REPORT

OF

BOARD OF ENGINEER OFFICERS

AS TO

MAXIMUM SPAN PRACTICABLE FOR SUSPENSION BRIDGES.

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OFFICE OF THE CHIEF OF ENGINEERS,
UNITED STATES ARMY,
Washington, D. C., October 25, 1894.

SIR: I have the honor to submit herewith a printed copy of the report of the Board of Engineer Officers convened in accordance with your order, dated January 27, 1894, to investigate and report their conclusions as to the maximum length of span practicable for suspension bridges. The report is a very valuable one, shows careful research, and I approve and concur in its conclusions.

Very respectfully, your obedient servant,

THOS. LINCOLN CASEY,
Brig. Gen., Chief of Engineers.

Hon. DANIEL S. LAMONT,
Secretary of War.

REPORT.

UNITED STATES ENGINEER OFFICE,
Philadelphia, Pa., September 29, 1894.

GENERAL: The Board of Officers of the Corps of Engineers appointed by Special Orders, No. 5, current series, Headquarters Corps of Engineers, U. S. Army, January 29, 1894, to make investigations as to certain bridges, in accordance with instructions of the Secretary of War, have the honor to submit the following report:

The instructions of the Board are contained in a letter from the Secretary of War to the Chief of Engineers, dated January 27, 1894, and in the indorsement of the Chief of Engineers thereon, dated January 30, 1894. Copies of this letter and the orders convening the Board are appended hereto.

The Secretary of War, in his letter of January 27, 1894, remarks that, "in view of the importance of questions arising in this Department in connection with the building of bridges over navigable streams, it is essential that it should be possessed of accurate and full information necessary to their intelligent and proper determination;" and directs the formation of "a Board of Officers of the Engineer Corps, who shall investigate and report their conclusions as to the maximum length of span practicable for suspension bridges and consistent with an amount of traffic probably sufficient to warrant the expense of construction."

The indorsement of the Chief of Engineers of January 30, 1894, directs the Board to include in its investigations "strength of materials, loads, foundations, wind pressure, oscillations and bracing."

The Board convened at New York City on February 13, 1894, and remained in session until February 15, 1894, when it adjourned to collect information. The Board held a session at Philadelphia, Pa., from March 6 to 10, 1894. An extended preliminary investigation of the subject under consideration had already been made when a Board of expert bridge engineers was appointed by the President, on June 15, 1894, under the provisions of the act approved June 7, 1894, to recommend what length of span, not less than 2,000 feet, would be safe and practicable for a railroad bridge to be constructed across the Hudson River between New York City and the State of New Jersey. The New York Board was composed of five engineers, four of whom were civil engineers of long and varied experience in the designing and construction of bridges and of the highest professional standing. It was therefore considered desirable to delay the completion of this report until the determinations of the New York Board could be ascertained and studied. The present Board has derived much assistance from the published report of the New York Board, as will be indicated below. At the last session, which was held at Philadelphia from September 20 to September 29, 1894, this report was unanimously adopted.

The question of the maximum practicable span may be investigated as a purely engineering problem, when certain preliminary conditions are established. The bridge will doubtless be a railroad bridge, since with the largest span the traffic capacity would not otherwise justify the cost. This assumed, the width must be at least sufficient to accommodate a double track. The number of double tracks required must be established so as to give a traffic capacity "probably sufficient to warrant the expense of construction."

The New York and Brooklyn bridge, the longest suspension bridge yet constructed, consists in reality of two similar bridges suspended side by side and braced together, the promenade being supported between the bridges as an extra weight on the interior cables. Following this idea, the Board in its preliminary investigation assumed a double-track railroad bridge as the unit bridge, bracing together side by side as many such bridges as were considered necessary to accommodate the traffic contemplated. The engineering problem was thus limited to the question of determining the maximum span for a double-track railroad bridge. It was found, however, that there are many serious practical objections to such an arrangement in a long-span bridge carrying very heavy loads. In this investigation, therefore, the loads will be assumed to be supported by only two sets of cables, one on each side of the bridge; an arrangement which was adopted as a basis of estimate by the New York Board.

In the various projects for long-span bridges across the Hudson River at New York the least traffic-capacity assumed was six tracks, and the New York Board adopted this number of tracks in its investigations. In this report, therefore, it is proposed to first consider the question of the maximum span for a six-track railway bridge as an engineering problem, after which the relations between span, traffic and cost of construction will receive such investigation as the nature of the subject will permit.

Since much of the information with reference to strength of materials, loads, etc., collected by the Board as directed by the Chief of Engineers, is necessary for the proper investigation of the question of the practical maximum span, this part of the subject will first receive attention.

STRENGTH OF MATERIALS.

The supporting cables of a suspension bridge of long span are made of steel. They are either chains composed of connected links or cables formed of parallel wires or twisted wire ropes. To obtain the longest span possible the weight of the cable must be a minimum as compared with its carrying capacity. The connections of a series of links add from 20 to 25 per cent to the dead weight of the chain, while in the wire cable the connections add, at most, only 2 or 3 per cent. Moreover steel in the form of wire has a minimum strength more than double its maximum strength in the form of bars suitable for the construction of a suspension chain. A link chain, therefore, will weigh about two and one-half times as much as a wire cable of equal carrying capacity; or in other words, a wire cable can be stretched about two and one-half times as far as a steel chain before being broken, other conditions being the same in both cases. Moreover, it is stated by Melan* that

*Handbuch der Ingenieurwissenschaften. Band II. Der Brückenbau—J. Melan, Leipzig, 1888.

considerable bending moments are sometimes produced by the friction between the links of chains, but the effects due to the stiffness of cables are so small that they may be neglected. It is therefore assumed that the cables are made of steel wires laid parallel to each other.

The strength of the suspension cable will depend upon the tensile strength of the steel employed in its construction and upon the number of wires it contains. The wire employed in the cables of the New York and Brooklyn bridge had a tensile strength of 170,000 pounds per square inch, and the cables were originally designed to contain each 6,188 wires of No. 7, B. W. G., but as some heavier wires were introduced during construction, the actual number of wires was only 5,400. These are the largest cables made up to the present time, having a diameter of 15⅝ inches. The cables of the Cincinnati suspension bridge have a diameter of 12 inches and each contains 5,200 No. 9 wires.

There is a practical limit to the number of wires which can be united in a cable, since as the number increases it becomes more and more difficult to adjust the wires so that each will bear its due proportion of stress under the varying conditions of temperature and loading. No unusual difficulties, however, were encountered in the manufacture of the cables above referred to, but it is believed that with the method employed for making the cables of the East River bridge the practical limit of the number of wires was very nearly, if not quite, attained. With improved methods the construction of much larger cables might be found practicable. An increase in the size of the wire does not materially increase the difficulty of construction. No. 3 wire having a tensile strength of 180,000 pounds per square inch, can now be readily obtained at a reasonable price. Indeed steel wire much stronger than this can be obtained (up to more than 300,000 pounds per square inch) but its present cost would prohibit its employment.

The Board therefore assume for the purposes of this investigation a suspension cable formed of 6,000 parallel steel wires, No. 3, B. W. G. The area of its cross-section will be 316 square inches without wrapping, and its breaking tensile strength will be 56,880,000 pounds, or 28,440 tons. With a safety factor of 3, which was adopted by the New York Board, and will be adopted in this investigation for reasons to be given hereafter, the working strength of this cable will be 18,960,000 pounds or 9,480 tons. Its diameter with wrapping will be 21½ inches. The New York Board have adopted a cable of about this size and strength in their estimates.

The total cable strength available for the support of the bridge depends upon the number of cables which can be practically combined as a single cable system on one side of the bridge. If many cables are employed it becomes difficult to distribute the strains among them so that each shall carry its proportionate load under the varying conditions of temperature and traffic. It is not easy to decide what is the practical limit of the number of cables to be assembled together. Where parallel wire cables are used they must be sufficiently separated horizontally and vertically to give room for the operation of the wire-wrapping machine and this requires intervals of at least 3 feet during construction.

The cables may cross the saddles on top of the towers side by side or they may be arranged in one or more vertical or nearly vertical planes. In the first case the cradling of the cables in converging planes, (which is desirable for lateral stability) requires considerable intervals between the saddles. In the investigations of the New York Board an interval of 20 feet was found to be necessary for such an arrangement. But the cables on one side of the bridge may also be arranged side by side in parallel inclined planes and held the same distance apart throughout their length by iron separators between the suspender clamps, in which case the saddles on the towers would be closer together. Still the number of cables suspended on each side cannot be made very large without increasing the dimensions of the towers and the piers supporting them far beyond the requirments of the roadway and suspended loads.

The vertical arrangement of the cables (or a combination of vertical and horizontal arrangements) certainly presents some very decided advantages. It requires less width at the top of the towers, and a large part of the stiffening of the bridge may be obtained by trussing the cables in a manner which will be again referred to. Moreover, with this arrangement the towers can be so constructed that new cables can be readily added to meet future demands for increased traffic capacity. This method, however, is not so simple as the other and with large loads and cables involves mechanical difficulties which can be properly dealt with only after an extended investigation of the problem as a special case. In this general investigation the Board consider it best to adopt the simpler arrangement, as has been done by the New York Board. Whatever arrangement is adopted, the Board are of the opinion that it would not be found convenient to work more than eight cables together as one cable system. For the purposes of this investigation, it is therefore assumed that the suspension bridge of maximum span is supported by sixteen 21½-inch cables. The following list giving the arrangements employed in a number of important bridges, may be of interest in this connection.

New York and Brooklyn Bridge, 1,595.5 feet span, has *one* 15⅝-inch cable on each side.

Niagara Bridge, 821.3 feet span, *two* 10-inch cables, one vertically over the other.

Wheeling Bridge, 1,010 feet span, *two* 8-inch cables, side by side.

Fairmount Bridge, 550 feet span, *seven* cables, 6 side by side and 1 above.

Freiberg Bridge, Switzerland, 870 feet span, *three* cables, 2 side by side and 1 above.

Dordogne Bridge, France, 350 feet span, *three* cables, side by side.

Niagara Bridge, at Lewistown, 1,400 feet span, had *four* cables side by side.

Menai Bridge, 600 feet span, *four* chains in the same vertical plane.

Tweed Bridge, Berwick, 450 feet span, *three* chains in the same vertical plane.

Tersing Bridge, over the Maas, *two* chains, one over the other.

Bridge Voconflans, St. Honorine, *two* chains, one over the other.

La Roche Bernard Bridge, over the Vilaine, 650 feet span, *two* cables, side by side.

Lambeth Bridge, England, *two* cables, side by side.

Donau Bridge, Pesth, 660 feet span, *two* chains, one over the other.

Moldau Bridge, Prague, *four* chains in pairs, over each other.

LOADS.

The total load supported by the bridge will consist of two parts, viz, the live load or weight of the passing traffic and the dead load or weight of the structure. It will be convenient to consider the load under these two heads.

1. *Live load.*—In another part of this report the difference in the character of the action of the live load upon a structure in stable equilibrium having a certain degree of flexibility from that of its action upon a more rigid structure in unstable equilibrium will receive due consideration. For the present we will only determine the magnitude of the live load. The live load per linear foot of span will be represented by q.

The greatest static effect upon the cable will be produced by the maximum load; that is, when the whole platform from tower to tower is covered with the heaviest possible railroad trains.

A suspension railroad bridge of very long span will as a rule be built only over a wide river or estuary navigable by ocean craft and therefore requiring a great height of the bridge above the water. To limit the expense of the shore extensions the approaches must be given as steep a grade as is admissible for a railroad bridge. It is, therefore assumed that the approaches will have a grade of 1 per cent.

The weight of a railroad train passing over the bridge need not be considered as any greater than that which the heaviest freight locomotive is capable of hauling up a 1 per cent grade. From a list published by the Baldwin Locomotive Works it appears that exceptionally heavy locomotives are built with 170,000 pounds on the drivers and a total weight of 192,500 pounds. The Baldwin Works allow 9 tons for each 1,000 pounds on the drivers as the maximum efficiency on a grade of 1 per cent. This extra heavy locomotive can therefore pull up on the bridge 1,530 tons including its own weight. Subtracting 96 tons for the weight of the locomotive, we have for the weight of the train, 1,434 tons. This is equal to 41 hopper-bottom gondola cars each 27 feet 2 inches long and weighing 35 tons. The length of the engine and tender being 54 feet, the total length of the train will be 1,168 feet, and the weight per linear foot of track will be 2,620 pounds, equal to 1.31 tons. If we suppose all 6 tracks to be loaded from end to end with such trains, the live load per linear foot will be 15,720 pounds, equal to 7.86 tons.

Any such loading as this, however, is so extremely improbable as to be a practical impossibility; indeed, at the height above water level at which such a bridge must be carried, the transportation of passengers, and not of freight, must be the main consideration. A purely freight traffic would in no conceivable location require six tracks over a bridge of very long span. Such a number of tracks would only be justified by the location of the bridge near a very large city and by a large passenger traffic. The trains passing over such a bridge would undoubtedly be controlled on the block system and not more than one train on each track would be allowed upon the bridge at the same time. If the stiffening girders could do their full duty the weights upon the bridge would be uniformly distributed and the live load per linear foot would be $\frac{3{,}060{,}000}{L} \times 6 = \frac{18{,}360{,}000}{L}$ pounds. The distribution, however, is not perfectly uniform, and there are occasionally other causes which produce an increase in local stresses. The Board consider it best to add 50 per cent to this estimate to cover these uncertainties. In computing the cable strength, therefore, the adopted value of the live load will be—

$$q = \frac{27{,}540{,}000}{L}$$

The live load assumed for the Niagara Suspension Bridge, which has a span of 800 feet, is 350 tons in a length of 450 feet of single track, which is equivalent to 1,600 pounds per linear foot of train, or less than 1,000 pounds per linear foot of the entire span, with a factor of safety of 4.41 in the cables and 4.0 in the stiffening girders.* The bridge, however, safely carries heavier trains in daily operation.

The largest existing railroad bridge is the Forth bridge, in Scotland. It has 2 tracks and 2 spans of 1,700 feet each. It was tested with 2 heavy trains side by side, each 1,000 feet long and weighing 900 tons. Each train was drawn by 2 locomotives each weighing 72 tons. This load was equivalent to 1,800 pounds per linear foot of track or 3,600 pounds per linear foot of span. These were considered extra heavy train loads very seldom occurring in actual operation on English roads.†

Making allowance for the heavier train loads of American railroads, it will be seen that 3,000 pounds per linear foot of track for a length of 1,500 feet. considered as distributed over the bridge from tower to tower (which is the value given by our formula), is an exceedingly safe assumption of the live load for a very long bridge in which the span exceeds the length of the maximum train. This value agrees very nearly with that assumed by the New York Board in its estimate for a "lighter structure."

It is very evident that the assumed live load per unit of track ought to diminish with the number of tracks and with the length of span. A single-track bridge of short span is strained nearly to its maximum every time a train goes over it. A 6-track bridge is strained to its maximum only when 6 maximum trains are abreast of each other; and when the span exceeds the maximum train length the maximum stress ought never to occur.

2. *Dead load.*—This produces at all times constant strains in the members of the bridge. It is composed of the weights of the following parts: The suspension cables with their wrapping, the platform, the stiffening girders, the wind and sway bracing, and the suspenders. The weights of these parts per linear foot of span will be represented as follows: Cables $= w$; cable wrapping $= w_0$; platform $= p_1$; girders $= p_2$; bracing $= p_3$; suspenders $= p_4$.

(1) *Weight of the suspension cables.*—The weight of a cable formed of 6,000 parallel steel wires, No. 3, B. W. G., having a diameter of $21\frac{1}{4}$ inches, without wrapping, will be 1,075 pounds, or 0.538 tons per running foot. If we assume 16 cables for the support of the entire bridge, the total cable weight per linear foot will be $w' = 17{,}200$ pounds $=$ 8.6 tons; and $w = 17{,}917 = 8.959$ tons.

(2) *Cable wrapping.*—The cable will be wrapped with iron wire of No. 9, B. W. G. The weight of this wrapping will be 26 pounds per linear foot, and for 16 cables, 416 pounds. The weight of wrapping per linear foot of span will therefore be $w_0 = 433$ pounds.

It will be found convenient for the purposes of this investigation to know the relation between the weight of the cable per linear foot of horizontal span (w) and its weight per foot measured in the direction of its axis (w'). The cable curve will be approximately a parabola, and its approximate length will be $\left(1 + \frac{2.67}{R^2}\right)$ L, in which L is the horizontal span in feet and R is the ratio of the versine of the cable to the span. Hence $w = \left(1 + \frac{2.67}{R^2}\right) w'$. For reasons which will be given in the sequel the value of R will be assumed as 8 in this investigation. For this value we have $w = 1.0417\ w'$ and $w' = 0.960\ w$.

* Proceedings Am. Soc. C. E., Vol. x, p. 195.
† Record of the Forth Bridge, p. 64.

(3) *Weight of the platform.*—In the arrangement adopted in this investigation for the purposes of estimate the cross-girders sustain the weight not only of the live load but also of the stiffening girders and lateral bracing. The distance between the cross-girders is 30 feet.

The platform further consists of longitudinal girders (stringers), the permanent way consisting of ties and rails and the covering. Its construction is the same as in any other railroad bridge.

The weights of the stiffening girders and lateral bracing, (which increase with the span), are imposed upon the cross-girders so very near the points of suspension that the weight of the platform may be considered practically independent of the span. The weight per linear foot of span for a 6-track platform, carrying stiffening girders and bracing, for a span of 3,200 feet, was determined by the New York Board to be 7,200 pounds. We may therefore adopt in this investigation the constant value $p_1=7{,}200$ pounds$=3.6$ tons.

(4) *Weight of the stiffening girders.*—It will be shown under the head of Vertical Bracing that the weight of the two stiffening girders per linear foot of 6-track bridge may be found in pounds from the formula $p_2=3{,}281+2.754\ L+0.0005312\ L^2$. This includes an allowance of material in the lower chords to provide for the stresses due to wind.

(5) *Weight of the lateral bracing.*—As will be shown under the head of Lateral Bracing, the weight of the wind and sway bracing per linear foot of 6-track bridge (so far as not provided for in the preceding paragraph) will be given in pounds by the formula $p_3=2{,}420+0.3889\ L$.

(6) *Weight of the suspenders.*—The suspended load is connected with the cables by 8 wire suspenders on each side of the bridge at each cross-girder. The suspenders at each girder are equal in length and are supposed to be adjusted so as to carry practically equal portions of the load. At the middle of the span the cables are 60 feet above the level of the suspension pins. The versine of the cable being $\frac{L}{8}$ the average length of a suspender is $\frac{L}{24}+60$. Asssuming a unit working stress of 30,000 pounds and adding 20 per cent for constructive details, the weight of the suspenders per linear foot of span will be

$$p_4=\frac{3.4\times 1.2}{30{,}000}\left(\frac{L}{24}+60\right)(p'-w_0)$$

in which $p'=q+p_1+p_2+p_3+p_4+w_0$.

For the purposes of this computation we may assume $p_4=1{,}300$ in the value of $p'-w_0$.

From values previously given we find $p'-w_0=14200+27540000\ L^{-1}+3.1429\ L+0.0005312\ L^2$; and $p_4=272+224726\ L^{-1}+0.10616\ L+0.00002215\ L^2+0.000000003\ L^3$.

(7) *Total suspended weight.*—The total suspended weight per linear foot of span exclusive of the cables will be $p'=q+p_1+p_2+p_3+p_4+w_0=13605+27764726\ L^{-1}+3.24906\ L+0.00055335\ L^2+0.000000003\ L^3$.

WIND PRESSURES.

The wind pressure upon a large bridge is of such magnitude as to require especial consideration. In the principal members of the Forth bridge (a cantilever construction), the maximum stresses due to wind have been stated by its engineer to be more than one-quarter greater than those due to the dead weight of the bridge and nearly three times as great as those due to the live load of passing trains. In suspension bridges these wind stresses, though they may be less than in other bridges, are still of very great importance, and must be carefully provided against by the introduction of metal which, while adding nothing to the carrying capacity of the bridge, does add considerably to its dead load, and therefore necessitates an increase in the strength of suspenders, main cables, towers, anchorages and foundations, and thus may add enormously to the total cost of the bridge. Under such circumstances, while it is, on the one hand, important to secure a sufficiency of wind bracing, it is, on the other hand, equally important not to use any more than is actually necessary.

Since the existing fund of information as to wind pressures, as to their effect on bridges, and as to the present most used methods of dimensioning wind bracing, is either quite limited or else is not easily accessible, it has been thought well to attach hereto a full history of past work in this direction with suggestions of rules for use in the dimensioning of large structures in places exposed to heavy winds. (*See* Appendix C.)

Past history shows the possibility, at almost any place, of an occasional tornado of power sufficient to destroy almost any existing engineering structure. Such tornadoes like violent earthquakes, are so rare that no large constructions of to-day are made thoroughly proof against them. In such a tornado, however, a suspension bridge would fare much better than any other form of bridge as it offers the least surface to the wind, as its overturning is almost a physical impossibility, and especially as the loss of large parts of its roadway and stiffening trusses would not necessarily destroy its main cables and towers (these being its essential and costly features). The Board have therefore, (for reasons stated in the Appendix) considered a maximum steady wind pressure of 30 pounds per square foot over the entire structure and over a continuous train, reaching entirely across the bridge, and also a similar 30-pound pressure over the unloaded bridge, accompanied by an added pressure of 20 pounds per square foot (making 50 pounds in all) over 1,000 feet of the unloaded bridge; this latter allowance being made to provide for occasional severe gusts.

The exposed surface of the bridge and load per running foot (by the method of calculation given in full in the Appendix) is taken at 30 square feet for the cables and suspenders, 49 square feet for the stiffening girder (including the upper chord, lower chord, web members, horizontal diagonals, and sway bracing), 18 square feet for the platform (including track, guard rails, ties, cross girders, and stringers), and 8 square feet for the train (excluding the portion sheltered by the high bottom chords and other adjacent parts of the stiffening girders). In view of the heavy weights and consequent great inertia of the cables and stiffening girders, the resulting wind pressure is treated as uniformly distributed over the entire bridge from end to end; though a more careful distribution, perhaps saving considerable metal, would be adopted in actual practice.

The cradling of the main cables and suspenders is considered sufficient to amply resist the 1,050 pounds per square foot of wind pressure due to the 30 square feet per linear foot of their own surfaces.

The wind bracing of the stiffening truss will then have to resist only the wind pressure on the stiffening truss, platform, and train; which amounts to 2,345 pounds per linear foot for the unloaded bridge, and 2,250 pounds per foot for the loaded bridge.

As the stiffening truss is hinged at its middle, the wind bracing (at least near the ends of the half trusses) must be arranged to carry all the wind stresses to the bottom chord; so that this bracing is taken as composed of a very light upper horizontal truss (not theoretically necessary), of a strong vertical sway bracing, and of a heavier lower horizontal truss. Since the wind trussing may be combined with the adjacent parts of the stiffening girder and platform, this lower wind truss will be built up by increasing, where necessary, the dimensions of the cross girders of the platform, and the lower chords of the stiffening truss, and by inserting cross diagonals between them.

Because of the great size of these cross girders and lower chords, and therefore the great excess of strength in this lower truss when the bridge is unloaded, the Board regard the 2,250 pounds per linear foot of wind pressure on the loaded bridge as the one which throws the greatest strain upon its members, and therefore take this value as the one to be used in combination with the other loads upon a bridge of maximum length.

In case further lateral stiffness against wind should, at any time, be thought desirable, it may be obtained by the use of horizontal wind cables under the platform; and small main cables may at any time be added, if found necessary, to support the added weight of such wind cables; but the Board consider the use of such wind cables unnecessary.

OSCILLATIONS.

In considering the character and importance of small motions in bridges it is necessary to distinguish carefully between stability and rigidity. A suspension bridge is the most stable of all bridge structures. The locus of the centers of gravity of its vertical cross-sections lies far below the points of support. The live load, which is the main cause of its vertical oscillations, always moves below the gravity-line, thereby increasing the lateral stability of the structure. As the span is increased the gravity-line rises, but the resulting slight decrease in stability is more than compensated for by the diminution in the ratio of the live to the permanent load. The small motions of erect-arch and deck bridges must be carefully confined within small limits to insure the safety of the structures, but there is no such necessity in the case of suspension bridges, where the system is in stable equilibrium and sure to return to its position of rest whatever may be the magnitude of the displacement.

The lateral oscillations are due mainly to the action of the wind. These are met not only by the great weight of the structure, but also by the cradling of the cables, which much increases the lateral stability.

It is possible to construct a suspension bridge so that it will have any degree of rigidity desired, but it will appear from the above that rigidity is in this case of much less importance than it is in most other kinds of bridges; indeed, it may be shown that a certain small flexibility is a positive advantage in suspension bridges.

The Board do not consider it necessary to give in this report an elaborate development of the theory of bridge oscillations, because it is perfectly easy to stiffen a suspension bridge so as to reduce both its vertical and lateral deflections, and consequently the duration of its oscillations, within any desired limits; and in bridges of very long span, where the ratio of live to dead load is comparatively small, no difficulty from this cause need be anticipated. The following brief remarks on this subject relate to suspension bridges of comparatively small weight and span.

As before remarked, oscillations in suspension bridges are mainly produced by the impulses of the moving load and by the pressure of the wind. The magnitude of the oscillations is sometimes increased by the lengthening of the central part of the cable, due to the straightening of the chains of the side span under the action of a load on the central span. Theoretically an infinitely small impulsive force may produce an infinitely small amplitude of oscillation of finite duration. If such impulses are repeated a summation of their effects may result. This will occur when the interval between two impulses is equal to the time of oscillation, and may occur when it is greater. Under these circumstances small impulsive forces, by many repetitions, may produce a great oscillation in an elastic or suspended body. In this way the wind has been known to raise waves in the platform of a light suspension bridge.

In the case of a bridge truss it is important that the time of oscillation produced by a load acting impulsively should not exceed a certain amount, in order that the oscillations may not be cumulative.

In highway bridges it is especially the measured step of pedestrians which gradually augments the amplitude of the oscillation when the time of oscillation is equal to or greater than the time of a step, which may be assumed as from 0.6 to 0.7 second. If the time of oscillation of the structure is greater, it can adjust itself to the time of a step by the formation of centers of oscillation. The result is that by the gradual accumulation of energy changes of form are produced which are considerably greater than those produced by an equivalent static load.

These oscillations occur not only in elastic or stiffened systems, but also in slack systems. A freely suspended heavy chain moved from the position of equilibrium in its vertical plane will assume oscillating motions which will gradually increase if new impulses occur in the time intervals corresponding to the time of oscillation or a fraction thereof. It is shown by Melan that the time of oscillation of a slack chain is materially greater than that of even a very elastic construction, which explains the well-known fact that unstiffened suspension bridges can very easily be brought into great oscillations by a few pedestrians.

Prof. Melan in his treatise on Bridge Construction deduces the equations $t = 1.806\sqrt{f}$ and $t = 2.0063\sqrt{u + 0.8\,u_0}$ for the times of oscillation in a slack and a stiffened system, respectively; in which t is the time of oscillation in seconds, f is the versine of the cable in meters, u_0 is the deflection in the middle due to the uniformly distributed dead load, and u is the increase of deflection due to a concentrated load in the position of rest in meters.

From these formulas it follows that the time of oscillation of a bridge structure depends only upon the magnitude of its deflection in the position of rest, no matter what may be the character or size of the structure. Hence the deflection must be kept within certain limits, in order that the bridge may not be set in vibration by the steps of pedestrians or other regularly repeated impulses.

The time of oscillation and consequently the deflection must be the smaller the more rapidly the repetition of impulses occurs. Hence greater stiffness is required in a railroad suspension bridge than in a similar highway bridge.

As the deflection u_0 due to the uniformly distributed load is in general proportional to the fourth power of the span, it follows that the time of oscillation varies approximately as the square of the span. Therefore, this time may be diminished by fastening the structure at intermediate points to the shore or ground so as to form a number of centers of oscillation.

The occurrence of cumulative oscillations may also be prevented by employing together several systems for stiffening the cables, these systems having different periods of oscillation; and where a number of cables are assembled on each side of the bridge, the same result may be accomplished by employing different versines for the cable curves. Thus, in the bridge designed by Mr. Gustav Lindenthal for the North River Bridge Company, the stiffening is obtained by trussing between the cables and by continuous longitudinal platform girders. As these two systems have different periods the chance of cumulative oscillations is greatly reduced. In the Niagara Suspension Bridge the two systems of cables have versines of 54 and 64 feet, respectively, and consequently different periods of oscillation.

The arrangements to prevent deflections due to the moving load and the wind will receive consideration in other parts of this report.

BRACING.

The bracing required to stiffen the bridge may be conveniently considered under two heads, vertical bracing and lateral bracing.

1. *Vertical bracing.*—It is the object of this bracing to confine within definite and small limits the oscillations and deflections caused mainly by the rolling load. Various methods have been employed for this purpose, the principal of which are as follows:

(1) Stays extending from the tops of the towers to the platform.

(2) Stays extending from the bottoms of the towers to the cables.

(3) Longitudinal stiffening girders connected with the platform and extending over the whole length of the bridge.

(4) Bracing between two cables hanging in the same vertical plane.

(5) Trussing the cable on its concave side.

(6) Trussing between the cable and the platform.

Two or more of these methods of stiffening are frequently employed together in the same bridge. For example: Methods (1) and (3) are employed at the East River Bridge; method (3) at the Niagara Bridge; method (4) at the Allegheny River Bridge at Seventh street, Pittsburg; method (5) at the Point Bridge over the Monongahela at Pittsburg; method (6) at the Lambeth Bridge, England. Experience has proved all these methods to be effective, and for some of them special advantages of economy are claimed. Thus the over-floor stays of method (1) not only prevent the development of large vertical oscillations in the platform, but also relieve the suspension cables of a considerable part of the load. In method (2) the stays add to the weight on the cables instead of relieving them, and in this respect it is not as good as method (1). It has been objected, however, to the use of stays in bridges of large span that they complicate the conditions of equilibrium, as it is difficult to adjust them so as to bear a definite portion of the stresses under the varying conditions of load and temperature. By bracing two cables—method (4)—we utilize the cables as the chords of the stiffening girder and save the material otherwise required for the chords. Still greater economy is claimed for combinations of the stiffening systems.

The simplest and most employed stiffening system is the longitudinal stiffening girder. Such girders are convenient for supports to the lateral bracing and for side guards, and at none of the existing bridges where other methods were employed was the girder entirely dispensed with as a part of the stiffening system. The girder rests upon the floor beams and is thus suspended from the cable. It does not support any load, but merely distributes it, hence it is absolutely a dead weight, adding nothing to the strength of the bridge.

The Board consider it very probable that for a given special case a lighter and better stiffening system than that supplied by the simple longitudinal platform girder could be worked out by combining the trussed cables and longitudinal girder systems, as has been done by Mr. Gustav Lindenthal in his elegant design for the North River Bridge. In applying this method, however, to wire cables carrying very heavy weights over a very long span some new questions of constructive detail will require solution, and for the purposes of a general investigation it seems best to follow the usual method of stiffening. It will, therefore, be assumed that vertical stiffness is obtained entirely by longitudinal platform girders with parallel chords.

These girders are usually of the open frame or lattice type. While their rigidity does not affect the actual safety of the cables which carry the entire dead and live load, it does determine the suitability of the bridge for railroad purposes. There is no doubt that a suspension bridge can be made as rigid for railroad trains as a bridge of any other system.

If a flexible cable be loaded ununiformly it will be depressed on the side of the heaviest load and will rise on the opposite side. It is the object of the stiffening girder to reduce the distortion of the cable to a practical minimum. There are two practical ways in which the girder may be constructed:

(1) As a continuous girder, loosely supported at the ends with reactions in both vertical directions, but permitting horizontal motion.

(2) As a girder loosely supported at the ends, as in the first case, and hinged in the middle.

For the first case the problem of equilibrium is statically indeterminate; that is, the conditions of equilibrium can not be formulated without including the elastic forces developed in the girder. In the second case we have only to deal with static forces, and the stresses in the girder can be calculated with a close degree of approximation by the simple law of the lever.

In the designing of stiffening girders the formulas given by Prof. Rankine have generally been employed. The formulas for the continuous girder are deduced in his Applied Mechanics (p. 370). The formulas for the hinged girder are given, but not deduced, in his Civil Engineering (p. 579). Rankine's methods, which are approximate in character, have been extended to a high degree of accuracy by subsequent investigators. Probably the most complete investigation of the straight stiffening girder is that of Prof. J. Melan, of the Technical High School at Brünn.

Rankine's values of the maximum bending moments for the hinged and continuous girders are as follows: Hinged girder, $M=0.0156qL^2$; continuous girder, $M=0.01786qL^2$; while Melan, by more accurate methods, obtains for the hinged girder $M=0.01883qL^2$, and for the continuous girder $M=0.01652qL^2$. It will be noticed that with Rankine's values the maximum bending moment is smaller for the hinged girder than for the continuous girder, while with Melan's values the reverse is the case.

Although the hinged girder presents very decided theoretical advantages, especially in the determination of the stresses, it has some disadvantages which have prevented its employment in any important practical case. The introduction of the middle joint has for its principal object the attainment of a static determination, but, as Melan has pointed out, the theoretical conditions can not be fully satisfied unless the girder and cable have a common joint at the middle. If the girder alone is jointed, increased bending strains must be produced in the cable directly over the joint. The arrangement of the wind-bracing becomes more troublesome, for, since the upper chords of the girders are cut, all the wind stresses must be transferred to the lower chords. The wind-bracing would doubtless be heavier than for a continuous girder.

Mr. Lindenthal has shown that while there are no temperature stresses in a three-hinged arch at the middle hinge they do exist for any change from the normal temperature in the connected half-arches. His investigation of this important question (which originally appeared in the Engineering News of March 10, 1888, and which has been revised by him for the Board) will be found in Appendix D. For the purposes of this investigation, however, there is no question that the hinged girder ought to be adopted, in order to avoid the complicated formulas which would be required in the other case. It will give results as accurate as the nature of the inquiry permits and on the safe side as regards weight of metal, which could be considerably reduced for any given case in practice by the use of continuous girders, or by other methods requiring more extended computations. The New York Board employed the same method.

It will appear hereafter that when considerable rigidity is required, as in railroad bridges, the stiffening truss becomes the greatest single element of weight and therefore its economical designing is a matter of the highest importance. The Board therefore appends to this report a translation of Prof. Melan's complete investigation of the straight stiffening girder, which it is believed is unknown to most American bridge engineers. A simpler investigation, but sufficiently rigorous for all practical purposes, covering both the hinged and continuous girders, has been attached by the New York Board as an appendix to its report.

In the case of the hinged girder the greatest distortion of the cable occurs when the moving load covers the platform from one tower to a point at a distance 0.105 L from the middle of the bridge. It is here assumed that there will be practically no temperature strains and the simple statical conditions will enable us to express with sufficient accuracy the weight of the girder in terms of the span.

The values of the bending moments and shears which will be used in determining the weight of the girder are based upon the following assumptions:

1. The stiffening girders are supposed to have a height sufficient to prevent great vertical flexure. So far as the vertical strains due to loading are concerned it is most economical to make the height as great as is practically possible, and with the hinged girder this may be done, since changes of temperature are without material influence. The height of the girder will therefore be assumed at 120 feet, as adopted by the New York Board.

2. The tensions on the suspenders are supposed to be always equal; that is, the vertical reaction between the cable and the girder is uniformly distributed over the whole length of the span; and the tensions on the cable are assumed to be invariable. It has been shown by M. Boulongne* that in the case of the suspenders the values obtained on

* Note sur les Ponts Suspendus, Annales des Ponts et Chaussees, 7 Série. Tome 1, 1892, p. 742.

this hypothesis can not be in error more than 5 per cent, and for the cables the error is on the safe side, since it increases the amount of work required of the girder.

3. The effects of the elastic elongation of the suspenders, due to live load and temperature changes, are neglected. As remarked by the New York Board, these disturbances can be avoided by omitting the suspenders for a short distance next the towers.

The discussion of the values of the maximum bending moments and shears is omitted from this report, as it is given fully in Melan's investigation (Appendix E), and also in Appendix E to the report of the New York Board.

The curve of maximum bending moments covering the half girder, whose length is $\frac{1}{2}$ L, is very nearly a parabola, and its area is very nearly $\frac{2}{3}\times\frac{L}{2}\times 0.01883\, q_1\, L^2$. The average maximum bending moment will therefore be

$$M_m = 0.01255\, q_1\, L^2.$$

The average maximum chord-stress is found by dividing this moment by the height of the girder; the area of the cross-section of the chord in square inches is obtained by dividing this chord stress by the assumed working unit stress; and the theoretical weight of the chord per linear foot by multiplying this area by 3.4 pounds. To obtain the practical weight the theoretical weight must be increased by about 25 per cent for constructive details. Although the chords are subject to reversal of strains, the Board have assumed a unit working stress of 15,000 pounds, for reasons which will be fully explained elsewhere.

The weight of the upper chord in pounds per linear foot will therefore be—

$$\frac{0.01255\, q_1\, L^2\times 3.4\times 1.25}{15{,}000\times 120} = 0.00000002963\, q_1\, L^2$$

The dimensions of the lower chord will have to be considerably increased, as it serves also as a chord in the wind girder. Assuming for computation a wind pressure of 2,250 pounds per linear foot, for reasons before given, the average maximum bending moment due to the wind will be—

$$M_{mw} = 187.5\, L^2$$

The unit working stress will be assumed at 30,000 pounds per square inch for reasons which will be given elsewhere. Remembering that the width of the track-platform between the axes of the girders is 100 feet, we obtain for the weight per linear foot of material to be added to the lower chord to take care of the wind-stresses

$$\frac{187.5\, L\times 3.4\times 1.25}{30{,}000\times 100} = 0.0002656\ L^2.$$

The average total weight per linear foot of the lower chord is therefore

$$(0.0002656 + 0.00000002963\, q_1)\, L^2$$

The shearing stresses due to a continuous moving load are greatest at the origin of the span and diminish to 0, changing from upward to downward stresses at a distance $\frac{L}{4}$ from the origin. The average maximum shear without regard to sign for all positions from the origin to the middle pin is $S = 0.1038\, q_1 L$.

The theoretical weight of the web must be increased by about 50 per cent for constructive details and to provide for the transfer of weight from the upper to the lower chord. The lattice bars being placed at an angle of 45°, the weight of the web per linear foot of span will be

$$\frac{0.10381\, q_1 \mathrm{L} \times 3.4 \times 1.5 \times 2}{15{,}000} = 0.00007058\, q_1 \mathrm{L}.$$

For the total weight per linear foot of each stiffening girder we finally obtain by addition

$$\frac{p_2}{2} = q_1\,(0.00007058\ \mathrm{L} + 0.0000005926\ \mathrm{L}^2) + 0.0002656\ \mathrm{L}^2.$$

The value of q_1, which is to be employed in this formula, must now be determined. It will be remembered that the stiffening girders carry no weight, not even their own. They simply distribute the inequalities of the live load and limit the deflections in the cables and floor system. The girders must be dimensioned with special reference to those positions of the live load which correspond to the greatest deflection. The maximum bending moment in either half-girder corresponds to a continuous live load extending from the origin towards the middle of the span and covering a distance 0.395 L. For all spans up to 2,957 feet (equal to the maximum train length divided by 0.395), the single freight train on each track (all six trains being supposed to advance together with their engines abreast) will produce the maximum bending effects, the length of such a train being 1,168 feet. For greater spans it is evident that the maximum effect will not be thus produced. Accordingly, for spans greater than 2,957 feet we should divide the maximum train load by 0.395 L to obtain the live load per linear foot of span to be used in determining the weight of the stiffening girders.

The weight of the train is 3,060,000 pounds; hence for each girder we obtain

$$q_1 = \frac{3060000 \times 3}{0.395\ \mathrm{L}} = \frac{23240506}{\mathrm{L}}$$

and by substitution in the preceding equation

$$\frac{p_2}{2} = 1640.3 + 1.377\ \mathrm{L} + 0.0002656\ \mathrm{L}^2$$

The total girder-load per linear foot for the whole bridge will therefore be $p_2 = 3281 + 2.754\ \mathrm{L} + 0.0005312\ \mathrm{L}^2$.

This formula leaves entirely out of consideration the fact that part of the live load is taken up directly by the cables, yet it is certain that a considerable part of the action of the live load may be thus absorbed. In his reconstruction of the Niagara Suspension Bridge, Mr. L. L. Buck, the engineer in charge, provided for a maximum deflection of 15 inches in 500 feet, and thus reduced the value of $2q_1$ in his formulas from 0.8 ton to 0.6 ton. Mr. W. Hildenbrand, in his reports relative to a proposed suspension bridge across the Hudson River, which are appended to the report of the New York Board, provides for a maximum deflection giving a grade not exceeding 1 per cent. He thereby reduces $2q_1$ from 9,000 pounds to 7,800 pounds. The Board do not doubt that within narrow limits a certain degree of flexibility is an advantage to the bridge. Deflections in a system of stable equilibrium do not impair the safety of the structure as they do in an unstable system like the upright arch, and they may exert a very beneficial influence in modifying the dynamic effects of a rapidly varying live load. On the other hand, it is to be remarked that any increase in the grade of the track-platform is accompanied for fast trains by a certain increase in the dynamic action of the live load.

The proportion of live load absorbed by the cables increases as the catenary becomes flatter, but the cables must be made heavier. It is not easy to determine satisfactorily the resultant effect of these deflections, and in order to be on the safe side the Board make no allowance for them in this investigation.

Lateral bracing.—The principal duty of the lateral or sway bracing is to resist the action of the wind. The top lateral system is a light riveted lattice connecting the top chords of the stiffening girders. Since these chords are cut at the middle, the entire work of resisting wind pressure is done by the bottom lateral system. The top system, in conjunction with the cross-frames and hangers in a vertical plane above each cross-girder, serve simply to transfer the v ind stresses and a portion of the load to the bottom system. We may assume for the weight of the top system 500 pounds per linear foot, and for the cross-frames and hangers, 1,920 pounds per linear foot, as computed by the New York Board. These weights may be considered constant for all values of the span within the limits of this investigation.

In the bottom system the chords are the bottom chords of the stiffening girders, in the dimensioning of which the wind stresses have already been provided for. The cross girders of the floor system form the lateral struts, and the diagonals are strained in tension. It only remains to determine the weight per linear foot of the diagonals. The depth of the wind-girders (d) is 100 feet, and the theoretical panel length (b) is 120 feet. The number of panels (n) on each side of the middle point is therefore $\frac{L}{2b}$. The theoretical panel load is 2,250 b; the assumed unit stress is 25,000 pounds; and 25 per cent is added for constructive details. The weight of the diagonals per linear foot of span will then be

$$\frac{2\times1.25\times3.4\times2250\ b\ L^2\ (b^2+d^2)}{25000\ d\ L\times4\ b^2}=0.3889\ L$$

For the weights in pounds per linear foot of the lateral or sway bracing, including the cross-frames and hangers, we therefore have $p_3=2420+0.3889\ L$.

WORKING STRESSES.

In determining the weights of the two most important members of the bridge—the cables and the stiffening girders—the Board have assumed working stresses which are greater than those generally adopted in truss or arch bridges of moderate span, and which, therefore, require explanation.

The most approved formulas for the determination of working stresses are based upon the experiments of Herr Wöhler, made for the Prussian Ministry of Commerce, and published at Berlin in 1870.* These experiments not only confirmed the earlier result obtained by Sir W. Fairbairn and others, that with repeated applications of a load a bar breaks under less than its static breaking load, but they also showed that the breaking load varies inversely with the difference between the maximum and minimum stresses. Furthermore, it wa found that a bar may be broken by a still smaller fraction of the static

* Uber die Festigkeitsversuche mit Eisen und Stahl.

breaking load if it is alternately strained in opposite directions, the stress alternating between a positive and a negative quantity.

The principal formulas representing these results are based upon two radically different interpretations of the observed facts. In one case it is assumed that the repeated alternations of stress produce an actual weakening of the material which has been called "fatigue." This view is represented by the Launhardt-Weyrauch formulas, which are as follows:

For stress in one direction—

$$a = u(1 + \frac{t - u}{u} \phi)$$

For alternating positive and negative stresses—

$$a = u(1 - \frac{u - s}{u} \phi)$$

in which $\phi = \frac{\text{min. s}}{\text{max. s}}$ = the ratio of the least to the greatest stress; a = breaking strength under the assumed conditions, which is to be divided by the factor of safety (generally 3); t = breaking strength under a static load; u = the limiting strength (Ursprungsfestigkeit), measured by the greatest load the bar will bear with an indefinite number of alternations between 0 and u without reversal; and s = vibration-resistance, which is the limiting strength for alternations of equal magnitude with reversal.

In the other case it is assumed that the alternations produce no change whatever in the molecular condition of the material, but that the increased effects are produced entirely by an increase in the stresses due to dynamic action, the stress being equal to the load only when all the forces acting are in static equilibrium. This view is represented by the so-called dynamic formula, which is

$$a = \text{max. S} + \eta \text{ (max. S} - \text{min. S)}$$

In this formula η is a coefficient depending upon the violence and time-rate of the load-changes.

The Launhardt-Weyrauch formulas are based entirely upon Wöhler's experiments, and do not take into consideration variations in the rate or violence of the dynamic action. Prof. Fidler says that in these experiments the load was applied about four times per minute.

Prof. Fidler has shown that when the alternations are rapid (for which case $\eta = 1$) the dynamic formula represents Wöhler's experiments as accurately as the Launhardt-Weyrauch formulas. No satisfactory determination, however, has been made of the value of η for the various cases occurring in practice. In the case of cross-girders and vertical suspenders, which receive the full action of the elastic vibrations due to a sudden imposition of the load, Prof. Fidler assumes $\eta = 1$, and he adopts the same value for the diagonals of the web bracing. For the flanges of a girder or in the principal members of an arch or suspension bridge, in which the stress-changes take place more gradually, he recommends a reduced value bearing some unknown relation to the length of span. He considers the value $\eta = \frac{1}{2}$ to be large enough for all spans down to about 100 feet.

It is not possible in this report to make an extended comparison of the merits of these formulas. It may be remarked, however, that the effects of variations in dynamic action certainly play an important part in the determination of ultimate strength, although there are as yet no experiments showing how far this strength is affected by the frequency as distinguished from the mere number of the stress-changes, nor whether a period of rest after "fatigue" restores strength. These effects of dynamic variations, which are entirely unrepresented in the Launhardt-Weyrauch formulas, really exist, and are of special importance in the theory of suspension bridges. A clear and able discussion of the whole subject will be found in Chapter XIII, of Prof. Fidler's Practical Treatise on Bridge Construction.

The Board have adopted for the cables a working stress of 60,000 pounds, which is one-third of the static breaking load. Prof. Melan says that, owing to the lack of experiments with steel wires, we can consider the laws of Wöhler only so far as to allow for large spans a somewhat greater value of the working stress. For ordinary spans he adopts a working strength of about one-fourth the ultimate strength of the wire. The Board believe that a safety factor of 3 is amply sufficient to cover both the effects of variations in stress and the imperfections of manufacture and adjustment in the cables. As regards variations in stress, it is to be remarked that there are no reversals, the wire being always in tension; that considerable deflections correspond to relatively slight changes in stress; and that the stresses are slowly and gradually applied, and well within the high elastic limit.

This latter point is of special importance, for it is probable that Wöhler's law of reversals does not hold good for stresses well within the elastic limit. For example, in the balance spring of a watch, tension and compression succeed each other some 150,000,000 times in a year, and the spring works for years without apparent injury.* In this connection it may be remarked that, although cables which have been long in use have been frequently examined, no deterioration of strength which could be attributed to variations of stress has ever been discovered. If we use the Launhardt formula, we are justified in making u very nearly equal to t.

If we employ the dynamic formula, the factor max. S—Min. S will be very small, for the reasons just given. As for the coefficient η, we only know that it diminishes as the span increases, and, according to Prof. Fidler, it need not be greater than $\frac{1}{2}$ for a span as small as 100 feet; it must therefore be a very small fraction for spans as large as those now under consideration. The variation-term of this formula will probably be so small that it may be safely neglected.

As regards imperfections of manufacture and adjustment, which are covered in general practice by the safety factor 3, the following points are to be noted. The uniformity of strength is greater for wire than for any other form of steel. The process of manufacturing a wire cable is in itself a test of the material and insures a more nearly uniform distribution of stress over the cross-section than can be obtained in any other structure formed of a very great number of parts. If a factor of 3 is sufficient to cover the defects of material and construction of a riveted bridge-member, a somewhat less factor ought to be sufficient for a wire cable.

For the reasons above given, the Board are of the opinion that a safety factor of 3 is sufficient, and have therefore adopted 60,000 pounds per square inch as the working stress for the cables.

The New York Board adopted the same working stress, giving as their reason that this is "the same proportion of the ultimate strength that the 20,000 pounds adopted in the cantilever structure bears to the probable strength of eye-bar steel."

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* Prof. Ewing, Strength of Materials, Enc. Brit.

The Board have adopted 15,000 pounds per square inch for the working strength of the stiffening girders. The New York Board limited the stresses due to a moving load to 12,500 pounds, because there is a reversal of strains, but allowed the stresses from the combined effects of moving load and wind to run up to 22,500 pounds. The reasons of this Board for adopting a higher working stress in the stiffening girders are as follows:

Although there is a theoretical reversal of strains, it will rarely and perhaps never occur with the maximum stress, since this would require six of the heaviest freight trains, abreast of each other, to cross and recross the bridge, first in one direction and then in the other. This would probably never happen on a bridge devoted principally to passenger traffic, and it could be prevented by the simplest police regulations. Again, the lower chords of the girders have been made of sufficient strength to resist the combined maximum stresses of the live load and the wind; but the maximum chord-stresses could never occur at the same time, since with the maximum wind pressure no trains could cross the bridge. Some allowance has been made for this by the adoption of a working strength of 30,000 pounds for that part of the material added to resist wind. The only duty of these girders is to distribute the live load and thus prevent inconvenient deflections. It is not necessary to give them the margin of strength which they would require if they were essential to the stability of the bridge.

The Board are of the opinion that the great distinction between the stable equilibrium of a suspension bridge, which can not break down from the failure of any stiffening member, and the unstable equilibrium of a truss, arch, or cantilever bridge, in which the failure of a member may involve the collapse of the entire bridge, ought to receive full recognition in the adoption of unit stresses and safety factors. The weight of the stiffening girders constitutes the most important single element in the suspended weight of the bridge, being for the maximum span about one-half the entire permanent load. It should be made no greater than is absolutely necessary, for the structure ought not to be kept under a continuous stress to provide a larger margin for stresses which may never occur. The Board believes that the working stresses adopted are amply sufficient for the members of the bridge.

TOWERS.

The weight of the towers forms no part of the suspended load, and therefore is only indirectly connected with the question of the maximum span. There is, of course, a practical limit to the height to which the towers can be carried, and the relation between their cost per vertical linear foot and the cost of the suspension system per linear foot of span will be an important element in determining the most economical versine for the cables.

The towers will be supposed to be formed of steel columns braced together, and will start from the upper surface of the masonry, 165 feet below the lowest point of the cables. An empirical formula, giving the approximate weight of metal (W_t) in the towers, has been deduced by Mr. Lindenthal from various estimates of designs for suspension bridges, as follows:

Since the section of the cable is throughout the same, the tangent to the cable at the tower in the end span should intersect a horizontal line tangent to the cable curve at a distance from the axis of the tower not greater than that of the intersection of the similar tangent in the middle span; hence the end spans should each be at least one-fourth of the main span, and the entire length of the bridge from face to face of anchorage should be at least 1.5 L. Subject to this condition, the end spans should be made as short as convenient to save cable weight. This is also important, when the backstays carry any directly suspended load, because the bending moments from the live load in their stiffening girders may otherwise become greater than in the main span. In the present investigation, however, the loads in the side spans are supposed to be supported from beneath, and the backstays have simply to transmit the suspended load of the main span to the anchorages, the pressure on the top of each tower being equal to the total dead and live load of the main span.

Let L_a = length of the bridge exposed to wind pressure reacting laterally on the towers, in this case equal to L.

h_t = height of the metallic portion of the towers from bedplate to cable bearing.

W_s = suspended dead load plus maximum live load per unit of span.

R_t = reaction at top of towers, $= 2\ L\ W_s$ (for both towers).

w_1 = weight of steel per linear foot of square inch cross section = 3.4 pounds.

S = factor of safety. This will be assumed as 3.

u = ultimate strength of steel per square inch, corresponding to S = 1.

a = coefficient of practice, including stairways, housings, cable bearings, etc., deduced from actual designs = 1.65 (Lindenthal).

Steel having an ultimate strength of from 90,000 to 100,000 pounds per square inch, and an elastic limit from 56,000 to 60,000 pounds, is considered by Mr. Lindenthal more suitable and economical for heavy towers than a forgeable or punchable steel, with an ultimate strength of 60,000 pounds. All rivet holes in such high steel must be drilled and not punched and reamed.

The metal in the towers is proportional to the reaction R_t and the height h_t. The weight of metal in the towers, exclusive of bracing, will therefore be $\frac{R_t\ h_t\ s\ a\ w_1}{u}$.

The towers require bracing against wind pressure and bending from temperature changes in the cables. The metal in the braces will be proportional to the square of the height of the towers and to the length L exposed to wind pressure and temperature changes; hence the weight of the bracing in tons will be $L\ h_t^2\ S\ b$, in which b is the coefficient of proportional weight deduced from actual designs = 0.001 (Lindenthal).

The weight of the towers will therefore be

$$W_t = \frac{R_t\ h_t\ s\ a\ w_1}{u} + L\ h_t^2\ s\ b = h_t\ \frac{S\ (R_t\ a\ w_1 + L\ h_t\ b)}{u}$$

Making $s=3$, $w_1=3.4$, $a=1.65$, $b=0.001$, $R_t=2\ W_s L$, and $u=60000$, we obtain for the weight of the towers in pounds

$$W_t = \frac{L\ h_t\ (0.187\ W_s + h_t)}{333}$$

This is on the assumption that the towers are constructed on the plan followed by the New York Board, so that the cables may be arranged side by side, and that steel of an ultimate strength of 60,000 pounds per square inch is employed in their construction.

BACKSTAYS.

The length and consequently the weight of the backstays will depend entirely upon the arrangement of the end spans, and this will in every case be determined by the local conditions. If l represents the length of a single backstay, the total weight (W_b) of the backstays for the whole bridge will be $W_b=2\ l\ w'$ in which w'=the weight of all the cables per linear foot.

If the backstays intersect the horizontal plane, tangent to the cable-curve at a distance $\frac{1}{4}$ L from the axis of the towers (which is the most economical arrangement so far as the total amount of cable metal is concerned), the length of each stay from the floor level to the top of the tower (the floor being considered horizontal and 60 feet below the lowest points of the cables) will be $5\left(\frac{L}{8}+60\right)$, to which should be added a constant length of about 100 feet to carry the end of the stay to its point of connection with the anchor chain, which should be well below the floor level. For this case the formula becomes

$$W_b=(468.3+0.559\ L)\ w'=(449.6+0.537\ L)\ w.$$

ANCHOR CHAINS AND PLATES.

The anchor chains are formed of steel eye bars and connect with the cables outside of the masonry of the anchorages, and with bearing plates of rolled steel at their lower ends. They are proportioned for a stress of 20,000 pounds per square inch with an allowance of 40 per cent for the weight of pins and constructive details. The tension on each backstay is 18,960,000 pounds. The weight of the anchor-chains per linear foot for each back stay will therefore be

$$\frac{18{,}960{,}000\times 3.4\times 1.4}{20{,}000}=4{,}512 \text{ pounds.}$$

The length of each chain may be assumed as 200 feet. The weight of chains for each backstay will therefore be 902,400 pounds. The weight of steel in each anchorage plate may be assumed as 100,000 pounds, making the total weight of anchorage metal for each backstay 1,002,400 pounds. If n represents the number of standard cables in the bridge, the total weight of the anchorage metal (W_a) will be $W_a=2{,}004{,}800\ n$.

In this formula no deduction of weight is made for the diminution of the tension due to the friction of the chains on their supports.

For the bridge of maximum span with 16 standard cables we have W_a=32,076,800 pounds=16,038.4 tons.

MASONRY AND FOUNDATIONS.

Anchorages.—As the anchorage masonry acts merely as a weight, an inexpensive class of masonry can be used everywhere except, perhaps, in the immediate vicinity of the bearings of anchorage cables and plates. The foundations need go no deeper than necessary to obtain a soil giving sufficient resistance to horizontal sliding, and therefore will be of comparatively simple and easy construction.

Tower foundations.—The lower portion of the towers above ground and all that below ground will naturally be built of masonry, and may be all treated as constituting the tower foundations. Being proportioned directly to the weight which they have to carry, such foundations for suspension bridges differ from those of other bridges only so far as affected by the great height of the towers proper and their consequent great weight and leverage; and therefore are like other foundations except that they must be given more cross-section, more care, and a better footing upon their beds.

In proportioning such piers, the New York Board adopted the following limits of stress:

The pressure between the metallic bed-plates and the top of the masonry should not exceed 20 tons to the square foot. The pressure within the masonry and on the foundation should not exceed 10 tons to the square foot; but in determining these pressures, the weight of materials displaced by the pier is to be deducted.

The New York Board remark that "while these pressures have been exceeded in some structures, they are higher than in usual practice, and call for masonry of good quality and of more than ordinary cost."

The method of foundation construction will depend greatly upon local considerations. For a bridge of maximum span these foundations should rest upon solid rock, if possible, and at least upon hard, incompressible impermeable soil. Modern methods have already established foundations at a level of 162 feet below the water surface, and provide means for going still deeper, if necessary, and for obtaining a properly leveled surface in the rock when found; so that the question of foundations affects to-day only the economy and not the engineering practicability of bridge construction.

THE ENGINEERING PROBLEM OF MAXIMUM SPAN.

It is now proposed to investigate the maximum length of span practicable for a suspension bridge entirely from an engineering point of view, leaving the question of the relation between traffic capacity and cost of construction for subsequent consideration.

If we suppose the cable-curve to be referred to rectangular axes through the lowest point as an origin, we have from the construction of the funicular polygon—

$$\frac{dy}{dx}=\frac{1}{Q}\int P\,dx$$

in which Q represents the constant horizontal tension at any point of the polygon, and P the total suspended load per linear foot of span. The units are the pound and the foot.

The load P is composed of the weights per unit of span of the live load, track-platform, bracing, cables and suspenders, some of which vary slightly with x; but the Board has found by a careful analysis that even with the unusual weights and spans considered in this investigation, the error involved in the assumption that P is constant is too small to be of any practical consequence. It is therefore assumed that the load is uniformly distributed, in which case we obtain by integration for the equation of the cable-curve $y=\frac{p\,x^2}{2Q}$ which represents a parabola. If L represents the span and R the ratio of the span to the versine of the cable we obtain $Q=\frac{P\,L\,R}{8}$.

The greatest stress on the cables is at their highest points, and for this maximum tension the strength of the cables must be proportioned. Its horizontal component is the constant horizontal tension Q and its vertical component is the weight on the half span, $\frac{P\,L}{2}$. If T represents this maximum stress we obtain

$$T=\sqrt{\frac{P^2\,L^2}{4}+Q^2}=\frac{P\,L}{8}\,\sqrt{R^2+16}=\frac{(p'+w)}{8}\,L\,\sqrt{R^2+16}$$

If we represent by L_1 the limiting span, that is, the span at which the cable will carry its own weight with a given stress per unit of cross-section without carrying any other load whatever, we obtain from the above equation, by making $p'=O$ and $L=L_1$

$$T=\frac{w\,L_1}{8}\,\sqrt{R^2+16}$$

hence

$$\frac{p'+w}{w}=\frac{L_1}{L}$$

From the above equations we obtain

$$L_1=\frac{8\,T}{w\,\sqrt{R^2+16}}$$

For metallic towers and large spans the value R=8 will be generally about the most economical value for the ratio of the span to the cable-versine, when the cost of the foundations is taken into consideration. If we make R = 8 and substitute for T the working strength per square inch of the material of the cable, which we have assumed as 60,000 pounds, and for w the weight of a linear foot of the cable measured horizontally and having a cross-section of one square inch, which is 3.54 pounds, we obtain $L_1=15160$ feet;
and

$$\frac{p'+w}{w}=\frac{15160}{L}$$

The values of p' and w in pounds, as determined previously, are as follows:

$$p'=13605+27764726\,L^{-1}+3.24906\,L+0.00055335\,L^2+0.000000003\,L^3$$
$$w=17917.$$

Substituting these values and reducing we obtain

$$31522\,L+3.24906\,L^2+0.00055335\,L^3+0.000000003\,L^4=243856994,$$

the solution of which gives for the practical maximum span

$$L=4335 \text{ feet.}$$

This span is measured between the highest points of the cables at opposite ends of the bridge.

The Board consider this a conservative value of the maximum span, as it is based upon assumptions well within the limits of theory and experience.

The elements of the bridge, deduced by the preceding formulas, are as follows:

Span between tops of cables	feet	4,335
Height of towers above masonry	do	707
Number of cables		16
Diameter of each cable with wrapping	inches	21½

Suspended weight per linear foot of span.

	Pounds.
Live load	6,353
Platform	7,200
Stiffening girders	25,202
Wind bracing	4,106
Cables	17,917
Cable wrapping	433
Suspenders	1,445
Total suspended weight per linear foot =31.328 tons.	62,656

	Tons.
Suspended weight for whole middle span=	135,807
Weight of backstays	24,882
Weight of anchor chains and plates	16,038
Weight of towers	57,172
Total weight of metal in the bridge	233,899

THE RELATIONS BETWEEN SPAN, TRAFFIC AND COST.

In the preceding pages the Board have determined to the best of their ability the maximum span practicable for a suspension bridge from a purely engineering point of view. Their instructions further require them to investigate the maximum span "consistent with an amount of traffic probably sufficient to warrant the expense of construction." This involves the consideration of two subjects; the cost of construction and the traffic capacity of the bridge.

The cost of a suspension bridge can not be determined simply as a function of the span and traffic. In the construction of every such bridge there are elements of cost which depend almost entirely upon local conditions, and can not be estimated even with the roughest approximation until these conditions are fully known. For example, the cost of the piers will depend upon the depth of the solid foundation below the bottom of the stream or the surface of the ground; the cost of the towers and anchorages will depend upon the height at which it is necessary to elevate the roadway above the water surface, which again will depend upon the character of the river navigation; the cost of spaces for anchorages, approaches and terminal facilities will depend upon the local land values.

By examining the detailed costs of several very large bridges, it is found that these indeterminate local elements constitute on the average more than 60 per cent of the cost of such bridges in cities. It has been stated that in the case of the New York and Brooklyn Bridge, the cost of the bridge structure proper was only one-third of the expenditure for the entire work. In the case of the suspension bridge to cross the Hudson at New York, estimated for by the New York Board, the local elements determine about 54 per cent of the whole estimated cost, although the cost of approaches, terminal facilities and land are not included.

The determination of any relation between traffic capacity and the cost of construction warranted thereby is equally difficult. It may, of course, be assumed that the bridge of maximum span will be constructed only in a locality where the conditions of commerce justify the belief that the traffic capacity of the bridge will be fully utilized. But in the general case it is impossible to determine what charges the traffic will bear. Moreover, the construction of such a bridge might be desirable even if the traffic were not likely to give a sufficient return for the vast sum invested. By the combined action of railroad companies, such a bridge might be built for the general benefits resulting from increase and facility of traffic, even though it might fail to earn directly a reasonable interest on its cost; and the enterprise might be assisted by adjacent cities, as was done in the case of the New York and Brooklyn Bridge.

But while the Board have been unable to arrive at definite conclusions in the general case, they believe that much may be learned from the study of the problem as limited by the conditions of a special locality, and the material for such an inquiry is furnished by recent investigations in connection with proposed bridges across the Hudson River at New York.

It is said that the number of ferry passengers crossing from New Jersey to New York City now exceeds 85,000,000 per year, and the passenger and freight traffics are growing rapidly. It can scarcely be doubted that a bridge in this locality would be used to its full capacity. Such a bridge would, however, be employed principally for passenger traffic, the facilities for moving freight on floats at water level to any point on the water fronts being ample and convenient.

The Hudson River at New York forms the most important part of the interior harbor. Its mid-channel depth of at least 49 feet, and its clear width of at least 2,800 feet between pier-head lines, make it one of the finest roadsteads in the world. It is navigated by an enormous commerce. Strong protests against its obstruction by a pier in the channel have been made by the commercial interests of the port. The least objectionable location for such an obstruction would be not far from the middle point, between the pier-head lines, where it would divide the upstream and downstream traffic, but this location is prohibited by the great depth to a firm foundation.

The New York Board reported that it is safe and practicable to cross the river with a single span, and estimated the cost of a suspension bridge for that purpose, its New York pier being between Fifty-ninth and Sixtieth streets, at $30,743,000. This is the estimate for their *Lighter Structure*, but it provides for a bridge amply sufficient for the purposes for which it is intended. Moreover, the estimate was made for the purposes of comparison, and the report of the Board distinctly states that it is not to be taken as an absolute estimate of cost. This Board considers this estimate perfectly satisfactory for the purpose for which it was made, but they think it desirable to determine a minimum as well as a maximum estimate, to show the variations to which such estimates are liable and how much they are affected by legitimate differences in the assumptions upon which they are based. An estimate has therefore been made on the following assumptions:

The cost of structural steel is taken at 4 cents per pound, in accordance with the views of a majority of the New York Board, as indicated in their report.

The cost of wire work is taken at 7 cents per pound, which is based upon prices given by leading manufacturers and upon actual experience in the case of the New York and Brooklyn Bridge.

The weights of metal are determined by the formulas given in this report.

The bridge is supposed to be located near Sixty-ninth street, New

York, and the cost of the substructure is assumed to be the cost at the lower location (between Fifty-ninth and Sixtieth street), as estimated by the New York Board, less $2,900,000, which they state would be saved by adopting the upper location. The minimum estimate is as follows:

Structural steel:		
Suspended weights	pounds..	90, 870, 000
Towers	do....	52, 313, 000
Chains and anchor plates	do....	18, 324, 000
Total	do....	161, 507, 000
At 4 cents per pound		$6, 460, 280
Wire work:		
Main cables and wrapping	pounds..	30, 358, 000
Backstays and wrapping	do....	22, 738, 000
Suspenders	do....	3, 222, 000
Total	do....	56, 318, 000
At 7 cents per pound		$3, 942, 260
Cost of superstructure		10, 402, 540
Cost of substructure		11, 784, 000
Total cost of bridge		22, 186, 540

The final plans for a work of such magnitude would only be adopted after the most extended theoretical and experimental investigations, and the estimated cost would undoubtedly be much reduced by such studies. Assuming the most favorable location and the most competent engineering management, the Board believe that $23,000,000 is a reasonable estimate for a six-track railroad suspension bridge 3,200 feet long, and they consider the amount of traffic which such a bridge would accommodate sufficient to warrant the expense of construction. They believe, however, that the bridge should be so constructed that its capacity can be readily increased, and with the suspension system this can be provided for by giving suitable dimensions to the towers and anchorages.

If sufficient inducements were offered to competent engineers to prepare competitive designs and estimates for a single-span bridge at this locality, the Board do not doubt that perfectly satisfactory plans would be obtained within the limit of cost of the estimate given above.

The Board desire to express their obligations to Mr. Gustav Lindenthal, C. E., Mr. W. Hildenbrand, C. E. and Mr. L. L. Buck, C. E. for information and valuable suggestions.

The following appendices accompany this report:

Appendix A.—Orders and instructions.

Appendix B.—Correspondence with wire manufacturers.

Appendix C.—Wind pressure.

Appendix D.—Temperature Strains in Three Hinged arches, by Gustav Lindenthal, C. E.

Appendix E.—The Theory of the Stiffening Girder, by Prof. J. Melan.

Respectfully submitted.

C. W. Raymond,
Major, Corps of Engineers.
Wm. H. Bixby,
Captain, Corps of Engineers.
Edw. Burr,
Captain, Corps of Engineers.

Brig. Gen. Thomas L. Casey,
Chief of Engineers, U. S. A.

Appendix A.

ORDERS AND INSTRUCTIONS.

War Department,
Washington, D. C., January 27, 1894.

Sir: In view of the importance of questions arising in this Department in connection with the building of bridges over navigable streams, it is essential that it should be possessed of accurate and full information necessary to their intelligent and proper determination.

I have therefore to direct that you convene a Board of Officers of the Engineer Corps, who shall investigate and report their conclusions as to the maximum length of span practicable for suspension bridges and consistent with an amount of traffic probably sufficient to warrant the expense of construction.

Very respectfully,

Daniel S. Lamont,
Secretary of War.

The Chief of Engineers, U. S. Army.

[First indorsement.]

Office Chief of Engineers,
U. S. Army,
January 30, 1894.

Respectfully referred to Col. C. B. Comstock, Corps of Engineers, in connection with Special Orders No. 5, Headquarters, Corps of Engineers, January 29, 1894.

Attention is invited to the within instructions.

The investigations of the Board will include strength of materials, loads, foundations, wind pressure, oscillations, and bracing.

A detailed report is desired.

* * * * * * *

By command of Brig. Gen. Casey:

H. M. Adams,
Major, Corps of Engineers.

[Third indorsement.]

Office Chief of Engineers,
U. S. Army,
February 3, 1894.

Respectfully referred to Maj. Charles W. Raymond, Corps of Engineers, senior member of the Board, attention being invited to instructions contained in first indorsement.

By command of Brig. Gen. Casey:

H. M. Adams,
Major, Corps of Engineers.

Special Orders, No. 5.

Headquarters, Corps of Engineers,
United States Army,
Washington, D. C., January 29, 1894.

A Board of Officers of the Corps of Engineers, to consist of Colonel Cyrus B. Comstock, Captain William H. Bixby, First Lieut. Edward Burr, will assemble at New York City, on the call of the Senior Member, to make investigations as to certain bridges in accordance with instructions of the Secretary of War to be transmitted.

Upon the completion of the duty assigned them the members of the Board will return to their proper stations.

The journeys required under this order are necessary for the public service.

By command of Brig. Gen. Casey:

John G. D. Knight,
Captain, Corps of Engineers.

Special Orders, No. 7.

HEADQUARTERS, CORPS OF ENGINEERS,
UNITED STATES ARMY,
Washington, D. C., February 7, 1894.

By direction of the Secretary of War Colonel Cyrus B. Comstock is relieved and Major Charles W. Raymond is detailed as a member of the Board of Officers of the Corps of Engineers appointed by Special Orders, No. 5, current series, from these Headquarters.

By command of Brig. Gen. Casey:

JOHN G. D. KNIGHT,
Captain, Corps of Engineers.

APPENDIX B.

CORRESPONDENCE WITH WIRE MANUFACTURERS AND OTHERS WITH REFERENCE TO STEEL WIRE.

UNITED STATES ENGINEER OFFICE,
Norfolk, Va., June 19, 1894.

DEAR SIRS: The Secretary of War has appointed a Board of Engineer Officers (of which Maj. C. W. Raymond, Corps of Engineers, at Philadelphia, Pa., is the senior member) to investigate certain questions with regard to suspension bridges of long span, and the board has assigned to me the duty of collecting information with regard to steel wire suitable for use in the cables of such bridges. As manufacturers of steel wire of high grade, I beg to submit to your consideration the following questions and to request such answers thereto as you may be disposed to give:

(1) It is understood that the steel wire used in the cables of the East River Bridge was No. 7 and No. 6, and had a tensile strength of not less than 170,000 pounds per square inch, and that steel wire with a tensile strength of 170,000 pounds per square inch and upward is used at present in the manufacture of steel wire ropes. In the present condition of the industry what is the maximum tensile strength that it would be possible or practicable to guarantee in a contract for the wire of a suspension bridge of very large span, the contract being presumed to be of such magnitude as to warrant the introduction of such special machinery as might be necessary to obtain the best results? Would it be possible or practicable to furnish wire with a tensile strength of 300,000 pounds per square inch that would satisfy the other conditions as to ductility, cost, etc.?

(2) Other than tensile strength, what tests would you recommend for such wire as could be furnished as per paragraph (1)?

(3) Under the conditions named in paragraph (1), could wire of the given quality be furnished of the size of No. 3 B. W. G.?

(4) Can you name, approximately, what would be a reasonable price for such wire and could it be furnished, under the conditions named in paragraph (1), at a price not exceeding 10 cents per pound?

Answers to the above inquiries and any other information relating to the subject that you may feel disposed to furnish will be highly appreciated and duly acknowledged.

Very truly, yours,

EDW. BURR,
First Lieutenant, Corps of Engineers.

The JOHN A. ROEBLING'S SONS COMPANY,
Trenton, N. J.

JOHN A. ROEBLING'S SONS COMPANY.

TRENTON, N. J., *June 22, 1894.*

DEAR SIR: We have your favor of the 19th instant with reference to wire suitable for suspension bridges of long span. In the cables of the East River bridge Nos. 7 and 8 wires, having a tensile strength of 170,000 pounds per square inch, were used, costing about 7½ cents per pound. To-day we can make a purer and higher grade of wire in No. 3 gauge, galvanized, and properly prepared, having an ultimate strength of 180,000 pounds per square inch, for about 4½ cents per pound. This would about be the class of wire commercially suitable for bridge purposes. For special cases a higher grade of wire can be made that will stand 225,000 pounds per square inch at a cost of about 8¼ cents per pound. Both classes of wire to stretch 4 per cent in 12 inches and to bend around their own diameter. Wire can be made to stand the above tensile strengths for less money, but there will be a lack of uniformity and fine physical properties. As to making wire with a strength of 300,000 pounds per square inch, we have seen it, and make it, but always in small sizes, say one-eighth to one-sixteenth of an inch in diameter. We have even made wire as high as 350,000 pounds. To make wire in one-quarter of an inch diameter size, to stand over 225,000 pounds, with proper ductility tests, is beyond our skill and knowledge, and for suitable wire we place the limit at 225,000 pounds.

In either case special machinery will have to be constructed, and samples can not be made on the spur of the moment.

Yours, truly,

JOHN A. ROEBLING'S SONS COMPANY,
C. E. R.

Lieut. EDWARD BURR,
First Lieutenant, Corps of Engineers, U. S. Army, Norfolk, Va.

WASHBURN AND MOEN MANUFACTURING COMPANY.

WORCESTER, MASS., *July 6, 1894.*

DEAR SIR: Replying to your communication of June 19, we have given your communication careful investigation and from the information we have been able to collect we beg to submit the following report, and at the same time ask you for more data regarding your requirements:

First. The size of the wire used for the East River Bridge, we believe, was No. 8 instead of Nos. 6 and 7. The tensile strength was supposed to be 160,000 pounds to the square inch, instead of 170,000 pounds. We had been led to believe that quite a percentage of the wire, however, which was used in the bridge was under 160,000. The only other requirement was wrapping the wire cold around a three-quarter-inch mandril.

The present standard of steel rope wire is somewhat as follows:

	Crucible-steel rope wire; strain per square inch.	Plow-steel rope wire; strain per square inch.
	Pounds.	*Pounds.*
No. 6 to No. 8	160,000 to 180,000	225,000 or over.
No. 9 to No. 16	160,000 to 180,000	240,000 or over.
No. 16 to No. 18	170,000 to 190,000	270,000 or over.
No. 18 and finer	180,000 or over.	270,000 or over.

"In the present condition of the industry" we can undertake to give at least 230,000 pounds per square inch, No. 8 and finer, or even a little higher, with a mechanical test very much more severe than the Brooklyn bridge wire; i. e., to wrap around its own diameter three times and unwrap again without breaking, whereas the Brooklyn Bridge wire only had to wrap around a three-quarter inch bar. If it is desired by the Engineers to have a still harder wire but less ductility we can certainly give a much higher strength, but this is all in the direction of the more brittle wire and one that will stand less bending test.

We sent samples of No. 8½ and No. 9½ to the World's Fair showing a tensile strength of 230,000 to 250,000 pounds per square inch, with good elongation and bending test. Please find herewith a printed card showing official test of the Ordnance Department at Watertown Arsenal, Mass.

In answer to your question, "Would it be possible or practicable to furnish wire with a tensile strength of 300,000 pounds to a square inch, etc." With special machinery and appliances and a large contract we have little doubt of our ability to produce wire of a tensile strength of 300,000 to the square inch, but necessarily the requirements of elongation and bending test must be somewhat lower. The larger the size the greater the difficulty in accomplishing the result of high tensile strength with high elongation and bending properties.

Eminent engineers differ in regard to the requirements for suspension-bridge purposes; some favor wire with high elongation and low tensile strength, while others require high tensile strength and low elongation.

Second. We should recommend three tests, tensile strength, elongation, and wrapping. If the wire is hard and at the same time large in diameter the torsion test is not reliable. We think the three tests named are sufficient for all practicable purposes.

Third. Wire could be furnished of No. 3 B. W. G., but this would necessitate special machinery and a large outlay, and would necessarily be more expensive to produce than No. 7 size. We advise that, taking everything into consideration, that 300,000 pounds per square inch would be too high, unless your Board of Engineers consider tensile strength in preference to other requirements.

Fourth. The answer to this question depends entirely upon what conditions your Board of Engineers ask for, and we submit the following questions:

What tensile strength will be decided upon?

What other mechanical tests will be required in addition to the tensile strength, and how severe?

The maximum and minimum weight of pieces required?

Amount of leeway which will be allowed in drawing from one end of the piece to the other?

How large a quantity of wire will be required?

What will the size of the wire be?

We wish to call your attention to tests which have been made for us at U. S. Arsenal, Watertown, Mass., which are no doubt on file, and to which we invite your attention.

Very truly,

WASHBURN & MOEN MANUFACTURING COMPANY,
F. H. DANIELS,
Superintendent.

Lieut. ED. BURR,
Corps of Engineers, U. S. A., U. S. Engineer Office, Norfolk, Va.

THE TRENTON IRON COMPANY.

TRENTON, N. J., *July 10, 1894.*

DEAR SIR: Replying further to your favor of the 19th ultimo, as stated in my letter of the 30th owing to an absence of two weeks at the time your letter was received and continued rush of business since then, it has really been impossible to take up the question with any reasonable care until the present moment. Now, replying to your several inquiries, I should say that under the present condition of the wire industry a maximum tensile strength of 200,000 pounds per square inch for Nos. 6 and 7 wire could be easily obtained without any very great increase in the cost; that is to say, wire of that tensile strength could be obtained at figures varying from 8 to 9 cents per pound. If higher tensile strengths than this were required, say up to 300,000 per square inch, the cost of the machinery and the cost of the necessary extra plant for making such wire, might make its use almost impossible, from a financial point of view. I doubt whether wire of that tensile strength could be obtained at less than 15 to 16 cents per pound.

Answering your second query, would say that in addition to tensile strength, I would recommend that tests for elongation and also for torsional strain should be adopted.

Answering your third question, would say that under the conditions named in question 1, I should say that wire of a quality of 180,000 to 190,000 pounds could be furnished of No. 3 Birmingham wire gauge. But in order to do this, quite considerable changes would have to be made in the plant of any existing wire mills, as the work of drawing would have to commence at very much larger sizes than any we now use. I am not prepared to say that any tensile strength exceeding 200,000 pounds per square inch could be obtained, commercially, for wire of No. 3 gauge; it certainly could not be obtained for 10 cents per pound.

In regard to the question of galvanizing wires of these sizes, would say that while the cost would only be somewhere in the neighborhood of three-quarters of a cent per pound extra for this, I should imagine that the wires would be very much injured by this galvanizing. In order to make the zinc adhere properly to the iron it would be necessary to heat these large sizes to such an extent as to practically destroy the effects of the hardening by drawing, which would have given them their tensile strength. Consequently, I should certainly recommend that the wires should not be galvanized in the ordinary way, but should be coated electrically with aluminium. This can be done cold, and without injuring the strength of the material at all; and while it has never yet been done on wire to a large extent, it has been used on the public buildings tower in Philadelphia with considerable success, so far as the writer is informed. If there are any other questions which you would like to put to us regarding this matter, we shall be happy to take the matter up, and if possible give you intelligent replies.

Yours, truly,

TRENTON IRON COMPANY,
E. GYBBON SPILSBURY,
Managing Director.

Lieut. ED. BURR,
U. S. Army, Norfolk, Va.

LETTER OF GEN. EDWARD W. SERRELL.

8 LIVINGSTON PLACE, STUYVESANT PARK,
New York, September 23, 1894.

SIR: * * * As to the Bullivant strand, these are the leading facts: The strand is made of 61 wires; the central wire is soft iron; the others are of a very high grade of steel. It is crucible metal, made with boxwood charcoal, so fine and hard that I have had several knife blades made of the parts of the strands broken at Watertown that can be made sharp as a razor. I send you inclosed a small chisel made of the end of a wire broken in the testing machine testing a strand. There was no hold-fast attachment to this strand, and it was fastened in the machine by the grip with a million of pounds pressure. You can see the marks made by the helices mashing into each other, and yet the chisel is moderately sharp and has cut cast iron. This wire is very hard and unusually stiff, but will bend and wind upon its own diameter, and will recover from a bend of 180 degrees closed without flaw. Strands can be had made of skeins of the length of the strand without joint in the wires, and the strand 2,800 feet long. The helices of the strand alternate in opposite directions, and the angle of the pitch increases from the core to the outside one in such a way that the strain upon each wire is more uniform than upon separate skeins, at least so a Board of Engineers, of which I was chairman, decided after several weeks of investigation. The Board consisted of Lieut. Col. Paine, C. E., U. S. V.; Lieut. Col. George V. Fosberg, R. A., V. C.; Mr. Sloane, C. E., of Chicago; Mr. James Rowan, C. E., of the Canadian Pacific Railroad; Mr. S. N. Haight, C. E., and myself. It was found that the stretch, which was at first feared, was a very small fraction more than that of a single straight wire. The stretch was continued to rupture; the exact difference between the strand and the straight wire was $\frac{2}{33600}$. (Extract from Col. Fosberg's report.)

The tests for fatigue of this metal show good results; up to two-thirds of elastic limit no fatigue was apparent. At the end of three days' continuous strain (Col. Parker's report), where the stress was final at 250,500 pounds to the square inch, the elastic limit was 196.100 (N. O. Olsen's reports), but it is proper to say that doubt as to the elastic limit exists in this case, as it seems to me it should in most cases. The diameter of the wire, which is round, averages 0.160 inches. (*See* report of Col. Fosberg, Col. Parker, Mr. Olsen, and Col. Paine.) A single wire usually broke at about 5,000 pounds' dead pull, and so uniform is the lay of the strand that the longest rupture of a 61-wire rope was never more than 14 inches, and many were not more than 3½ inches; some not over 2 inches. * * * Metal to sustain a stress of tension can be had in any quantity that will sustain 245,000 pounds to the square inch, and in lengths of 4,000 feet without joint. This is ultimate, of course, but such metal has an elastic limit as high as 200,000 pounds to the square inch. * * *

Yours, truly,

EDWARD W. SERRELL,
Brevet Brigadier-General, U. S. Volunteers.

Capt. ED. BURR,
Corps Engineers, U. S. Army.

LETTER OF MAJ. J. W. REILLY, ORDNANCE DEPARTMENT.

WATERTOWN ARSENAL,
Watertown, Mass., October 15, 1894.

SIR: In reply to your letter of the 4th, I have to say that some tests of steel wire, said to have been manufactured by Bullivant & Co., London, England, were made for Mr. Edward W. Serrell, C. E., the tensile strength of which ranged from 209,810 to 241,360 pounds per square inch, the average strength of 12 specimens being 228,430 pounds per square inch.

These wires were about 0.16 inch diameter, corresponding nearly to No. 8, B. W. G.

Other tests on wire substantially this size have been made on metal of domestic manufacture, the average tensile strength of 24 specimens being 231,720 pounds per square inch, while the highest and lowest gave 238,610 and 222,060 pounds, respectively, per square inch.

No published reports of these tests have been made, at least none emanating from this arsenal. The usual manuscript report of the results were furnished the parties for whom the tests were made.

For the detailed results of the latter tests you are respectfully referred to the Washburn & Moen Manufacturing Company, Worcester, Mass.

Copies of these tests can be had from this arsenal, but in each of these cases the tests were made for private parties and paid for by them, and therefore the results, as in all similar cases, are considered as the property of the parties defraying the cost of the tests, and copies of the reports of the results would be subject to their orders.

Respectfully, your obedient servant,

J. W. REILLY,
Major, Ordnance Department, U. S. Army, Commanding.

Capt. EDWARD BURR,
Corps of Engineers, Norfolk, Va.

APPENDIX C.

WIND PRESSURES.

[Compiled and prepared for the Board by Capt. W. H. Bixby, Corps of Engineers.]

INDEX.

As this question of wind pressures is now attracting the attention of engineers more than formerly, it is thought well to run over hastily the history of past experiments and past and present literature on the general subject of fluid motion against plates and solids, giving brief summaries of the most important sets of experiments (and reports on the same) which might be desirably referred to by any one wishing to examine further and in detail into such subject.

Since the accuracy of observations and also of deductions therefrom depend greatly upon the previous knowledge of each experimenter or writer, and also on the perfection of the methods and apparatus employed, it is quite important to bear always in mind the date of the experiments or treatises. Accordingly the following list is here inserted to show approximately the dates of such of the principal experiments or treatises as may either be referred to below or may be met in consulting other works on this general subject:

Experimenter or writer.	Year.
Tartaglio (ballistics)	1537
Galileo	1590–1638
Toricelli	1643
Hooke (pendulum anemometer)	1667
Mariotte	1686
Newton	1687
Daniel Bernouilli (fluids and ballistics)	1718–1738
S. Gravesande	1720
Leupold (gravity anemometer and tube anemometer)	1724
Robins (fluids and ballistics)	1742–1781
d'Alembert	1744–1752
Rumford (ballistics)	1751
Rouse and Smeaton	1759
Pitot (tube anemometer)	1760
Borda	1763
Lind (tube anemometer)	1775
Bossut	1777
Dubuat	1780–1786
Coulomb	1780–1790
Edgeworth	1783
Woltmann	1785–1790
Hutton (fluids, ballistics)	1786–1790
Vince	1798
Eytelwein	1801
Benzenburg	1802
Venturoli	1808
Swedish Commission	1810–1815
Beaufoy	1821–1834
Thibault	1826–1828
Bessel	1826–1828
Poncelet	1826–1851
Sabine	1827–1829
Duchemin	1829–1842
Rennie	1831
Baily	1832
Piobert, Morin, and Didion, at Metz (fluids and ballistics)	1835–1838
Osler (spring anemometer)	1837
Robinson (rotary cup anemometer)	1846
Stokes (treatise)	1851
J. B. Francis	1855
Helie (ballistics)	1861
Bashforth (ballistics)	1865–1870
Wenham and Browning	1867–1870
Hagen	1873–1874
Dohrandt	1873–1878
Robinson	1878
A. M. Wellington (trains)	1878
London Engineering (on Tay bridge)	1880
C. Shaler Smith (on bridges)	1880–1881
Collingwood (on bridges)	1880–1881
American Society Civil Engineers (on bridges)	1880–1881
Unwin (fluids) Encyclopedia Britannica	1880
Maitland (ballistics) Encyclopedia Britannica	1880
Recknagel	1880
Schellbach	1885
Thiesen	1885
Ferrel (atmosphere)	1885
Allen	1886
Hazen	1886
Abbe (apparatus)	1887
St. Venant	1887
Dines	1888–1889
Crosby	1890
Langley	1893
Kernot	1893

The usually recognized authorities on wind pressures to-day in engineering construction appear to be Poncelet in France, Unwin in England (*see* article on Hydromechanics in last edition of Encyclopedia Britannica), and Shaler Smith (*see* Proceedings American Society Civil Engineers) in the United States.

Our general theoretical treatment of the resistance of plates to the motion of fluids against them dates back to the time of Galileo, in 1590; but it was reserved to Sir Isaac Newton to give them practical form, which he did in his Principia, in about 1687, in terms which are expressed to-day by the general formula

$$p = d\, a\, H^2 = d a \frac{v^2}{2g},$$

in which p is the pressure per unit of surface, d is the density of the fluid, a is the maximum resisting area of the body in question, v is the velocity of relative movement of the fluid, g is the coefficient of gravity and $H = \frac{v^2}{2g}$ is the height required for gravity to give to a freely falling body the velocity v; the particles of the fluid being supposed perfectly free to move past each other in all directions. This formula, modified so as to apply specially to motions through air, becomes [1]

$$p = \frac{0.0027}{1 + .003665\, t} \quad \frac{P}{P_1} v^2$$

[1] See Abbe, p. 246, Annual Report Chief Signal Officer, U. S. Army, 1887, in part 2, vol. 4, Annual Report Secretary of War, 1887.

in which p is the resistance in pounds per square foot of the exposed surfaces, t is the temperature in centigrade degrees, P is the barometric pressure at the place of observation, P_1 is the barometic pressure at sea level on the forty-fifth parallel of latitude when the temperature is zero centigrade, and v is the velocity of the fluid or surface in miles per hour. This same formula, under temperatures of zero, and barometric pressures of sea level on the forty-fifth parallel, further reduces to $p=K\,0.0027\,v^2=K\frac{1}{270}\,v^2$ in which $K=1.0$ is a constant introduced for comparison of Newton's theoretical formula with the later formulæ of practice and experiment. In this latter formula, the value of p must be decreased about 1 per cent for each 4.5° F. (each 2.5° C.) in excess of the freezing point, and must be increased about 1 per cent for each 0.36-inch excess of barometric pressure above the normal of 27.795 inches (760 mm.), with other minor corrections for latitude, height of apparatus above sea level, and dryness of air, at the place and time of experiment. The average conditions of barometer and thermometer in the middle of Europe and of the United States will therefore require these values to be decreased about 5 per cent in fine summer weather and increased about 10 per cent in cold winter storms.[1]

Since the days of Newton almost all investigators have sought by practical experiments either to prove the above theoretical formula or to determine the extent of the differences between theory and practice. Most of the differences so far proved have been merely those obtained by varying the values of K, leaving the formula otherwise unchanged. In the majority of cases of flat surfaces normal to moving air, K has been found to be somewhere between 1.3 and 1.8, the gain of from 0.3 to 0.8 being ascribed mainly to the effects of the partial vacuum on the rear side of plates and of solids exposed to the pressure of moving fluids. In some few cases, experiments have added a small constant, so as always to give a result slightly in excess of that called for by $p=K\,0.0027v^2$, and in still other cases they have also added terms to express variations with the first and third powers of the velocities, these formulæ then taking the general shape of $p=A+Bv+Cv^2+Dv^3$, in which A, B, C, and D remain constant for the same surface and different values of v, some experimenters obtaining formulæ in which A and D may be always zero, others obtaining those in which B and D may be zero, or A and B may be zero. In still other cases experimenters have added terms to indicate a change in the pressure per unit according to the outline, or else to the size, of the plate. However, such changes are the exceptions and not the rule, and their necessity can hardly be regarded as yet proven; many of these changes having been recently found due partly or perhaps mainly to want of proper allowance for the errors or constants of the apparatus or methods used in the experiments.

The following brief summaries will show the general nature of these experiments, and their results, and will refer the reader to such printed reports and papers as might be of special interest to him, should he desire to go more fully into their examination.

In 1742, Robins' experiments, with projectiles at high velocities, suggested to him the idea that the resistance varied with an increasing power of the velocity.[2]

In 1759, a Mr. Rouse, in a paper read by John Smeaton,[3] submitted to the Royal Society of England a table of wind pressures based mainly upon his own experiments with windmills, whose sails were moved bodily through the air by machinery, the resistances being deduced from the weights which either moved the wheel, or were lifted by it, during varying velocities of movement. Smeaton, in communicating the tables to the society, advised them that its resistances for velocities over 50 miles per hour did not seem as well authenticated as for those of less than 50 miles per hour. This table, which has been quoted as authority in numerous handbooks, text-books, and the U. S. Signal Service books, since then, gives results which may be deduced for any special velocity from the formula $p=0.00492\,V^2$ or $p=\frac{1}{203}v^2$ (usually quoted for easy use as $p=\frac{1}{200}v^2$), this being deducible from the Newtonian formula by making $K=1.82$.

In 1787–'88, Hutton, in England, made some experiments[4] with thin plates of 4 by 8 inches, on whirling tables, from which he deduced the value $\sin a^{1.842\cos a}$, for the ratio of the resistance (parallel to the motion) of thin inclined plates to that of similar plates turned normally to the direction of the motion, a being the angle between the plate and the direction of the motion. This formula gives values as follows: 1.00 for 90°, 0.99 for 80°, 0.96 for 70°, 0.87 for 60°, 0.72 for 50°, 0.52 for 40°, 0.33 for 30°, 0.16 for 20°, and 0.05 for 10°. (For values of pressures normal to the moving plate or normal to the direction of motion, see farther on, under head of Encyclopedia Britannica, 1880.)

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[2] See Maitland, in article on Gunnery in Ency. Britannica.

[3] For summary, see pp. 173–176, article by F. Collingwood, in vol. 10, Trans. Am. Soc. C. E. for May, 1881; and full article, see Transactions Royal Society, England, 13 May and 14 June, 1759.

[4] See Duchemin, p. 216, vol. 5, Mem. d'Artillerie, 1842; also Unwin, article Hydromechanics, Ency. Britannica, 1880; also Langley, p. 63, No. 801, Smith. Contr., 1891.

His experiments with projectiles under high velocities suggested to him a formula for normal resistance of the form of $p = Av + Bv^2$; in which A and B were constants.[2]

In about 1798 Samuel Vince[5] made a series of experiments as to the resistances of different surfaces to motion through water, using various plane and curved surfaces of about 1 inch square area, moving in circles of 7.57 inches radius on the bars of a small whirling table; the table being turned by a pair of equal weights pulling in opposite directions on two cords wound around a vertical drum on the vertical axis of rotation of the whirling table. His results showed that in water and on whirling tables the resistances of surfaces in directions normal to their motion was closely proportional to the square of their velocity; that when the plane surfaces were inclined to the direction of their motion, the resistance in the direction of the motion varied somewhere between the sine and the $\sin^2$ of the angle of its inclination such that, the resistance at 90° being taken 1.00, that at 80° would be about 0.97, 70° about 0.92, 60° about 0.81, 50° about 0.67, 40° about 0.52, 30° about 0.33, 20° about 0.16, and 10° about 0.05; and that the resistance offered by the convex front of a hemisphere was only about 0.4 times that of its flat base front.

In 1828 Thibault also, experimented with thin plates moved normally to their surfaces;[6] using a whirling table of about 4.5 feet radius, velocities of from 1 to 22 miles per hour, and plates of from 0.5 to 1.00 feet square. His results gave formulæ similar to those of Newton, except that K varied from 1.66 to 1.90; being larger for the larger plates and also for those having their longer sides placed parallel to the radius of the whirling table.

In about 1829 Col. Duchemin, in France, made some valuable original experiments and investigations on the laws of resistance of fluids, these results being published in 1842. In his published article[7] he reviews the progress of similar investigations up to that time, referring to the experiments of Bernouilli in 1738, Rouse in 1759, Borda in 1763–'67, Dubuat in 1780, Coulomb in 1780–'90, Hutton in 1788, Samuel Vince in 1798, Venturoli in 1808, Thibault in 1826, and a dozen other investigators prior to himself. These experiments were made with all kinds of apparatus—falling bodies, swinging pendulums, whirling tables, towage, and exposure to currents—but the velocities rarely exceeded 20 miles per hour.

In this article he shows especially that, as to resistances, the air acts almost identically as do water and other fluids, due regard being made for its less density and freer fluidity; that the resistance of thin plates to being moved against fluids is only 0.62 of the pressure on their front surface, and only 0.67 of the pressure upon them when held still against the moving fluid; that the resistance of cylinders held end on to the direction of the motion, if three times as long as their diameter, is only 0.66 of that of their front face, being the same whether they or the fluid be in motion, but if shorter than three diameters, their resistances vary with this shortness, and are much greater when the water is in motion than when the cylinder is in motion, so that for cylinders so short as to be called thin plates the resistance is 0.932 of that of their front face when the water is in motion, and only 0.627 of that of the front face when the plate be moved against the water; that similarly shaped surfaces moved with different velocities exert resistances closely proportional to the square of the velocities, to the densities of the fluid, and to the area of the surfaces; that these resistances are much affected by the shape of the front surface and a little by its size and outline, and, in the same way, though less, by the rear surface of the body or plate moved; that whirling tables (on account of the centrifugal forces brought into action) gave resistances much greater[8] than those obtained by the rectilinear movement of surfaces, this increase being the greater as the surfaces were larger and the diameter of the whirling table smaller; that small surfaces gave slightly greater resistances per unit of surface than similarly shaped large surfaces; that lead bullets, by deformation or through other causes at high velocities of 15 to 900 miles per hour, gave results far in excess of the true resistances;[9] and that inclined surfaces gave resistances which were considerably in excess of the product of the pressure on non-inclined surfaces by the sine of the angle of inclination.

The most important laws thus brought out by Duchemin are expressed by him in formulæ which, after reduction to English measures, read as follows: For ordinary air pressures on thin stationary plates,[10] $p = 0.00942\ v^2 = \frac{v^2}{203}$, ($p$ being the pressure in pounds avoirdupois per square foot, and v the velocity in miles per hour), this being the Newtonian formula, except that $K = 1.82$; for pressures on circular and

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[5] For items, see Duchemin, pp. 214 and 236, vol. 5 of Memorial d'Artillerie 1842, and Abbe, p. 234, as above; and for full article see Philosophical Transactions, 1798.

[6] See Abbe, p. 234, as above.

[7] See pp. 65-380, Memorial d'Artillerie, vol. 5, 1842.

[8] See p. 206, same.

[9] See p. 270, same,

[10] See pp. 300–302, same.

triangular flat thin plates, practically the same results as for square plates; for air pressures on inclined flat thin plates $p=0.00492\ v^2 \frac{2 \sin^2 a}{1+\sin a}$ for the pressure parallel to the wind, $0.00492\ v^2 \frac{2 \sin a \cos a}{1+\sin^2 a}$ for lifting or depressing pressures perpendicular to the wind, and $0.00492\ v^2 \frac{2 \sin a}{1+\sin^2 a}$ for pressures normal to the surface,[11] a in each case being the angle between the wind and the surface (for values for each 10°, see farther on under head of Encyclopedia Britannica, 1880); for spherical-headed surfaces,[12] 0.4, and cylindrical headed,[13] 0.6, of the pressure on flat plates of the same diameter; for conical and wedge-headed surfaces pointed at the wind,[13] $p=0.00492\ v \sin^2 i$, in which i is the angle between the wind and the conical surface or between the wind and the sides of the wedge; and for stationary flat-headed but solid cylindrical bodies,[14] less pressures amounting to 80, 72, and 71 per cent of the pressure on thin flat plates, according as the length of the solid bodies was 1, 2, and 3 or more times their front diameters.

Duchemin also refers, as with importance, to the supposition first advanced by d'Alembert[15] in about 1750, and afterward developed more fully by Dubuat in about 1780, that plates and solid bodies in moving fluids, and especially under high velocities, act as if they were partly protected in front by a false prow made up of compressed articles of the fluid itself.[15]

In 1835–'38 a commission of French officers, composed of Piobert, Morin, and Didion, made at Metz a series of experiments[16] with whirling tables, falling bodies, and flying bullets, through both air and water, at both low and high velocities.

Their results seemed to indicate a pressure slightly in excess of that due to the square of the velocity, and gave a formula[17] for thin flat plates exposed to air pressures $p = 0.0073 + 0.0034\ v^2$ (p in pounds and v in miles per hour); which, however, at velocities of over 10 miles per hour returned practically to the old Newtonian formula, except that $K = 1.25$. Amongst other results they found for wedge-headed projectiles[18] the pressure equal to that on flat heads multiplied by $\frac{a}{90}$, in which a is the angle between the wedge surface and the direction of the motion; for parachutes[19] whose height was 0.3 their diameter, a pressure in ordinary position 1.9 times, and up-side down 0.77 times, that on flat plates of the same diameter; and for spheres[20] only 0.15, for cubes 0.65, and for long cylinders 0.30 times the pressures on thin disks of the same diameters.

Later experiments with projectiles in 1840 suggested to Didion the usefulness for normal resistances at high velocities of a formula $p = Av^2 + Bv^3$, in which A and B were constants.[2]

The U. S. Ordnance Manual (1861)[21] gives for the then standard formula for the resistance of air to projectiles a French formula of $p=0.0011\ v^2 \left(1 + \frac{1}{103} v\right)$ for spherical projectiles, which would, by inference, give $p=0.0022\ v^2 \left(1 + \frac{1}{103} v\right)$ for flat-headed projectiles, in which p is in pounds per square foot and v in miles per hour; this latter formula for flat-heads being the Newtonian formula, in which $K=1$ for velocities of 22 miles per hour, $K=1.25$ for 51 miles velocity, $K=1.50$ for 100 miles velocity, and $K=1.80$ for 130 miles velocity. The main difference between this formula and others is therefore that it gives for low velocities a less pressure than is deduced from the mass of experiments with low velocities, leaves the pressure at velocities between 100 and 130 miles approximately those of the other most usual formulæ, and gives rapidly increasing pressures for velocities of over 130 miles.

In 1861 Helie, from experiments with projectiles, concluded that the normal resistance at high velocities was best expressed by a formula of the form of $p=Av^3$, in which A was a constant.[2]

In 1865–1870 Bashforth, from experiments with projectiles, concluded that the normal resistance at high velocities was best expressed by the formula $p=Av^2$ up to

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[11] See p. 213, Memorial d'Artillerie, vol. 5, 1842.

[12] See p. 237, same.

[13] See pp. 248–253, same.

[14] See p. 76, same.

[15] See p. 336, same. This idea has been strengthened in the last few years by the appearance of the waves of compression in air in front of flying bullets, as shown in recent instantaneous photographs of such bullets.

[16] For full details, see pp. 553–632, same; and pp. 197–292, vol. 7 (1852) of the Memorial d'Artillerie.

[17] See p. 279, vol. 7, same.

[18] See p. 291, same.

[19] See p. 289, same.

[20] See p. 237, same.

[21] See p. 482.

values of $v=1,100$ feet per second (about 750 miles per hour); then by $p=Bv^3$ up to values of $v=1,350$ feet per second (about 930 miles per hour), and then by $p=Cv^2$ for higher velocities, in which A, B, and C were constants. Modern ballistics in England are still (1894) based upon these formulæ.[2]

In 1867–1870 Wenham and Browning made some experiments for the London Aeronautical Society with plates of 1 to 2 feet square, placed in a wind blast issuing from an 18-inch square pipe under a 0.6 to 1 inch water pressure.[22] Their experiments tended to confirm Duchemin's formula for normal, parallel, and lateral pressures on inclined thin plates.

In 1878 A. M. Wellington, at or near Cleveland, Ohio, made an extensive set of experiments[23] with locomotives, cars, and different grades or slopes of railroad track, in order to determine all the various resistances to the haulage of trains, making use of the drop test—that is, starting the cars from a state of rest on a known grade and deducing the resistances from the velocities acquired. The passage of the trains past eleven different points was noted by electricity to within one-twentieth of a second, the experiments covering velocities from 0 to 30 miles per hour.

So far as the air resistances were concerned, the results of these experiments were to show that for velocities of 10 miles per hour p is less than $\frac{1}{500}v^2$ (less than $0.0020\ v^2$) for the combined head resistance and tail suction of a train whose cross section was $10\times14=140$ square feet, and p is less than $\frac{1}{4500}v^2$ (less than $0.00023\ v^2$) for the combined side and end resistance of each succeeding car, the gaps between the ends of two adjacent car bodies being 6 feet—p in these formulæ being the resistance of the air in pounds per square foot, and v the velocity in miles per hour—so that the formula becomes that of Newton in which $K=0.74$.

In these experiments he also discovered that fully half the previously reported air resistance in such cases was not really due to air pressures, but was due to the effects of oscillation and concussion between the various individual parts composing the entire train. The remarks of Mr. Wellington give rise to the inference that similar friction, oscillation, and concussion in whirling tables and other apparatus may be the cause of the high values of the constants deduced from such past experiments. This suggestion, moreover, is evidently in accord with conclusions of many other investigators.

In about 1880–'81, just after the fall of the Tay bridge, the matter of wind pressures naturally attracted the special attention of bridge engineers of England as well as of the United States, and the London Engineering of 1880–'81 contains very numerous articles and discourses on this subject with special reference to its relation to bridges and other engineering structures. These articles are too numerous to itemize, but their most important features are the wide divergencies of opinion between engineers both as to the amount of such pressures, the allowance to be made for them, and the methods of calculating the supposably exposed surfaces of the structures.

Probably the most valuable part of these discussions is the recommendation of the Tay bridge commission of 1881,[24] which may be briefly stated as follows: On solid girder bridges an allowance of 56 pounds per square foot over all the girder (and also over as much of a train as may rise above the girders); and on open or lattice girders an allowance of 56 pounds per square foot over the train and over the actual area of ironwork of so much of the windward girder as projects above or below the train, and also an allowance of 28, 42, or 56 pounds per square foot over the actual area of the ironwork of the like projecting portions of the leeward girder, according as the open spaces of this leeward girder form less than two-thirds, from two-thirds to three-fourths, or over three-fourths, respectively, of the total area of the outline of the girder. The factor of safety against such pressures should be 4 against tension and compression and 2 against overturning. The same committee also stated their opinion[25] that the pressures to be expected from winds of any given locality might very nearly be expressed by the formula $p=\frac{1}{100}v^2$, in which p is in pounds per square foot, and v is in miles per hour as given by the usual cup velocity anemometer records of that locality.

In 1880 Messrs. A. Welch, C. Shaler Smith, and F. Collingwood read papers upon the general subject of wind pressures before the American Society of Civil Engineers,[26] and Mr. Smith submitted a summary of his personal observations of the

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[22] Unwin, Ency. Britannica, 1880, article Hydromechanics; also p. 244, Abbe on Meteorological Apparatus, 1887; also Annual Report London Aeronautical Soc., 1871.

[23] See pp. 493, 518, 522, and 911, Railway Location, A. M. Wellington; and for full article, see Transactions Am. Soc. C. E., Feb., 1879.

[24] See Burr, Stresses in Bridge and Roof Trusses, p. 368; and Fidler, Bridge Construction, p. 416.

[25] See Burr, p. 370.

[26] See pp. 391-400, vol. 9, Oct., 1890, and pp. 139 to 186, vol. 10, of May, 1881, Trans. Am. Soc. Civil Engineers.

force and action of wind and tornadoes upon bridges and structures, adding a statement of the rules followed by him in his own bridge practice. These papers, with their discussion before the society, give the reasons for the present ideas and practice of American bridge engineers. The most important features of these papers are the many descriptions by eyewitnesses of the force and violence of the wind in actually overturning houses, water tanks, bridges, and even locomotives, the small width of the path of most violent action, the varying ideas of the pressure actually necessary to produce these results, and the practical rules submitted by Mr. Smith as to what he deems necessary allowances for wind pressures.

The most violent records of wind force thus given were the blowing down of three bridges (between 1866 and 1880) at computed pressures of from 18 to 27 pounds per square foot; several cases of train derailment at computed pressures of 30.5 pounds per square foot; the destruction of brick houses at computed pressures of from 58 to 84 pounds per square foot; the overturning of a barrel of tar at a computed pressure of 52 pounds per square foot; the overturning of a locomotive at a computed pressure of 93 pounds per square foot; the lifting of a piano, transporting it 270 feet, and then replacing it on its feet without apparent injury; the bursting of houses by blowing outward their ends and sides, and the perforation of a half-inch pine board (originally the side of a dwelling house) by wheat straw flung against it end first (by a Wisconsin tornado).

As to the upward tendency of the wind through the bottom of the bridge, enough evidence was presented to suggest the decided advisability of not making the flooring continuous unless so left as to be easily ripped up by the wind without damage to the rest of the bridge.

As to the actually observed velocity and force of the wind, as measured by anemometers, the discussion brought out the records of the Bidston Observatory near Liverpool, England, showing isolated cases of 92 miles total velocity in a single hour and 80 pounds per square foot maximum pressure in 1868, 82 miles velocity and 65 pounds pressure in 1870, 79 miles velocity and 90 pounds pressure in 1871, 81 miles velocity and 70 pounds pressure in 1875, 80 miles velocity and 64 pounds pressure in 1877, and 59 miles velocity and 38 pounds pressure in 1879; the velocities being measured by Robinson cup anemometers (giving only the totals per hour), and the pressures being measured by Osler spring-pressure anemometers (giving maximum and not average pressure); but, it was also stated by a resident of that place that these velocities and pressures must have been of limited extent and duration, as no houses nor cars nor freight trucks appeared to have been blown over or damaged by these winds.[27]

As regards the destruction of the Tay bridge, the discussions show that the London Engineering of January 2, 1880, computing the exposed surfaces of the bridge as being that of the windward girder, plus that of the train, plus also 0.5 that of the lee girder, deduced a possible pressure of the wind of from only 23 to 36 pounds and expressed the belief that the maximum pressure did not exceed 25.5 pounds. Others, basing their computations on the non destruction of the adjacent signal boxes, etc., deduced pressures of from 15 to 40 pounds. The doubtfulness of deductions from the exposed surfaces of the bridge is evident however from the fact that such surface, as measured by C. Shaler Smith according to his American rules, was 2,576 square feet against only 800 square feet as measured by English engineers, or as 3.2 to 1; so that definite results and accord can not be expected unless the various computers will agree better as to what constitutes the so-called exposed surface of a bridge.

As to the width of path of the greatest violence of tornadoes, Mr. Smith, having followed up the path of several cyclones, found but one case where 60 feet width was not sufficient to cover the pathway within which the computed pressures exceeded 30 pounds per square foot; and numerous instances were given by others to show that in their opinion even the gusts of both ordinary and severe winds were of very limited reach in side directions. Moreover, a cyclone whose center actually crossed a 320-feet-length span of railroad bridge, showing maximum pressures of at least 52 pounds per square foot at one point on the bridge and 84 pounds per square foot elsewhere, did not destroy this span, although its wind bracing had been built by Mr. Smith with allowances of only 30 pounds per square foot of exposed surface and a strain of 20,000 pounds per square inch on the braces.

However, the most important feature of these papers is the set of empirical rules used by Mr. Smith in dimensioning wind bracing on high and exposed bridges, which may be briefly stated as follows: Spans must be proportioned to stand a horizontal wind pressure of either 30 pounds per square foot on both structure and train, or else 50 pounds on the structure alone, using whichever of these pressures is the greatest. In measuring the structure, both trusses are to be measured, using 1.5 times the front surface of ties, 2 times that of chords, and 1 time that of the

[27] See later on, as to 40 per cent reductions to be applied to the small plate-pressure records to give pressures on large surfaces, and the 25 per cent reductions to be applied to all cup rotary anemometers.

other parts of the truss; and the train is to be considered as a continuous front of 10 feet height over its full length. The train pressure is moreover to be treated as a moving or live load and the truss pressures as dead loads. Stresses are to have a factor of safety of 4. Piers are to be proportioned in the same general way as spans, except that they shall be given a base broad enough to prevent any tension strains in their columns. If the bridge is on a curve, the spans and piers must be proportioned to stand a centrifugal force due to the maximum load moving at 40 miles per hour, in addition to the wind strains. In addition to all the above, an allowance of 10,000 pounds in each member must be made to provide for the initial stress produced in screwing up rods during the final adjustment of the bracing.

In support of the above views Mr. Smith and others expressed opinions that 30 pounds per foot wind pressure rarely if ever extended over more than 60 feet continuous horizontal length at one time and place; that even this wind was but rarely directed squarely against the side of a bridge; that in the case of at least one bridge he has assured himself of the whole bridge having received as much as 1.8 times the wind pressure on 1 truss; that as fully loaded passenger and freight trains would leave the track at pressures of 31 and 56 pounds per square foot, and as then the bridge would be destroyed by derailment, it was useless to protect it against heavier pressures under train loading; that with no tension in the piers under 30 pounds wind pressure the piers would safely stand all excess pressure that might come upon it; and that even if the bridge were overstrained occasionally by wind pressures such cases would be so rare and the fatigue of metal so slight that the material resisting this strain would not be seriously injured.

The new edition of Encyclopedia Britannica (in vol. 12, published about 1880) contains an article on hydromechanics, by Unwin and Greenhill, which may fairly be regarded as representing English ideas of that date.

In this the relations of pressure and velocity are given by the Newtonian formula, with $K=1.3$ for plates moving in still air or water, and $K=1.8$ for water or air moving against still plates.

The pressure of horizontally-moving air on thin inclined plates is stated at either $H=P\ (\sin a)^{1.842 \cos a}$, according to Hutton, or as $H=P\ \frac{2 \sin^2 a}{1+\sin^2 a}$, $L=P\ \frac{2 \sin a \cos a}{1+\sin^2 a}$, and $N=P\ \frac{2 \sin a}{1+\sin^2 a}$, according to Duchemin, in which H is the horizontal resistance, L is the lateral resistance (or resistance to lifting or sinking), $N=\sqrt{L^2+R^2}$ is the normal resistance, P is the pressure on the same plane when normal to the motion of the air, and a is the angle between the wind and the inclined surface. (See also Hutton[4] and Duchemin.[7]) From Duchemin's formula for H, the pressures, parallel to the wind, become 1.00 for 90°, 0.99 for 80°, 0.94 for 70°, 0.86 for 60°, 0.74 for 50°, 0.59 for 40°, 0.40 for 30°, 0.21 for 20°, and 0.06 for 10°. From Duchemin's formula for N, the pressures, normal to the inclined plate, become: 1.00 for 90, 80, or 70°, 0.99 for 60°, 0.97 for 50°, 0.91 for 40°, 0.80 for 30°, 0.61 for 20°, and 0.34 for 10°. From Duchemin's formula for L, the lateral (lifting, depressing, or side moving) pressures normal to the wind become: 0.00 for 90° (where the plane is normal to the wind), 0.17 for 80°, 0.34 for 70°, 0.50 for 60°, 0.62 for 50°, 0.70 for 40°, 0.70 for 30°, 0.58 for 20°, 0.33 for 10°, and 0.00 for 0° (where the plane is parallel to the wind). For angles of from 90° to 50° the Hutton and Duchemin formulæ for H, N, and L give results in accord with each other to within 0.02.

In 1885, William Ferrel, of the office of the Chief Signal Officer, U. S. Army, prepared an extensive report[28] on meteorology in general, based upon meteorological research and observations from all over the world during the preceding twenty-five years, giving a thorough theoretical discussion of the whole subject of atmospheric conditions and movements, together with many practical illustrations. The chapters on cyclones and tornadoes contain much matter of special interest to the subject of wind pressures in general. Numerous bibliographical references make the report of much additional value to those studying this subject. Amongst other things, the following of his statements are specially interesting in connection with our present investigation:

The normal atmospheric pressure being 14.7 pounds per square inch, or over 2,100 pounds per square foot, it is not surprising that small differences of barometric pressures may cause winds of high velocities and great pressures. If the barometric pressure in a tornado be 3 inches below the general pressure (which has sometimes been the case in the past) the resulting wind may have a velocity of over 300 miles per hour and a pressure of over 240 pounds per square foot. Moreover, if the center of such a tornado passed suddenly over a building with closed windows and doors, the sudden expansion of the air in the building due to this 3 inches differences of barometric pressure might reach 211 pounds per square foot of surface, plenty enough to blow cellar doors from their fastenings, blow out windows and ordinary doors,

[28] See Ferrel, pp. 1-443, part 2, vol. 4, Annual Report Secretary of War, 1885.

burst out walls, blow up roofs, and wreck buildings generally, as is known actually to have been done. Such an example as this is by no means an extreme supposition, for it is possible for the differences of barometric pressures in tornadoes to be twice as much as this and even more. In the great storm of January 26, 1884, in Scotland, a reading of the barometer near Crieff showed only 27.222 inches [29] when reduced to sea level, and in this case the difference of barometric pressures between its center and some points within the area of the storm must have been at least 3 inches. In the hurricane of January 24, 1868, also in Scotland, barometric observations at Aberdeen and Culloden showed an actual difference of 1 inch in 138 miles, accounting easily for extended wind disturbances with velocities of from 71 to 93 miles.[30] These two actually observed cases are merely cases of cyclones or severe but general storms extending over large areas. In the cases of tornadoes, which are more local and at the same time more severe within their smaller area, these differences of barometric pressure, and the consequent wind velocities and wind pressures may be very much greater. In 600 tornadoes observed in the United States (see Professional Papers No. 7, U. S. Signal Service, by Finley), the width of the path of destruction, supposed to measure the disturbance between the areas of sensible winds on the north and south sides of the storm's center, varied from 40 to 10,000 feet, the average being 1,085 feet.[31] Under the most favorable circumstances (when over deep and narrow valleys) 1.2 inches of low barometer in the middle of a tornado might give to the ascending current of air a velocity of as much as 57 miles per hour;[32] but ordinarily, on fairly level ground, this velocity would be much less by reason of the friction of the incoming air upon the adjoining earth surface around the tornado center.

As to the relation between velocities and pressures of air on varying surfaces, the discussions of pure theory indicate that horizontal winds should give horizontal pressures of $p=0.0027\,v^2$ against vertical plane surfaces normal[33] to the wind, in which p is the pressure in pounds per square foot of the exposed surface, and v is the velocity in miles per second, strictly according to the Newtonian formula with $K=1$; and that this pressure should be multiplied by 0.66 for horizontal pressures against vertical cylinders,[33] by 0.50 against spheres,[33] by sin a^2 for horizontal pressures against inclined vertical plane surfaces, and by sin a for the normal pressures and sin a cos a for the lateral pressures on such planes,[34] a being the angle between the wind and the plane. In all these cases p must, of course[35] be decreased about 1.0 per cent for each 4.5° F. excess of temperature above the freezing point, and must be increased about 1 per cent for each 0.36-inch excess of barometer reading above the normal 27.795 inches, with other slighter corrections for latitude and height of apparatus and dryness of air at the place and time of the experiments. However, the values obtained from the above theoretical formulæ should all be increased in order to give results in accord with actual experiment, this increase for thin square plates[36] normal to the wind being perhaps as much as 35 per cent, giving $p=0.00365\,v^2$ or $p=\frac{v^2}{274}$ (with perhaps other lesser variations according to the shape and size of the plate), and this increase for hollow spheres[37] of 1 square foot maximum cross section being perhaps as much as 10 per cent, giving $p=0.002\,v^2$ or $p=\frac{v^2}{500}$; such increase being probably due to the effects of air friction and other minor causes not allowed for by the Newtonian formula, and to the effects of friction, oscillation, and inertia of apparatus not completely allowed for by the experimenter.

Attention is also called to the fact that the actual velocity of wind must necessarily vary very greatly according to its altitude above the ground and also above sea level, according to the proximity and shape of the adjacent hills and valleys, and must almost invariably on land be also exceedingly variable from moment to moment; and to the fact that the values of wind velocities and wind pressures as deduced from the records of the most used anemometers may often be very greatly in error, the ordinary cup anemometer, with a coefficient of 3.0, while approximately correct for light winds of from 5 to 10 miles per hour, giving (unless specially corrected therefor) velocities from 20 to 25 per cent too large for winds of 25 to 30 miles recorded values, and perhaps greater excess errors for winds of still higher velocities.[38]

In 1886, H. Allen Hazen made a set of original experiments in Washington,[39] with a whirling bar, using plates from 4 inches to 2 feet square, moved in circles of 4, 8, and 16 feet radius, with velocities of from 4 to 16 miles per hour, in a closed room about 50 feet square having, however, only about 40 square feet of clear space. The plates were hung from the ends of the bar by delicate threads of from 4 to 8 feet length so as to swing freely back under the air pressure, the bar was turned by the untwisting of ropes, the revolutions of the bar were recorded by a chronograph, and the

[29] See Ferrel, p. 255, part 2, vol. 4, Annual Report Secretary of War, 1885.
[30] See p. 285, same.
[31] See p. 288, same.
[32] See p. 312, same.
[33] See p. 305, same.
[34] See p. 298, same.
[35] See p. 302, same.
[36] See pp. 407–8, same.
[37] See pp. 305 and 315, same.
[38] See pp. 403–8, same.
[39] See pp. 241–248, vol. 134, Am. Journal of Science, 1887.

resistances were computed from the observed angular swing of the plates under the air pressure, the extent of swing being noted by an observer standing at the center of revolution. The experiments with 16 feet radius circles gave no signs of increased pressures for the larger plates. In addition to using the results of his own experiments, Mr. Hazen consulted the work and the results of Borda in 1763, Piobert, Didion, and Morin in 1835, Hagen in 1873, and compared his results with theirs, with final conclusions as follows: That for velocities up to 7 miles per hour with small plates, and up to 4 miles per hour with large plates, $p = .0034\ v^2 = \frac{v^2}{295}$, in which p is the pressure in pounds per square foot, and v the velocity in miles per hour; this being the Newtonian formula, except that $K = 1.25$; that the increase of pressures heretofore ascribed to large plates is mainly due to the centrifugal forces of the whirling tables; that for accurate results in such matters the whirling tables must be of greater radius than 16 feet; and that pressures for higher velocities need special further experiment. As the radii of the whirling tables used by Vince were less than 8 inches, those by Thibault less than 5 feet, those by Bordan and Hagen less than 8 feet, while those of Hazen were as great as 16 feet, Mr. Hazen's conclusions are entitled to special consideration as regards the increased length of radius of whirling tables necessary to secure reliable results.

In 1887, Cleveland Abbe, of the office of the Chief Signal Officer, U. S. Army, prepared an extensive report[40] on meteorological apparatus and methods. The chapters on measures of wind velocity and on anemometers are of special value in relation to the subject of wind velocities and pressures, as they contain careful descriptions of all the most used air-velocity and air-pressure apparatus, and equally careful digests of all past work in connection with the determination of their possible errors and their constants. Chapter 12 contains a theory of vanes, with descriptions of those most used in determining the directions of the horizontal and vertical components of the wind. Chapter 14 contains tables of the scales of wind velocities as used by various foreign nations, and their comparison with that of the International Bulletin as now published by the U. S. Signal Service. Chapters 15, 16, and 17 contain a theory of anemometers with description of the various direct-pressure anemometers (the pendulum or swinging plate, the normal plate, and the tubular), the suction anemometers (with horizontal tubes and with vertical tubes), and the rotation anemometers (windmills, screw propellers, and cups). Incidentally also to the discussion of the theory of such anemometers, these chapters contain a brief but careful description of the various experiments of Mariotte, Wolt mann, Dubuat and Thibault, as to the pressure of moving fluids on stationary solids; and of Mariotte, Newton, Benzenberg (with falling bodies), of Robins, Hutton, Bashforth, Mayevski (ballistic pendulum), of Bouguer, Borda, Ulloa, Buchanan, French commission, Swedish commission of 1810–'15, Dubuat, Piobert, Morin, and Didion, (towage through water), as to the resistance of a stationary fluid to a solid moving rectilinearly through it; and of Robins, Schober, Rouse, Borda, Coulomb, Edgeworth, Hutton, Vince, Thibault, Beaufoy, Rennie, Prechtel, Dohrandt, Hagen, Rechnagel, Schellbach, and Thiesen, as to the resistance of a stationary fluid to a solid moved circularly through the fluid (water or air) by the use of whirling tables of some sort; and of Borda, Dubuat, Coulomb, Bessel, Sabine, Baily, and Stokes as to the resistance of a stationary fluid to both swinging and revolving spherical pendulums. From these descriptions and discussions the following items are selected as of special interest in connection with wind pressures and velocities.

In order to obtain reliable results from the comparison of the records of various experiments, great accuracy is necessary as to the conditions under which these experiments were made, and as to the constants, so called, of the apparatus used; but in matter of fact, a large proportion of the more accurately made experiments are not recorded in such way as to be accurately comparable with each other. The temperature effects are often quite great, but such observations are not always taken, and even then are often not reliable to within one or two degrees. The measures of the wind directions are often erroneous to several degrees, and the wind velocities and pressures to large per cents, from mere absence of continuous automatic records showing the variations at short intervals. The constants by which the displacement records of the pendulum pressure anemometers[41] are converted into pounds per square foot may be easily erroneous to 10 per cent in ordinary winds and more in high winds; those of the normal pressure plate anemometer[42] may occasionally be erroneous to as much as 10 per cent at ordinary velocities, though the improved forms of the Osler anemometer reduce this percentage very greatly; those of the most used velocity anemometers[43] (the Robinson hemispherical cup anemometer, usually

[40] See Abbe, pp. 1–392, plates 1–36, part 2, vol. 4, Annual Report Secretary of War, 1887.
[41] See p. 245, same.
[42] See p. 248, same.
[43] See p. 285, same.

assumed to rotate with one-third the velocity of the wind), may, unless each anemometer be carefully compared with the standard, easily be erroneous to from 15 per cent for low up to 25 for high velocities, since the ratio between the actual velocity of the wind and the revolution of the anemometer varies with the size of the cups, the lengths of the arms, the total weight and inertia of the moving parts, the surfaces of friction, the density of the outside air, and finally the velocity of the wind. Heavy anemometers of the pressure plate or rotary cup form may also, by the great inertia of the moving parts, show much larger instantaneous pressures and much greater hourly velocities than actually exist. Moreover, it is to be remembered that all these pressure and velocity anemometers, as ordinarily used, show only the horizontal component of the wind and not its total force. We must always, therefore, bear in mind that many allowances must be made for the errors of apparatus of all kinds, and that reports of past and present work must be judged accordingly.

As to the general direction of winds on land, statements have been made[44] that the wind, in Europe at least, has a general downward inclination of from 10° to 20°; but it does not seem likely that this is anything but the effects of occasional and strong gusts, so that except for such gusts the direction of the wind may be assumed as ordinarily horizontal.

As to the relation between pressures and velocities, the evidence so far seems to be still decidedly in favor of the use of the Newtonian formula (with proper coefficients for K) in preference to all others yet suggested. For the case of solids moving through stationary liquids the evidence up to 1887 seems to be in favor of $K = 1.3$, which for the pressure on thin plates, when moving through stationary air would give $p = 0.0035\, v^2$ or $p = \frac{v^2}{285}$ (subject to the usual modifications[1] for temperature, barometric readings, and latitude) in which p is in pounds per square foot, and v in miles per hour (equivalent to $p = 0.0075\, v^2$ or $p = \frac{v^2}{613}$ if p be in pounds and v in feet per second); and this relation is indorsed by Poncelet in France and Unwin in England.[45] For the case of liquids moving against stationary solids (as winds against anemometers, houses, and bridges), the evidence up to 1887 seems to show a value of K somewhere between 1.3 and 1.85, and nearer 1.85 than 1.3, so that for apparent fear of underestimation Poncelet in France and Unwin in England indorse the use of $K = 1.85$ with a suggestion that more accurate experiments are still needed.[46] For $K = 1.85$, the Newtonian formula, in the case of air striking normally on stationary thin plates, would give $p = 0.0050\, v^2$ or $p = \frac{v^2}{200}$ in which p is in pounds per square foot and v is in miles per hour (or $p = \frac{v^2}{430}$ if v is in feet per second), this being also practically the same as the Rouse-Smeaton formula of 1759, which is that adopted by the U. S. Signal Service in its work in connection with the preparation of the International Bulletin of Meteorology.[47] As to the differences between pure theory and actual practice in the above cases, it is at present impossible to decide whether the differences between $K = 1.3$ and $K = 1.85$ for moving plate and moving fluid are due to errors of observation or represent laws of nature, but it may be fair to assume for them equal weights and take $K = 1.66$ as a common factor applicable alike to both cases.[48] The experiments of Dubuat, showing by actual direct measures that the vacuum on the back of the plate, due to imperfect fluidity or imperfect continuity in the disturbed fluid, added 0.433 units of pressure to each 1 unit of direct pressure on the front of the plate,[49] are in thorough keeping with the above assumption. For $K = 1.66$, the Newtonian formula, in case of air on thin plates, either being stationary, would give $p = .0045\, v^2$ or $p = \frac{v^2}{222}$, in which p is in pounds per square foot and v in miles per hour. However, this value is not yet indorsed by Poncelet, Unwin, and other engineering authorities.

As to the determination of such relations up to 1887 by falling bodies, by the ballistic pendulum and by flying bullets, the results, while of use for purposes of comparison among themselves, are hardly accurate enough for use in determining the direct relation of pressure to velocity.

As to the use of whirling machines in determining fluid pressures on flat surfaces, it appears that the experiments of Borda, Robins, Vince, Coulomb, Hutton, Edgeworth, Beaufoy, Rennie, and Hagen (all that are quoted as made prior to 1886), were made with small plates of only from 0.01 to 0.30 square feet area, in circular paths of only from 0.6 to 8 feet radius, with velocities of only from 0.6 to 16 miles per hour, so that their results must be regarded as somewhat doubtful even for velocities of 16 miles per hour, and certainly quite doubtful for greater untested velocities.[50] More-

[44] See Abbe, p. 195, part 2, vol. 4, Annual Report Secretary of War, 1887.
[45] See p. 229, same.
[46] See pp. 224 and 227, same.
[47] See p. 207, same.
[48] See p. 224, same.
[49] See p. 224, same.
[50] See pp. 224, 225, 229–243, same.

over, the experiments of Scott in 1872, Robinson in about 1873, Dohrandt in 1873, 1877, and 1878, and Stelling in 1882, with the Robinson cup rotary anemometer, whirled on tables of from 9 to 10 feet radius with from 7 to 42 miles per hour velocity (in endeavoring to determine the constants of the Robinson rotary anemometer), showed at times the existence of induced circular currents of from 1 to 2.2 miles per hour velocity in the air adjacent to the edge of the whirling table (from 10 to 7.5 per cent of the velocity of the edge of the table, according as this latter velocity varied from 10 to 30 miles per hour), showed also the existence of decided radial currents whose amount and effect has not yet been thoroughly discussed and allowed for, and gave much other good evidence that the history of past experiments with small whirling machines is not to be greatly relied upon except so far as it is confirmed by other more reliable testimony; and that reliable results are not to be obtained in this way in the future except by the use of much larger machines and many further precautions against errors of the apparatus.[51]

Although almost all experimenters feel certain that the pressure per unit on large surfaces must be much less than on smaller ones,[52] and that the shape of the perimeter of the surface must also to some extent influence the pressure on the plate, still the law of such variation is not yet satisfactorily determined.[53] The formula of Hagen (1874) is regarded as the latest and best of its kind, but its value is diminished by the fact that it was deduced from the movement of plates of only from 0.02 to 0.30 square feet area, moved at velocities of only from 1 to 4 miles per hour, on a whirling table of only about 8 feet radius, and in a small room.[54] This formula, however, reduced to sea level at the forty-fifth parallel and to the temperature of freezing becomes[55] $p=0.00306\,(1+0.048\,C)\,v^2$ or closely $p=0.0031\left(1+\frac{C}{20}\right)v^2$, in which p is in pounds per square foot, v is in miles per hour, and C is the circumference (or perimeter) of the surface in feet.[56]

To anyone desirous of studying in detail the many obscure points of fluid resistance, Prof. Abbe calls attention to the valuable work of St. Venant, Mem. Inst. of France, on Resistance des Fluids, published in 1888, which was received by him too late to be used in his own work.

In 1887 experiments were made in France[57] with small counterweighted plates, suspended laterally from the side of a moving train. The train was run at increasing velocities until the resistance of the air overturned the weighted plates. The velocity of the train at that moment was noted and the ratio of the two computed therefrom. For plates of 0.11 square foot area, at velocities of 44.5 miles per hour, the pressure was thus determined to be in accordance with the formula $p=0.0054\,v^2$, which is the Newtonian formula, with $K=2$. The measurements were, of course, rendered somewhat uncertain by the nearness of the sides of the cars. Within the limits of their experiments no practical effect was obtained from varying the size of the plates.

Between 1884 and 1890, during the construction of the Forth bridge, experiments were made[58] to determine the probable pressure liable to come upon this bridge, as well as the relations between the pressures on large surfaces and the records of the ordinary small plate anemometers. For this purpose a large fixed board gauge (or anemometer), 15 by 20 feet, having 300 square feet area, was erected on top of the old castle on the island of Inchgarvie near the middle of the bridge, was so placed as to be parallel to the length of the bridge, and was further provided with a small disk gauge at its center and another small disk gauge at one of its upper corners. About 8 feet on one side of the large gauge was also placed another small fixed gauge of about 1.5 square feet area, and also a second disk gauge of the same size but differing from the others in that it was free to turn and by means of a vane was kept pointed at the wind. All these vanes were read at about 9 a. m. each day for six years. In the interval from December 11, 1883, to January 25, 1890, the heaviest gales were fourteen in number, the records of the first two being rejected because of imperfect arrangement of the apparatus. Of the remaining twelve, only four, those of March 20, 1885, December 4, 1885, November 17, 1888, and January 21, 1890, came squarely against the fixed gauges. The records for these days, respectively, show pressures of 30, 25, 35, and 36 pounds per square foot on the small revolving plate gauge; of 25, 27, 41, and 38 on the small fixed gauge; but of only 17, 19, 27, and 15 on the

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[51] See Abbe, pp. 269–293, part 2, vol. 4, Annual Report Secretary of War, 1887.

[52] See p. 211, same.

[53] See p. 258, same.

[54] See Hazen, p. 242, vol. 134, Am. Journal of Science, 1887.

[55] See p. 236, Abbe; and p. 408, Ferrel.

[56] This formula reduced to the Newtonian form would give $K=1.16$ for plates of 1 inch square, $K=1.35$ for plates of 1 foot square, $K=3.4$ for plates 10 feet square, $K=10$ for the side of a house of 40 feet square, and K over 20 for the side of a building 200 feet long and 50 feet high. Evidently this formula is not intended to apply to large surfaces, as under the pressures given by it no large building would be able to stand even against light breezes.

[57] See Hazen, p. 246, vol. 134, Am. Journal of Science, 1887; and R. R. and Engineering Journal, February, 1887.

[58] See p. 221, London Engineering for February 28, 1890.

large fixed gauge; that is, the large gauge showed pressures 31, 29, 34, and 60 per cent less than on the small fixed gauge, and also 43, 24, 23, and 58 per cent less than on the revolving plate gauge. The other eight of the severe gales came from directions making about 45° with the normal to the fixed plates, so that the normal pressures on the fixed plates were probably only about 90 per cent of the actual wind pressures, and should accordingly be increased by about one-ninth, or 11.1 per cent, in order to give the actually supposed force of the wind itself in the direction of its motion. The actual gauge records of these eight storms were as follows: On October 27 and 28, 1884, March 31, 1886, February 4, 1887, January 5, 1888, November 2, 1889, January 19 and 25, 1890, the pressures were 29, 26, 26, 26, 27, 27, 27, 27 pounds per square foot on the small revolving plate gauge; 23, 29, 31, 41, 16, 34, 28, 24 on the small fixed gauge; and only 18, 19, 19, 15, 7, 12, 16, 18 on the large fixed gauge; but adding one-ninth (as above stated necessary to correct for the inclination of the wind), the wind pressures become 25.5, 32.2, 34.4, 45.5, 17.8, 37.7, 31.1, 26.7 as deduced from the small fixed gauge, and 20, 21.1, 21.1, 16.7, 7.8, 13.3, 17.8, 20 as deduced from the large fixed gauge; that is to say, the pressures on the large gauge, if normal, would have been 21, 34, 38, 63, 56, 65, 43, 25 per cent less than on the small fixed gauge if also normal, and also 31, 19, 19, 35, 71, 49, 34, 25 per cent less than on the small revolving plate gauge. On all twelve observations together, the large plate thus gave an average of 41 per cent less readings than the small plate gauge and an average of 36 per cent less than the revolving plate gauge. On two days, March 31, 1886, and January 25, 1890, the large gauge was compared further with the two small gauges at its own center and upper corner, giving readings of 19 and 18 pounds on the large plate, 28.5 and 23.5 on the center gauge, and 22 and 22 on its corner, showing on the large plate average pressures of 26 per cent less than at its own center and 13 per cent less than at its own upper corner. Moreover, as will be seen, during all these six years the maximum pressure on the large plate was only 27 pounds per square foot (against 35 on the small revolving plate and 41 on the small fixed plate).

In addition to these comparisons on a large scale Mr. Baker also made small comparative tests with a wind blast against small models of open-work trusses and of lattices, and satisfied himself that an allowance of 1.8 of the exposed actual surface of the ironwork of the front girder, was sufficient to cover the total wind pressures on this and another similar girder in its rear.

As this bridge was proportioned to stand 56 pounds per square foot of actual surface on each girder, the results of the above-described six years subsequent wind measurements seem to show an unnecessarily large margin of safety.

On the strength of the above records Mr. Burr, of Columbia College, New York, feels justified[59] in saying that a 50-pound per square foot pressure over the entire length of a 500-foot-long bridge span will probably be as rare as a cyclone.

From 1884 to 1888 the records of the Bidston Observatory, near Liverpool, England, show ten severe storms, whose pressures were measured by a plate pressure anemometer of 2 square feet area, and velocities by the ordinary cup rotary anemometer.[60] These storms and their records arranged in order of their severity are as follows: On January 23, 1884, May 20, 1887, January 26, November 20, and May 3, 1888, March 30, 1886, October 26, 1884, December 9, 1886, February 3 and November 1, 1887, the pressure plate anemometers registered, respectively, 70.2, 65.2, 49.2, 49, 44.4, 41.9, 40.6, 40.4, 40.1, and 40 pounds per square foot, while the cup velocity anemometer registered 78, 78, 74, 71, 66, 62, 64, 69, 66, and 57 miles per hour. According to the Newtonian formula of $p = K\,0.0027\,v^2$, the values of K deduced from these records are, respectively, 4.3, 4, 3.4, 3.6, 3.8, 4.1, 3.7, 3.2, 3.4, and 4.6; giving an average of $K = 3.8$ or $p = 0.01\,v^2$ for the ratio of the registered pressures and velocities. But assuming that the small plate pressure records were 40 per cent in excess of the pressures against large surfaces (according to the Forth bridge tests as above given) and assuming that the cup velocity anemometer records were the actual velocities of the wind (having already been reduced according to recent investigation), the actual pressures of the wind would reduce to about 42, 39, 30, 30, 27, 25, 24, 24, 24, and 24 pounds per square foot; the actual velocities of the wind would remain 78, 78, 74, 71, 66, 62, 64, 69, 66, and 57 miles per hour; and the resulting values of K would reduce to 2.6, 2.4. 2, 2.2, 2.3, 2.4, 2.2, 1.9, 2, and 2.7, averaging $K = 2.3$ and $p = 0.0061\,v^2$ or $p = \frac{v^2}{164}$ for the actual pressure on large surfaces in terms of the velocities taken from recent records of the usual cup rotary anemometers. As these values of p and K are from 2.4 to 1.4 times as great as those deduced from experiments in which the pressures are far more accurately measured and the velocities are very uniform and measured with exceeding accuracy, it is evident that there is yet much to be investigated either as to actual wind velocities and pressures or the accuracy of the recording apparatus in ordinary use.

It is quite possible that the large plate pressure records will in turn have to be

[59] See p. 475, Burr, Stresses on bridge and roof trusses.
[60] See p. 333, London Engineering of March 14, 1890.

reduced 40 per cent more to allow for the effects of inertia under the action of severe gusts. Two successive 40 per cent reductions (64 per cent in all) of the small plate pressure records (making the average steady pressure on a large surface only 36 per cent of that of the maximum unreduced small plate pressure records) combined with a 10 per cent reduction of carefully rated velocity records (making the average steady velocity only 90 per cent of the maximum records of rotary cup anemometers whose coefficient is 2.3 or which are otherwise carefully rated) seems at present the least unreasonable method of reconciling the present apparent differences between the results of experiment with meteorological apparatus and those with other more perfected apparatus. (For the records of the Bidston Observatory from 1868 to 1879, see paragraph above under head of American Society of Civil Engineers, papers of 1880.)

In 1887 T. C. Fidler, in his book on Bridge Construction, [61] devotes one chapter to the subject of wind pressure [62] with special reference to its bearing on bridge construction. This chapter is perhaps the best single, short, concise, comprehensive, and practical review of the whole subject yet in print. From it are taken the following points, not as well brought out elsewhere above.

In bridges of large span the wind stresses become of great importance since they may be as great as those produced by the entire dead and live load. In the Forth bridge, where the allowance for wind stress was 56 pounds per square foot over the exposed surfaces of both girders, the maximum stresses were estimated by the engineer as 2.9 times that of the live load and 1.3 times that of the dead load.

In a bridge whose sides are solid girders, the horizontal wind pressure on the whole bridge will depend greatly on the connection of those girders. If the side girders are connected top and bottom by solid ceilings and floors so as to make the combination act like a cylindrical solid instead of like two thin plates, then the total pressure on the bridge will be less than on one single solid thin girder standing alone; if connected by only a solid floor without roof, then it is possible that the shelter of the floor may equalize its skin friction with results that the total pressure on the bridge will be only as much as on one solid girder alone; if connected by an open platform, then the total pressure on the bridge will be something between 1 and 2 times that of a single solid girder standing alone.

The wind pressure on the side of a railway car is only about 0.8 that on a similar thin surface if standing alone, and that of a house or monument touching the ground is probably still less; so that in computing wind pressures from the overturning of cars or monuments we must ordinarily increase the computed pressure by at least 25 per cent in order to get the pressure which would have been exerted on a thin pressure gauge surface of the same area. In the same way it is found that the pressure calculated from the breaking of ordinary window panes in the side of a closed house must be increased by 50 per cent in order to get the corresponding pressure on a thin plate alone or on the ordinary wind pressure gauge.

When the wind blows upon a grating or lattice girder, the pressure on the grating or girder is somewhere between that of the entire area inclosed by its perimeter and that of the actual front surface of the bars or plates that compose it; and the pressure on a second girder in rear of the first would usually be less than that on the first. Gaudard is quoted as expressing these pressures by the formulæ.

$$P=pS\left(1-k\frac{s}{S}\right);\quad P_1=Pk\frac{s}{S};\text{ and } P+P_1=pS\left[1-\left(k\frac{s}{S}\right)^2\right];$$

in which P is the total pressure on the front girder, P_1 is the total pressure on the rear girder, $P+P_1$ is the total pressure on both girders, p is the wind pressure per unit of surface, S is the total area inside the perimeter of the girder, s is the total combined areas of the openings in the girder, and k is a constant of contraction, assumed by Gaudard to vary between $k=0.65$ for small orifices and 1 for large openings. Fidler, however, suggests the need of practical tests of this formula, stating that while some such formula may be true for special girders it is very probable that the amount of shelter varies somewhat with the shape and arrangement of the lattice bars, and very certain that it varies very greatly with the distance between the girders.

In girders of wide open panels and comparatively narrow bars, any particular bar on the front of the windward girder will shelter almost completely another similar bar close to it (as when bars are close in pairs); will also slightly shelter it at from 1 to 2 diameters distance; but will not shelter it at all, appreciably, at 6 to 10 diameters distance. Mr. Baker, of the Forth bridge, found that if two similar disks were placed exactly in rear of each other but at distances apart of 1, 2, 3, and 4 diameters, the total pressure on the combination amounted respectively to 1, 1.4, 1.6, and 1.8 times the pressure on the front disk alone; and that this total pressure was but little, if any, increased by the insertion of intermediate disks. Experiments by Thibault,

[61] T. Clayton Fidler, Practical Treatise on Bridge Construction, 432 pages, London, 1887.
[62] See pp. 402-418, same.

however, show as much as 1.7 pressures at single-diameter distances; so that further experiment is needed in this direction.

Allowances should be made for the dynamic effect of sudden gusts, especially in high winds, as a gust which lasts as long as the time of oscillation which is thereby given to the bridge may produce the same stress in the bridge as that directly deducible from the extreme pressure given at the same time by a pressure-gauge anemometer. This stress may, moreover, be much further increased should the interval between gusts happen to be in rhythm with the time of oscillation of the bridge. In consulting the records of wind anemometers this dynamic effect must be borne in mind, as the records may show 1 to 2 (or even more) times the actual pressures and velocities, according as the wind is steady or very gusty. The general results of practice so far tend to show that in a squally wind the dynamometer or pressure-plate gauge may record from 40 to 75 pounds per square foot, while the actual pressure upon a thin plate might only be about 37 pounds, that against overturned carriages be only 30 pounds, and that against broken windows only about 25 pounds.

In most bridges the wind pressures are provided against by horizontal wind girders, lying in the plane of the bridge floor, assisted in some cases by another similar wind girder overhead. In most cases these wind girders are of uniform width, equal to that of the bridge, and are therefore calculated like any other truss or girder with parallel chords and lattice or openwork posts and diagonals. In special cases the wind girders may be constructed as entirely independent girders, so as to have no work or duty except the resistance of the wind pressures; but whenever, as in most cases, they are combined with the other parts of the bridge, any piece so subjected to both kinds of stress must be proportioned so as to resist the maximum stress that may result from the most unfavorable combinations of wind, dead load of bridge, and live load of moving vehicles.

As it is hardly possible for the wind pressure to be always uniform over the entire length of bridge, nor to be blowing with full force on one half and with no force at all on the other half, it is advisable, especially on long bridges, to consider one half the wind pressure on the girders as a dead load uniform over the whole bridge, and the other half of such pressure on the girders, as well as all the pressure on a moving train, as a rolling load.

As cars are liable to be blown over with pressures of from 26 to 34 pounds per square foot pressure, no train is therefore likely to enter a bridge under such wind pressures; still, as one may be caught upon it (as in that of the Tay bridge), it is advisable to consider such possibility.

The Board of Trade Commission, shortly after the Tay bridge disaster, recommended an allowance of 56 pounds per square foot on once to twice the actual front surface of the material in one girder, with a factor of safety of 2 against overturning and 4 against rupture; and the Forth bridge was built in accordance therewith.

However, although up to about 1880 the general best practice of English engineers was to provide against winds pressures of from 30 to 40 pounds per square foot, and of American engineers to provide against from 30 to 50 pounds, yet the actual cross sections of the wind bracing of many American bridges show that, even if these pressures were allowed on but one girder alone, the resulting stress would be very high and sometimes approach, if it did not exceed, the elastic limit of the materials of the bridge; while in many English bridges there has sometimes been no wind bracing at all, and it has at other times been totally insufficient to stand anything like pressures of 30 to 40 pounds, yet these bridges (and many other structures, such as houses, etc., of far less resistance) have stood for a great many years, and have proved conclusively that no such actual pressures as 30 to 50 pounds per square foot have been experienced in the localities where they have been situated.

In conclusion, attention is called to the fact that, even after all has been said as to wind pressures, it is really impossible at the present day by any general rule to estimate the wind stresses on a long-span bridge to within 100 per cent of their real value. In such cases the responsible engineer will generally be disposed to err on the safe side. But as this margin of error may make tens or hundreds of thousands of dollars difference in the cost of the bridge, and as on the one hand extra material not actually needed can only serve to unnecessarily burden and therefore weaken the bridge, and on the other hand a lack of sufficient material will give far more disastrous results, it is to-day more than ever specially desirable that further practical experiments and observations should be made to get facts by which to reduce this present large margin.

In 1888–'89 W. H. Dines, at Hersham, England, made some experiments[63] upon the resistance of plates to moving air, from which he deduced the formula $p=0.0035\, v^2$, in which p is in pounds per square foot, and v is in miles per hour, this being the Newtonian formula with $K=1.25$.

In 1889, at the Eiffel tower in Paris, measurements were made during several

[63] See p. 333, London Engineering, March 14, 1890; and p. 63, Langley, in Smithsonian Contributions, No. 801.

months[64] upon the relation between the velocities of wind at the top and bottom of the tower at stations 995 feet apart vertically, the lower station being taken 69 feet above the ground, so as to be clear of the adjacent buildings. In the design of this tower the wind pressure was allowed for by assuming a pressure of 61.4 pounds per square foot over the entire height, or 41 pounds at the base, increasing to 82 pounds at the top, according to which gave the highest resulting total pressure at each joint.

The observations of 1889 covered 12 days in June, 28 in July, 31 in August, and 30 in September, 101 days in all, during which time the wind averaged 4.9 miles per hour at the bottom and 15.7 miles at the top, while the top velocities exceeded 17 miles for 39 per cent of the whole time and over 22 miles for 21 per cent of the time. The ratio between the top and bottom velocities did not remain constant, as the velocities at each place varied, but was 5 for bottom velocities of about 3 miles, 4 for bottom velocities of about 4 miles, 3 for bottom velocities of about 5 miles, and 2 for bottom velocities of about 7 miles.

In 1890 some experiments were made on top of Mount Washington, New Hampshire, by C. F. Marvin, with surfaces of 4 square feet and 9 square feet area,[65] from which he deduced the formula $p=0.004\ v^2$, in which p is in pounds per square foot and v is in miles per hour. The unit pressures were found practically the same for both large and small surfaces.

In 1890 O. T. Crosby, late lieutenant, Engineer Corps, U. S. Army, and B. F. Dashiell made a series of experiments[66] with plates of from 1 to 2 square feet cross section moved with velocities of from 10 to 130 miles per hour (those of 30 miles being numerous), in circles of 5.5 feet radius in a room about 40 feet long, 13 to 19 feet wide, and 12 feet high. The apparatus was belted to and driven by a 90-horse-power steam engine, whose speed could be regulated to less than half a revolution. The velocity of the plates was computed in most cases from actual count (electrically or otherwise) of the revolutions of the whirling table. The plates revolved at distances of only from 2.5 to 5 inches above the platform of the whirling table, but at much greater distances from the sides of the room. The pressures were measured by the compression of springs in rear of the plates and continuous automatic records of the same. Great care was taken in first measuring the resistance of the apparatus itself, this resistance being afterwards deducted from that shown by the plates and apparatus together, in order to obtain the net resistance of the plates alone. The marked radial and rotary motion of the air in the room due to the rotation of the apparatus, though not carefully measured, was allowed for by adding 10 per cent to the recorded pressures. On account of the apparent large errors from friction and inertia at low velocities no accuracy is claimed for velocities of less than 30 miles per hour. The result of these experiments indicated that at high velocities the resistance to normal pressure of air was much less than called for by the Newtonian formula, and that at velocities from 30 to 130 miles per hour and for short solid bodies with flat heads of 1 to 2 feet square cross section the relation of pressure to velocity is expressed by the formula $p=0.144\ v$, or $p=\frac{v}{7}$, in which p is in pounds per square foot and v is in miles per hour. These resistances were found to be about 28 per cent less in the case of the addition of a wedge-shaped head whose height was equal to its base, about 30 per cent for a pyramidal head whose height was twice its base, about 45 per cent for a wedge head whose height was twice its base, and about 50 per cent for a parabolic wedge whose height was equal to its base.

These results, especially that the pressure varies with the first power of high velocities, being of so surprising a nature, were checked by further experiments with a small flat-headed car of 5.1 square feet cross section, driven by an electric motor around a track of about 2 miles circumference at a speed of about 50 miles per hour; the pressures being measured in much the same way as before by the use of compressed springs and automatic records. The total registered pressure on the entire 5.1 square foot front surface was less than 40 pounds; that is, less than 8 pounds per square foot.

In connection with his report on these experiments Lieut. Crosby also reviews the work of his predecessors in this line of research. His review, interesting in itself, shows that his own experiments were entered into by him understandingly, and his results are therefore entitled to all the more consideration. Attention is called to the almost unanimous opinion of all investigators up to date, that the practical law of pressure-velocity variation between any two definite limits of speed must be deduced from actual and carefully made experiments between these same limits, and that the relations deduced from experiments with the lower velocities can not safely be rigorously applied to higher velocities. As to the effect of the size of plate upon the pressure per unit of surface, Crosby infers the trend of authority toward an increase of this pressure as the surface increases in the case of moving

[64] See p. 12121, Sc. Am. Supplement for July 19, 1890.
[65] See Johnson, Modern Framed Structures, p. 33; also Engineering News of December 13, 1890.
[66] See pp. 663, 664, 689, 690, and 715-718, London Engineering, May 30, June 6, and June 13, 1890.

plates; and perhaps the reverse in the case of stationary plates. As to the pressure on inclined plates, he tabulates the results of a dozen formulæ obtained by other investigators and gives as the result of his own experiments a value of $p = \frac{\sin a + \sin^2 a}{2}$ for the horizontal pressure per square foot of plate, and $p = \frac{1 + \sin a}{2}$ for the horizontal pressure per square foot of the vertical base of the plate; both formulæ being for plates moved horizontally, a being the angle between the surface of the plate and the direction of the motion. He calls attention further to the fact that as his moving plate was quite close to the stationary platform of his apparatus, his recorded resistances may have been increased (but at any rate not diminished) thereby.

In view of the radical differences between his normal-pressure results and those of prior investigators, and in view of the evident errors of formulæ for low velocities when applied to high ones, he urges the importance of further careful and extended experiments with velocities as high as (or higher than) those used by him.

In 1888 and 1890 S. P. Langley, of the Smithsonian Institution, made a set of very interesting experiments at the Allegheny (Pa.) Observatory,[67] with plates of from 0.25 to 1 square foot area, with lengths of from 0.15 to 6.2 times their breadth, moved at velocities of from 10 to 70 miles per hour in circles of 30 feet radius in the open air and at distances of at least 8 feet above the ground, the whirling table being driven by a 10-horse power steam engine under 90 pounds of steam, steadied by a fly wheel making about 120 revolutions per minute; the velocities of the plates being measured by an electric chronograph recording every quarter revolution of the whirling table, and the pressures being recorded automatically by the compression of carefully calibrated springs connected to the plate holder. Friction wheels were introduced wherever it was desirable to allow motion, so as to reduce the friction as much as possible, and many other methods were adopted to minimize the amount of unnecessary instrumental friction and oscillation. With respect to the pressures on thin plates moved through air, these experiments give results as follows: For plates held normally to their line of motion and at temperatures of 50° F. and barometric pressures of 736 mm. $p = .0036\, v^2$ or $p = \frac{v^2}{280}$,[68] in which p is in pounds per square foot and v is in miles per hour. If reduced to freezing temperatures and the normal of 760 mm. (as in previously described formulæ) this becomes $p = .0039\, v^2$ or $p = \frac{v^2}{260}$; being the Newtonian formula, in which $K = 1.44$. This result is deduced from the records of sixty-eight different observations[69] whose velocities were well scattered between 10 and 70 miles per hour, and whose deduced pressures rarely differ by as much as 10 per cent from that of the average as given by the above formula. As to the effect of varying size of plates, the 6-inch square plates gave results slightly more than the 8-inch square plates, but slightly less than the 12-inch square plates, so that the result was not indicative of any special law governing such variation. As to the pressure on inclined thin moving plates, it was clearly shown that the resultant pressure on the inclined plate is normal to the inclined surface,[70] and hence that the effects of skin friction, viscosity, and the like, are practically negligible in such experiments; that this normal pressure upon the inclined surface is much greater (even twenty times in the case of angles as small as 5°) than that deducible from Newton's original discussion, and agrees very closely[71] (within 0.02) with that of Duchemin's formula of $\frac{2 \sin a}{1 + \sin^2 a}$, in which a is the angle between the plate and the line of motion; that the center of this resultant and normal pressure is not exerted at the center of the inclined plate, but always a little forward of the same toward the advanced edge of the plate,[72] so that the forward half of the plate always carried more than half this total pressure, the amount of these variations depending on the angle of inclination of the plate (as also shown by Joessel in 1870 and Kummer in 1875–'76); that this pressure, being a fluid pressure and increasing rapidly with the velocity, has in the case of horizontal winds at high velocities under small angles of inclination a lifting power far greater[73] than it is ordinarily given credit for; and also that inclined rectangular planes whose surfaces are inclined from 0° to 30° to the line of motion[74] may experience much greater or much less pressures than square plates according, respectively, as their longer or shorter side is

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[67] Langley, Experiments in Aerodynamics, 115 pages, 10 plates, 1891; published as Smithsonian Contributions No. 801.
[68] See p. 99, same.
[69] See p. 98, same.
[70] See p. 104, same.
[71] See p. 24, same.
[72] See pp. 89 and 114, same.
[73] See pp. 25 and 105, same.
[74] See pp. 61, 62, same.

in advance, while those whose surfaces are inclined from 30° to 90° may experience exactly the reverse effects, thus showing that not only the shape but the orientation of the inclined surfaces is of great importance in determining the resistance offered by them to their own motion or to that of the fluid in which they are moved. The results of these experiments, both on account of the great radius (30 feet) of the whirling table, the great care taken to eliminate the difficulties and errors of prior experimenters, and the high velocities actually obtained, must necessarily be allowed great weight in comparison with those of other like prior experiments, and seem to be sufficient proof of the theory that air pressures on moving plates under velocities up to about 70 miles per hour are (to within at least 10 per cent) proportional to the square of the velocity; leaving the question of the discordant results obtained from ordinary wind-pressure and velocity gauges and the question of the pressure of moving air upon stationary plates, to be further investigated, if necessary, by methods in which the motion shall be that of the air and not that of the plate.

On April 29, 1892, a cyclone crossing the Champ de Mars, Port Louis, Mauritius, struck an obelisk of 50 feet height and 5 feet square cross section (called the Tombeau Malartic), tearing off and throwing down its upper half.[75] The portion thus thrown down was 26 feet high and 5 feet 3 inches square at its surface of rupture, and was built of hard blue basalt facing, backed with rubble bedded in mortar. The appearance of the fracture indicated that the upper portion did not slide off from the lower, but was lifted clear from it. Calculations from the exposed surface and weight of the fallen portion showed that the mere overturning of the upper half of the obelisk would require a force of 142 pounds per square foot of its exposed surface. The director of the observatory at that place reported the maximum velocities of his records as 112 miles per hour for the greatest hourly record and 123 miles per hour for the greatest velocity for a few seconds. If the velocity be taken at 112 miles per hour and the pressure at 142 pounds per square foot, the ratio of the two would be expressed by $p=\frac{v^2}{88.}$

In 1887 and 1893 S. P. Langley, of the Smithsonian Institution, made a series of experiments,[76] first at the Allegheny (Pa.) Observatory and then at the Smithsonian Institution, Washington, D. C., to determine the internal irregularities and variations of ordinary winds. At Allegheny he used an ordinary rotary Robinson cup anemometer, with metal cups of about 3 inches diameter, moving in circles of 6.75 inches radius, recording every twenty-fifth revolution, placed on a mast, so as to be 32 feet above the top of the hill and 453 feet above the adjacent Ohio River Valley. The observations at Washington were made with three different-sized anemometers; the first being about the same as that used at Allegheny, but recording every fifth revolution; the second being made with paper cups of only 0.3 the weight of the first one and recording every revolution; and the third being made of half size, and with paper cones weighing only 0.2 of the first and recording every half revolution; these anemometers being mounted 153 feet above the ground (11 feet above the tower parapet). The third anemometer, whose moment of inertia was thus less than 0.01 of that of the first one, would start and stop almost instantaneously with the wind. These anemometers showed that even in ordinary winds (and more so in high winds) the air moves in a tumultuous mass, the velocity at a single fixed point sometimes jumping almost instantaneously from one extreme to the other. The general characteristics of these variations is well shown by the record of the six minutes from ten to sixteen minutes past noon on February 4, 1893, at Washington, when an ordinary high wind of from 20 to 27 miles an hour was blowing. The record of the ordinary anemometer showed a velocity of about 23 miles per hour at the beginning of this interval, dropping to about 20 miles at the end of the first mile, rising to about 27 miles at the end of the second mile, and continuing at this rate for sometime afterwards. The record of the light anemometer, registering each revolution, shows however, starting at 23 miles, a rise to 33 miles in ten seconds, then a fall to 23 miles in ten seconds; then up to 36 miles in thirty seconds, and so on, passing through eighteen notable maxima and as many minima in these six minutes, the average time of each rise or each fall being a little over ten seconds and the average change in velocity being about 10 miles an hour. The most rapid change in this interval was a drop from 24 miles down to nothing and a rise back to 22 miles, all in less than fifteen seconds. In like manner the observations of July 16, 1887, with an average velocity of 18 miles, showed a rise and fall of 33 per cent each way from the mean, occurring within two minutes of each other; those of January 14, 1893, with 13 miles average velocity, 40 per cent oscillation within 0.5 minutes; those of February 20, 1893, with 27 miles average velocity, 75 per cent in 0.3 minutes, and holding a 60 per cent increase for nearly a whole minute; while during all

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[75] See Engineering Record for July 23, 1892.

[76] See Langley, Internal Work of the Wind, 1893, published as Smithsonian Contributions No. 884.

these observations the average oscillation was about 20 per cent up or down; and occurred once or twice a minute. Under such circumstances, because of their weight and great inertia, the ordinary rotary anemometers, while failing to show the maximum velocities, would record average velocities considerably in excess of the truth; while the ordinary plate anemometers would record instantaneous pressures far in excess of the truth.

In 1893 Mr. Kernot, of the Melbourne University, Australia, made a series of experiments [77] with a steady jet of air of 12 by 10 inches cross section, directed upon small models. The blast was furnished by a screw propellor of 28 inches diameter and 48 inches pitch, driven by a gas engine at speeds of from 400 to 800 revolutions per minute. The helical currents of air from this propeller were collected by means of a radial diaphragm and a conical mouthpiece having its axis tangential to the helical direction of the air, so as to secure a steady jet. In front of this jet was placed the object to be tested, supported on a very delicately arranged carriage running on an accurately leveled surface plate; the force exerted being measured by a delicate spring balance, the accuracy of which had been verified by means of standard weights. A large number of experiments were made with this apparatus. From the small size of the models and the necessarily delicate arrangement of the apparatus, results could not be expected sufficiently accurate to be used in the revision of the general pressure and velocity formulæ, but very interesting comparisons could be made. Comparing various surfaces, exposing to the blasts equal areas and assuming the pressure to be 1 on a square unit of thin plate normal to the blast, it was found that the pressure was 0.90 on the side of a cube normal to the blast, 0.90 also on the cube turned edgewise, 0.80 on the side of a square pyramid whose axis was normal to the blast, 0.96 on the same pyramid turned edgewise, 0.60 on octagonal prisms, 0.50 on cylinders and cones, 0.36 on spheres, 0.36 one way and 1.15 the other on spherical cups, and 0.80 on lattice work whose openings were 55 per cent of the whole exposed area. The pressure on cylinders and spheres could be increased 20 per cent by bringing near them a plain surface parallel to the wind, the lateral escape of the wind around the cylinder being thereby cut off. In the case of a hollow building turned with open side toward the wind the pressure to lift the roof and force out the sides was equal to the pressures on the exposed end. The variations of pressure caused by the proximity of other surfaces was very marked; each surface appeared to affect the pressure on other surfaces to a distance in front equal to once, and in rear equal to several times, its own breadth. The pressure on a 6-inch disk was reduced 66 per cent by the presence of a 9-inch disk placed 2 inches in its rear, and this reduction of pressure diminished as the 9-inch plate was moved back, disappearing only when the rear disk was about 9 inches (its own diameter) in rear of the 6-inch disk. The pressure on a 7-inch disk was diminished 120 per cent (that it was urged forward against the direction of the wind with 20 per cent pressure) by the interposition of a 9-inch disk placed 4 inches to its front. A parapet of 0.16 of the height of a roof, reduced the pressure on the roof by 33 per cent. A wall connecting the edge of the roof with the ground in its front reduced the pressure on the roof to very much less than it was when the wind could blow freely under the roof. Mr. Kernot concludes his paper by a recommendation that 20 pounds per square foot be regarded as sufficient allowance for wind pressures in positions of full exposure in Australia whenever the exposed surface measures over 300 square feet, and that 30 pounds be allowed only upon smaller surfaces of exposure; and recommends, further, that the factors of safety be taken as 2 against overturning and 3 against rupture.

RECENT PRACTICE.

The practice of to-day in England is presumably in accord with the recommendation given under the heads of the Tay bridge commission and London engineering articles (1880–'81) and Fidler (1887); which is, to estimate the maximum wind pressure at each place by the formula $p = 0.01v^2$, in which v is taken from the rotary cup anemometer records; and in the most exposed places to allow for 56 pounds per square foot wind pressure over from 1 to 2 times the effective area of one bridge girder.

The practice of to-day in France is presumably in accord with the rules laid down by the text-books of the corps of bridges and highways [78] which estimate the wind velocities of storms at 100 miles per hour, and which allow for pressure of 60 pounds per square foot over the effective area of 1 truss of a solid truss bridge, or of 1.5 trusses of an open-work truss bridge.

These recent English and French allowances seem more than necessary in long bridges, where the cost of the wind bracing is a question of hundreds of thousands of

[77] For abstract of paper, see p. 176, vol. 29, Engineering Record for February 10, 1894.
[78] See p. 799, E. Collignon, Cours de Mechanique, Resistance des Materiaux, 3d ed., 1885.

dollars; especially in view of the fact that so many bridges with so much less wind bracing have stood so well for years, and that one (*see* Shaler Smith, 1880, above) dimensioned for only 30 pounds pressure stood safely even when one of its spans was actually crossed by the very center of a tornado.

The American practice, as it stands in 1894, may best be gauged by the rules laid down in the last editions of the text-books[79] of Johnson, Bryan, and Turneaure, of A. J. Du Bois, and of W. H. Burr.

In these works the relation of pressures to velocities is variously expressed by the formula[80] $p=0.0040v^2$, $p=0.0054v^2$, and $p=0.0100v^2$, respectively, in which p is in pounds per square foot and v is in miles per hour, being the Newtonian formulæ with $K=$ 1.5, 1.9, and 3.7. However, they all agree in the ordinary allowances of 30 pounds per square foot for wind pressures on large surfaces,[81] while increasing this to 45 (Johnson) or 50 (Du Bois) or 40 (Burr) pounds for small surfaces and unloaded bridges. The ratio of the normal pressure upon inclined surfaces to the normal pressure on normal surfaces is given by both Johnson and Du Bois[82] as $\sin a^{1.84 \cos a - 1}$ (quoted from Unwin, 1869, as ascribed by him to Hutton), in which a is the angle of the surface with the wind; this giving 1 for angles of from 90° to 60°, 0.95 for 50°, 0.88 for 45°, 0.83 for 40.°, 0.66 for 30°, 0.45 for 20°, 0.24 for 10°, and 0.13 for 5° (these results for angles less than 50° being all considerably smaller than those of Duchemin's and Langley's formulæ).

The rules for the application of these wind pressures to the calculation of large railroad bridges are mainly as follows:[83] The exposed area of spans is figured—first, of the unloaded bridge, the wind pressure being treated as a dead load; second, of the loaded bridge, the wind pressure on the train being considered as a live load; and then each portion of the wind bracing is dimensioned to stand the maximum strain that may come upon it under either condition of load. The exposed area of a girder is measured by taking the front surface of each part (such as the ordinary upper chords and posts) which stands by itself, 1.5 times the front surfaces of those parts (as ordinary ties) which are in pairs, and 2 times the front surface of those parts (as ordinary lower chords) which are composed of sev ral bars, one behind the other.[84] The wind pressure on the unloaded bridge is calculated at 30 to 50 pounds per square foot on the exposed area of the floor system and of either both open girders or one closed or plate girder.[85] The train is treated[86] as a continuous surface of 10 feet height with its bottom 2.5 feet above the rails. The exposed area of the loaded bridge[87] is measured by adding together the exposed surfaces of all the windward girder of the floor system, of the train, and of all the leeward girder (or at least of so much of it as is not closely and completely sheltered by the train). The wind bracing is usually composed of one horizontal truss under the floor and one between the top chords of all through bridges, or between the bottom chords of all deck bridges; and of vertical sway bracing at every panel point of high through bridges or at the panel points of all deck bridges. The dead load of the wind pressure is assumed ordinarily as divided equally between the upper and lower wind trusses, where both exist; and the live load is assumed ordinarily as all carried by the wind truss of the loaded chord (the lower in a through bridge, the upper in a deck bridge). In ordinary long railroad bridges these wind loads are ordinarily assumed[88] at 600 pounds (300 dead, 300 live) per linear foot of bridge, of which three-fourths (as above explained) goes to the wind truss next the flooring and one-fourth to the other wind truss. In calculating the stresses on each wind truss, the loads are assumed as exerted on the windward side. The chords and end posts of the main trusses are not usually stiffened or increased in size on account of the assumed wind stresses, except, first, when such stress alone, or in combination with a temperature strain, may change the strain from tension to compression in a chord built to stand only tension; and second, when the wind stress on any member exceeds 25 per cent of the sum of the maximum stress due to the dead load added to that due to the live load.[89]

Members subject to alternate tensile and compression stresses are built to resist each and proportioned so as to stand each stress alone, increased by an amount

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[79] Johnson, Bryan, and Turneaure, Theory and Practice of Modern Framed Structures, 527 pp., N. Y., 1893; A. J. Du Bois, Strains on Framed Structures, 540 pp., N. Y., 9th ed., 1893; W. H. Burr, Stresses in Bridge and Roof Trusses, 475 pp., N. Y., 8th ed., 1893.

[80] See p. 33, Johnson; p. 66, Du Bois; pp. 370 and 475, Burr.

[81] See pp. 33, 34, Johnson; pp. 65-405, Du Bois; p. 370, Burr.

[82] See p. 34, Johnson; p. 65, Du Bois.

[83] See pp. 109-118, Johnson; pp. 405-414, Du Bois; pp. 366-394, Burr. (Johnson, in his work, quotes at length the bridge specifications of F. H. Lewis, while Du Bois, in like manner, quotes those of Theodore Cooper.)

[84] See p. 405, Du Bois.

[85] See p. 109, Johnson; p. 406, Du Bois.

[86] See p. 393, Johnson.

[87] See p. 110, Johnson; p. 405, Du Bois; p. 370, Burr.

[88] See p. 110, Johnson; p. 405, Burr.

[89] See p. 472, Du Bois.

equal to 0.8 of the lesser.[89] Where the wind acts the bridge is necessarily bent somewhat to the leeward side, and the tension chord of that side of the bridge must be made strong enough to stand this extra stress. Under the same circumstances the tension chord of the windward side is compressed, sometimes in excess of the tension due to the dead load, and therefore precautions must be taken to prevent its buckling under such excess of compression. As the wind may blow in either direction, both directions of wind must be considered.

The sway bracing which is placed between the vertical posts of the main trusses at each panel point is introduced in order to prevent independent lateral vibration and swaying of the vertical trusses; also to stiffen the long vertical posts, as well as to assist in carrying some of the wind stresses to the leeward girder. In double-track railway bridges the sway bracing also helps to prevent lateral distortion of the cross section of the bridge under a load on a single track; but the extra stress due to such work is usually comparatively slight. The main stress on the sway bracing comes usually when the wind is blowing on the unloaded bridge, in which case the web members of the lateral system, belonging to the chord supporting the floor, are only about one-third loaded, while the web members of the other lateral system are fully loaded, giving rise naturally to unequal lateral deflections and the transfer of some stress from the weaker system through the sway bracing to the stiffer system. The parts of the sway bracing are therefore usually dimensioned so as to be able to carry 0.5 the wind load due to each panel of the bridge.[90]

On ordinary heavy railway bridges the exposed areas per linear foot may be roughly estimated at 10 square feet for the train, 1 square foot for the ends of the ties and sides of the guard rails, 4 square feet for the longitudinal floor girders (or stringers), and 5 square feet per linear foot for each truss (or girder), or a total of 10 to 14 square feet for the dead wind load on the two trusses and floor, or 2.5 to 3.5 square feet for each chord.[91]

In ordinary double-track railroad bridges, with vertical sway bracing, the weight of this bracing[92] for both trusses may be roughly estimated at $\left(\frac{6\,Nl}{170} + \frac{1136}{p}\right)\frac{b}{15}$ in pounds per linear foot of track, in which l is length in feet, N is number of panels, p is panel length in feet, and b is width of bridge in feet.

When bridges are built on a curve they must also be stiffened laterally against the centrifugal force of the moving train.[93] This extra stress is estimated at $0.0000117\ v\ DP$, in which v is the velocity in miles per hour, P is the weight of the train in tons, D is the degree of the curve or the angle subtended at the center by a chord of 100 feet of track. For a speed of 30 miles per hour this amounts to, approximately, 0.01 of the weight of the train for each degree of curvature; and each additional 10 miles of speed is assumed to add as much more. This centrifugal force is assumed to act at a level of 5 feet above the rails. This stress is carried by the floor beams to the posts of the outer trusses and thence through the lateral trusses to the rest of the bridge. As one of the lateral systems usually lies close to the flooring, most of the stress comes upon such system and its amount is usually added to that of the wind stresses and treated as if it were an extra and live wind load on one side of the bridge.

All the rods of the wind trusses must be further strengthened to cover the initial tension (ordinarily given them by turnbuckles in bringing all pieces to their proper bearings); and this initial tension is usually estimated at 1 ton for 1 inch diameter round rods plus 0.25 tons for each additional 0.125 inch of diameter, or 1.125 tons for 1 inch square rods plus 0.281 ton for each additional 0.125 inch of side, or equal allowances for equal areas of other shapes.[94]

In the calculation of wind stresses on piers or towers the same rules apply, with slight modifications, as follows: The wind pressure on the whole pier[95] is calculated at that of the front surfaces of the train, and of all floor systems of the pier, added to twice that of the trussing of the front face of the tower, assuming the wind pressure on the pier and train together at 30 pounds[96] per square foot, and on the unloaded pier at 50 pounds[97] per square foot, and providing against the most unfavorable of the two cases. In figuring the weight of the loaded pier the train weight is to be taken at that of the lightest train that would not be blown over by a 30-pound pressure wind. The pier must then be given base enough to prevent its overturning and also enough to prevent any tensile stresses in its posts.[98]

On the open iron work piers of the average railroad bridge, the exposed area of each side truss of the pier is roughly estimated at 4.5 square feet[99] per linear foot of height. The inclined and horizontal bracing of piers and trestles is to be made

[90] See pp. 114–116, Johnson; p. 409, DuBois.
[91] See pp. 406, 414, Du Bois.
[92] See p. 418, Du Bois.
[93] See pp. 117, 393, Johnson; p. 407 Du Bois.
[94] See p. 351, Du Bois.
[95] See pp. 391–399, Johnson; pp. 414–423, Du Bois.
[96] See p. 414, Du Bois.
[97] See p. 393, Johnson.
[98] See p. 416, Du Bois.
[99] See p. 415, Du Bois; p. 399, Johnson.

strong enough to resist the stresses of all wind and centrifugal forces and of the resistances to horizontal sliding on the foundations, while the columns are to be made strong enough to carry the vertical components of the stresses due to wind and centrifugal forces, as well as the loads of the train, track, girders, and pier itself.

Ordinarily no allowances are made for any longitudinal wind stresses on the bridge. The piers, however, need and receive longitudinal bracing to resist stresses resulting from a longitudinal force of about 0.20[100] the dead load of the train (or 800 pounds per linear foot), due to a possible sudden application of the train brakes, and this allowance is also expected to cover any longitudinal wind stresses; but usually the columns of the towers are not given any extra cross section or strength to carry these infrequent stresses, since such stresses will probably not occur during high wind or during other maximum loads.[101]

Where the end of the bridge slides on a bedplate during extension of the bridge under temperature strains, the stress of sliding friction is usually taken at 0.25 of the dead load on the bedplate;[102] and special arrangements may sometimes have to be made to take special care of such stresses.

In ordinary wrought-iron railroad bridges (in which the wrought iron has an ultimate strength of 50,000 pounds per square inch, with a stretch of 12.5 to 18 per cent and an elastic limit of 26,000 pounds) the stresses[103] allowed per unit of structure are from 5,000 to 6,000 pounds per square inch of net area on tension floor-beam hangers and other members liable to irregular and sudden strain; 8,000 pounds on floor beams, stringers, and plate girders; 15,000 pounds on tension pieces of wind and other lateral bracing; 9,000 to 10,000 pounds (reduced for length) on compression posts of lateral and other wind bracing; 7,500 to 8,000 pounds for other live loads and 15,000 to 16,000 pounds for dead loads, on tension chords; 7,500 to 8,000 pounds for live loads and 15,000 to 16,000 pounds for dead loads (reduced for length), on compression chords and posts of the main trusses; 4,000 pounds for shearing of web plates; and 7,500 pounds for shearing, 12,000 pounds for crushing, and 15,000 pounds for bending, of rivets and pins. The reductions for length, to be deducted from the above limiting stresses in compression pieces, vary from $30\frac{l}{r}$ to $40\frac{l}{r}$ for ordinary live loads, from $50\frac{l}{r}$ to $60\frac{l}{r}$ for wind strains, and from $60\frac{l}{r}$ to $80\frac{l}{r}$ for ordinary dead loads, in which l is the length of the compressed piece in inches and r is the least radius of gyration of its section in inches. No compression member is allowed to have a length greater than 45 times its least diameter or width. Girders are given depths of from one-tenth to one-twelfth their span.

Soft steel may ordinarily be considered as from 10 to 15 per cent stronger than wrought iron, and medium steel as from 20 to 22 per cent stronger than wrought iron.[104]

Although the above represents fairly the average practice of to-day in the dimensioning of bracing, there is a rapidly increasing tendency[105] amongst bridge engineers towards determining the dimensions of tension and compression members of all trusses, including those of wind bracing, by the use of new formulæ, which take into consideration for each piece its ultimate breaking strength (b) under a single applied tensile stress, its limit of elasticity (e) beyond which a single tensile stress may produce a permanent set, its safe limit for simple repeated srains (p) of either tension or compression alone, its safe limit for repeated reversal of strains (v) from tension to compression, or *vice versa*, and its maximum (m) and minimum (n) strains in case of simple repetitions of compression alone or tension alone, or its maximum (m) of the kind of strain which is the greater and the maximum (n) of the other kind which is the lesser, in case of repeated reversals of strains. The repetition limit has been found by experiment to be a little less than the limit of elasticity, and at the same time to vary between one-half and two-thirds of the breaking strength of the metal; and the reversal limit (called vibration resistance by Wöhler) has been found in the same way to vary between one-half and two-thirds of the repetition limit. In the case of an ordinary wrought iron whose ultimate strength, or ordinary breaking limit, is 50,000 pounds per square inch, the elastic limit may be about 60 per cent or 30,000 pounds; the repetition limit about 52 per cent or 26,000 pounds, and the reversal limit about 32 per cent or 16,000 pounds.

From the results of the experiments of Wöhler and others (as published in 1870 and following years), Prof. Launhardt has deduced formulæ of the form of $s=\frac{1}{c}p\left(1+\frac{b-p}{p}\cdot\frac{n}{m}\right)$ for repeated stresses of either tension or compression alone; and Prof. Weyrauch has deduced formulæ of the form of $s=\frac{1}{c}p\left(1-\frac{p-v}{p}\cdot\frac{n}{m}\right)$

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[100] See p. 404, Johnson.
[101] See p. 404, Johnson.
[102] See p. 397, Johnson.
[103] See p. 395, Burr; pp. 342 and 471, Du Bois.
[104] See p. 488, Johnson.
[105] See pp. 242–245, 322, 488, Johnson; pp. 332–340, Du Bois; pp. 238–259, Fidler.

for repeated stresses of alternate tension and compression; in which s is the safe load, c is a constant of safety taken usually at about 3.5, and the other quantities are as above described; all values being in the same unit, usually pounds per square inch.

If $p=\frac{b}{2}$ and $v=\frac{p}{2}$, as in ordinary grades of wrought iron, then these formulæ reduce to

$$s=\frac{1}{c}\cdot\frac{b}{2}\left(1+\frac{n}{m}\right)$$ for repeated stresses or either tension or compression alone,

$$s=\frac{1}{c}\cdot\frac{b}{2}\left(1-\frac{1}{2}\cdot\frac{n}{m}\right)$$ for repeated stresses of alternate tension and compression,

and

$$s=\frac{1}{c}\cdot b$$ for unvarying dead-load stresses,

$$s=\frac{1}{c}\cdot\frac{1}{2}\cdot b$$ for repeated stresses of equal intensity, if unreversed, and

$$s=\frac{1}{c}\cdot\frac{b}{4}$$ for repeated reversed stresses of equal intensity.

If $p=\frac{2}{3}\cdot b$ and $v=\frac{2}{3}\cdot p$, as in some stronger grades of wrought iron, then these formulæ reduce to

$$s=\frac{1}{c}\cdot\frac{2}{3}\cdot b\left(1+\frac{1}{2}\cdot\frac{n}{m}\right)$$ for simple repetitions,

$$s=\frac{1}{c}\cdot\frac{2}{3}\cdot b\left(1-\frac{1}{3}\cdot\frac{n}{m}\right)$$ for repeated reversals, and

$$s=\frac{1}{c}\cdot b$$ for unvarying dead-load stresses as before,

$$s=\frac{1}{c}\cdot\frac{2}{3}\cdot b$$ for repeated stresses of equal intensity, if unreversed,

$$s=\frac{1}{c}\cdot\frac{4}{9}\cdot b$$ for repeated reversed stresses of equal intensity.

In the case of long struts, the safe load must of course be further reduced by the usual formulæ applicable to such cases; either Rankine's or Gordon's, or that given in the preceding paragraph. The use of these new formulæ is therefore equivalent to that of the old formulæ modified by a variable coefficient of safety such as, in the most disadvantageous combinations of live loads, to allow on ordinary metal only one-fourth and on the best metal only four-ninths of the stress that would be allowed upon it, if the stress were unvariable. Such formulæ seem to be far better than those of the older method for the safe and advantageous dimensioning of the different parts of trusses in ordinary work. The Pennsylvania Railroad formulæ for bridge trusses may be deduced from the above by making $p=\frac{b}{2}$, $v=\frac{p}{2}$, and $c=3.5$; which allows no metal, even under quiet dead loads, to receive a stress of over two-sevenths of its ultimate strength, and which allows metal under the worst combination of live loads (alternate and equal, tension and compression) to receive a stress of only one-fourteenth of its ultimate strength. The most serious objection that can be made to these new formulæ of Launhardt and Weyrauch is that they are deduced under the supposition that metal by repeated strain within the elastic limit is fatigued in the same general way as when strained beyond such limit, a supposition that still awaits decided proof.

Fidler,[105] however, suggests that the results of the Wöhler experiments may be thoroughly accounted for by the well-known laws of dynamic action, in accordance with which a load instantaneously applied will produce stresses twice as great as will the same load if quiet and constant, and any instantaneously applied increment of load will cause an increment of stress equal to twice that due to an equal increment of quiet and constant load. In accordance with this point of view (whose justness is very fully proved by him) Fidler has deduced formulæ which he calls "dynamic" formulæ, and which (using the same nomenclature as in the preceding paragraph) give:

m = the stress produced by m acting quietly and constantly,

$2m$ = the stress produced by m acting instantaneously,

$2(m-n)$ = the stress produced by $(m-n)$ acting instantaneously,

$n+2(m-n)=m+(m-n)$ = the maximum stress resulting from instantaneously increasing the stress from n up to m, and

$n+(1+a)(m-n)=m+a(m-n)$ = the general expression for maximum stress in all cases; a being a factor introduced to allow for dynamic action of the force $(m-n)$, equal to 1 when the application is instantaneous, to some intermediate value between 1 and 0 in case of gradual application of the force, and to 0 only when the force is applied so gradually as to be considered quiet and constant. This "dynamic" formula arranged for comparison with the Launhardt and Weyrauch formulæ, therefore becomes

$s=\frac{1}{c}.b.\frac{m}{m+a(m\mp n)}$ for all cases of varying stresses, the minus sign of n being used for unreversed and the plus sign for reversed stresses; thus giving,

$s=\frac{1}{c}.b$ for unvarying dead load stresses.

$s=\frac{1}{c}.b.\frac{1}{2}$ for repeated stresses of equal intensity, if unreversed, and

$$s=\frac{1}{c}.b.\frac{1}{3}$$

for repeated reversed stresses of equal intensity; all of which are in perfect harmony with the results of the Wöhler experiments. For ordinary use in bridge construction, the new "dynamic" formula takes the form of

$$A=\frac{m+a\ (m\mp n)}{\frac{1}{c}.b}$$

in which A is the desired area for safe loading of the member under consideration, and the other quantities are as before given; the minus sign of n being used as before for unreversed and its plus sign for reversed stresses, and a being taken at 1 for floor beams (both stringers and cross beams), floor-beam hangers, web bracing, and all other pieces subject to very irregularly and suddenly applied strains, being taken at intermediate values of from 1 to 0.5 for main girders of spans of from 20 to 100 feet length, and at 0.5 for main girders of ordinary spans of more than 100 feet length. This new "dynamic" formula recommends itself by its thorough rationality, its comparative simplicity, its easy adaptability to the varying conditions of actual practice, and its division of the old single coefficient of safety into two distinct parts, of which one, c, depends merely on conditions of manufacture, inspection, and accident, and the other, a, depends on the more easily foreseen conditions of application of its definite rolling or live loads. It now seems quite probable that this new "dynamic" formula will before many years be used in preference to all the others.

Whenever either the new Launhardt and Weyrauch formulæ or the new Fidle "dynamic" formulæ are carefully applied, it would however seem safe to diminish the coefficient of safety from 3.5 to nearly 2, so as to utilize the strength of the metal up to nearly its elastic limit, in the case of unchangeable loads, taking care, of course, to see that the maximum allowed load $\left(\frac{b}{c}\right)$ never exceeds the minimum elastic limit (e) of such metal as might be expected to actually get into the finished structure, due allowance being made for the ordinary unevenness of manufactured products, the ordinary differences between the results of tests upon small pieces and those upon full-sized members, and the ordinary failures of attempted careful inspection.

Wind trusses are, of course, subject to the same variations of temperature as the rest of the bridge, and must be arranged with due regard to such changes. The usual allowance for elongation of iron is 1 inch per 100 feet under 150 degrees temperature change.[106]

In analyzing the wind stresses in the laterals, it is well to consider the various wind loads as being transmitted through intermediate parts to the points of final support by such routes as require the least internal work, that is, routes such that the internal work done in producing strain will be minimum.[107] In some cases the weakness or accidental loosening of some pieces of the main or lateral girders may cause the stresses to be transmitted by unexpected routes, and to throw unusual and excessive strains on other parts; and such opportunities must be considered and provided against if possible.

INFERENCES TO BE DRAWN FROM PAST EXPERIENCE.

The above review of past experiment and observation brings out prominently several features specially worthy of note.

The wind is now known to be far more powerful and also far more variable than it was credited with being in the early days of bridge building and of other engineering construction; and the cost of safely protecting against it is so great that its

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[106] See p. 469, Du Bois.   [107] See p. 196, Johnson.

exact power and variation is of far more importance now to engineers than ever before.

The discordant results so far obtained from the cup rotary velocity anemometers and even the ordinary small plate pressure anemometers show that more accurate instruments are necessary before their records can be much utilized in the accurate determination of actual wind velocities and wind pressures. At present their records must be reduced by from 10 to 64 per cent before the results of the velocity and pressure anemometers will be in accord with those of other more accurate apparatus.

The normal pressure of the wind on the ends of cylinders of somewhere about 3 diameters' length is unanimously considered less than on either longer cylinders or on shorter cylinders or on thin plates of the same head surface. The increased pressure thus acknowledged on short cylinders and thin plates is generally considered to be due to the partial vacuum which is formed by the rush of the wind away from behind the short cylinder or plate; and therefore will diminish with any convexity or pointing of the front head that reduces the scattering of the striking wind, or with any convexity or pointing of the rear surface that will fill up the place otherwise occupied by the partial vacuum, or with any arrangement (like that of numerous small holes all over the front plate) that will allow a little air to go through the plate in quantity sufficient to reduce the partial vacuum. This partial vacuum must also be dependent to some extent upon the shape of the perimeter of the plate, as well as upon the ratio of the perimeter to the area of the plate: but the amount of this effect is evidently small, or its general effect at least would have long ago been settled beyond question. Combining the results of experiment with the ideas of theory, it would seem probable that for each different velocity of the wind there is some ratio of the perimeter to area of plate, at which the partial vacuum per unit of surface is a maximum; so that for a given velocity of wind, the pressure per unit of area will be least for both the largest and smallest surfaces and greatest for some intermediate area.

In only one case (that of Crosby, 1890) has experiment indicated results departing markedly from the Newtonian law that the normal pressure of the wind is directly proportional to the square of the velocity; and considering that this result is not confirmed by other experiments (those of Langley, 1890) with more perfected apparatus at very nearly the same high velocities, it would appear that we must continue to admit the Newtonian law of variation with the square of the velocity, at least for the present and until it be disproved by the concordant results of many experimenters. At the same time, it is evident that any future change in the old formula is to be toward a reduction of the pressure and not toward its increase.

The difference between the results of moving air on still plates and of moving plates against still air, which should be nothing, according to theory, but which was in the first place very considerable, now seems to be rapidly diminishing as experimental apparatus is perfected, and as the errors of such apparatus become better known; so that it is to-day quite probable that this difference will soon be proved so small as to be practically nothing.

The extension of wind-pressure formulæ to other conditions markedly different from those under which these formulæ were deduced is now pretty well admitted to be unsafe; and consequently the real pressures of wind at high velocities is still a matter of considerable doubt, even upon small surfaces, and still more upon very large surfaces. The best that can be done under such circumstances is to temporarily accept the best established results of the past, checking them as far as possible with outside collateral evidence, and to stand by these results while endeavoring to secure others more satisfactorily proven. What is now needed is observation of actual wind pressures at high velocities on large surfaces, such as actual large houses, sheds, or bridges, with measurements of the actual pressure, or deflection, or oscillation thereby produced, and comparison of these results with measurements of the actual wind velocities obtained by some method more certain than that of the ordinary instruments of to-day.

The component and resultant pressures upon circular and square inclined surfaces seem already known well enough for all practical purposes (see Duchemin, Langley); but the diminution of this pressure per unit of area due to the increased length of such surfaces in the direction of the wind still needs much further investigation. About all that is as yet definitely known as to this diminution is that under small angles (less than 30 degrees) of inclination of the plate to the wind, the forward portion of the plate so diverts the wind that the rear portion of the plate does not receive as much pressure, nor offer as much resistance as if it stood alone; while under high angles of inclination, the effect is sometimes apparently reversed (see Langley).

When the wind is so cornered that its free escape is impossible, as when it blows into the open side of an otherwise closed house or shed or sphere, then the mass of air thus cornered naturally acts as does every fluid under such circumstances, and its pressure is practically equalized in all directions and is directed normally to all its confining surfaces. Hence, under such circumstances, it is able in many cases to lift roofs and blow out sides of buildings, that are parallel, or even somewhat inclined toward, the general direction of the wind, and in this way to exhibit a force which at first thought appears far in excess of that actually exerted.

The pressures upon, and resistances of, spheres, cylinders, cones, pyramids, cups, etc., are not yet as well determined as desirable, the results of various experiments differing sometimes by 50 per cent of the corresponding normal pressure upon flat surfaces of the same area. In many cases, however, this difference is due to want of clearness in the statements of the experimenters, some of whom report the total normal pressure upon the surface itself, while others report only such component of the pressure as is parallel to the direction of the wind and which therefore represents the resistance of the surface to the wind's motion. In such cases it is usually well not to adopt extreme reported values, at least not until the accuracy of their report has been further checked by careful examination of the original records or by comparison with those of other experiments.

The extent of shelter given by one body to another adjacent body at its side or in its rear, as well as the increased pressure that can be thrown upon one body by the near presence of another body, is something as to which the results of past experiment show wide differences. In both cases not only the form but also the roughness of the two bodies should be considered as materially affecting the results. Still, such results as are already attained are yet of considerable value, though much further careful investigation is desirable.

APPLICATION TO BRIDGES AND OTHER ENGINEERING STRUCTURES.

In reviewing the above experiments with respect to their application to building construction, and weighing the value of each observer's results, according to the circumstances of their method and records, it would seem that the following are safe deductions:

As to meteorological records, all maximum wind velocity records of the past derived from the use of the ordinary cup rotary anemometer whose constant of reduction is 3 (almost all those prior to 1873, most of those prior to 1885, and even many since) should be reduced at least 20 per cent for 30 mile velocities and perhaps a little more for higher velocities, to correct for the now known past error in the constants of the apparatus (see Ferrel and Abbe); and even after such correction all these records may have to be still further reduced 10 per cent on account of the unevenness of ordinary and high winds (see Langley and Forth bridge experiments) to get the average wind. But on the other hand, it is to be remembered that all high wind velocities (see Langley, 1893 experiments) may be subject to repeated oscillations of from 30 to 40 per cent each way from the average at half minute intervals for several minutes, and sometimes even to an increase of 60 per cent for nearly a whole minute; so that it is not unreasonable to assume the velocities of occasional gusts to be as high as those given by the unreduced records of the ordinary cup rotary anemometer (estimating the wind velocity as three times that of the cups), and to assume the steady wind velocity during the same time as not over 0.70 of that given by the same records. Again, all maximum small (say 1.5 square feet area) plate pressure anemometer records (see Forth bridge experiments) should be reduced at least from 36 to 40 per cent in order to give the corresponding maximum pressures on large surfaces of 300 square feet area, and about 40 per cent more to allow for the effects of gusts or something else; so that on bridges and houses, it is not unreasonable to assume the actual maximum wind pressure of gusts as not over 0.6 of that registered by the ordinary small plate pressure gauge anemometers, and to assume the actual steady wind pressure during the same time as not over 0.36 times that given by the same records.

As to the direct relation of wind velocities to the corresponding wind pressure upon thin plates, normal to the wind, it seems reasonable to assume this as given, for freezing temperatures, by the formula (see Langley, 1890 experiments) $p = 0.0039 v^2 = \frac{v^2}{260}$ (or K = 1.44); in which p is in pounds per square foot and v is in miles per hour. For cold winter storms this would become $p = 0.0043 v^2 = \frac{v^2}{233}$, giving resulting pressures as follows: 4 pounds pressure for 30 miles velocity, 7 pounds for 40 miles, 11 pounds for 50 miles, 15.5 for 60 miles, 21 pounds for 70 miles, 27.5 for 80 miles, 35 for 90 miles, 43 pounds for 100 miles, 52 pounds for 110 miles, 62 pounds for 120 miles, 73 pounds for 130 miles, 84 pounds for 140 miles, and 97 pounds for 150 miles. (While some experiments show very decidedly that the pressures are no greater than the above, yet in view of the experiments made by Langley at nearly as high velocities as the others and under more favorable conditions as to the whirling table, it does not seem wise to adopt the lesser pressures until confirmed by further experiment.)

As to pressure on thin inclined surfaces in terms of the pressure of the same wind velocities on thin surfaces normal thereto, these pressures seem best expressed by the formulæ—

$$H \frac{2 \sin^2 a}{1 + \sin^2 a}, L = \frac{2 \sin a \cos a}{1 + \sin^2 a}, \text{ and } N = \frac{2 \sin a}{1 + \sin^2 a}$$

in which H is the component parallel to the wind, L the lifting, depressing, or side moving component perpendicular to the wind, and N is the normal pressure on the inclined surface (see Duchemin and Langley). These values give $H = 1.00$, $L = 0.00$, $N = 1.00$ for $a = 90°$; $H = 0.94$, $L = 0.32$, $N = 1.00$ for $a = 70°$; $H = 0.86$, $L = 0.43$, $N = 0.99$, for $a = 60°$; and $H = 0.67$, $L = 0.67$, $N = 0.94$ for $a = 45°$. (For values at intermediate and other angles, see under previous heading of Encyclopædia Britannica, 1880.)

As to the pressures parallel to the wind upon solids and curved surfaces in terms of the pressure of the same wind upon thin surfaces normal to the wind, these pressures seem best expressed as follows: 90 per cent on cubes turned either square or edgewise; 80 per cent on the side of octagonal prisms; 70 per cent on the ends of cylinders of 3 diameters (or more) length; 60 per cent on the outside of cylinders and 40 per cent on the outside of spheres and deep cups (the pressures on convex surfaces being subject to as much as one-fifth increase in case of their close proximity to any single plane surface which might prevent the easy flow of the wind around the cylinder); 1.15 to 1.30 per cent on the concave side of cylinders, channels, and flat cups; and 1.30 to 1.70 on the concave side of spheres, and deep cups.

As to the effective surface of resistance of lattice work and gratings it is to be remembered that the first result of scattering a great number of very small holes over the surface of a large thin plate is to allow the wind to go through the plate in sufficient quantity to reduce the partial vacuum otherwise existing on the back of the plate and thus to immediately reduce somewhat its total resistance, so that this reduced resistance will probably not be much greater than that of a cylinder of 3 diameters length, after which the enlargement of the holes tends merely to change the condition from that of a single large isolated plate to that of a large number of individual small plates touching on two sides only. Moreover, as long as each opening is of small diameter and the total front area of the openings is much less than the total front area of the solid parts, the wind passing through each opening is in the condition of a contracted vein of fluid, and the effective area of resistance to its flow is much greater than the total front area of the solid parts of the lattice. On the other hand, when each opening is of large diameter and the total front area of the openings is very much greater than that of the solid parts, the contraction becomes comparatively very slight and the effective area of resistance to its flow approaches very nearly that of the exact total front area of the solid parts alone. From this point of view and in fair accordance with the theorem of Gaudard (see Fidler) and the experiments of Kernot (see Kernot), and in default of more accurate knowledge, it would seem reasonable to assume the effective area of a half open or very open lattice to be at least equal to the front area of its solid portions plus a percentage equal to the ratio of the frontage of the solid portions to the frontage of the entire lattice (opening and solid portions together), that is to say, plus 1 per cent if the frontage of the solid parts were only 1 per cent of that of the whole lattice, by 5 per cent if the frontage of the solid parts were 5 per cent of that of the whole lattice, and so on, until, when the frontage of the solid parts equals the frontage of the openings, the total effective area of resistance of the lattice would be 150 per cent of the frontage of the solid parts, or 75 per cent of the frontage of the entire lattice, after which, as the area of the openings diminished and as long as the openings were large enough to prevent the formation of a partial vacuum in rear of the lattice, the effective area of resistance of the lattice would remain practically constant and neither much more nor much less than 75 per cent of its total front area. In case that the ratio of openings to solid parts in a lattice is not tolerably uniform over all the different portions of the lattice, then for purposes of exact computation of its effective area of resistance against wind pressure the lattice should be divided up into sections such that the above ratio shall be tolerably constant throughout each separate section, and each section should then be computed separately; after which the results may be combined as may be judged best. While the above rule is thought to be as accurate as present data will justify, further experiment in this direction seems very desirable.

As to the effective shelter of one surface by another, or the increased pressure thrown upon one surface by the proximity and position of another, their amount must depend greatly upon the shape, form, smoothness, and position of each surface; so that only the most general rules are practicable and these must be modified according to the local circumstances in each particular case. The following rules seem to go fully as far as is authorized by present knowledge, and are given more as suggestions than as actually proven facts.

A flat plate may have its own effective area of resistance to the wind considerably diminished by the presence of another larger parallel flat plate directly in its rear, so long as the distance between the two plates is not greater than once the least width of the rear and larger plate, this diminution amounting in exceptional cases to as much as 66 per cent of the normal resistance of the front plate or 30 per cent of that of the rear plate; but in such case the resistance of the rear plate is also diminished so that the total resistance of the two plates is not much if any greater than that of the rear and larger plate if standing alone. As the distance between the plates is increased the resistance of each increases until when the distance between the two plates becomes from three to six times the least width of the front plate, then the resistance of the two plates together becomes practically what it would have been if each of the two plates had stood entirely alone. On the other hand a flat plate may have its own effective area of resistance to the wind slightly increased by the presence of another smaller parallel flat plate in its rear, so long as the distance between the two plates is not greater than once the least width of the front and larger plate, this increase amounting in exceptional cases to perhaps as much as 20 per cent of the normal resistance of the rear plate, or 12 per cent of that of the front plate; but in such case the resistance of the rear plate is reversed so that the total resistance of the two plates is not much if any greater than that of the front and larger plate if standing alone. As the distance between the plates is increased the resistance of each increases, until when the distance between the two plates becomes from three to six times the least width of the front plate, then the resistance of the two plates together becomes practically what it would have been if each of the two plates had stood entirely alone. If the two plates be connected together by web plates at their center or side so as to form an I beam, channel beam, or hollow beam, the web plates being in each case solid surfaces parallel to the wind, the total resistance of the combination will remain practically unchanged in the case of the I beam, will be reduced to about that of the larger plate alone in the case of the hollow beam, and will be about half as much reduced in the case of the channel beam. The above rules apply of course only to the case of winds normal to the front surfaces of two plates whose centers are in a line with the motion of the wind. Cylindrical and spherical plates may shelter each other in similar manner, but to a very much less extent; but it does not seem probable that either the front or rear shelter of convex smooth semicylindrical plates will extend to more than one-half the distance nor more than one-half the amount of that produced by flat plates. With convex, smooth hemispherical plates the shelter will probably be less than one-third that of flat plates.

Plane surfaces, parallel to the wind, placed at the sides of either flat plates or convex smooth cylindrical plates may greatly diminish or increase the resistance of the latter according as they assist or obstruct the flow of the wind around the edge of the plates. If put in front or in rear of plates and opposite their center lines the effect will be practically nothing; if put on the sides of flat plates and in their front the resistance of the plate may be slightly increased, and if in their rear it may be slightly diminished; if put on the sides of convex cylindrical plates and in front the resistance may be increased up to that of a flat plate under similar circumstances, but if in their rear it may be slightly diminished over what it was before. Channel bars and concave semicylindrical bars will usually offer at least as much resistance as flat bars of the same front, and perhaps more.

Plane surfaces, normal to the wind, such as fences or parapets, may also, by their proximity to other surfaces also normal to the wind, either add to or reduce the resistance of the latter surfaces according as they deflect the wind away from the latter surface or as they obstruct it passage around the latter surfaces. In such cases, however, the combination of the two surfaces acts usually as a single inclined rough surface of considerable depth, and should be treated accordingly.

The advanced portions of large surfaces nearly parallel to the wind and long in its direction often serve to a considerable extent as a shelter to the retired portions, in which case the advanced portions may receive more than their average share of the entire pressure, and the retired portions very much less, so much so that the total pressure against such a surface, which is long in the direction of the wind, may be much less per unit of surface than if the surface were wider and less deep; in other words, may act toward the wind much the same as a convex cylindrical surface, so that its total resistance will be less than that of a normal plane surface of the same frontage to the wind. On the other hand, if such surface be nearly normal to the wind it may act much the same as a concave cylindrical surface, so that its total resistance may be a little greater than that of a normal plane surface of the same frontage to the wind. If, however, the plane surface is long across the wind and short in the direction of the wind, the preceding results may be reversed, so as when nearly normal to the wind to act like a convex cylinder, and when nearly parallel to the wind to act like a concave cylinder.

Gratings and lattices afford shelter to objects in their rear in much the same way as do solid plates, though to a less extent, largely dependent upon their greater or less solidity. The sheltered object is exposed, firstly, over its entire surface to a reduced wind pressure from the wind that comes through the openings of the grating, and, secondly, near its edges (and sometimes nearly everywhere) to a nearly full wind pressure from the wind that swings around the outer edges of the grating, this first pressure being at least equal to such a percentage of that on the grating as represents the ratio of the effective area of the openings to the full area of the grating, and the second pressure being at least equal to that which would exist if the grating were replaced by a solid plate. Consequently, in default of more accurate knowledge, the rear surface (whether solid or open) may reasonably be considered as sheltered by the front grating only to the same extent as if this grating were replaced by a smaller solid plate whose frontage was equal to the effective front area of the solid parts of the grating. Where the grating is not uniform in its openings it should be treated as composed of several smaller gratings, each of which is practically uniform as to its openings.

From the above rules it is fairly easy to deduce the approximate wind pressures upon the different parts of a bridge or other engineering structure by taking up each part in detail. In this application, however, it is well to bear in mind the modifications due to diagonal directions of the wind, that is, to those not exactly horiontal and not exactly normal to the bridge axis. It is hardly necessary in modern, well-stiffened ordinary and long bridges to consider any uplifting wind pressures on the bridge, for general observation testifies to these being of small amount except in peculiar gorge-like localities or during cyclones or tornadoes. Even in such cases, all that is apparently necessary to the safety of the bridge is to so arrange the flooring that it may be torn up from the floor girders by the wind before the upward lifting wind strain becomes great enough to throw serious strain upon the main bridge girders or trusses. In the earlier-built suspension bridges, which were very lightly built, whose suspension cables were not cradled, and which were not stiffened by diagonal stays, the roadway was so easily disturbed from its normal position by slight movements of rolling loads that it was in a state of constant undulation when subjected to even light-traffic loads or moderate winds. Under such circumstances winds slightly inclined upward from the horizontal could easily send undulations across the bridge and back and put successive portions of the roadbed in positions inclined to the horizontal enough to allow the wind pressure to lift the roadbed on its windward side and to thus throw greatly increased pressure upon the leeward main cables. This might even happen in case where the main cables were cradled, provided the diagonal stays and stiffening trussing were omitted. But in the modern suspension bridge, using both strong stiffening trusses and strong diagonal stays, no such action need be feared. Downward pressing winds are of still less frequent occurrence and still less force and therefore need not be considered at all. The greatest wind pressure upon the ordinary bridge will therefore come from horizontal winds, and the greatest strain upon the bridge will be naturally from such as are either normal to the axis of the bridge, or nearly so. However, as the wind may be inclined 20° either way from the normal before the normal pressure on the surface diminishes perceptibly, 45° before it diminishes as much as 6 per cent, and 60° before 20 per cent, a diagonal wind, by blowing in behind the front surfaces of vertical and diagonal bracing of all vertical trusses and by thus reaching all their rear and otherwise protected parts, may throw on the bridge a much greater strain than that due to a normal wind. Therefore, if the strain on the bridge be computed from that of a normal wind, the effective area of all vertical trusses must include at least all surfaces that may be reached by diagonal winds of about 45° angle. This of itself will therefore usually prevent the necessity of considering the question of any possible shelter afforded to each other by paired tie-rods or other verticals or diagonals, in the vertical trusses.

As to the inequalities of the winds, that is to say gusts, and the sudden pressure, thereby brought upon the bridge, it seems reasonable to assume that such gusts will not extend at one time over more than 600 to 1,000 feet length of any bridge, and that such pressure should be treated like any other live load. However, it should be borne in mind that the maximum pressure of gusts during storms of maximum intensity will occur so rarely, and the strain of the metal will therefore be so slight that in such cases it is reasonable to allow the stress to reach very nearly the elastic limit of such metal. Moreover, it is necessary to bear in mind the fact that before any gust can act upon the wind bracing of a bridge, it must take the time and use up the energy necessary to overcome the great inertia of the bridge, this inertia moreover increasing very rapidly with the length of the bridge span. Consequently in long bridges, where periodic oscillation of the bridge has been duly provided against, it would seem that wind pressures would be amply provided for by allowing for first a dead-load wind pressure equal to the average steady pressure of high winds over the entire effective area of the bridge, increased by a live-load wind pressure equal to the added effect of 30 per cent added wind velocity (or 70 per cent added wind pressure) over from 600 to 1,000 feet length of the bridge; and that the value of the wind velocities may be taken at 70 or 90 per cent (according as the anemometer constant is 3 or 2.3) of the maximum of the ordinary cup rotary anemometer records of the neighborhood, converted into wind pressure by the use of the formula $p=\frac{v^2}{233}=0.0043\ v^2$; or the value of the wind pressures of gusts on the large bridge surface may be taken at 60 per cent, and the average steady wind on the bridge at 36 per cent of the maximum of the small plate pressure anemometer records of the neighborhood. Acting upon this basis and the records of 1868 to 1884 of the Bidston Observatory, long bridges near that observatory should be prepared to resist the theoretical pressure due to steady winds (computed at 0.9 of the velocity anemometer records) of as much as 83 miles per hour once in sixteen years, of 83 to 64 miles perhaps once per year, of 64 to 54 miles perhaps twice in one year, and of less than 54 miles at shorter intervals; or to resist steady pressures (computed at 0.36 of the small-plate pressure anemometer records) of from 33 to 29 pounds per square foot twice in sixteen years, of from 29 to 18 pounds perhaps once per year, of from 18 to 15 pounds perhaps twice in one year, and of less than 15 pounds at shorter intervals; or to resist steady pressures (as deduced from velocity anemometer records and the formula $p=\frac{v^2}{233}$) of as much as 30 pounds once in sixteen years, of 30 to 18 pounds perhaps once per year, of from 18 to 13 pounds perhaps twice in one year, and of less than 13 pounds at shorter intervals; so as to make necessary an allowance of at least 30 pounds per square foot steady wind pressure over the whole bridge front, and 50 pounds per square foot gusty-wind pressure over from 600 to 1,000 feet length. These same limits appear large enough for all ordinary localities in the United States, except in those regions where occasional tornadoes are to be naturally expected and specially provided for.

The computation of wind stresses and bracing upon a long single-track railroad bridge, adapted to the heaviest trains, and having panels of 25 feet length and 28 feet depth and spans of 150 feet length, would therefore proceed about as follows:

The effective wind area of the roadbed (including the guard rails, ends of the cross-ties, and track girders) may be obtained by treating the fronts of tie ends as forming a continuous grating with a 75 per cent coefficient, the front guard rails as short solids with a 90 per cent coefficient, the front track girders as H beams or short solids with a 90 per cent coefficient, considering the length of the ties as of neutral effect, considering the rear guard rails as unsheltered, and considering the rear-track girders as adding about 10 per cent to the area of the front girder; so that the total effective resistance of the roadbed per foot length of bridge may, as a rule, be taken at about 2 square feet for the two guard rails and the ties, and about 5 square feet for the two track girders, or about 7 square feet in all.

The effective wind area of the main girders or trusses of the bridge may be obtained by treating each of its round vertical or diagonal tie rods as a cylinder with a 60 per cent coefficient (none of them considered as receiving or giving any shelter to the other), each of its vertical or diagonal tie bars as a flat plate with a 100 per cent coefficient (none of them considered as receiving or giving any shelter to the other), each horizontal bar or plate (at 8 or more breadths distance in front of or behind another) and each group of horizontal tiebars or plates (one behind the other and within two breadths distance of the broadest plate) as a single plate of the breadth of the broadest with a 100 per cent coefficient, each group of horizontal tiebars or tie plates (one behind the other and within about 4 breadths distance of the broadest plate) as a single plate of the breadth of the broadest and with a 150 per cent coefficient, square solid-sided hollow beams as short solids with a 90 per cent coefficient, I beams and square lattice-sided hollow beams, when horizontal, as a single flat plate with a 100 per cent coefficient, the same I and lattice beams when vertical or diagonal and with solid sides and open front as a single flat plate with a 100 per cent coefficient, the same I and lattice beams when vertical or diagonal and with solid front and open sides as 2 flat plates of sizes equal to their front and rear, each with a 75 per cent coefficient, other open built beams and octagonal beams as flat plates of the size of the greatest cross section parallel to the track with an 80 per cent coefficient, rough surfaced cylinders as octagonal beams, smooth surfaced cylinders, and wire ropes, and wire-wrapped cables as cylinders with a 60 per cent coefficient, horizontal cylinders as completely sheltering other similar cylinders touching them in their rear and as giving 50 per cent shelter to those at 2 diameters distance and nothing at 4 diameters distance, vertical and diagonal cylinders and octagonal posts as giving no shelter to others in their rear, and the rear main girders or trusses (if lattice work or low solid work) as receiving no shelter from front girders or trusses, but both front and rear trusses being considered as sheltered by so much of the track girders (or stringers) as may be directly in front or rear of their actual surfaces; so that the total effective area of resistance of the main girders may be assumed per foot length of bridge at about 0.8 square feet for each tension chord, about 1.4 square feet for each compression chord, about 1 square foot for both diagonals (after one-sixth reduction for shelter by track girders), about 1.5 square feet for each post (after one-sixth reduction for shelter by track girders), or about 4.5 square feet for each girder, or about 9 square feet for both girders.

The effective wind area of the two trusses of the horizontal bracing of heavy bridges (so far as not already provided for above) may be taken at that of the diagonals of a single truss, one truss being usually entirely sheltered by the compression chords of the main girders, the other truss, however, being where its posts are sheltered but its diagonals unsheltered; and these diagonals may be treated as short solids with a 90 per cent coefficient; so that the total effective area of the horizontal bracing per foot length of bridge may be assumed at about 0.2 square foot.

The effective wind area (due to diagonal wind pressure) of the vertical sway bracing may be taken at that of the diagonals alone, the other pieces either being sheltered or already provided for above; and these diagonals may be treated as cylinders with a 70 per cent coefficient, and as not sheltering each other; so that the total effective area of the sway bracing per foot length of bridge may be assumed at not over 0.05 square foot.

For a heavy single-track railroad bridge, the total effective area of wind resistance may therefore amount to $7.0 + 9.0 + 0.2 + 0.05 = 16.25$ square feet per foot length of bridge.

For double-track bridges, the effective wind area of the roadbed will be increased about 40 per cent by the presence of the added guard rails and track girders, and the effective area of the other parts of the bridge will be increased about the same percentage by the added size of each piece; so that unless specially computed (as, however, should be done in each case in the final revision of plans) the effective area of the double-track structure may be taken approximately as 40 per cent greater than that of a single-track bridge, or as about 23 square feet per foot length.

The effective wind area of the piers or towers may be determined in the same general way as that of the spans, being equal to that of its front truss, its rear truss, the front surface of its various platforms, and the front surface of the diagonals of its sway bracing; so as to amount per foot height to fully 2.5 square feet per corner, or 5 square feet per front or rear side, or 10 square feet per foot height for the entire pier of a single-track bridge, or 15 square feet per foot height for the entire pier of a double-track bridge. In calculating the stability of a pier against wind, it is to be remembered that the piers are held down upon their foundations by not only their own weight but also by that of their share of the bridge and its load, and that they are pressed laterally by not only their own wind load but also by that of their share of the bridge and its load.

The effective wind area of trains may be determined by regarding the train as 10 feet high with its bottom 2.5 feet above the rails and treating it as a short solid with a 90 per cent coefficient, and considering it as sheltering its own height of the verticals and diagonals of the two trusses and all trains in its rear; so that the added wind area due to the train will be only about 7 to 8 feet per foot length of bridge occupied by trains. This load, however, must be regarded as a moving load, traveling gradually across the bridge. As some car of a train would probably be blown over by an 80-mile wind and as a train, therefore, would not enter the bridge at such time, it is not necessary to consider the wind pressure on the train at times of maximum wind velocities, although such train area should be considered under the gusty effect of moderate winds.

In suspension bridges, the effective area of resistance of the main cables and suspenders may be obtained by treating each main cable as a cylinder with a 60 per cent coefficient and considering it as completely sheltering other cables directly in rear if touching, one to the other, or giving 50 per cent shelter at a 2 diameters distance, center to center, but no shelter at 4 diameters distance to either other cables or other parts of the bridge; and by treating each suspender as a cylinder with a 60 per cent coefficient and considering it as giving no shelter to any other suspenders or other parts of the bridge; so that in bridges of 4,000 feet length of span the effective wind area of the main cables and suspenders alone may be as much as 8 square feet per foot length of single track bridge, or 12 square feet per foot length of double-track bridge, and 4 square feet per foot length for each additional track.

Having found the effective wind area of the span and train and piers, the total pressure to be provided against may be next figured at 30 pounds per square foot over the entire bridge with a train upon it, or 50 pounds per square foot over from 600 to 1,000 feet length of the unloaded bridge (including piers), increased by 30 pounds per square foot over the rest of the unloaded bridge, and then using in subsequent computations whichever of these totals produces the greatest stress upon the particular members under consideration.

The wind bracing for ordinary bridges may then be arranged in the form of two horizontal trusses, one above and the other below the roadway, as in the usual arrangement of modern bridges, stiffened by vertical sway bracing. In case of very long bridges, it may be desirable to supplement these by horizontal cables passing under the bridge floor and around the piers and out to the shore anchorages, the versine of the cable being from two-thirds to three-fourths the breadth of the piers. In suspension bridges, these two methods are advisably further supplemented by the swinging in or "cradling" of the main cables to such extent that this cradling may be not only enough to support all the wind pressure on the cables and suspenders, but also considerable of that on the stiffening truss and roadway, besides otherwise stiffening the bridge against lateral movement.

The dimensioning of the various parts of the wind-bracing may next be computed by the usual methods, the use of the new formulæ of Launhardt and Weyrauch being recommended as preferable to the older methods, and that of the new dynamic formula of Fidler as preferable to either. In these computations, especially under either of the new formulæ, it seems hardly necessary, because of the infrequency of the maximum stresses, to use a coefficient of safety much greater than that actually necessary to make certain that the maximum stress is not allowed at anytime to go beyond the minimum elastic limit of such metal as may be naturally expected to actually get into the finished structure, due allowance being made for the ordinary unevenness of manufactured products and the ordinary failures of attempted careful inspection, this minimum elastic limit being deducible from the tests of full-sized members, as well as from those of a large number of smaller pieces, of such metal as it is proposed to use. With ordinary grades of wrought iron and ordinary circumstances of manufacture and inspection the coefficient of safety may best be taken at 3.5, and the repetition limit at one-half and the reversal limit at one-fourth the ultimate strength of the metal, thus allowing dead-load stresses of 29 per cent and maximum reversal stresses (those under most unfavorable conditions) of 7 per cent of the ultimate strength of the metal; but in exceptional cases of the best steel (bar or wire) and the most careful manufacture and inspection (admitted, however, only after careful tests), it may be allowable to reduce the coefficient of safety to 2 and to increase the reversal limit to one-third of the ultimate strength, thus allowing dead-load stresses of 50 per cent and maximum reversal stresses (those under most unfavorable conditions) of 17 per cent of the ultimate strength of the metal.

In the dimensioning of piers or towers, the breadth of the pier at each of its horizontal sections must be sufficient to prevent the overturning of the pier about either edge of such section, a coefficient of safety of 2 being used in the computation of such stability, and the breadth must also be such that no tensional strains shall ever occur in any of the vertical or main posts of the pier.

APPENDIX D.

TEMPERATURE STRAINS IN THREE-HINGED ARCHES.

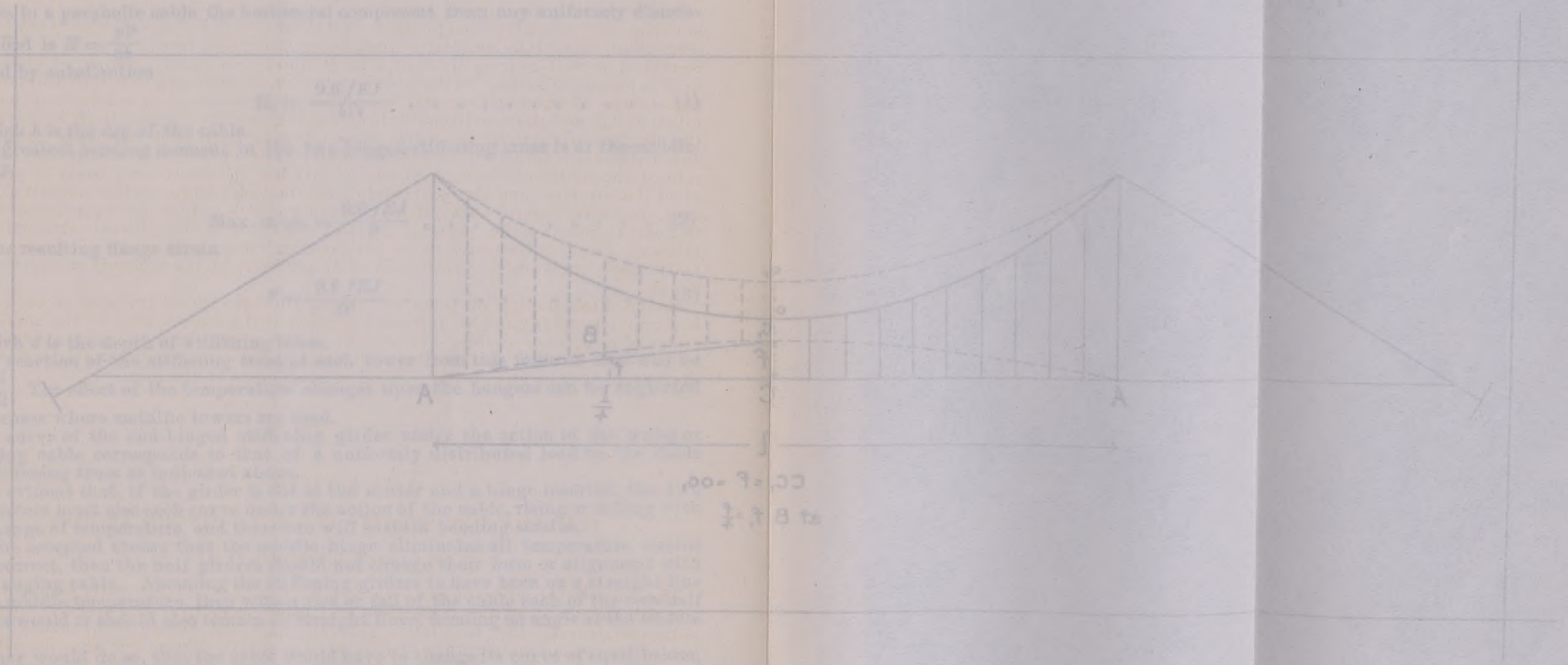
[By GUSTAVE LINDENTHAL, C. E.]

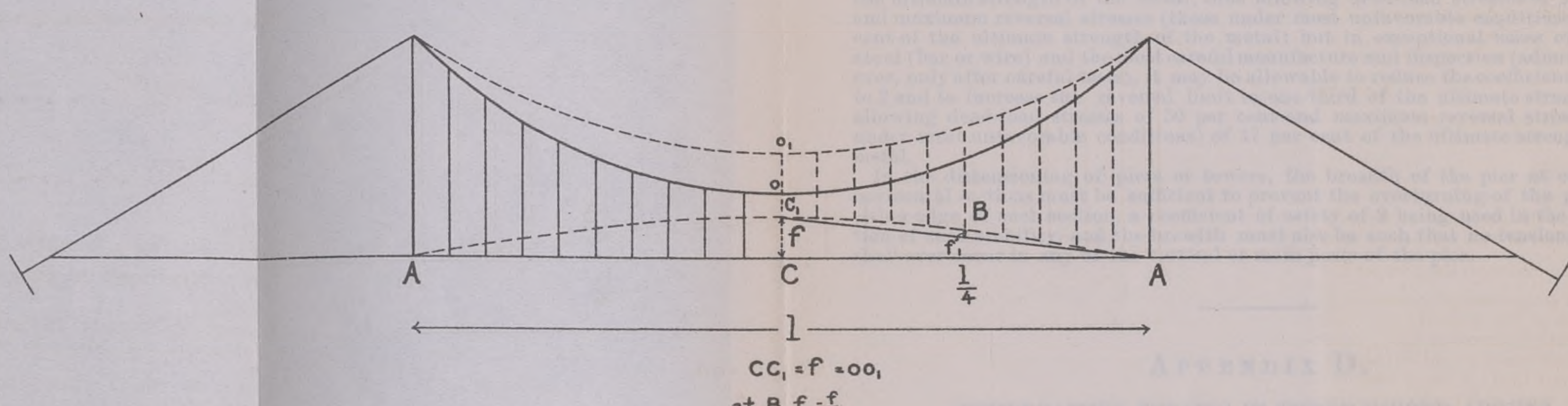
The theoretical advantages of three-hinged arches are generally assumed to be the calculation of strains from loads on statical principles and freedom from temperature strains. But this latter generally accepted theory is erroneous. There are, of course, no temperature strains at the middle hinge, but they do exist for any change from the middle temperature in the arches between the end and middle hinges, and are too large to be neglected in the computations. In this paper it is proposed to restrict the investigation to the temperature strains in a suspension bridge with three-hinged trusses, which is a special case of the three-hinged arch.

The value of the force H_t is the horizontal component in the cable, caused solely by the resistance of the stiffening trusses to the change of curvature of cable from a middle temperature. H_t represents tension or compression according as the cable shortens or lengthens, and must correspondingly be added or deducted from the horizontal component H, caused by the loads in the cable.

In a suspension bridge having one main span and two side spans, in which the cables carry no load (except their own), the change of length due to temperature will be proportionate to the length of cables from anchorage to anchorage, and the effect caused by such change will be concentrated in the middle span.

It will be best in each given case to numerically compute this change of length in the cables and the resulting deflection or rise in the middle span.





A
C
A
o1
o
C1
f
B
f1
l/4
l
CC1 = f = OO1
at B f1 = f/4

The numerical calculation of this deflection or rise $= f$, which is then known, should precede the following investigation. First, assuming a stiffening truss without a middle hinge and having only end hinges at the towers, we will find its resistance to bending from the formula for deflection in trusses, namely $f = \frac{5}{384} \frac{pl^4}{EJ}$, in which p is the load per linear foot, E is the modulus of elasticity of steel, J is the moment of inertia of the truss, and l is the length of the two-hinged stiffening truss in the middle span. This formula is deduced from the elastic deformation of a solid beam of uniform section, but is applicable to low-framed trusses, the web of which has little influence on the deflection. In trusses of greater height than one-tenth of the span, the above formula will give values too small, because the effect of the elastic deformation of the web upon the deflection then becomes marked. The stiffening trusses in a suspension bridge are never so high as ordinary self-supporting trusses, and the above formula therefore can be used for them without material error.

H_t will act on the stiffening girder precisely as a uniformly distributed load would, the value p_t of which per linear foot can be obtained from the above formula for deflection, namely,

$$p_t = \frac{384\ fEJ}{5l^4}$$

Since in a parabolic cable the horizontal component from any uniformly distributed load is $H = \frac{pl^2}{8h}$

we find by substitution

$$H_t = \frac{9.6\,fEJ}{hl^2} \quad \ldots\ldots\ldots\ldots \quad (1)$$

in which h is the dip of the cable.

The greatest bending moment in the two-hinged stiffening truss is at the middle, namely,

$$\text{Max. mom.} = \frac{9.6\,fEJ}{l^2} \quad \ldots\ldots\ldots\ldots \quad (2)$$

and the resulting flange strain

$$F = \frac{9.6\,fEJ}{dl^2} \quad \ldots\ldots\ldots\ldots \quad (3)$$

in which d is the depth of stiffening truss.

The reaction of the stiffening truss at each tower from this force or load will be $\pm p_t \frac{l}{2}$ The effect of the temperature changes upon the hangers can be neglected in all cases where metallic towers are used.

The curve of the end-hinged stiffening girder under the action of the rising or dropping cable corresponds to that of a uniformly distributed load on the cable and stiffening truss as indicated above.

It is evident that, if the girder is cut at the center and a hinge inserted, the two half girders must also each curve under the action of the cable, rising or falling with the change of temperature, and therefore will sustain bending strains.

If the accepted theory that the middle hinge eliminates all temperature strains were correct, then the half girders should not change their form or alignment with the changing cable. Assuming the stiffening girders to have been on a straight line at the middle temperature, then with a rise or fall of the cable each of the two half girders would or should also remain on straight lines, forming an angle at the middle hinge.

If they would do so, then the cable would have to change its curve of equilibrium, whatever form it may be, in each half span. It can not do so without an outer force, which in this case is the resistance of the half girders to bending.

The force bending the half girders can be again assumed as equivalent to a load p_b, distributed over the half girder. Strictly speaking this is not correct, as the following consideration will show:

Let $A\ C$ (in diagram) be the one-half girder, fastened at A, and free at C (the middle hinge) to move up or down.

Then, if the tension in the suspenders were the same as for a middle temperature, the half girder $A\ C$ would be moved from $A\ C$ into the position $A\ C_1$, remaining a straight line. But as the action of the changing cable curve will force the half girder to assume the curved form $A\ B\ C_1$, the tension in the suspenders can not be uniform; it will be largest at B, gradually decreasing toward A and C. Whether the decrease in tension (depending in reality upon the variations of J) from the middle toward both ends of the half girder is uniform or not is for the small cambers usual in practice not very material to the investigation, for which it will be sufficiently accurate, for obtaining the bending moments, to assume as an equivalent a uniformly distributed load p_h, acting on the half girder and bending it, just as if there were equal reactions at A and C_1.

The cable is in equilibrium under uniform load when the curves of the half girders, meeting at the middle hinge, have the same tangent. Of course it may happen that the curve or camber of the half girder may be less, but it can not be more. The value f_1, at $\frac{l}{4}$ is therefore the distance from the chord line A C_1 to B for the curve corresponding to the cable curve in equilibrium.

For flat catenaries it can be assumed that $f_1 = \frac{f}{4}$ without sensible error.

As the deflections of beams vary for the same load per lineal unit as the fourth power of the spans, all other things being equal, and as the chord sections (moment of inertia) at the middle of a continuous stiffening girder do not differ greatly for the same height of girders from the chord sections at the middle of each half in a three-hinged stiffening girder, we can, approximately, write,

$$\frac{f_1}{f} = \frac{1}{4} = \frac{p_h \left(\frac{l}{2}\right)^4}{p_t\, l^4};\ p_h = 4\, p_t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

which means that the force acting upon the stiffening girders, having a middle hinge, is four times as great as in a continuous (or two-hinged) girder, and since the bending moments are as the square of the span, it follows that the maximum bending moment at the middle of the half girder is as large as in the middle of the continuous girders.

In the case of p_h becoming larger than the dead load suspended from the cable, the half girders will kink in at the middle hinge, and the effect from live load upon the stiffening truss would be found greater than ordinarily computed.

No allowance has been made in the above investigation for the elongation of the cables caused by the horizontal component H_t. H_t is not a negligible quantity as far as the cable sections are concerned, but can be neglected for the stiffening trusses. When the dip of the cable is large and the stiffening trusses comparatively small in height H_t will be small. For a flat cable with a high-stiffening truss H_t would have to be allowed for also in the proportioning of a two-hinged stiffening truss.

It does not follow, however, that because p_h in a three-hinged girder is four times the value of p_t in a two-hinged girder, the value H_t is four times as great in the cable for the three-hinged girder. It merely indicates that the dead load on the cable is no longer evenly distributed for a rise or fall from the normal position. The suspenders near the quarter of each span will carry a comparatively larger share of the dead load than the suspenders near the towers or near the center of the span when the cable rises, and vice versa when the cable drops.

While the above method of obtaining the temperature strains in three-hinged stiffening girders is based on approximations (as are also, for instance, the formulæ for temperature strains in arches) it gives sufficiently close values for all practical purposes. As may be seen the values are too large to be neglected in designing an important structure.

All forms of three-hinged arches are subject to temperature strains, which therefore do not offer the supposed advantage in this respect over the end-hinged arches; but the different forms of three-hinged arches will be differently affected.

Application to the design discussed by the New York board:

l = 3,200 feet length of span;

A = 3,800 square inches metal section of four chords;

$J = \frac{Ad^2}{4}$ = 13,680,000 moment of interia of two stiffening trusses;

d = 120 feet = depth of stiffening truss;

f = $\pm$ 1.91 feet deflection solely from temperature;

for $\pm$ 60° F. (changes in cables).

Maximum flange strain $F = \frac{9.6\, f\, E\, J}{dl^2}$ for E = 28,000,000 and substituting above values, F = 2,857.8 tons.

Maximum stress from temperature alone per square inch of flange = 3,008 pounds at the middle of each half stiffening truss.

The corresponding $p_h = \frac{4 \times 384\, fEJ}{5l^4} = \frac{307.2\, fEJ}{l^4}$

Substituting the above values, p_h = 2,143 pounds.

The dead load per lineal foot of bridge suspended from the cables being given by the board as 17,000 pounds, which is more than eight times p_h (the force bending the half girder), no kinking in at the middle hinge from temperature changes would take place.

APPENDIX E.

THE THEORY OF THE STIFFENING GIRDER.

[By J. MELAN, professor at the Technical High School at Brünn. Translated by permission from Handbuch der Ingenieurwissenschaften, Zweiter Band. Der Brückenbau.]

A.—THE SLACK ARCH AND UNSTIFFENED SUSPENSION BRIDGE.

§3. *The cable- or archpolygon for any assumed vertical loading.*

When a supposed weightless cable, supported at two points, is subject to simple vertical forces, it assumes a definite polygonal form which is dependent upon the relation between these forces.

Let A represent the vertical component of the tension at one of the points of support, and $P_1 \ldots P_m$ the forces acting to the mth side of the polygon; then the vertical component of the stress in the latter is $V_m = A - P_1 - P_2 \ldots - P_m$, while the horizontal component H remains constant for all sides of the polygon. The angle τ_m which the mth side of the polygon forms with the horizontal is determined from,

$$tg\tau_m = \frac{V_m}{H} = \frac{A - P_1 - P_2 - \ldots - P_m}{H}, \quad (1)$$

and the resulting stress from

$$T_m = \sqrt{V_m^2 + H^2} = H \sec \tau_m \quad (2)$$

If, furthermore, $x_m\ y_m$ and $x_{m+1}\ y_{m+1}$ represent, with the point A (fig. 8) as the origin, the coordinates of the vertices of the polygon adjoining its mth side, we then have

$$y_{m+1} - y_m = \frac{A - P_1 - P_2 \ldots - P_m}{H} \cdot (x_{m+1} - x_m) \quad (3)$$

or, $\Delta y_m = \dfrac{A - P_1 \ldots - P_{m-1} - P_m}{H} \cdot \Delta x_m$.

In a similar manner we have for the preceding side of the polygon,

$$\Delta y_{m-1} = \frac{A - P_1 - P_2 \ldots - P_{m-1}}{H} \cdot \Delta x_{m-1}.$$

If we now place $\Delta y_m - \Delta y_{m-1} = y_{m+1} - 2y_m + y^{m-1} = \Delta^2 y_m$, and if furthermore $\Delta x_m = \Delta x_{m-1} = a$, that is, if the horizontal distances between points of the polygon are equal, we also have

$$\Delta^2 y_m = -\frac{P_m}{H} \cdot a \quad (3^a)$$

In general, we have

$$(tg\tau_m - tg\tau_{m-1}) = \frac{P_m}{H} \quad (3^b)$$

From the forces P and the horizontal tension H the corresponding cablepolygon can be determined analytically by known methods as well as graphically.

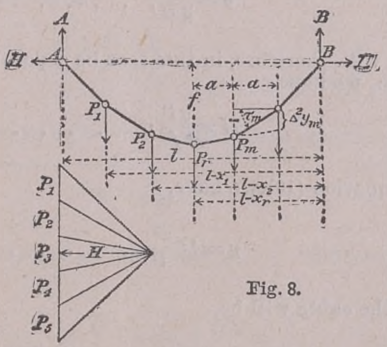


Fig. 8.

NOTE.—*Seilpolygon* and *stabpolygon* have been translated as *cablepolygon* and *archpolygon*, as referring to the form of the cable or arch, excepting where used in connection with the graphical method, in which cases they are read as *equilibrium polygon*.

If the points of support of the cable lie in a horizontal plane, A may be determined, like the abutment reaction of a freely supported girder, from the moment of the vertical loads.

$$A = \frac{\Sigma P\,(l - x)}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If in this case f is the versin of the cable polygon, and r is its lowest corner, we have

$$H = \frac{M_r}{f} = \frac{A\,x_r - P_1\,(x_r - x_1) - P_2\,(x_r - x_2) - \,.\,.\, P_{r-1}\,(x_r - x_{r-1})}{f} \quad . \quad . \quad (5)$$

If, instead of the dip f, the length of the cable is given and if the connecting points of the suspenders 1, 2, ... and thereby the lengths $e_0, e_1, e_2, \ldots\ldots$ of the separate parts of the cable are known, the horizontal tension is determined from

$$\frac{e_0}{\sqrt{H^2 + A^2}} + \frac{e_1}{\sqrt{H^2 + (A - P_1)^2}} + \frac{e_2}{\sqrt{H^2 + (A - P_1 - P_2)^2}} + \ldots = \frac{l}{H} \quad . \quad . \quad (6)$$

It is evident that the strains T in the separate members of the polygon increase as we approach the supports and in the first and last members reach their greatest values. That, furthermore, for forces due to downward acting loads only *tension* can occur in a suspended or inverted polygon and only *compression* can occur in an erect polygon.

When the forces due to the loading are distributed according to a fixed plan, the cable- or archpolygon assumes a definite form. If q is the load per horizontal unit of length at any assumed point whose abscissa is x, we have from equation (3),

$$dy = \frac{A - \int q\,dx}{H}\,.\,dx;$$

from which we have as the differential equation of the curve of the cable,

$$H\frac{d^2 y}{d x^2} = -q \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Since, if r is the radius of curvature of the cable curve $\frac{d^2 y}{d x^2} = -\frac{1}{r}\,sec^3\,\tau$, we have from equation (2),

$$T_x = q\,r\,cos^2\,\tau \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and for the load q_0 and the radius r_0 at the vertex of the curve,

$$H = q_0 r_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

For a uniformly distributed load, that is when q is constant, we obtain by transferring the origin of coordinates to the vertex of the curve and by the integration of equation (7),

$$y = \frac{q\,x^2}{2\,H}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The curve of the cable will, therefore, in this case be a parabola. If l represents the span and f the versin, we have

$$H = \frac{q\,l^2}{8f}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

and consequently from equation (10) we have,

$$y = 4f\,\frac{x^2}{l^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The greatest stress in the cable will be,

$$T_{max} = \sqrt{H^2 + \left(\frac{1}{2}\,q\,l\right)^2} = \frac{q\,l^2}{8f}\sqrt{1 + \left(\frac{4f}{l}\right)^2} \quad . \quad . \quad . \quad . \quad (13)$$

If the loading is not constant per horizontal unit of length but per unit of length of the cable, the cable curve becomes a common catenary.

Then from equation (7) and with the vertex of the curve as origin of coordinates we have,

$$H\frac{d^2y}{dx^2} = g \sec \tau = g\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{1}{2}}; \qquad 13a$$

from which we have for the equation of the curve when $\frac{1}{c} = \frac{H}{g}$ represents the parameter of the catenary,

$$y = \frac{1}{2c}(e^{cx} + e^{-cx} - 2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

For the length s of the catenary we have,

$$s = \frac{1}{c}\sqrt{2cy + c^2y^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

§ 4. *The unstiffened suspension bridge.*

1. *Form of the chain and magnitude of the horizontal tension.*—Assume the weight of the roadway to be suspended from the chain, together with the supposed uniformly distributed live load of q_0 per unit of length, and make the practically closely approximate supposition that the weight of the chain or cable is distributed as though the curve of the cable were a parabola and that the cross section of the chain throughout is proportional to its stress for the maximum loading. Let the weight of a unit of length of the chain at its center be g_0; then at the distance x from the center the weight of a horizontal unit of length of the chain is

$$g_x = g_0 \sec \tau^2,$$

and, as from equation (12) for the parabola $\sec \tau^2 = 1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{64 f^2 x^2}{l^4}$ we have, $g_x = g_0\left(1 + \frac{64 f^2 x^2}{l^4}\right)$.

Let the weight of the suspenders for a horizontal unit of length be

$$\mathfrak{g}\, y = \mathfrak{g}\frac{4 f x^2}{l^2},$$

so that by transferring the origin of coordinates to the center of the curve, equation (7) becomes

$$H\frac{d^2y}{dx^2} = q_0 + g_0 + \left(g_0\frac{64 f^2}{l^4} + \mathfrak{g}\frac{4 f}{l^2}\right)x^2.$$

If, for abbreviation, we place

$$\left.\begin{aligned} q_0 + g_0 &= q \\ \frac{2}{3}\frac{f}{l^2}\left(16\frac{f}{l^2}g_0 + \mathfrak{g}\right) &= \varkappa, \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

we have by twice integrating the above differential equation, between the limits of x and 0

$$y = \frac{x^2}{2H}(q + \varkappa x^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

and from this, by substitution of the values $x = \frac{l}{2}$ and $y = f$

$$H = \frac{l^2}{8f}\left(q + \varkappa\frac{l^2}{4}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

As a rule the length of roadway l' is somewhat less than the distance l between the points of support of the chains. If this be taken into consideration and it be assumed that the weight of the suspenders is uniformly distributed and included in the weight of the roadway, g_1 per linear meter, we have, if p represents the live load per linear meter, for the horizontal stress the expression

Origin at one end as in Fig. 8

$$H = g_0 \left(1 + \frac{8}{3} \frac{f^2}{l^2}\right) \frac{l^2}{8f} + (p + g_1)\, \frac{l^2 - (l - l')^2}{8f} \quad . \quad . \quad . \quad . \quad (18^a)$$

If F_0 represents the cross section of the chain at the center in square centimeters, γ its specific weight (kilograms per cubic decimeter), s its stress under greatest loading (kilograms per square centimeter), then will $g_0 = F_0 \frac{\gamma}{10}$ (kilograms per linear meter) and $F_0 = \frac{H}{s}$; from which also $g_0 = H. \frac{\gamma}{10 . s}$. Likewise we have $\mathfrak{g} = q_0 \frac{\gamma}{10 . s}$.

This substituted in equations (16) and (18) gives,

$$H_{\max} = \frac{q_0 l^2}{8f} \frac{1 + \frac{1}{6} f \frac{\gamma}{10 . s}}{1 - \frac{\gamma}{10 . s} \left(\frac{l^2}{8f} + \frac{1}{3} f\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

The weight of the half chain becomes, $G = \int_0^{\frac{l}{2}} g_x \, d x$

$$G = H_{\max} \cdot \frac{\gamma}{10 . s} \frac{l}{2} \left(1 + \frac{16}{3} \frac{f^2}{l^2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

From this the vertical component of the abutment reaction becomes,

$$V_{\max} = \frac{1}{2} q_0 l \left(1 + \frac{1}{3} f \frac{\gamma}{10 . s}\right) + G \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

and the greatest stress in the chain is,

$$T_{\max} = \sqrt{V^2_{\max} + H^2_{\max}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

The angle which the chain makes with the horizontal at the point of support is determined from $tg\ \tau_0 = \frac{V}{H}$, or is approximately,

$$tg\ \tau_0 = \frac{4f}{l} \left(1 + \frac{1}{3} f \frac{\gamma}{10 . s}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

The length of chain is obtained from,

$$L = 2 \int_0^{\frac{l}{2}} \sqrt{1 + \left(\frac{d y}{d x}\right)^2}\, d x = 2 \left\{ x + \frac{1}{\frac{d^2 y}{d x^2}} \left(\frac{d y}{d x}\right)^3 \left[\frac{1}{6} - \frac{1}{40} \left(\frac{d y}{d x}\right)^2 + \ldots \right] \right\}_0^{\frac{l}{2}},$$

in which $\frac{d y}{d x}$ and $\frac{d^2 y}{d x^2}$ are to be deduced from equation (17). Substituting we have,

$$L = l \left\{ 1 + \frac{16 f^2}{l^2} \frac{\left(q + \varkappa \frac{l^2}{2}\right)^3}{\left(q + \varkappa \frac{l^2}{4}\right)^2 \left(q + \frac{3}{2} \varkappa l^2\right)} \left[\frac{1}{6} - \frac{2 f^2}{5\, l^2} \left(\frac{q + \varkappa \frac{l^2}{2}}{q + \varkappa \frac{l^2}{4}}\right)^2 + \ldots \right] \right\} . \ (24)$$

For small values of $\varkappa$, that is, for small values of the versin in terms of the span, we have an approximate expression for the length of chain,

$$L = l\left\{1 + \frac{8}{3}\frac{f^2}{l^2} - \frac{32}{5}\frac{f^4}{l^4}\right\} \quad \ldots \ldots \ldots \quad (25)$$

which will also apply to flat parabolic curves.

The maximum span for a suspension bridge of assumed material and unit stress is determined by the condition that the denominator of equation (19) shall remain positive, that is, that

$$8f > \frac{\gamma}{10\,s}\left(l^2 + \frac{8}{3}f^2\right)$$

from which we have,

$$l < \frac{\frac{80\,s}{\gamma}\,\frac{f}{l}}{1 + \frac{8}{3}\,\frac{f^2}{l^2}} \quad \ldots \ldots \ldots \ldots \quad (26)$$

From this we obtain the theoretical maximum spans of suspension bridges for the following stresses and relations between versin and span:

$\frac{f}{l} =$	$\frac{1}{10}$	$\frac{1}{12}$	$\frac{1}{14}$	$\frac{1}{15}$	$\frac{1}{16}$	$\frac{1}{17}$	$\frac{1}{18}$
l_{max} for $\gamma = 7.79$ and $s = 700$ kg	700	588	507	473	444	418	396 m
l_{max} for $\gamma = 7.79$ and $s = 1{,}600$ kg	1,600	1,344	1,158	1,083	1,016	957	905 m

2. *Most economical relation between versin and span.*—Upon the versin of the chain is dependent not only its own weight and the weight of the suspenders, but also the cost of the piers. Disregarding its influence upon the backstays and upon the anchorages, that relation of versin to span is easily computed, for which the cost will be a minimum. If we designate this relation by $\frac{f}{l} = n$, and let P represent the relation of the cost per meter of height of piers of an assumed width to the cost of 1 kilogram of ironwork and, for abbreviation, place $\frac{\gamma}{10\,.\,s} = \varepsilon$, the equation of condition from equation (20) becomes, with a little permissible simplification,

$$\frac{\varepsilon\left(1 + \frac{16}{3}n^2\right)}{8\frac{n}{l} - \varepsilon\left(1 + \frac{8}{3}n^2\right)}\,q_0\,l + \frac{1}{3}\,\varepsilon\,n\,q_0\,l^2 + P\,n\,l = min.$$

Differentiating and solving for n by placing the very small quantity

$$\frac{2\,\varepsilon\,q_0\,l + 3\,P}{\varepsilon\,q_0\,l + P}\,\frac{\varepsilon\,l}{12} = \alpha$$

we have,

$$n = \alpha + \sqrt{\alpha^2 + \frac{1}{8}\,\frac{\varepsilon\,q_0\,l}{\varepsilon\,q_0\,l + P}} \quad \ldots \ldots \ldots \quad (27)$$

Neglecting α we have,

$$n = \sqrt{\frac{1}{8}\,\frac{\varepsilon\,q_0\,l}{\varepsilon\,q_0\,l + P}} \quad \ldots \ldots \ldots \ldots \quad (28)$$

For instance, for $P = \frac{1{,}100}{0.22} = 5{,}000$, $\varepsilon = 0.00065$, $q_0 = 6{,}000$ kilograms, and $l = 100$ meters, n would be $= 0.085 = \frac{1}{12}$.

3. *Changes of form.*—For simplification in the following investigations we will assume the form of the chains in an unloaded bridge, that is, in one subject only to a dead load, to be a parabola, which supposition is permissible when the proportion of the versin to the span is small and the approaches are short. We will also disregard the resistance to changes of form due to friction between the links of the chain or to the stiffness of the wire cable.

Let $g =$ the total dead load of the bridge per linear meter.

Let $p =$ the assumed uniformly distributed live load per linear meter.

1. *Maximum deflection* at the center due to change of form when loaded. This will occur when a certain central part of a length 2ξ is loaded. We will disregard for

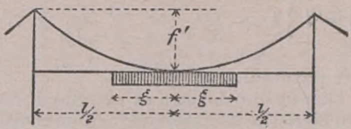


FIG. 9.

the present the possible sliding of the saddles on the piers and assume, therefore, that the supports of the chains are perfectly rigid. Assuming f' to be the deflection of the chain caused by the load, we have from equation (5)

$$f' = \frac{1}{8}\frac{gl^2}{H} + \frac{1}{2}\frac{p\xi(l-\xi)}{H} \quad \ldots\ldots\ldots\ldots (\alpha)$$

We further find that when we introduce for the length of the parabolic curve the approximate expression equation (25) with omission of the third member, we have for the half chain length the two values (for the unloaded and for the loaded conditions)

$$\left.\begin{array}{l}\frac{L}{2} = \frac{l}{2}\left(1+\frac{8}{3}\frac{f^2}{l^2}\right) = \frac{l}{2} + \frac{1}{H^2}\left\{\frac{1}{6}(p+g)^2\xi^3 + \frac{1}{4}\left[p\xi + \frac{1}{4}g(l+2\xi)\right]^2 \right. \\ \qquad\qquad \left. (l-2\xi) + \frac{1}{24}g^2\left(\frac{l}{2}-\xi\right)^3\right\}\end{array}\right\} \ldots (\beta)$$

From this results,

$$H = \frac{1}{4f}\sqrt{l\left\{\frac{1}{4}g^2l^3 + \frac{3}{2}pgl^2\xi + 3p^2l\xi^2 - (4p^2+2pg)\xi^3\right\}} \quad \ldots (\gamma)$$

and then from equation (α)

$$\frac{f'}{f} = \frac{2p\xi(l-\xi) + \frac{1}{2}gl^2}{\sqrt{l\left\{\frac{1}{4}g^2l^3 + \frac{3}{2}pgl^2\xi + 3p^2l\xi^2 - (4p^2+2pg)\xi^3\right\}}} \quad \ldots (29)$$

The maximum value of f' is found by differentiating the preceding expression for ξ, giving the equation of condition,

$$2\left(\frac{\xi}{l}\right)^4\left(2\frac{p}{g}+1\right)\frac{p}{g} + 2\left(\frac{\xi}{l}\right)^3\left(\frac{p}{g}-1\right)\frac{p}{g} + \frac{3}{2}\left(\frac{\xi}{l}\right)^2\left(1-\frac{p}{g}\right) - \frac{\xi}{l} + \frac{1}{8} = 0 \quad \ldots\ldots\ldots\ldots (30)$$

The value of $\frac{\xi}{l}$ from the above when substituted in equation (29) gives as the maximum center deflection, $\Delta f_1 = f' - f$.

We find

for $\frac{p}{g}=$	0	$\frac{1}{2}$	1	2	3
$\frac{\xi}{l}=$	0.5	0.141	0.126	0.113	0.107
$\Delta f_1=$	0	0.028	0.045	0.067	0.079 .f.

Within the limiting values of $\frac{1}{4}$ and 4 for $\frac{p}{g}$ the following approximate values may be determined:

$$\left.\begin{array}{l}\frac{\xi}{l} = 0.1 + 0.025\,\frac{g}{p} \\ \Delta f_1 = 0.007 + 0.046\,\frac{p}{g} - 0.0075\,\frac{p^2}{g^2}\end{array}\right\} \quad \ldots\ldots\ldots (31)$$

2. *Deflection at the center due to changes in length of the chain.*—The differentiation of equation (25), when the relation $\frac{f}{l}=n$ is introduced, gives

$$\Delta f_2=\frac{15}{16\,(5\,n-24\,n^3)}\,\Delta L \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

Changes in the length of the chain may be due either to its elongation under stress, to variations in temperature, or to the compression or sliding of the anchorages.

If s represents the specific stress on the chain due to the greatest live load (p per unit of length), the elongation due to the dead load g of the bridge may be detrmined from,

$$\Delta L_g=\frac{s}{E}\,\frac{g}{g+p}.\,L \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

and the increased elongation due to the effect of the live load from,

$$\Delta L_p=\frac{s}{E}\,\frac{p}{g+p}.\,L \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

The change in length due to variations of temperature of $\pm\, t°$ is, when the coefficient of expansion $\omega=0.0000124$,

$$\Delta L_t=\pm\,\omega\, t\, L \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

When the main chains are continued as backstays on both sides to the anchorages (fig. 10), and when a sliding of the chains over the firmly fixed saddles is pos-

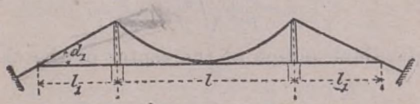


FIG. 10.

sible the whole length of the chain between the anchorages is to be introduced for L in equations (32) to (35); hence,

$$L=l\left(1+\frac{8}{3}\,n^2-\frac{32}{5}\,n^4\right)+2\,l_1\,sec\,\alpha_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

If, on the other hand, a shifting of the saddles occur before a sliding of the chains, we have, when $\Delta\, L$ represents the change in length of the main chain, and $\Delta\, L_1$ the change in the length of the backstays,

$$\Delta f_2=\frac{15}{16\,(5\,n-24\,n^3)}\,\Delta L+\frac{2}{\sqrt{1+n^2}}\,\frac{15-8\,(5\,n^2-36\,n^4)}{16\,(5\,n-24\,n^3)}\,\Delta L_1.$$

If in general we place

$$\Delta L=c.\,L=c.\left(1+\frac{8}{3}\,n^2-\frac{32}{5}\,n^4\right)l$$

$$\Delta L_1=c\,L_1=c.\sqrt{1+n^2}.\,l_1,$$

in which c is known from the coefficients of L in equations (33) to (35), we have

$$\Delta f_2=\left\{\frac{1}{2}\,l-l_1+\frac{15+96\,n^4}{16\,(5n-24\,n^3)}\,(l+2\,l_1)\right\}.\,c \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

3. *Deflection at the center due to a shifting of the chain saddles.*—When, in the case of a chain with fixed ends, the span of the main chain is reduced by $\Delta\, l$ by a shifting of the chain saddles without a simultaneous change in the length of the main chain, the center of the chain is depressed an amount,

$$\Delta f_3=\frac{15-8\,(5\,n^2-36\,n^4)}{16\,(5\,n-24\,n^3)}\,\Delta l \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38$$

If, simultaneously with the shifting of the chain saddles a sliding of the chain occurs in such a manner that its total length between anchorages (fig. 10) remains constant, the deflection at the middle of the chain will be,

$$\Delta f_3 = \frac{15 - 8\,(5\,n^2 - 36\,n^4) - 15\sqrt{1+n^2}}{16\,(5\,n - 24\,n^3)}\,\Delta l \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

4. *Maximum longitudinal motion of the center of the chain.*—For the lowest point of

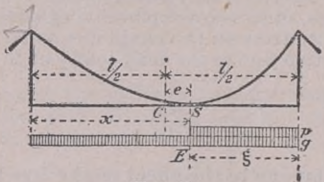


FIG. 11.

the curve of the cable $\frac{dM}{dx} = 0$; accordingly, as is evident from fig. 11, as long as S lies to the left of E,

$$\frac{1}{2} g\,(l - 2x) + \frac{1}{2} p \frac{\xi^2}{l} = 0.$$

Hence x has its maximum value, when ξ has its maximum; therefore, when ξ becomes $= l - x$. This substituted in the above equation gives,

$$\frac{\xi}{l} = -\frac{g}{p} + \sqrt{\frac{g}{p} + \frac{g^2}{p^2}},$$

and the greatest longitudinal motion of the center of the chain from the center of the bridge becomes,

$$\frac{e}{l} = \frac{1}{2} + \frac{g}{p} - \sqrt{\frac{g}{p} + \frac{g^2}{p^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

The versin of the chain may be assumed as constant for the relations of $g : p$ occurring in practice. The lifting of the chain in the middle of the bridge then is,

$$\Delta f_4 = \left(\frac{2e}{l + 2e}\right)^2 . f \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

from which we have,

for $\frac{g}{p} =$	$\frac{1}{3}$	$\frac{1}{2}$	1	2	3
$e =$	0.167	0.134	0.086	0.051	0.036 . l
$\Delta f_4 =$	0.0625	0.0445	0.0214	0.0084	0.0045 . f.

Example.—A steel wire cable bridge is to be designed for a street 12 meters wide. Span $l = 100$ meters, versin $f = 8$ meters. Two cables are to be used. The loads per cable and per linear meter are: Dead load of the roadway, 1,400 kilograms; live load $400 \times 6 = 2{,}400$ kilograms; therefore, $q_0 = 3{,}800$ kilograms. Assuming the stress $s = 2{,}000$, we have from equation (19) the horizontal tension $H = 632.2$ t.

Computing from this the weight of the cable as $g_0 = 260$ kilograms per meter and the weight of the suspenders as $g = 4$ kilograms, we have from equation (16), $q = 3{,}800 + 260 = 4{,}060$ and $\varkappa l^2 = 39$, with which from equation (18) the horizontal tension can be more accurately determined, as $H = 635.9$ t. Furthermore, equation (24) gives for the length of the chain $L = 101.672$ meters and equation (23) for the angle at the support $tg\,\tau_0 = 0.32034$.

Neglecting the stiffening due to the roadway or the inclined stays which are always employed, the changes of form may be computed as follows:

Maximum deflection for partial loading $\left(\text{from equation (31) for } \frac{p}{g} = \frac{2{,}400}{1{,}671}\right)$ $\Delta f_1 = 0.057 . f = 0.456$ meters; center deflection due to elastic elongation for total loading, from equations (37) and (34) when the horizontal length of the backstay is assumed as $l_1 = 25$ meters, $\Delta f_2 = 0.208$ meters; deflection due to a variation of temperature of $\pm\, 30°$ C., $\Delta f_t = \pm\, 0.144$ meters. Finally the greatest longitudinal motion of the center of the bridge will be from equation (40), $e = 0.110\,l = 11$ meters, and the greatest lifting of the center of the bridge will be from equation (41) $-\Delta f_4 = 0.0325\,f = 0.260$ meters. The vertical motion of the center of the bridge when the greatest live load is crossing would then be, if the bridge is wholly unstiffened, approximately $+0.46$ and -0.26; that is, 0.72 meters.

4. *Secondary strains.*—The preceding theory of slack suspension construction assumes that the chain or the cable can always assume the position of equilibrium corresponding to the load; that is, the resistance of friction between the links of the chain or of stiffness in the cable was neglected. Although the latter is actually so small that it has practically no influence upon the stress on the cable, this in general is not true as to the resistance of friction between the links of a chain. Experiments made by Steiner and Fränkel on existing chain bridges with the aid of Fränkel's elongation indicator have proven the existence of considerable bending stresses in the individual links of the chains.

Let T represent the force acting in the axis of a link, d the diameter of a link pin, and φ the coefficient of friction; then the moment produced in the link by the action of friction may reach a value $M\rho = \varphi\, T.\frac{d}{2}$. For approximation, we may place $\varphi\, T = \varphi'\, H$ if we let $\varphi' = \varphi\left(1 + 8\frac{f^2}{l^2}\right)$ for the link subject to the greatest strain. In order now to obtain the maximum value of the bending moment we must determine H for such an unequal loading as will produce a moment tending to change the form of the chain and equal to the moment of friction $\varphi'\, H\, \frac{d}{2}$. For this purpose we may consider the live load p per unit of length divided into two parts, p' and p'', the former of which, only partially applied, produces the moment which tends to change the form of the chain and may be approximately determined from equation (86[a]) as $0.0165\ p'\, l^2$, while the second part p'' is to be taken over the entire span. Then, neglecting the effect of the load p' in the determination of H, we have

$$0.0165\ p'\, l^2 = \varphi' . \frac{1}{8}\,(p'' + g)\,\frac{l^2}{f} . \frac{d}{2};$$

from which,

$$p'' + g = \frac{0.264}{0.264 + \varphi'\,\frac{d}{f}}\,(p + g);$$

and the greatest bending moment is,

$$M_\rho = \frac{\varphi' . d}{16}\,\frac{0.264}{0.264 + \varphi'\,\frac{d}{f}}\,(p + g).\frac{l^2}{f}$$

From this, knowing the cross sections of the links of the chain, the bending stresses are also easily determined. In old chain bridges, particularly on account of the possible presence of rust, it will probably be necessary to take for φ, the coefficient of friction, a rather high value—at least 0.20.

Example.—In an existing chain bridge of 83.7 meters span and 5.673 meters versin, the dead load on one main chain is $g = 968$ kilograms and the live load is $p = 1{,}218$ kilograms per linear meter. For this we calculate $H_{g+p} = 327.2$ t. The diameter of the link pin is $d = 0.05$ meters. Placing $\varphi' = 0.25$, then $\frac{0.264}{0.264 + \varphi'\,\frac{d}{f}} = 0.99$ and $M\rho = 0.25 \times 0.025 \times 337.2 \times 0.99 = 2.086$ t.

The chain consists of five links 13.2 centimeters wide and 3.1 centimeters thick; accordingly we have for one link a bending moment of $\frac{208{,}600}{5} = 41{,}720$ kilogram, centimeters and therefore with a resisting moment of a link of 90 cm³ the maximum bending stress with most unfavorable loading is $\frac{41{,}720}{90} = 462$ kilograms per square centimeter.

B.—Stiffening of the Archpolygon by a Straight Girder.

§ 5. *Approximate theory.*

In order to limit the magnitude of the changes of form in the archpolygon and slack cable, which have been computed in § 4, they have been combined, as previously mentioned, with a straight girder by means of suspenders, and thereby a generally statically indeterminate truss system has been formed. We will next give the approximate theory of this system which, though approximate, is fully sufficient for practical application in most cases and in which both of its modifications, represented in figs. 2 and 3, will be treated together.

1. *Conditions of equilibrium of the archpolygon.*—Equations (1) to (5) in § 3 also hold good here when we substitute therein for the forces P the stresses S of the suspenders increased at the joints by the forces K resulting from the dead load of the archpolygon. If the suspenders are placed at equal distances a, we have from equation (3^a).

$$-H.\Delta^2 y_m = (S_m + K).a \qquad (42)$$

If the points of the polygon lie upon a parabola with a vertical axis whose versin

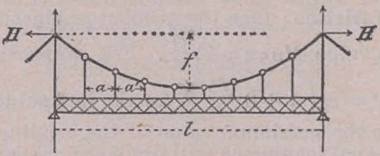


FIG. 12.

is f, and if the suspenders adjoining the mth suspender are distant therefrom a and a' respectively, we have from equation (3^b)

$$H.\frac{4f(a+a')}{l^2} = S_m + K \qquad (43)$$

If, therefore, in a parabolic polygon the horizontal distances between joints are equal, the stresses in the suspenders also all become equal, and in general the girder seems to be assigned the duty of distributing the varying live loads to the suspenders in such a manner as is necessitated by the form of the polygon of the chain or cable.

For a continous curve the vertical forces acting on an arch for a horizontal unit of length are determined from,

$$s_m + k = -H.\frac{d^2y}{dx^2}; \qquad (44)$$

and for a parabolic curve more especially from,

$$s + k = H.\frac{8f}{l^2} \qquad (45)$$

We will now assume for the approximate theory that the foregoing equations (42) to (45) are true also after the change of form; that is, we will suppose this change to be so small that the distribution of the stresses in the suspenders, due to the form of the archpolygon, remains practically the same. This supposition seems the more admissible since the stiffer is the girder constructed, the smaller are its changes of form, due to elasticity, which are transferred through the suspenders to the archpolygon.

2. *External forces of the stiffening truss.*—These are composed of the weight of the truss and the parts suspended from or supported upon it, the live load, and the stresses in the suspenders which will act as vertical forces directed upward.

If we neglect the last-mentioned forces we may readily determine the moment $\mathfrak{M}$ and the shear $\mathfrak{Q}$ for any cross-section of the girder, distant x from the support, as in an ordinary unsupported girder (simple or continuous, according as it rests upon two or more supports). This moment and shear would exist if the chain or the arch were not present and the whole load were carried by the girder alone. If M_s represents the bending moment for the cross section under consideration due to the forces acting through the suspenders, the resulting total moment in the stiffening girder is,

$$M = \mathfrak{M} - M_s.$$

Draw a straight line (fig. 13) through the points of support, A and B, of the chain or arch as axis of abscissas and measure from it the ordinates of the archpolygon. We have, from a well-known property of the equilibrium polygon, and on the supposition that the dead load of the chain (k per unit of length) is uniformly distributed, for the girder with length of span of $A\,B = l$ and supported at two points,

$$M_s + \frac{1}{2}\,k\,x\,(l - x) = H\,.\,y\,;$$

from which,

$$M = \mathfrak{M} + \frac{1}{2}\,k\,x\,(l - x) - H\,y.$$

By neglecting the dead load of the chain or by including it in the forces acting in

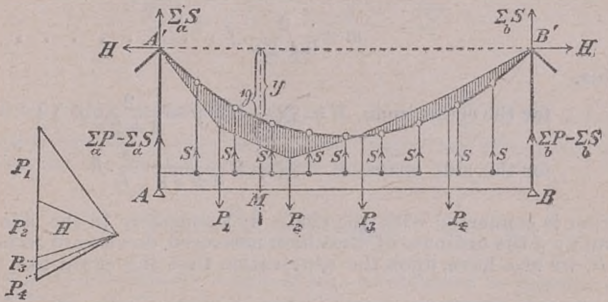


Fig. 13.

the stiffening truss, if we represent $\mathfrak{M}$ by the ordinates $\mathfrak{y}$ of an equilibrium polygon or equilibrium curve which was constructed for the external forces of the girder and for a pole distance $= H$, we have,

$$M = \mathfrak{M} - H\,y = H\,(\mathfrak{y} - y) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

The moment in the stiffening girder is, therefore, proportional to the vertical distance between the equilibrium polygon for the external forces constructed through the points A and B, and the axis of the archpolygon (fig. 13).

For a continuous stiffening girder extending over several spans

$$M_s = H\,y - M''\,\frac{x}{l} - M'\,\frac{l - x}{l} = H\left(y - m''\,\frac{x}{l} - m'\,\frac{l - x}{l}\right),$$

whence we have,

$$M = \mathfrak{M} - H\left(y - m''\,\frac{x}{l} - m'\,\frac{l - x}{l}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

when M' and M'' represent the moments at the points of support of a continuous girder, the points of support being at the suspenders and adjacent to the correspond-

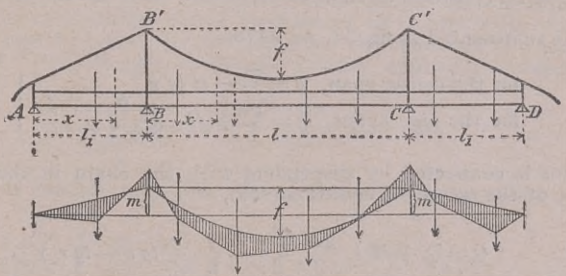


Fig. 14.

ing spans. M' and M'', with a given form of the archpolygon, can always be easily computed or constructed as distances m' and m'', for a pole distance $H = 1$.

In the case represented in fig. 14, where, with a symmetrical arrangement, the girder is prolonged over the two smaller side spans, but is not suspended from the backstays, we have,

$$\left.\begin{array}{l}\text{for the center span, } M = \mathfrak{M} - H(y - m) \\ \text{for the side spans, } M = \mathfrak{M} + H.m\frac{x}{l_1}\end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad (48)$$

For a uniform distribution of the stresses in the suspenders, that is, for a parabolic chaincurve, and approximately, also, for a parabolic archpolygon, we have with the relation $\nu = \frac{l_1}{l}$ between the spans,

$$m = \frac{2}{3+2\nu}.f \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

and from this,

$$\left.\begin{array}{l}\text{for the center span, } M = \mathfrak{M} - H\left(y - \frac{2}{3+2\nu}f\right) \\ \text{for the side spans, } M = \mathfrak{M} + H\frac{2}{3+2\nu}\cdot\frac{x}{l_1}.f\end{array}\right\} \quad . \quad . \quad . \quad . \quad (49^a)$$

If the girder is connected with the chain by suspenders in the side spans, and if we designate by y the ordinate of the chain measured downward from the connecting line $A\ B$, we also have, upon the supposition that the chain has the curve of a parabola,

$$m = \frac{2\,(1+\nu^3)}{3+2\nu}f; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

and from this,

$$\left.\begin{array}{l}\text{for the center span, } M = \mathfrak{M} - H\left(y - \frac{2\,(1+\nu^3)}{3+2\nu}f\right) \\ \text{for the side spans, } M = \mathfrak{M} - H\left(y_1 - \frac{2\,(1+\nu^3)}{3+2\nu}\cdot\frac{x}{l_1}f\right)\end{array}\right\} \quad . \quad . \quad (50^a)$$

For the computation of the shears Q, the following formulas may be used:
When the girder extends over only one span,

$$Q = \mathfrak{Q} - H.\,tg\,\tau \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (51)$$

When the girder is continuous over several spans

$$Q = \mathfrak{Q} - H\left(tg\,\tau - tg\,\sigma + \frac{m' - m''}{l}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

in which τ represents the angle which the side of the archpolygon cut by the cross section, or the tangent to the chain curve, makes with the horizontal; and σ the angle which the connecting line between the points of support of the chain makes with the horizontal (positive when directed downward). m' and m'' are the same as above.

For the case represented in fig. 14, we have,

$$\left.\begin{array}{l}\text{for the center span, } Q = \mathfrak{Q} - H\,tg\,\tau \\ \text{for the side spans, } Q = \mathfrak{Q} + H\,\frac{2}{3+2\nu}\,\frac{f}{l_1}\end{array}\right\} \quad . \quad . \quad . \quad . \quad (53)$$

If the girder is connected by suspenders with the chain in the side spans we have, in place of the second of equations (53),

$$Q = \mathfrak{Q} + H\left(\frac{2\,(1+\nu^3)}{3+2\nu}\,\frac{f}{l_1} + tg\,\sigma - tg\,\tau\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

3. *Determination of the horizontal stress H.*—In the truss systems treated above the horizontal stress can only be statically determined when, through the method of construction, another condition is given that will remove the static indetermi-

nation existing with reference to this stress. Thus, for instance, we might in the first place produce in the chain a constant horizontal stress by substituting an attached weight for one of the anchorages. The stiffening girder would in this case be subject to a constant uplifting due to constant forces acting upward. Another arrangement which, however, has so far only been proposed, consists in balancing the forces in the abutment; that is, in bringing into a definite relation the vertical forces in the chain and in the girder at the abutment by supporting both of these

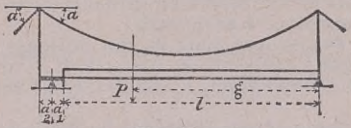


FIG. 15.

systems upon a balancing contrivance (fig. 15.). If $\varkappa = \frac{a_1}{a_2}$ be the relation of the lever-arm suppporting the girders to the arm supporting the chain, we have by this arrangement and the loading indicated in fig. 15,

$$H = \frac{P.\,\xi}{2l}\;\frac{\varkappa}{1+\varkappa}\,cotg\;\alpha \;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\quad (55)$$

Finally, a statically determinate form may also be obtained by introducing a hinge in the stiffening truss. In this case we must place the moment M for the cross section through the hinge equal to zero, whereby we obtain a determinate equation for H. If we place the hinge at the center of the truss and designate the moment of the free-ended girder with reference thereto by $\mathfrak{M}_0$, we have, from equation (46) for a truss extending only over a single span,

$$H = \frac{\mathfrak{M}_0}{f};\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\quad (56)$$

and for a continuous truss, from equation (48),

$$H = \frac{\mathfrak{M}_0}{f - m}\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\quad (57)$$

But if none of the arrangements just discussed is chosen we must, in accordance with the general statement previously made with reference to statically indertermi-nate systems, deduce the missing determinate equation for the horizontal stress from the elastic changes of form of the system. For this purpose we can follow a different method. We can place the change of the ordinates of the joints of the archpolygon, increased by the change in length of the suspenders, equal to the deflections of the stiffening truss, and develop therefrom the expression for the horizontal stress. But we arrive at this in a shorter way by the application of the "theorem of least work to produce change of form."

Let P represent the axial force, M the bending moment which acts upon the cross sections of the separate parts of the system whose length we will represent by s; let F be the area of cross section and J its moment of inertia with reference to an axis through its center of gravity; and let E be the coefficient of elasticity; then the expression for "work to produce change of form," evidently becomes,

$$A = \int \frac{P^2}{2EF}\,ds + \int \frac{M^2}{2EJ}\,ds = min.\;.\;.\;.\;.\;.\;.\;.\quad (58)$$

and the equation of condition deduced therefrom with reference to the case in question becomes,

$$\frac{dA}{dH} = \int \frac{P}{EF}\,\frac{dP}{dH}\,ds + \int \frac{M}{EJ}\,\frac{dM}{dH}\,ds = 0\;.\;.\;.\;.\;.\quad (59)$$

To apply this equation it is necessary to express the axial force and the moment for the separate parts of the system in terms of the external forces and the unknown horizontal strain H.

We will next consider the truss system (shown in fig. 12) of a suspension bridge with a stiffening girder covering only one span and represented by—

F, the area of any cross section of the chain, and by F_0, that at the center of the chain;

λ, the length of a chain link, that is, one side of the polygon;

a, the distance between two suspenders;

$2F_1$, the area of the cross section of the stiffening girder (in case of a framed truss, the areas of both chords);

J, the moment of inertia of the stiffening girder;

F_2, the area of cross section of a suspender;

y', the length of the suspenders, that is, the ordinate between the axis of the girder and the archpolygon;

y, the ordinate of the archpolygon, measured from the chord joining the end points;

h, the height of the stiffening truss (distance between flanges of the girder);

f, the versin of the chain;

f', the height of the end verticals; F_3, their cross section;

l_1, the horizontal projection of the backstays;

α_1, their angle of inclination to the horizontal;

α, the angle of the main chain at the point of support;

G, the total load acting on the stiffening girder;

A and B, the abutment reactions due to it;

K, the load at the joints due to the dead load of the archpolygon, assumed uniformly distributed and equal to k per linear meter.

If we consider that the stress on the chain is

$$T_m = H\frac{\lambda}{a}, \qquad (60)$$

and if we make its cross section proportional to the stress in each member, then will $F = F_0\frac{\lambda}{a}$ and the cross section of the backstays will be $F' = F_0 \sec.\alpha_1$.

The expressions for the separate parts of the system given by equation (59) may now be computed, for which purpose the following table will be useful. In it are introduced the stresses on the suspenders from equation (42), and it is also assumed either,

(a) that the stiffening truss is not independently supported, but suspended with its ends on the points of support of the chain; or,

(b) that the stiffening truss has an independent support.

Part of system.	P and M respectively.	$\frac{dP}{dH}$ and $\frac{dM}{dH}$ respectively.	F and J respectively.	$\int ds$.	$E\frac{dA}{dH}$
Main chains	$H\frac{\lambda}{a}$	$\frac{\lambda}{a}$	$\frac{\lambda}{a}F_0$	λ	$\frac{H}{aF_0}\sum_0^l \lambda^2$
Backstays	$H \sec \alpha_1$	$\sec \alpha_1$	$F_0 \sec \alpha_1$	$2l_1 \sec \alpha_1$	$\frac{2H}{F_0} l_1 \sec^2 \alpha_1$
Suspenders	$H\frac{\Delta^2 y}{a} - K$	$\frac{\Delta^2 y}{a}$	F_2	y'	$\frac{H}{a^2 F_2}\sum_0^l y'(\Delta^2 y)^2 - \frac{K}{aF_2}\sum_0^l y'\Delta^2 y$
(a)-End verticals.	$A - H\,tg\,\alpha$ $B - H\,tg\,\alpha$	$-tg\,\alpha$	F_3	$2f'$	$\frac{2H}{F_3} f'\,tg^2\alpha - \frac{G}{F_3} f'\,tg\,\alpha$
(b)-Piers...	$H(tg\,\alpha + tg\,\alpha_1)$	$(tg\,\alpha + tg\,\alpha_1)$	F_3	$2f'$	$\frac{2H}{F_3} f'(tg\,\alpha + tg\,\alpha_1)^2 \cdot \frac{E}{E'}$
Stiffening girder.	$\mathfrak{M} + \frac{1}{2}kx(l-x) - Hy$	$-y$	J	$\int dx$	$H\int_0^l \frac{y^2 dx}{J} - \int_0^l \frac{\mathfrak{M}}{J} y\,dx - \frac{1}{2}k\int_0^l \frac{yx(l-x)dx}{J}$

If we consider that

$$\sum_0^l \lambda^2 = \sum_0^l a^2 + \sum_0^l (\Delta y)^2 = la - \sum_0^l y\,\Delta^2 y,$$

and that for an approximately parabolic archpolygon we may write

$$\int \frac{yx(l-x)}{J}dx = \frac{l^2}{4f}\int \frac{y^2 dx}{J},$$

we have from $\frac{dA}{dH}=0$, the following expressions for the horizontal stress:

For case (a)—

$$H=\frac{F_0\int_0^l \frac{\mathfrak{M}y}{J}\,dx+f'\,tg\,\alpha\,\frac{F_0}{F_3}G+\left(+\frac{F_0}{F_2}\sum_0^l y'\Delta^2 y+\frac{F_0\,l^2}{8f}\int_0^l\frac{y^2\,dx}{J}\right)k}{l-\sum_0^l\frac{y}{a}\,\Delta^2 y+2\,l_1\sec^2\alpha_1+\frac{F_0}{F_2}\sum_0^l\frac{y'}{a^2}(\Delta^2 y)^2+2f'\,\frac{F_0}{F_3}\,tg^2\alpha+F_0\int_0^l\frac{y^2\,dx}{J}}, \quad (61)$$

For case (b)—

$$H=\frac{F_0\int_0^l\frac{\mathfrak{M}\,y}{J}dx+\left(+\frac{F_0}{F_2}\sum_0^l y'.\,\Delta^2 y+\frac{F_0\,l^2}{8f}\int_0^l\frac{y^2\,dx}{J}\right)k}{l-\sum_0^l\frac{y}{a}\,\Delta^2 y+2\,l_1\sec^2\alpha_1+\frac{F_0}{F_2}\sum_0^l\frac{y'}{a^2}(\Delta^2 y)^2+2f'\,\frac{F_0}{F_3}\,\frac{E}{E_1}(tg\,\alpha+tg\,\alpha_1)^2+F_0\int_0^l\frac{y^2\,dx}{J}} \quad (62).$$

Equation (61) also gives, with $l_1=0$, the horizontal thrust of an arch stiffened by a straight girder (fig.16).

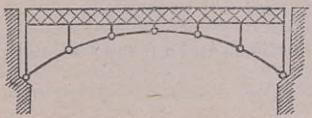


Fig. 16.

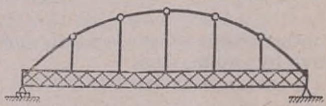


Fig. 17.

Equation (61) is also applicable to the system represented in fig. 17, when l_1 and $f'=0$, and $y'=y$ and the denominator of the expression is increased by the term $\frac{F_0}{2F_1}\,l$ deduced from the axial force in the girder.

The same term, with simultaneous substitution of $l_1=0$, is to be added also to the denominators of equations (61) and (62) if, instead of the backstays, a straining beam with cross section $=2F_1$ is applied.

If we divide both numerator and denominator of the preceding equations by F_0 and include those terms not containing the moment of inertia J under the designations S and s, the general expression for H becomes,

$$H=\frac{\int_0^l\frac{\mathfrak{M}\,y}{J}\,dx+\frac{1}{F_0}\,S+\frac{k\,l^2}{8f}\int_0^l\frac{y^2\,dx}{J}}{\int_0^l\frac{y^2\,dx}{J}+\frac{s}{F_0}} \quad \ldots\ldots \quad (63)$$

Since the second term of the numerator, as compared with the first, can be neglected in all cases occurring in practice, we have a close approximation,

$$H=\frac{\int_0^l\frac{\mathfrak{M}\,y}{J}\,dx}{\int_0^l\frac{y^2\,dx}{J}+\frac{s}{F_0}}+\frac{k\,l^2}{8f}\,(1-\varkappa), \quad \ldots\ldots \quad (63^a)$$

in which $\varkappa$ represents the quantity

$$\frac{\frac{8}{l}}{F_0 \int\limits_0^l \frac{y^2\, dx}{J}}$$

which is always very small.

We will now suppose that the moment of inertia J of the girder can be assumed constant for a sectional length a and assume J_m to be the moment of inertia for any assumed section between the $(m-1)$ th and mth suspenders. Then the two definite integrals in the above formula can be resolved into a series, which will be obtained by extending the integration to the different sectional lengths a_m. Therefore, if we measure the abscissa x from the $(m-1)$ th suspender, we may place,

$$y = y_{m-1} + \frac{x'}{a_m}(y_m - y_{m-1}) \text{ and } \mathfrak{M} = \mathfrak{M}_{m-1} + \frac{x'}{a_m}(\mathfrak{M}_m - \mathfrak{M}_{m-1}),$$

when $\mathfrak{M}_{m-1}$ and $\mathfrak{M}_m$ are the simple moments of the girder with reference to the joints $m-1$ and m. Then we obtain,

$$\frac{1}{J_m}\int\limits_0^{a_m} \mathfrak{M}\, y\, dx = \frac{a_m}{6\,J_m}\left[\mathfrak{M}_{m-1}(2\,y_{m-1} + y_m) + \mathfrak{M}_m(2\,y_m + y_{m-1})\right],$$

$$\frac{1}{J_m}\int\limits_0^{a_m} y^2\, dx = \frac{a_m}{6\,J_m}\left[y_{m-1}(2\,y_{m-1} + y_m) + y_m(2\,y_m + y_{m-1})\right].$$

If we now further assume a mean moment of inertia J_0 and a mean sectional length a_0 we have by summation,

$$\int\limits_0^l \frac{\mathfrak{M}\, y\, dx}{J} = \frac{a_0}{6\,J_0}\Big[\ldots + \frac{a_m}{a_0}\frac{J_0}{J_m}\Big\{\mathfrak{M}_{m-1}(2\,y_{m-1}+y_m) + \mathfrak{M}_m(2\,y_m + y_{m-1})\Big\} +$$

$$+ \frac{a_{m+1}}{a_0}\frac{J_0}{J_{m+1}}\Big\{\mathfrak{M}_m(2\,y_m + y_{m+1}) + \mathfrak{M}_{m+1}(2\,y_{m+1} + y_m)\Big\} + \ldots\Big]$$

$$= \frac{a_0}{6\,J_0}\sum_0^l \mathfrak{M}_m\left[\frac{a_m}{a_0}\frac{J_0}{J_m}(2\,y_m + y_{m-1}) + \frac{a_{m+1}}{a_0}\frac{J_0}{J_{m+1}}(2\,y_m + y_{m+1})\right];$$

and also,

$$\int\limits_0^l \frac{y^2\, dx}{J} = \frac{a_0}{6\,J_0}\sum_0^l y_m\left[\frac{a_m}{a_0}\frac{J_0}{J_m}(2\,y_m + y_{m-1}) + \frac{a_{m+1}}{a_0}\frac{J_0}{J_{m+1}}(2\,y_m + y_{m+1})\right].$$

Placing for abbreviation,

$$v_m = \frac{a_m}{6\,a_0}\frac{J_0}{J_m}(2\,y_m + y_{m-1}) + \frac{a_{m+1}}{6\,a_0}\frac{J_0}{J_{m+1}}(2\,y_m + y_{m+1}), \quad \ldots \quad (64)$$

we finally obtain, by substitution in equation (63^a), for the horizontal force due to the external forces, (not weight of cable for which $H_c = \frac{kl^2}{8f}(1-\varkappa)$)

$$H = \frac{\sum\limits_0^l \mathfrak{M}_m v_m}{\sum\limits_0^l y_m v_m + \frac{8}{a_0}\frac{J_0}{F_0}} \quad \ldots \quad (65)$$

If the truss is subject only to a single load G (fig. 18) which lies at a distance ξ from the support, the two summations of the preceding expression can be given a

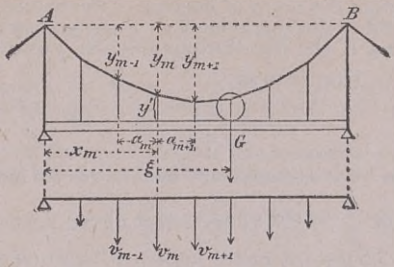


FIG. 18.

static significance which will make their determination possible also by graphical methods. That is to say, if x_m represents the abscissa of any assumed joint,

$$\sum_0^l \mathfrak{M}_m v_m = G\left[\frac{l-\xi}{l}\sum_0^\xi x_m v_m + \frac{\xi}{l}\sum_\xi^l (l-x_m)\, v_m\right] = G\, m_\xi$$

$$\sum_0^l (m_\xi)\, v_m = \mu,$$

in which m_ξ represents the moment of a girder with free ends, the forces v being considered as acting in the verticals at the joints, and the moment being referred to the line of application of the load G. μ is the static moment, with reference to the chord AB, of these forces v which are considered as acting horizontally through the joints of the archpolygon. These moments can also be readily represented by the ordinates of equilibrium polygons.

If the horizontal distance a between joints is constant we have,

$$v_m = \frac{J_0}{6J_m}(2y_m + y_{m-1}) + \frac{J_0}{6J_{m+1}}(2y_m + y_{m+1});$$

and for a constant moment of inertia J,

$$v_m = \frac{1}{6}(y_{m-1} + 4y_m + y_{m+1});$$

or with sufficiently close approximation, $v_m = y_m$.

The graphical construction is to be made in precisely the same manner as in fig.

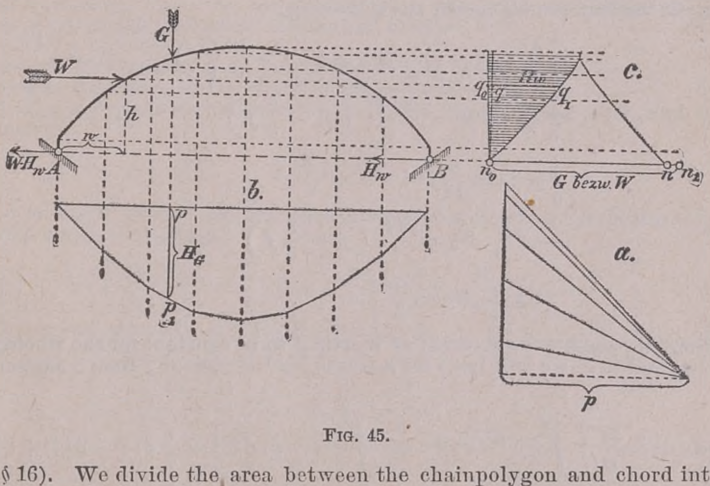


FIG. 45.

45 (see § 16). We divide the area between the chainpolygon and chord into strips of a width a, which can be assumed as equal to the distance between suspenders,

and compute the load element v either from equation (64) or assume them approximately equal to y. Then with the pole distance p the two equilibrium polygons b and c are constructed for the vertical and horizontal applications of the forces v. The ordinate drawn through the point of loading of the equilibrium polygon b gives the horizontal stress H when the single load G is represented by a length which is composed of a section $n_0 n$ of the equilibrium polygon c on the chord AB, and the length $n\, n_1 = c = \frac{s}{ap}\, \frac{J}{F_0}$. That is to say,

$$H = \frac{m\xi}{\mu + cp}.\ G \quad \ldots \ldots \ldots \quad (66)$$

(α) In a computed example l was assumed $=50$ meters, $f = 6.5$ meters, $\Sigma \frac{y}{a} \Delta^2 y = 4.506$ meters, $l_1 = 15$ meters, and $\frac{J}{F_0} = 2.049$; the effect of the suspenders was neglected; it was assumed that $a = 2.5$ meters, $p = 35$ meters, wherefrom $c = 2.167$ meters. For the load at the center of the bridge the construction gave $H = \frac{19.3}{15.0} = 1.286\ G$.

(β) If the stiffening girder is a framed truss the preceding equations (63) to (66) can also be used. J is then the moment of inertia of both upper and lower chord; that is, with a cross section F_1 for each chord and a distance h between their centers of gravity, $J = F_1 \frac{h^2}{2}$. J may be assumed constant for each panel length a or, in the first approximate computation, for the whole length of the truss. If we also wish to consider the effect of the web members we must, with reference to equation (51), increase the numerator in expressions (61) and (62) for H, by $\frac{h}{a} sec^3\, \gamma \Sigma\, \mathfrak{D}\, \frac{F_0}{F_4} \Delta y$, and the denominator by $-\frac{h}{a^2}\ sec^3\, \gamma\ \Sigma \frac{F_0}{F_4}\, y\, \Delta^2 y$, in which F represents the cross secton of a web member, and γ the angle of inclination of the member to the vertical. We must therefore place in case (a), equation (63), and in the following equations,

$$S = G\,.f'\, tg\, \alpha\, \frac{F_0}{F_3} - k \frac{F_0}{F_2} \sum_0^l y'\, \Delta^2 y + \frac{h}{a}\ sec^3\, \gamma \sum_0^l \mathfrak{D}\ \frac{F_0}{F_4} \Delta y$$

$$s = l - \sum_0^l \frac{y}{a}\, \Delta^2 y + 2 l_1\, sec^2\, \alpha_1 + \frac{F_0}{F_2}\, \Sigma\, \frac{y'}{a^2} (\Delta^2 y)^2 + 2 f'\, \frac{F_0}{F_3}\, tg^2\, \alpha - \frac{h}{a^2}\, sec^3 \gamma \sum_0^l \frac{F_0}{F_4}\, y\,.\ \Delta^2\, y.$$

But the influence of the web members of the stiffening truss, as well as the suspenders, so far as concerns the total change of form, is so small that these members may be neglected without appreciable error. For practical conditions, their influence upon the value of H amounts to not more than 0.3 to 0.5 per cent.

(γ) If the chain or arch has the form of a *parabola* and the distance a between suspenders be assumed to be infinitely small, we have, when $h_1 = f' - f$ represents the distance of the center of the arch from the axis of the girder, for the summations appearing in the expressions for H, the following:

$$\sum_0^l \frac{\lambda^2}{a} = l - \sum_0^l \frac{y}{a}\, \Delta^2 y = l \left(1 + \frac{16}{3} \frac{f^2}{l^2}\right)$$

$$\sum_0^l \frac{y'}{a^2} (\Delta^2 y)^2 = \frac{64\, f^2}{l^3} \left(f' - \frac{2}{3} f\right)$$

$$\int_0^l y^2\, dx = \frac{8}{15} f^2\, l$$

$$-\Sigma \frac{y'}{a} \Delta^2 y = \frac{8 f}{l} \left(f' - \frac{2}{3} f\right)$$

$$-\Sigma \frac{y}{a} \Delta^2 y = \frac{16}{3} \frac{f^2}{l}.$$

(δ) If, further, we assume the moment of inertia J to be constant for the whole length of the beam and $= J$, we will have for a single load G distant ξ from a support,

$$\int_0^l \mathfrak{M}\, y\, dx = G\, \frac{4f}{l^2} \left[\int_0^\xi x^2 (l - x)\, dx + \xi \int_\xi^{\frac{l}{2}} x\, (l - x)\, dx \right] = \frac{1}{3}\, \xi\, (\xi^3 - 2\, l\, \xi^2 + l^3)\, \frac{f}{l^2}\ G.$$

Substituting this in equation (61) and designating by F_2 the area of cross section of the suspenders corresponding to a linear meter of the girder, the horizontal stress becomes,

$$H=\frac{\left[\frac{1}{3}\xi(\xi^3-2l\xi^2+l^3)+4f'l\frac{J}{F_3}\right]G+\left[8f\left(f'-\frac{2}{3}f\right)\frac{J}{F^2}+\frac{l^4}{15}\right]kl}{\frac{8}{15}fl^3+\frac{l^3}{f}\left(1+\frac{16}{3}\frac{f^2}{l^2}\right)\frac{J}{F_0}+\frac{2l_1l^2}{f}sec^2\alpha_1\frac{J}{F_0}+64\frac{f}{l}\left(f'-\frac{2}{3}f\right)\frac{J}{F_2}+32ff'\frac{J}{F_3}} \qquad (67)$$

Neglecting the effect of the elongation of the suspenders and leaving out of consideration the weight of the chain itself we have,

$$H=\frac{5\frac{\xi}{l}\left(\frac{\xi^3}{l^3}-2\frac{\xi^2}{l^2}+1\right).G}{8\frac{f}{l}+15\frac{J}{fl^2F_0}\left(l+\frac{16}{3}\frac{f^2}{l}+2l_1sec^2\alpha\right)} \quad .\ .\ .\ .\ \ (67^a)$$

The preceding equation will be sufficient, as a rule, for the first approximate computation, with which as a basis we can determine the variation of the moment of inertia J and then repeat the computation with the more accurate equations (63) and (65). Of course, even in the first computation, a supposition in regard to the relation $\frac{J}{F_0}$ is necessary; but this value lies within certain limits for which the variations in H is not very important. If v represents the relation $\frac{J}{F_0f^2}$ and Δv its variation, then, approximately, $\frac{\Delta H}{H}=-3.\Delta v$. With a height h and an area of cross section $2F$ for the stiffening truss, $v=\frac{F_1}{F_0}\frac{h^2}{4f^2}$, and this relation may be assumed in extreme cases as between 0 and $\frac{1}{20}$. Between these limiting values, the variation in the horizontal stress would not amount to more than about 15 per cent.

Applying the same method of computation to the development of equation (61,)

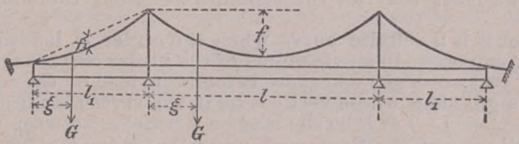


FIG. 19.

we may also deduce the expression for the horizontal stress in a girder continuous over the three spans (fig. 19). We will here content ourselves with the approximate equation depending upon the assumption of a parabolic weightless chain and a stiffening girder with a constant moment of inertia.

In the following let—

f represent the versin of the chain in the middle span l;

f_1 represent the versin of the chain in the side spans l_1;

J and J_1 the respective moments of inertia of the stiffening truss, and place for abbreviation,

$$\frac{2(l^3+l_1^3)}{l^2(3l+2l_1)}=\varepsilon.$$

Then for a single load G on the middle span at a distance ξ from the support we have, with a parabolic chain,

$$H=\frac{\left(\frac{\xi}{l}\right)^3-2\left(\frac{\xi}{l}\right)^2+1-\frac{3}{2}\varepsilon\left(1-\frac{\xi}{l}\right)-\frac{3}{2}\varepsilon\frac{l^3}{l^3+l_1^3}\left(1-\frac{\xi}{l}\right)\left[\left(1-\frac{3}{2}\varepsilon\right)+\right.}{\left(\frac{8}{5}-4\varepsilon+3\varepsilon^2\right)\frac{f}{l}+2\frac{J}{J_1}\left(\frac{8}{5}\frac{f_1^2}{f^2}-2\varepsilon\frac{f_1}{f}+\varepsilon^2\right)\frac{f}{l}\frac{l_1}{l}+3\frac{J}{F_0fl}\left[1+\frac{16}{3}\frac{f^2}{l^2}+\right.}$$

$$\frac{\left.\left(\frac{f_1}{f}-\varepsilon\right)\frac{l_1}{l}\frac{J}{J_1}\right]}{\left.2\frac{l_1}{l}\left(1+\frac{16}{3}\frac{f_1^2}{l_1^2}\right)\right]}\frac{\xi}{l}G \quad .\ .\ .\ .\ .\ .\ .\ .\ \ (68)$$

If the single load G is on a side span at a distance ξ from the abutment the horizontal stress in the chain becomes,

$$H=\frac{\left\{\left[\left(\frac{\xi}{l_1}\right)^3-2\left(\frac{\xi}{l_1}\right)^2+1\right]\frac{f_1}{f}-\frac{1}{2}\varepsilon\left(1-\left(\frac{\xi}{l_1}\right)^2\right)-\frac{1}{2}\varepsilon\frac{l^3}{l^3+l_1^3}\left(1-\left(\frac{\xi}{l_1}\right)^2\right)\right.}{\left(\frac{8}{5}-4\varepsilon+3\varepsilon^2\right)\frac{f}{l}+2\frac{J}{J_1}\left(\frac{8}{5}\frac{f_1^2}{f^2}-2\varepsilon\frac{f_1}{f}+\varepsilon^2\right)\frac{f}{l}\frac{l_1}{l}+3\frac{J}{F_0 f l}}$$

$$\frac{\left.\left[\left(1-\frac{3}{2}\varepsilon\right)\frac{J_1}{J}+\left(\frac{f_1}{f}-\varepsilon\right)\frac{l_1}{l}\right]\right\}\frac{J}{J_1}\frac{l_1^2}{l^2}\frac{\xi}{l_1}}{\left[1+\frac{16}{3}\frac{f^2}{l^2}+2\frac{l_1}{l}\left(1+\frac{16}{3}\frac{f_1^2}{l_1^2}\right)\right]}G \quad . \; . \; . \; . \; . \quad (68^a)$$

The effect of the elongation of the suspenders is not considered in the above.

If in the two preceding equations we place $\varepsilon=0$, we obtain the expressions for the horizontal stress in a stiffening truss extending over three spans, but *not continuous* over the center supports. If, on the other hand, we place $l_1=0$ in equation (68) we obtain the horizontal stress for a stiffening truss extending over only one opening and fixed horizontally at the ends.

To determine *the effect of moving loads* it is preferable to make use of the principles of the method of moments.

For this investigation and the later applications of this method, the general principles of the method of moments may be given here. In order to obtain the diagram of moments for a given weight, whose effect as a moving load we wish to determine, we must draw the ordinates of that weight at the points and with values corresponding to the different positions of the load. The connecting curve then indicates those points at which the effect of the load is greatest and least, and with its assistance we can also determine the most unfavorable position of any number of loads. Thus, if the diagram of moments is constructed for a moving load of a unit of weight, the value to be determined for any number of weights is obtained by taking the sum of the products of the loads into the corresponding ordinates Y of the diagram of moments. If, instead of the unit of weight, the diagram of moments is constructed for the force G, we must take for the separate weights P the sum $\frac{1}{G}\,\Sigma\, P\, Y$. The diagram of moments is a definite curve when the girder is loaded uniformly. If, however, loading can occur in single points of the girder only, as is the case in main girders on which rest cross girders, there is found instead of the curve an inscribed polygon with straight sides whose vertices lie in the verticals through the points of application of the loads. Let P be the load on a panel a, between two cross beams, for which the ordinates of the diagram of moments for the moving load of a unit of weight are $Y_{,}$ and $Y_{,,}$, and let e be the distance of this load from the first cross beam; then the ordinate of the diagram of moments at the point of application of the load is found from the equation

$$P.\,Y=Y_{,}\frac{P.\,(a-e)}{a}+Y_{,,}\,\frac{Pe}{a},$$

n which Y corresponds to the ordinates of the straight lines passing through the points $Y_{,}$ and $Y_{,,}$. In order to determine the effect of a system of separate moving loads, which will be considered hereafter, we will suppose the separate sides of the diagram of moments pertaining to a loaded section to be inclined to the horizontal axis of abscissas at angles $\alpha_1\ \alpha_2\ \alpha_3, \ldots$ while the loads $P_1\ P_2\ P_3\ \ldots$ are located upon their projections. Then the total effect of these loads is represented by the sum $P_1\,Y_1+P_2\,Y_2+P_3\,Y_3+\ldots$ and the variation in this value due to a movement of $\Delta\,\xi$ by single loads of the system, if we presuppose that during this variation no new loads enter or leave the girder or pass a cross beam, will be,

$$\Delta\,\xi\,(P_1\,tg\,\alpha_1+P_2\,tg\,\alpha_2+P_3\,tg\,\alpha_3+\;.\;\;.\;\;.),$$

since the ordinate Y, pertaining to any assumed load for a movement of $\Delta\,\xi$ changes its value by $\Delta\,\xi\,tg\,\alpha$.

This expression serves also for the case of a uniform load on the girder when α_1 $\alpha_2\ldots$ represent the angles with the horizontal made by the tangents to the diagram of moments at points on the verticals through the loads, and $\Delta\,\xi$ represents a very small movement. We now ascertain whether, in order to have the most unfavorable position of the load, motion is required to the right or the left, according as the expression in parenthesis has a positive or a negative sign, and also that in general for a prominent value, a load will be at a crossbeam, etc. It is better to determine the expression in parenthesis by construction, representing the loads upon the line of forces of the stress polygon and, starting with the beginning of the first load, constructing a polygon whose sides form the angles $\alpha_1\ \alpha_2$ . . . . with the vertical, and have the lengths $P_1\ P_2$ . . . . as vertical projections. The position of the extremity of this polygonal stress-line to the right or left of the line of forces, decides as to the movement to be made.

We finally obtain from the diagram of moments also the effect of a fixed, uniformly-distributed load. This is proportional to the area between the axis of abscissas and the diagram of moments corresponding to the loaded section.

In order to obtain the *diagram of the horizontal stress* or the *horizontal-stress curve*, we must compute the diagram of moments from the previously developed equations for the different possible positions of a single load (which we will assume equal to the unit of weight) and lay off their values as ordinates on the verticals through the points of application of the loads. From this we find the horizontal stress for any assumed loading either by a system of single loads or by a fixed load. That is to say, if we designate by $\eta_1\ \eta_2$ . . . . . the ordinates of the horizontal-stress curve at the points of the loads $P_1\ P_2$ . . . . . we obtain the horizontal stress from

$$H = P_1\,\eta_1 + P_2\,\eta_2 + \ldots$$

If, on the other hand, the load q per unit of length over the section $x_2 - x_1$ is distributed uniformly, the horizontal stress is,

$$H = q\int_{x_1}^{x_2}\eta\,dx = q\,\Phi,$$

in which Φ represents the area of the horizontal-stress curve between the ordinates which limit the loaded section. If a hinge is introduced at the center of the stiffening truss, then from equations (56) and (57) the horizontal-stress curve is given by the diagram of moments referred to the center cross section of the girder. For a simple girder, therefore, the horizontal-stress curve becomes a triangle with a middle ordinate $\frac{1}{4}\,G\,\frac{l}{f}$.

Example.—(a) In a suspension bridge stiffened by a continuous truss (fig. 19), let $l_1 = 0.4\,l$, $f = 0.1\,l$, $f_1 = 0.04\,l_1$, $J = 5\,J_1$, and $\frac{J}{F_0\,f\,l} = \frac{1}{600}$; then, $\varepsilon = \frac{2\,(1 + 0.4^3)}{3 + 2 \times 0.4} = 0.56$, and there result from equations (67) and (68) the following values of the horizontal stress:

Load in a side span,

$\frac{\xi}{l_1} =$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$H =$	−0.0635	−0.1285	−0.1942	−0.2566	−0.3095	−0.3431	−0.3453	−0.3012	−0.1930 . G.

Load in the middle span,

$\frac{\xi}{l} =$	0.1	0.2	0.3	0.4	0.5
$H =$	0.6206	1.2056	1.6783	1.9837	2.0801 . G.

Fig. 20 gives the horizontal-stress curve drawn according to these values.

(b) If the truss is not continuous over the middle supports we obtain with the same suppositions as above—

For a load in a side span,

$\frac{\xi}{l} =$	0.1	0.2	0.3	0.4	0.5
$H =$	0.0690	0.1305	0.1791	0.2092	0.2197 . G;

For a load in the center span,

$\frac{\xi}{l} =$	0.1	0.2	0.3	0.4	0.5
$H =$	0.5388	1.0195	1.3991	1.6347	1.7166 . G.

As is evident from the expressions for H the magnitude of the horizontal stress is dependent upon the factor $\frac{J}{F_0 f l}$. But the value of this factor lies for practical purposes within narrow limits, probably between one two-hundredth and one eight-hundredth, for which, in the example treated above as (a), the horizontal stress

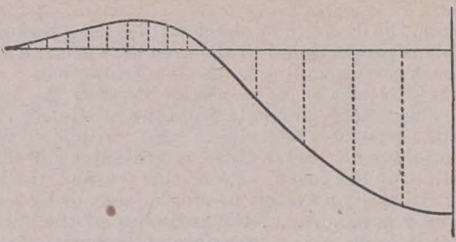


Fig. 20.

would be between 0.85 and 1.021 times the computed value. The maximum values of H would occur when $\frac{J}{F_0}=0$ and would reach 1.093 times the computed value in the above example.

§ 6. *Maximum stresses in the system due to the live load.*

1. *Maximum horizontal stresses.*—The greatest tension in the chain or the greatest thrust in the arch occurs when the center span is entirely covered by a uniformly distributed load. For a system of single loads the most unfavorable condition of loading is to be determined from the horizontal-stress curve according to the principles of the diagram of moments.

Making the same suppositions for simplification and neglecting the same quantities as in equations (67^a) and (68), the greatest horizontal stress due to a total uniform load (p per unit of length) is found to be:

For a truss extending over one span,

$$\mathfrak{H}_{tot}=\frac{p\,l}{N} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (69)$$

and by the introduction of a hinge at the center of the girder,

$$\mathfrak{H}_{tot}=\frac{1}{8}\frac{l}{f}\cdot p\,l; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (69^a)$$

for the continuous truss extending over three spans,

$$\mathfrak{H}_{tot}=\frac{l+4\,l_1-5\,l_1\left(1+\frac{f_1}{f}\frac{J}{J_1}\right)\varepsilon}{10\,(3\,l+2\,l_1)\,N_1}\,pl, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (70)$$

in which N and N_1 represent the denominators of the expressions (67^a) and (68) respectively.

2. *Stresses in the stiffening truss.*—For the determination of the maximum forces acting upon the stiffening truss, therefore for the determination of its dimensions,

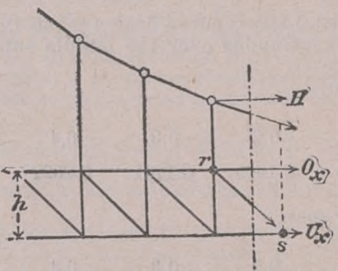


Fig. 21.

a knowledge of the maximum bending moments and shearing stresses is required. For a plate girder we can from these determine in the usual manner the necessary areas of cross section; for a framed girder with parallel chords the chord stresses are obtained from the moments, the stresses in the web system from the shears. According to equations (46) and (47), the moment M can, in general, be represented by the product $H\,z$, in which z is the vertical distance between the equilibrium polygon of the external forces and the axis of the archpolygon. The moments are to be taken with reference to the joints of the suspension system, which are opposite the corresponding panel points of the stiffening truss, and accordingly (see fig. 21) we have,

$$\left.\begin{aligned} O_x &= -\frac{M_s}{h} = -\frac{z_s}{h}\,H \\ U_x &= +\frac{M_r}{h} = +\frac{z_r}{h}\,H \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (71)$$

If, as in the system shown in fig. 22ª, the horisontal stress of the arch is taken up by the stiffening truss the axial strain produced thereby is, of course, to be considered in determining the dimensions of the truss. In a framed girder the stresses in the upper and lower chords in this case become,

$$\left.\begin{aligned} O_x &= -\frac{M_s}{h} + \frac{H}{2} = -\left(z_s - \frac{h}{2}\right)\frac{H}{h} \\ U_x &= +\frac{M_r}{h} + \frac{H}{2} = +\left(z_r + \frac{h}{2}\right)\frac{H}{h} \end{aligned}\right\} \quad \ldots\ldots \quad (72)$$

In general we can therefore write,

$$\left.\begin{aligned} O_x &= -\frac{e_o}{h}\,H \\ U_x &= +\frac{e_u}{h}\,H \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (73)$$

if we understand by e_o and e_u the distances of the equilibrium polygon from the axis of the archpolygon, assumed to be one of the two lines (in system shown in fig. 17, and fig. 23ª) which are obtained by moving the axis of the archpolygon vertically a distance $\frac{h}{2}$ up or down.

3. *Curve of pressure; most unfavorable condition of loading.*—To determine the most unfavorable action of the live load upon the stiffening truss it is advisable to first investigate the effect of a single moving load. If for this purpose we use the graphical method the problem becomes the construction of the equilibrium polygon of the external forces for any assumed position of the load. This presents no difficulties after we can obtain from the horizontal-stress curve the value of the horizontal stress upon which the construction of the equilibrium polygon is to be based.

For a stiffening truss extending only over one span the equilibrium polygon must pass through the points of upports A and B of the chain. In this case we find the directions of the sides of the equilibrium polygon simply as the resultants of the horizontal tension and the vertical abutment reaction due to the load, which latter is determined for a single load lying at a distance ξ from a support A by $G\frac{l-\xi}{l}$, hence by the ordinate of the straight line bc (fig. 22). Carrying out the construction of the equilibrium polygon for the several positions of the load in the manner shown in the figure, we obtain for the locus of the point E, in which the sides of the equilibrium polygon cut each other in the load vertical, a curve $K\,E\,K$ which we will call the *curve of pressure* from analogy with the rigid arch, to be treated hereafter. Analytically the ordinate of this curve measured from the chord $A\,B$ is found from,

$$\eta = \frac{G\,(l-\xi)}{H.\,l}\,\xi \quad \ldots\ldots\ldots\ldots \quad (74)$$

By the introduction of the approximate expression (67ª) for H we have,

$$\eta = \frac{N.\,l^3}{5\,(l^2 + l\,\xi - \xi^2)} \quad \ldots\ldots\ldots\ldots \quad (75)$$

From this last equation, for $\xi = O$, we obtain also the ordinate of the point K, while for the determination of this point by construction the relation $A_1\,K = G\,cotg\,\alpha$ is of assistance, α representing the angle of inclination of the tangent to the horizontal-stress curve at a.

If the stiffening truss is *hinged* at the center, the equilibrium polygon of the external forces must always pass through the point C_1, on the axis of the arch polygon and vertically over the hinge, since the moment of this cross section is equal

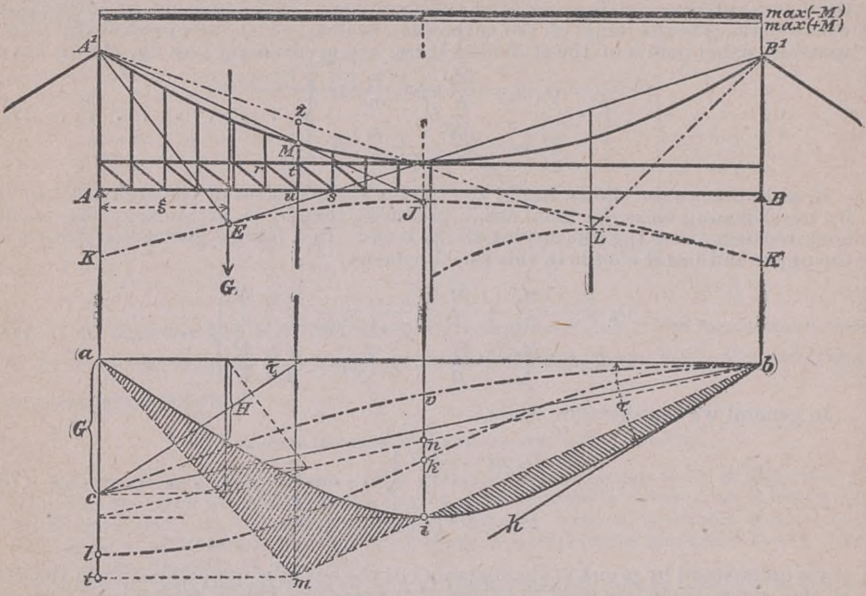


Fig. 22.

to zero. The curve of pressure in this case consists of two straight lines, which are the prolongations of the chords $A\ C$ and $B\ C$ (fig. 23).

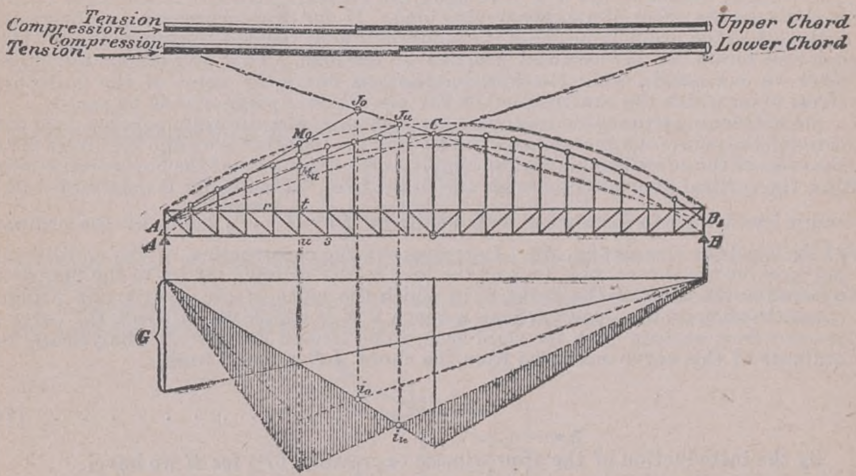


Fig. 23.

The equations of these straight lines, with reference to the axis of abscissas $A\ B$ are,

$$\left.\begin{aligned} \eta &= \frac{2f}{l}(l-\xi) \\ \eta &= \frac{2f}{l}\xi \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (76)$$

Since from the preceding (equation 71) the stresses in the upper chord *rt* (fig. 22) and the lower chord *su* are determined from the distances of the equilibrium polygon from the point M of the chain, we obtain by means of the point J, which is the intersection of the connecting line $A_1\ M$ with the curve of pressure, that position of the single load for which a reversal of stress occurs in the upper and lower chords. For all loads lying to the right of J, the intersection of the equilibrium polygon with the vertical through M will be above the chain, and z will, therefore, be negative; while for all loads lying to the left (up to the finally determined point of intersection J_1 of the connecting line $B_1\ M$ and the curve of pressure), this intersection will fall below M, and z will therefore be positive. Therefore the loading of the section $J\ B$ produces the maximum *tension* in the *upper* chord $r\ t$ and the maximum *compression* in the *lower* chord $s\ u$; while loading the other part $A\ J$ of the truss produces the maximum *compression* in the *upper* chord and the maximum *tension* in the *lower* chord.

In those systems in which the horizontal stress is taken up by the chords of the stiffening truss (fig. 17) and to which, therefore, equations (72) apply, two lines are to be used, as stated above, in place of the axis of the chainpolygon, and distant from the latter $\pm\frac{h}{2}$, the upper line corresponding to the upper chord and the lower line to the lower chord (fig. 23).

Analytically the abscissa ξ_1 of the limit of the load or the neutral point J, is determined from,

$$\xi_1 = \frac{x}{y}\eta_1 \quad \ldots\ldots\ldots\ldots \quad (77)$$

in which x and y represent the coordinates of the point M referred to the chord $A_1\ B_1$. If for η_1 we substitute the approximate expression (75), we obtain the definite equation,

$$-\xi_1^3 + l\xi_1^2 + l^2\xi_1 = \frac{Nl^3}{5}\cdot\frac{x}{y} \quad \ldots\ldots\ldots\ldots \quad (78)$$

In case the stiffening truss is hinged at the center we obtain from equation (76) for the abscissa of the load limit,

$$\frac{\xi_1}{l} = \frac{2fx}{l.y+2fx} \quad \ldots\ldots\ldots\ldots \quad (79)$$

If the truss is also continuous over the side spans a complication arises, since the equilibrium polygon can not be drawn through the points of suspension of the chain, due to the fact that the moment at the center supports does not become equal to zero.

If $\mathfrak{M}_1 = H\,\mathfrak{m}_1$ and $\mathfrak{M}_2 = H\,\mathfrak{m}_2$ represent the moments of the vertical loads for a continuous truss at the two middle supports, the points of intersection of the equilibrium polygon with the verticals at the supports (according to fig. 14) are to be placed $(\mathfrak{m}_1 - m)$ and $(\mathfrak{m}_2 - m)$ respectively, above the points of support of the chain, in which the length m is determined from equations (49) and (50). The quantities $\mathfrak{M}_1$ and $\mathfrak{M}_2$ and $\mathfrak{m}_1$ and $\mathfrak{m}_2$ are to be determined by the theory of continuous trusses either analytically or graphically.

In fig. 24 the graphical construction is shown. F and F_1 are the fixed points in the center span to be determined in the usual manner. Let us place $C_{,,}\ E = B_{,,}\ C_{,,} - 3\,F_1\ C_{,,}$ and $E\ G = B\ C\frac{F\ F_1}{B_{,,}\ F}$; also place $G\ H = \frac{1}{H}.\ G.\frac{\xi(l-\xi)}{l}$. This latter length is obtained in in the section $M\ T$ of the simple equilibrium polygon constructed on the load vertical with the horizontal stress H. The straight connecting line $E\ H$ gives in the section $M\ L$ on the load vertical the length $\mathfrak{m}_1$ and on the vertical placed symmetrically as to the center, the length $\mathfrak{m}_2$. If we now lay off these lengths on the verticals at the supports and the height of the equilibrium polygon $M\ T$ on $P\ K$, we have in $N\ K\ O$ the equilibrium polygon in its correct position with regard to the chain curve.

If we construct the equilibrium polygon for different positions of a single load we obtain again the *curve of pressure* $K_1\ K_{,,}$ as the locus of the point K in which the sides of the equilibrium polygon intersect on the vertical through the position of the load. Moreover the sides of the different equilibrium polygons circumscribe a curve $U\ U_1$, which we may designate as *the inscribed curve of pressure.* We also construct the curve $A_{,,}\ Q$, whose ordinates correspond to the moments at the supports, reduced for H, for the various positions of the single load in a side span. If for the single load acting at Q, Q be horizontally projected to Q_1, the straight line drawn through

Q_1 to the fixed point F_1 represents the line of the equilibrium polygon in the center span corresponding to this position of the load.

There is now no difficulty in determining for any cross section of the stiffening truss the most unfavorable conditions of loading with regard to the moments. If S represents the point on the curve lying in the vertical through the cross section, the tangent drawn through S to the inscribed curve of pressure by its intersection J with the curve of pressure determines the limit of loading in the middle span, while that in the left-side span is given through the point Q.

In fig. 24, schemes for loading are drawn for three different cross sections.

4. *Determination of maximum moments and maximum chord stresses.*—(a) *Method of diagram of moments.*—In a girder extending over only one span the moment produced in the cross section M by a single load is expressed by,

$$M = \mathfrak{M} - H\,y = y\left[\frac{\mathfrak{M}}{y} - H\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (80)$$

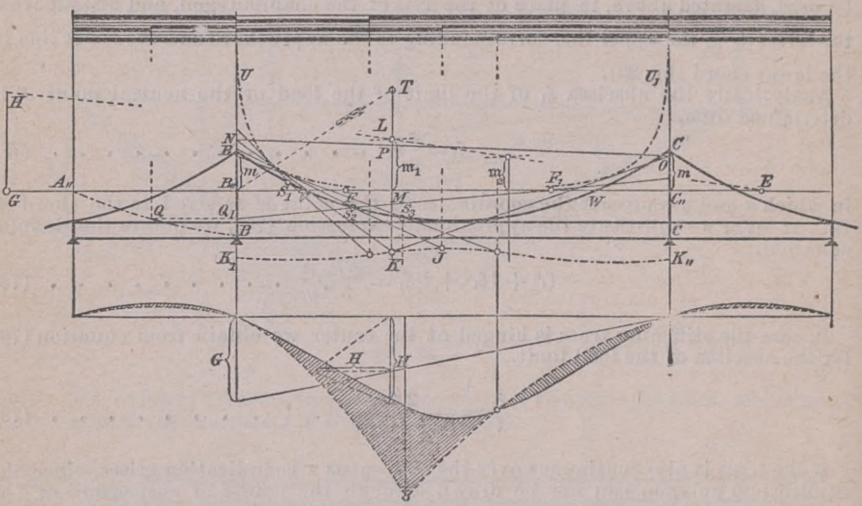


FIG. 24.

For a moving load $\frac{\mathfrak{M}}{y}$ represents the diagram, constructed with the pole distance y, of the moments of a truss with free ends and therefore the moment M is proportional to the difference of the ordinates of this diagram and the horizontal-stress curve.

The diagram of simple moments is, as is known, given by a triangle whose height in the vertical at the cross section is $= G\,\frac{x\,(l-x)}{l\,.\,y}$. But for this construction can also be used the neutral point J (figs. 22 and 23), which, as shown previously, is obtained by the aid of the curve of pressure and is projected to i in the horizontal-stress curve. If Φ_1 and Φ_2 represent the areas lying above and below the horizontal-stress curve and bounded by it and the above-mentioned diagram (in fig. 22, surfaces $i\ b\ h$ and $a\ m\ i$), then for a uniformly distributed load of p per unit of length we have,

$$\left.\begin{aligned} max\,(+M) &= \frac{p}{G}\,\Phi_2\,y \\ max\,(-M) &= \frac{p}{G}\,\Phi_1\,y \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (81)$$

In order to ascertain the areas Φ it suffices to determine the areas of the horizontal-stress curve either analytically or graphically and, reduced to the basis $\frac{l}{2}$, to lay them off as ordinates of a curve bkl, so that the area $\overline{b\ i'\ i\ h} = \overline{i'\ k} - \frac{l}{2}$ (fig. 22). Then the ordinate $i'\ k$ corresponds to the horizontal-stress for a uniform loading of the section $B\ J$, and at the same time we have $a\ c = G = \frac{pl}{2}$. The reduction of the surface of the triangle $i\ i'\ b$ to the basis of $\frac{l}{2}$ gives the length $i'\ n$; therefore, $\Phi_1 = \overline{kn}\ \frac{l}{2}$ and,

$$max\ (-\ M) = \overline{kn}\ .\ y.$$

Similarly the effect of a total loading is obtained from

$$M_{\text{tot}} = \overline{t\,l}\ .\ y$$

and the greatest positive moment becomes $max\ (+\ M) = (\overline{kn} + \overline{lt})\ y$, in which the lengths $k\,n$ and $t\,l$ are to be laid off with a unit of force determined from $a\,c = \frac{p\,l}{2}$.

This method can also be applied for the continuous stiffening truss (fig. 24) for which, of course, the diagram of the moments $\mathfrak{M}$ is not so simple a figure as a triangle, but (upon the basis $y - m$) must be deduced from the theory of continuous girders. For this purpose the equilibrium polygons which served in the construction of the curve of pressure are of assistance. For those cross sections lying near the points V and W and for which therefore $y - m$ becomes very small, this method is, however, not applicable; but for these cross sections the moment M is very nearly (for the point V and W exactly) equal to the moment $\mathfrak{M}$ for the ordinary continuous truss.

(b) *Determination of maximum moments from the equilibrium polygon of partial loading.*—If η represents the distance of the equilibrium polygon of loading from the axis of the arch measured on the vertical through the cross section M, and $\mathfrak{H}$ the horizontal stress, then according to the theorem applied to §5, section 2, the moment for the cross section M of the stiffening truss is, in general,

$$M = \mathfrak{H}\ .\ \eta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (82)$$

The quantities $\mathfrak{H}$ and η can be determined either analytically or graphically. For a uniformly distributed load we obtain the horizontal stress by the integration between proper limits of the expressions deduced for a single load. If the loading extend from a point whose abscissa is ξ_1 to the right hand end of the truss, we have for a simple truss with a center hinge $\left(\xi_1 < \frac{l}{2}\right)$

$$\mathfrak{H} = \frac{p}{8f}\left(l^2 - \xi_1^2\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (83)$$

For a jointless truss (from equation (67^a) when the denominator of that expression is placed $= N$) we have,

$$\mathfrak{H} = \frac{p\,l}{N}\left[1 - \left(\frac{\xi_1}{l}\right)^5 + \frac{5}{2}\left(\frac{\xi_1}{l}\right)^4 - \frac{5}{2}\left(\frac{\xi_1}{l}\right)^2\right] \quad . \quad . \quad . \quad . \quad (84)$$

In a similar manner the values of the horizontal stress for the continuous truss under partial, uniform loading are found from the integration of expressions (68) and (68^a).

As previously mentioned the curve of the horizontal stress $\mathfrak{H}$ can also be obtained graphically from the area of the horizontal-stress curve H. If in addition to this the parabola bc of the vertical pressures be constructed (fig. 22), the resultant of the lengths $i'v$ and $i'k$, which correspond to the abutment reactions for a loading of the part of the truss $J\,B$, gives the direction of the side of the polygon passing through A. Its intersection with the vertical through M determines the quantity $M\,z = \eta$. The determination of this quantity can be simplified by noting the locus of the point L; that is, the so-called *second curve of pressure*, in which the prolongation of the side of the equilibrium polygon $A_1\,L$ intersects the center line of the loaded section. Therefore the maximum negative moment with reference to the cross section M is found by the multiplication of the two lines $i'k$ and $M\,z$, of which the former is to be measured by the scale of force $\left(a\,c = \frac{p\,l}{2}\right)$, and the latter by the scale of length.

Analytically, the moment of a load not extending beyond the cross section under consideration is given by,

$$M = \mathfrak{B}\,x - \mathfrak{H}\,y;$$

in this expression we have for the negative maximum or for for the minimum moment,

$$\mathfrak{B} = \frac{p\,(l - \xi_1)^2}{2\,l}$$

$\mathfrak{H}$ is found from equations (83) and (84) and ξ_1, is determined from equations (79) and (78).

If M_{tot} represents the moment for a total, uniform loading we have, for a parabolic form of the archpolygon or chaincurve and with the use of the approximate expression (67^a) for the horizontal stress, the following expressions for the maximum moment acting upon the stiffening truss:

For a truss with a center hinge,

$$\left.\begin{aligned} M_{tot} &= 0 \\ M_{\substack{max\\min}} &= \pm \frac{p\,x\,(l-x)\,(l-2x)}{2\,(3l-2x)} \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (85)$$

The absolute maximum moment occurs when $x = 0.234\ l$ and becomes,

$$absol.\ max\ M = \pm\, 0.01883\, p\, l^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (85^a)$$

For a truss without hinge,

$$\left.\begin{aligned} M_{tot} &= \frac{Nl-8f}{N} \cdot \frac{p\,x\,(l-x)}{2\,l} \\ M_{min} &= -\frac{2\,p\,x\,(l-x)}{N} \cdot \frac{f}{l}\left[2-5\,\frac{\xi_1}{l}+10\left(\frac{\xi_1}{l}\right)^3-10\left(\frac{\xi_1}{l}\right)^4+3\left(\frac{\xi_1}{l}\right)^5\right] \\ M_{max} &= M_{tot} - M_{min}. \end{aligned}\right\} \quad . \quad (86)$$

In the above, ξ_1 is determined by equation (78). Let N represent the denominator of the approximate expression for H (67^a). The absolute maximum moment is dependent upon this quantity; if we place, as an approximation, for its smallest value $N = 8\,\frac{f}{l}$, the absolute maximum moment occurs when $x = 0.250\,l$, and will be,

$$absol.\ max\ M = 0.01652\ p\ l^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (86^a)$$

Equation (86) holds good for cross sections from $x = 0$ to $\frac{x_1}{l} = \frac{N}{20}\,\frac{l}{f}$; to produce the minimum moment for the middle cross-sections from x_1 to $l - x_1$, a part of the span next the left support is also to be loaded, and therefore the minimum moment is composed of two parts, which are given by equation (86) for the two points x and $l - x$, symmetrically placed with respect to each other.

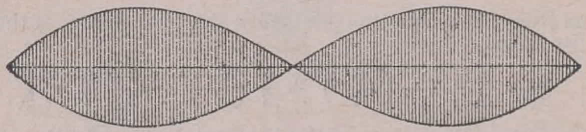


FIG. 25 *a*.—Maximum moment for girder, with hinge at center.

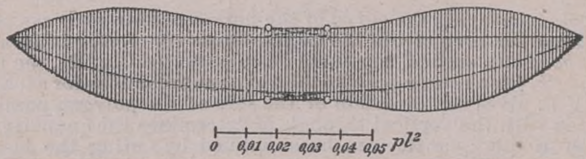


FIG. 25 *b*.—Maximum moment for girder, without hinge.

$\frac{f}{l} = 0{,}13 \qquad N = 1{,}215.$

Accordingly in figs. 25 *a* and 25 *b*, the lines of maximum moments, are represented for a truss with and without a center hinge.

For the latter case the example on page 88 was chosen. In it $l = 50$ meters, $f =$ 6.5 meters, $l_1 \sec \alpha^2\ \alpha_1^2 = 19.05$ meters, $\frac{J}{F_0} = 2.049$, $F_0 = 0.03$ square meters, hence $N =$ 1.215; hence the moment at the center of the bridge under the total load becomes $0.0180\ p\ l^2$.

5. *Determination of maximum shears.*—The most unfavorable conditions of loading with regard to shears are obtained from a consideration of equation (51). From it we find that position of the single load for which a change of sign occurs in the shear, by finding the point of intersection J of the curve of pressure with the side of the equilibrium polygon, whose angle of inclination is τ; that is, the side parallel

to the tangent at the point of the axis of the archpolygon that lies in the vertical through the cross section. Moreover Q changes its sign at the cross section itself, so that the most unfavorable loading represented in fig. 26 occurs. The same rule also applies for the center span of a truss continuous over three spans, for which case the side span lying upon the side of the cross section is not to be loaded at all for the maximum positive shear, while the other side span is to be totally loaded; or, for cross sections in the center of the truss, is to be partially loaded out from the end.

For the determination of the maximum shears the two above-mentioned methods can again be applied.

(a) *Method of diagrams of moments.*—The shear in any assumed cross section of the stiffening truss is determined by equation (51) from $Q = \mathfrak{Q} - H\,tg\,\tau = tg\,\tau\,(\mathfrak{Q}\,cotg\,\tau - H.)$ If we consider a framed truss with parallel chords and let δ be the angle of inclination of the web member cut by the cross section, the stress in this web member follows from

$$S = Q\,sec\,\delta = tg\,\tau\,sec\,\delta\,(\mathfrak{Q}\,cotg\,\tau - H) \quad . \quad . \quad . \quad . \quad . \quad . \qquad (87)$$

The values which the quantity in parenthesis of the foregoing expression has for different positions of a single load can be simply represented. That is to say, they appear as the difference between the ordinates (multiplied by $cotg\,\tau$) of the diagrams of the shears $\mathfrak{Q}$ for the truss with free ends and the ordinate of the horizontal-stress

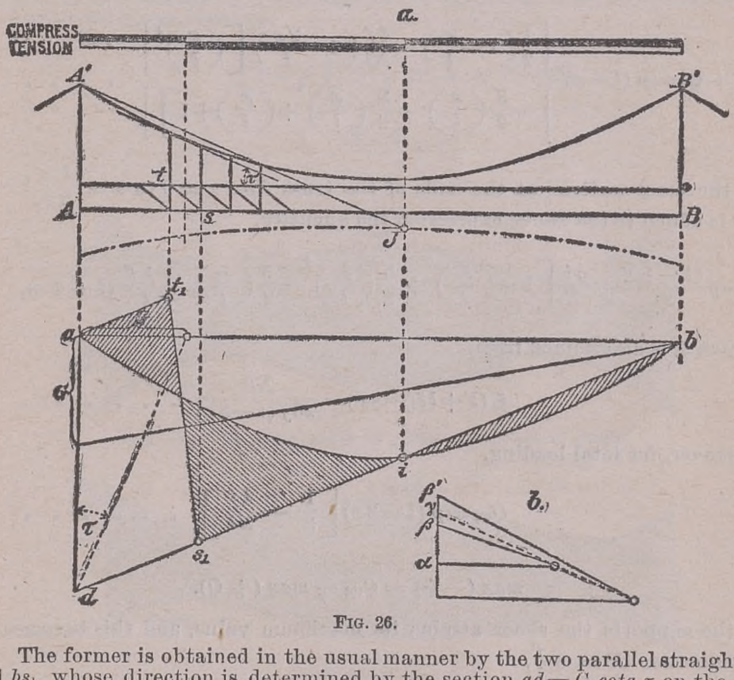


FIG. 26.

curve. The former is obtained in the usual manner by the two parallel straight lines at_1 and bs_1, whose direction is determined by the section $ad = G\,cotg\,\tau$ on the verticals at the supports. The corners t_1 and s_1 lie upon the verticals through the joints corresponding to the lateral member under consideration. The point of intersection i with the horizontal-stress curve, which should lie vertically under J, gives us control over this. The maximum shears and the stresses in the diagonals, due to a uniformly distributed load, are again determined by the areas between the diagram of the shears $\mathfrak{Q}$ and the horizontal-stress curve. Those areas lying below the latter are to be introduced as positive and those lying above as negative. These areas are to be multiplied by the factors $\frac{p}{G}\,tg\,\tau$ and $\frac{p}{G}\,tg\,\tau\,sec\,\delta$ in order to obtain the maximum shears and stresses in the diagonal members, respectively.

Equation (87) applies also for the center span of a truss continuous over four supports, except that in this case the diagram of the shears $\mathfrak{Q}$ is no longer represented by straight lines.

For cross sections near the middle of the girder the shear becomes nearly equal to that $\mathfrak{Q}$ of the girder with free ends.

(b) *Determination of shears from the equilibrium polygon for partial loading.*—With the aid of the second curve of pressure we may again draw, for partial loading, the equilibrium polygon and the stress polygon belonging to it (fig. 26b). In the latter we have $\alpha\beta = \mathfrak{Q}$, $\alpha\gamma = H \tan \tau$, when $O\gamma$ is drawn at an angle τ and therefore $\gamma\beta = Q$ for the loading of the part JB of the truss. Similarly $\gamma\beta'$ gives the shear for the loading of the section MB, consequently $\beta\beta'$ is the maximum positive shear $+Q_{max}$ in the cross section M.

Upon the supposition that the curve of the chain is a parabola the following expressions for the maximum shears may be developed:

For a truss with a center hinge,

$$\left.\begin{aligned} x=0 \quad \text{to } 0.25\, l \quad max\,(\pm Q) &= \pm \frac{p\,(l^2-3\,lx+4\,x^2)^2}{2\,l^2\,(3\,l-4\,x)} \\ x=0.25 \text{ to } 0.5\ l \quad max\,(\pm Q) &= \pm \frac{px^2\,(3\,l-4\,x)}{2\,l^2} \end{aligned}\right\} \quad \ldots\ldots \quad (88)$$

The absolute maximum shears occur at the supports and at the center of the truss, and their actual values are $max\,(\pm Q) = \frac{1}{6}\,pl$ and $\frac{1}{8}\,pl$, respectively.

For a truss without a hinge we have,

$$max\,(+Q) = p\,(l-x) \left\{ \begin{aligned} &\tfrac{1}{2}\left(1-\tfrac{x}{l}\right) - \tfrac{4f}{Nl}\left(1-\tfrac{2x}{l}\right)\left[\left(\tfrac{x}{l}\right)^4\right. \\ &\left. -\tfrac{3}{2}\left(\tfrac{x}{l}\right)^3 - \tfrac{3}{2}\left(\tfrac{x}{l}\right)^2 + \left(\tfrac{x}{l}\right) + 1\right] \end{aligned} \right\} \quad \ldots\ldots \quad (89)$$

In the cross sections at the ends of the truss from $x = 0$ to $x = \frac{l}{2}\left(1 - \frac{Nl}{20f}\right)$ there is added to the above expression the quantity,

$$-p\,\frac{(l-\xi_2)^2}{2\,l} + \frac{p\,l}{N}\left[1 - \left(\frac{\xi_2}{l}\right)^5 + \frac{5}{2}\left(\frac{\xi_2}{l}\right)^4 - \frac{5}{2}\left(\frac{\xi_2}{l}\right)^2\right]\frac{4\,f}{l^2}\,(l-2\,x), \quad . \quad (90)$$

in which ξ_2 is determined from,

$$\xi_2\,(l^2 + l\,\xi_2 - \xi_2^{\,2}) = \frac{Nl^5}{20\,f\,(l-2\,x)} \quad \ldots\ldots\ldots \quad (91)$$

Moreover, for total loading,

$$Q_{tot} = p\,(l-2\,x)\left[\frac{1}{2} - \frac{4f}{Nl}\right] \quad \ldots\ldots\ldots \quad (92)$$

and,

$$max\,(-Q) = Q_{tot} - max\,(+Q).$$

At the supports the shear attains its maximum value, and this becomes for the approximation $N = 8\,\frac{f}{l}$,

$$absol.\ max\,(+Q) = 0.1523\,pl.$$

For the center of the truss,

$$max\,(+Q) = \pm\,\frac{1}{8}\,pl.$$

In figs. 27a and 27b the curves of maximun shear are represented.

6. *Effect of changes of temperature.*—If t represents the change of temperature with reference to the unstrained condition and ω the coefficient of expansion, then the definite equation for the horizontal stress H, replacing equation (59), becomes,

$$\int\left(\frac{P}{EF} + \omega\,t\right)\frac{d\,P}{d\,H}\,d\,s + \int\frac{M}{EJ}\,\frac{d\,M}{d\,H}\,d\,s = 0 \quad \ldots\ldots \quad (93)$$

If the denominators of expressions (61), (62), etc., be represented by N_{61}, N_{62}, etc., the development of equation (93) in a method similar to that used on pages 84 and 85, gives for the corresponding cases the following expressions for H,

$$\left.\begin{aligned} H_t &= -\frac{\left[l + 2\,l_1 \sec^2 \alpha_1 - f'\left(\Sigma \dfrac{\Delta^2 y}{a} + 2\,tg\,\alpha\right)\right] F_0 E\,\omega\,t}{N_{61}} \\ H_t &= -\frac{\left[l + 2\,l_1 \sec^2 \alpha_1 - f'\left(\Sigma \dfrac{\Delta^2 y}{a} + 2\,(tg\,\alpha - tg\,\alpha_1)\right)\right] F_0 E\,\omega\,t}{N_{62}} \end{aligned}\right\} ; \quad . \quad . \quad (94)$$

and by neglecting the effect of the suspenders,

$$H_t = -\frac{(l + 2\,l_1 \sec^2 \alpha_1)\,F_0 E\,\omega\,t}{F_0\,N_{63}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (95)$$

In the truss system represented in fig. 17 a uniform change of temperature in all the parts of the system produces no stresses. On the other hand, if the temperature of the arch should change by t, and that of the stiffening truss and suspenders by t', the horizontal stress becomes,

$$H_t = +\frac{\left(l - \Sigma \dfrac{y}{a}\,\Delta^2 y\right) F_0 E \,.\, \omega\,(t - t')}{N_{61} + \dfrac{F_0}{2\,F_1}\,l} \quad . \quad . \quad . \quad . \quad . \quad . \quad (96)$$

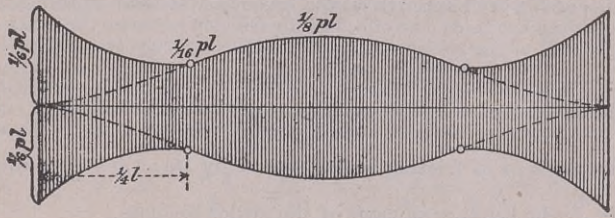


FIG. 27*a*.—Maximum shear for girder with hinge at center.

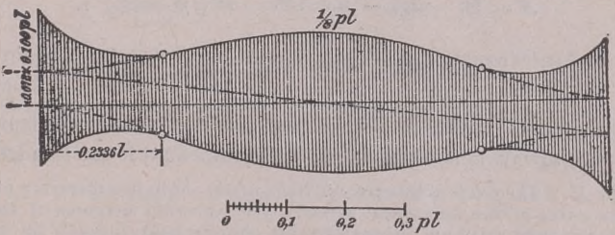


FIG. 27*b*.—Maximum shear for girder without hinge.

In stiffening trusses hinged at the center changes of temperature have, of course, no effect.

For the determination of moments and shears we may use

$$M = -H_t\,y \qquad\qquad Q = -H_t\,tg\,\tau.$$

In a similar manner we can also determine the effect of a shifting of the chain supports which may occur either by a compression of the anchorages or by the backstays becoming stretched taut by loading. If, from these causes, the span is diminished by δl the variation ΔH in the horizontal stress is determined from,

$$\Delta H = -\frac{F_0 E\,\delta l}{F_0 N_{63}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (97)$$

For the approximate assumption of a parabolical chain the denomimators of expressions (94), (95), and (97) are to be replaced by

$$\frac{1}{15} f l^2 \frac{F_0}{J} N,$$

in which N represents the denominator of expression (67^a).

In a truss continuous over three spans the horizontal stress due to a uniform change of temperature in the whole system becomes

$$H_t = -\frac{3 E J \omega t \left[\left(1 + \frac{16}{3} \frac{f^2}{l^2}\right) 2 + \frac{l_1}{l} \left(1 + \frac{16}{3} \frac{f_1^2}{l_1^2}\right) \right]}{f.\ l.\ N_{68}} \quad . \quad . \quad . \quad (98)$$

In the example for which the maximum moments due to the live load ($p = 5\ t$) are shown in fig. 25 we obtain from equation (95) for the horizontal stress due to a difference of temperature of $\pm$ 30°,

$$H_t = \frac{(50 + 38.1)\ 0.03 \times 20\,000\,000 \times 0.0000118 \times 30}{642.39} = \mp 29.1 \text{ t}.$$

The resulting moment in the middle of the stiffening truss reaches a value of $\pm$ 189.1 t m.

The stresses due to changes of temperature or yielding of the anchorages can therefore reach rather high values in the sytem of stiffened suspension bridges under consideration. The arched construction (fig. 16 or 17) has advantages in this respect. Material advantages are gained by the removal of these stresses of deformation and this removal is attained by the introduction of a hinge in the stiffening truss.

7. *Secondary strains.*—In the theory of stiffened suspension bridges developed heretofore joints without friction were presupposed in the chain as well as in the stiffening truss. As to the effect of the firm riveting of the latter we may call attention to the general discussion in Chapter IX on the secondary stresses in framed trusses, and we need investigate here only the effect of the friction between the links of the chain upon the magnitude of the horizontal stress and the external stresses of the stiffening truss.

In wire-cable bridges this resistance of friction is replaced by the stiffness of the cables, but this latter is of so little significance that it may be left out of consideration.

If, with a diameter of pin d and a coefficient of friction φ and with a stress T in the chain, the moment of friction be designated by $M\rho = \varphi T \frac{d}{2}$ or approximately by $M\rho = \varphi H \frac{d}{2}$, the bending moment of the girder becomes,

$$M = \mathfrak{M} - Hy - M\rho = \mathfrak{M} - H\left(y + \varphi \frac{d}{2}\right).$$

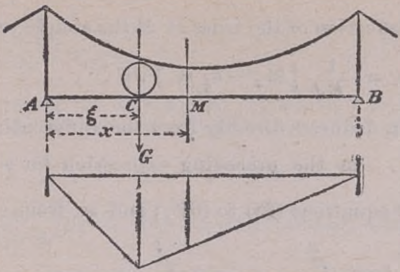
The expressions for H (equations 61–65) developed in § 5 therefore remain unchanged, except that the ordinate y of the arch is to be increased by $\varphi \frac{d}{2}$; but since this quantity as compared with y is always very small (with a coefficient of friction $\varphi = 0.2$, $\varphi \frac{d}{2}$ will always amount to only a few millimeters) it can be neglected in the expression for H. *The friction between the links of the chain has therefore no appreciable effect upon the value of the horizontal stress.* The external stresses of the stiffening truss also experience only an inconsiderable change and, indeed, the moments are diminished by the moment of friction, $\varphi H \frac{d}{2}$. We can, however, account for this circumstance by considering that the axis of the chainpolygon is moved $\varphi \frac{d}{2}$ downward.

As a rule, however, the bending stress in the links of the chain produced by the moment of friction is not to be neglected. Its computation is made in the usual manner.

§ 7. *Computation of deflections.*

(a) *Deflection due to loading.*—According to the theorem of *Castigliano* we obtain the motion of any assumed point on an elastic system from the differential coefficient of the work of deformation due to the action of a force presupposed to be acting at the point under consideration and in the direction of the motion.

In fig. 28 let a single load G act upon the truss $A\ B$ at a distance ξ from the support Let the axial forces and moments produced thereby be represented by P_ξ and



M_ξ. Then the deflection in a cross section whose abscissa is x is found from,

$$\eta = \frac{d\,A}{d\,G_x} = \int \frac{P_\xi}{EF} \cdot \frac{d\,P_x}{d\,G_x}\,ds + \int \frac{M}{EJ}\,\frac{d\,M_x}{d\,G_x}\,ds, \quad \ldots \ldots \quad (99)$$

when P_x and M_x represent the axial force and moment for a load G_x acting at x Write the horizontal forces corresponding to the positions ξ and x of the load as $H_\xi = h_\xi\ G$ and $H_x = h_x{}^1 G$, in which the coefficients h_ξ and h_x are to be taken from equations (61) to (67ª) developed above, or from the horizontal-stress curve, and are therefore to be assumed as known. We have $\frac{d\,P_x}{d\,G_x} = \frac{d\,P_x}{d\,H_x} \cdot h_x$, $\frac{d\,M_x}{d\,G^x} = \frac{d\,M_x}{d\,H_x} \cdot h_x$ and obtain for a truss without hinge and extending over one span:

Main chains, $\frac{d\,A}{d\,G_x} = \sum_0^l H_\xi . \frac{\lambda}{a} \cdot \frac{\lambda}{EF} . h_x \frac{\lambda}{a} = \frac{1}{E\,F_0}\ h_\xi \cdot h_x \ \sum_0^l \frac{\lambda^2}{a} . G$

Backstays, $\quad " = \Sigma\ H_\xi . sec\ \alpha_1 \frac{2\ l_1\ sec\ \alpha_1}{E\ F_0 sec\ \alpha_1} h_x\ sec\ \alpha_1 = \frac{2}{E\ F_0} h_\xi .\ h_x . l_1 sec^2 \alpha_1 . G$

Suspenders, $\quad " = \Sigma\, H_\xi \cdot \frac{\Delta^2 y}{a} \cdot \frac{y'}{E F_2} . h_x . \frac{\Delta^2 y}{a} = \frac{1}{E\,F_2} h_\xi\ h_x\ \Sigma\ y' \left(\frac{\Delta^2 y}{a}\right)^2 . G$

Girder, $\quad " = \frac{1}{E\ J} \int (\mathfrak{M}_\xi - H_\xi\ y) . \left(\frac{d \mathfrak{M}_x}{d\,G_x} - y\ .\ h_x\right) d\,x$

$$= \frac{1}{E\ J}\ [.\mathfrak{m}_x + h_\xi\ h_x\ \mu - h_x\ m_\xi - h_\xi\ m_x] . G .$$

In which $\mathfrak{m}._x$ represents the moment, with reference to the point M, of a simple truss for which the moment area corresponding to the position of the unit of load at C, represents the load area. Therefore we have the expressions,

$$\text{for } x < \xi \qquad .\mathfrak{m}_x = [2\,l\,\xi - \xi^2 - x^2]\ \frac{x\,(l-\xi)}{6l} \quad \ldots \quad (100)$$

$$\text{for } x > \xi \qquad .\mathfrak{m}_x = [2\,lx - x^2 - \xi^2]\ \frac{\xi\,(l-x)}{6\,l} \quad \ldots \quad (100^a)$$

Furthermore, m_ξ and m_x represent the moments for the verticals through C and M, respectively, which occur in a simple truss whose load area is formed by the area between the chainpolygon and chord. μ is the double static moment of this area with reference to the chord. Now, since from equation (66), $h_\xi = \frac{m_\xi}{\mu + cp}$ and $h_x = \frac{m_x}{\mu + cp}$, we have $m_\xi h_x = m_x\, h_\xi$, and from the summation of the values of $\frac{d\,A}{d\,G_x}$

$$\left.\begin{aligned}\eta = \Big[\frac{h_x\, h_\xi}{EF_0}\Big(l - \sum_0^l \frac{y}{a} \Delta^2 y + 2\,l_1\ sec^2\,\alpha_1 + \frac{F_0}{F_2}\,\Sigma\, y' \left(\frac{\Delta^2 y}{a}\right)^2 \Big) \\ + \frac{1}{EJ}\ (.\mathfrak{m}_x + h_x h_\xi \mu - 2\ h_\xi\, m_x)\ \Big] . G\end{aligned}\right\} \quad . \ . \quad (101)$$

If we consider that the first quantity in parenthesis in the preceding expression according to the notation introduced becomes $=\frac{F_0}{J}\,c\,p$, and that $m_x = h_x\,(\mu + c\,p)$, we finally obtain for the deflection of the truss at M the simple relation,

$$\eta = \frac{1}{EJ}\left[\mathfrak{m}_x - h_\xi\, m_x\right] . G, \quad \ldots\ldots\ldots \quad (101^a)$$

which could also have been deduced directly from the differential equation of the elastic line $\frac{d^2\eta}{dx^2} = \frac{M}{EJ}$. In the preceding expression for η, $\mathfrak{m}_x$ is determined from equation (100); h_ξ by equations (63) to (67ª); and m_x from,

$$m_x = \frac{l-x}{l}\int_0^x x\,y\,dx + \frac{x}{l}\int_x^l y\,(l-x)\,dx \quad \ldots\ldots \quad (102)$$

and for a parabolic chain from

$$m_x = \frac{f\,x}{3\,l^2}\,(x^3 - 2\,l\,x^2 + l^3) \quad \ldots\ldots\ldots \quad (102^a)$$

From this a simple graphical construction may be obtained for the ordinates of the elastic line. In its character it is similar to that used in figs. 1ª to 1ᶠ on Pl. I, for the arch hinged at the abutments. That is, if we write the expression for η in the form $\eta = \frac{h_\xi}{EJ}\left(\frac{\mathfrak{m}_x}{h_\xi} - m_x\right)G$, it may be considered as the difference between the ordinates of two equilibrium polygons, of which one (I) corresponds with the horizontal-stress curve, while the other (III) was constructed for the simple area of moments (fig. 1ᵈ) as a load area. The latter is drawn from the polygon of forces (fig. 1ᵉ) with H_ξ as the pole distance. If ζ_1 and ζ_3 be the ordinates of the two equilibrium polygons measured by the scale of length, a the width of the strips into which the load area is divided, p the pole distance of the force polygon (fig. 1ᵇ and 1ᶠ), we have $\eta = \frac{a\,p}{EJ}\,(\zeta_3 - \zeta_1)\,H_\xi$ and the scale upon which the ordinates of deflection are to be measured is constructed from $\frac{E\,J}{H_\xi\,a\,p}$ times the unit of length.

In the example on p. 88, $J = 0.0625$ m⁴, $a = 2.5$ m, $p = 35$ m, $H_\xi = \frac{11.45}{15.0}\,G = 0.764\,G$, $E = 20{,}000{,}000\ t$ per square meter. If the scale of 1 meter = 2 millimeters is chosen, the scale for the ordinates of deflection becomes 1 millimeter = 37.4 millimeters.

With the aid of the deflections for a unit load represented in cross section C no further difficulty is found in obtaining the resultant deflection in cross section C due to any assumed system of single loads, or to a uniformly distributed load. On the theory of the reciprocity of motion the deflection in C for a load lying in M equals the deflection in M for an equal load in C. In order, therefore, to obtain the total deflection in C we must take the sum of the ordinates of the deflection polygon measured on the load verticals and multiplied by the corresponding loads.

The above construction, which presupposes a constant moment of inertia of the stiffening truss, may also be applied with a variable moment of inertia. If J_0 represents a definite moment of inertia and J the moment of inertia at any assumed point, we must reduce the ordinates of the load areas (that is, the areas between the axis of the arch and chord) and the simple moment areas in the proportion $\frac{J_0}{J}$ and then carry out the construction of these reduced areas as above. In the arched truss exemplified on Pl. I the variation of the moment of inertia was taken into consideration.

In the stiffening truss hinged at the center we must introduce in equation (101), which determines the ordinates of deflection, $h_\xi = \frac{\xi}{2f}$ and $h_x = \frac{x}{2f}$ in which we assume ξ and x as $< \frac{l}{2}$. Replacing the first expression in the parenthesis, which gives the effect of the axial strains, by Σ for abbreviation, we obtain,

$$\eta = \frac{G\,x\,\xi}{4\,f^2\,E\,F_0}\,\Sigma + \frac{G}{EJ}\left[\mathfrak{m}_x + \frac{x\,\xi}{4\,f^2}\,\mu - \frac{1}{2\,f}\,(m_x\,\xi + m_\xi\,x)\right] \quad \ldots \quad (103)$$

The quantities $\mathfrak{m}_x$, μ, m_x, and m_ξ have the same significations as mentioned above. For $x > \frac{l}{2}$ we have,

$$\eta = \frac{G(l-x)\xi}{4f^2 E F_0}\Sigma + \frac{G}{EJ}\left[\mathfrak{m}_x + \frac{(l-x)\xi}{4f^2}\mu - \frac{1}{2f}\left[m_x\xi + m_\xi(l-x)\right]\right] \quad . \ (103^a)$$

By the introduction of the values of m_x, μ, and m_ξ which are applicable for a parabolic chain, we obtain the following expressions:

For $x < \xi < \frac{l}{2}$:

$$\eta = \frac{G x \xi \Sigma}{4f^2 E F_0} + \frac{G x}{30 E J l^2}\left[4 l^3 \xi - 15 \xi^2 l^2 + 15 \xi l (x^2 + \xi^2) - 5 \xi (x^3 + \xi^3) - 5 x^2 l^2\right] \quad . \ (104)$$

For $x > \xi < \frac{l}{2}$:

$$\eta = \frac{G x \xi \Sigma}{4f^2 E F_0} + \frac{G \xi}{30 E J l^2}\left[4 l^3 x - 15 x^2 l^2 + 15 x l (x^2 + \xi^2) - 5 x (x^3 + \xi^3) - 5 \xi^2 l^2\right] \quad . \ (105)$$

For $x > \frac{l}{2} > \xi$:

$$\eta = \frac{G \xi (l-x)\Sigma}{4f^2 E F_0} + \frac{G \xi (l-x)}{30 E J l^2}\left[5 \xi^2 (l-\xi) + 5 x (l-x)^2 - l^3\right] \quad . \ . \ . \ (106)$$

We can also apply the graphical method to the determination of the ordinates of deflection. Equation (103) may be written:

$$\eta = \frac{G}{EJ}(\mathfrak{m}_x - h_\xi m_x) + \delta \frac{2x}{l} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ (107)$$

when,

$$\delta = \frac{G \xi l \Sigma}{8f^2 E F_0} + \frac{G l}{4 E J f}\left(\frac{\xi}{2f}\mu - m_\xi\right) \quad . \ . \ . \ . \ . \ . \ (108)$$

The expression in parenthesis in equation (107) can again, as above, be represented as the differences between the two equilibrium polygons I and III (fig. 1[a]–1[f], Pl. I) in which, however, the simple moment area [fig. 1[d]) is to be drawn from the force polygon (fig. 1[e]) based on a pole distance $H_\xi = G\frac{\xi}{2f}$. In order to obtain the deflections the vertical distances of the two equilibrium polygons are to be increased by the ordinates of a triangle whose greatest height at the center of the bridge is $\delta \frac{EJ}{H_\xi a p}$. What has been said applies also to the effect of a system of loads or of a fixed load.

(b) *Deflection due to changes of temperature or moving of the chain supports.*—Equation (101[a]), which determines the deflection ordinate at a distance x from the end of the truss, applies also for the case in which the changes of form are due not to loading, but to effect of temperature changes or to the shifting of the chain saddles. In this case $\mathfrak{m}_x = 0$ and $G h_\xi = H_t$, the horizontal stress due to these influences. Hence,

$$\eta = -\frac{1}{EJ} H_t m_x \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ (109)$$

In which H_t is to be deduced from equations (94) to (96) for changes of temperature and from equation (97) for a yielding of the anchorage end of the chain, while m_x is determined from equations (102) and (102[a]). As the quantity m_x is proportional to the horizontal stress due to a single load lying at the point x, the deflections of the stiffening truss are determined also from the ordinates of the horizontal-stress curve; that is, from the equilibrium polygon I (fig. 1[a] Pl. I) for which (using the above notation) the unit of scale to be used shall be $\frac{EJ}{H_t a p}$ times the unit of the scale of length.

In a stiffening truss hinged at the center there occurs at the point x $\left(< \frac{l}{2}\right)$ a depression or rise, due to a change in length of the chain, which may be determined from $\eta = \Delta f. \frac{2x}{l}$ in which Δf is determined by equations (32) to (37).

§8. *More accurate theory of the archpolygon with straight stiffening girder.*

The theory developed in the preceding paragraphs and based upon the application of the theory as to the work necessary to produce change of form, gives satisfactory results only for systems in which very small (elastic) changes of form occur. Therefore, in the systems treated above, the admissibility of this approximate method of computation depends upon the degree of completeness with which the stiffening truss performs its task of overcoming the slackness of the system. If this truss is weakly proportioned (as is the case in such half-stiffened suspension bridges as the older structures), that is, if it possesses but a small moment of inertia, the static changes of form of the archpolygon can not be left out of consideration, and a closer method of computation must be employed.

Let H_g be the horizontal stress in the chain in its normal position of equilibrium, for which a curve of parabolic form may be assumed. We may then consider H_g as the effect of the constant load of the weight of the structure. Let the stresses in the suspenders under this load be s_0 (per unit of length) and the dead load due to the chain be k, then from equation (44) the differential equation of the curve of the cable becomes,

$$H_g \frac{d^2 y}{d x^2} = -(s_0 + k) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\alpha)$$

Let a load q (per unit of length) upon the girder change the stress in the suspenders to s_1, the horizontal stress to $(H_g + H)$, the ordinate of the curve of the cable to $y + \eta$. and then there again exists between these quantities the relation,

$$-(H_g + H) \frac{d^2 (y + \eta)}{d x_2} = s_1 + k \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\beta)$$

But by the addition of the load q a deflection of the stiffening truss is also produced. If we neglect the elongation of the suspenders this additional deflection of the truss can be placed equal to the depression η of the chain at the points lying in the same verticals, so that η is the ordinate of the elastic line of the girder for which we have the differential equation,

$$E J \frac{d \eta^4}{d x^4} = q - (s_1 - s_0) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\gamma)$$

The addition of α, β, γ gives

$$E J \frac{d^4 \eta}{d x^4} - (H_g + H) \frac{d^2 \eta}{d x^2} - H \frac{d^2 y}{d x^2} = q.$$

If for abbreviation we place,

$$\frac{H + H_g}{E J} = c^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (110)$$

the integration of the above differential equation, under the supposition that J and q are constant within the limits of integration, gives,

$$\frac{d^2 \eta}{d x^2} = A e^{cx} + B e^{-cx} - \frac{H}{H + H_g} \left(\frac{q}{H} + \frac{d^2 y}{d x^2} + \frac{1}{c^2} \frac{d^4 y}{d x_4} + \quad . \quad . \right)$$

or, since with a parabolic form of the chain $\frac{d^2 y}{d x^2} = -\frac{8f}{l^2}$ and $\frac{d^4 y}{d x^4} = 0$,

$$\frac{d^2 \eta}{d x^2} = A e^{cx} + B e^{-cx} - \frac{H}{H + H_g} \left(\frac{q}{H} - \frac{8f}{l^2} \right) \quad . \quad . \quad . \quad . \quad . \quad (\delta)$$

From the forces $q - (s_1 - s_0)$ acting upon the girder, there results a moment $M = \mathfrak{M} - (H + H_g)(y + \eta) + H_g y$, that is,

$$M = \mathfrak{M} - (H + H_g) \eta - H y, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (111)$$

when $\mathfrak{M}$ represents the moment of the load q for a truss resting upon two supports.

Hence we have,

$$\frac{d^2 \eta}{d x^2} = -\frac{M}{E J} = + c \eta - \frac{c^2}{H + H_g} (\mathfrak{M} - H y) \quad . \quad . \quad . \quad . \quad . \quad (\varepsilon)$$

Equating (δ) and (ε) and substituting $y = \frac{4 f}{l^2} x (l - x)$ we obtain when we finally introduce the constants $C_1 = \frac{A E J}{H}$ and $C_2 = \frac{B E J}{H}$ instead of A and B,

$$\eta = \frac{H}{H + H_g} \left(C_1 e^{c x} + C_2 e^{-c x} + \frac{8 f}{c^2 l^2} - \frac{4 f}{l^2} x (l - x) + \frac{\mathfrak{M}}{H} - \frac{q}{H c^2} \right) \quad . \quad (112)$$

The constants C_1 and C_2 of the preceding equation are determined from the condition that for $x = 0$ and $x = l$ we must have $\eta = 0$. If the live load q extends only over a part of the span, different values of the constants for the loaded and unloaded sections are to be introduced in the equation of the elastic line. The necessary determinate equations follow from the condition that for the limiting points the same values of η and $\frac{d \eta}{d x}$ must be obtained from the two adjoining sections.

The substitution of the value of η in equation (111) gives for the moment the equation,

$$M = - H \left[C_1 e^{c x} + C_2 e^{-c x} + \frac{1}{c^2} \left(\frac{8 f}{l^2} - \frac{q}{H} \right) \right] \quad . \quad . \quad . \quad . \quad (111^a)$$

From equation (111) we see that the moment M due to the live load q is no longer proportional to that load, but that its value is also influenced by the horizontal strain H_g which was present in the structure before the application of the load q. Compared with the approximate theory, $-(H + H_g) \eta$ gives the effect of the change of form and of the initial strain H_g of the chain. This acts advantageously in a stiffened suspension structure since in the case of loading which produces the greatest positive moment, η becomes positive and the moment is therefore diminished; and this holds good also for the numerical value of the negative moment. In an arched-rib structure, however, the reverse is the case, since the deflection of the truss produces an increase in the moment. The proportion of the member $-(H + H_g) \eta$ to the approximate value of the moment $\mathfrak{M} - H y$ can, of course, under certain circumstances, become quite large even in a rigid truss; for instance, for cross sections for which $\mathfrak{M} - H y$ becomes very small or equal to zero. On the other hand, we have for those cases of loading which correspond to the maxium moments, an appreciable effect from this change of form only when the coefficient c in the preceding equations attains a certain value, as is the case with a small moment of inertia J.

From equation (111^a) the moments of the stiffening truss under a definite loading can be computed exactly only when the horizontal stress H is known. As the changes of form of the chain will in general be very small, it may be ascertained in advance that this horizontal stress will vary but little from the value which is obtained from the preceding approximate formulas (§ 5.) We can therefore generally content ourselves with computing the moments from formula (111^a) with this approximate value of the horizontal stress.

However, if it is desired to determine H more accurately, it can be done with the help of the condition that for small changes of form the work of the forces $s_1 + k$ acting upon the chain must equal the work of the internal strains in the chain for changes of form; therefore, retaining the above notation,

$$\int_0^l (s_1 + k) . \eta \, d x = \frac{(H + H_g) H}{E F_0} \left(\Sigma \frac{\lambda^2}{a} + 2 l_1 \sec^2 \alpha_1 \right)$$

From equation (β) we may write with close approximation,

$$s_1 + k = - (H + H_g) \frac{d^2 y}{d x^2} = \frac{8 f}{l^2} (H + H_g)$$

and further for abbreviation writing $\Sigma \frac{\lambda^2}{a} + 2\, l_1 \sec^2 \alpha_1 = L$, in which for a parabolic chain,

$$L = l\left(1 + \frac{16}{3}\frac{f^2}{l^2}\right) + 2\, l_1 \sec^2 \alpha_1,$$

there results by the substitution of η from equation (112),

$$\int_0^l \left[C_1 e^{cx} + C_2 e^{-cx} + \frac{1}{H}\left(\mathfrak{M} - \frac{q}{c^2}\right) + \frac{8f}{l^2 c^2} - \frac{4fx(l-x)}{l^2} \right] dx$$

$$= c^2 \frac{J}{F_0}\frac{l^2 L}{8f},$$

from which the horizontal stress H is computed:

$$H = \frac{\int_0^l \left(\mathfrak{M} - \frac{q}{c^2}\right) dx}{-\int_0^l (C_1 e^{cx} + C_2 e^{-cx})\, dx + \frac{2}{3} fl - \frac{8f}{lc^2} + c^2 \frac{J}{F_0}\frac{l^2 L}{8f}} \quad . \; . \; . \; (113)$$

To apply this formula it is necessary first to determine the approximate value of H according to the preceding method, that is, from $H = \frac{\int_0^l \mathfrak{M}\, y\, dx}{\int y^2\, dx + \frac{JL}{F_0}}$, and with with this value to compute the quantities c, C_1, and C_2 of the above formula.

Equations (111[a]) and (113) apply for any assumed condition of loading. In the following let us apply them first to the case in which the loading consists of only a single load lying at distances ξ and $l - \xi = \xi'$ from the supports. For this case the following values of the constants are determined:

For the length from 0 to ξ,

$$C_1 = \frac{1}{e^{cl} - e^{-cl}} \left\{ \frac{P}{2cH}(e^{-c\xi'} - e^{c\xi'}) - \frac{8f}{c^2 l^2}(1 - e^{-cl}) \right\}$$

$$C_2 = \frac{1}{e^{cl} - e^{-cl}} \left\{ \frac{P}{2cH}(e^{c\xi'} - e^{-c\xi'}) + \frac{8f}{c^2 l^2}(1 - e^{cl}) \right\}$$

For the length from ξ to l,

$$C_3 = \frac{1}{e^{cl} - e^{-cl}} \left\{ \frac{P}{2cH}(e^{-c\xi} - e^{c\xi}) - \frac{8f}{c^2 l^2}(1 - e^{-cl}) \right\}$$

$$C_4 = \frac{1}{e^{cl} - e^{-cl}} \left\{ \frac{P}{2cH}(e^{c\xi} - e^{-c\xi}) + \frac{8f}{c^2 l^2}(1 - e^{cl}) \right\}$$

Whence we obtain,

$$H = \frac{P\xi\xi'}{\frac{2}{c^2}\left[K - \frac{8f}{l}\right] + \frac{2}{3} fl + c^2 \frac{J}{F_0}\frac{l^2 L}{8f}}, \quad . \; . \; . \; . \; . \; . \; (114)$$

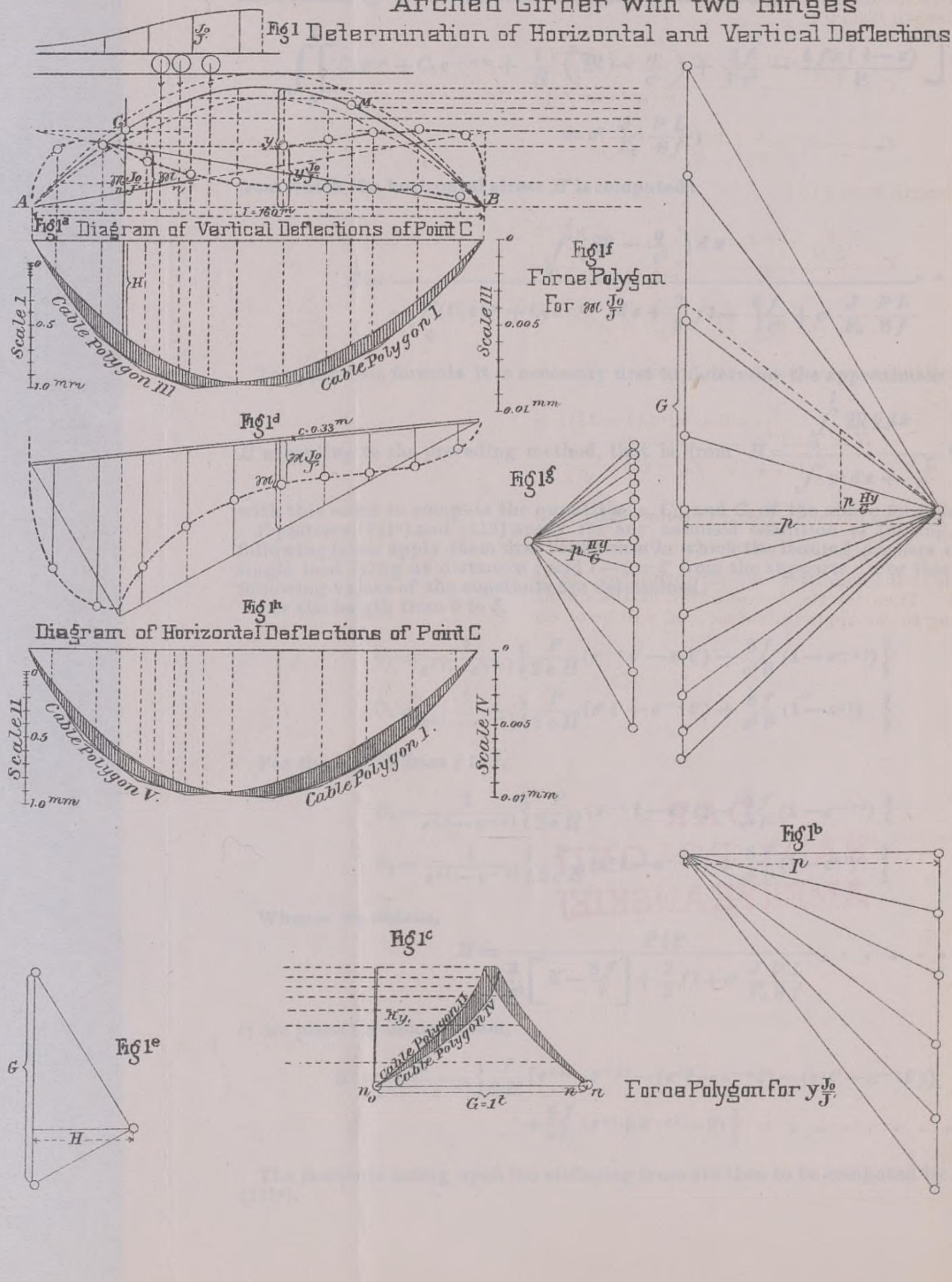
if we place for abbreviation,

$$K = \frac{2}{e^{cl} - e^{-cl}} \left\{ \frac{P}{2H}\left[e^{cl} - e^{-cl} - (e^{c\xi} - e^{-c\xi}) - (e^{c\xi'} - e^{-c\xi'})\right] + \frac{8f}{cl^2}(e^{cl} + e^{-cl} - 2) \right\} \quad . \; . \; . \; . \; . \; . \; (114^a)$$

The moments acting upon the stiffening truss are then to be computed by equation (111[a]).

PLATE 1.

Arched Girder with two Hinges



According to this more accurate theory, as already mentioned above, no relation exists between the loads and the internal strains. The method of moments is therefore not applicable, and the horizontal stress and moments and shears which produce the internal strains must be computed separately for every case of loading.

Below are given the formulas for that case in which the loading consists in a uniform load p per unit of length covering the entire span, and a load q per unit of length extending from the left point of support to the point ξ. We obtain— for the length from 0 to ξ,

$$C_1 = \frac{\frac{q}{2H}(e^{c\xi'} + e^{-c\xi'} - 2) + \left(\frac{p+q}{H} - \frac{8f}{l^2}\right)(1 - e^{-cl})}{c^2(e^{cl} - e^{-cl})}$$

$$C_2 = -C_1 + \frac{1}{c^2}\left(\frac{p+q}{H} - \frac{8f}{l^2}\right);$$

for the length from ξ to l,

$$C_3 = \frac{\frac{q}{2H}(e^{c\xi} + e^{-c\xi} - 2) + \left(\frac{p}{H} - \frac{8f}{l^2}\right)(1 - e^{-cl})}{c^2(e^{cl} - e^{-cl})}$$

$$C_4 = -C_3 + \frac{1}{c^2}\left(\frac{p}{H} - \frac{8f}{l^2}\right);$$

and substitution in equation (113) gives,

$$H = \frac{\frac{1}{12}[p\,l^3 + q\,\xi^2(3l - 2\xi)] - \frac{1}{c^2}(p\,l + q\,\xi)}{\frac{1}{c}(C_1 + C_3 - C_2 - C_4) - \frac{8f}{c^2 l} + \frac{2}{3}f\,l + c^2\frac{J}{F_0}\frac{l^2 L}{8f}} \quad . \quad . \quad . \quad (115)$$

Here H is the horizontal stress produced by the load $p+q$ when, according to the above, $c = \frac{H + H_g}{EJ}$ and H_g represents the horizontal stress in the chain which corresponds to the condition of equilibrium of the chain when the loads p and q are omitted. Then for the computation of the quantity c the value of H determined according to the approximate method will be of service.