Closed-Form Design Formulas for Adhesively Bonded Composite Beams, Paweł Szeptyński; Cracow University of Technology, Faculty of Civil Engineering; Professor

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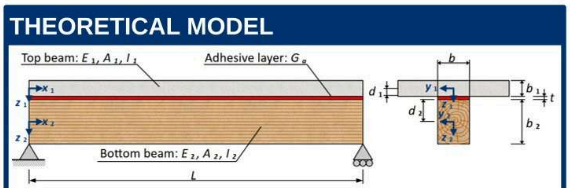
Closed-form design formulas for adhesively bonded composite beams

BACKGROUND

Adhesively bonded composite beams are becoming increasingly popular in civil engineering. However, there is still limited support for the design of such beams in terms of standardization, code regulations, and design guidelines. In particular, determining deflections as well as the distribution of stress and strain often requires the use of sophisticated FEM software. The aim of this work is to develop a simple yet sufficiently precise tool for structural engineers. This research employs the author's analytical model of composite elastic beams to derive closed-form design formulas for extreme deformations and stresses. These theoretical predictions are compared with more accurate 3D FEM models, as well as with experimental data from 36 different test types (13 TCC beam types) performed on a total of 60 composite beams.

THEORETICAL MODEL



Similarity numbers for adhesively bonded composite beams

$$\alpha=\frac{d_1+d_2}{L}\qquad \beta=\frac{G_a L^2 b}{E_1 A_1 t}\qquad \gamma=\frac{G_a L^2 b}{E_2 A_2 t}\qquad \delta=\frac{G_a L^3 b(d_1+d_2+t)}{t(E_1 I_1+E_2 I_2)}$$

$$\varepsilon=\frac{L^3(q+A_1 g_1+A_2 g_2+b\,t\,g_a)}{E_1 I_1+E_2 I_2}\qquad \lambda=\sqrt{\alpha\,\delta+\beta+\gamma}$$

Governing equations:

$$\begin{cases}\dfrac{d^2\tilde{u}_1}{d\chi^2}+\beta\left[\left(\tilde{u}_2-\tilde{u}_1\right)+\alpha\dfrac{d\tilde{w}}{d\chi}\right]=0\\[2mm] \dfrac{d^2\tilde{u}_2}{d\chi^2}-\gamma\left[\left(\tilde{u}_2-\tilde{u}_1\right)+\alpha\dfrac{d\tilde{w}}{d\chi}\right]=0\\[2mm] \dfrac{d^4\tilde{w}}{d\chi^4}=\varepsilon+\delta\left[\left(\dfrac{d\tilde{u}_2}{d\chi}-\dfrac{d\tilde{u}_1}{d\chi}\right)+\alpha\dfrac{d^2\tilde{w}}{d\chi^2}\right]\end{cases}$$

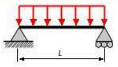
Volkersen shear lag theory

coupling terms

Bernoulli – Euler beam theory

DESIGN FORMULAS - example

SIMPLY SUPPORTED BEAM
UNIFORMLY DISTRIBUTED LOAD



Mid-span deflection:

$$w_{\max}=\varepsilon L\frac{768\alpha\delta e^{\frac{\lambda}{2}}+(e^{\lambda}+1)\left[5\lambda^4(\beta+\gamma)+48\alpha\delta(\lambda^2-8)\right]}{384\lambda^6(e^{\lambda}+1)}$$

Mid-span stress distribution in the top beam:

$$\sigma_{1,span}=\varepsilon E_1\left[-\alpha\beta\left(\frac{2e^{\frac{\lambda}{2}}}{\lambda^4(e^{\lambda}+1)}+\frac{\lambda^2-8}{8\lambda^4}\right)+\frac{z_1}{L}\left(\frac{\alpha\delta\left(e^{\frac{\lambda}{2}}-1\right)^2}{\lambda^4(e^{\lambda}+1)}+\frac{\beta+\gamma}{8\lambda^2}\right)\right]$$

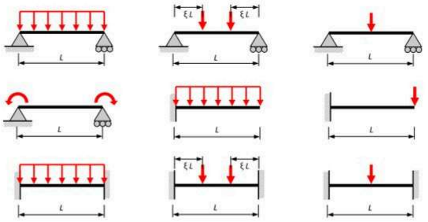
Mid-span stress distribution in the bottom beam:

$$\sigma_{2,span}=\varepsilon E_2\left[\alpha\gamma\left(\frac{2e^{\frac{\lambda}{2}}}{\lambda^4(e^{\lambda}+1)}+\frac{\lambda^2-8}{8\lambda^4}\right)+\frac{z_2}{L}\left(\frac{\alpha\delta\left(e^{\frac{\lambda}{2}}-1\right)^2}{\lambda^4(e^{\lambda}+1)}+\frac{\beta+\gamma}{8\lambda^2}\right)\right]$$

Maximal shear stress in the adhesive layer in the supported cross-section:

$$\tau_{\max}=\varepsilon\alpha G_a L\frac{\lambda(e^{\lambda}+1)-2(e^{\lambda}-1)}{2t\lambda^3(e^{\lambda}+1)}$$

Elaborated load-support layouts:



RESULTS

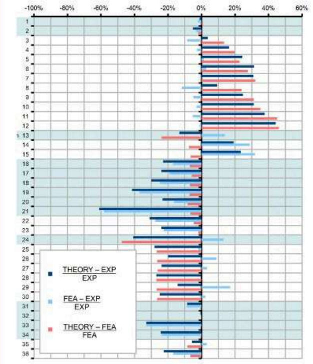


FIG 1. Relative error of FEA and theoretical predictions of the mid-span deflection compared with the recorded experimental results [3].

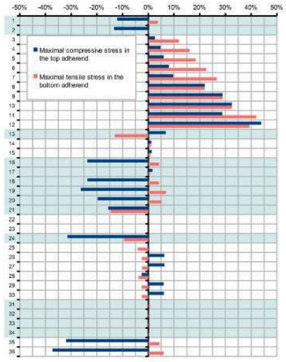


FIG 2. Relative error of theoretical predictions of the extremal normal stresses in edge fibers of the mid-span cross-section compared to the FEA results [3].

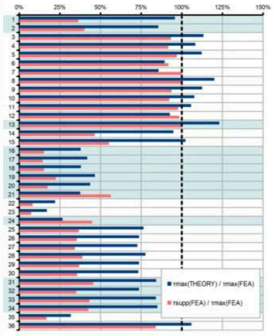


FIG 3. Comparison of FEA and theoretical predictions of maximal shear stress in the bondline and shear stress in the supported cross-section [3].

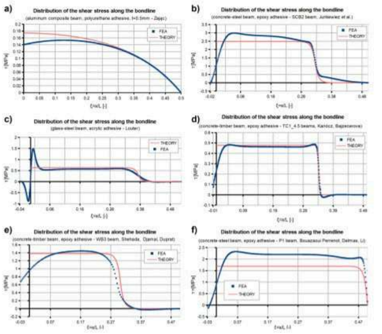


FIG 4. Distribution of shear stress aong the bondline – comparison of the results of the FEA and predictions of the theoretical model [3].

CONCLUSIONS

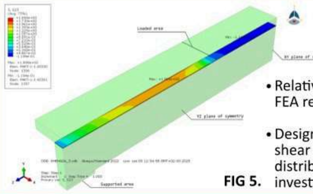


FIG 5.

- Relative error of deflection estimate compared to the experimental result in almost all cases is less than ca. 30%.
- Relative error of normal stress estimate compared to the FEA results in the most of cases in less than ca. 30%
- Design formulas significantly underestimate the extremal shear stress in the bondline. Transverse shear stress distribution and influence of concentrated loads should be investigated (cf. FIG 5. [3]).

REFERENCES

[1] **THEORETICAL MODEL**
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[3] **DESIGN FORMULAS AND EXPERIMENTAL VALIDATION**
P. Szeptyński, *„ Design formulas for adhesively bonded composite beams "*, Structures, 79 (2025), 109550