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Adiabatic gas transport in the long flow channels

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Abstract The paper present the determination of the state parameters of natural gas at the pipeline inlet based on knowledge of the pressure and temperature at the receiving point. Natural gas transport will be carried out through an offshore section of a transmission pipeline. The equations of the Fanno flow model will be used to describe the thermodynamic parameters of the gas in the flow lines. The mathematical equations of the flow mentioned above models have been derived from an analysis of the mass, energy and momentum balance equations. They also take into account the viscous friction forces in the transported gas. Based on the carried out calculations, changes in the Mach number, pressure and velocity of methane transported along the analysed pipeline were determined. In addition, the total entropy gain in the analysed methane flow was determined. The novelty of the calculations presented is the use of the Fanno flow model, which considers a realistic adiabatic gas flow. This is in contrast to the isothermal flow model, which assumes an unchanging temperature of the transported gas. In the case under consideration, the adopting model was possible because of the similar temperature values of the gas flowing in the pipeline and the corresponding temperature values of the surrounding seawater. The fundamental advantage of the Fanno flow model is that it satisfies the mass balance of the flowing gas in each cross-section. Thus, the product of the velocity and density of the gas in a pipeline of constant diameter assumes a constant value.

Keywords: Natural gas pipeline; Fanno flow; Adiabatic flow; Mach number; Redlich-Kwong state equation

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Nomenclature

a	–	speed of sound, m/s
D	–	pipeline inner diameter, m
k	–	wall roughness, m
L	–	length, m
L_*	–	critical pipeline length, m
M	–	Mach number
p	–	pressure, Pa
p_{01}, p_{02}	–	stagnation pressure for the pipeline inlet and outlet sections respectively, Pa
$\dot{Q}_m$	–	mass flow rate, kg/s
$\dot{Q}_v$	–	volumetric flow rate, m^3/s
R	–	individual gas constant, J/(kgK)
r	–	residual
Re	–	Reynolds number
S	–	constant cross-sectional area, m^2
s	–	specific entropy, J/(kgK)
T	–	temperature, K
v	–	specific volume, m^3/kg
w	–	velocity, m/s

Greek symbols

η	–	dynamic viscosity, Ns/m^2
κ	–	isentropic exponent
λ	–	coefficient of friction losses
ρ	–	density, kg/m^3

Subscripts

1	–	inlet parameter
2	–	outlet parameter
c	–	critical
m	–	mass
v	–	volumetric
*	–	denotes that state where $M = 1$

Superscript

r	–	real gas

1 Introduction

Natural gas is one of the fastest-growing energy sources for thermal and chemical plants. The need to reduce carbon dioxide emissions into the atmosphere or stabilise them has created significant demand for this resource.

Natural gas is a mineral resource formed from the anaerobic decomposition of organic substances accumulated deep beneath the Earth's surface. The composition of natural gas is variable and depends primarily on the location from which it is extracted. It comprises hydrocarbon components, the main (CH_4) and other so-called heavy hydrocarbons, usually not in significant quantities. In addition, carbon dioxide (CO_2), hydrogen (H_2), hydrogen sulphide (H_2S), nitrogen (N_2), and noble gases such as helium (He) and argon (Ar) may be present in natural gas [1,2].

It should be noted that the content of individual hydrocarbons varies in natural gas from different fields. Gas that consists almost exclusively ($> 90\%$) of methane and ethane is called dry gas. A typical dry gas is a Norwegian gas containing nearly 97% CH_4 and C_2H_6 in its composition. In contrast, a gas that contains hydrocarbons with a higher molecular weight than methane is called condensate gas or wet gas. This gas is generally found together with oil fields.

It is worth noting that high molecular weight hydrocarbons and non-hydrocarbon components should be removed from natural gas (up to the content specified by the relevant standards) before transport or processing [2].

Natural gas distribution systems are highly complex installations and are not infrequently used to transport other gases like hydrogen. This makes the design and operation of natural gas distribution systems increasingly complex, requiring multi-criteria optimisation. Koo [3] proposed a novel implicit method of characteristics (MOC) that provides efficient and reliable transient flow analyses for natural gas pipelines. Zhang *et al.* [4] proposed the transient pressure simulation method to determine the pressures caused by the movement of the smart isolation device in subsea gas pipeline maintenance technology. The technique was verified experimentally, presenting its good accuracy in simulating the moving process caused in gas pipelines by isolation devices. Glot *et al.* [5] numerically and experimentally investigated processes of propagation of deformation waves in the underground gas pipeline. The numerical simulation of dynamic processes in the underground pipelines allowed the development of deformation wave propagation and decay patterns. Koo [6] analyzed two numerical methods of modelling transient flow in a single natural-gas pipeline – the pressure-based finite-volume method and the conventional implicit method of characteristics depending on the nature of the gas flow. Zhou *et al.* [7] developed a dynamic model to simulate the behaviour of a single pipeline and pipeline network according to the actual geographical topology.

Ensuring flexibility in the operation of the electricity grid with renewable energy sources (RES) and energy storage can be achieved by transporting natural gas supplemented with hydrogen. This hydrogen is usually produced by electrolysis processes realised using excess electricity from RES. Witek and Uilhoorn in [8], particularly determined the permissible hydrogen content that can be blended with natural gas in existing pipeline systems and then investigated the impact on linepack energy for both compressor start-up and shut-down scenarios.

In addition to natural gas, liquefied natural gas (LNG) can be used in power generation systems. Szczygieł and Rutczyk [9] analysed the change in the parameters of LNG during its transportation by ship and the regasification process. It is mainly used for power generation, but some facilities allow the use of LNG cryogenic exergy for other tasks, such as in the food industry for freeze-drying food. Wieczorek [10] proposed using liquefied petroleum gas (LPG) to remove inert gas from the tank. For this purpose, she developed a thermodynamic model of this process, which made it possible to determine the required concentration and amount of LPG gas to accomplish this process. The developed thermodynamic model was tested for various conditions [11].

Trawinski [12] noted the need to identify working fluids when developing the mathematical models used to describe thermodynamic systems. In particular, the paper [12] analysed the semi-ideal gas model as an extension of the ideal gas model for selected gases in extended ranges of thermodynamic parameters. The Redlich-Kwong, Peng-Robinson, Soave-Redlich-Kwong, and Lee-Kesler models were analysed. The Redlich-Kwong and Lee-Kesler equations of state proved to be the most accurate for describing the gases under study: humid atmospheric air, high-methane natural gas and flue gas.

Generally, it can be said that the equations of the isothermal model or the corresponding equations of the Fanno flow model [8,12,13] describe the thermodynamic parameters of the gas in transport pipelines. The mathematical equations of the aforementioned flow models were derived from the analysis of the mass, energy and momentum balance equations, and they also take into account the viscous friction forces in the transported gas [13–15]. Classical methods of calculating the parameters of gas flowing in a pipeline cannot be applied to the case under consideration due to the large parameter variations. These parameter changes are due to the expansion process of the transported gas.

Since the Fanno model equations describe gas flows under realistic adiabatic conditions, they will be used in the further analysis included in this paper. Theoretical considerations and experimental studies show that the Fanno model equations accurately describe both subsonic and supersonic gas flows [14, 15]. An essential advantage of subsonic flow in the Fanno model under consideration is that there is no restriction on the length of the flow pipe (Fanno tube). It is worth noting that the Fanno model equations in question often assume a constant value for the coefficient of frictional loss λ. In Fanno flow, the temperature of the gas changes and therefore the value of the Reynolds number also changes. An analysis of the formula describing the Reynolds number $\left(\mathrm{Re} = \dfrac{\rho w D}{\eta}\right)$ shows that in a pipe of constant diameter, its value is only affected by a change in gas viscosity. Given that in the flows under consideration, the gas temperature changes only slightly, the assumption mentioned above of invariability of the friction loss coefficient $\lambda = \lambda\left(\mathrm{Re}, \dfrac{k}{D}\right)$ is justified. When larger gas temperature differences are present, it is recommended that the averaged value of the frictional loss coefficient along the length of the Fanno tube be included in the calculation [13, 15].

2 Estimation of gas parameters at the inlet to pipeline

Due to the high methane content (98%) of the transported natural gas in the Baltic Pipe pipeline, the calculations are limited to the case of methane flow. Due to this, the thermophysical properties of the transported gas will be determined as for pure methane. Based on the assumption according to [16] that the transmission capacity of the Baltic Pipe pipeline is $\dot{Q}_v = 8 \cdot 10^9$ nm^3/year with a density of $\rho = 0.7167$ kg/nm^3 the corresponding methane mass flow rate in the pipeline under consideration can be calculated:

$$\dot{Q}_m = \dot{Q}_v \rho = 8 \cdot 10^9 \, \frac{\mathrm{nm}^3}{\mathrm{year}} \cdot \frac{\mathrm{year}}{365 \cdot 24 \cdot 3600 \ \mathrm{s}} \cdot 0.7167 \, \frac{\mathrm{kg}}{\mathrm{nm}^3} = 181.711 \, \frac{\mathrm{kg}}{\mathrm{s}} \,. \quad (1)$$

The calculations included for transported methane: $T_n = 273.15$ K, $p_n = 1013.25$ hPa, 1 year $= 365 \cdot 24 \cdot 3600$ s.

Considering the characteristic dimensions of the offshore section of the Baltic Pipe pipeline (from Denmark to Poland): $L = 275$ km, $D = 900$ mm and $k/D = 0.0001$, the parameters of transported methane in the flow region can be determined. The working pressure of the Baltic Pipe pipeline given by the operator is between 5 MPa and 12 MPa, depending on the volume of gas to be transported [16]. It should be noted that in further calculations, the values of thermodynamic parameters of transported methane in the outlet section of the pipeline were assumed. They are $T_2 = 288$ K, and $p_2 = 170$ kPa. It can be stated that the temperature and pressure values adopted are based on the customer's needs. In addition, the calculations take into account: $R = 518.33$ J/(kg K) – the individual gas constant of methane, $\kappa = 1.30$ – the isentropic exponent, $\eta(T = 298\ \text{K}) = 0.01 \cdot 10^{-3}$ N s/m^2 – dynamic viscosity of methane [17]. The adopted calculation scheme according to the Fanno model for the Baltic Pipe pipeline under consideration is shown in Fig. 1.

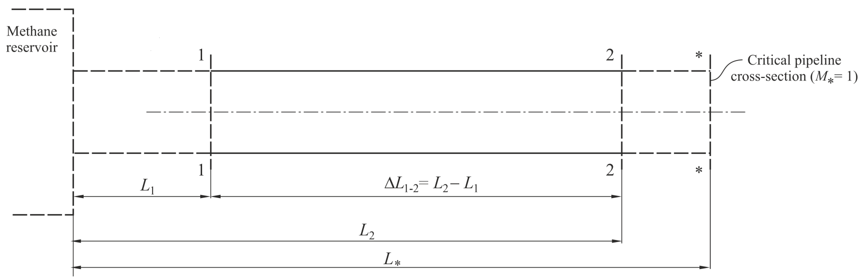

Figure 1: Fanno pipeline diagram with marked sections L_1, L_2, and L_*.

Because the values of the methane parameters in the outlet section of the pipeline section are close to ambient conditions, the density ρ_2 in this section is determined from the equation of the state of an ideal gas:

$$\rho_2 = \frac{p_2}{RT_2} = 1.139\ \frac{\text{kg}}{\text{m}^3}\,. \tag{2}$$

The velocity of methane in the outlet section of the pipeline is calculated from the flow continuity equation:

$$\dot{Q}_m = w_2 \rho_2 S_2\,, \tag{3}$$

where: w_2 – methane velocity in the outlet cross-section, ρ_2 – density of methane in the outlet cross-section, $S_2 = S$ – constant cross-sectional area of the pipeline.

According to the aforementioned values of the transported gas parameters and the pipeline dimensions, the value of the velocity and the Mach number in the outlet section (2–2) will be determined:

$$S_2 = \frac{\pi D^2}{4} = 0.6362\ \mathrm{m}^2,$$
$$w_2 = \frac{\dot{Q}_m}{\rho_2 S_2} = 250.9\ \frac{\mathrm{m}}{\mathrm{s}}\,.$$

The following relation describes the Mach number at the outflow:

$$M_2 = \frac{w_2}{a_2}\,, \tag{4}$$

where $a_2 = \sqrt{\kappa R T_2}$ is the speed of sound in methane calculated for the parameters in the outlet section.

Assuming the methane temperature $T_2 = 288$ K in the outlet cross-section, the corresponding speed of sound will take the value $a_2 = 440.53$ m/s. After substituting the corresponding values into Eq. (4), is obtained

$$M_2 = 0.56954\,.$$

The coefficient of friction loss $\lambda = \lambda\left(\mathrm{Re}, \dfrac{k}{D}\right)$ for methane flow in the pipeline is determined from the formula

$$\frac{1}{\sqrt{\lambda}} = -1.8\log\left[\left(\frac{\frac{k}{D}}{3.7}\right)^{1.11} + \frac{6.9}{\mathrm{Re}}\right], \quad 4000 \le \mathrm{Re} \le 10^8. \tag{5}$$

Because of the significant value of the Reynolds number,

$$\mathrm{Re} = \frac{\rho w D}{\eta} = \frac{\rho_2 w_2 D_2}{\eta_2} = 2.572 \cdot 10^7, \tag{6}$$

the Haaland formula will be used to determine the value of the friction loss coefficient. After substituting the appropriate values, this yields $\lambda = 0.01237$. The low λ value is because the inside of the gas pipeline is coated with a special layer of high-smoothness material to minimise pressure losses caused by viscous friction.

The value of Mach number M_1 in the inlet section of the Baltic Pipe pipeline under consideration is calculated from the Fanno formula [13–15]:

$$\frac{1}{\kappa}\frac{\left(1 - M_1^2\right)}{M_1^2} + \frac{(\kappa+1)}{2\kappa}\ln\left[\frac{\left(\frac{\kappa+1}{2}\right)M_1^2}{1 + \left(\frac{\kappa-1}{2}\right)M_1^2}\right] = \lambda\frac{(L_* - L_1)}{D}\,. \tag{7}$$

It is worth mentioning that the value of the Mach number M_1 is calculated by knowing the value of the parameter of the right-hand side of Eq. (7). To calculate the parameter $\lambda\dfrac{(L_* - L)}{D}$ the known value of the Mach number M_2 will be used. After substituting the value of M_2 into the Fanno formula, it will result in the following:

$$\frac{1}{\kappa}\frac{(1-M_2^2)}{M_2^2} + \frac{(\kappa+1)}{2\kappa}\ln\left[\frac{\left(\dfrac{\kappa+1}{2}\right)M_2^2}{1+\left(\dfrac{\kappa-1}{2}\right)M_2^2}\right] = \lambda\frac{(L_* - L_2)}{D}\,. \tag{8}$$

Figure 1 shows the segments L_1, L_2 and L_* appearing in the formulae (7) and (8). It is seen from Fig. 1 that the difference in length of the pipeline sections $L_2 - L_1 = \Delta L_{1-2} \cong L$, which is equal to the length of the undersea section of the Baltic Pipe. This difference in the length of the pipeline sections will be used in the further calculations. After substituting into Eq. (8) values of Mach number $M_2 = 0.56954$ and isentropic exponent $\kappa = 1.30$, is obtained

$$0.6881 = \lambda\frac{(L_* - L_2)}{D}\,. \tag{9}$$

Assuming a constant value of the friction loss coefficient λ, with a constant diameter D of the pipeline, the following will be written from Fig. 1:

$$\lambda\frac{\Delta L_{1-2}}{D} = \lambda\frac{(L_* - L_1)}{D} - \lambda\frac{(L_* - L_2)}{D}\,. \tag{10}$$

From Eq. (10) is obtained

$$\lambda\frac{(L_* - L_1)}{D} = \lambda\frac{\Delta L_{1-2}}{D} + \lambda\frac{(L_* - L_2)}{D}\,. \tag{11}$$

After the substitution of the corresponding parameter values in Eq. (11), the parameter of the right-hand side of Eq. (7) takes the value of

$$\lambda\frac{(L_* - L_1)}{D} = 3780.4103.$$

Having calculated the value of the parameter $\lambda\dfrac{(L_* - L_1)}{D}$ in Eq. (7) results in a non-linear algebraic equation from which the value of the Mach number M_1 is determined:

$$\frac{1}{1.3}\frac{(1-M_1^2)}{M_1^2} + \frac{(1.3+1)}{2\cdot 1.3}\ln\left[\frac{\left(\dfrac{\kappa+1}{2}\right)M_1^2}{1+\left(\dfrac{\kappa-1}{2}\right)M_1^2}\right] = 3780.4103. \tag{12}$$

The non-linear algebraic Eq. (12) was solved using Newton's method, considering the subsonic flow range in the pipeline section under consideration. The value of the determined Mach number is $M_1 = 0.01425$ ($r = 5 \cdot 10^{-5}$). According to the Fanno flow concept, the ratio of methane pressure in the inlet and outlet sections is determined from the formula [14, 15]

$$\frac{p_1}{p_2} = \frac{M_2}{M_1}\left[\frac{2+(\kappa-1)M_2^2}{2+(\kappa-1)M_1^2}\right]^{0.5}. \tag{13a}$$

After substituting the corresponding values in Eq. (13), the pressure ratio is calculated $\dfrac{p_1}{p_2} = 40.922$. Taking into account the assumed value of the outlet pressure $p_2 = 170$ kPa, the value of the methane pressure in the pipeline inlet section is calculated, which is $p_1 = 40.922 p_2 = 6956.742$ kPa.

Equation (13a) can also be written in the form

$$\frac{p_1}{p_2} = \frac{M_2}{M_1}\left(\frac{T_1}{T_2}\right)^{0.5}, \tag{13b}$$

from which the temperature of the methane in the inlet section of the pipeline is determined:

$$T_1 = \left(\frac{p_1}{p_2}\right)^2 \left(\frac{M_1}{M_2}\right)^2 T_2 . \tag{14}$$

After substituting the relevant parameter values into Eq. (14), the value of the methane temperature in the inlet section is obtained and is $T_1 = 302$ K.

Due to the slight temperature difference of methane in the inlet and outlet sections of the pipeline, the value of the friction loss coefficient will remain constant. The methane velocity at the inlet cross-section is determined using a known value of the Mach number M_1:

$$M_1 = \frac{w_1}{a_1} . \tag{15}$$

where a_1 is the speed of sound in methane under thermodynamic conditions of the inlet section.

Speed of sound a_1 is determined from the known formula

$$a_1 = \sqrt{\kappa R T_1} . \tag{16}$$

By substituting the relevant parameter values into Eq. (16), the following result is obtained: $a_1 = 451.11$ m/s. The calculated speed of sound a_1 to determine the methane velocity in the inlet section (1–1) (Fig. 1), i.e., $w_1 = a_1 M_1 = 6.43$ m/s.

Notably, the general dependence of the variation of gas velocity in individual sections of the Fanno tube is described by the relation [13, 15]

$$\frac{w_2}{w_1} = \frac{M_2}{M_1}\left[\frac{2+(\kappa-1)M_1^2}{2+(\kappa-1)M_2^2}\right]^{0.5}. \tag{17}$$

To compare the results obtained with the Fanno model, corresponding calculations were additionally conducted with the isothermal flow model. Taking into account the model of isothermal flow in the pipeline under consideration, the Mach number in the inlet section was determined from the relation [14]

$$\lambda\frac{L}{D} = \frac{1}{\kappa}\frac{\left(1-M_1^2\right)}{M_1^2} + \ln\left(\kappa M_1^2\right). \tag{18}$$

Considering the calculated values of the friction loss coefficient λ and the dimensions of the pipeline, the value of Mach number was obtained from relation (18) $M_1 = 0.1424847$ ($r = 6.5 \cdot 10^{-4}$).

The corresponding value of the Mach number M_2 in the outlet section for the isothermal flow model was determined from the formula

$$\lambda\frac{L}{D} = \lambda\frac{L}{D}\bigg|_1 - \lambda\frac{L}{D}\bigg|_2 = \frac{1}{\kappa}\frac{\left(1-M_1^2\right)}{M_1^2} - \frac{1}{\kappa}\frac{\left(1-M_2^2\right)}{M_2^2} + \ln\left(\frac{M_1}{M_2}\right)^2. \tag{19}$$

By substituting the relevant data, the Mach number value of the outlet section was obtained $M_2 = 0.87750$ ($r = 6.5 \cdot 10^{-4}$).

It is easy to see that the calculated values of the Mach number M_2 in the pipeline outlet section show a significant difference $\Delta M_2 = 0.87750 - 0.56954 = 0.30796$. It is worth mentioning that the methane flow under consideration in the pipeline is a subsonic flow for both the Fanno and the isothermal models.

The corresponding static pressure and flow velocity changes for the isothermal model were determined from the relationship [14]:

$$p_2 = p_1\frac{M_1}{M_2}, \tag{20}$$

$$w_2 = w_1\frac{M_1}{M_2}. \tag{21}$$

Due to the very similar values of the Mach number M_1 in the pipeline inlet cross-section, it was assumed that the static pressure in the inlet cross-section is identical for both models considered. The calculations also considered that the temperature in the inlet section T_1 of the transported methane is similar for both models.

The Mach number, static pressure and velocity distribution along the pipeline determined from Eqs. (19)–(21) are shown as dashed lines in Figs. 2 to 4, respectively. Figures 2–4 have also shown the changes in the Mach number, static pressure and velocity of the flowing methane as a function of the Baltic Pipe pipeline length calculated from Eqs. (12), (13), and (17) – the Fanno model. It should be noted that the beginning of the pipeline is located on Danish territory, while the end is situated respectively on the Polish coast.

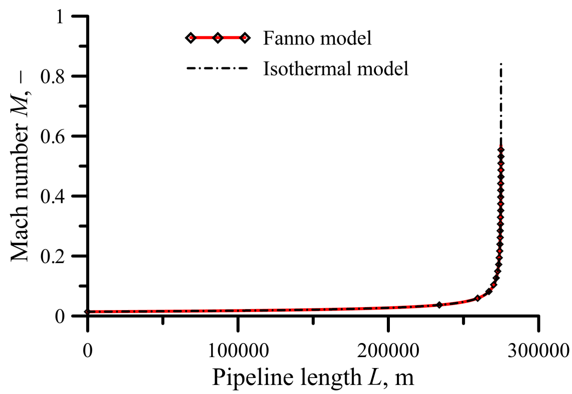

Figure 2: Change in Mach number value along the Baltic Pipe undersea section.

The significant changes in the Mach number values describing the methane flow in the final part of the pipeline, i.e. at the Polish coast, can be seen in Fig. 2. As can be seen, for most of the pipeline length, the methane flow has the features of incompressible flow.

It can be seen from Fig. 3 that the highest static pressure value is assumed in the flowing methane in the pipeline inlet section located in Denmark. At half the length of the pipeline, the pressure is about 30% lower than the corresponding pressure at the mentioned inlet cross-section. The static pressure value in the transported methane (CH_4) decreases by 50% at about 70 km from the outlet cross-section. Due to the decreasing value of the discussed pressure of flowing methane, the Baltic Pipe pipeline wall thickness could be considerably reduced over a large part of the pipeline.

Determined form Fanno model the corresponding changes in the flow velocity of the transported gas are shown in Fig. 4. The velocity of flow-

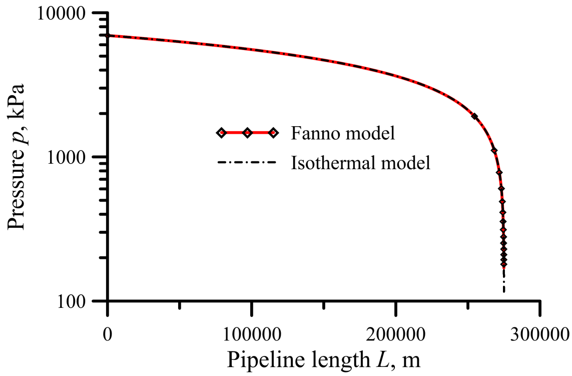

Figure 3: Change in static methane pressure along the Baltic Pipe undersea section.

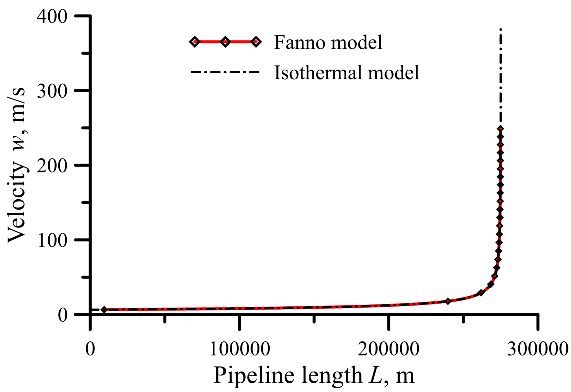

Figure 4: Change in methane velocity along the Baltic Pipe pipeline undersea section.

ing methane in the outlet section of the pipeline reaches the highest values. Approximately one kilometer before the outlet section, the transported methane (CH_4) velocity reaches $w = 100$ m/s. Evident compressibility of flowing methane ($M \geq 0.3$) is marked at the end of the pipeline, i.e. approximately 400 m upstream of the outlet cross-section.

Due to the unchanging temperature of the flowing gas in the isothermal model, the obtained values of the methane parameters in the outlet cross-section show significant differences compared to the corresponding results obtained in the adiabatic Fanno model. Because of the very small value of the Mach number calculated for the methane flow in the inlet section ($M_1 = 0.01425$), the considered Fanno flow, as well as isothermal flow features incompressible flow over a large part of the pipeline section.

In particular, it is only at a distance of approximately 400 m upstream of the pipeline outlet cross-section (Fig. 5) that the Mach number reaches a value of $M_3 = 0.3$. In the mentioned final section of the pipeline, there is an intensive expansion of the flowing methane, characterised mainly by the Mach number value in the outlet section $M_2 = 0.56594$.

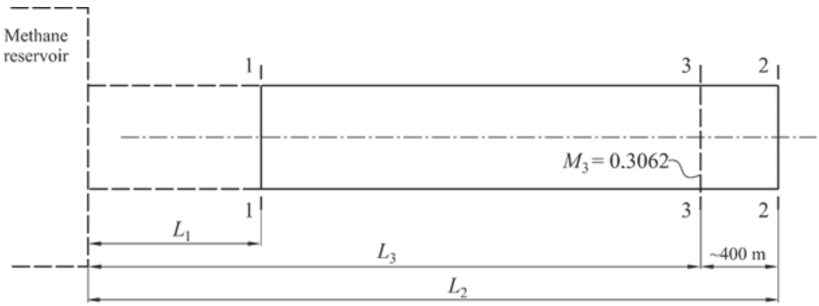

Figure 5: Diagram of section 3–3 location in Fanno flow.

The considered methane flow velocity in the subsea section of the Baltic Pipe increases continuously from a value of $w_1 = 6.43$ m/s in the inlet section to a value of $w_3 = 134.9$ m/s in the cross-section, where the Mach number is $M_3 = 0.3$ (Fig. 5), reaching a value of $w_2 = 250.90$ m/s in the outlet section.

The methane density in the pipeline's inlet section in Fanno model is described by the following formula [13, 15]:

$$\frac{\rho_1}{\rho_2} = \frac{M_2}{M_1} \left[\frac{2 + (\kappa - 1)M_1^2}{2 + (\kappa - 1)M_2^2} \right]^{0.5} . \tag{22}$$

Substitution of the Mach number and isentropic exponent values into Eq. (22) gives $\dfrac{\rho_1}{\rho_2} = 39.0247$ from which $\rho_1 = \rho_2 \cdot 39.0247 = 44.449$ kg/m^3.

Based on calculated from Fanno model values of the thermodynamic parameters of methane in the pipeline inlet section, the mass flow rate is determined:

$$\dot{Q}_m = \rho_1 w_1 S. \tag{23}$$

Taking into account the values of the individual quantities in Eq. (23) gives $\dot{Q}_m = 181.83$ kg/s.

By comparing the calculated value of the methane mass flow rate in the inlet section of the pipeline with the corresponding value in the outlet section, it can be said that a high leak tightness characterises the pipeline.

In the considered flow, furthermore, the differential stagnation pressures at the inlet and outlet sections, respectively, will be determined:

$$\Delta p_0 = p_{01} - p_{02}\,. \tag{24}$$

The individual stagnation pressures for the inlet and outlet sections are determined from the following equations [14, 15]:

$$\frac{p_{01}}{p_1} = \left(1 + \frac{\kappa - 1}{2} M_1^2\right)^{\frac{\kappa}{\kappa-1}}, \tag{25}$$

$$\frac{p_2}{p_{02}} = \frac{1}{\left(1 + \frac{\kappa - 1}{2} M_2^2\right)^{\frac{\kappa}{\kappa-1}}}. \tag{26}$$

After substitution data into Eqs. (25)–(26) it results in: $p_{01} = 6.598 \cdot 10^6$ Pa, $p_{02} = 208.856 \cdot 10^3$ Pa from which $\Delta p_0 = 6389$ kPa.

Considering that the Fanno flow is non-isentropic, the entropy gain between the outlet and inlet sections will be determined using the equation [15]

$$s_2 - s_1 = R \ln\left(\frac{p_{01}}{p_{02}}\right). \tag{27}$$

After the substitution of the calculated stagnation pressures, the entropy gain in the considered flow is $\Delta s = s_2 - s_1 = 1789.37$ J/(kg K).

The difference in stagnation pressure Δp_0 determines the approximate value of the excess pressure in the inlet cross-section needed to transport the assumed mass flow of methane in the pipeline under consideration.

3 Analysis of the calculation results obtained with the Fanno model

It is worth noting that the Fanno model equations are designed to describe the adiabatic flow of gas, considering internal friction in a flow pipe with a constant cross-sectional area. They are consistent with a flowing gas's mass, momentum and energy balance equations. However, it should be noted that the Fanno model equations include the equation of the state of an ideal gas. Thus, the Fanno model equations can be used without limitations to describe the motion of gases whose thermodynamic parameters (p, T) differ little from the corresponding ambient parameters.

In the case of the calculation results obtained in this work, especially in the pipeline inlet section, the differences in the mentioned parameters are considerable. It is easy to see that the calculated pressure in the pipeline inlet section is $p_1 = 6.957$ MPa, which is about 40.9 times higher than the corresponding pressure in the outlet section. From the ideal gas equation of state, the density of flowing methane in the inlet section is $\rho_1 = 44.449\ \mathrm{kg/m^3}$.

To validate the obtained calculation results, the equation of state of the real gas written in the form of the Redlich-Kwong equation was used [17,18]:

$$p = \frac{RT}{v-b} - \frac{a}{T^{0.5}v(v+b)}\,, \tag{28}$$

where $a = \dfrac{0.42748R^2T_c^{2.5}}{p_c}$ and $b = \dfrac{0.08664RT_c}{p_c}$ are constants in the Redlich-Kwong real gas state equation, p_c and T_c are the critical pressure and temperature for gas, $v = 1/\rho$ is specific volume.

In the case of methane, the critical pressure and temperature are $p_c = 45.992\cdot 10^5$ Pa and $T_c = 190.6$ K [17,18]. After substituting into Eq. (28) the values of the relevant parameters, i.e., $R = 518.33$ J/(kgK), $T_1 = 302$ K, $v_1 = 1/\rho_1 = 0.022498\ \mathrm{m^3/kg}$, the corresponding pressure value was calculated $p_1^{(r)} = 6270244$ Pa ≈ 6.27 MPa. It is easy to see that the relative deviation of the results obtained does not exceed 10%. The Redlich-Kwong equation was tested in this work using tabulated values of methane thermodynamic parameters $p_a = 6$ MPa, $v_a = 0.02343\ \mathrm{m^3/kg}$, and $p_b = 8$ MPa, $v_b = 0.01706\ \mathrm{m^3/kg}$ [18]. The respective relative deviations of the results obtained were: for data with subscript (a) $\delta_{R-K} = 0.2\%$ and (b) $\delta_{R-K} = 0.1\%$.

Apart from the aforementioned test of the Redlich-Kwong model, an additional corresponding examination of the Lee-Kesler model was carried out in this study. The general form of the equation describing the Lee-Kesler model is of the form [12]

$$Z = \left(\frac{p_r v_r}{T_r}\right) = 1 + \frac{B}{v_r} + \frac{C}{v_r^2} + \frac{D}{v_r^5} + \frac{c_4}{T_r^3 v_r^2}\left(\beta + \frac{\gamma}{v_r^2}\right)\exp\left(-\frac{\gamma}{v_r^2}\right). \tag{29}$$

Coefficients of Lee-Kesler equation of state are given as

$$B = b_1 - \frac{b_2}{T_r} - \frac{b_3}{T_r^2} - \frac{b_4}{T_r^3}\,, \quad C = c_1 - \frac{c_2}{T_r} - \frac{c_3}{T_r^3}\,, \quad D = d_1 - \frac{d_2}{T_r}\,,$$

$$p_r = \frac{p}{p_c}\,, \quad T_r = \frac{T}{T_c}\,, \quad v_r = \frac{v p_c}{R T_c}\,,$$

where constants b_1, b_2, b_3, b_4, c_1, c_2, c_3, c_4, d_1, d_2, β, γ are the constants of the Lee-Kesler model, whose numerical values were taken from [12].

The corresponding relative deviations for the Lee-Kesler model were obtained by performing similar calculations as for the Redlich-Kwong model: for data with subscript (a) $\delta_{L-K} = 1.63\%$ and (b) $\delta_{L-K} = 2.74\%$. As the calculated relative deviations of the analysed results for the Redlich-Kwong model are smaller than the corresponding deviations obtained for the Lee-Kesler model, the real gas model described by Eq. (28) was used for further calculations.

The methane flow considered in the Baltic Pipe pipeline is a single-phase flow of a medium exhibiting the characteristics of superheated steam. Therefore, changes in the coefficients describing the specific heat c_p and c_v for methane are negligibly small in the considered flow case. Analysing the change in specific heat at constant pressure c_p for methane in the temperature range 275 K to 310 K and pressure of 8 MPa, the relative difference in values is in the range 7% [17, 18].

Since the velocity of methane in the inlet section v_1 was determined using the calculated Mach number M_1, the speed of sound in methane, a_1 was also determined by the equation

$$a_1 = \sqrt{\kappa \left[\frac{R T_1}{\left(1 - b\rho_1\right)^2} - \frac{a}{T_1^{0.5}} \frac{\rho_1 \left(2 + b\rho_1\right)}{\left(1 + b\rho_1\right)^2} \right]}\,. \tag{30}$$

Equation (30) is derived from Eq. (28), in which the inverse of density replaces specific volume ($v_1 = 1/\rho_1$). In particular, Eq. (30) was achieved after determining the partial derivative from pressure after density at a constant temperature [15]. After substituting the values for the quantities appearing in Eq. (30), the $a_1^{(r)} = 409.72$ m/s was obtained.

Using the previously calculated value of the Mach number in the inlet section $M_1 = 0.014251$, the corresponding methane velocity under real gas conditions is $w_1^{(r)} = M_1 a_1^{(r)} = 5.84$ m/s.

Calculated velocity $w_1^{(r)} = 5.84$ m/s is less than the corresponding velocity calculated for the ideal gas model ($w_1 = 6.43$ m/s).

From the analysis of the calculation results obtained with the Fanno model and the related results obtained with the real gas model, it can be concluded that they differ slightly. It also appears that the deviations noted cannot pose a threat to the correct operation of the pipeline.

4 Conclusions

According to the calculations, it can be concluded that the isothermal model shows greater simplicity compared to the adiabatic Fanno model. However, the adiabatic Fanno model allows for more precise results describing the transported gas in the pipeline under consideration.

Based on the above, the Fanno flow model equations were used to calculate the thermodynamic parameters of methane in the subsea section of the Baltic Pipe pipeline. It is worth remarking that the Fanno model equations are valid for adiabatic flow involving viscous friction of the gas. It can also be stated that the methane flow in the Baltic Pipe pipeline with a multilayer wall of thickness $g = 87.5{-}137.5$ mm, located on the seabed, meets the conditions for adiabatic flow. The calculations indicate that the temperature of the transported gas varies between $T_1 = 302$ K and $T_2 = 288$ K.

Based on the carried out calculations, the respective values of the pressure ratios in the corresponding pipeline cross-sections were also determined, i.e., ratio of inlet and outlet pressure $p_1/p_2 = 40.925$, and ratio of pressure the inlet and approximately 400 m from the pipeline outlet $p_1/p_3 = 21.192$.

The pressure values in the corresponding pipeline cross-sections can be estimated from the quoted pressure ratios. For the outlet pressure $p_2 = 170$ kPa, the pressure in the inlet section is $p_1 = 6.957$ MPa, and the corresponding pressure in the pipeline section where the Mach number reaches $M_3 = 0.3$ takes the value $p_3 = 328.277$ kPa. It can be found that the value of pressure p_3 is only about twice the outlet section pressure p_2. A good agreement can be found by comparing the obtained results of calculating the pressure of methane transported in the pipeline inlet section with the corresponding working pressure values given by the Baltic Pipe pipeline operator.

The calculated difference in stagnation pressures determined for the inlet and outlet sections is $\Delta p_0 = 6389$ kPa. The calculated values of the stagnation pressures were also used to calculate the total entropy increase in the methane flow under analysis, and it is $\Delta = 1789.37$ J/(kgK).

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