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Artificial Intelligence overview for optimizing production scheduling in a picture framing company

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Abstract

Artificial Intelligence is significantly transforming manufacturing and impacting warehouse planning, production, operations, and logistics. This article explores AI applications in these areas, analyzing its role in optimizing scheduling and comparing established methods for efficiency. In warehouse planning, AI optimizes inventory placement, automates processes, and enables predictive maintenance. In production, it improves scheduling, predicts machine failures, and personalizes products. AI enhances operations by analyzing sensor data to predict failures and schedule maintenance. In logistics, it optimizes transport routes, automates shipping, and manages inventory. The study examines scheduling using priority rules, complete search, and genetic algorithms. These methods were tested on data from a picture framing company with make-to--order production. Scheduling quality was measured by minimizing makespan and comparing computational efficiency. The genetic algorithm outperformed complete search, scheduling 70 orders on 10 machines in just over 500 ms, significantly reducing computational time. This efficiency is crucial for larger problems where complete search becomes impractical. The findings highlight AI's potential to improve scheduling in manufacturing, making it a valuable tool for complex production. AI-driven solutions can enhance efficiency across industries, providing policymakers with a pathway to support advanced manufacturing technologies.

Keywords: algorithms, efficiency, logistics, manufacturing, optimisation, scheduling, Artificial Intelligence

1. Introduction

Production engineering is a multidisciplinary field that deals with the planning, design, implementation, management and protection of production and logistics systems. It employs sophisticated methodologies and instruments for the management of voluminous data sets, encompassing their acquisition, collection, storage, and analysis, to facilitate precise decision-making in the face of challenges. The overarching objective is to ensure the efficiency and reliability of these systems. These methods support engineers, who are pivotal to the operations of manufacturing enterprises, in their efforts to enhance efficiency, uphold quality standards, and curtail costs (Lipski, Świć and Bojanowska, 2014).

Methods for production planning and selecting a production plan that meets market demands while aligning with available resources have been a subject of research since the 1950s. A pioneer in this field is considered to be Johnson, who, in his work at the time, analyzed job scheduling for two and three production stations (Johnson, 1954). During that period, research on optimal solutions for production scheduling was divided into two main approaches. This division is crucial as it highlights the different perspectives and methodologies that researchers have applied to address the complexity of production planning. The first approach focused on finding optimal solutions based on artificial test problems. Researchers following this path applied numerous simplifications, which meant that the developed methods were effective only in specific, mostly theoretical, cases. The second group of researchers adopted a different approach. They aimed to develop methods that produced results comparable to those of the first group while addressing problems with a complexity level closer to real-world production planning challenges. This division is particularly important as it demonstrates how different approaches have emerged to handle the evolving challenges in production scheduling, which will be further explored in the next chapter regarding the evolution of production scheduling techniques. Given this complexity, this topic has been widely explored in numerous publications, with authors presenting various methods for production scheduling (Sobaszek, Świć and Gola, 2016).

The utilisation of artificial intelligence (AI) tools and methods has witnessed a marked increase in their application to provide this support, owing to their rapid development and economic utility (Huang, Shen, Li, Fey and Brecher, 2021). The development of AI can be categorised into five primary domains (Fig. 1), with machine learning serving as the foundational technology for other advancements in the field (Purta, Boniecki, Marciniak, Szarek and Krok, 2017).

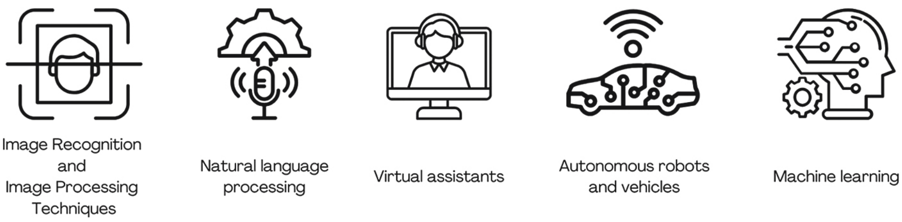

Fig. 1. Main areas of AI development (Purta, Boniecki, Marciniak, Szarek and Krok, 2017)

The concept of artificial intelligence in computers can be traced back to 1950, when Alan Turing proposed a test that was later named after him. This test was designed to assess whether a computer could communicate in a manner that would convince a human observer that it was interacting with another human being. For a considerable period, the fields of artificial intelligence and digital technology have been the subject of scientific research, with limited commercial applications. However, recent years have witnessed a paradigm shift in this landscape. The advent of AI can be viewed as a natural progression from earlier production planning methodologies, as it has been designed to address

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the intricacies inherent in such methods, such as the prediction and reaction to variable factors in production. The potential of AI to enhance operational efficiency can be categorized into four distinct domains of the value chain, as illustrated in Fig. 2 (Purta, Boniecki, Marciniak, Szarek and Krok, 2017).

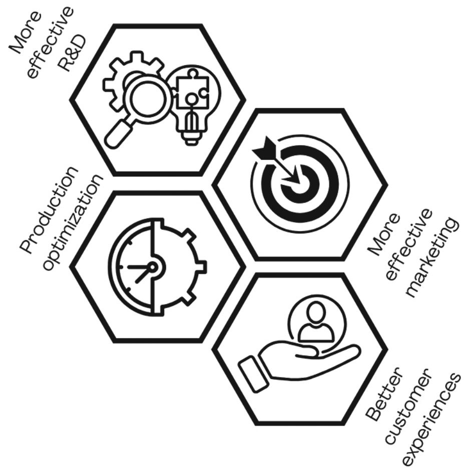

Fig. 2. Main areas of the value chain (Purta, Boniecki, Marciniak, Szarek and Krok, 2017)

At present, artificial intelligence methods are extensively employed in production for a variety of purposes, including machine and device monitoring, predictive maintenance, automation of production processes, warehouse management, analysis of large data sets and chatbots. In the domain of production engineering, AI facilitates the following (Purta, Boniecki, Marciniak, Szarek and Krok, 2017; Korbiel, Czerwiński and Kania, 2023):

- the integration of manufacturers' and suppliers' systems (the utilisation of AI technology to facilitate the exchange of data has been shown to enhance procurement processes and cost control).
- the integration of manufacturers' and suppliers' systems is a key aspect of the optimisation of production processes (the utilisation of AI technology facilitates the exchange of data, thereby enhancing procurement processes and cost control).
- the optimisation of the production process encompasses various aspects, including the calculation of the optimal shipping time of components, the analysis of the thickness of coatings applied throughout the production cycle, the prediction of the quality of produced gases in real time, the creation of more accurate forecasts based on large data sets from machines, and the prediction of failures.
- the reduction of production costs and the enhancement of efficiency are achieved by decreasing the time required for maintenance work and by making ongoing decisions that help balance the load on the power grid.
- the adaptation of products to customer needs is facilitated by tools that analyse previous customer transactions and the products they search for, as well as by adapting production plans to current trends.

The objective of the present paper is dual: firstly, to present and discuss the advantages, disadvantages and potential areas of application of artificial intelligence and advanced algorithms in the context of optimising the production schedule; and secondly, to do so based on a selected industry example. The problem was addressed through the utilisation of various methodologies, including priority rule, complete search and genetic algorithms. These methods have been integrated into a software framework and evaluated using data from a picture framing company that operates on an individual, make-to-order basis.

1.1. Minimizing Processing Time in Diverse Production Systems

In the ScienceDirect database, the term "production scheduling" appeared almost 1800 times in 2024 in the context of engineering. The authors of the article (Han, Cheng, Meng, Zhang, Gao, Zhang and Duan, 2024) utilized process duration minimisation as an optimisation criterion in a company operating under a flexible job-shop system. The authors combined two methods: integer programming and a genetic algorithm with a double population. The experimental results demonstrated that the proposed methodology yielded enhanced solutions in comparison to the reference points established in extant studies.

The solution to the flexible job-shop problem, based on the processing time minimisation criterion, was also the subject of research in paper (Li, Li, Wang and Han, 2024). The authors elected to enhance the genetic algorithm by multiplying selection mechanisms, utilising double crossover and mutation operators, and assessing population diversity. While the researchers obtained satisfactory results, they also identified the need for further development, including multi-criteria optimization, a common requirement in real production companies.

The paper (Umam, Mustafid and Suryono, 2022) explored the application of the genetic algorithm in conjunction with a tabu search method to address the task scheduling problem in a flow-shop system, focusing on the minimisation of processing time as the primary criterion. The authors observed that while genetic algorithms have been shown to yield effective results in global searches, they have a tendency to become trapped in local optima during local searches. To address this, the authors proposed a hybrid approach that integrates the genetic algorithm with the tabu search method. This combination was implemented to prevent the genetic algorithm from becoming trapped in local optima during local searches. The paper also presented a novel population initialisation method for the genetic algorithm, employing partial opposites, which enabled the authors to enhance 115 out of 120 analysed schedules.

The objective of the present study was to examine the reduction of processing time in open-shop systems. The authors conducted a comparative analysis of the complete search method with four approximate methods: GIA, BRKGA, CSA, and IABC, in addition to a novel BRKGA-IG method that integrates a genetic algorithm with a random key and an iterated greedy local search procedure. The experimental results demonstrated that the BRKGA-IG algorithm yielded superior solutions in comparison to the other methods examined.

1.2. Application of Artificial Intelligence and Deep Learning in Production Scheduling

Beyond traditional optimization techniques, artificial intelligence and deep learning have emerged as powerful tools for tackling complex production scheduling problems. In papers (Zhao, Luo and Zhang, 2024; Ren, Ye and Yang, 2021), the authors employed deep learning algorithms in conjunction with neural networks in order to address issues pertaining to the scheduling of work in flow-shop systems. A similar approach was applied to job-shop scheduling by (Serrano-Ruiz, Mula and Poler, 2024), who used a digital twin of the workshop built using deep learning to solve the problem.

In the present study (Huang, Gong and Lu, 2024), the authors applied a Type-2 fuzzy system to minimise energy consumption and processing time in an eco-friendly job-shop environment producing made-to-order items, where the processing time of orders is uncertain. The employment of a fuzzy system enabled the authors to characterize this uncertainty. Subsequent research involved the employment of a memetic algorithm, augmented by a hierarchical heuristic neighbourhood search, enabling the acquisition of optimal solutions.

In their paper (Agrawal, Gans and Goldfarb, 2022), the authors contemplated a shift in approach from a predictive to a reactive model in agent-based programming, thereby incorporating signals from the environment. The authors noted that production processes are exposed to various factors, the negative effects of which can be minimised in the predictive approach by implementing protective and safety measures. However, there is a concern that shifting the scheduling approach from predictive to reactive could compromise reliability and reduce productivity due to the absence of the aforementioned measures. Consequently, special attention and adaptation of production processes to dynamic environmental changes are required.

1.3. Practical Applications of Artificial Intelligence in Production Engineering

Artificial intelligence extends beyond scheduling optimization, offering a wide range of tools to enhance various aspects of production engineering. The practical use of artificial intelligence tools in production engineering has been widely presented in scientific publications.

The authors of the publication (Skoczypiec and Małopolski, 2024) present ways to use artificial intelligence and elements of the Industry 4.0 concept in the field of electrical discharge machining (EDM). The authors indicate that the use of AI tools allows, among other things, for significant improvements in order fulfilment times, productivity, process repeatability and the quality of manufactured parts. The authors of the article (Huang and Lee, 2021) also addressed topics related to machining, proposing an approach to estimating tool wear and surface roughness using deep learning and sensor fusion based on a one-dimensional convolutional neural network (1D-CNN). This enabled the evaluation of a range of sensor combinations, allowing the identification of those that were critical to the process. The solution was subsequently implemented in a practical setting for the purpose of monitoring tool wear. The research conducted by the authors (Liao, Zhang, Chen and Song, 2024) focused on the scheduling of operations for numerically controlled (CNC) machining centres that share identical capabilities, with consideration given to both machine and tool availability. The objective was to minimise delays, and to this end, a new two-stage adaptive algorithm utilising genetic algorithms was proposed. The results of five out of the six tests indicated that the ABC II algorithm demonstrated superior performance compared to other heuristic algorithms. In (Hangjoo, 2023), the author presents a novel scheduling method tailored for non-identical parallel CNC machines. The objective of this approach is to enhance operational efficiency by assigning a single operator to oversee and perform multiple manual tasks concurrently, with the primary goal of reducing delays. To improve the current system, the author utilised constraint programming techniques to optimise scheduling within the company. The article (Abubakar, Almeida and Gemignani, 2021) presents the potential applications of artificial intelligence in the detection and diagnosis of faults in photovoltaic systems. The research methods employed are classified into two categories: visual-thermal methods and electrical methods. The former utilise changes in the colour of panels, contamination or cracks as indicators, while the latter are primarily concerned with the identification of diverse forms of short circuits. The primary focus of research in this domain is the utilisation of neural networks, machine learning and decision trees to address the aforementioned challenges. The authors of (Kudelina, Vaimann, Asad, Rassõlkin, Kallaste and Demidova, 2021) also conducted research on machine fault diagnosis (MFD) methods, identifying machine learning, neural networks and decision trees as effective diagnostic tools. The authors highlight the benefits of utilising AI tools, with a particular emphasis on the ability to process big data sets. The application of artificial intelligence in production engineering, including techniques such as deep learning and neural networks, has been shown to reduce order fulfilment times, enhance productivity and improve the quality of manufactured parts.

2. Evolution of production scheduling

The first mentions of the development of scheduling can be found in history as early as the seventeenth century when Robert Hooke introduced the first scheduling diagrams in his publication. In the eighteenth century, Joseph Priestley took a more innovative approach by presenting the "Bibliography Chart" (Fig. 3), which was the first bar chart. This chart was an addition to his lectures on history and politics (Weaver, 2006). These pioneering approaches to scheduling laid the foundations for later developments in the field.

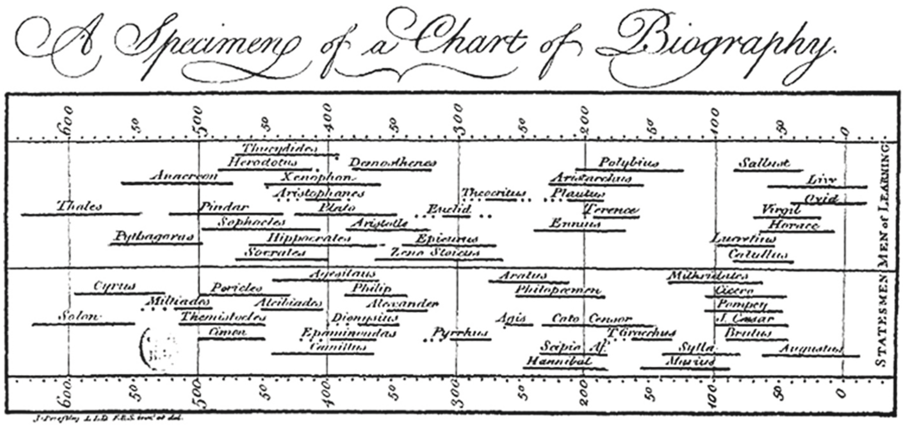

Fig. 3. Bibliography Chart Created by Joseph Priestley (Weaver, 2006)

The subsequent pivotal juncture in the evolution of planning methodology materialised with the publication of William Playfair's seminal work, the Commercial and Political Atlas, which was released at the close of the eighteenth century. This work is notable for its introduction of the first pie charts and time series graphs relating to trade in goods, such as imports and exports (Spence, 2006).

Subsequent developments in the nineteenth century were characterised by the seminal contributions of Karol Adamiecki, who pioneered a novel method of labour scheduling (Weaver, 2006). His inspiration came from observing the low productivity of the rolling mill where he worked. He decided to create a graphical representation of the interdependence of work, which brought enormous benefits. Thanks to this innovative method, it was possible to almost double the efficiency of the rolling mill (Kosieradzka, 2018). In parallel with Adamiecki's work on scheduling, Henry Gantt also contributed to the development of this field (Grześ, 2014). At the beginning of the twentieth century, he presented in his publication the first balance and work schedule for a foundry (Wilson, 2003). It is worth noting that at that time, knowledge of the Polish language was not as widespread internationally as knowledge of the English language. As a result, Gantt's work gained greater popularity, which led to his scheduling methods being widely adopted worldwide (Grześ, 2014).

The advent of formal production planning strategies coincided with the turn of the nineteenth century, marking the establishment of institutions specialising in the development of plans, inventory management, and operation monitoring. In 1956, Du Pont pioneered the use of automatic project scheduling utilising the Critical Path Method (CPM) with the Remington Rand UNIVAC device. Subsequently, in 1958, Polaris introduced a calculator based on the PERT method. These events signalled the inception of the development of computer systems for self-scheduling production tasks, utilising advanced algorithms and optimisation techniques (Sobaszek, Świć and Gola, 2021; Woźniak, 2002; Jeffrey, 2005). In the subsequent decades, MRP, MRP II, and the now widely utilised ERP systems evolved, offering modules for efficient production scheduling (Sobaszek, Świć and Gola, 2021; Wozniak, 2002; Jeffrey, 2005). The introduction of computer technology had the greatest impact on the evolution of production scheduling processes (Sobaszek, Świć and Gola, 2021; Woźniak, 2002; Jeffrey, 2005).The development of scheduling methods over the years is presented in Fig. 4.

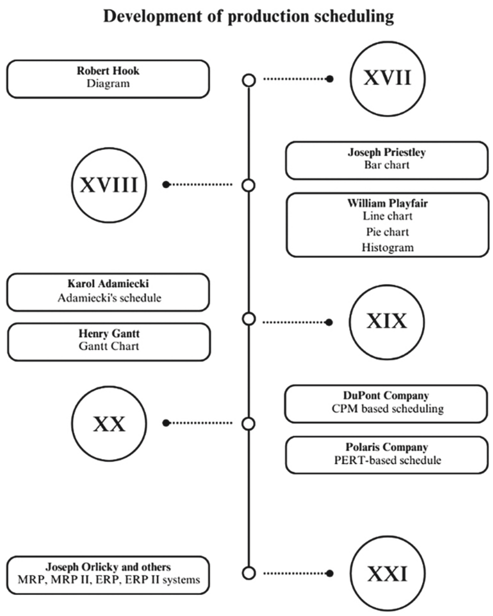

Fig. 4. Development of production planning (Weaver, 2006; Priestley, 1795; Spence, 2006; Kosieradzka, 2018; Grześ, 2014, 195-216; Wilson, 2003, 430-437; Sobaszek, Świć and Gola, 2021; Woźniak, 2002; Jeffrey, 2005)

3. Production scheduling methods and tools

Production scheduling, defined as the process of developing a detailed production plan to achieve set goals while considering defined constraints, is of critical importance in industrial settings (Pająk, 2006). It is a pivotal component of the production system, interconnecting with other stages of the process through feedback loops and exerting a substantial influence on the overall production activity. The implementation of production schedules facilitates the coordination of activities, thereby enhancing efficiency and reducing operational costs. Schedules facilitate the identification of resource conflicts, the tracking of task flow to the workshop, and the timely ordering of necessary raw materials. They also allow for the evaluation of the likelihood of meeting delivery deadlines and determining available times for machine maintenance and servicing (Jeffrey, 2005).

The challenges associated with production scheduling can be effectively addressed through the implementation of various task scheduling methods. These methods are classified and grouped into categories according to the algorithm used (Sobaszek, 2012).

Exact methods, otherwise referred to as methods seeking exact solutions, are techniques where the computational calculation continues until the extreme of the objective function is reached. Due to their high computational complexity, especially time consumption, these methods are rarely applied in practice. Exhaustive search involves the identification of all potentially acceptable schedules in the first stage, followed by the selection of active schedules. Random search involves the generation of active and inactive schedules at random. Integer programming involves the conversion of decision variables into integers, rendering it complex and applicable only to solving simple theoretical problems. Branch and bound method employs state-

-space tree analysis to illustrate the possible paths of the algorithm, ultimately leading to the solution of the problem (Sobaszek, 2012; Pawlak, 1999; Ławrynowicz, 2006).

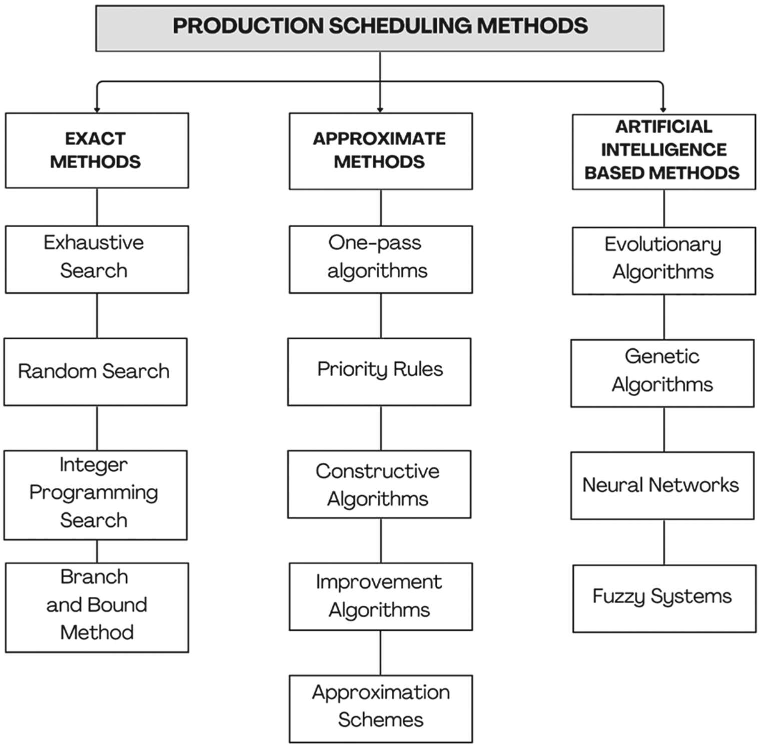

Fig. 5. Methods for scheduling production tasks (Pawlak, 1999)

The subsequent group of methods is that of approximate methods, which allow for the achievement of sufficiently accurate solutions within an optimal timeframe. These methods include the following (Pawlak, 1999; Zawadzka, Badurek and Łopatowska, 2012; Demir and Yilmaz, 2021; Klimek and Łebkowski, 2015; Bachman, Janiak, Kozik and Winczaszek, 2002; Winczaszek, 2006):

- dispatching systems, where operations on a given machine are arranged in increasing order according to the plan, with new operations added to the end of the queue.
- priority rules, which are easy to use, require low computational power, and do not require deep specialised knowledge, making them very popular.
- constructive algorithms, which use priority or insertion algorithms. These algorithms provide rapid analysis but are primarily employed to identify initial solutions that are subsequently refined using local search algorithms.
- improvement algorithms, which refine the solutions obtained via constructive algorithms by applying optimization techniques to enhance performance.
- approximation schemes, which generate solutions with a predetermined level of accuracy but involve high computational complexity, limiting their competitiveness compared to other methods.

Methods based on artificial intelligence includes various approaches such as evolutionary algorithms, which mimic biological evolution through selection, reproduction and mutation. Genetic algorithms, a subset of evolutionary algorithms, use random search within the space of possible solutions. Optimisation is achieved by evolving initial solutions. Neural networks are used either alone or in combination with other methods. Recurrent networks are particularly useful for scheduling problems. Fuzzy systems are used when processing time, setup time and constraints are imprecise. They are also used when historical data is not available (Pawlak, 1999; Zawadzka, 2007; Akyol and Bayhan, 2007; Hureira and Vartanian, 2019; Odii, Okpalla and Ejem, 2016).

The dynamic development of artificial intelligence has led to its increasingly widespread application, also as a tool enabling prediction and scheduling (Agrawal, Gans and Goldfarb, 2022).

4. Production restrictions and disruptions

In the context of manufacturing systems, production restrictions and disruptions represent a pivotal consideration. These factors encompass elements that either impose limitations on the capacity of the production process or give rise to unanticipated interruptions. Such disruptions have the potential to compromise the efficiency and predictability of operations, whether their origins lie within or outside the manufacturing system. The management of such disruptions is of paramount importance to ensure the timely completion of projects and the maintenance of product quality. In addressing these challenges, various simplifying assumptions are frequently made in the modelling of production processes. However, these assumptions may not fully reflect the complexity and variability of real-world manufacturing environments. Consequently, production planners must be prepared to adjust schedules and processes to mitigate the effects of these disruptions and maintain operational efficiency. A substantial number of studies have sought to formulate an optimal solution to the scheduling problem, frequently relying on simplified assumptions, thereby rendering the presented models largely theoretical in nature. The most commonly adopted assumptions include (Sobaszek, Świć and Gola, 2021; Pawlak, 1999; Bożejko, Uchroński and Wodecki, 2009):

- It is not possible for individual operations within a single order to be performed simultaneously. However, in practice, different production stages can be carried out concurrently and subsequently integrated into a final product.
- It is an established fact that a single machine cannot execute more than one operation concurrently. While this assumption is generally accurate, it does not account for modern CNC machines, which enable simultaneous processing.
- The composition of each task is defined by a specific number of operations allocated to designated machines. However, in reality, a machine may process multiple operations within the same order.
- It is imperative that each task is completed without interruption. This is a common practice in real-world production systems.
- The duration of operations is independent of the established schedule. Consequently, delays in initiating an operation do not adversely affect the overall completion time of the task.
- The possibility exists for waiting for a machine to become available. However, in practice, companies often implement buffers that allow for more flexible process management.
- Furthermore, it is assumed that each machine type is present in a single instance. However, this assumption is rarely realistic, since manufacturing facilities typically possess multiple identical machines.
- It is acknowledged that periods may occur during which machines are not fully utilised. In practice, many machines operate in a non-continuous manner.
- It is also postulated that a machine is incapable of performing two operations concurrently. This assumption is analogous to the constraint that prevents the simultaneous execution of two tasks on a single machine.

- Furthermore, the assumption that machines are continuously available and resistant to failure is erroneous. However, in reality, equipment requires maintenance and may suffer unexpected breakdowns.
- The aforementioned limitations are recognised and addressed through technological modifications. While this is largely accurate, minor modifications or disruptions may occasionally occur.
- The models do not take randomness into consideration. However, in practice, unforeseen events such as sudden order changes have the potential to disrupt production schedules.
- The variability of technology is not accepted. Consequently, each operation is assigned to a specific machine. However, in real production processes, the same operation can often be performed on different machines.

The aforementioned assumptions are representative of the common constraints that are often present in the context of scheduling systems for job-shop production (Sobaszek, Świć and Gola, 2021). In response to these limitations, various methods have been developed over the years to reduce or eliminate certain constraints, forming the basis of ongoing research (Wojakowski, 2012).

In a real production environment, various types of disruptions occur, directly affecting the production schedule. These disruptions render the schedule outdated and necessitate adjustments in production organization. Due to their significant impact on production flow, they should be considered at all stages of the manufacturing process. Disruptions can be categorized into several types (Klimek, 2010; Kowalska, Sikora and Hadaś, 2017):

- Upstream disruptions – characterized by issues pertaining to the supply of materials and components, quality problems, or delays in deliveries. These factors have the capacity to exert a significant influence on production continuity.
- Downstream disruptions – vary depending on the production model. In a make-to-order system, they arise from changes in customer orders, fluctuations in quantities, and modifications to order specifications. In contrast, disruptions in a make-to-stock system are frequently attributed to forecasting inaccuracies, seasonal fluctuations in production, delays in the delivery of finished products, or inadequate inventory monitoring.
- Internal disruptions – attributed to issues inherent within the production process itself, including operator errors, machine failures, technology misalignment with quality requirements, low process repeatability, or problems with the flow of information between different stages of production.
- External disruptions – related to factors that are beyond the company's direct control, including labour market fluctuations, economic cycles, and changes in demand.
- Random disruptions – an inherent feature of the manufacturing process. Such disruptions may be attributed to factors such as vibrations or variations in machine setup, which can result in inconsistencies in production.
- Quality-related disruptions – arise due to variations in the quality of delivered components or incorrect quality assessment. These issues can lead to defective products and increased waste.
- Cost-related disruptions – attributable to dynamic changes in raw material prices, miscalculations of customer profitability, or unexpectedly high service costs, all of which have the potential to impact production budgets and profitability.
- Operational disruptions – refer to instances where the execution of specific operations is impeded while other tasks at a given workstation proceed without interruption.

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- Machine-related disruptions – result in a complete cessation of production at a specific workstation, thereby preventing any operations from being conducted.
- Process-related disruptions – lead to the withdrawal of an entire process. Such disruptions may be attributed to substantial errors in the technological workflow, necessitating a comprehensive reassessment and redesign of the production process.

It is therefore vital that effective management strategies are implemented to minimise the impact of such disruptions and ensure the smooth running of production operations.

5. Case study

5.1. Company Profile

The company for which the production schedule is created is specialised in the field of picture framing. A sales consultant is employed, working a single shift from Monday to Friday, and production workers are organised in a three--shift, four-team system. The production process is conducted at ten distinct stations. The sales consultant's primary responsibility is the accurate and precise input of orders into the company's system. Subsequent to order acceptance, the responsibility for checking stock levels and, if necessary, placing an order for semi-finished products with the distributor, lies with the production employee. Once all the necessary components have been collected, the operator initiates the processing of the order. The order execution process is depicted in Fig. 6.

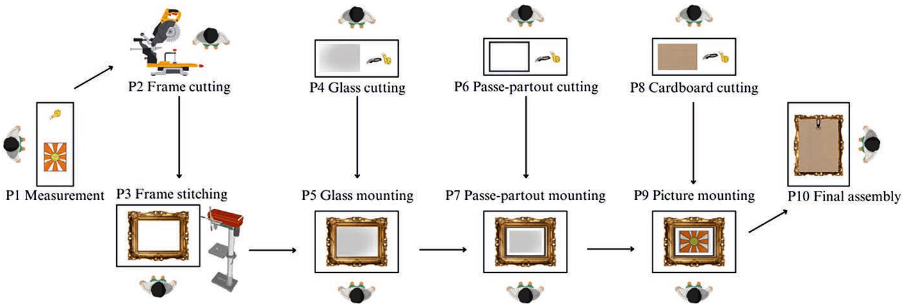

Fig. 6. The course of the process through the assembly stations in the analysed enterprise (source: own elaboration)

5.2. Problem Description

As demonstrated in Fig. 6, specific production processes within the workflow are inherently interdependent. While some tasks can be executed independently, others require strict adherence to a predetermined sequence. Specifically, the cutting of individual components – namely the frame, glass, passe-partout, and cardboard – can be carried out autonomously without interference from one another. However, it is imperative to emphasise that the prerequisite for this step is the completion of the measurement phase, which ensures that each component is cut to the appropriate dimensions.

Conversely, subsequent operations impose constraints on the order in which they must be executed. For instance, the assembly of the frame is contingent upon the completion of the frame-cutting stage. Similarly, the installation of the glass is only possible after both the glass has been cut to size and the frame has been assembled. The passe-partout, once cut, can only be installed after the preceding frame and glass installation steps have been finalized. This interdependency extends further throughout the production process, necessitating a structured workflow that adheres to these sequential constraints. The aforementioned dependencies can be formally expressed through mathematical equations that define the relationships between tasks. The employment of such equations facilitates the modelling of the logical flow of operations, thereby ensuring that each step is executed at the appropriate stage while maintaining efficiency and structural integrity in the final assembly. The formulation of these constraints provides a clear framework for the optimisation of the production sequence, the minimisation of delays, and the avoidance of potential inefficiencies.

$$T_1 = \textit{start date of order execution}$$

$$T_2 = \max(E_1)$$

$$T_3 = \max(E_2)$$

$$T_4 = \max(E_1)$$

$$T_5 = \max(E_3, E_4)$$

$$T_6 = \max(E_1)$$

$$T_7 = \max(E_5, E_6)$$

$$T_8 = \max(E_1)$$

$$T_9 = \max(E_7, E_8)$$

$$T_{10} = \max(E_9)$$

where: $T_i - P_i$ process start time; $E_i - P_i$ process end time.

In order to ascertain the duration of the production process with a high degree of accuracy, the Critical Path Method (CPM) can be employed. This technique, which is employed in a variety of industries, including construction, aviation, software development, research projects and engineering, is one which is particularly useful in cases where task interdependencies play a crucial role in project planning and execution. The efficacy of CPM is contingent upon the existence of sequential dependencies between tasks, rendering it particularly advantageous in structured workflows where the completion of certain operations is a prerequisite for the initiation of subsequent operations. The methodology employed to ascertain the critical path is illustrated in Fig. 7 (Khare, Khare, Nema and Baredar, 2019), providing a step-by-step representation of the process. The critical path is determined through a series of calculations based on predefined task durations and dependencies, allowing for the identification of the longest sequence of dependent tasks that dictates the total project duration.

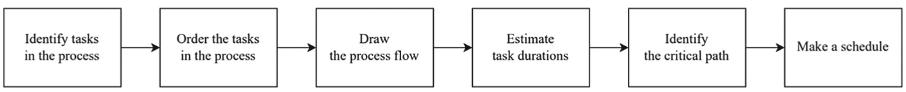

Fig. 7. The sequence of activities that represent the critical path (source: own elaboration)

The following formula may be used to perform these calculations in accordance with the CPM method:

$$C_{10} = T_1 + d_1 + \max(d_2, d_3) + \max(d_4, d_5) + \max(d_6, d_7) + \max(d_8, d_9) + d_{10}$$

where: $d_i - P_i$ process duration; $T_i - P_i$ process start time; $C_i - P_i$ process completion time.

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The employment of this method facilitates the precise estimation of the completion time for a singular product within a production cycle. However, it is important to note that real-world manufacturing environments frequently involve the concurrent production of multiple products, thereby introducing additional complexity. In order to effectively manage such conditions, it is essential to develop a comprehensive production schedule using alternative scheduling methods that account for resource allocation, machine availability, and workflow optimisation. It is imperative to comprehend the interdependencies between processes in order to construct an efficient and synchronised production plan, ensuring smooth operations and minimising delays.

5.3. Methodology

The problem under analysis is of a flow shop nature, in that each task has the same sequence of operations on the assigned machines. The task is subject to the following limitations:

- sequence restrictions: given operations must start after the previous ones have been completed;
- capacity limitations: only one task can be processed on one machine at a time;
- eligibility restrictions: operations are assigned to specific positions.

The company's current inventory management approach is based on the First In First Out (FIFO) method. This method entails the processing of orders in accordance with the order in which they are received. In order to ascertain the most efficacious method for creating a production schedule for the analysed company, it was decided to compare several methods, taking into account both the order execution time and the time needed to obtain the schedule (Ombati Momanyi, Oduol and Musyoki, 2014). The following methods were analysed:

- a complete search was conducted, whereby all possible configurations were checked and the one that best met the criteria was selected (Mohammadi and Sheikholeslam, 2023).
- the short processing time (SPT) priority rule was implemented, whereby the operation with the shortest execution time was selected from the queue of operations waiting to be performed (Uzorh and Innocent, 2014).
- the genetic algorithm was used to optimise schedules based on principles found in the evolution of living organisms (Rivera, Cisneros, Sánchez--Solís, Rangel-Valdez and Rodas-Osollo, 2020).

The implementation of each of these methods is illustrated in Fig. 8.

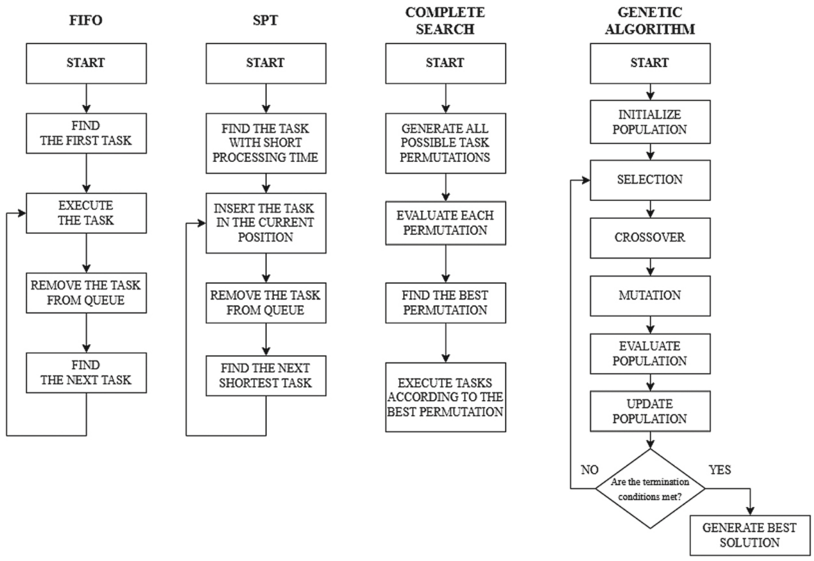

Fig. 8. The block diagrams of the FIFO, SPT, complete search, and genetic algorithm methods (source: own elaboration)

The decision was taken to conduct a simulation for a schedule of 10 orders, with the simulation to be repeated seven times, given that the company analysed has approximately 70 orders per day. A program was developed in C#, capable of generating a production schedule according to the FIFO, SPT, complete search and genetic algorithm methods for specified data. The experimental phase was conducted utilising equipment with the parameters enumerated in Table 1. Due to the magnitude of the problem – 10 workstations (machines) – it was only feasible to generate a schedule for 10 orders, hence the division into seven trials. Attempts to extend this to a greater number of orders resulted in unsatisfactory waiting times for a solution due to the extended execution time of the complete search. Table 2 provides a summary of the order processing times and solution wait times for each trial. Fig. 9 illustrates the makespan values obtained in specific tests through the utilisation of the aforementioned analytical methodologies. Fig. 10 presents a comparative analysis of the time requirements for the execution of the schedules.

Table 1. Details of the apparatus utilised in the experimental procedures

Parameter	Information
The quantity of random-access memory (RAM)	16 GB
The processing speed of the central processing unit (CPU)	Intel(R) Core(TM) i5-8300H CPU 2.30 GHz
The type of system	64-bit operating system, x64 processor

Table 2. The results of seven tests for the FIFO, SPT, complete search and genetic algorithm methods

	FIFO	SPT	Complete search	Genetic algorithm
Test		**1**		
Makespan [min]	253	255	234	234
Time to solutions [ms]	0	1	4829	79
Test		**2**		
Makespan [min]	238	252	227	227
Time to solutions [ms]	0	1	5349	76
Test		**3**		
Makespan [min]	236	243	211	211
Time to solutions [ms]	0	1	5277	113
Test		**4**		
Makespan [min]	232	236	211	211
Time to solutions [ms]	0	1	5585	83
Test		**5**		
Makespan [min]	224	230	216	216
Time to solutions [ms]	0	1	5093	78
Test		**6**		
Makespan [min]	249	253	228	228
Time to solutions [ms]	0	1	5095	81
Test		**7**		
Makespan [min]	252	252	232	232
Time to solutions [ms]	0	1	5027	88
Makespan sum [min]	**1684**	**1721**	**1559**	**1559**

5.4. Results and Discussion

The results obtained from the experiments are presented and analysed below. As demonstrated in Fig. 9, the makespan times demonstrate minimal variation. Nevertheless, the complete search and the genetic algorithm consistently yield the same value, which is the lowest observed.

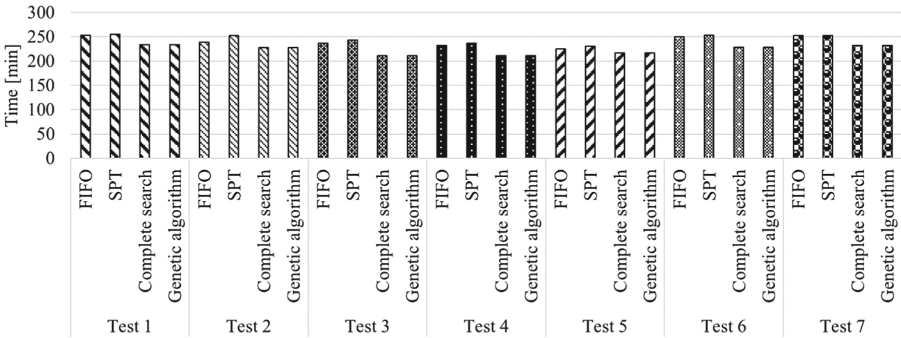

Fig. 9. The makespan values obtained in specific test (source: own elaboration)

As demonstrated in Fig. 10, the complete search method is remarkably time-consuming when attempting to generate a solution. The SPT and FIFO priority rules facilitate rapid solution generation, although they are not the optimal scheduling configurations. The genetic algorithm, while requiring a greater investment of time, is considerably faster than a complete search. Notably, the genetic algorithm generates results that are equally effective to those produced by complete search methods.

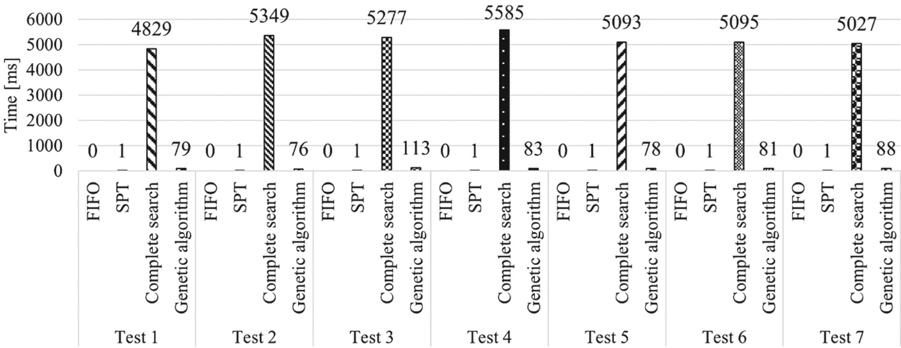

Fig. 10. The time required to obtain a solution in a specific test (source: own elaboration)

As illustrated in Fig. 11, the resulting order of operations in the production schedule for Test 1, as generated by a genetic algorithm program, is presented. The commencement and cessation times of a given order are indicated within parentheses.

```
Schedule generated by genetic algorithm:
Job 2:
P 1: [0 - 17] , P 2: [17 - 30] , P 3: [30 - 42] , P 4: [17 - 32] , P 5: [42 - 51] , P 6: [17 - 30] , P 7: [51 - 63] , P 8: [17 - 26] , P 9: [63 - 80] , P 10: [80 - 99]

Job 10:
P 1: [17 - 37] , P 2: [37 - 52] , P 3: [52 - 62] , P 4: [37 - 52] , P 5: [62 - 69] , P 6: [37 - 54] , P 7: [69 - 83] , P 8: [37 - 44] , P 9: [83 - 98] , P 10: [98 - 117]

Job 8:
P 1: [37 - 56] , P 2: [56 - 74] , P 3: [74 - 86] , P 4: [56 - 72] , P 5: [86 - 93] , P 6: [56 - 71] , P 7: [93 - 99] , P 8: [56 - 67] , P 9: [99 - 116] , P 10: [116 - 136]

Job 3:
P 1: [56 - 75] , P 2: [75 - 90] , P 3: [90 - 102] , P 4: [75 - 94] , P 5: [102 - 106] , P 6: [75 - 89] , P 7: [106 - 117] , P 8: [75 - 83] , P 9: [117 - 129] , P 10: [129 - 153]

Job 6:
P 1: [75 - 96] , P 2: [96 - 109] , P 3: [109 - 116] , P 4: [96 - 108] , P 5: [116 - 123] , P 6: [96 - 111] , P 7: [123 - 133] , P 8: [96 - 109] , P 9: [133 - 146] , P 10: [146 - 169]

Job 4:
P 1: [96 - 116] , P 2: [116 - 131] , P 3: [131 - 139] , P 4: [116 - 136] , P 5: [139 - 143] , P 6: [116 - 128] , P 7: [143 - 150] , P 8: [116 - 123] , P 9: [150 - 165] , P 10: [165 - 188]

Job 9:
P 1: [116 - 134] , P 2: [134 - 146] , P 3: [146 - 153] , P 4: [134 - 154] , P 5: [154 - 160] , P 6: [134 - 148] , P 7: [160 - 171] , P 8: [134 - 138] , P 9: [171 - 189] , P 10: [189 - 214]

Job 1:
P 1: [134 - 140] , P 2: [140 - 156] , P 3: [156 - 162] , P 4: [140 - 157] , P 5: [162 - 169] , P 6: [140 - 154] , P 7: [169 - 176] , P 8: [140 - 155] , P 9: [176 - 187] , P 10: [187 - 211]

Job 5:
P 1: [140 - 150] , P 2: [150 - 164] , P 3: [164 - 169] , P 4: [150 - 170] , P 5: [170 - 179] , P 6: [150 - 167] , P 7: [179 - 193] , P 8: [150 - 164] , P 9: [193 - 210] , P 10: [210 - 232]

Job 7:
P 1: [150 - 173] , P 2: [173 - 187] , P 3: [187 - 192] , P 4: [173 - 187] , P 5: [192 - 198] , P 6: [173 - 190] , P 7: [198 - 201] , P 8: [173 - 181] , P 9: [201 - 213] , P 10: [213 - 234]


Genetic algorithm execution time: 79 ms
Genetic algorithm makespan: 234 minutes
```

Fig. 11. Schedule generated according to the Genetic algorithm for Test 1 (source: own elaboration)

An alternative solution was also attempted. A schedule was created for all 70 orders, circumventing the exhaustive search, which was constrained by the computing capabilities of the equipment. The outcomes of this experiment are presented in Table 3 and compared in the graph in Fig. 12.

Table 3. The results of the FIFO, SPT and genetic algorithms methods without division into smaller sections

	FIFO	SPT	Genetic algorithm
Makespan [min]	1202	1214	1182
Time to solutions [ms]	0	1	508

It can be concluded that the utilisation of the genetic algorithm enables a daily time saving of 20 minutes. However, it is evident that the utilisation of this method results in a substantially greater time investment compared to approaches that employ priority rules.

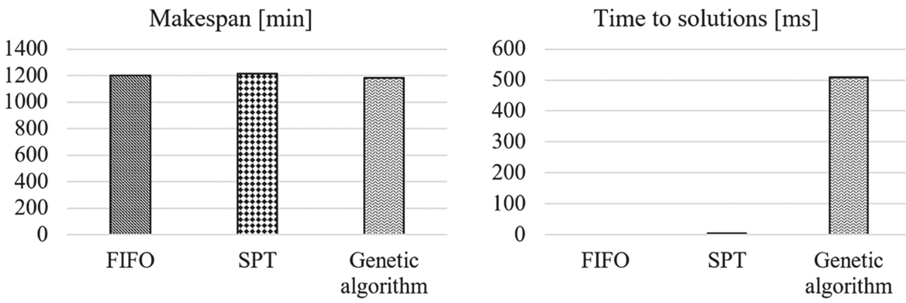

Fig. 12. Comparison of the results of FIFO, SPT and genetic algorithms without division into smaller sections (source: own elaboration)

The exhaustive search and the genetic algorithm produced the optimal solutions in the studied group of cases. The algorithmic approach yielded identical results, albeit with a notable reduction in execution time. In instances where the problem is particularly complex, such as instances with more than 10 orders on 10 machines, a standard computer is unable to perform a complete search. By circumventing the complete search, a single daily production schedule for 70 orders executed on 10 machines was derived. In this experiment, the genetic algorithm achieved the optimal result in just over 500 milliseconds, which is approximately 10 times faster than the time taken to obtain a schedule of 10 orders on 10 machines using a complete search.

It should be noted, however, that the performance of genetic algorithms may vary between runs due to their stochastic nature, and they do not always guarantee finding the global optimum.

Additionally, the effectiveness of the solution may depend on appropriate parameter tuning, such as population size, mutation rate, and selection method.

The company in question is a small to medium-sized enterprise, with a limited machine park and a daily order volume of 70. Consequently, the introduction of a substantial production support program is deemed to be an economically unfeasible proposition. The solution to these challenges lies in the implementation of selected artificial intelligence tools, such as the genetic algorithm. The employment of this algorithm has been demonstrated to enhance the efficiency of production processes, leading to a substantial reduction in the implementation time of the plan. In comparison with the previously utilised FIFO method, the implementation time has been shown to be reduced by more than an hour per day. This enhancement in operational efficiency, characterised by a reduction in production time and costs alongside an augmentation in customer satisfaction through accelerated order fulfilment, signifies a substantial benefit to the company. It is noteworthy that the integration of the genetic algorithm did not require the allocation of

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resources in the form of new equipment or the augmentation of machinery. The implementation of the algorithm is facilitated by a medium-class computer.

6. Direction of production scheduling developement

Recent research in the field of production scheduling is increasingly focused on developing hybrid algorithms that combine different optimization methods. For instance, a combination of integer programming and genetic algorithms has been employed to achieve enhanced solutions for complex scheduling problems, particularly in environments such as flexible job-shop systems. This approach has been shown to engender greater flexibility and efficiency, especially in dynamic and variable production settings. The integration of diverse techniques has emerged as a key strategy to circumvent the constraints imposed by individual methods, thereby facilitating the development of more comprehensive solutions.

Another significant direction of development is multi-criteria optimization, which addresses the need to consider multiple factors when scheduling production. Conventional scheduling methodologies predominantly prioritise a solitary objective, such as minimising processing time. However, in practical scenarios, decision-makers are required to balance various factors, including cost, quality, and resource utilisation. Recent studies emphasise the importance of optimising multiple criteria simultaneously, as this approach facilitates the management of trade-offs that arise in complex production environments. The incorporation of multi-criteria optimization within contemporary manufacturing systems is therefore pivotal in ensuring optimal outcomes that align with the evolving demands of such environments.

However, while genetic algorithms (GAs) are powerful for global optimisation, they tend to struggle with finding local optima. To address this issue, there has been a growing trend to support GAs with local search methods, such as tabu search. This combination ensures that the algorithm avoids getting stuck in local maxima and can provide higher-quality solutions. This hybrid approach enhances the efficiency of the algorithm and enables it to solve more complex scheduling problems with better results.

The application of artificial intelligence in production scheduling is also evolving, particularly with the integration of deep learning and neural networks. These methods are becoming increasingly prevalent in the resolution of scheduling problems within flow-shop systems, where traditional methodologies may not achieve optimal performance. Researchers are exploring the use of AI to predict and adapt to dynamic changes in production environments, allowing systems to adjust to real-time variations (Jones et al., 2023). This adaptation has the potential to enhance the accuracy and efficiency of scheduling processes in industries characterised by rapidly evolving demands and production conditions.

Another promising area of research involves the application of fuzzy logic systems to manage uncertainties in production processes, such as variations in processing times or setup times. These systems facilitate the modelling and management of uncertainties, thereby ensuring the provision of more robust scheduling solutions. Furthermore, the field is seeing increased interest in memetic algorithms, which integrate genetic algorithms with local search methods, enhancing the quality of solutions. These algorithms offer an efficient way to handle the complexities of real-world production environments by refining solutions through iterative improvements.

Finally, it is important to note the emergence of a shift from predictive to reactive scheduling approaches. Predictive scheduling is predicated on the assumption of a stable environment and is focused on pre-planning, while reactive scheduling involves real-time adjustments based on changes in the environment. This approach is particularly useful in highly dynamic production systems where unforeseen disruptions or changes are common. However, it is important to note that this transition necessitates meticulous management. A failure to implement this shift effectively can result in a decline in reliability and productivity. Consequently, researchers are exploring ways to ensure that reactive scheduling maintains efficiency while responding to external changes, emphasising the need for adaptable and resilient production systems.

7. Conclusion

Contemporary research in the domain of production process optimisation encompasses a range of methodologies. The most frequently chosen optimization criterion is the minimization of processing time. In pursuit of this objective, researchers engage in experimentation, combining diverse techniques and methodologies in accordance with the designated type of production system. The field of production scheduling using artificial intelligence methods is developing dynamically, leading to the continuous improvement of existing methods by introducing new mechanisms and considering increasingly complex aspects such as multi-criteria optimization and dynamic environmental changes.

In the context of the experiments conducted in this study, the genetic algorithm was found to significantly reduce the time needed to generate production schedules, offering a time-saving of 20 minutes per day compared to the FIFO method. While the genetic algorithm demonstrated optimal results, it required more computational time than priority rule methods, such as FIFO and SPT. However, it was still considerably faster than the complete search method, which showed significant execution delays. Despite its strengths, the genetic algorithm is not free of limitations. The performance of the algorithm can vary depending on parameter settings, such as mutation rate and population size, and it may not always guarantee the global optimum, especially in more complex production environments.

In conclusion, the implementation of selected artificial intelligence tools, such as the genetic algorithm, allows for a significant improvement in production efficiency while maintaining low operating costs, which is crucial for small and medium-sized enterprises.

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