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Elastic pattern transformation in microstructure of cellular auxetic materials in compression test

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Abstract

The goal of this paper is to investigate the pattern transformation in auxetic material microstructure in compression loading. Nonlinear behavior resulting from large microstructural beams deflection and buckling is analysed. Deformation study in compression test is performed using ABAQUS by conducting finite element calculation. Timoshenko beam elements are used for material microstructure discretization. Calculations lead to obtaining nonlinear equilibrium paths. The analysis allow to asses values of critical stress and critical strains. Specimens of increasing size consisting of elemental segments are analyzed. Influence of geometric microstructural parameters on the type of pattern transformation and response of material as equivalent continuum is studied.

Keywords: auxetic cellulars, pattern transformation, microstructural instability, critical load

1. Introduction

Mechanical metamaterials refer to a group of manmade structures with certain specific mechanical properties resulting mainly from the geometry of microstructure and the properties of the skeleton material. These materials reveal enhanced properties such as energy absorbtion, resilience, changeability of elastic stiffnesses. These qualities can be designed to improve mechanical behavior. The main goal for providing new modern materials is to tailor microstructures with specified unique response. For cellular materials it is possible to achieve this purpose by various structural arrangements (Mitschke et al., 2016).

The concept of creating of anisotropy in lattice materials by special cellular shapes is crucial in the construction of these materials. Specialy designed properties can be achieved mainly by adjusting material anisotropy (Ren et al., 2018).

Cellular materials provide enhanced energy absorbtion possibilities because of effective nonlinear stess-strain behavior, and the ability to achieve large deformation at near constant stress (Gibson and Ashby, 1997). Elastic energy is absorbed as a result of cell walls bending. Strong effective nonlinearity originates from the buckling instability of microstructural beams. Energy absorbing effective material behavior is described and discussed for two-dimensional honeycomb structures in works by Papka and Kyriakides (1994; 1999), Triantafyllidis and Schnaidt (1993), Ohno et al. (2002). Pattern transformation in microstructure takes place when compressive load is applied and when this load reaches its critical value. As a result of the transformation the incremental effective properties of the material can substantialy change (Jiang et al., 2019).

One of these specific effective property is deformability in the direction perpendicular to compression described by Poisson's ratio. Manmade materials can reveal auxeticity, the property of negative value of Poisson's ratio. Word 'auxetikos', means 'that which tends to increase'. Auxetic materials expand in the direction lateral to longitudinal stretching. First such mechanical behavior was described by Lakes in 1987. Main works on cellular auxetics were started by Gibson and Ashby (1997) and developed by Lakes, Kolpakov, Almgren in the end of the twentieth century. Now are continued by Evans, Scarpa and Alderson. Auxetic cellular microstructures with variety of structural arrangements were described by Lakes, Theocaris, Smith, Gaspar, Grima and Yang. The review on negative Poisson's ratio materials is given by many authors for example by Prawoto (2012).

Mechanical metamaterials having both positive and negative Poisson's ratios depending on elastic range are described Mitschke (2016). They can be tailored for example to bioengineering applications (Fu, 2017a; 2017b; Ren, 2018). This property can be achieved by instability pattern transformation leading to auxetic behavior of materials with periodic microstructure (Holmes, Douglas, 2019; Nguyen et al., 2019). This phenomenon is explained by Bertoldi et al. (2010) on the example of 2D structure with circular holes.

The main issue of this paper is to describe homogeneous pattern transformations in inverted honeycomb class of periodic cellular structures. These pattern transformations are triggered by reversible elastic microstructural beam instability. Energy absorbtion is achieved by large deformation which originates in buckling of a member in the cell microstructure.

Several approaches can be applied to investigate bifurcations occurring in infinite periodic cellulars. Saiki et al. (2002) proposes the block diagonalization method for determination of the rve size and lowest critical eigenvalue. This approach is also used by Triantafyllidis and Schnaidt (1993). Okumura et al. (2004) employed the method of Bloch wave perturbation. Micro-macro transition scale theory is used by Saiki et al. (2002) to analyze the microscopic bifurcation and post-bifurcation behavior of elastoplastic, periodic cellular solids as equivalent continuum.

Laroussi et al. (2002) obtained nonlinear strain-stress paths for open cell--foams applying two methods. The first one is based on traditional eigenvalue instability analyses on representative volume elements of increasing size. The second method is based on periodic and non-periodic displacements variation on a minimum unit cell.

For most of cellular microstructures and variety of boundary conditions some structural members are subjected to inplane bending combined with compression, which leads to their large deflections. It results in strongly nonlinear strain-stress path for eqiuvalent continuum without necessity of critical load estimation.

The aim of this work is assesment of critical compressive stress and strain and obtaining nonlinear equilibrium path of inverted honycomb structured cellulars.

The objective is achieved by establishing increasing segments, for which FEM calculation is carried out. Size effect study is performed to predict values of critical stress and strain for infinite equivalent continuum. Numerical tests by means of ABAQUS FEA are executed for cellular two materials with different sets of geometric parameters.

2. Perfect auxetic cellular microstructure

Fig. 1. presents cellular material with infinite regular plane structure of inverted honeycombs. Material skeleton is composed of flexible struts connected in stiff joints. The following parameters describe microstructure geometry: H – vertical strut length, L – sloping strut length, γ – angle between adjacent struts, t – width of the struts. The structure thickness in direction perpendicular to the plane is assumed to be unity: $g = 1$.

Classical theory of homogenized media presented by Nemat-Naser (1999a; 1999b) gives solution of linear elastic behavior described by stiffness matrix and strength of equivalent material. Analysis is based on the response of the smallest repetitive element called Representative Volume Element (RVE). Determination of equivalent continuum properties (stiffness or compliance matrix and material constants) based on the idea of micromechanical framework is given by Janus-Michalska (2009).

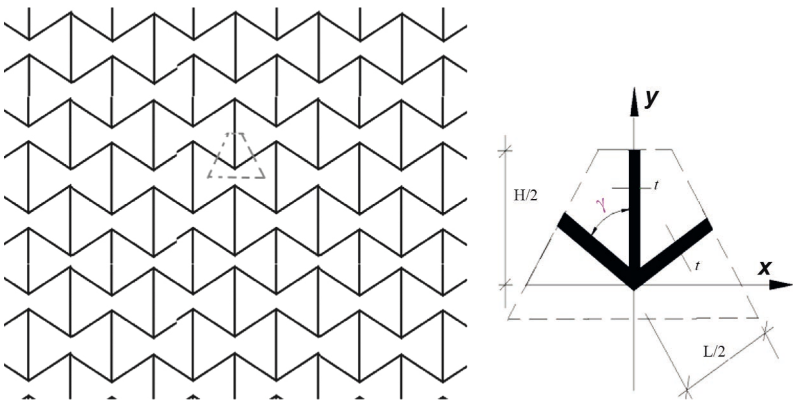

Fig. 1. Representative unit cell of perfect microstructure and its geometric parameters (own elaboration)

3. The concept of finite sized segment for instability analysis

Stability analysis of periodic solids peformed by Geymonat et al. (1993) shows that the eigenmodes of microscopic bifurcation can have a longer periodic length than can be described within the area of single unit cell. That is the reason for using enlarged unit cell (Muller, 1987). This unit is called segment or finite--sized specimen. Different bifurcation modes depending on the size segments of increasing size can occur. The critical value of the load parameter is then defined as the infimum of the load parameter for periodic instability deformation mode. This is infinimum of on all possible periodic segments. For each eigenvalue relevant eigenmode is interpreted as deformation pattern. This method can not be applied for infinite structure, but it gives an insight into the deformation mechanism. It is also helpful in estimation of buckling load and explanation of the material behavior. Microscopic instabilities in infinite periodic solids are invesigated by the method proposed by Laroussi et al. (2002) considering eigenmode analyses on representative volume elements of increasing size (increasing number of repeating units in the RVEs).

In case of cellular microstructured materials finite size increasing specimens should be also considered due to size effect which may influence course of nonlinear strain stress path. Only periodic boundary conditions guarantee the same nonlinear path for increasing structural segments. For other boundary conditions the size segment influences the obtained results.

The method of increasing size specimens is crucial for evaluating critical load by experiments, where only finite dimension of specimen can be considered. Size effect study unable to predict values of critical stress and strain for infinite equivalent continuum.

Lets define elemental structural segment of dimensions $\mathrm{D1}_0 \times \mathrm{D2}_0$ as shown in bold line in Fig. 2 (upper right corner). Specimens $S1, S2, S3\ldots$ of scale ratio $d=1, 2, \ldots$ of the increasing size $\mathrm{D1}\times\mathrm{D2}$, where: $\mathrm{D1}=d\cdot D1_0$, $\mathrm{D2}=d\cdot \mathrm{D2}_0$, are of the same boundary conditions in each specimen. Specimens of increasing sizes are shown in left upper corner of microstructure in Fig. 2.

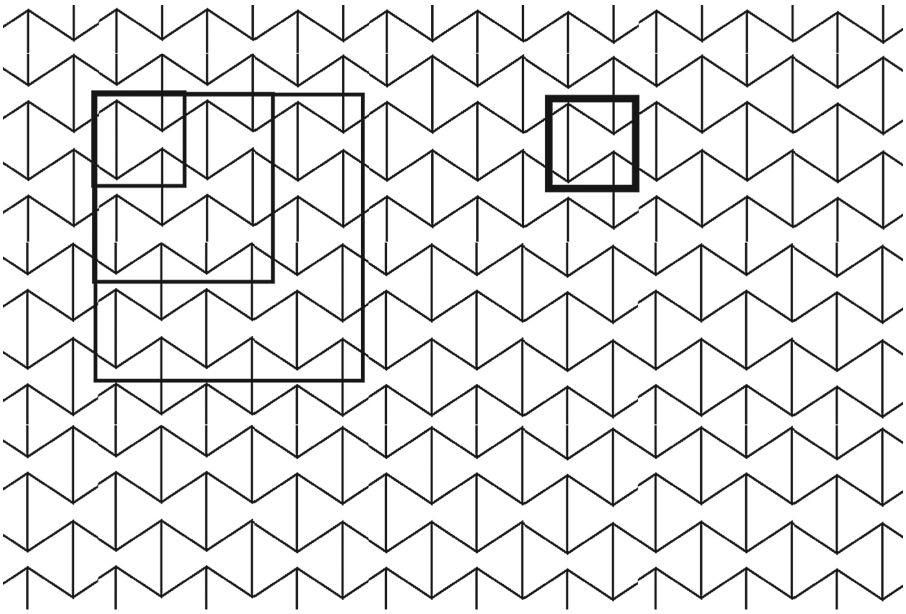

Fig. 2. Cellular microstructure with finite--sized periodic specimens of dimensions $\mathrm{D1}\times\mathrm{D2}$ and elemental segment (own elaboration)

4. Compression test

To simulate the experimental tests for the finite-sized periodic specimen the following boundary conditions are introduced: bottom and upper edge are fixed in the vertical direction. The specimen is uniformly compressed in the horizontal direction. Boundary conditions relative to the compression test of the specimen are illustrated in Fig. 3. To obtain the nominal stress, the total sum of forces on the edge is divided by horizontal edge area. For nominal strain calculation the edge displacement should be devided vertical edge length. Simulations of compression test for finite specimen of auxetic materials are known in

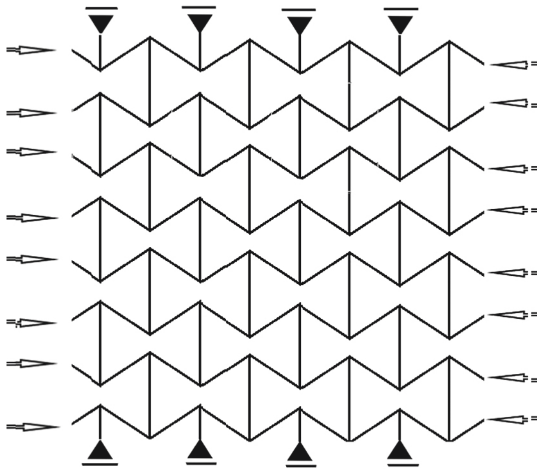

Fig. 3. Compression test (own elaboration)

literature and were conducted for example by Dong et al. They lead to obtaining the transformed pattern and the critical nominal strain.

In this work author presents very simple method leading to estimation of critical loads and nonlinear path. All the load displacement nonlinear paths for increasing segments can be obtained without assuming any imperfection. The reason for that is that compressive load produces combined bending (bending moment and compressive force) in microstructural beams. It leads to large displacements at nearly constant load value. Then the critical load relevant to displacement pattern can be calculated as maximum on strain-stress path. Then infimum of the set of critical loads for finite size should be chosen to define material critical load (the method presented in work by Combescure et al., 2016).

5. Numerical simulation

Load–displacement analyses for the finite-sized segments were performed with ABAQUS/Standard. The numerical simulations are carried out using a nonlinear finite element code. All nonlinear paths are computed for increasing scale ratios. Microstructural instabilities enforce the pattern transformation which gives nonlinear equivalent material response depending on the geometry and type of applied boundary conditions applied. Constrained boundaries in compression test introduce boundary layers with different displacement pattern or restricted pattern transformation than in the bulk of the material (Ameen et al., 2018). These effects strongly depend on the size of elemental segment relative to the size of the specimen, which is a typical size effect.

The instabilities can be also investigated and confirmed by the use of a linear perturbation procedure to confirm the observed pattern transformation arising from elastic instability corresponding to the lowest eigenmode of the structure. Then an imperfection was introduced into the mesh in the form of this eigenmode, which enabled load–displacement analysis for the finite-sized structures.

5.1. Microstructural data

Two microstructures made of PA6 polyamid are chosen as examples. Their microstructural properties are listed in Table 1. Examples of their finite size specimens are shown in Fig. 4.

Table 1. Specification of microstructures

Type	Geometric parameters of skeleton [mm]	Skeleton material Young modulus
1)	H = 10.0 L = 7.5 t = 1.0 β = 60°	E_S = 1800 MPa
2)	H = 16.0 L = 8.0 t = 1.0 γ = 80°	E_S = 1800 MPa

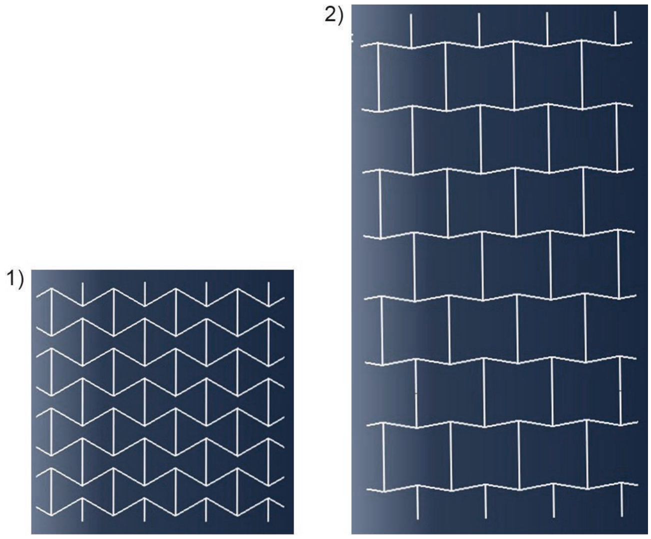

Fig. 4. Two microstructures (own elaboration)

5.2. FEM discretization of microstructure

Numerical investigations were performed on both structures finite-sized specimens using the nonlinear finite element code ABAQUS. Each mesh was constructed using Timoshenko beam elements. The assumed accuracy of 2% was maintained by mesh densification.

6. Results of numerical analysis

Transformations in patterns as shown in the snapshots of deformed configurations in specimens of increasing size for material with microstructure 1) are illustrated in Fig. 5. Transformation to a pattern strongly depends on the geometry of the initial microstructure.

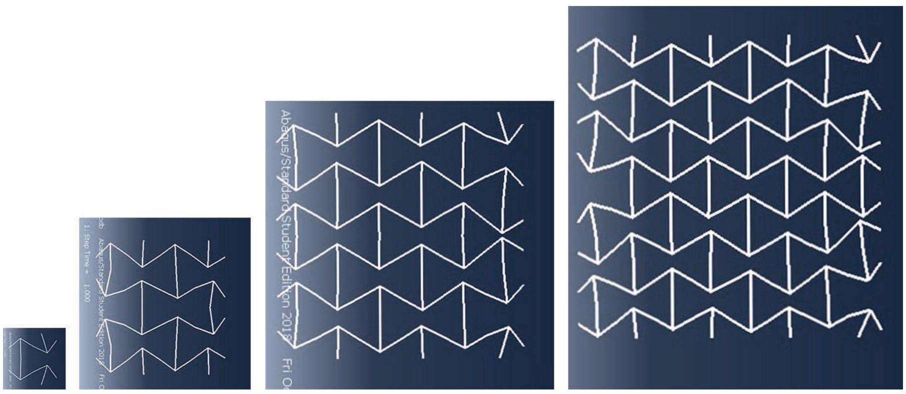

Fig. 5. Deformation modes on the example of first structure (own elaboration)

In Fig. 6 and Fig. 8 the nominal stress is plotted against the applied nominal strain for the first and second microstructure for segments with different scale ratios $S1$, $S2$, … . Upon reaching a critical stress level, that deformation

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progress through the structure at relatively constant stress up to large strains. The presented curves exhibit initial linear elastic behavior which change into strongly nonlinear and attain maximum. For nominal values it is not plateau stress as for true strain-stress plot, because of decreasing width of specimen.

The plots generated for each structure show sensitivity of size effect and show asymptotic decreasing tendency as specimen size increases as shown in Fig. 7 and Fig. 9. Numerical analyses show that the pattern transformations are the result of local elastic instabilities and due to elasticity transformations are reversible.

The stress-strain path also provide assessment of energy absorption opportunities. For the first structure maximum of stress 450 kPa is attained at strain 0.06, whereas for the second structure maximum stress 200 kPa is attained at strain 0.009, which gives approximately fifteen times less energy absorption capacity. This indicates the interesting possibility of tailoring the initial microstructure to obtain the desired energy adsorption.

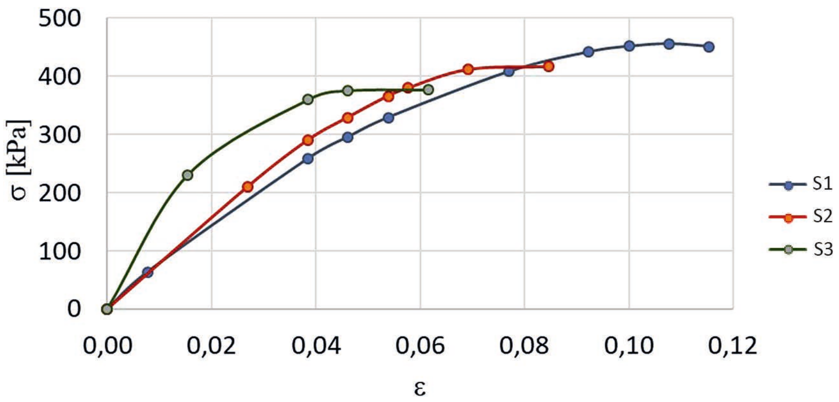

Fig. 6. Nominal stress versus to nominal strain for increasing specimens of first structure (own elaboration)

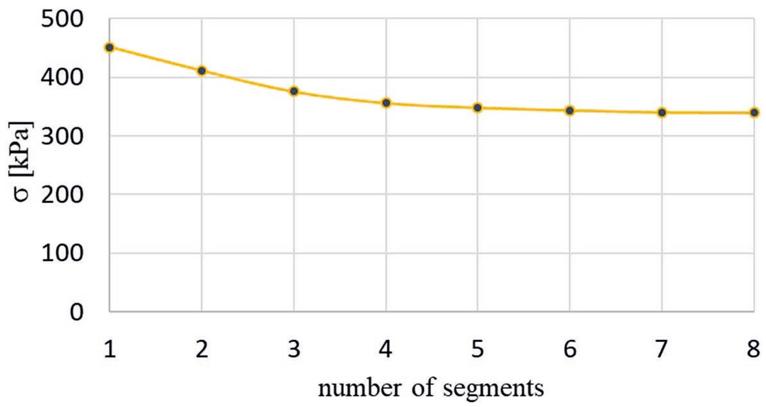

Fig. 7. Critical stress plotted against the scale ratios corresponding to increasing specimen sizes for the first structure (own elaboration)

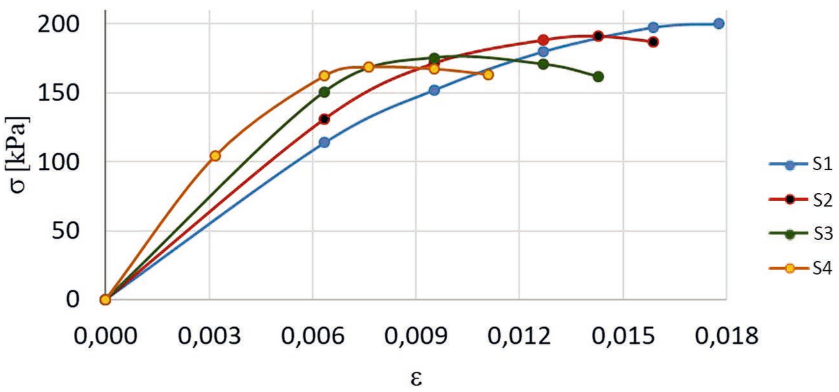

Fig. 8. Nominal stress versus to nominal strain for increasing specimens of the second structure (own elaboration)

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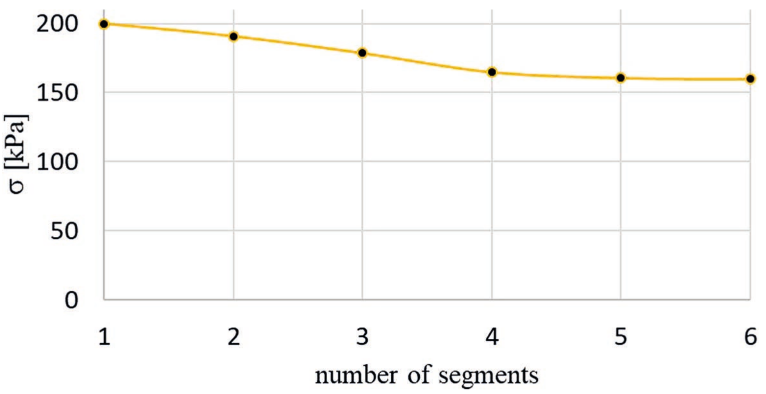

Fig. 9. Critical stress plotted against the different scale ratios corresponding to increasing specimen sizes (own elaboration)

7. Conclusions

The mechanics of nonlinear deformation of two 2-dimensional periodic auxetic cellular structures have been studied. Deformation which results from pattern transformations originates from large deflections caused by combined compressive force and bending moments in microstructural beams.

For evaluating critical compressive stress for infinite microstructure an approach based on enlarged volume element simulations are carried out. In compression test the constrained boundaries introduce boundary layers in which the pattern transformation is restricted. When relative thickness of these boundary layers increases the size effect can be observed.

Finite element analysis in ABAQUS code is applied to obtain nonlinear paths and estimate critical compressive stress and also evaluate elastic energy. Numerical tests indicate the dependence of the results on microstructural geometric parameters. The mechanical properties calculated this way should be validated using experimental results. Studies similar to those carried out by Dong et al. (2019) are recommended.

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