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Study on defect influence on elastic stifnesses and strength of auxetic cellular materials

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Abstract

The goal of this paper is to investigate the effects of randomly removed structural members of auxetic cellular material on the evaluation of elastic constants. Statistical volume element SVE size ensuring prescribed accuracy is adopted by giving relation with the size of representative volume element RVE considering number of statistical random defects. Statistical analysis is carried out within the Mathcad code. Inverted honeycomb structures with randomly removed structural members under the typical statistical boundary conditions are considered. The study is achieved by conducting finite element calculation by means of ABAQUS FEA of the auxetic structure leading to obtaining the stiffness matrix and strength. Timoshenko beam elements are used for material microstructure discretization with special attention paid on relation of mesh density with results accuracy. Influence of geometric microstructural parameters on examples of variety of inverted honeycomb microstructures is studied. As result dependence of residual effective elastic moduli and strength on defects density for each microstructure is specified.

Keywords: auxetic cellulars, defected lattice materials, statistical volume element

1. Introduction

Cellular materials of various microstructural arrangements play important role in recent design trends and manufacturing in the material industry. Cellulars are used in different engineering areas such as aircraft, aerospace or transportation industry and also as structural elements in civil engineering structures and sport devices. Lattice materials offer special mechanical properties, therefore they are used not only as structures carrying loads but also for energy adsorbtion, heat dissipation or impact resistance (Evans, 1998; Fleck, 2004; Deshpande, 2001; Gu and Evans, 1998; Wadley, 2006). These materials are finding their applications in structural elements such as panels, shells of various shapes or cores of sandwich panels. The effectice mechanical behavior of ideal lattice materials described by effective stiffness and strength is presented in works by Gibson (1997), Wang and McDowell (2004) and Janus-Michalska (2009). All described cellulars reveal advantageous strength and stiffness-to-weight ratios. Specific or specialy tailored properties can be achieved by material anisotropy. The concept of creating of anisotropy in lattice materials by special cellular shapes is becoming crucial in the construction of modern materials.

One of these specific effective material properties is auxeticity, in other words the property of negative value of Poisson's ratio. Materials, which expand in lateral direction when they are stretched logitudinally exhibit this property and are called 'auxetics'. The concept of such mechanical behavior was described for the first time by Lakes in 1987. The word 'auxetic', which reffers to negative Poisson's ratio material was used first in 1991. It was derived from the word 'auxetikos', that in Greek means 'that which tends to increase'. First cellular auxetics were widely investigated and described by Gibson and Ashby, Lakes, Kolpakov, Almgren in 80-ties of twentieth century. Research on auxeticity was continued by Evans and Alderson. A number of auxetic cellular configurations with variety of structural symmetry were proposed by Lakes, Theocaris , Smith, Gaspar, Grima and Yang. The extensive review on negative Poisson's ratio materials is given by Prawoto (1998), Yang et al. (2004) and Caneiro et al. (2013).

Cellulars, available in practice, contain various kinds of microstructural manufacturing defects such as misalinged, thinner or fractured cell walls, stochastic dispersion of nodes and lattice nodes imperfections. An attempt of the classification of defected materials is given for example by Gaydachuk et al. (2006).

The presence of defects in cellular materials influences the overall behavior of material as the equivalent continuum. It also results in change of material anisotropy. A number of works are devoted to mechanical properties of defected cellulars. Most recent researches are carried out by Zhu et al.,(2011,2012), Wang and McDowell (2004), Li et al.,(2005), Symons and Fleck (2008), Mukhopadhyay and Adhikari (2016), Ajdari et al., (2008), Cui et al., Chen (2010), Lu (2006) and Fleck (2004). Chen Ozaki (2009) considered single defect, and calculated the interaction between several defects.

Missing bars defects have the greatest influence on cellulars behavior and leads to degradation in equivalent mechanical properties and because of this reason is obviously a practical research subject. Effects of defects on in--plane properties of periodic metal honeycombs is investigated by Guo (1999), Wang and McDowell (2003), Wang et al. (2004), Wallach and Gibson (2001). The research based on detailed study of random defects led to the conclusion that some cellular microstructures such as honeycombs are sensitive to degradation of their mechanical properties whereas some such as triangular are not sensitive.

Analysis of a randomly defected microstructure requires simulations on sample called statistical volume element, which should adopted to predict material constitutive properties. Size determination of the statistical volume element for randomly defected cellular material is of crucial importance. It is the smallest material volume element of the material for which macroscopic

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constitutive representation is a sufficiently accurate model to represent constitutive response (Drugan and Willis, 1996). In practice the size of the SVE can be determined for a given accuracy of obtained properties. The concept of SVE are extensively used in material science for analysis of random microstuctures, Yin et al. (2008), Gitman et al. (2004). The scaling from statistical to representative volume element is the subject of works by Ostoja-Starzewski (1998,1999,2002), Valavala et al. (2009), Sena et al. (2013), Torquato (2002) and Xiangdong (2006).

The aim of this work is an assessment of in-plane properties of inverted honycomb structured cellulars with missing microstructural beams. Comparison of the obtained results to intact (that is defect free) material stiffnesses is made. The objective is achieved in following steps: for each microstructure establishing smallest SVE for which no bias of the results is observed; generation of various realizations of randomly defected microstructure of the given defect density; FEM calculation leading to stiffness matrix and material strength; calculating accuracy of the obtained results. Numerical tests performed by means of ABAQUS FEA are carried out for several materials with different combinations of geometric parameters of their microstructure.

2. Intact and defected auxetic cellulars

Cellular material with intact infinite two-dimensional structure of inverted honeycombs is presented in Fig. 1a. Microstructural skeleton is composed of beams connected in stiff joints. The geometry can be described by the following parameters: H – length of vertical beams, L – length of sloping beams, γ – angle between vertical and sloping beam, t_{H} – width of the vertical beams, t_{L} – width of the sloping beams , as shown in Fig. 2. The structure with random missing bar defects is presented in Fig. 1b.

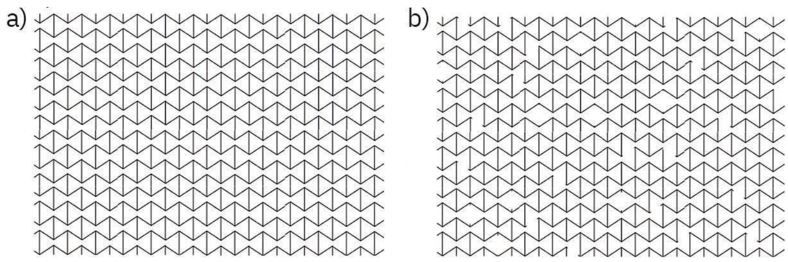

Fig. 1. a) Intact auxetic two-dimensional regular material microstructure, b) auxetic microstructure with random missing bars (own elaboration)

3. Compliance tensor for intact microstructure

On the basis of classical theory of homogenized media presented by Nemat Naser (1998) representative volume element RVE is adopted for the analysis of linear eleastic behavior described by stiffness matrix and strength of equivalent material. Determination of equivalent continuum properties (stiffness or compliance matrix and material constants) is performed by the FEM calculation based on the idea of micromechanical framework (Janus-Michalska, 2009).

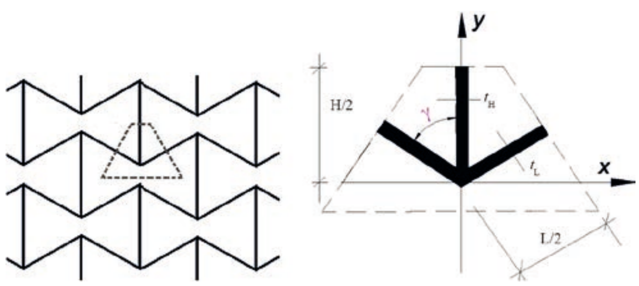

Fig. 2. Representative unit cell of intact microstructure and its geometric parameters (own elaboration)

Hooke law for equivalent continuum written in Voigt notation is as follows:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{xy}}{E_y} & \frac{\eta_{xy/x}}{G_{xy}} \\ -\frac{\nu_{yx}}{E_x} & \frac{1}{E_y} & \frac{\eta_{xy/y}}{G_{xy}} \\ \frac{\eta_{x/xy}}{E_x} & \frac{\eta_{y/xy}}{E_y} & \frac{1}{G_{xy}} \end{bmatrix} \cdot \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \tag{1}$$

where engineering constants are: E_x, E_y – Young moduli in x and y direction, G_{xy} – Kirchoff modulus in xy plane, $\eta_{xy/x}$, $\eta_{xy/y}$ – coefficients describing influence of shear stress on normal strain in x and y direction, $\eta_{x/xy}$, $\eta_{y/xy}$ – coefficients describing influence of normal stress in x and y direction on shear strain.

These constants can be obtained in tests of elongations in two directions and shear in xy plane. Due to symmmetry only six constant are independent: E_x, E_y, G_{xy}, ν_{xy}, $\eta_{xy/x}$, $\eta_{xy/y}$.

4. The concept of statistical volume element

In the case of material microstructure with nonperiodic disorder the representative volume must be a scale larger than the microscale to ensure a homogenization. Microstructure with random missing beams reveals statistical fluctuations of the effective properties over finite domains. In this case the effective linear properties of randomly defected material can be determined by numerical simulations on volume elements on mesoscale of material called statistical volume element, which is much larger than representative volume element and also much smaller than the material specimen. The concept of statistical volume element is based on mean values of effective properties of volumes assuming that a sufficient number of realizations is considered Kanit et al. (2005), Yin et al. (2008). The response of the SVE must be independent of the type of boundary conditions. The volume of statistical element should be large enough to ensure given accuracy of material properties obtained by spatial averaging of stress, or strain in a given domain. When considering a material as a realization of a randomly defected structures, we abandon the idea that there exists one single possible minimal SVE size. The overall moduli can be obtained by averaging over small domains of the material, using greater number of realizations or over the larger volumes using fewer realizations. The mean properties computed on finite size domains are called apparent (Jin, 2005; Kanit, 2003) or residual (Wang and McDowell, 2004). If the domain size is too small these properties do not coincide with the effective ones. The smallest SVE called critical should be sufficiently large that no bias occurs in the estimation. This phenomenon is generated by edge effects (Jiang et al., 2001). For a given volume size one can predict the minimal number of realizations that must be considered for a given volume size in order to estimate the effective property for a given precision. Conversely in practice, for given volume size and number of realizations one can estimate the appearent properties precision.

5. Statistical volume element simulations

For the determination of the precision corelated with minimal number of realizations and choice of critical sve, the proposed algorithm is as follows:

- choose mesoscale starting from greater value of mesoscale parameter, and generate given number n of different realizations of the microstructure for considered volume size;
- submit each microstructure to loading with boundary conditions related to tension-compression test and shearing test and calculate the apparent properties;
- compute mean value and variance of apparent property for the considered sve;
- calculate the precision for the estimation of effective property and check that the number of realizations was sufficient to achieve given accuracy if the precission is insufficient increase the n number;
- increase the number of realizations and check if it influences significantly the precision, decrease the number and check the precision;

for given sve indicate the minimal number of realization necessary for given precision

- change the mesescale (half value of mesoscale parameter) and repeat from the beginning;
- increase the number of realization to obtain the same accuracy as previously $>N * n$;
- point out to the minimal number of realizattion neccesary for given precision;
- compare with results from previous mesoscale;
- if comparison does not satisfy given precision it means that bias occurs and current mesoscale is not acceptable.

We consider fluctuations of the average values over different realizations of the randomly defected material inside the area A. For sufficiently large area A, the apparent moduli do not depend on the type of boundary conditions and coincide with the wanted effective properties of the medium (Sab, 1992).

SVE can be defined for the given material property, given precision and given number of realizations we are ready to generate. The size of SVE of an estimated property can be related directly to the precision of the mean value.

The variance of computed apparent properties for each volume size is used to define the precision of the estimation.

According to the theory of samples the relative error on the mean value $\bar{C}$ of the apparent modulus C obtained with n realizations of area A can be calculated by the following formula:

$$\varepsilon_{\mathrm{rel}} = \frac{\varepsilon_{\mathrm{abs}}}{\bar{C}} = \frac{2D_C(A)}{\bar{C}\sqrt{n}}$$

where: D_C^2 variance of the apparent modulus, $[\bar{C}-2D_C;\bar{C}+2D_C]$ the interval of confidence.

Alternatively we can determine number of necessary realizations by the formula: $n = \dfrac{4D_C^2(A)}{\bar{C}^2\varepsilon_{\mathrm{rel}}^2}$.

The apparent elastic moduli are obtained in windows of decreasing sizes. When the mean value for small area with periodic conditions is different from the effective one obtained for large specimens bias is observed.

The different RVE sizes found for the different properties are estimated.

6. Simulations of defected microstructures

Two intact structures made of PA6 polyamid are chosen as examples. Their microstructural properties are listed in Table 1.

Table 1. Specification of microstructures

Type	Geometric parameters of skeleton [mm]	Skeleton material Young modulus
1)	H = 10.0 L = 7.5 t_H = 0.75 t_L = 1.0, β = 60°	E_S = 1800 MPa
2)	H = 8.0 L = 16.0 t_H = 1.0 t_L = 0.5 γ = 80°	E_S = 1800 MPa

The structures with their repetitive structural segments being windows of dimensions $D1_0 \times D2_0$ are presented in Fig. 3. Addition of these segments to specimens guarantee the same boundary conditions in each specimen.

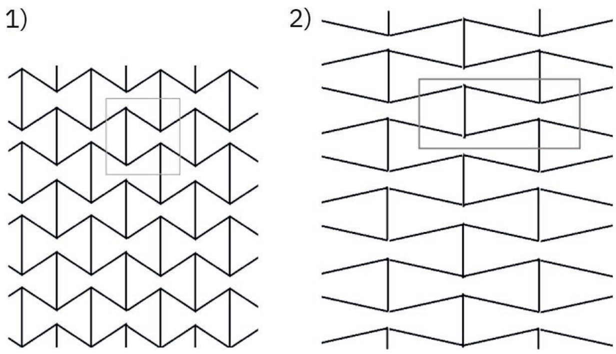

Fig. 3. Two microstructures with repetitive structural segments of dimensions $D1_0 \times D2_0$ (own elaboration)

Mathcad subroutine generating Random Numbers create vectors of random numbers that are respectively uniformly distributed over an interval. These random beam memebers of skeleton are removed.

Calculations are conducted in rectangular domains called windows, which size is characterized by nondimensional parameter (Vernerey [33]) $\delta = D1/D1_0 = D2/D2_0$, which defines the mesoscale of observation.

7. FEM analysis of microstructure

In order to calculate apparent properties series of numerical calculations are performed by means of ABAQUS FEA. Material microstructure is discretized by the plane network of Timoshenko beam elements. Mesh size convergence studies were initially performed, leading to the selection of number of beam elements for each cell wall for given accuracy. It should be checked also that this mesh density is sufficient to get a precision better than given on the statistical fluctuations and variance of the results when many realizations are considered.

In some simulations, large size of area cannot be handled. This means that it will be necessary to use meshes with a huge amount of degrees of freedom. Many realizations of defects were simulated for decreased area sizes.

The areas called windows are subjected to deformation relevant to elongation and homogeneous shearing strain. These loadings correspond to typical experiments.

For elongation experiment simulation nodes on opposite edges of the mesh at left and right are moment free. The stress is calculated by dividing the sum of the reaction forces on the boundary nodes by the edge area. The uniaxial stiffness is calculated as the ratio of the calculated stress divided by the imposed strain.

For shearing test the specimen behavior is analyzed under the state of stress relevant to pure shear deformation shown in Fig. 4b. The shear stress is calculated by dividing the sum of the reaction forces on the boundary nodes by the edge area. The macroscopic shear stress divided by shear strain gives the macroscopic shear stiffness. The dimension of the specimens D1 and D1, relative to the segment size is defined by mesoscale coefficient.

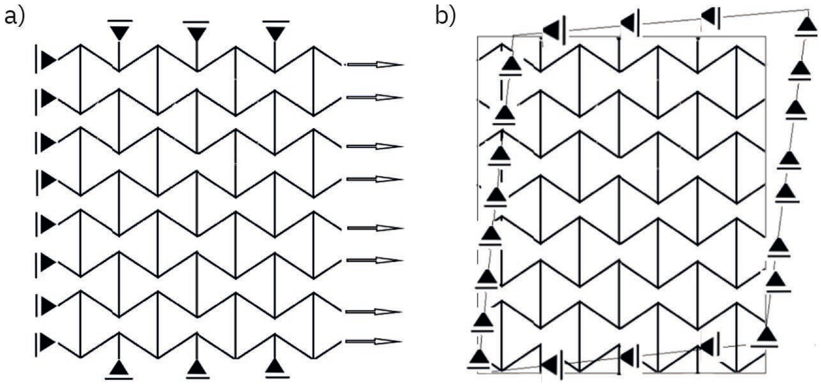

Fig. 4. Homogeneous deformations in tests a) elongation, b) pure shearing (own elaboration)

Simple shear and uniaxial elongation tests are performed on samples with different mesoscale.

8. Results of numerical analysis

Scale dependent response leading to determination of apparent moduli and their statistics is studied numerically. For the same microstructure, it will be shown that the analysis leading to obtaining the RVE size and number of realizations differs if Young modulus or shear modulus or strength is considered.

Density of material of intact structure can be calculated as follows:

$$\rho_{\text{int}} = \frac{(2\text{L}t_{\text{L}} + \text{H}t_{\text{H}})}{2(\text{H} - \text{L}\cos\gamma)\text{L}\sin\gamma}.$$

The densities of considered intact microstructures are:

$$\rho_{1\text{int}} = 0.277,\ \rho_{2\text{int}} = 0.146.$$

For numerical simulations samples with the same loss of relative density are considered and the results are compared.

For the considered structures with their geometric parameters relative foss of densities $\frac{\Delta\rho_{i\text{int}}}{\rho_{i\text{int}}}$ are the same for each microstructure if the number of defected beams m, is the same.

The following realizations are considered:

realizations for $\frac{\Delta\rho_{\text{int}}}{\rho_{\text{int}}} = 16.17\%$

m 1 4 9 16 25 … … 144
δ 1 2 3 4 5 … … 12

realizations for $\frac{\Delta\rho_{\text{int}}}{\rho_{\text{int}}} = 9.37\%$

m 9 36 81
δ 4 8 12

realizations for $\frac{\Delta\rho_{\text{int}}}{\rho_{\text{int}}} = 7.41\%$

m 4 16 36 64
δ 3 6 9 12

realizations for $\frac{\Delta\rho_{\text{int}}}{\rho_{\text{int}}} = 4.16\%$

m 1 4 9 16 36
δ 2 4 6 8 12

realizations for $\frac{\Delta\rho_{\text{int}}}{\rho_{\text{int}}} = 1.18\%$

m 1 4 9 16
δ 3 6 9 12

realizations for $\frac{\Delta\rho_{\text{int}}}{\rho_{\text{int}}} = 1.04\%$

m 1 4 9
δ 6 8 12

Some ralizations begins for large SVE, since one defected beam gives required percentage of loss of density.

8.1. Apparent moduli

Numerical simulations on samples of the microstructure are carried out. Number of random samples or realizations were evaluated for each case using FE analysis, leading to computed estimates of effective elastic stiffness for elongation and shear loading conditions. For minimal number of specimens, increasing number has no noticeable effects on the averages.

Constants for materials of intact microstructures are as follows:

$$E_{X1} = 118{,}5\ \text{MPa},\ \nu_{xy1} = -1{,}687,\ G_{xy1} = 0{,}0263\ \text{MPa},\ E_{X2} = 198{,}5\ \text{MPa},$$

$$\nu_{xy2} = -1{,}231,\ G_{xy2} = 0{,}0879\ \text{MPa}.$$

Normalized material constants for defected structures as functions of loss of density are presented in figures below.

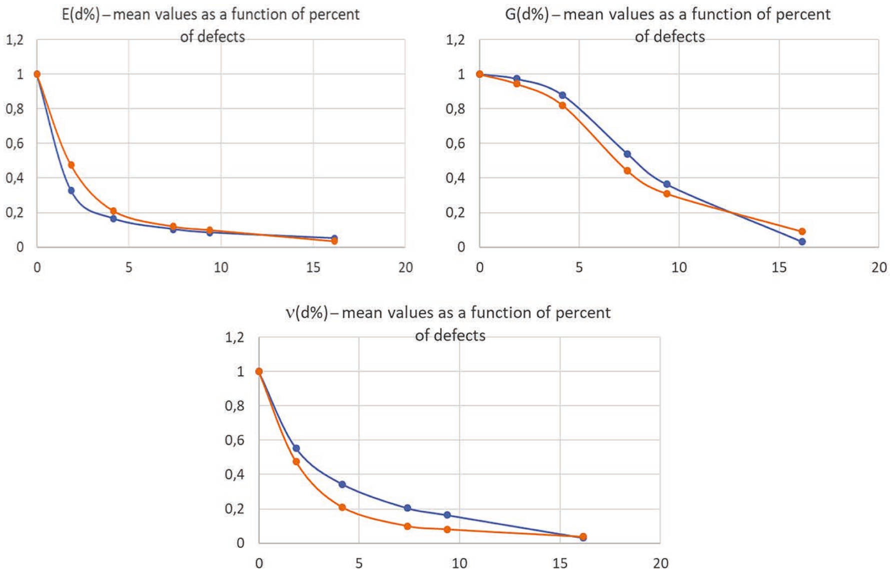

Fig. 5. Normalized apparent Young's modulus, Poisson's ratio and shear modulus by effective properties of intact structure, plotted against the reduction in density (own elaboration)

The reductions of effective Young's modulus and Poisson ratio are much more sensitive to the number of defected beams in the SVE than the reduction of shear modulus. This is because the shear modulus mainly results from from bending of cell walls, which is more compliant, whereas Young modulus results from compression or bending, which is stiffer. The loss of stiffer elements results in greater loss of density.

Minimal number of realizations for chosen precision ε = 2% relevant to Young modulus, Poisson's ratio and shear modulus is visualized in Fig. 6, 7, 8 (subsequent curves for 1.04%, 1.85%, 4.16%, 7.41%, 9.37%, 16,17% loss of density).

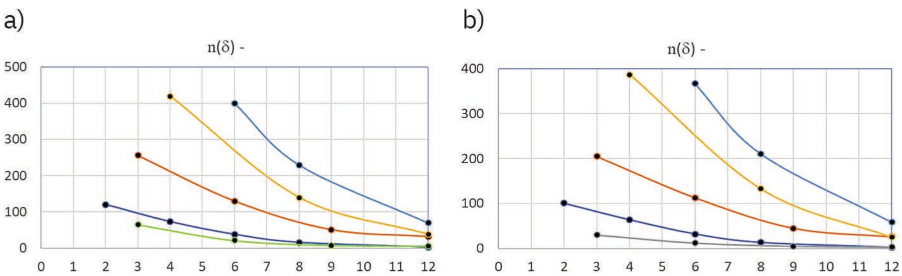

Fig. 6. Young modulus – minimal number of realization in dependence on mesoscale coefficient and defects, a) structure 1, b) structure 2 (own elaboration)

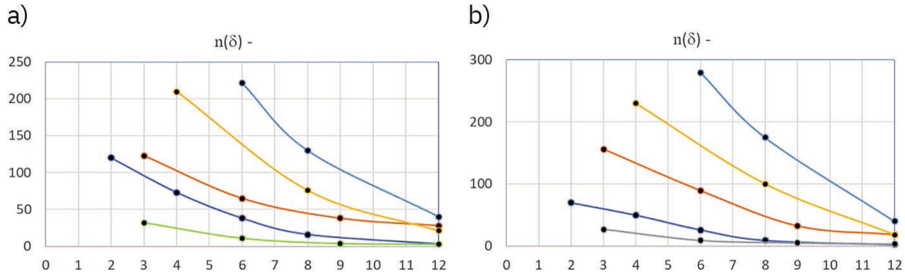

Fig. 7. Shear constant – minimal number of realization in dependence on mesoscale coefficient and reduction of density
a) structure 1, b) structure 2 (own elaboration)

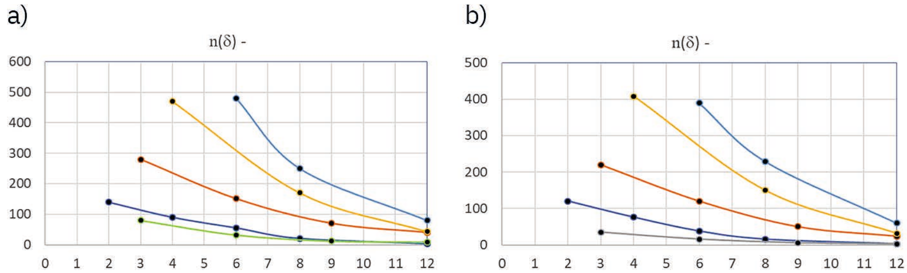

Fig. 8. Poisson's ratio – minimal number of realization in dependence on mesoscale coefficient and reduction of density
a) structure 1, b) structure 2 (own elaboration)

Simulations for abtaining statistically representative properties should be conducted using SVE of greater arrays than 4X4 segments for loss of density 9.37%, or 6X6 for loss 16.17%, otherwise bias occurs. The scatter of results or interval of confidence for the effective Young's modulus among the various realizations for each SVE size is greater than interval corresponding shear realizations. Generally all results depend on geometric microstructural parameters.

9. Conclusions

An approach based on statistical analysis is proposed to study the influence of random material microstructure defects on material constitutive properties of auxetic cellulars. Statistical volume element simulations are carried out to predict apparent material constants corresponding to various realizations of random microstructure defects. A computing framework based on finite element analysis on randomly defected microstructure samples has been developed. As a result the number of realizations associated with a given precision of the estimation of the apparent material property and mescsale coefficient can be obtained. It is shown that it depends on the investigated material property. Defects density influence on apparent constitutive moduli is investigated. Numerical tests show the dependence of the results on microstructural geometric parameters. The variety of results is obtained due to missing bars defect. The greater influence is observed for deformation switching from stretching dominated to bending dominated. The mechanical properties calculated this way can be compared with experiments.

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