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Calculation of brake-force distribution on three-axle agricultural trailers using simulation methods

Paweł Radzajewski
radzajewskilech@wp.pl | https://orcid.org/0000-0003-4158-5853
Department of Mechanical Engineering
Bialystok University of Technology

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Abstract

The paper presents a new methodology for calculating the optimal linear distribution of braking forces for a three-axle trailer with "walking beam" and "bogie" suspension of the rear axle assembly that will meet the requirements of the new European legislation, EU Directive 2015/68. On this basis, a computer program for selecting the linear distribution of braking forces between axles has been developed. The presented calculations and simulation results of the braking process can be used in the design process to select the parameters of the wheel braking mechanisms and then the characteristics of the pneumatic valves of the braking system. The adaptation of the braking system of agriculture trailers is a very important factor for improving the safety of the transportation systems.

Keywords: Brake-force distribution, optimisation, agricultural vehicles, braking systems

1. Introduction

Currently, in trailers and towed agricultural machines, the most frequently installed braking systems are pneumatic or hydraulic powered and are operated from an agricultural tractor (Forrer, 2019; Haldex, 2015; Knorr, 2015; Wabco, 2013, 2016, 2017). The use of inertial overrun brakes is only possible on slow-moving towed vehicles (v≤40 km/h) with a total mass of less than 8000 kg and on high-speed vehicles (v>40 km/h) with a total mass of less than 3500 kg (Commission Delegated Regulation, 2014).

Depending on the weight and design of tractor, the tractor's service brakes are operated by mechanical, hydraulic or pneumatic systems. Low- and medium-power tractors use simple and inexpensive hydraulic braking systems without power assistance (Keyser, Hogan, 1992). Mechanically actuated brakes are also still attractive for cost reasons. Higher-power tractors use hydraulic braking systems powered by the tractor's hydraulic and pneumatic braking systems (Lin, Zhang, 2007).

The cooperation between the tractor and trailer braking systems is ensured by the tractor-mounted trailer-brake control valve (pneumatic or hydraulic). Depending on the type of tractor service brakes applied, trailer-brake control valves are actuated mechanically, pneumatically or hydraulically (Kamiński, 2014; Kamiński, Kulikowski, 2015; Khaled, Mahmoud, 2005).

Since 2016, the new EU Regulation on agricultural vehicles (Commission Delegated Regulation, 2014) has imposed a number of more strict and previously unused requirements on agricultural machinery manufacturers in terms of the braking performance of tractors and trailers, compatibility, safety and stability standards, as well as the introduction of ABS for vehicles moving at speeds above 60 km/h.

For all categories of towed vehicles, the required braking rate has been increased and for vehicles with a total mass of more than 3500 kg (agricultural trailers of categories R3 and R4 and towed agricultural machinery of category S2) and moving at a speed of more than 40 km/h, a specific distribution of braking forces between the axles of vehicle is required. In this way, it is possible to meet the requirements of achieving a sufficiently large relative deceleration (braking rate, i.e. the quotient of deceleration to earth acceleration $z=d/g$), conditioning the achievement of a short stopping distance, and ensuring the directional stability of the braked vehicle in all traffic conditions for safety reasons. As in the regulations for wheeled vehicles (ECE, 2001), no separate recommendations were drawn up for vehicle combinations, treating the individual units as if they were single vehicles.

The adoption of new European legislation in the field of agricultural vehicles imposes high requirements on the manufacturers of agricultural tractors, trailers and machinery with regard to the design of braking systems (Glišović, et al., 2015). The design calculations should take into account many parameters related to the characteristics of the braking process, such as the mass of individual vehicle combinations and load (the above requirements apply to unladen and fully laden vehicles) and their arrangement, as well as the overall dimensions and structural parameters of the vehicle, including the axle system and the type of suspension used. It results that meeting the combined requirements with regard to the efficiency, stability and compatibility of the braking of agricultural trailers depends on the proper selection and calculation of the individual components of the braking system: braking mechanisms, brake actuators, valves and regulators of braking forces (Andrew, 2014; Kamiński, 2012; Khaled, Mahmoud, 2005).

This article has been created as a result of the demand by agricultural machinery companies for new solutions for the selection of braking force distribution in multi-axle trailers. The current methods (Andrew, 2014; Kamiński, 2005; Kamiński, Kulikowski, 2019; Kamiński, Miatluk, 2005; Ondrus, et al., 2018; Ren, Zhecheng , 2019; Tayanovsky, Balsay, 2015; Wang, et al., 2011; Vrabel, et al., 2017) of selecting the distribution of braking forces between the front axle

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and rear axles, treating the rear axle assembly as a single axle, do not meet the requirements of the new EU Directive (Commission Delegated Regulation, 2014) for three-axle trailers. This result in axle blockage in full braking and, consequently, reduced braking efficiency of the rear axle assembly. This paper presents the methodology of design calculations and graphic results of a simulation program in the Matlab environment related to the selection of the optimal distribution of braking forces between the axles of three-axle agricultural trailers equipped with a pneumatic braking system. The manuscript describes a new method for selecting the distribution of braking forces between the front axle and the rear axles in three-axle agricultural trailers, including rear axle suspension, using the example of the two most commonly used suspension variants. In addition, the article contains a new, not previously developed method for selecting the distribution of braking forces between the axles of rear suspension together with the correction of the distribution of braking forces between the front axle and the rear axle assembly resulting from the braking process characteristics. The distribution of braking forces of the rear axle assembly has also been optimised. The presented results have been developed using a program to simulate the braking process of three-axle trailers created in the Matlab environment. On the basis of the presented methodology, it is possible to develop in the computer program a mathematical model for engineering calculations supporting the design of braking systems of trailers and towed agricultural machinery of R3, R4 and S2 categories, aimed at obtaining and shaping the desired functional-utility properties. The correct distribution of braking forces between the axles of a vehicle determines the proper efficiency and stability of braking and is an essential basis for the further design of braking system components.

2. Braking efficiency and stability requirements for braking systems of vehicles of categories R3 and R4

In the process of the selection of braking-force distribution between the axles of trailers (towed machine), it is necessary to aim of an ideal distribution. In this case, the adhesion utilized rates f_i of all axles are the same throughout the braking process and are thus equal to the braking rate (braking intensity) z of vehicle:

$$\frac{T_1}{R_1} = \frac{T_2}{R_2} = \ldots = \frac{T_i}{R_i} = f_i = z \tag{1}$$

where: T_i – braking force of wheels of i axle, R_i – vertical reaction of road surface on wheels of i axle, z – braking rate of vehicle $z = \Sigma T_i / \Sigma R_i$.

This distribution of braking forces was assumed to be optimal because on a homogeneous surface, the highest possible braking intensity is obtained under the given conditions and the requirements for braking performance are easily met (Table 1).

Table 1. Required braking efficiency of service brakes of agricultural trailers (Commission Delegated Regulation, 2014)

Vehicle category	Braking rate *z* [%] at p=6.5 bar	
	v<30 km/h	v>30 km/h
Trailers R2, R3, R4	35%	50%
Towed machines S2	35%	50%

Due to the variable loading level of trailers, it is practically impossible to achieve a perfect distribution of braking forces, even with the use of braking--force regulators. Therefore, for high-speed agricultural vehicles (speed above

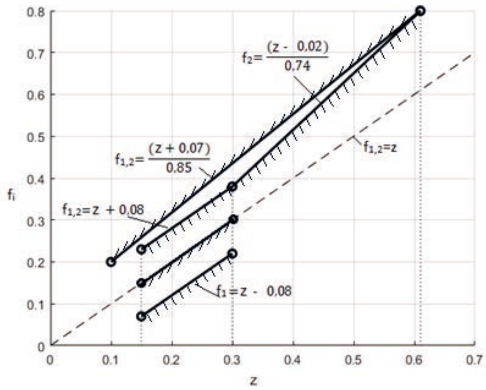

Fig. 1. Adhesion utilization limits for both solutions

40 km/h), permissible limits have been set for the deviation of the adhesion utilization rates of individual axles from the ideal distribution. Since 2016, the two solutions shown in Fig. 1 (Commission Delegated Regulation, 2014) have been are allowed.

First solution: The adhesion utilization rate for each axle must meet the minimum required braking performance (Commission Delegated Regulation, 2014) (short braking distance):

$$\left.\begin{matrix} f_1 \\ f_2 \end{matrix}\right\} \le \frac{z+0.07}{0.85} \quad \text{for} \quad z=0.1-0.61 \tag{2}$$

and the condition of the earlier blocking of the front-axle wheels of the vehicle to ensure heading stability is:

$$f_1 > z > f_2 \quad \text{for} \quad z=0.15-0.30 \tag{3}$$

Second solution: for $z = 0.15 \div 0.30$, the adhesion utilised rates by both axles should be within a certain range, and then the limits of wheel blocking are determined by dependencies:

$$\begin{matrix} f_1 \ge z-0.08 \\ f_{1,2} \le z+0.08 \end{matrix} \quad \text{for} \quad z=0.15-0.30 \tag{4}$$

However, the lower limit of $z - 0.08$ does not apply to the utilization of rear-axle adhesion. In addition, for $z \ge 0.30$, the following relation applies:

$$f_2 \le \frac{z-0.3}{0.74}+0.38 \quad \text{for} \quad z \ge 0.3 \tag{5}$$

The requirements described above also apply to trailers with more than two axles. The wheel-lock sequence requirements are regarded as met if for a range of braking rates between 0.15 and 0.30, the adhesion utilized by at least one front axle is greater than that utilized by at least one rear axle.

$$f_{1i} > f_{2i} \quad \text{for any} \quad i \tag{6}$$

3. Determination of the permissible brake-force distribution area

In three-axle agricultural trailers, the two rear axles are positioned very close to each other and operate in tandem (Bengt, 2016; Colaert Essieux, 2017; Van Stralen, 1983). The braking force must be distributed between the front axle and

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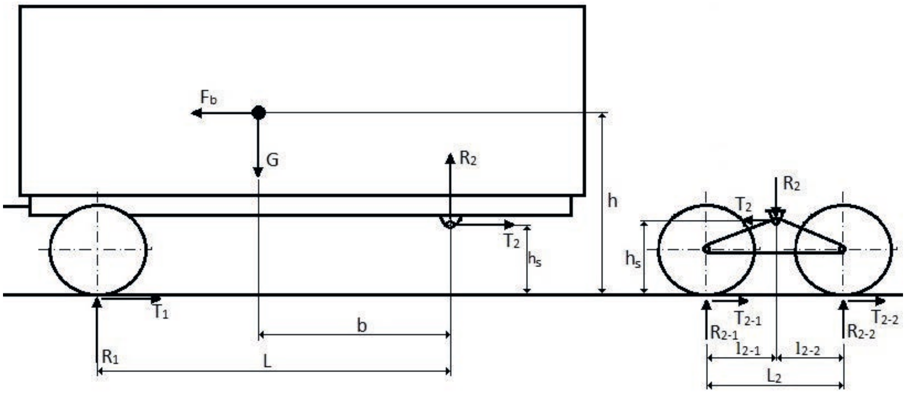

Fig. 2. Diagram of forces acting on a three-axle trailer with a walking beam suspension where: L – wheelbase of trailer, h – height of the centre of gravity position above the ground, b – distance of the centre of gravity from the vertical plane passing through the suspension mounting, h_s – height of the suspension mounting position above the ground, L_2 – wheelbase of the rear axles of the trailer

the rear axle assembly according to the load distribution between the axles. The system of forces acting on a three-axle agricultural trailer with a walking beam tandem suspension is shown in Fig. 2. The adopted design model assumes the omission of the unsprung mass of the tandem axle, which means omitting the gravity and suspension inertia forces.

The balance of forces and moments acting on the trailer has the form of:

$$\sum F_x = T_1 + T_2 - G \cdot z = 0 \tag{7}$$

$$\sum F_y = R_1 + R_2 - G = 0 \tag{8}$$

$$\sum M_2 = G \cdot b + G \cdot z \cdot (h - h_s) + T_1 \cdot h_s - R_1 \cdot L = 0 \tag{9}$$

If the above system of equations is solved, the dependencies describing the reactions of R_1 and R_2 on the trailer are obtained:

$$R_1 = G\left(\frac{b}{L} + \frac{(h-h_s)}{L}z\right) + T_1 \cdot \frac{h_s}{L} \quad \text{or} \quad R_1 = G\left(\frac{b}{L} + \frac{h}{L}z\right) - T_2 \cdot \frac{h_s}{L} \tag{10}$$

$$R_2 = G\left(1 - \frac{b}{L} - \frac{(h-h_s)}{L}z\right) - T_1 \cdot \frac{h_s}{L} \quad \text{or} \quad R_2 = G\left(1 - \frac{b}{L} - \frac{h}{L}z\right) + T_2 \cdot \frac{h_s}{L} \tag{11}$$

On the model without suspension mass, the horizontal and vertical forces acting on the suspension mount correspond to the sum of reaction and braking forces of the rear-axle wheels. Knowing that:

$$T_2 = T_{2-1} + T_{2-2} \qquad R_2 = R_{2-1} + R_{2-2} \tag{12}$$

and that the adhesion utilized rates applied to the rear axles are:

$$f_{2-1} = \frac{T_{2-1}}{R_{2-1}} \qquad f_{2-2} = \frac{T_{2-2}}{R_{2-2}} \tag{13}$$

In this case, the adhesion utilized rates used by the rear axle assembly are calculated from the dependency:

$$f_2 = \frac{\sum (f_{2i} \cdot R_{2i})}{\sum R_{2i}} \tag{14}$$

From the above formulas, the relative (related to the G-weight of trailer) braking forces of the front axle and rear axle assembly can be determined for the braking intensity limits depending on the braking rate z:

$$\gamma_1 = \frac{T_1}{G} = \frac{R_1 \cdot f_1}{G} = \frac{\left(b + z\left(h - h_s\right)\right) f_1}{L - f_1 \cdot h_s} \qquad \gamma_2 = \frac{T_2}{G} = \frac{R_2 \cdot f_2}{G} = \frac{\left(L - b - z \cdot h\right) f_2}{L - f_2 \cdot h_s} \tag{15}$$

In three-axle trailers, the relative braking force of the rear axle assembly can be written as the sum of relative braking forces of the tandem suspension axle:

$$\gamma_2 = \gamma_{2-1} + \gamma_{2-2} \tag{16}$$

The relative braking force of the rear axles can be written as follows:

$$\gamma_2 = \frac{T_2}{G} = \frac{\left(R_{2-1} + R_{2-2}\right) f_2}{G} = \frac{R_{2-1} \cdot f_{2-1} + R_{2-2} \cdot f_{2-2}}{G} \tag{17}$$

Under ideal braking conditions, the adhesion utilized rates used by the front and rear axle assemblies of the trailer are identical and equal to the braking intensity $f_1 = f_2 = z$, and the brake-force distribution is described by the parametric equation:

$$\gamma_1 = \frac{\left(b + z\left(h - h_s\right)\right) z}{L - z \cdot h_s} \qquad \gamma_2 = \frac{\left(L - b - z \cdot h\right) z}{L - z \cdot h_s} \tag{18}$$

By substituting dependencies (2), (3), (4), (5) from Section 2 to the formulas for relative braking forces (15), the limits of brake-force distribution between the front axle and rear axle assembly of a vehicle can be determined, and the limitation areas will be obtained in which the values of vehicle axle adhesion utilized rates should be found. Table 2 lists the limitations for the first and second solution. The relation $\gamma_2 = z - \gamma_1$ applies to all ranges.

Table 2. Braking efficiency and stability requirements

Curve	Dependence $f_{1,2}$–z	Scope of z	γ_1
A-B	$z \geq 0.85 \cdot f_{1,2} - 0.07$	$0.1 \div 0.61$	$\gamma_1 = \frac{\left(b+z\left(h-h_s\right)\right)\left(\left(z+0.07\right)/0.85\right)}{L-\left(\left(z+0.07\right)/0.85\right)h_s}$
C-D	$z \leq f_{1,2}$	$0.15 \div 0.30$	$\gamma_1 = z - \frac{\left(L-b-z\cdot h\right)z}{L-z\cdot h_s}$
A'-C'	$z \geq 0.85 \cdot f_{1,2} - 0.07$	$0.1 \div 0.15$	$\gamma_1 = z - \frac{\left(L-b-z\cdot h\right)\left(\left(z+0.07\right)/0.85\right)}{L-\left(\left(z+0.07\right)/0.85\right)h_s}$
D'-B'	$z \geq 0.85 \cdot f_{1,2} - 0.07$	$0.3 \div 0.61$	$\gamma_1 = z - \frac{\left(L-b-z\cdot h\right)\left(\left(z+0.07\right)/0.85\right)}{L-\left(\left(z+0.07\right)/0.85\right)h_s}$
J-K	$z \geq f_{1,2} - 0.08$	$0.15 \div 0.30$	$\gamma_1 = \frac{\left(b+z\left(h-h_s\right)\right)\left(z+0.08\right)}{L-\left(z+0.08\right)h_s}$
M-N	$z \leq f_{1,2} + 0.08$	$0.15 \div 0.30$	$\gamma_1 = \frac{\left(b+z\left(h-h_s\right)\right)\left(z-0.08\right)}{L-\left(z-0.08\right)h_s}$
K-L	$z \geq 0.3 + 0.74\left(f_{1,2} - 0.38\right)$	$0.1 \div 0.61$	$\gamma_1 = \frac{\left(b+z\left(h-h_s\right)\right)\left(\left(z-0.3\right)/0.74+0.38\right)}{L-\left(\left(z-0.3\right)/0.74+0.38\right)h_s}$
K'-L'	$z \geq 0.3 + 0.74\left(f_{1,2} - 0.38\right)$	$0.1 \div 0.61$	$\gamma_1 = z - \frac{\left(L-b-z\cdot h\right)\left(\left(z-0.3\right)/0.74+0.38\right)}{L-\left(\left(z-0.3\right)/0.74+0.38\right)h_s}$
J'-K'	$z \geq f_{1,2} - 0.08$	$0.15 \div 0.30$	$\gamma_1 = z - \frac{\left(L-b-z\cdot h\right)\left(z+0.08\right)}{L-\left(z+0.08\right)h_s}$
M'-N'	$z \leq f_{1,2} + 0.08$	$0.15 \div 0.30$	$\gamma_1 = z - \frac{\left(L-b-z\cdot h\right)\left(z-0.08\right)}{L-\left(z-0.08\right)h_s}$

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After substituting appropriate values to Table 2, values were obtained in the γ_2–γ_1 system depending on braking intensity for extreme limit points. The coordinates of the characteristic points are shown in Table 3.

Table 3. Coordinates of characteristic points

Point	z	$f_{1,2}$	γ_1	γ_2
A	0.1	0.2	$\gamma_1 = \frac{(b+0.1(h-h_s))0.2}{L-0.2\cdot h_s}$	$\gamma_2 = 0.1 - \gamma_1$
B	0.61	0.8	$\gamma_1 = \frac{(b+0.61(h-h_s))0.8}{L-0.8\cdot h_s}$	$\gamma_2 = 0.61 - \gamma_1$
C	0.15	0.15	$\gamma_1 = 0.15 - \gamma_2$	$\gamma_2 = \frac{(L-b-0.15\cdot h)0.15}{L-0.15\cdot h_s}$
D	0.3	0.3	$\gamma_1 = 0.3 - \gamma_2$	$\gamma_2 = \frac{(L-b-0.3\cdot h)0.3}{L-0.3\cdot h_s}$
A'	0.1	0.2	$\gamma_1 = 0.1 - \gamma_2$	$\gamma_2 = \frac{(L-b-0.1\cdot h)0.2}{L-0.2\cdot h_s}$
C'	0.15	0.259	$\gamma_1 = 0.15 - \gamma_2$	$\gamma_2 = \frac{(L-b-0.15\cdot h)0.259}{L-0.2588\cdot h_s}$
D'	0.3	0.435	$\gamma_1 = 0.3 - \gamma_2$	$\gamma_2 = \frac{(L-b-0.3\cdot h)0.435}{L-0.435\cdot h_s}$
B'	0.61	0.8	$\gamma_1 = 0.61 - \gamma_2$	$\gamma_2 = \frac{(L-b-0.61\cdot h)0.8}{L-0.8\cdot h_s}$
J	0.15	0.23	$\gamma_1 = \frac{(b+0.15(h-h_s))0.23}{L-0.23\cdot h_s}$	$\gamma_2 = 0.15 - \gamma_1$
K	0.3	0.38	$\gamma_1 = \frac{(b+0.3(h-h_s))0.38}{L-0.38\cdot h_s}$	$\gamma_2 = 0.3 - \gamma_1$
M	0.15	0.07	$\gamma_1 = \frac{(b+0.15(h-h_s))0.07}{L-0.07\cdot h_s}$	$\gamma_2 = 0.15 - \gamma_1$
N	0.3	0.22	$\gamma_1 = \frac{(b+0.3(h-h_s))0.22}{L-0.22\cdot h_s}$	$\gamma_2 = 0.3 - \gamma_1$
L	0.61	0.799	$\gamma_1 = \frac{(b+0.61(h-h_s))0.799}{L-0.799\cdot h_s}$	$\gamma_2 = 0.61 - \gamma_1$
J'	0.15	0.23	$\gamma_1 = 0.15 - \gamma_2$	$\gamma_2 = \frac{(L-b-0.15\cdot h)0.23}{L-0.23\cdot h_s}$
K'	0.3	0.38	$\gamma_1 = 0.3 - \gamma_2$	$\gamma_2 = \frac{(L-b-0.3\cdot h)0.38}{L-0.38\cdot h_s}$
M'	0.15	0.07	$\gamma_1 = 0.15 - \gamma_2$	$\gamma_2 = \frac{(L-b-0.15\cdot h)0.07}{L-0.07\cdot h_s}$
N'	0.3	0.22	$\gamma_1 = 0.3 - \gamma_2$	$\gamma_2 = \frac{(L-b-0.3\cdot h)0.22}{L-0.22\cdot h_s}$
L'	0.61	0.799	$\gamma_1 = 0.61 - \gamma_2$	$\gamma_2 = \frac{(L-b-0.61\cdot h)0.799}{L-0.799\cdot h_s}$

Using the relations between the limit values of the adhesion utilized rates of the axles (2), (3), (4), (5) and the technical data of the trailer, the limit values of the permitted distribution of the braking forces can be determined in the graph of relative braking forces $\gamma_2 = f(\gamma_1)$. A graphical interpretation of the above dependencies is presented in Figure 3 and Figure 4. The requirements of the braking efficiency of Table 2 in $f_{1,2}$-z system from the described recommendations according to the first solution are illustrated by AB and CD lines in Figure 3a and Figure 4a. The corresponding limits of braking forces in the $\gamma_2 - \gamma_1$ coordinate system for an example trailer in the laden condition are shown in Figure 3b and the unladen condition in Figure 4b. The limit curves are calculated by substituting (2), (3) adhesion utilized rates f_1, f_2 to the relation (15). The lower limit is set by the AB line, which meets the requirements of the front axle adhesion utilized rate. The upper limit is determined by A'B' lines for the rear axle and CD with

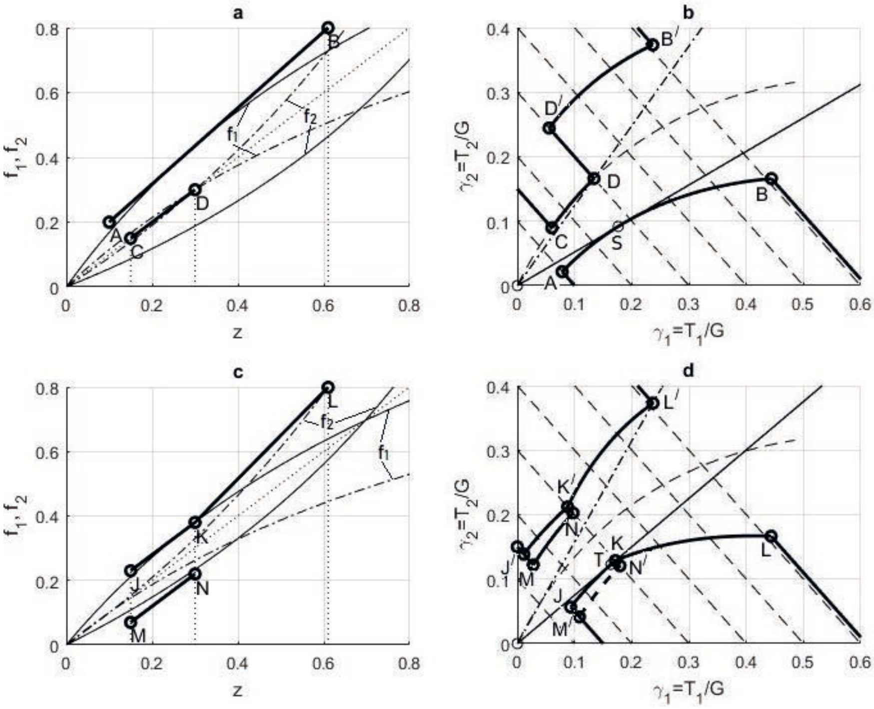

Fig. 3. Determination of parameters of constant brake-force distribution between laden trailer axles with parameters: m=24 000 kg, L=5.15 m, b=1.85 m, h=1.8 m, L_2=1.2 m, $l_{2\text{-}1}$=0.62 m, $l_{2\text{-}2}$=0.58 m, h_s=0.54 m: a and c – limit values of adhesion utilized rates, b and d – limit values of braking distribution between vehicle axles

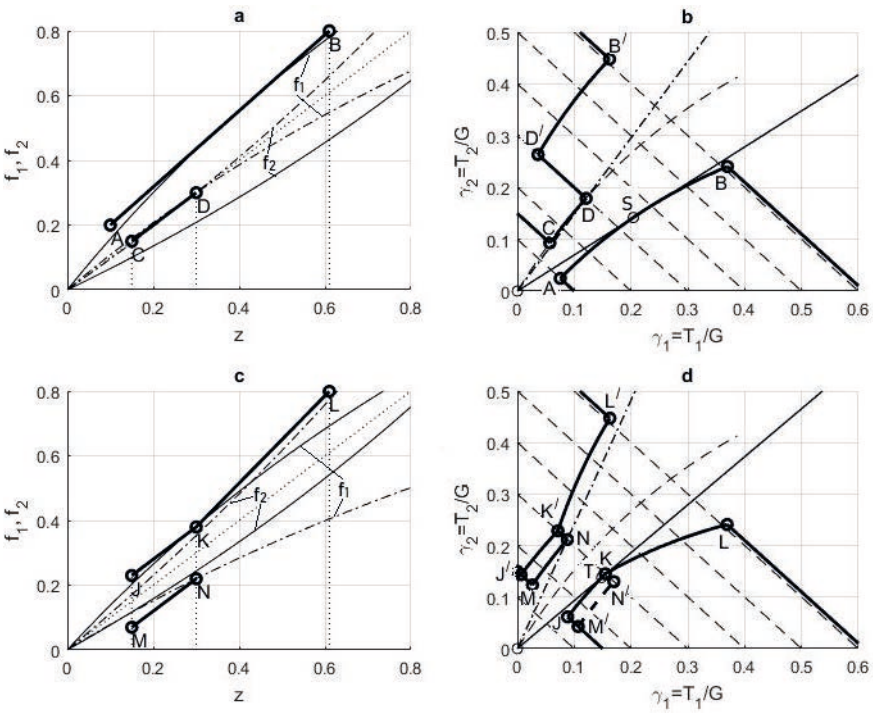

Fig. 4. Determination of parameters of constant brake force distribution between unladen trailer axles with parameters: m=7700 kg, L=5.15 m, b=1.85 m, h=1.08 m, L_2=1.2 m, $l_{2\text{-}1}$=0.62 m, $l_{2\text{-}2}$=0.58 m, h_s=0.54 m: a and c – limit values of adhesion utilized rates, b and d – limit values of braking distribution between vehicle axles

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the wheel lock sequence condition (3). In the case of three-axle trailers, the condition of not exceeding the limit line CD need not be met if the condition of the wheel-locking sequence (6) is met. In the second solution, the MN and JKL lines in Figure 3c and Figure 4c indicate the limits of the permissible area of the adhesion utilized rates. The corresponding relative braking-force areas of the second solution for laden and unladen trailers are shown in Figure 3d and 4d. The upper limit is defined by the lines J'K'L' for the rear axle and MN for the front axle. Due to the restrictive nature of condition (4) for the upper limit K'L' on the graph $\gamma_2 = f(\gamma_1)$, the scope of its application was limited to the range $z = 0.3 \div 0.61$. The lower limit is determined by the JKL line. The dependence of the impassability of line M'N' does not have to be met, which is a result of condition $z - 0.08$ and does not apply to the use of the adhesion of the rear axle (Commission Delegated Regulation, 2014).

3.1. Adjustment of brake-force distribution on three-axle trailers

In air-braking systems of agricultural trailers, the braking force correctors with radial (linear) characteristics (Haldex, 2015; Knorr-Bremse, 2015; Wabco, 2016) are usually applied. This characteristic is described by the equation of the straight line passing through the origin of the coordinate system and the second selected point in the graph of relative braking forces $\gamma_2 = f(\gamma_1)$, taking into account the area limitations described in the previous section (Section 3). The procedure for determining the permissible range of changes in the directional coefficient $i = T_2/T_1 = \gamma_2/\gamma_1$ of straight lines representing a constant distribution of braking forces must be carried out with the laden and unladen vehicle.

In the first solution, the permissible values of linear brake-force distribution coefficient ($i = T_2/T_1$) are determined by a directional line tangential to the limit curve AB at point S (Figs, 3b and 4b) and a line passing through point B' or D. A straight line with a lower value of the directional coefficient which does not cross the border lines of the braking distribution limits between the axles of the vehicle is selected (Figs. 3b and 4b in Section 3).

In the second solution, the lower limit of the permissible area is determined by the tangent line at point T with curve JK (Figs. 3d and 4d). If the point of contact T lies outside segment JK of the limit curve, the directional coefficient of limit line is determined from the coordinates of point K. The upper limit of the permissible range for the linear distribution of braking forces is determined by the line passing through the L', K' or N points (Figs. 3d and 4d), similar to the first solution.

The straight linear brake-force distribution passes through the origin of coordinate system $\gamma_2 = f(\gamma_1)$, and its directional coefficient is then calculated from the ratio of the ordinate to the cut-off of the characteristic point:

$$i = \frac{\gamma_2}{\gamma_1} = \frac{(L - b - z \cdot h) \cdot f_2 \cdot (L - f_1 \cdot h_s)}{\left(b + z(h - h_s)\right) \cdot f_1 \cdot (L - f_2 \cdot h_s)} \tag{19}$$

Using the relation , it is possible to describe the relative braking forces for each line with equations in which the braking rate *z* is a variable:

$$\gamma_1 = \frac{T_1}{G} = \frac{z}{1+i} \qquad \gamma_2 = \frac{T_2}{G} = i \cdot \frac{z}{1+i} \tag{20}$$

Based on the above formulas, the course of adhesion utilized rates by the axles depending on the braking rate *z* are determined by the formulas:

$$f_1 = \frac{T_1}{R_1} = \frac{z}{\left(\frac{b}{L} + \frac{(h - h_s)}{L} z + \frac{z \cdot h_s}{L(1+i)}\right) \cdot (1+i)} \tag{21}$$

$$f_2 = \frac{T_2}{R_2} = \frac{i \cdot z}{\left(1 - \frac{b}{L} - \frac{h}{L} z + \frac{i \cdot z \cdot h_s}{L(1+i)}\right) \cdot (1+i)} \tag{22}$$

Table 4 presents the calculation results of the limit values of directional coefficients i for the three-axle trailer in question.

Table 4. Limit values of the directional coefficient for the linear distribution of braking forces for a three-axle trailer

Solution option	Unladen trailer	Laden trailer
First variant according to (2), (3)	$i_{min} = i_S = 0.6958$	$i_{min} = i_S = 0.5204$
	$i_{max} = i_D = 1.4791$	$i_{max} = i_D = 1.2388$
Second variant according to (4), (5)	$i_{min} = i_T = 0.9312$	$i_{min} = i_T = 0.7509$
	$i_{max} = i_N = 2.4099$	$i_{max} = i_L = 1.5724$

The values of adhesion utilized rates f_1, f_2 for the axles corresponding to the individual limit lines of directional braking distribution coefficient (Table 4) for solution 1 and 2 are shown in Figs. 3a,c and 4a,c.

In order to calculate the optimal coefficient of the directional constant distribution of braking forces, the coefficient of vehicle weight utilization for braking is used (Kamiński, Kulikowski, 2015,):

$$\zeta(\mu) = \frac{T}{\mu \cdot G} = \frac{z}{\mu} \tag{23}$$

where: μ – adhesion coefficient to the ground

According to the above mentioned methodology (Kamiński, Miatluk, 2005), for further calculations the criterion of maximising the average value of the coefficient of adhesion utilization $\zeta(\mu)$ in the given range of the adhesion coefficient (Gredeskul, et al., 1975) is adopted:

$$\zeta_{[r} = \frac{1}{\mu_2 - \mu_1} \int_{\mu_1}^{\mu_2} \zeta(\mu) d\mu \tag{24}$$

The optimal value of the adhesion coefficient is then (Kamiński, Miatluk, 2005):

$$\mu_{op} = \mu_1 + \frac{b}{L}(\mu_2 - \mu_1) \tag{25}$$

where: for agricultural trailers, the following are adopted: $\mu_1 = 0.2$ and $\mu_2 = 0.5 \div 0.6$ (Kamiński, 2005)

The optimal value of the directional coefficient for the constant distribution of braking forces is calculated from the following relation:

$$i_{op} = \frac{L - b - \mu_{op} \cdot h}{b + \mu_{op}(h - h_s)} \tag{26}$$

The optimal value of the coefficient of directional constant distribution of braking forces should be between the limit values of the coefficient of brake-force distribution within the area permitted in the system.

Table 5 presents the optimal values for an example of an unladen and a laden trailer. The optimal values for the directional distribution of braking forces are between the limits for both solutions.

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Table 5. Optimal values of the directional coefficient for the linear distribution of braking forces for a three-axle trailer

Solution option	Unladen trailer	Laden trailer
Optimal ratio according to (26)	$i_{opt} = 1.4719$	$i_{opt} = 1.2271$

3.2. Adjustment of the brake-force distribution in the rear axle group of three-axle trailers

Two types of walking beam and boogie suspension are used in this trailer model, as shown in Fig. 5. The braking force of the rear axle assembly must be distributed according to the load distribution between the tandem suspension axles (Bengt, 2016; Colaert Essieux, 2017; Van Stralen, 1983). The adopted design model assumes the omission of the unsprung mass of the tandem axle, which means the omission of gravity and suspension inertia forces.

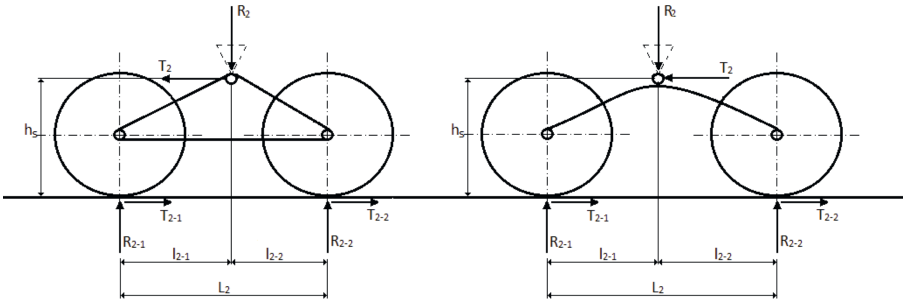

Fig. 5. Diagram of forces acting on the walking beam and boogie suspension

In both cases, the balance of forces and moments acting on the suspension is the same and has the form:

$$\sum F_x = T_2 - T_{2-1} - T_{2-2} = 0 \tag{27}$$

$$\sum F_y = R_{2-1} + R_{2-2} - R_2 = 0 \tag{28}$$

$$\sum M_{2-1} = T_2 \cdot h_2 - R_2 \cdot l_{2-1} + R_{2-2} \cdot L_2 = 0 \tag{29}$$

In order to take full advantage of the adhesion according to formulas (1) and (17), $f_{2-1} = f_{2-2} = f_2$ should be adopted. In this case, the distribution of braking forces between the rear axles can be written as the ratio of their normal reaction on the road surface:

$$i_r = \frac{T_{2-2}}{T_{2-1}} = \frac{R_{2-2}}{R_{2-1}} \tag{30}$$

By solving the system of equations (27)-(29), the forces acting on the tandem suspension wheels during braking are obtained:

$$R_{2-1} = R_2 \cdot \frac{l_{2-2}}{L_2} + T_2 \cdot \frac{h_s}{L_2} \qquad R_{2-2} = R_2 \cdot \frac{l_{2-1}}{L_2} - T_2 \cdot \frac{h_s}{L_2} \tag{31}$$

$$T_{2-1} = \frac{T_2}{1+i_r} \qquad T_{2-2} = \frac{i_r \cdot T_2}{1+i_r} \tag{32}$$

The adhesion utilized rates applied to the rear suspension axle, depending on the braking intensity, can be written as follows:

$$f_{2-1} = \frac{T_{2-1}}{R_{2-1}} = \frac{i \cdot z \cdot G}{R_{2-1} \cdot (1+i_r) \cdot (1+i)} \tag{33}$$

$$f_{2-2} = \frac{T_{2-2}}{R_{2-2}} = \frac{i_r \cdot i \cdot z \cdot G}{R_{2-2} \cdot (1+i_r) \cdot (1+i)} \tag{34}$$

Figure 6 shows the results of the simulation of the values of adhesion utilized rates $f_1, f_2, f_{2\text{-}1}, f_{2\text{-}2}$ used by the axles corresponding to the optimal value of directional braking-force distribution coefficients (Table 5) for laden and unladen vehicles. In the method described above, the most favourable values of the coefficient of the directional inter-axial distribution of braking forces are the optimal i_{opt} values for unladen and laden conditions. However, as shown in Fig. 6a, this method will not ensure optimal braking of the rear suspension axles, the adhesion utilized rate $f_{2\text{-}2}$ has significantly exceeded the adhesion limit value of the conditions (2), (3), (4), (5) described in Section 2, which may result in axle blockage at full braking.

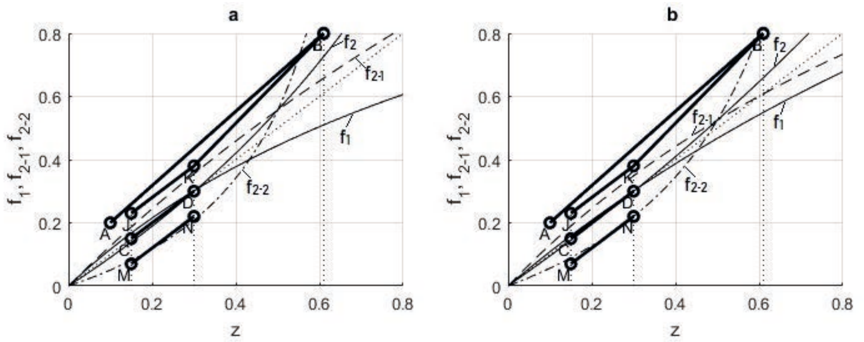

Fig. 6. Optimal course of the adhesion utilized rates used by the axles: a – laden trailer, b – unladen trailer

3.3. Correction of braking-force distribution between the front axle and rear axle assembly of three-axle trailers

In case of three-axle trailers, limits may be set for the distribution of braking forces between the front and one of the rear axles of vehicle. The calculation methodology is similar to that of Section 3 when selecting the distribution between the front axle and rear axle assembly on three-axle trailers. Knowing the distribution of braking forces between the rear axles (30), the limit values can be represented by the permissible ranges of relative braking forces for each rear axle relative to the front axle. In this case, the equations of relative braking forces for the braking-intensity limits depending on the braking rate *z* take the following form for the front axle limit (points A÷N):

$$\gamma_1 = \frac{T_1}{G} = \frac{R_1 \cdot f_1}{G} \qquad \gamma_{2-1} = \frac{z-\gamma_1}{1+i_r} \quad \text{or} \quad \gamma_{2-2} = i_r \cdot \frac{z-\gamma_1}{1+i_r} \tag{35}$$

and for the first rear axle (points A'÷N'):

$$\gamma_{2-1} = \frac{T_{2-1}}{G} = \frac{R_{2-1} \cdot f_2}{G} \qquad \gamma_1 = z-(1+i_r)\gamma_{2-1} \tag{36}$$

and for the second rear axle (points A'÷N'):

$$\gamma_{2-2} = \frac{T_{2-2}}{G} = \frac{R_{2-2} \cdot f_2}{G} \qquad \gamma_1 = z-\frac{(1+i_r)\gamma_{2-2}}{i_r} \tag{37}$$

Using the limits of the adhesion utilized rates used for the axles (35), (36), (37) and the technical data of the trailer, the lower and upper limits of the permissible distribution of the braking forces of the rear axles relative to the front axle can be determined from the graph of relative braking forces $\gamma_{2\text{-}1} = f(\gamma_1)$, $\gamma_{2\text{-}2} = f(\gamma_1)$. A graphical interpretation of the above dependencies on the basis of solution II (4), (5) is presented in Figure 7.

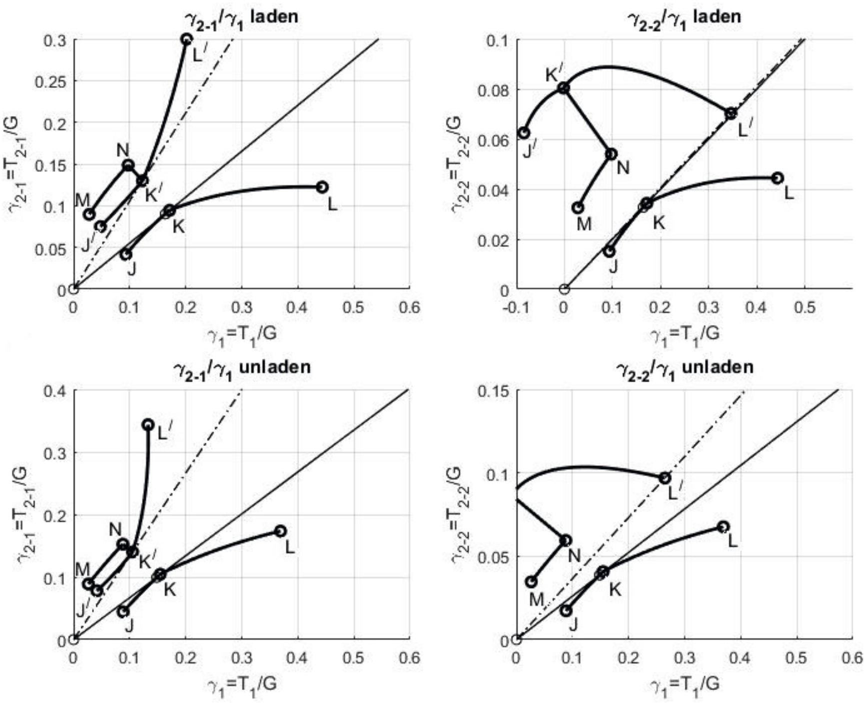

Fig. 7. Permissible ranges of relative rear-axle braking forces for laden and unladen trailers

Based on the brake-force distribution limits of the rear axles (Figure 7), it is possible to determine the brake-force distribution limits i between the axles of the vehicle so that the rear axles meet the braking stability requirements of Chapter 2. The limit values describe the formulas:

$$i(\gamma_{2-1}) = \frac{(1+i_r)\cdot\gamma_{2-1}}{\gamma_1} \qquad i(\gamma_{2-2}) = \frac{(1+i_r)\cdot\gamma_{2-2}}{i_r\cdot\gamma_1} \tag{38}$$

where: $\frac{\gamma_{2-1}}{\gamma_1}$ – permissible values of the coefficient range for linear distribution of braking forces of the first rear axle from Figure 7 $\gamma_{2\text{-}1} = f(\gamma_1)$, $\frac{\gamma_{2-2}}{\gamma_1}$ – permissible values of the coefficient range for linear distribution of braking forces of the second rear axle from Figure 7 $\gamma_{2\text{-}2} = f(\gamma_1)$,

If the coefficient of inter-axial brake-force distribution i (19) is not within the above permissible limits (38), its value must be changed. Knowing that the reaction of the rear axles depends on the rate i, it is necessary to recalculate the distribution of braking forces between the rear axles i_r and the values of permissible areas for the distribution of braking forces i (38) between the axles of the vehicle in the condition $\gamma_{2\text{-}1} = f(\gamma_1)$, $\gamma_{2\text{-}2} = f(\gamma_1)$. This operation must be repeated until the value i is close to the limit value.

Table 6 presents the limit values of directional coefficients of the linear inter-axial distribution of braking forces (38) from the first test calculated from optimal parameters (Table 5) with the coefficient of braking-force distribution between tandem suspension axles (30) for the laden trailer $i_r = 0.365$, and the unladen trailer $i_r = 0{,}39$.

Table 6. Limit values of directional coefficients for the linear inter-axial distribution of braking forces for a three-axle trailer

Solution option	Unladen trailer	Laden trailer
First variant according to Figure 7 $\gamma_{2\text{-}1}=f(\gamma_1)$	$i(\gamma_{2\text{-}1})_{min}=0.9312$	$i(\gamma_{2\text{-}1})_{min}=0.7509$
	$i(\gamma_{2\text{-}1})_{max}=1.8498$	$i(\gamma_{2\text{-}1})_{max}=1.4388$
Second variant according to Figure 7 $\gamma_{2\text{-}2}=f(\gamma_1)$	$i(\gamma_{2\text{-}2})_{min}=0.9212$	$i(\gamma_{2\text{-}2})_{min}=0.7509$
	$i(\gamma_{2\text{-}2})_{max}=1.3047$	$i(\gamma_{2\text{-}2})_{max}=0.7619$

Finally, after several simulation tests from the analysis of limit values of the brake-force distribution i for the laden and unladen vehicle axles on the basis of permissible values of the coefficient range of the directional linear distribution of the braking forces of the rear axles (38), for the above example, in Figure 8 the following has been assumed: $i = 0.88$ for a laden vehicle and $i = 1.34$ for an unladen vehicle.

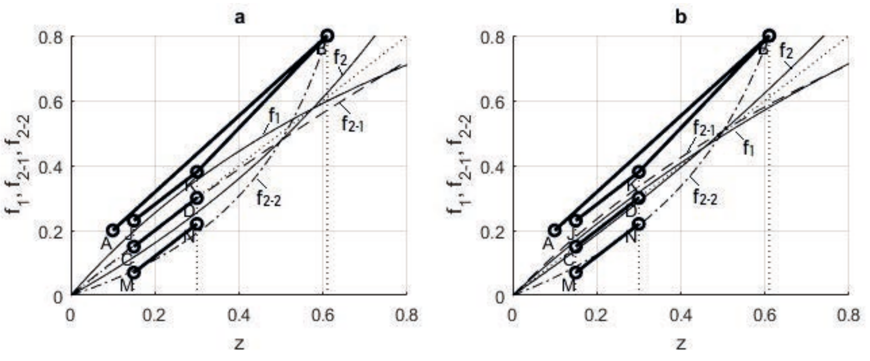

Fig. 8. Course of the adhesion utilized rates used by the axles: a – for a laden trailer, b – for an unladen trailer

Using formula (30), a graph of the coefficient for the distribution of braking forces of the rear axles, depending on the braking rate z was created for the inter-axial distribution of braking forces for laden and unladen trailers, adopted in Fig. 8.

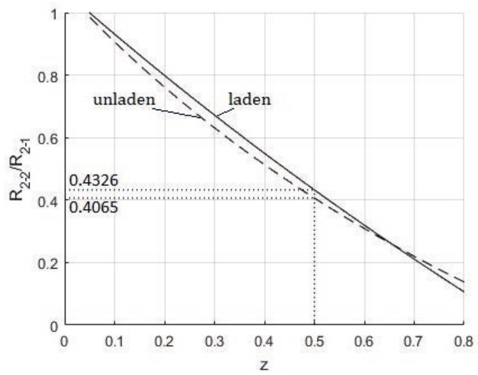

Fig. 9. Determination of optimal values for the constant distribution of braking forces on rear axles

The optimal value for the constant distribution of braking forces for the rear axles is determined with a minimum permissible braking rate (Table 1). Table 7 presents values for an example of an unladen and a laden trailer.

Table 7. Optimal value for constant brake distribution for rear axles

	Unladen trailer	Laden trailer
$i_r\ (z=0.5)$	0.4065	0.4326

Such a selected brake-force distribution of rear axles will be the most optimal and requires the use of expensive brake-force regulators between the rear axles. In fact, the distribution of forces between the rear axles is usually constant for all loading conditions.

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3.4. Optimising the distribution of braking forces on the rear axle assembly

In order to determine the constant distribution of the braking forces of rear axles, the braking-force distribution coefficient between the rear axles i_r should be optimised taking into account the extreme loading conditions of the trailer (Venkataraman, 2001). The most optimal distribution of braking forces between the rear axles will be achieved if, for the i_r coefficient, the course of adhesion utilized rates used by the rear axles is close to each other or to the optimal value of the adhesion utilized rate of the rear axle assembly (Kamiński, Radzajewski, 2019; Tang, et al., 2013). This means that the objective function Q, being their differences, for different values of the i_r coefficient tend to be minimal.

$$Q(i_r)=(f_{2-1}-f_{2-2})^2 \rightarrow min \quad \text{or} \quad Q(i_r)=(f_{2-1}-f_2)^2+(f_{2-2}-f_2)^2 \rightarrow min \tag{39}$$

In order to select representative (comparative) values, the maximum differences are sought for the course of adhesion utilized rates used by the axles for an unladen and a laden trailer for different values of the braking-distribution coefficient of the rear axles. Its scope has been limited to the range between the optimal values for the unladen and laden trailer $i_r \in (0.4065 \div 0.4326)$.

$$Q_{max}(i_r)=(f_{2-1}(z)-f_{2-2}(z))^2 \rightarrow max \tag{40}$$

In the second case, we can determine the sum of differences in the course of adhesion utilized rates used by the axles in braking rate range z for an unladen and laden trailer for different values of the coefficient of rear-axle braking distribution.

$$Q_{sum}(i_r)=\sum(f_{2-1}(z)-f_{2-2}(z))^2 \tag{41}$$

In both cases, the following condition can be verified for both loading conditions after obtaining the maximum values (40) or the sum (41) depending on the braking rate i_r:

$$Q_{max/sum\ laden}(i_r)+Q_{max/sum\ unladen}(i_r) \rightarrow min \tag{42}$$

In this case, the value of brake force distribution ratio i_r, which corresponds to the minimum function value of the relation (42), is the optimal value for constant brake-force distribution for all loading conditions. Based on the above dependencies, a computer program was developed for optimisation in the Matlab environment. The optimal course of adhesion utilized rates used by the axles is shown in Fig. 10 for a laden trailer and in Fig. 8b for an unladen trailer.

After optimisation, the constant value of braking rate for both extreme loading conditions was adopted as 0.4065. This means that it remains constant for the unladen trailer (Fig. 8b). As can be seen in Fig. 10a for a laden trailer

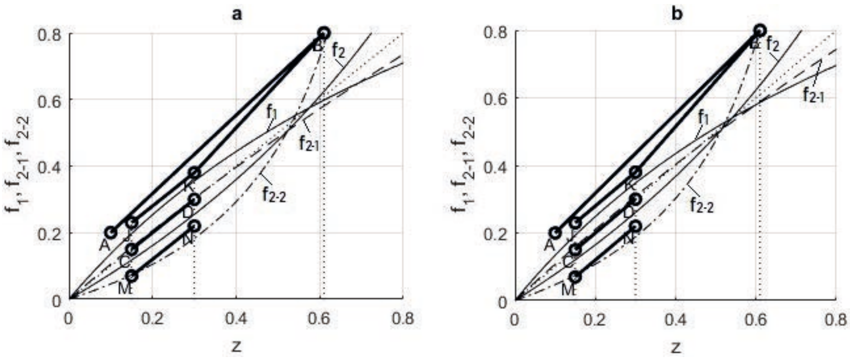

Fig. 10. Optimal course of the adhesion of utilized rates used by the axles for a laden trailer: a – $i_r = 0.4065$, $i = 0.088$; b – $i_r = 0.4065$, $i = 0.92$

after optimisation, the value of inter-axle braking-force distribution ratio i can be increased according to formula (38) so that the adhesion utilized ratio used by the second rear axle $f_{2\text{-}2}$ reaches the limit point L' or B, as shown in Figure 10b. This will ensure shorter braking distances as we reach the optimal value of braking distribution ratio i_{opt} (Table 5).

4. Summary and conclusions

The method described in this paper enables the selection of an optimal linear distribution of braking forces in the air braking systems of three-axle agricultural trailers, in which brake-force correctors with radial characteristics are used. This methodology takes into account the requirements of EU Directive 2015/68 on vehicle-braking efficiency and stability and uses braking-process characteristics to determine braking-force distribution limits between the front axle and rear axle assembly, and between the rear axles. The selection of brake-force distribution is the basis for calculations used in the design of pneumatic braking systems for agricultural trailers. In addition, it allows the selection of devices, brake actuators and pneumatic components (characteristics of brake valves) for different load conditions in trailers of any size.

As shown in this solution, the selection of the brake-force distribution coefficient between the front and rear axle assemblies may not be sufficient. In the case of a laden trailer with a weight of twenty-four tonnes, the rear axle wheels could lock with full braking (Fig. 6a). The aspects of brake-force distribution and the course of adhesion utilized rates by the tandem suspension axles must also be analysed. The second solution (4), (5) is most commonly used for three-axle trailers. The calculated coefficients of adhesion utilized by the axles for this solution are closer to a straight line, showing an ideal distribution of braking forces in which the coefficients of adhesion utilized by each axle are the same and equal to the braking rate.

On the basis of the presented methodology, it is possible to develop rules for the distribution of braking forces for trailers using brake-force correctors with different characteristics than radial (linear).

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