technical transactions

No. 2022/007

mechanics

sciendo

Model parameter on-line identification with nonlinear parametrization – manipulator model

Leszek Cedro

lcedro@tu.kielce.pl | https://orcid.org/0000-0002-2419-4044

Faculty of Mechatronics and Mechanical Engineering, Kielce University of Technology

Scientific Editor: Jacek Pietraszek, Cracow University of Technology
Technical Editor: Aleksandra Urzędowska, Cracow University of Technology Press
Language Verification: Timothy Churcher, Merlin Language Services
Typesetting: Małgorzata Murat-Drożyńska, Cracow University of Technology Press

Received: January 5, 2022
Accepted: May 30, 2022

Data Availability Statement: All relevant data are within the paper and its Supporting Information files.

Competing interests: The authors have declared that no competing interests exist.

Citation: Cedro, L. (2022). Model parameter on-line identification with nonlinear parametrization – manipulator model. *Technical Transactions, e2022007*. https://doi.org/10.37705/TechTrans/e2022007

Abstract

This paper presents an example of solving the parameter identification problem in the case of a robot with two degrees of freedom. In this study, a weighted recursive least squares algorithm was generalised to a case of nonlinear parameterisation in which the identified parameters did not satisfy the linear model. The generalisation involved linearising the model in the neighbourhood of current values of the parameter estimates. It was assumed that the estimates were updated every N steps of signal sampling. This method of identification can be applied whenever the parameters concerning a model need to be determined at the time of measurement. This is particularly useful in adaptive control when the plant parameters vary over time.

Keywords: on-line identification, nonlinear model identification, recursive LS algorithm

1. Introduction

Identification methods can be classified into on-line methods, when the model is required to be known in real time (e.g. in adaptive control (Niederliński, Mościński, Ogonowski, 1995; Woś, Dindorf, 2013)), and off-line methods, when the time of model creation is not significant (Vítečková, Víteček, 2013; Valasek, Pavliska, Perfilieva & Farana, 2013).

Precise control requires using digital systems and thus developing models for control purposes. It is essential that their properties change automatically in accordance with the input and output signals and that their structure is simple enough to allow on-line control.

One of the elements of an adaptive control system is an algorithm responsible for identifying the plant parameters (Bochnia, 2018; Graba, 2017; Miller, Adamczak, Świderski, Wieczorowski, Łętocha, Gapiński, 2017; Krzysztofik, Takosoglu, Koruba, 2017). As these may change over time and the process of identification is conducted concurrently with measurement, the identification algorithm is usually recursive in nature(Zorawski, Makrenek, Goral, 2016; Adamczak, Bochnia, 2016). A recursive identification algorithm is also called an on-line identification algorithm or a real-time identification algorithm.

There exists a large variety of on-line identification algorithms used in adaptive control systems (Fortescue, Kershenbaum, Ydsti, 1981; Wittenmark, Astrom, 1984; Wittenmark, Astrom, 1980). The majority of them are based on the classic least squares (LS) method. This algorithm can be used for models that are linear with respect to the parameters:

$$v_n = \boldsymbol{\varphi}_n^T \boldsymbol{\theta}, \tag{1}$$

where

$\boldsymbol{\theta}$ – a vector of unknown parameters,

v_n – a scalar signal,

$\boldsymbol{\varphi}_n$ – a vector signal dependent in a known way on the measured signals (the plant input and output),

$n = 0,1,\ldots$ – are the successive time instants.

The LS algorithm is based on the plant variables previously registered in a sufficiently long time interval $n = 0,1,\ldots,N-1$. The processing of measurement data is defined to be off-line because the identification process follows measurement of the input and output variables.

Frequently, it is necessary to identify time-varying plants for which the parameters change in time due to changes in external factors. In such cases, the LS algorithm does not provide us with correct results. Estimates derived through a classic method, in which the entire record of plant observation is taken into consideration with the same weight, are average estimates.

Similarly, the least squares recursive algorithm does not respond to changes in the plant parameters. By adapting the iterative algorithm to tracing time-dependent plant parameters, we arrive at a method with exponential forgetting. This method requires introducing the so-called forgetting factor into the equations which will be responsible for the algorithm memory length. To trace current changes in the values of the parameters, one should determine the estimates simultaneously while measuring the input and output variables. What happened in the past has no significant influence on the present state of the plant and should therefore be forgotten.

The method based on this concept is called a recursive LS algorithm with exponential forgetting or a weighted recursive least squares (WRLS) algorithm (Janecki, 1988).

The WRLS algorithm with exponential forgetting of the past data is obtained by the minimisation of the objective function

https://doi.org/10.37705/TechTrans/e2022007

$$J_n(\boldsymbol{\theta}) = \sum_{i=1}^{n} \lambda^{n-i} (v_i - \boldsymbol{\varphi}_i^T \boldsymbol{\theta})^2 + \lambda^n (\hat{\boldsymbol{\theta}}_0 - \boldsymbol{\theta})^T \boldsymbol{\Gamma}_0^{-1} (\hat{\boldsymbol{\theta}}_0 - \boldsymbol{\theta}) \tag{2}$$

with respect to the vector of the parameters $\boldsymbol{\theta}$, where λ is a constant satisfying the inequality $0 < \lambda < 1$. Let $\hat{\boldsymbol{\theta}}_n$ represent a value of the vector of the parameters $\boldsymbol{\theta}$ for which $\partial J_n(\boldsymbol{\theta}) / \partial \boldsymbol{\theta} = 0$. Using a simple transformation, one gets

$$\hat{\boldsymbol{\theta}}_n = \hat{\boldsymbol{\theta}}_{n-1} + \boldsymbol{\Gamma}_n \boldsymbol{\varphi}_n (v_n - \boldsymbol{\varphi}_n^T \hat{\boldsymbol{\theta}}_{n-1}), \tag{3}$$

$$\boldsymbol{\Gamma}_n^{-1} = \lambda \boldsymbol{\Gamma}_{n-1}^{-1} + \boldsymbol{\varphi}_n \boldsymbol{\varphi}_n^T, n = 1, 2, \ldots \tag{4}$$

In criterion (2), the components dependent on the past measurement data are multiplied by the factor λ^{t-i}. This implies that, when the estimate of $\hat{\boldsymbol{\theta}}$ is determined, the older measurement data are less important than the current data. The parameter λ is called a forgetting factor. Using the matrix inversion lemma, the algorithm equations can be written in such a form that there is no need to invert the matrix $\boldsymbol{\Gamma}_n$.

The value of the forgetting factor λ is selected according to the expected rate of changes in the parameters of the identified plant and the level of noise in the measurement data. In practice, parameter λ is selected in such a way that $0.9 < \lambda < 0.99$ to ensure attenuation of the measurement noise and plant disturbances.

A shortcoming of the algorithm is that the past data are also forgotten when, for example, $\boldsymbol{\varphi}_n = 0$, i.e. when there is no information about the plant. Note that from the condition $\boldsymbol{\varphi}_n = 0$, it is clear that $\boldsymbol{\Gamma}_n = \frac{1}{\lambda} \boldsymbol{\Gamma}_{n-1}$. Thus, the lack of information about the plant results in an exponential increase in the matrix $\boldsymbol{\varphi}_n$. An excessive increase in the eigenvalues of matrix $\boldsymbol{\varphi}_n$ is unfavourable as it may cause a considerable random change in the values of the parameter estimates, and consequently, the instability of the adaptive control system. Another drawback of the algorithm with the exponential forgetting of past data is that this algorithm cannot be used when the plant parameterisation is not unique.

In this paper, the WRLS algorithm was generalised into a case of nonlinear parameterisation, this is one in which the identified parameters did not satisfy the model (1). The generalisation involved linearising the model in the neighbourhood of the current values of the parameter estimates. It is assumed that the estimates are updated every N steps of signal sampling. This assumption is justified when the sampling period is small with regard to the expected rate of parameter variation. A modification is also applied to ensure that the matrix $\boldsymbol{\Gamma}_n$ is bounded irrespective of whether the signal $\boldsymbol{\varphi}_n$ is sufficiently exciting.

2. Model identification using nonlinear parameterisation

Suppose that the plant equation satisfies the relationship

$$f_n(\boldsymbol{\theta}) = 0 \tag{5}$$

where

$\boldsymbol{\theta}$ – is the parameter vector,

f_n – is a certain scalar function that is non-linear with respect to the parameters $\boldsymbol{\theta}$.

When the sampling period is small, information about the plant parameters obtained in one sampling step is generally insufficient. Thus, it is assumed that the parameter estimates will be updated after just N successive signal samples for the selected positive integer N. The aim is to derive a recursive equation that will allow determining parameter estimates $\boldsymbol{\theta}_m$ for the time instants lying in the interval $\alpha(m) = [(m-1)N, mN], m = 1, 2, \ldots$.

Define the following objective function

$$J_m(\boldsymbol{\theta})=\sum_{j=1}^{m}\lambda^{m-j}\left(\sum_{i\in\alpha(j)}\left(f_i(\boldsymbol{\theta})\right)^2\right)+\lambda^m(\hat{\boldsymbol{\theta}}_0-\boldsymbol{\theta})^T\boldsymbol{\Gamma}_0^{-1}(\hat{\boldsymbol{\theta}}_0-\boldsymbol{\theta}) \tag{6}$$

Let $\hat{\boldsymbol{\theta}}$ be a certain estimate of the parameter vector. The function $f_n(\boldsymbol{\theta})$ in the neighbourhood of point $\hat{\boldsymbol{\theta}}$ can be approximated by a linear form

$$f_n(\boldsymbol{\theta})\cong f_n(\hat{\boldsymbol{\theta}})+\left(\left.\frac{\partial f_n(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}}\right)^T(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}})=\nu_n(\hat{\boldsymbol{\theta}})-\boldsymbol{\varphi}_n(\hat{\boldsymbol{\theta}})^T\boldsymbol{\theta} \tag{7}$$

where

$$\boldsymbol{\varphi}_n(\hat{\boldsymbol{\theta}})=-\left.\frac{\partial f_n(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}},\ \nu_n(\hat{\boldsymbol{\theta}})=f_n(\hat{\boldsymbol{\theta}})-\boldsymbol{\varphi}_n(\hat{\boldsymbol{\theta}})^T\hat{\boldsymbol{\theta}}.$$

Substituting (7) into (6), we have

$$J_m\left(\boldsymbol{\theta},\hat{\boldsymbol{\theta}}\right)=\sum_{j=1}^{m}\lambda^{m-j}\left(\nu_k(\hat{\boldsymbol{\theta}})+\boldsymbol{\varphi}_k(\hat{\boldsymbol{\theta}})^T\boldsymbol{\theta}\right)+\lambda^m(\hat{\boldsymbol{\theta}}_0-\boldsymbol{\theta})^T\boldsymbol{\Gamma}_0^{-1}(\hat{\boldsymbol{\theta}}_0-\boldsymbol{\theta}).$$

Let $\hat{\boldsymbol{\theta}}_m$ denote a value of the vector of parameters $\boldsymbol{\theta}$, for which $\partial J(\boldsymbol{\theta},\hat{\boldsymbol{\theta}}_{m-1})/\partial\boldsymbol{\theta}=0$. As in the classic recursive algorithm with the exponential forgetting of past data, we obtain

$$\hat{\boldsymbol{\theta}}_m=\hat{\boldsymbol{\theta}}_{m-1}+\boldsymbol{\Gamma}_m\sum_{i\in\alpha(m-1)}\boldsymbol{\varphi}_i(\hat{\boldsymbol{\theta}}_{m-1})\left(\nu_i(\hat{\boldsymbol{\theta}}_{m-1})-\boldsymbol{\varphi}_i(\hat{\boldsymbol{\theta}}_{m-1})^T\hat{\boldsymbol{\theta}}_{m-1}\right), \tag{8}$$

$$\boldsymbol{\Gamma}_m^{-1}=\lambda\boldsymbol{\Gamma}_{m-1}^{-1}+\sum_{i\in\alpha(m-1)}\boldsymbol{\varphi}_i(\hat{\boldsymbol{\theta}}_{m-1})\boldsymbol{\varphi}_i(\hat{\boldsymbol{\theta}}_{m-1})^T,m=1,2,\ldots \tag{9}$$

Generally, it is assumed that $\boldsymbol{\Gamma}_0^{-1}=\varepsilon\mathbf{I}$ for ε ranging 10^3 – 10^6. Small initial values of the elements of the matrix $\boldsymbol{\Gamma}_0^{-1}$ ensure the immediate convergence of estimates to the real values of the parameters at the beginning of the algorithm.

To avoid an excessive increase in the inversion of the covariance matrix $\boldsymbol{\Gamma}_m$, when the signal $\boldsymbol{\varphi}_n$ is not sufficiently exciting, we will apply another modification of the algorithm (9). Let $G(x):[0.1]\to[0.1]$ be a differentiable function satisfying the conditions $x\le G(x)\le 1$, $G'(0)=1/\lambda$. Thus, function G approximates the straight line x/λ for small values of x. Let us then demand that $\boldsymbol{\Gamma}_m^{-1}\ge\varepsilon\mathbf{I}$ for each m. Equations (9) are modified as follows:

$$\boldsymbol{\Pi}_m^{-1}=\lambda\boldsymbol{\Gamma}_{m-1}^{-1}+\sum_{i\in\alpha(m-1)}\boldsymbol{\varphi}_i(\hat{\boldsymbol{\theta}}_{m-1})\boldsymbol{\varphi}_i(\hat{\boldsymbol{\theta}}_{m-1})^T, \tag{10}$$

$$\boldsymbol{\Gamma}_m=\varepsilon^{-1}G(\varepsilon\boldsymbol{\Pi}_m).$$

For instance, the function G can be a polynomial of the form

$$G(x)=\lambda^{-1}(x-(1-\lambda)x^2). \tag{11}$$

Other examples can be found in Ref. (Hunt, 1996).

In accordance with the scheme presented in Fig. 1, a test stand was prepared and an experimental procedure was conducted in order to validate the presented concept.

3. A mathematical model of a robot manipulator

In the next sections, the following problems will be solved: first, we will derive the equations for the DC motors, then we will define the kinetic and potential energy of the system (Fig. 1), and finally, we will symbolically derive the robot dynamic equations, using the second order Lagrange equations.

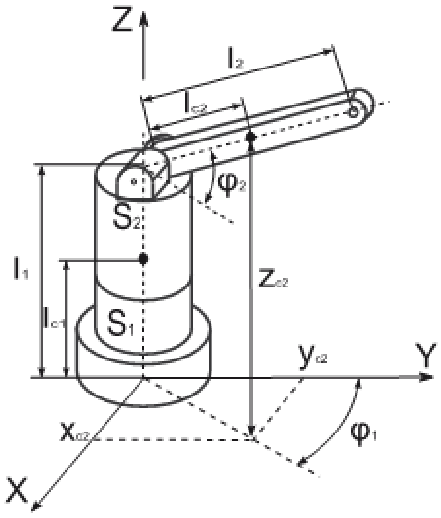

Fig. 1. An electrically-driven manipulator

Let $\boldsymbol{\varphi} = [\varphi_1\ \varphi_2]^T$ denote the vector of joint variables acting as generalised coordinates, m_j – the mass, l_j – the arm length, l_{cj} – the distance from the centre of gravity and S_j – the motor of the link j.

Using the typical equivalent diagrams (Fig. 2) of DC motors available in the literature and the second Kirchoff law, we can write the following electrical equation of the DC motor:

$$u_{z_j} = u_{R_j} + u_{L_j} + e_{e_j}, \quad \text{for} \quad j = 1,2 \tag{12}$$

where
u_{zj} – is the voltage supplied to the rotor.

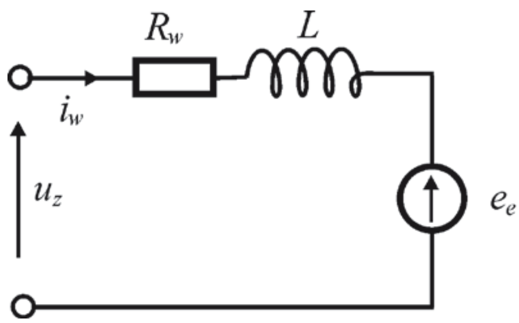

Fig. 2. Equivalent circuit of the DC motor

Due to the possibility of an open-loop system being difficult to control, it is essential that the identification be performed for a closed-loop system with properly selected PD controllers. Let us assume that the equations of the controllers have the following form:

$$u_{z_j} = K_{p_j}\left(\varphi_{z_j}(t) - \varphi_j(t)\right) - K_{d_j}\dot{\varphi}_j(t), \tag{13}$$

where
K_{pj}, K_{dj} – the parameters of the controllers,
$\varphi_{z_j}(t)$ – the control signals,
$\varphi_j(t)$ – the variables describing the position of the manipulator arms.

The voltage drops across the rotor winding resistance and inductance are:

$$u_{R_j} = R_{w_j} i_{w_j}(t), \tag{14}$$

https://doi.org/10.37705/TechTrans/e2022007

$$u_{Lj} = L_j \frac{di_{w_j}(t)}{dt}, \tag{15}$$

where
R_{wj} – is the equivalent rotor winding resistance,
L_j – is the equivalent rotor winding inductance,
i_{wj} – is the current flowing through the rotor windings.

The electromotive inductance force is

$$e_{e_j} = k_{e_j} \dot{\varphi}_j(t), \tag{16}$$

where
k_{ej} – is an electromotive constant.

Substituting the subsequent components to Eq. 12, we obtain:

$$L_j \frac{di_{w_j}(t)}{dt} + R_{w_j} i_{w_j}(t) + k_{e_j} \dot{\varphi}_j(t) = K_{p_j}[\varphi_{z_j}(t) - \varphi_j(t)] - K_{d_j} \dot{\varphi}_j(t), \text{ for } j = 1{,}2. \tag{17}$$

The rotor torque is:

$$M_{s_j} = k_{m_j} i_{w_j}(t), \tag{18}$$

where
k_{m_j} – is a mechanical constant.
Let us define the manipulator kinetic and potential energy. The following geometrical relations take place:

$$x_{c2} = l_{c_2} \cos(\varphi_2(t))\sin(\varphi_1(t)), \; y_{c2} = l_{c_2} \cos(\varphi_2(t))\cos(\varphi_1(t)), \tag{19}$$

$$z_{c2} = l_1 + l_{c_2} \sin(\varphi_2(t))$$

The velocity of the centre of gravity of the second arm of the manipulator is:

$$V_2 = \sqrt{\dot{x}_{c2}^{\;2} + \dot{y}_{c2}^{\;2} + \dot{z}_{c2}^{\;2}}. \tag{20}$$

Thus, the kinetic energy of the system is:

$$E = E_1 + E_2, \tag{21}$$

$$E_1 = \frac{J_{c1}\dot{\varphi}_1^{\;2}(t)}{2}, \; E_2 = \frac{m_2 V_2^2}{2} + \frac{J_{c2}\dot{\varphi}_2^{\;2}(t)}{2}, \; J_{cj} = \frac{m_j l_j^{\;2}}{12}, \; l_{c_j} = \frac{l_j}{2},$$

where
J_{cj} – are moments of inertia of the robot arms assumed for a uniform beam.

The potential energy of the system is:

$$U = U_1 + U_2, \tag{22}$$

$$U_1 = m_1 g l_{c_1}, \; U_2 = m_2 g(l_1 + l_{c_2} \sin(\varphi_2(t))),$$

where
g – is the acceleration of gravity.

Using the expressions for the kinetic and potential energy, we obtain two second-order Lagrange equations:

https://doi.org/10.37705/TechTrans/e2022007

$$\frac{d}{dt}\frac{\partial E}{\partial \dot{\varphi}_j}-\frac{\partial E}{\partial \varphi_j}+\frac{\partial U}{\partial \varphi_j}=M_{s_j}, \quad \text{for } j=1,2. \tag{23}$$

After substitution and simplification of all the variables, we have a system of two equations (where: $\varphi_j=\varphi_j(t)$, $\dot{\varphi}_j=\dot{\varphi}_j(t)$, $\ddot{\varphi}_j=\ddot{\varphi}_j(t)$, $\ddot{\varphi}_j=\ddot{\varphi}_j(t)$, $\varphi_{z_j}=\varphi_{z_j}(t)$):

$$\begin{gathered}
k_{e_1}\dot{\varphi}_1-\frac{l_2^2 m_2 R_{w_1}\sin(2\varphi_2)\dot{\varphi}_1\dot{\varphi}_2}{4k_{m_1}}-\frac{L_1 l_2^2 m_2\cos(2\varphi_2)\dot{\varphi}_1\dot{\varphi}_2^2}{2k_{m_1}}+\frac{R_{w_1}(l_1^2 m_1+3l_2^2 m_2[\cos(\varphi_2)]^2)\ddot{\varphi}_1}{12k_{m_1}}+\\
-\frac{L_1 l_2^2 m_2\sin(2\varphi_2)\dot{\varphi}_2\ddot{\varphi}_1}{2k_{m_1}}-\frac{L_1 l_2^2 m_2\sin(2\varphi_2)\dot{\varphi}_1\ddot{\varphi}_2}{4k_{m_1}}+\frac{L_1(l_1^2 m_1+3l_2^2 m_2[\cos(\varphi_2)]^2)\dddot{\varphi}_1}{12k_{m_1}}=u_{z_1}\\
\frac{gl_2 m_2 R_{w_2}\cos(\varphi_2)}{2k_{m_2}}+\frac{l_2^2 m_2 R_{w_2}\cos(\varphi_2)\sin(\varphi_2)\dot{\varphi}_1^2}{4k_{m_2}}+\left(k_{e_2}-\frac{gl_2 L_2 m_2\sin(\varphi_2)}{2k_{m_2}}\right)\dot{\varphi}_2+\\
+\frac{l_2^2 L_2 m_2\cos(2\varphi_2)\dot{\varphi}_1^2\dot{\varphi}_2}{4k_{m_2}}+\frac{l_2^2 L_2 m_2\sin(2\phi_2)\dot{\varphi}_1\ddot{\varphi}_1}{4k_{m_2}}+\frac{l_2^2 m_2 R_{w_2}\ddot{\varphi}_2}{3k_{m_2}}+\frac{l_2^2 L_2 m_2\dddot{\varphi}_2}{3k_{m_2}}=u_{z_2}
\end{gathered} \tag{24}$$

4. Model parameterisation

Parameterisation is a procedure of selecting parameters that are to be identified. It is important that this parameterisation is unique. If so, the model is said to be identifiable. The most suitable method of parameterisation is to assume that the parameters to be identified are the successive equation parameters: l_j, m_j, R_{wj}, L_j, k_{mj}, k_{ej}. However, even a superficial analysis suggests that such a model is non-identifiable because the number of parameters of the model can be reduced. Let us define the parameters:

$$\begin{gathered}
\theta_1=\frac{l_1^2 L_1 m_1}{k_{m_1}},\ \theta_2=\frac{L_1 l_2^2 m_2}{k_{m_1}},\ \theta_3=\frac{l_1^2 m_1 R_{w_1}}{k_{m_1}},\ \theta_4=\frac{l_2^2 m_2 R_{w_1}}{k_{m_1}},\ \theta_5=k_{e_1},\\
\theta_6=\frac{l_2 L_2 m_2}{k_{m_2}},\ \theta_7=\frac{l_2 m_2 R_{w_2}}{k_{m_2}},\ \theta_8=k_{e_2},\ \theta_9=l_2
\end{gathered} \tag{25}$$

For the parameters defined in the above way, Eq. 24 takes the following form:

$$\begin{gathered}
\frac{1}{12}(12\theta_5\dot{\varphi}_1-3\theta_4\sin(2\varphi_2)\dot{\varphi}_1\dot{\varphi}_2-6\theta_2\cos(2\varphi_2)\dot{\varphi}_1\dot{\varphi}_2^2+\theta_3\ddot{\varphi}_1+3\theta_4(\cos(\varphi_2))^2\ddot{\varphi}_1+\\
-6\theta_2\sin(2\varphi_2)\dot{\varphi}_2\ddot{\varphi}_1-3\theta_2\sin(2\varphi_2)\dot{\varphi}_1\ddot{\varphi}_2+\theta_1\dddot{\varphi}_1+3\theta_2(\cos(\varphi_2))^2\dddot{\varphi}_1)=u_{z_1}\\
\frac{1}{12}(6g\theta_7\cos(\varphi_2)+3\theta_7\theta_9\cos(\varphi_2)\sin(\varphi_2)\dot{\varphi}_1^2+12\theta_8\dot{\varphi}_2-6g\theta_6\sin(\varphi_2)\dot{\varphi}_2+\\
+3\theta_6\theta_9\cos(2\varphi_2)\dot{\varphi}_1^2\dot{\varphi}_2+3\theta_6\theta_9\sin(2\varphi_2)\dot{\varphi}_1\ddot{\varphi}_1+4\theta_7\theta_9\ddot{\varphi}_2+4\theta_6\theta_9\dddot{\varphi}_2)=u_{z_2}
\end{gathered} \tag{26}$$

Note that the proposed method of parameterisation is nonlinear with respect to the parameters $\boldsymbol{\theta}$.

Using the linearisation described in Eq. 7, we get:

$$v_j(t)=\boldsymbol{\varphi}_j^T(t)\boldsymbol{\theta}_j,\ j=1,2, \tag{27}$$

where for the first equation

$$v_1(t)=12\,(K_{p_1}\varphi_{z_1}-K_{p_1}\varphi_1-K_{d_1}\dot{\varphi}_1),$$

$$\boldsymbol{\varphi}_1(t)=[\dddot{\varphi}_1,-6\cos(2\varphi_2)\dot{\varphi}_1\dot{\varphi}_2^2-6\sin(2\varphi_2)\dot{\varphi}_2\ddot{\varphi}_1-3\sin(2\varphi_2)\dot{\varphi}_1\ddot{\varphi}_2+3(\cos(\varphi_2))^2\dddot{\varphi}_1,\ddot{\varphi}_1,$$
$$-3\sin(2\varphi_2)\dot{\varphi}_1\dot{\varphi}_2+3(\cos(\varphi_2))^2\ddot{\varphi}_1,12\dot{\varphi}_1],$$

$$\boldsymbol{\theta}_1=[\theta_1\ \theta_2\ \theta_3\ \theta_4\ \theta_5]^T,$$

and for the second equation

$$v_2(t)=12\,K_{p_2}\varphi_{z_2}-12K_{p_2}\varphi_2-12K_{d_2}\dot{\varphi}_2+\frac{3}{2}\theta_7\theta_9\sin(2\varphi_2)\dot{\varphi}_1^2+3\theta_6\theta_9\cos(2\varphi_2)\dot{\varphi}_1^2\dot{\varphi}_2+$$
$$+3\theta_6\theta_9\sin(2\varphi_2)\dot{\varphi}_1\ddot{\varphi}_1+4\theta_7\theta_9\ddot{\varphi}_2+4\theta_6\theta_9\dddot{\varphi}_2,$$

$$\boldsymbol{\varphi}_2(t)=[-6g\sin(\varphi_2)\dot{\varphi}_2+3\theta_9\cos(2\varphi_2)\dot{\varphi}_1^2\dot{\varphi}_2+3\theta_9\sin(2\varphi_2)\dot{\varphi}_1\ddot{\varphi}_1+4\theta_9\dddot{\varphi}_2,6g\cos(\varphi_2)+$$
$$+3\theta_9\cos(\varphi_2)\sin(\varphi_2)\dot{\varphi}_1^2+4\theta_9\ddot{\varphi}_2,12\dot{\varphi}_2,3\theta_7\cos(\varphi_2)\sin(\varphi_2)\dot{\varphi}_1^2+3\theta_6\cos(2\varphi_2)\dot{\varphi}_1^2\dot{\varphi}_2+$$
$$+3\theta_6\sin(2\varphi_2)\dot{\varphi}_1\ddot{\varphi}_1+4\theta_7\ddot{\varphi}_2+4\theta_6\dddot{\varphi}_2],$$

$$\boldsymbol{\theta}_2=[\theta_6\ \theta_7\ \theta_8\ \theta_9]^T.$$

5. Experimental results

Let us assume the following values of the parameters of the analysed simple robot system: $l_1 = 0.5$ m, $m_1 = 100$ kg, $l_2 = 1$ m, $g = 9.81$ m/s^2, $R_{wj} = 0.04\ \Omega$, $L_j = 0.004$ H, $K_{pj} = 191.304$, $K_{dj} = 31.6957$, $k_{mj} = 23$ Nm/A, $k_{ej} = 23$ Vs/rad. To observe the changes in the parameter estimates during on-line identification, we assume that the mass of the object carried by the manipulator is not constant. For $0 < t < 45$ s, the mass m_2 is 150 kg, and for $45 < t < 170$ s, $m_2 = 170$ kg.

The parameter estimates are updated every N samples ($N = 1900$), at $\Delta = 0.001$ s. The number of time intervals ($m = 87$) results from the total identification time ($t = 170$ s), the number of samples N, and the sampling period Δ.

The excitation of the plant caused by the set point should be sufficient to allow the identification of the parameters. The most common exciting signals are step signals, a combination of sinusoidal signals with different frequencies and random signals. In the experiment, we used the step signal shown in Fig. 3.

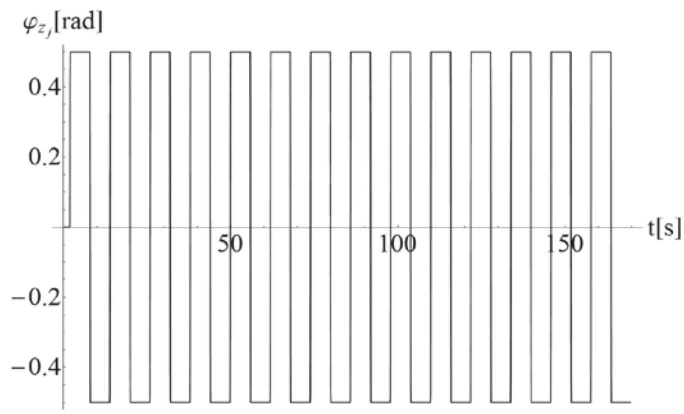

Fig. 3. Set-point angle $\varphi_{z_j}(t)$

The identification results obtained for the particular parameters are shown in Figs. 4-12. Along the horizontal axis, we have the numbers of time intervals m for which the estimation was performed.

https://doi.org/10.37705/TechTrans/e2022007

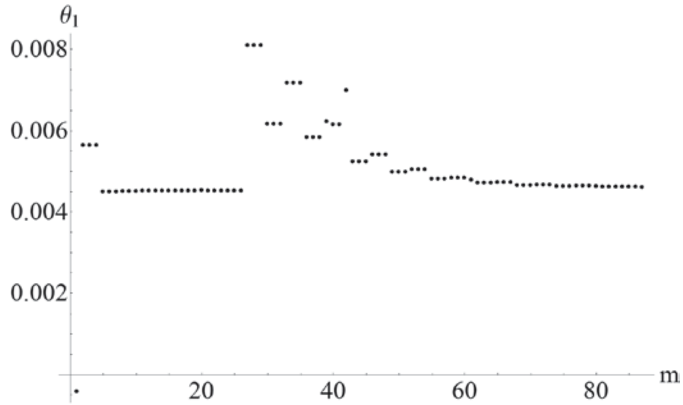

Fig. 4. Identification of the parameter θ_1

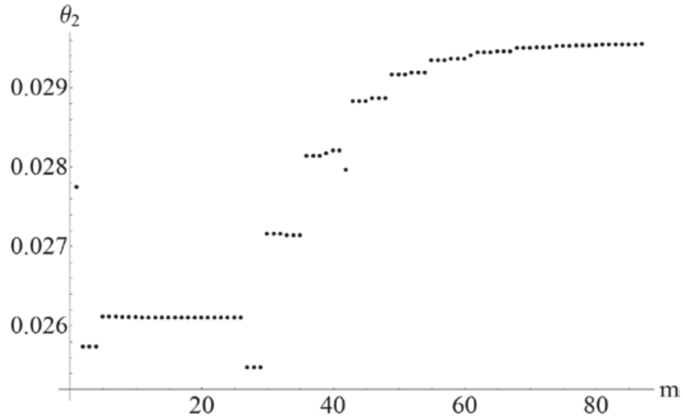

Fig. 5. Identification of the parameter θ_2

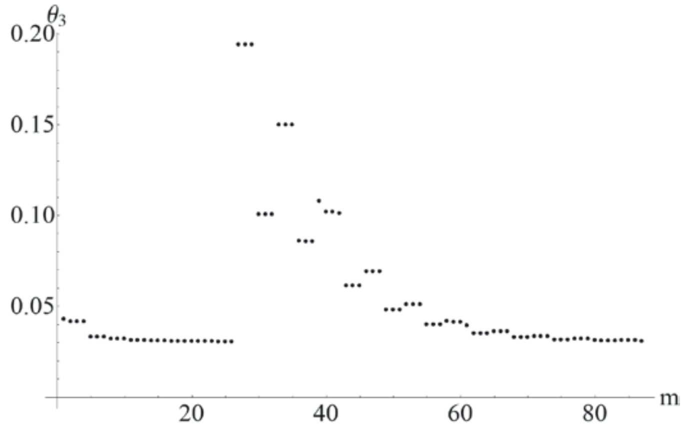

Fig. 6. Identification of the parameter θ_3

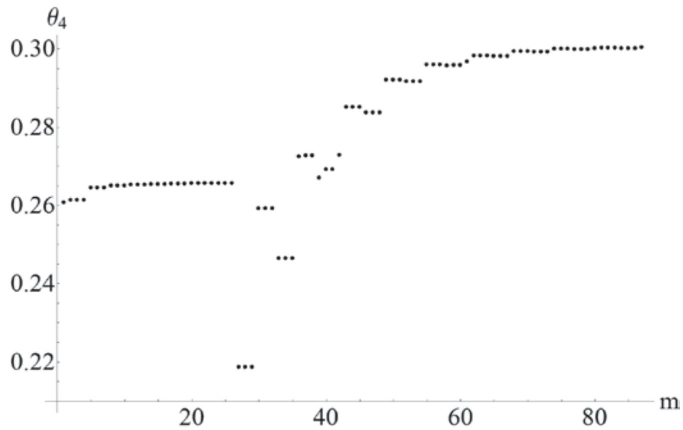

Fig. 7. Identification of the parameter θ_4

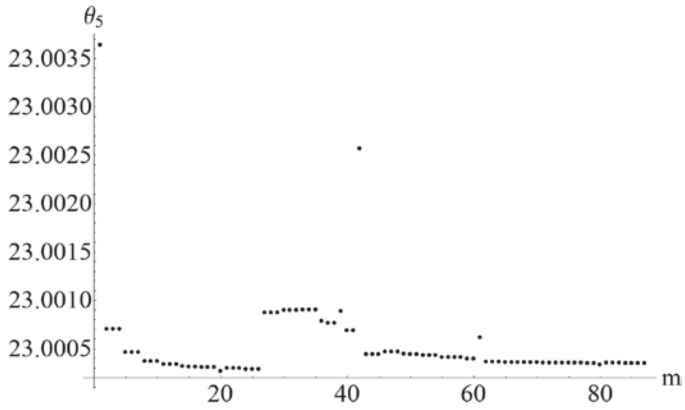

Fig. 8. Identification of the parameter θ_5

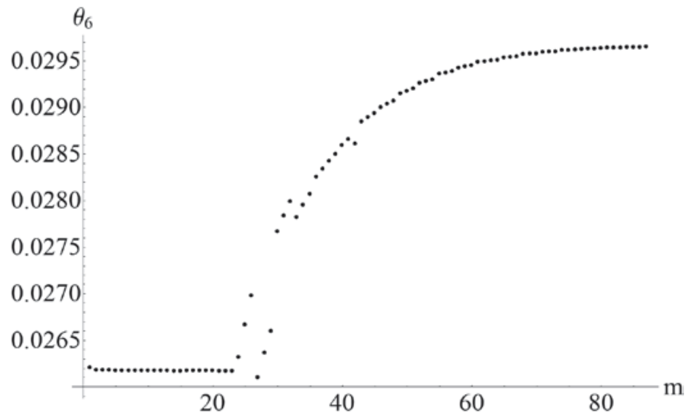

Fig. 9. Identification of the parameter θ_6

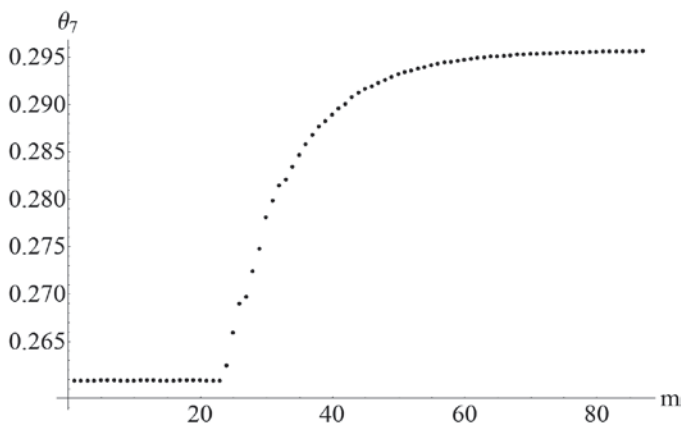

Fig. 10. Identification of the parameter θ_7

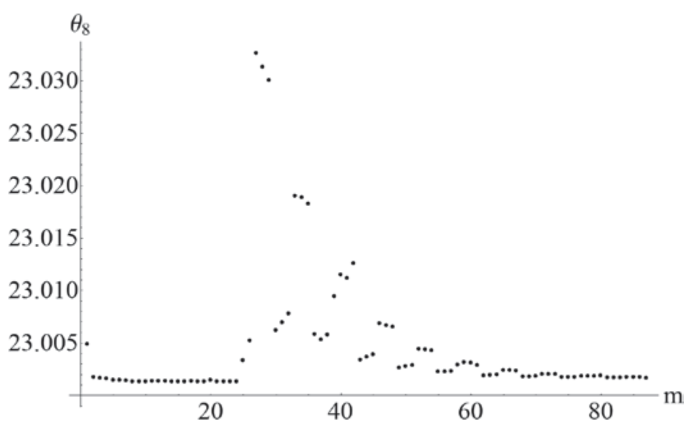

Fig. 11. Identification of the parameter θ_8

https://doi.org/10.37705/TechTrans/e2022007

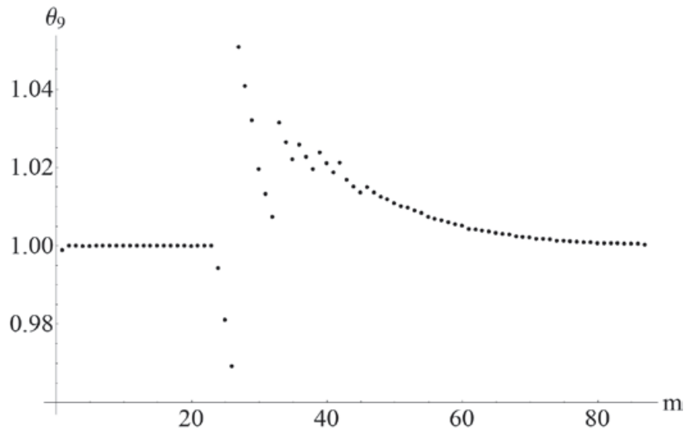

Fig. 12. Identification of the parameter θ_9

6. Conclusions

The values of the nine parameters $\boldsymbol{\theta} = [\theta_1 \ldots \theta_9]^T$ obtained through on-line identification are only slightly different from the real ones $\boldsymbol{\theta} = [0.00434783\ 0.026087\ 0.0434783\ 0.26087\ 23\ 0.026087\ 0.26087\ 23\ 1]^T$ for $0 < t < 45$ s and $\boldsymbol{\theta} = [0.00434783\ 0.0295652\ 0.0434783\ 0.295652\ 23\ 0.0295652\ 0.295652\ 23\ 1]^T$ for $45 < t < 170$ s. Decreasing the factor λ or the number of samples N used for the successive estimations causes the parameter estimates to reach the correct values sooner. For significantly time-varying systems, large changes in the plant parameters require use of an algorithm with a small value of the forgetting factor as this speeds up the forgetting of past data.

A rapid change in the manipulator mass causes a change in four of the identified parameters, θ_2 θ_4 θ_6 and θ_7, which reach the desired values after about a dozen iterations. Instantaneous changes in the values of the other identified parameters, θ_1 θ_3 θ_5 θ_8 and θ_9, are also observed. These are not only due to a sudden change in the mass but also to a step change in the external control signal.

On-line identification is a procedure aimed at increasing our knowledge after each new number N of measurements. It can be applied whenever it is essential to update a model concurrently with a measurement, as is the case with adaptive control systems when the plant varies over time.

Recursive methods is very important in the control systems when the control and signals filtration process is realized on the basis current model (Ljung, Gunnarson, 1990; Ljung, Söderstrom, 1987; Goodwin, Payne, 1977; Widrow, Stearns, 1985).

References

Adamczak, S., Bochnia, J. (2016). Estimating the Approximation Uncertainty for Digital Materials Subjected to Stress Relaxation Tests. *Metrology and Measurement Systems*, 23(4): 545–553, https://doi.org/10.1515/mms-2016-0048

Bochnia, J. (2018). Evaluation of relaxation properties of digital materials obtained by means of PolyJet Matrix technology. *Bulletin of the Polish Academy of Sciences: Technical Sciences*, No. 6, https://doi.org/10.24425/bpas.2018.125936

Fortescue, T.R., Kershenbaum, L.S., Ydsti, B.E. (1981). Implementation of self tuning regulators with variable forgetting factor. *Automatica*, 17: 831–835.

Goodwin, G.C., Payne, R.L. (1977). *Dynamic System Identification: Experiment Design and Data Analysis.* New York: Academic Press Inc.

Graba, M. (2017). Proposal of the hybrid solution to determining the selected fracture parameters for SEN(B) specimens dominated by plane strain. *Bulletin of the Polish Academy of Sciences: Technical Sciences*, No. 4, https://doi.org/10.1515/bpasts-2017-0057

Hunt, J.K. (1966). A Survey of Recursive Identification Algorithms. *Tras. Inst. MC*, 8(5): 273–278.

Janecki, D. (1988). New recursive parameter estimation algorithms with varying but bounded gain matrix. *Int. J. Contr.*, 47(1): 75–84.

Krzysztofik, I., Takosoglu, J., Koruba, Z. (2017). Selected methods of control of the scanning and tracking gyroscope system mounted on a combat vehicle. *Annual Reviews in Control*, No. 44: 173–182.

Ljung, L., Gunnarson, S. (1990). Adaptation and Tracing in System Identification – a Survey. *Automatica*, 26(1): 7–21.

Ljung, L., Söderstrom, T. (1987). *Theory and Practice of Recursive Identification*. Cambrigde Mass.: MIT Press.

Miller, T., Adamczak, S., Świderski, J., Wieczorowski, M., Łętocha, A., Gapiński, B. (2017). Influence of temperature gradient on surface texture measurements with the use of profilometry. *Bulletin of the Polish Academy of Sciences: Technical Sciences*, No. 1: 53–61, https://doi.org/10.1515/bpasts-2017-0007, 2017.

Niederliński, A., Mościński, J., Ogonowski, Z. (1995). *Adaptive control*. Warszawa: PWN.

Valasek, R., Pavliska, V., Perfilieva, I., Farana, R. (2013). Application of Fuzzy Transform for Noise Reduction in Helicopter Model Identification. (In) *Proceedings of the 2013 14th International Carpathian Control Conference (ICCC)* (pp. 400–405). Rytro: AGH University of Science and Technology.

Vítečková, M., Víteček, A. (2013). Simple Digital Controller Tuning, (In) *Proceedings of the 2013 14th International Carpathian Control Conference (ICCC).* Rytro: AGH University of Science and Technology.

Widrow, B., Stearns, S.D. (1985). *Adaptive Signal Processing*. Englewood Cliffs: Prentice Hall.

Wittenmark, B., Astrom, K.J. (1984). Practical issues in the implementation of self tuning controllers. *Automatica*, 20: 595–606.

Wittenmark, B., Astrom, K.J. (1980). *Simple self tuning controllers. In Methods and Application in Adaptive Control*. Berlin: Springer Verlag, Vol. 24: 21–30.

Woś, P., Dindorf, R. (2013). Adaptive control of the electro-hydraulic servo-system with external disturbance. *Asian Journal of Control*, 15(4): 1065–1080.

Zorawski, W, Makrenek, M., Goral, A. (2016). Mechanical Properties and Corrosion Resistance of HVOF Sprayed Coatings Using Nanostructured Carbide Powders. *Archives of Metallurgy and Materials*, 61(4): 1839–1846, https://doi.org/10.1515/amm-2016-0297