, 062314. https://doi.org/10.1103/PhysRevA.61.062314
Raschka, S., Mirjalili, V. (2017). *Python Machine Learning*. Birmingham: Packt Publishing.
Reza, F.M. (1994). *An Introduction to Information Theory*, Dover Publications.
Sagawa, T. (2014). Thermodynamic and logical reversibilities revisited. *Journal of Statistical Mechanics: Theory and Experiment*, *P03025*, 3. http://doi.org/10.1088/17425468/2014/03/p03025
Smith, P., *Category Theory: A Gentle Introduction*, Retrieved from https://www.logicmatters.net/categories (date of access: 10/10/2020).
Still, S. (2019). Thermodynamic cost and benefit of data representations. *Phys. Rev. Lett*. (in press); arXiv:1705.00612
Szilard, L. (1929). *Über* die Entropieverminderung in einem thermodynamischen System bei Eingriffen intelligenter Wesen (On the reduction of entropy in a thermodynamic system by the intervention of intelligent beings). *Zeitschrift für Physik*, *53*, 840–856. http://doi.org/10.1007/BF01341281 (English translation: NASA document TT F-16723).
Yan, L.L., Xiong, T.P., Rehan, K., Zhou, F., Liang, D.F., Chen, L., Zhang, J.Q., Yang, W.L., Ma, Z.H., Feng, M. (2018). Single-Atom Demonstration of the Quantum Landauer Principle. *Phys. Rev. Lett.*, *120*, 210601. https://doi.org/10.1103/PhysRevLett.120.210601

https://doi.org/10.37705/TechTrans/e2020042

Analiza ograniczenia Landauera i jej nieefektywność dla logiki wielowartościowej

Streszczenie

Opisujemy, używając niedawne badania związane z termodynamiką dla logiki wielowartościowej, problemy związane z zastosowaniem reguły Landauera dla termodynamicznego modelu pamięci komputera. Analiza jest oparta na wersji Szilarda demona Maxwella z termodynamiki równowagowej. Zostały zaprezentowane różne wersje termodynamicznej/mechanicznej pamięci – wersja gorąco-jedynkowa i implementacja bazująca na odwróconym eksperymencie Szilarda. Zaprezentowano również związek pomiędzy regułą Landauera i koneksją Galois.

Słowa kluczowe: reguła Landauera; entropia; logika multiwartościowa; kodowanie; druga zasada termodynamiki; termodynamiczna implementacja pamięci; koneksja Galois